Particle production close to threshold: a critical review

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Equation of state

EoS: **Symmetric**

$$\frac{E(\rho)}{N}=\frac{E(\rho_0)}{N}+\frac{K_0}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}+\frac{J_0}{162}\frac{(\rho-\rho_0)^3}{\rho_0^3}$$

$$P=\rho^2\frac{\partial E(\rho)}{\partial\rho}\qquad K_0=9\frac{\partial P}{\partial\rho}$$

Asymmetric NM

$$\frac{E(\rho,\beta)}{N}=\frac{E(\rho,\beta=0)}{N}+S(\rho)\beta^2\qquad \beta=\frac{\rho_n-\rho_p}{\rho}$$

$$S(\rho)=S(\rho_0)+\frac{L}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$

Sources:

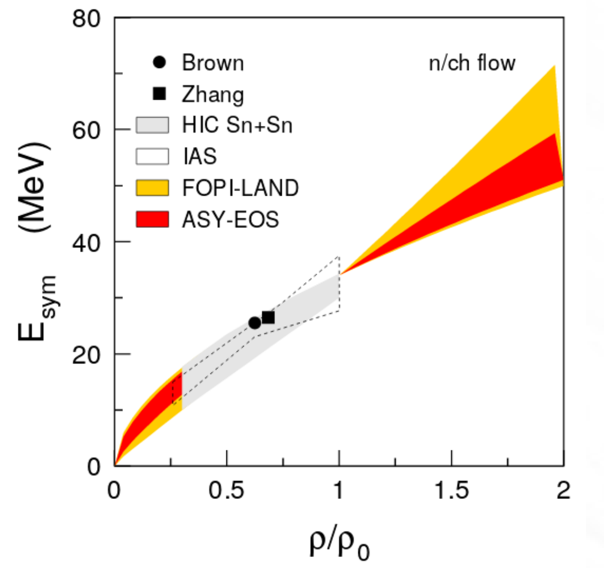
finite nuclei, low energy HIC	$\rho/\rho_0\leq 1$
heavy-ion collisions	$\rho/\rho_0\leq 3$
neutron stars	$\rho/\rho_0\leq 10$

Probes for supranormal densities:

Nucleon Flows: $\frac{dN}{d\phi}\sim 1+2v_1\cos\phi+2v_2\cos 2\phi$

v_1, v_2 – probes for symmetric EoS

Ratios, differences v_2 – probes for Symmetry Energy



P. Russotto et al., PRC 94, 034608 (2016)

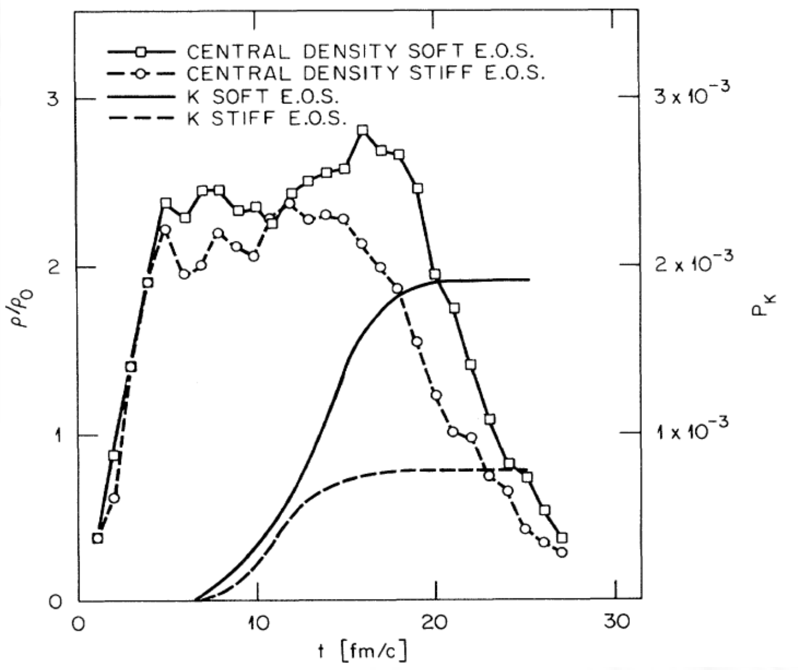
Particle Production: Multiplicity ratios heavy/light system - K_0

Single ratios, doubles ratios – symmetry energy

Particle Production as Probe of EoS

kaons

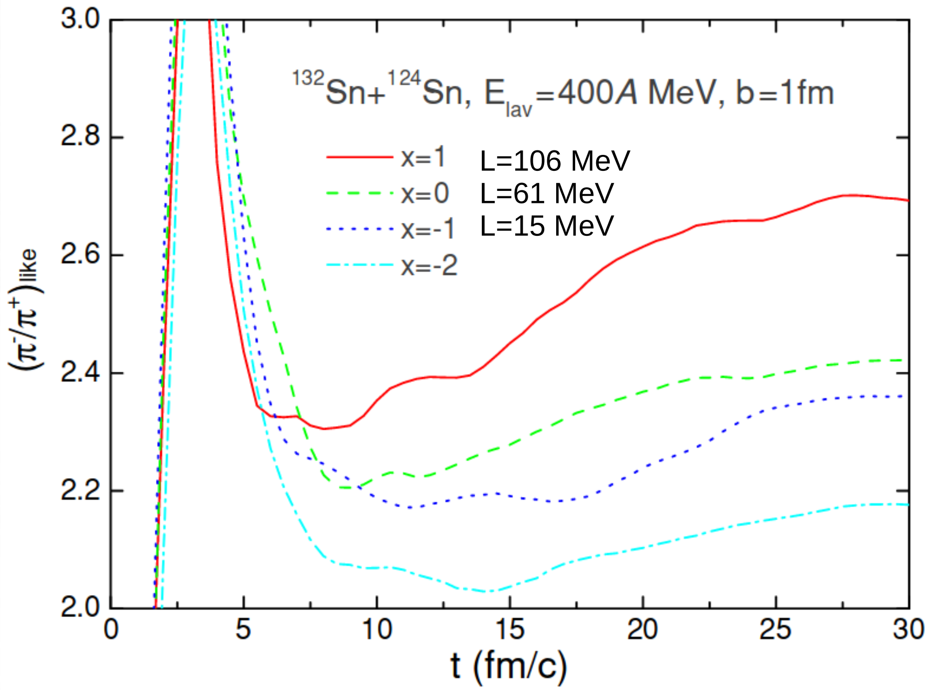
J.Aichelin, C.M. Ko PRL 55, 2661 (1985)



NbNb 700 MeV/A central collisions

pions

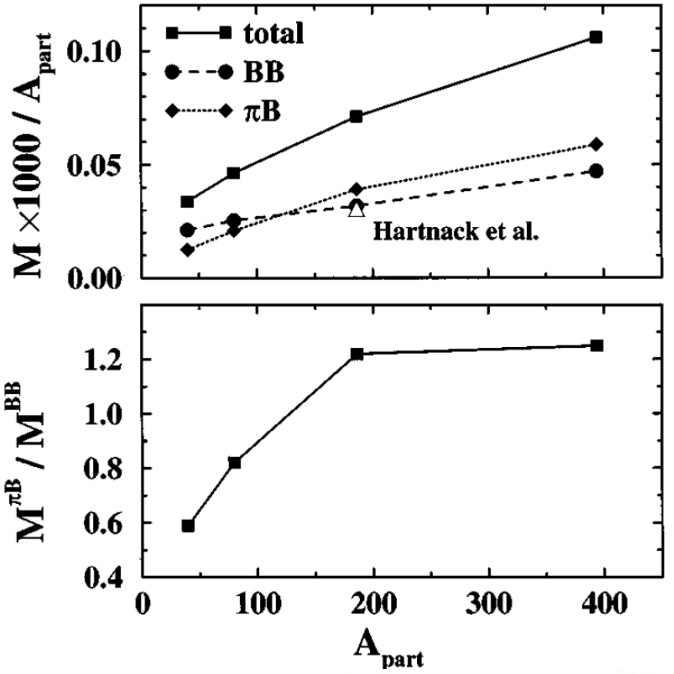
B.A. Li PRL 88, 192701 (2002); B.A. et al. PRC 71, 014608 (2005)



K^+ production below threshold

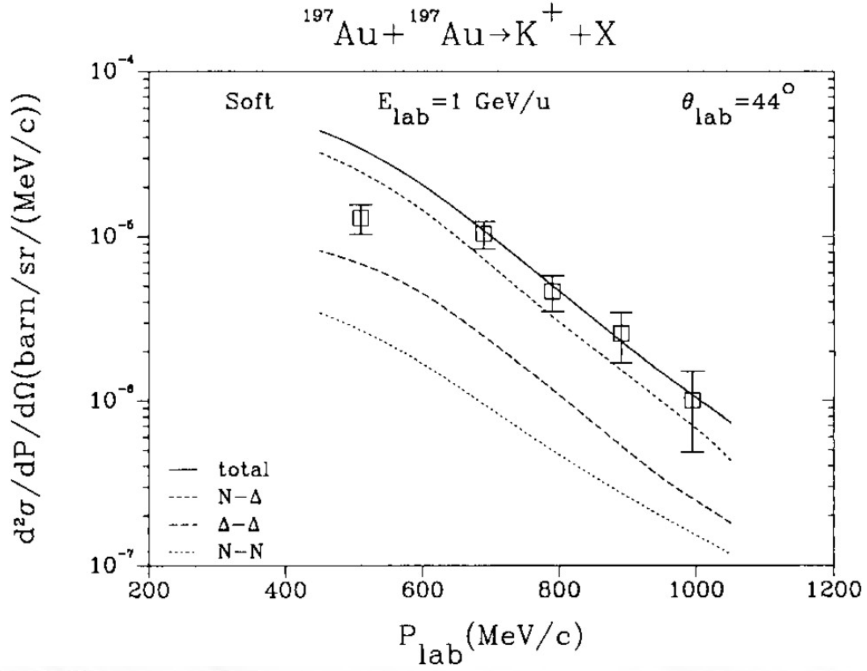
-production mechanism of K^+ is two-fold: $BB \rightarrow BYK^+$ (light systems) B=N,Δ; Y=Λ,Σ
$\pi B \rightarrow YK^+$ (heavy systems)

C. Fuchs et al PRC 56, R606 (1997)



Ne, Ca, Nb, Au @1AGeV

S.Huang et al. PLB 298, 41 (1993)



K$^+$ production below threshold (RQMD)

C. Fuchs et al PRL 86, 1974 (2001)
C. Fuchs 56, 1 (2006)

-production mechanism of K$^+$ is two-fold: BB → BYK$^+$ (light systems) B=N,Δ; Y=Λ,Σ
πB → YK$^+$ (heavy systems)

- treatment of final state phase-space that allows energy conservation (BB → BYK$^+$):

$$d\Phi_3(\sqrt{s}, m_B, m_Y, m_K) = d\Phi_2(\sqrt{s}, m_B, M)\, dM^2\, \Phi_2(M, m_Y, M_K)$$

$$\Phi_2(\sqrt{s}, m_1, m_2) = \frac{\pi\, p(\sqrt{s}, m_1, m_2)}{\sqrt{s}} \qquad p(\sqrt{s}, m_1, m_2) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}}$$

- if particles retain quasiparticles properties in dense medium (K$^+$)

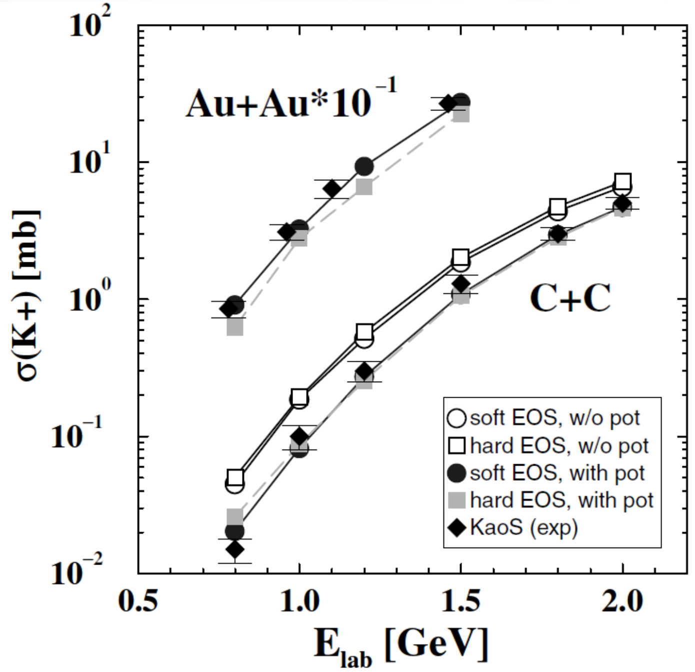
$$k_{1,\mu} + k_{2,\mu} = k'_{1,\mu} + k'_{2,\mu} \quad \text{equivalent} \quad k^*_{1,\mu} + k^*_{2,\mu} = k^{*'}_{1,\mu} + k^{*'}_{2,\mu}$$

then s,m_B,m_Y,m_K → s*,m_B*,m_Y*,m_K*

- if final state particle have different potentials or potentials are momentum dependent → modified on-shell condition

$$0 = k^{*2}_\mu - m^{*2}_K = k^2_\mu - m^2_K - 2 m_k U_{\text{opt}} = k^2_\mu - \tilde{m}^2_K$$

-threshold condition: $\sqrt{s} = \tilde{m}_B + \tilde{m}_Y + \tilde{m}_K$



K^+ production below threshold (IQMD)

C. Hartnack et al Phys. Rep. 510, 119 (2012)

In-medium KN potential (K^+)

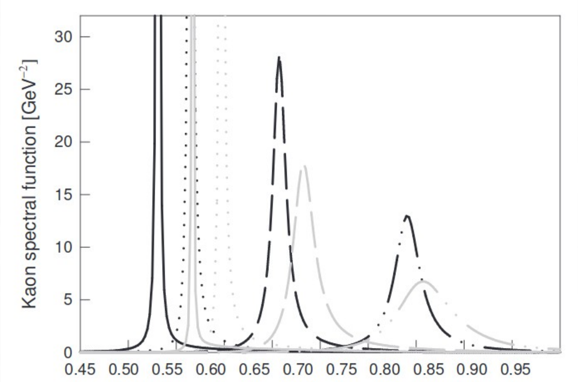
$$U_{opt} = \omega(\vec{k},\rho) - \sqrt{\vec{k}^2 + m^2}$$

$$\omega(\vec{k},\rho) = \sqrt{(\vec{k} - \vec{\Sigma}_v)^2 + m^2 + m\,\Sigma_s} + \Sigma_v^0$$

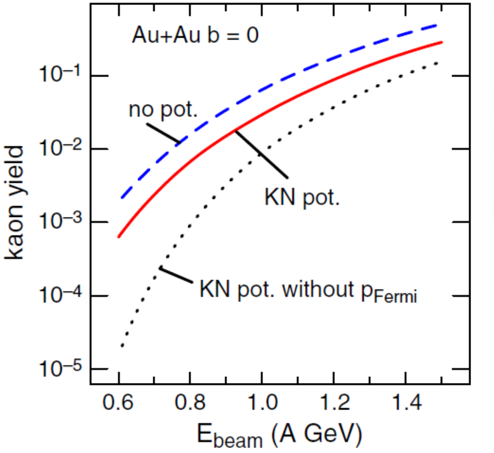
J. Schaffner-Bielich, J. Phys. G 27 (2001) 337

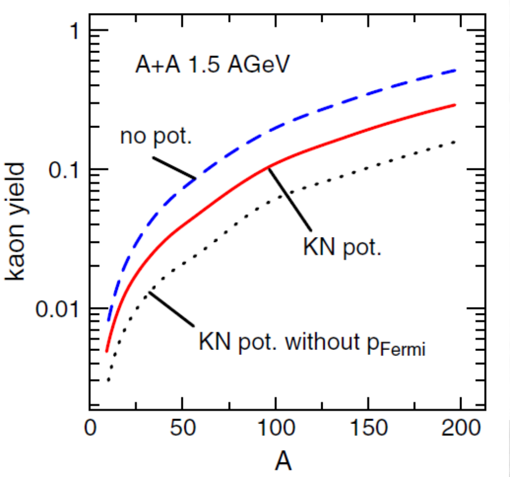
Can be approximated as:

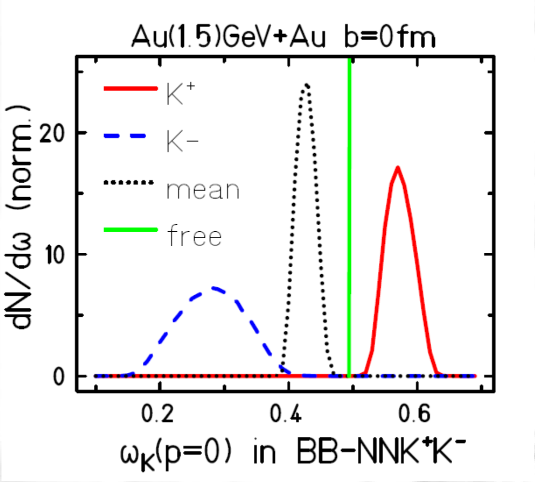
$$m_K(\rho) = m_K(\rho=0)\left(1 + \alpha\frac{\rho}{\rho_0}\right) \qquad \alpha = 0.08$$



K.L.Korpa et al., Acta Phys. Hun. A 22, 1 (2005)



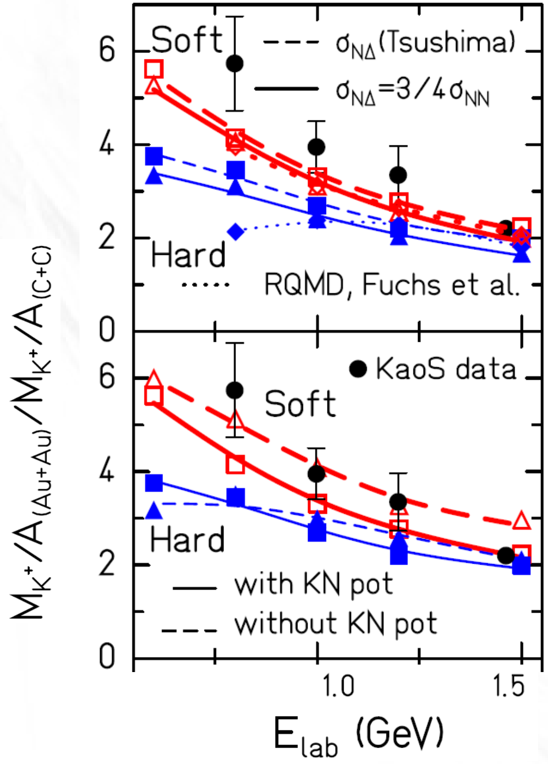




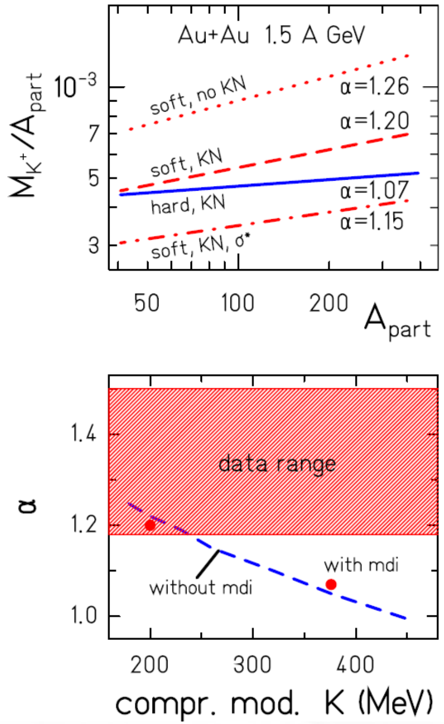
EoS of Symmetric Nuclear Matter

C. Hartnack et al PRL 86, 012302 (2006)

ratio of K^+/A multiplicities



A_{part} dependence of multiplicities $M_{K^+} \approx A_{part}^{\alpha}$



- comparison theory-experiment for a much wider set of experimental data exists: see C. Hartnack et al Phys. Rep. 510, 119 (2012)

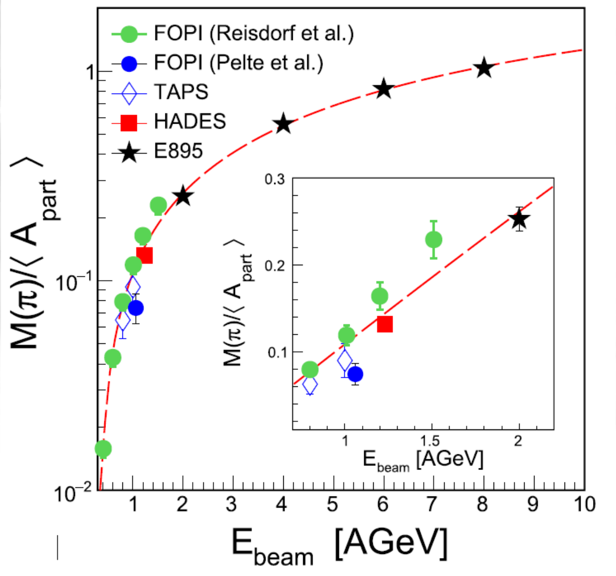
particle spectra in rapidity, transverse mass (K^+,K^-,Λ), v_1 and v_2 flows (K^+,K^-,Λ): results for IQMD and HSD

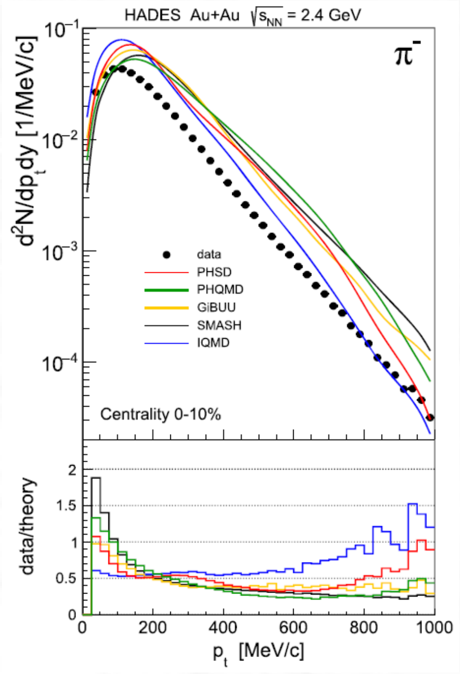
Pion production at E_{lab}=1.23 AGeV (HADES)

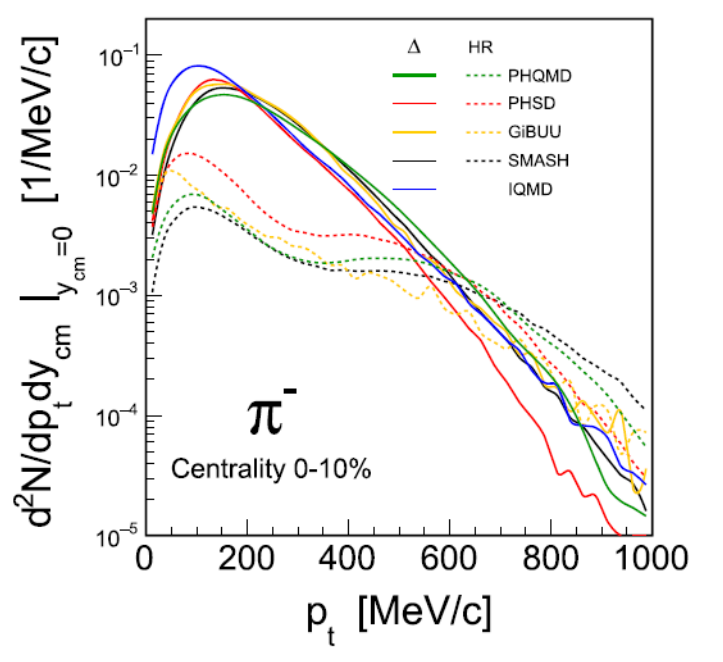
J. Adamczewski-Musch et al., EPJA 56, 259 (2020)

- high statistics data for 4 centrality bins in the range of 40% most central collisions
- data sets as function of transverse momentum, transverse mass, rapidity, and polar angle

2.5σ difference with respect to FOPI data point at 1.2 AGeV







for a very recent possible explanation: see K. Godbey et al. Arxiv:2107.13384

Symmetry energy effects in Kaon Production

G. Ferini et al. NPA 762, 147 (2005)
G. Ferini et al. PRL 97, 202301 (2006)

RBUU model based on RMF used to describe HIC

$$s_{in}=(k_1^\mu+k_2^\mu)^2=(k_3^\mu+k_4^\mu)^2=s_{out}$$

$$k^{*\mu}=k^\mu-\Sigma^\mu \qquad m^*=m+\Sigma_s$$

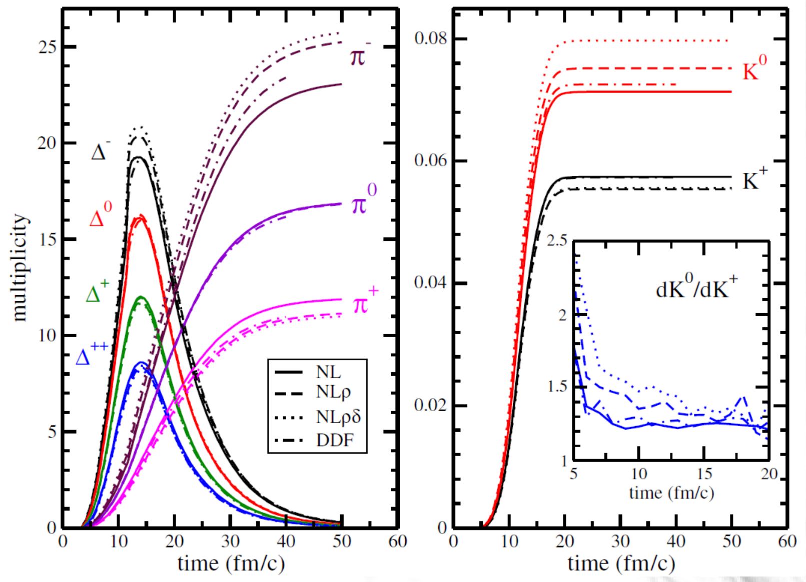
$$\Sigma_s(p/n)=-f_\sigma\rho_s\pm f_\delta\rho_{s3}$$

$$\Sigma^\mu(p/n)=f_\omega j^\mu\mp f_\rho j_3^\mu$$

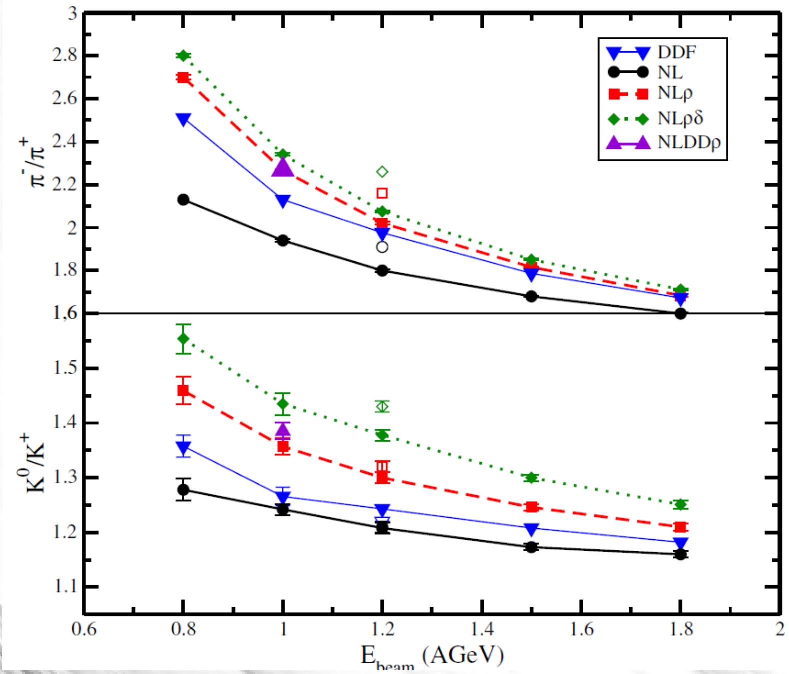
medium modified threshold condition

$$s_{in}\geq(m_3^*+\Sigma_3^0+m_4^*+\Sigma_4^0)^2-(\vec{\Sigma}_3+\vec{\Sigma}_4)^2$$

Symmetry energy: $$S(\rho)=\frac{1}{6}\frac{k_F^2}{E_F}+\frac{1}{2}\left[f_\rho-f_\delta\frac{m^{*2}}{E_F^2}\right]\rho$$



central AuAu collision 1 AMeV



Threshold effects in pion production

T.Song et al. PRC 91, 014901 (2015)

RVUU transport model + NLρ, NLρδ RMF models

threshold condition

$$\sqrt{s_{\rm th}} = \sqrt{(m_3^* + \Sigma_3^0 + m_4^* + \Sigma_4^0)^2 - |\boldsymbol{\Sigma}_3 + \boldsymbol{\Sigma}_4|^2}$$

for static nuclear matter ($\vec{\Sigma}_i = 0$)

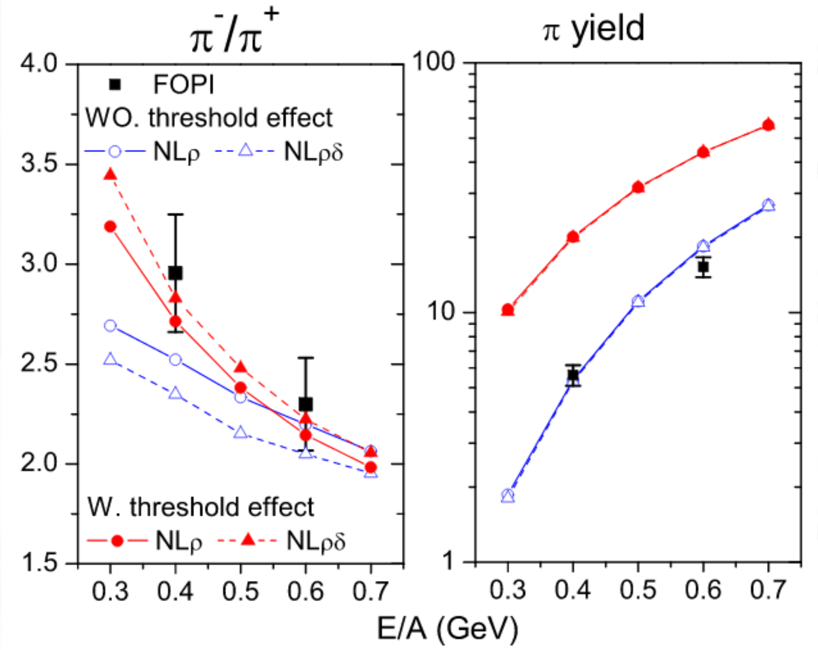
$$\sqrt{s_{\rm in}} - \sqrt{s_{\rm th}} \simeq E_1^* + E_2^* + \Sigma_1^0 + \Sigma_2^0 - m_3^* - m_4^* - \Sigma_3^0 - \Sigma_4^0$$

non-relativistic limit

$$\sqrt{s_{\rm in}} - \sqrt{s_{\rm th}} \simeq m_1 + m_2 - m_3 - m_4 + \Sigma_1^s + \Sigma_2^s - \Sigma_3^s - \Sigma_4^s + \frac{|\mathbf{p}_1^*|^2}{2m_1^*} + \frac{|\mathbf{p}_2^*|^2}{2m_2^*} + \Sigma_1^0 + \Sigma_2^0 - \Sigma_3^0 - \Sigma_4^0 \quad (30)$$

$$\Sigma_i^s = m_i^* - m_i .$$

L=83/98 MeV (NLρ, NLρδ)
$S(\rho_0)$=30 MeV



to describe experimental values of multiplicies

$$\sigma_{NN\to\Delta N}(\rho_N) = \sigma_{NN\to\Delta N}(0)\exp(-A\rho_N/\rho_0)$$ A=1.65

Impact of the pion potential

Z. Zhang et al, PRC 95, 064604 (2017)

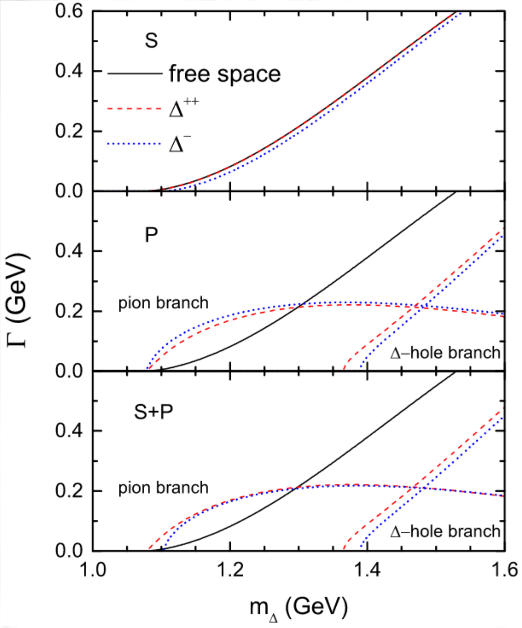
S-wave self-energy: ChPT up to two loops N.Kaiser et al. PLB 512, 283 (2001)
P-wave self-energy: Delta-hole model+short range corrections

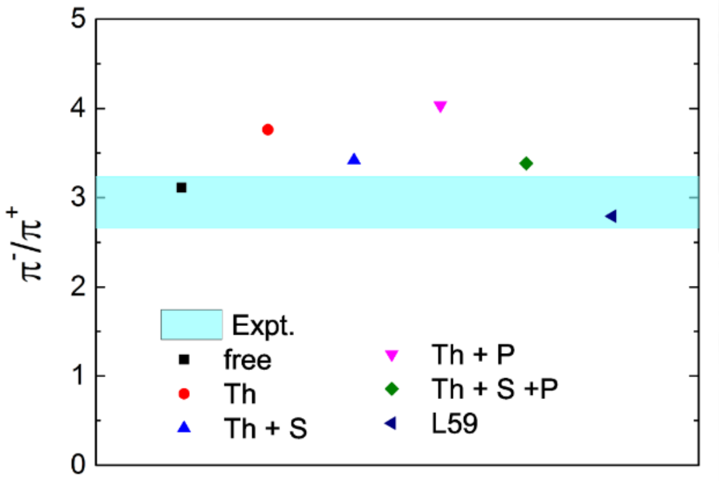
$$\Gamma(m_\Delta,\rho) = \frac{1}{2m_\Delta}\sum_{m_t}\int \frac{d^3\boldsymbol{p}_N}{(2\pi)^3 2E_N}\frac{d^3\boldsymbol{k}d\omega}{(2\pi)^3} \times \delta\left(\omega^2 - k^2 - m_\pi^2 - \Pi_S^{m_t} + \Pi_P^{m_t}(\omega,\boldsymbol{k})\right)|\mathcal{M}|^2 \times (2\pi)^4\delta^3(\boldsymbol{p}_N+\boldsymbol{k})\delta(E_N+\omega-m_\Delta)$$

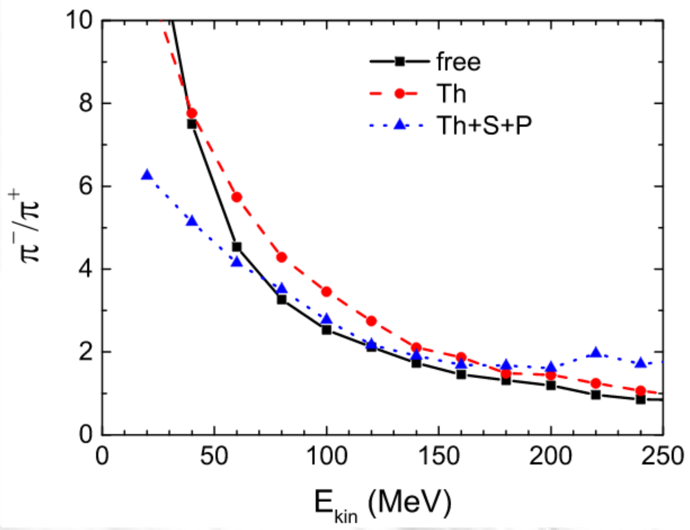
$$\Rightarrow \sum_{m_t,i}\frac{k_{m_t,i}^2}{8\pi\sqrt{m_N^2+k_{m_t,i}^2}\,m_\Delta\omega_{m_t,i}}S(k_i,\rho)|\mathcal{M}|^2 \times \left|\frac{\boldsymbol{k}_{m_t,i}}{\sqrt{m_N^2+k_{m_t,i}^2}} + \frac{d\omega_{m_t,i}}{d\boldsymbol{k}_{m_t,i}}\right|^{-1}$$

$\rho=2\rho_0$
$\delta=0.2$

NLρ: L=83 MeV







$$\sigma_{NN\to\Delta N}(\rho_N) = \sigma_{NN\to\Delta N}(0)\exp(-A\rho_N/\rho_0)$$

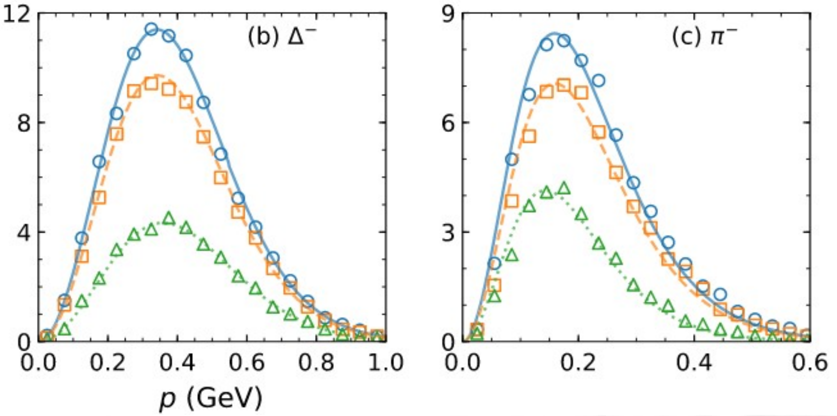
A=-2.35,1.5,1.4,1.65,1.5,1.5

Thermodynamic consistency

Z.Zhang et al, PRC 97, 014610 (2018)

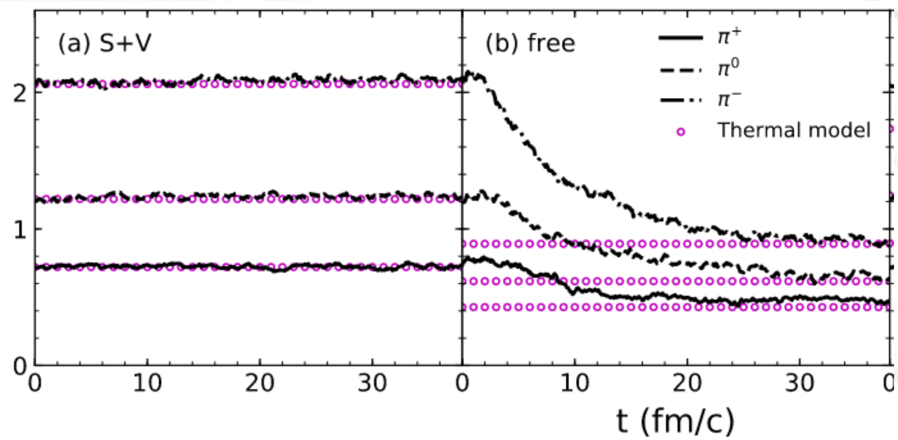
- RVUU+NLρ RMF model
- initial numbers of N,Δ,π are determined from equilibrium conditions for T=60 MeV, $\rho=1.5\rho_0$, δ=0.4
- detailed balance in the medium for NN↔NΔ and Δ↔Nπ account for in-medium modification of masses and momenta
- vacuum decay width and spectral function are used for Δ(1232)

Momentum distributions at t=40 fm/c



Thermal model – curves
Transport model - symbols

Time evolution of pion multiplicities



DC, PLB 753, 166 (2016)

Threshold effects in a QMD model

Elastic scattering:

$$\sqrt{s_f} \approx \sqrt{s_i}$$

$$\sqrt{s^*} = 0.5\left(\sqrt{s_f} + \sqrt{s_i}\right)$$

Resonance excitation:

$$\sqrt{s_f} - \sqrt{s_i} \approx 25\,\mathrm{MeV}$$

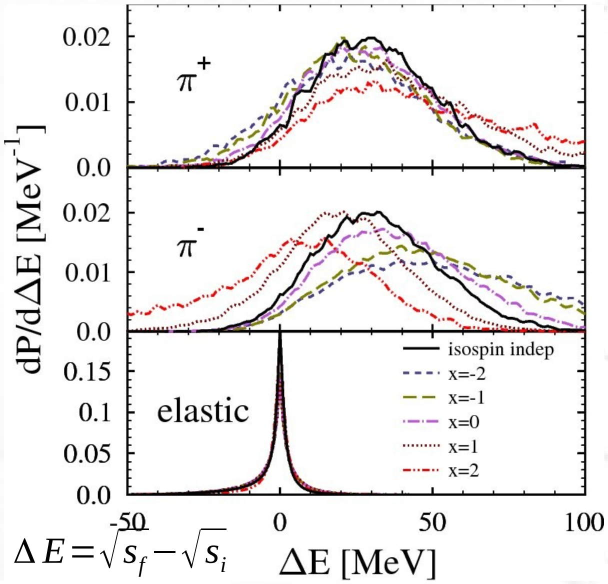
$$\sqrt{s^*} = \sqrt{s_f}$$

Resonance absorption: detailed balance

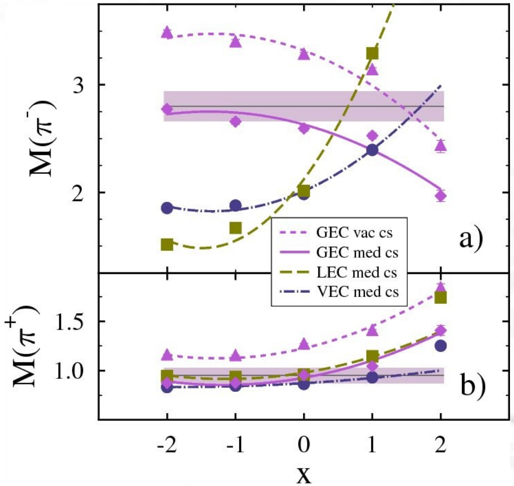
P. Danielewicz et al., NPA 533, 712 (1992)

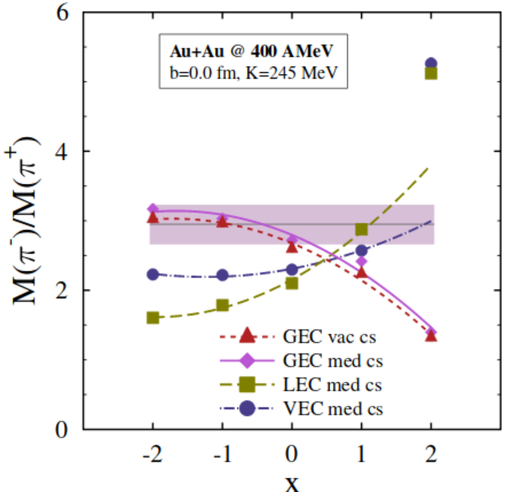
$$\frac{d\sigma^{NR\to NN}}{d\Omega} = \frac{1}{4}\frac{m_R\, p_{NN}^2}{p_{NR}}\frac{d\sigma^{NN\to NR}}{d\Omega} \times \left[\frac{1}{2\pi}\int_{m_N+m_\pi}^{\sqrt{s_i}-m_N} dM\, M\, p'_{NR}\, A_R(M)\right]^{-1}$$

$$\sqrt{s_i} \to p_{NR}, \quad \sqrt{s_f} \to p_{NN}$$



$$\Delta E = \sqrt{s_f} - \sqrt{s_i}$$

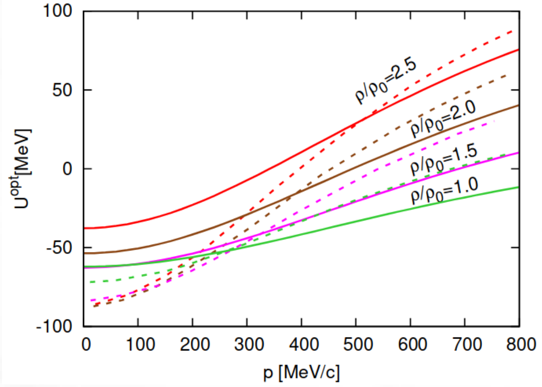


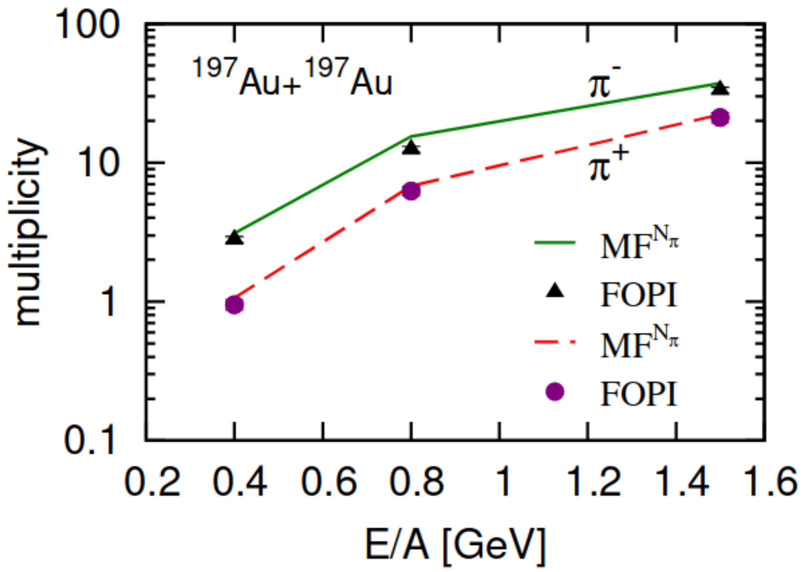


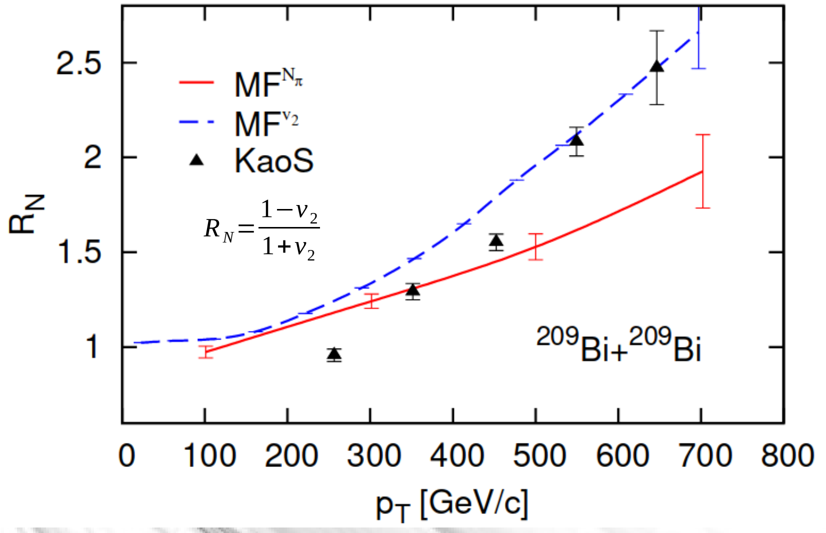
Impact of the optical potential

J. Hong et al. PRC 90, 024605 (2015)

- pBUU transport model shows no sensitivity to SE for multiplicity ratios for various impact energies
- includes the effect of S-wave pion optical potential
- propagates light cluster degrees of freedom
- Huber et al. parametrization of NN→NΔ cross-section
- momentum indepent/dependent (stadard) MF overpredict/underpredict pion multiplicities at the lower impact energies



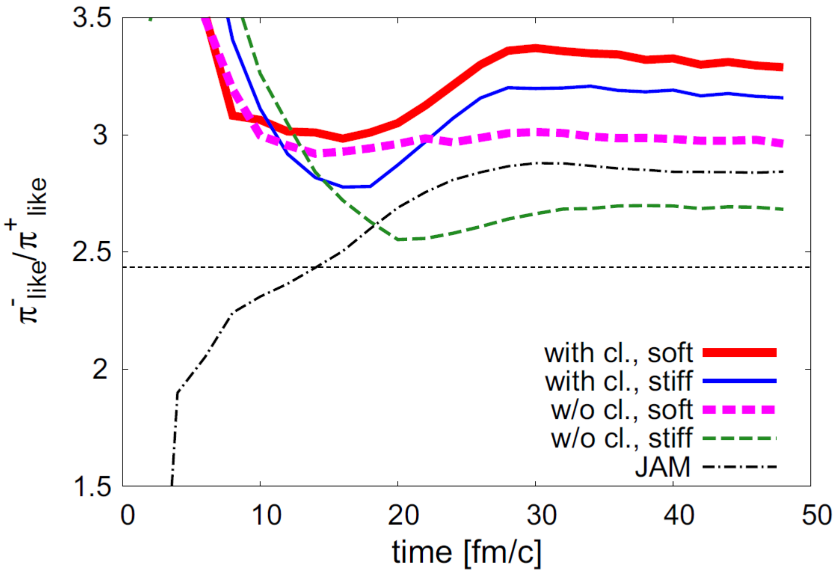




Cluster correlations/detailed balance

Cluster correlations: AMD+JAM

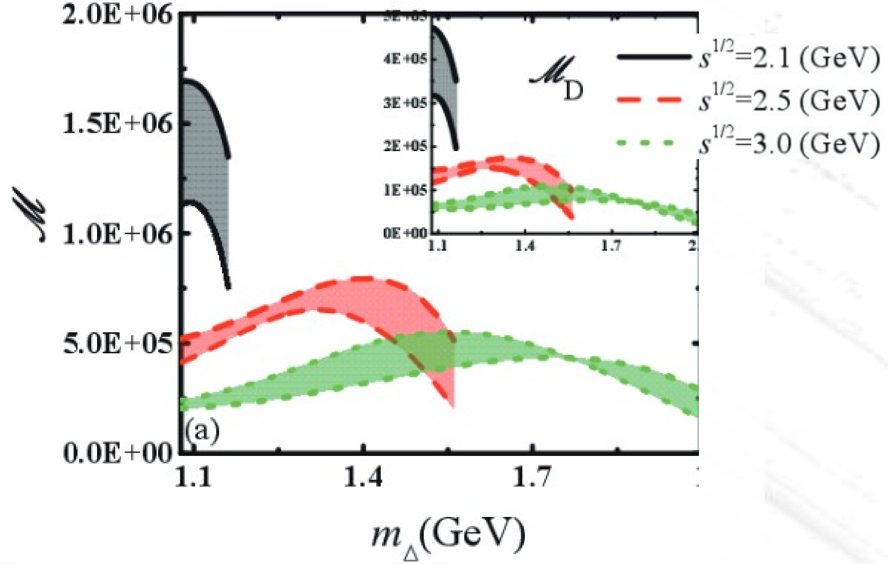
132Sn124Sn 300 MeV/A b<1.0 fm



N. Ikeno et al. PRC 93, 044612 (2016)PRC 97, 069902 (2018)

Detailed balance assumption: scattering amplitude M is independent of m_Δ

determined from a fit of a OBE model to exp data



Y.Cui et al., Chin. Phys. C 44, 024106 (2020)

Cluster correlations/detailed balance

Cluster correlations: AMD+JAM

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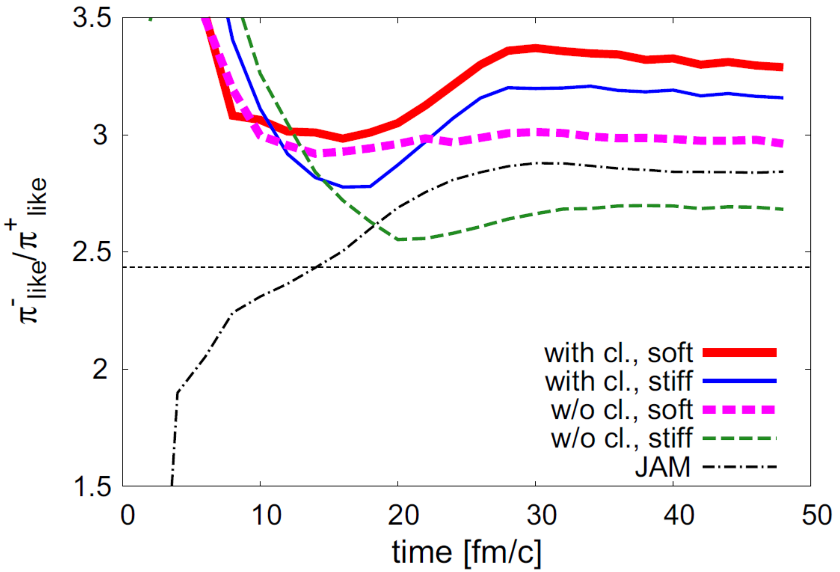

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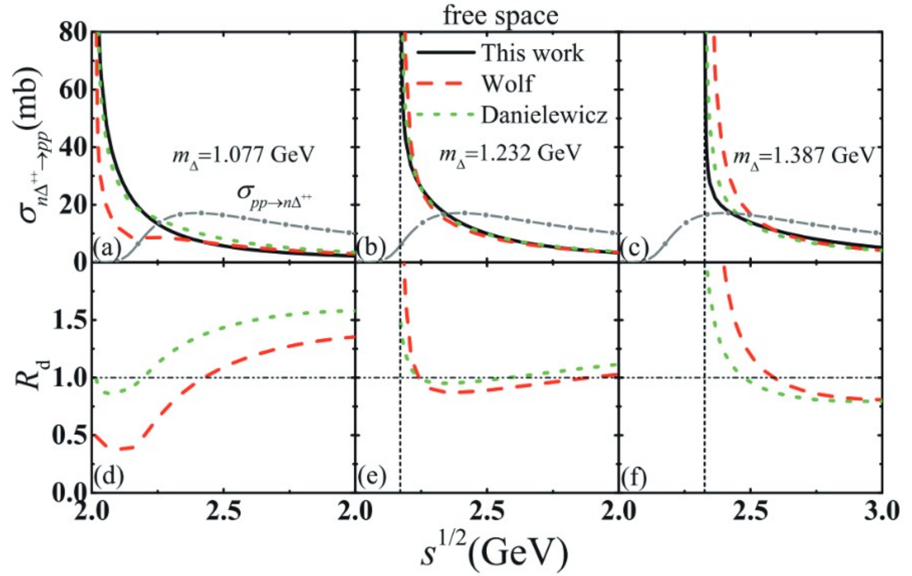


Y.Cui et al., Chin. Phys. C 44, 024106 (2020)

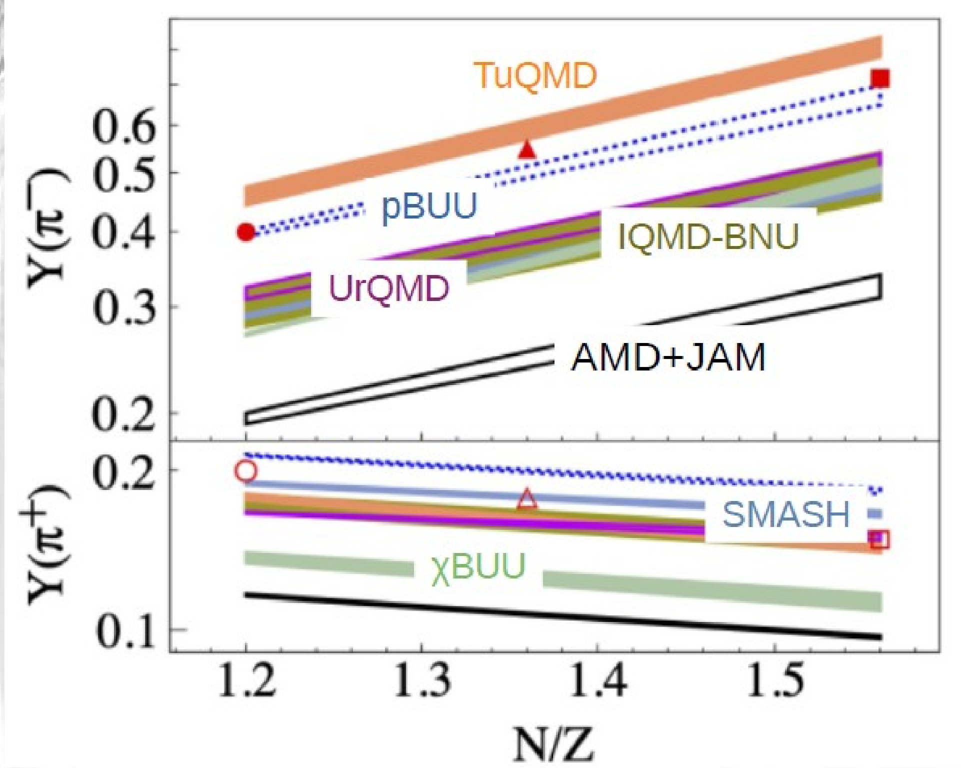
Model for in-medium $\sigma_{NN\to N\Delta}$: A. Larionov et. al. NPA 728, 135 (2003)

comparison to pion production data (first generation FOPI, TAPS)

Theoretical predictions for SπIRIT

G. Jhang et al., PLB 813, 136016 (2021)

Multiplicities



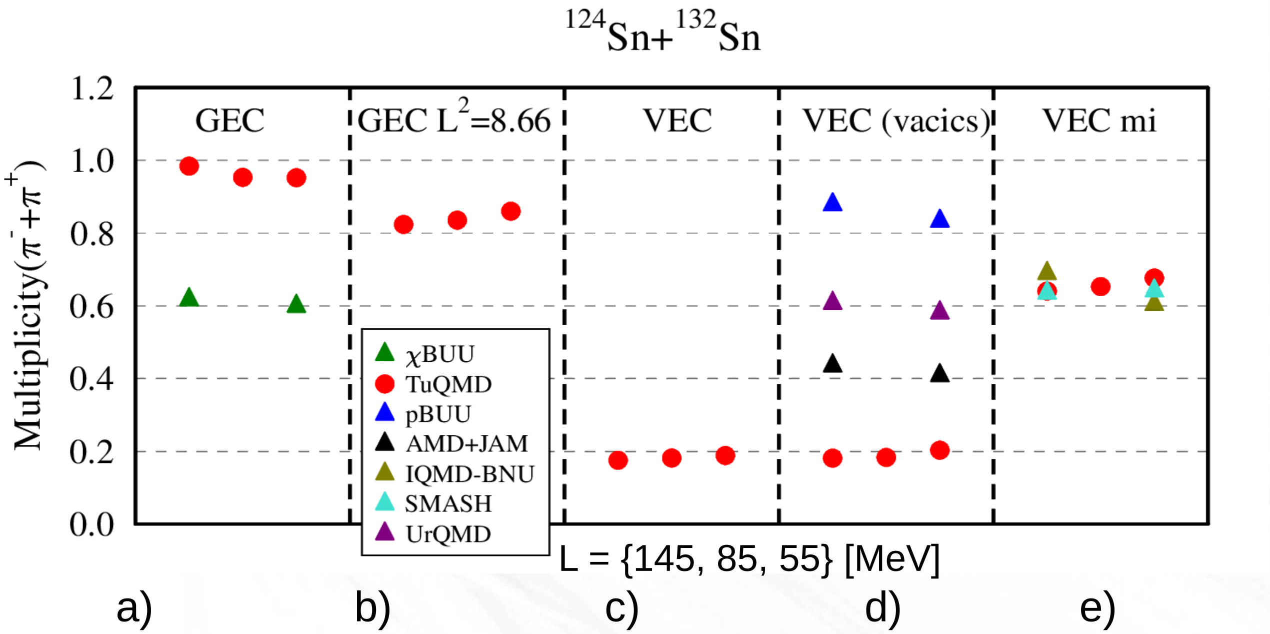
Total charged multiplicities vary by a factor of 2.5
Pion multiplicity ratios and double ratios vary by factors of 2.0 and 1.2 respectively

Model	**Features**
χBUU	Threshold Effects; MDI Skχm* ; Bertsch elastic CS; Huber Inelastic CS;
TuQMD	Threshold Effects; MDI2 (Gogny); Li-Machleidt + Cugnon+ med corrections; Huber inelastic CS+ med corr
pBUU	MDI – Landau EF; medium modified elastic CS (stopping); Huber inelastic CS
AMD+JAM	SLy4+mom dependence; cluster correlations; med modified elastic CS; Randrup inelastic CS
IQMD-BNU	MI interaction; Cugnon elastic CS +Li-Machleidt med corr; Bertsch inelastic CS
SMASH	MI interaction; Cugnon elastic CS; Dmitriev OBE inelastic CS; no Coulomb
UrQMD	MDI interaction (old HA); PDG elastic CS+med corrections; modified Huber OBE model (mass dependent amplitudes)

Pion production in Sn+Sn @ 270 MeV

5 scenarios to address model depedence / differences between various models:

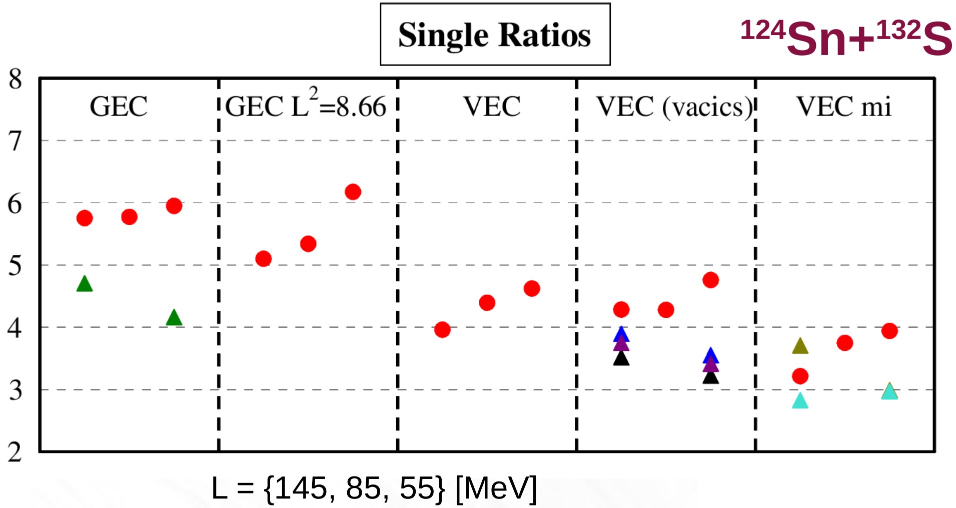
a) threshold effects included, MDI, L^2=4.33 fm^2
b) threshold effects included, MDI, L^2=8.66 fm^2
c) no threshold effects, MDI, in-medium inelastic cross-sections
d) no threshold effects, MDI, vacuum inelastic cross-sections
e) no threshold effsctds, MI, vacuum inelastic cross-sections



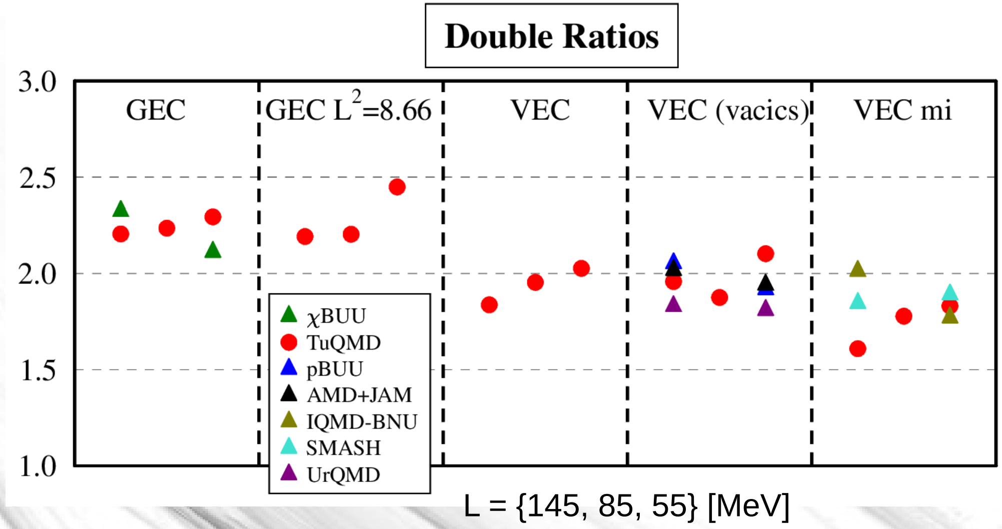
Note: χBUU model normalized to AuAu 400 MeV/nucleon; TuQMD overpredicts those data by 30%

Pion production in Sn+Sn @ 270 MeV

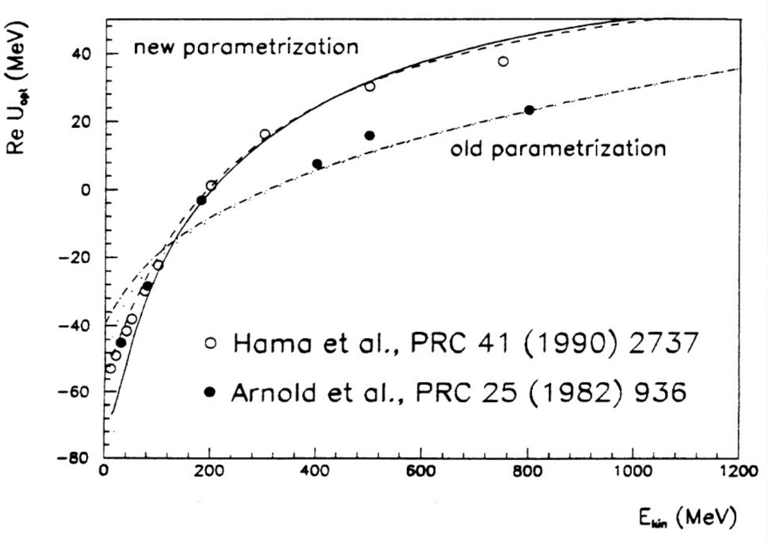
$^{124}Sn+^{132}Sn$



Note: Model dependence of the Pauli blocking algorithm would reduce single ratios by ~10%



Momentum Dependence of Potentials

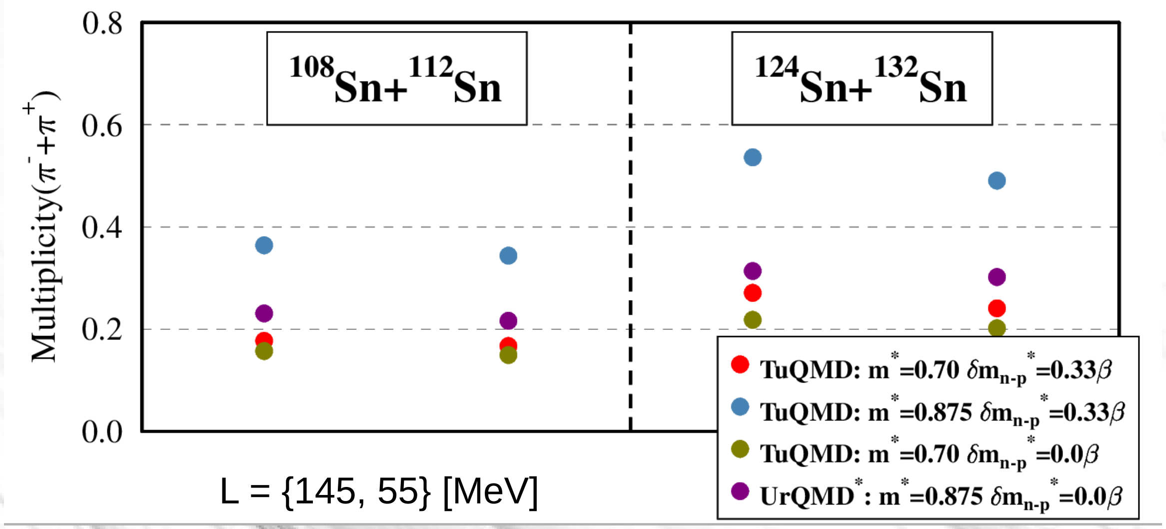


C. Hartnack et al., Phys. Rev. C 49, 2801 (1994)

UrQMD – employs "old parametrization"

pBUU – weaker momentum dependence of the optical potential needed in order to Describe pion multiplicities for AuAu 400 MeV/nucleon (J. Hong et al., PRC 90, 024605)

- additional sensitivity to $\sigma_{NN\rightarrow N\Delta}$ and $\Gamma_{\Delta\rightarrow N\pi}$ (factor of two for each system)
- impact on single ratios reduced, in particular for the ^{108}Sn^{112}Sn system

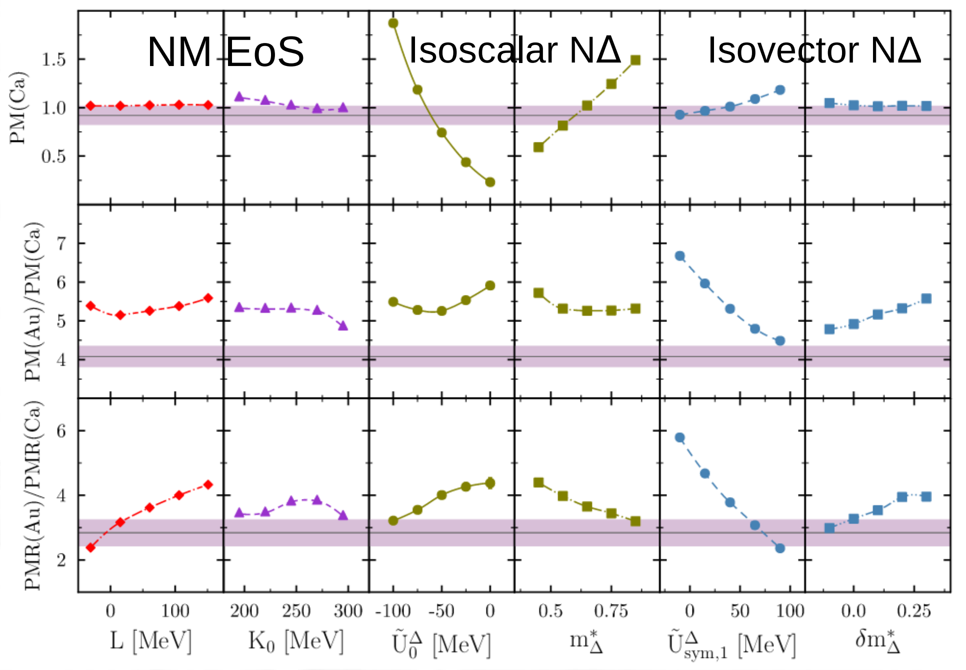


Impact of the Δ(1232) potential

D.C. et al. ArXiv:2101.08679

- threshold effects included

horizontal bands – experimental FOPI data (b_0<0.15) at 400 MeV/nucleon



-additional residual sensitivity of the NΔ interactionat supranormal densities (quantified in terms of potential's strength at $2\rho_0$)

Pion production channels close to threshold

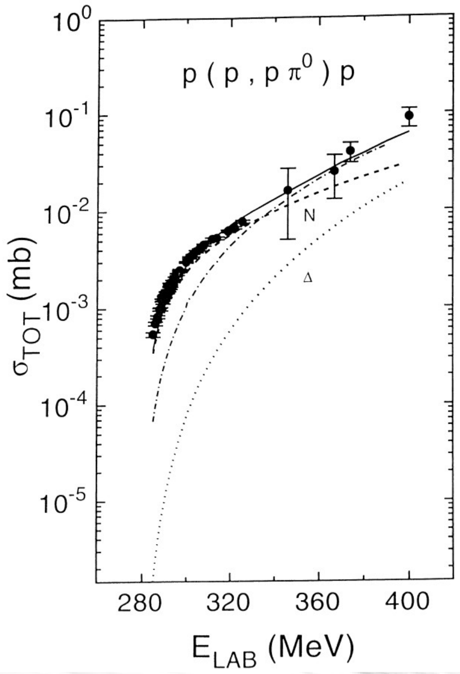
two-step mechanism: - resonance excitation+ resonance decay
- input for transport: parametrization of NN→NR cross-sections

popular choice: OBE model of Huber et al: single, double production of Δ(1232) and N(1440) NPA 573, 587 (1991)
fitted cut-off parameter values to NN→NΔ and NN→NN* cs for p_{lab}>1.0 GeV/c

close to threshold: - non-resonant production becomes important/dominant

OBE model of Engel et al.

A. Engel et al. NPA 603, 387 (1996)
R. Shyam et al. PLB 426, 1 (1998)



physics content: similar to Huber et al OBE model
- VNN (V=π,ρ,ω,σ) – elastic pp, pn scattering data (energy dep couplings)
- πNΔ – Δ→Nπ decay
- ρNΔ – mass differential cross-sections NN→NΔ at 1-2.5 GeV

allows description of experimental data of total, double and triple differential cross-sections for pp→nπ^+p and pp→pπ^0p in the energy range 0.3-2.0 GeV

Summary & Conclusions

Subtreshold Kaon (K^+) production:

- probe sensitive to the EoS of symmetric matter close $2\rho_0$
- consistency of constraints for K_0 with a limited number of models (2) that are very similar
- recent comparison to HADES pion spectra evidenced important difference between models, including high energy pions that are important for K^+ production (momentum dependence of the interaction?)

Pion production close to threshold:

- several strong competing effects: threshold effects, momentum dependence of the interaction (consistency with nucleonic observables needed), medium modification of cross-sections
- for integrated multiplicity ratios extracting the density dependence of SE is not a one parameter problem
- close to threshold non-resonant production channels need to be included