Light clusters in dynamic evolution of heavy-ion collisions

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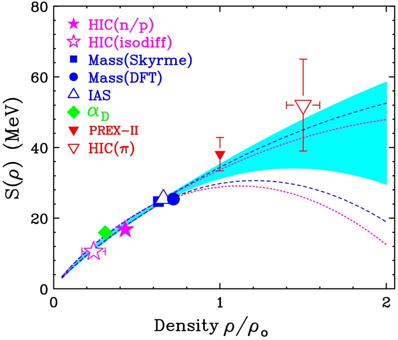
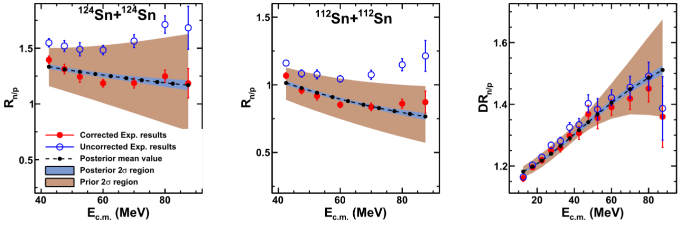
Symmetry energy constraints and clusters



Lynch, Tsang, arXiv:2106.10119.



★ HIC(n/p) is a result from "coalescence-invariant (CI)" neutron and proton spectra from Sn + Sn central collisions at 120*A* MeV. [Morfouace et al., PLB 799 (2019) 135045.]

$$"n" = n + {}^2H + 2 \times {}^3H + {}^3He + 2 \times {}^4He$$

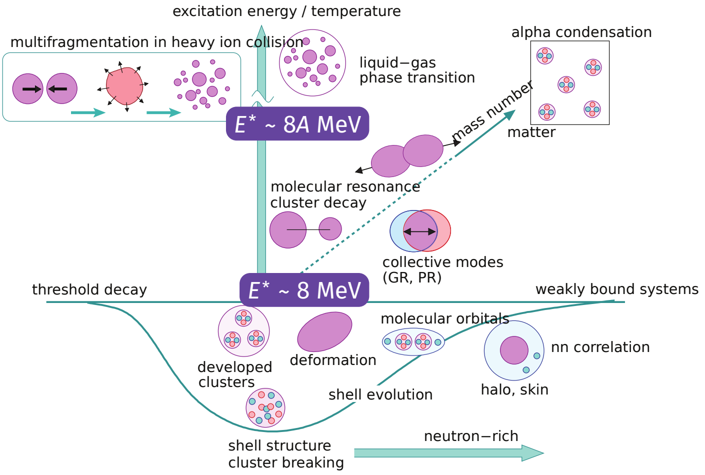
$$"p" = p + {}^2H + {}^3H + 2 \times {}^3He + 2 \times {}^4He$$

"CI" spectra may (or not?) be affected by cluster formation if many clusters are formed, e.g., through energy conservation.

We have to understand cluster formation.

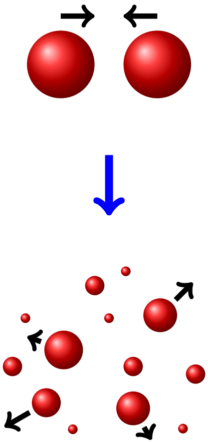
Clustering phenomena in excited states of nuclear systems

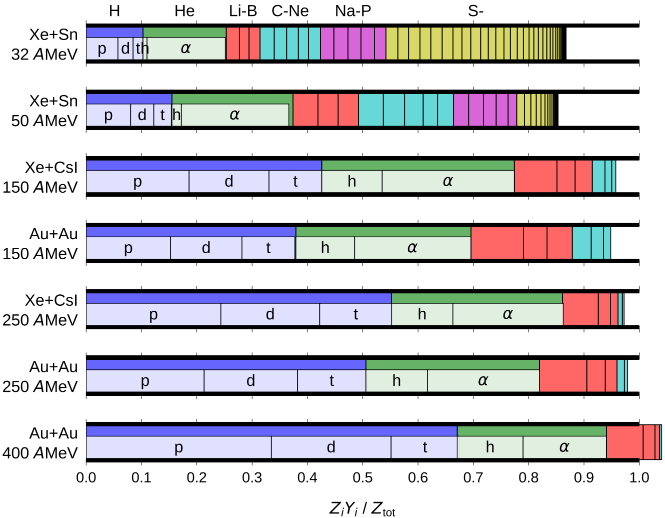
$E^* \sim 80A$ MeV Gas of clusters at higher energies



Kanada-En'yo, Kimura, Ono, Prog. Theor. Exp. Phys. 2012 01A202 (2012)

Fraction of protons in clusters and fragments in heavy-ion collisions



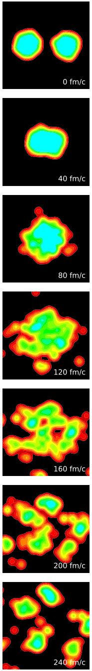


INDRA: Hudan et al., PRC67 (2003) 064613. FOPI: Reisdorf et al., NPA 848 (2010) 366.

Figure in Ono, PPNP 105 (2019) 139.

This is a challenge to theorists and transport models.

Cluster recognition and cluster dynamics in transport approaches



- Calculate $e^{-iHt/\hbar}|\Psi(t=0)\rangle$, and find clusters.

- Cluster correlation in dynamics — Find whether a nucleon moves together with some other nucleons for a while.
  - Clusters will not necessarily be emitted.
  - Correlations affect the time evolution.

- SACA, FRIGA [Le Fèvre et al., PRC 100 (2019) 034904.], for QMD.
- Coalescence prescription, to predict clusters in BUU.

- At large t (e.g. $t = 200$–1000 fm$/c$), there is nothing controversial in finding clusters and fragments.
  - ..., if the state at this time is predicted by the transport model reasonably well.
  - The decay of excited fragments should be calculated by a statistical decay code.

BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

Coupled equations for $f_n(\mathbf{r},\mathbf{p},t)$, $f_p(\mathbf{r},\mathbf{p},t)$, $f_d(\mathbf{r},\mathbf{p},t)$, $f_t(\mathbf{r},\mathbf{p},t)$, $f_h(\mathbf{r},\mathbf{p},t)$ are solved by the test particle method.

$$\frac{\partial f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}}\cdot\frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\rm coll}[f_n, f_p, f_d, f_t, f_h]$$

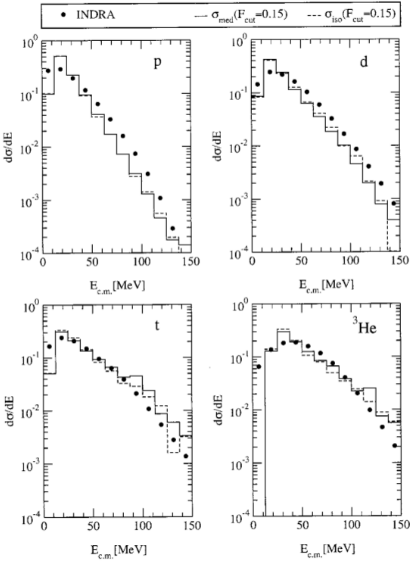
$$\frac{\partial f_p}{\partial t} + \mathbf{v}\cdot\frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}}\cdot\frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\rm coll}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}}\cdot\frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\rm coll}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v}\cdot\frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}}\cdot\frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\rm coll}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}}\cdot\frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\rm coll}[f_n, f_p, f_d, f_t, f_h]$$

Similar transport models for relativistic collisions:
Oliinychenko et al, PRC 99 (2019) 044907. KJ Sun et al., arXiv:2106.12742 [nucl-th].



Renormalized cluster spectra
Kuhrts et al., PRC63(2001)034605.

BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

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$$\frac{\partial f_p}{\partial t} + \mathbf{v}\cdot\frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}}\cdot\frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\mathrm{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}}\cdot\frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\mathrm{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v}\cdot\frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}}\cdot\frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\mathrm{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}}\cdot\frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\mathrm{coll}}[f_n, f_p, f_d, f_t, f_h]$$

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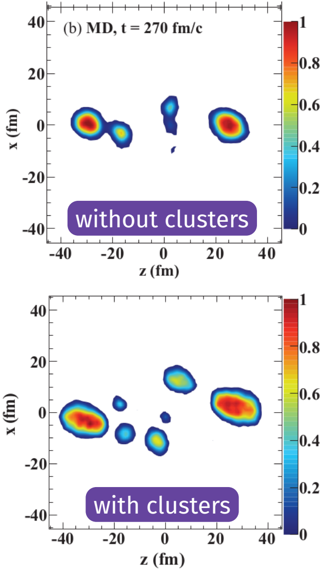
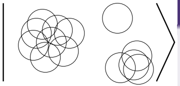


Isospin diffusion and fragmentation

Coupland et al., PRC 84 (2011) 054603.

Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, H\}_{\mathrm{PB}} \quad + \quad \text{(NN collisions)}$$

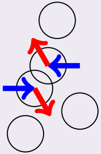
$\{\mathbf{Z}_i, H\}_{\mathrm{PB}}$: Motion in the mean field

$$H = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

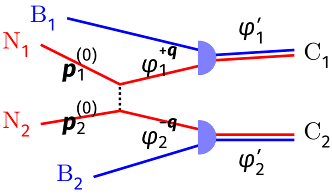


- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

$$N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2$$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



$$\boldsymbol{p}_{\rm rel} = \tfrac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2) = p_{\rm rel}\hat{\boldsymbol{\Omega}}$$

$$\boldsymbol{p}_1 = \boldsymbol{p}_1^{(0)} + \boldsymbol{q}$$

$$\boldsymbol{p}_2 = \boldsymbol{p}_2^{(0)} - \boldsymbol{q}$$

$$\varphi_1^{+\boldsymbol{q}} = \exp(+i\boldsymbol{q}\cdot\boldsymbol{r}_{N_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\boldsymbol{q}} = \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_{N_2})\varphi_2^{(0)}$$

Transition probability

$$W(\mathrm{NBNB} \rightarrow \mathrm{CC}) = \frac{2\pi}{\hbar}|\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2\delta(E_f - E_i)$$

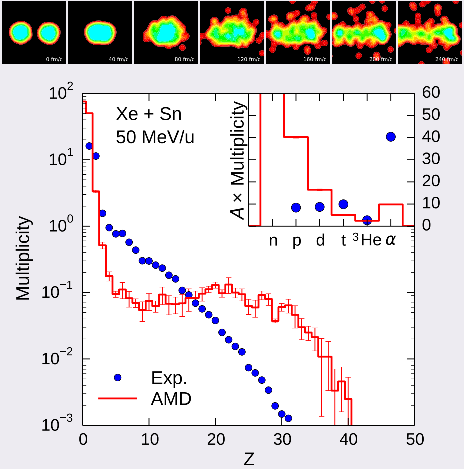
$$v\,d\sigma \propto |\langle\varphi_1'|\varphi_1^{+\boldsymbol{q}}\rangle|^2|\langle\varphi_2'|\varphi_2^{-\boldsymbol{q}}\rangle|^2|M|^2\delta(E_f - E_i)p_{\rm rel}^2\,dp_{\rm rel}\,d\Omega$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

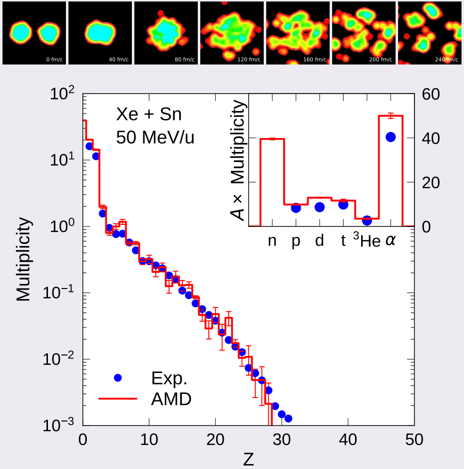
$\Leftarrow (d\sigma/d\Omega)_{\rm NN}$ in free space (or in medium)

Ono, J. Phys. Conf. Ser. 420 (2013) 012103.
Ikeno, Ono et al., PRC 93 (2016) 044612.
Ono, JPS Conference Proceedings 32 (2020) 010076.

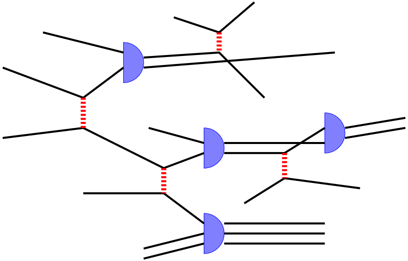
Without clusters

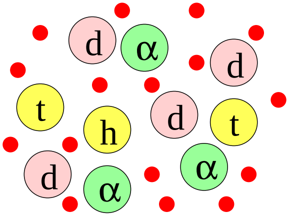


With clusters



INDRA data: Hudan et al., PRC67 (2003) 064613.

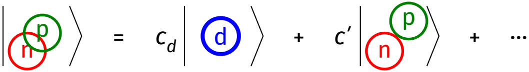




- A cluster in AMD is still composed of nucleon wave packets. The many-body state is a Slater determinant of nucleons.
- Clusters are not only created but also broken.
- Clusters are in medium, so the existence and the properties may be modified.
- For a collision, there are many possible configurations (C_1, C_2) of cluster formation. Non-orthogonality is treated suitably.
- Correlations to bind several clusters (like $\alpha + t = {}^7\mathrm{Li}$) are also important and taken into account.

- $n + p + X \leftrightarrow d + X'$
- $d + n + X \leftrightarrow t + X'$
- $d + p + X \leftrightarrow h + X'$
- $t + p + X \leftrightarrow \alpha + X'$
- $h + n + X \leftrightarrow \alpha + X'$
- $d + d + X \leftrightarrow \alpha + X'$
- $2n + p + X \leftrightarrow t + X'$
- $n + 2p + X \leftrightarrow h + X'$
- $d + n + p + X \leftrightarrow \alpha + X'$
- $2n + 2p + X \leftrightarrow \alpha + X'$
- $d + d \leftrightarrow p + t$
- $d + d \leftrightarrow n + h$
- $p + t \leftrightarrow n + h$
- $d + t \leftrightarrow n + \alpha$
- $d + h \leftrightarrow p + \alpha$
- $d + t \leftrightarrow 2n + h$
- $d + h \leftrightarrow 2p + t$
- $d + \alpha \leftrightarrow t + h$

One proton(↑) and one neutron(↑) (both described by Gaussian wave packets)



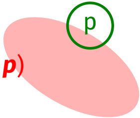
$$\varphi_p(\boldsymbol{r}_1)\,\varphi_n(\boldsymbol{r}_2) = \left[c_d\underline{\psi_d(\boldsymbol{r})} + (\text{continuum states})\right]\varphi_{\mathrm{cm}}(\boldsymbol{R}), \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \quad \boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)$$

$$P_d(\boldsymbol{P}) = \left|\langle e^{i\boldsymbol{P}\cdot\boldsymbol{R}/\hbar}\underline{\psi_d}|\varphi_p\varphi_n\rangle\right|^2 = \int \frac{d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{p}}{(2\pi\hbar)^3}\underline{\rho_d^{\mathrm{W}}(\boldsymbol{r}_1 - \boldsymbol{r}_2, \boldsymbol{p})}\, f_p(\boldsymbol{r}_1, \tfrac{1}{2}\boldsymbol{P} + \boldsymbol{p})\, f_n(\boldsymbol{r}_2, \tfrac{1}{2}\boldsymbol{P} - \boldsymbol{p})$$

One proton(↑) and many neutrons(↑)

$$P_d(\boldsymbol{P}) \overset{???}{=} \int \frac{d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{p}}{(2\pi\hbar)^3}\underline{\rho_d^{\mathrm{W}}(\boldsymbol{r}_1 - \boldsymbol{r}_2, \boldsymbol{p})}\, f_p(\boldsymbol{r}_1, \tfrac{1}{2}\boldsymbol{P} + \boldsymbol{p})\, f_n(\boldsymbol{r}_2, \tfrac{1}{2}\boldsymbol{P} - \boldsymbol{p})$$

LW Chen, CM Ko, BA Li, NPA 729 (2003) 809. Mattiello et al., PRC 55 (1997) 1443.



$f_n(\boldsymbol{r}, \boldsymbol{p})$

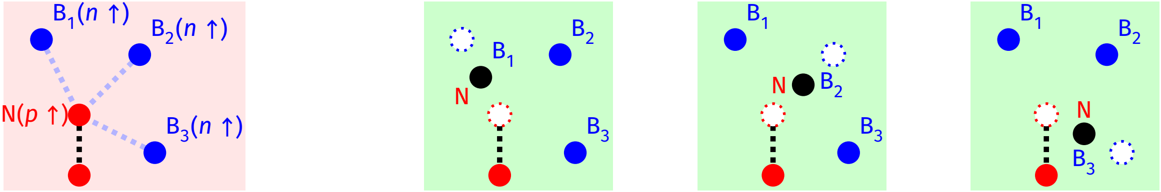
This is valid only in the dilute limit. In general, e.g, the "probability"

$$N_d = \int \frac{d\boldsymbol{P}}{(2\pi\hbar)^3} P_d(\boldsymbol{P}) \quad \text{can be } > 1.$$

- Identifying clusters in a many-body system is a fundamental problem.
- Another question is how the identified clusters are propagated.

Construction of Final States in AMD/Cluster

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$|\Phi^{\boldsymbol{q}}\rangle$
After $\boldsymbol{p}^{(0)} \to \boldsymbol{p}^{(0)} + \boldsymbol{q}$

$|\Phi'_1\rangle$
$N + B_1 \to C_1$

$|\Phi'_2\rangle$
$N + B_2 \to C_2$

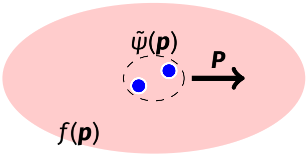
$|\Phi'_3\rangle$
$N + B_3 \to C_3$

Final states are not orthogonal: $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

The probability of cluster formation with one of B's:

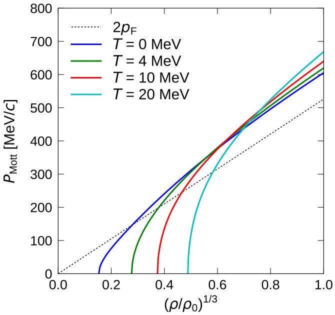
$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N^{-1}_{ij} \langle \Phi'_j|, \qquad P = \langle \Phi^{\boldsymbol{q}} | \hat{P} | \Phi^{\boldsymbol{q}} \rangle \qquad \neq \sum_i |\langle \Phi'_i | \Phi^{\boldsymbol{q}} \rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$



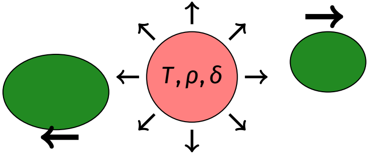
Equation for a deuteron in uncorrelated medium

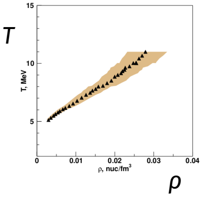
$$\left[e(\tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p})+e(\tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p})\right]\tilde{\psi}(\boldsymbol{p}) + \left[1-f(\tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p})-f(\tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p})\right]\int\frac{d\boldsymbol{p}'}{(2\pi)^3}\langle\boldsymbol{p}|v|\boldsymbol{p}'\rangle\tilde{\psi}(\boldsymbol{p}') = E\tilde{\psi}(\boldsymbol{p})$$



Formula from Röpke, NPA867 (2011) 66.

- A bound deuteron cannot exist inside the Fermi sphere, except at very low densities.
- A deuteron can exist if its momentum is high enough.
- When there is no bound solution, is it still possible that nucleons are correlated in the continuum like a resonance?
- What happens when the medium is correlated?





Qin et al.,
PRL108(2012)172701
Ar, Zn + Sn @ 47*A* MeV

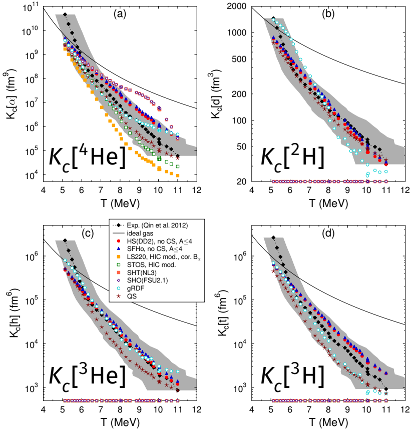
Assumption

The particles emitted at the same velocity v_{surf} were emitted at the same time from the same source characterized by (T, ρ, δ).

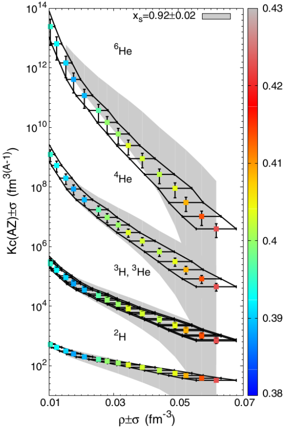
Equilibrium Constants

$$K_c(N,Z) = \frac{\rho(N,Z)}{\rho_p^Z \rho_n^N} \quad \text{for cluster } (N,Z)$$

Hempel et al., PRC 91 (2015) 045805.



Paris, Bougault et al.,
PRL 125 (2020) 012701.
Xe + Sn @ 32*A* MeV



Condition to switch on/off clusters

With or without clusters

$$N_1 + N_2 + B_1 + B_2 \to C_1 + C_2 \qquad \text{or} \qquad N_1 + N_2 \to N_1 + N_2$$

The condition to switch on clusters

$$\rho' < \rho_c, \qquad \rho_c = 0.125\ \text{fm}^{-3} \text{ or } 0.060\ \text{fm}^{-3} \text{ etc.}$$

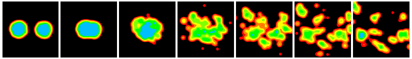
Density with a momentum cut for the nucleon N_i ($i = 1, 2$)

$$\rho_i'^{(\text{ini})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i - \mathbf{P}_k|\big)\, e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

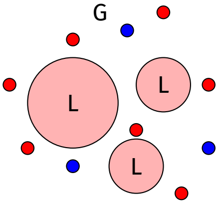
$$\rho_i'^{(\text{fin})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i^{(\text{fin})} - \mathbf{P}_k|\big)\, e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

$$\rho' = \left(\rho_1'^{(\text{ini})} \rho_1'^{(\text{fin})} \rho_2'^{(\text{ini})} \rho_2'^{(\text{fin})}\right)^{\frac{1}{4}}$$

An energy-dependent momentum cut was chosen, $p_{\text{cut}} = (375\ \text{MeV}/c)\, e^{-\epsilon/(225\ \text{MeV})}$, where ϵ is the collision energy (i.e. the sum of the kinetic energies of N_1 and N_2 in their c.m. frame).



In a late stage of reaction



Fractionation/Distillation

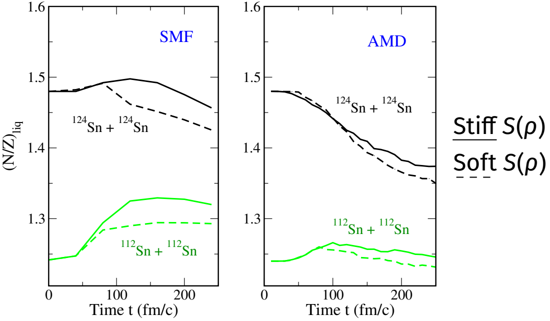
$$\frac{N}{Z}\,(\text{Liquid}) < \frac{N}{Z}\,(\text{Gas})$$

- Gas = $\sum(A \le 4$ particles)
- Liquid = $\sum(A > 4$ fragments)
- Total = Gas + Liquid

^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn, E/A = 50 MeV, $b \approx 0$.

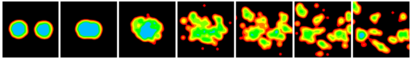
Colonna and Ono, PRC 82 (2010) 054613.

N/Z of the Liquid Part

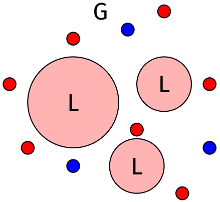


Symmetry energy effects in Fermi energy domain

- Fractionation and related dynamical effects
- Isospin drift
- Isospin diffusion



In a late stage of reaction



Fractionation/Distillation

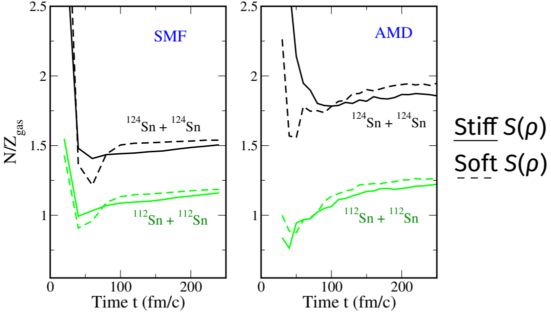
$$\frac{N}{Z}\text{(Liquid)} < \frac{N}{Z}\text{(Gas)}$$

- Gas = $\sum(A \leq 4$ particles)
- Liquid = $\sum(A > 4$ fragments)
- Total = Gas + Liquid

$^{124}Sn + {}^{124}Sn$ and $^{112}Sn + {}^{112}Sn$, $E/A = 50$ MeV, $b \approx 0$.

Colonna and Ono, PRC 82 (2010) 054613.

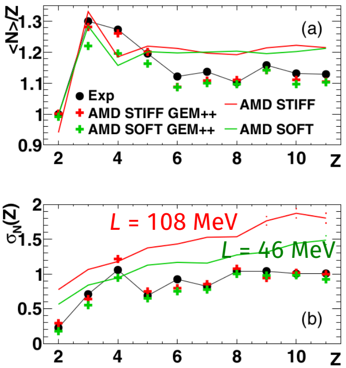
N/Z of the Gas Part



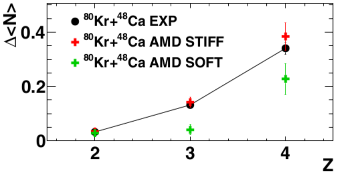
Symmetry energy effects in Fermi energy domain

- Fractionation and related dynamical effects
- Isospin drift
- Isospin diffusion

Fragments in $v_{cm} < v < v_{QP}$

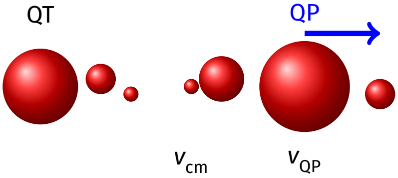


$(v_{cm} < v < v_{QP})$ − $(v > v_{QP})$



Piantelli et al. (FAZIA Collaboration), PRC 103 (2021) 014603.

^{80}Kr + ^{48}Ca at 35 MeV/nucleon (INFN-LNS)



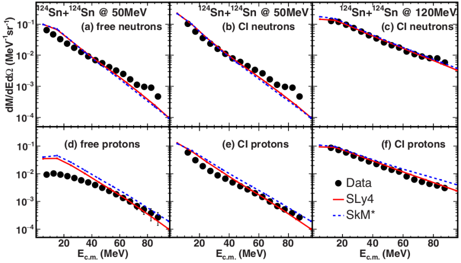
- In the width of the isotope distribution $\sigma_N(Z)$, clear dependence on the stiffness of the symmetry energy is found, for the excited fragments produced in AMD.
  - After statistical decays calculated by GEMINI++, the sensitivity in $\sigma_N(Z)$ becomes weaker.
- For the final He, Li and Be fragments (emitted backward in the QP frame), the yields of neutron-rich isotopes are sensitive to the stiffness of the symmetry energy.

L = 108 MeV (strong isospin drift) is more favored than L = 46 MeV.

MSU data:

Coupland et al., PRC 96 (2016) 011601(R).

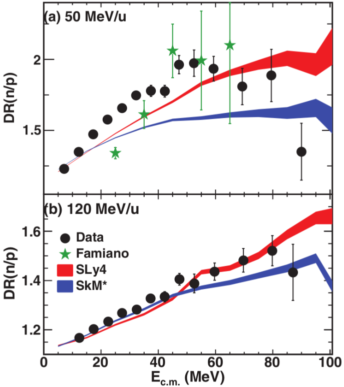
Sn + Sn, E/A = 50 and 120 MeV, $b \lesssim 3$ fm



CI = (free nucleons) + (nucleons in clusters)

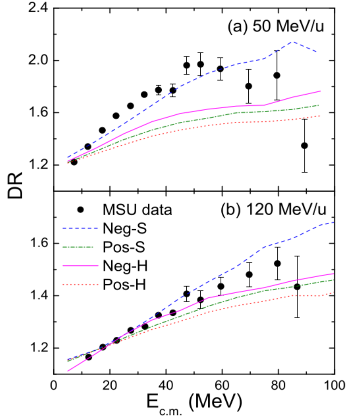
$$\mathrm{DR} = \frac{\text{“}n/p\text{” in } {}^{124}\mathrm{Sn} + {}^{124}\mathrm{Sn}}{\text{“}n/p\text{” in } {}^{112}\mathrm{Sn} + {}^{112}\mathrm{Sn}}$$

compared with ImQMD



DR comapred with an IQMD

J. Su et al., PRC 94 (2016) 034619.



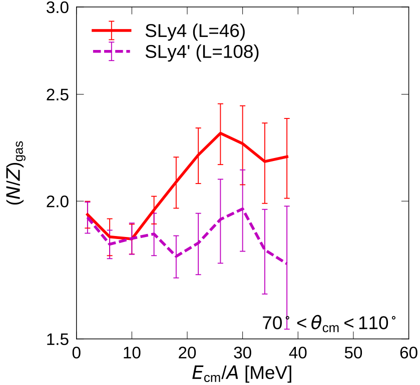
Similar result in
YX Zhang et al., PLB 732 (2014) 186.

Bayesian analyses at E/A = 120 MeV by Morfouace et al., PLB 799 (2019) 135045.

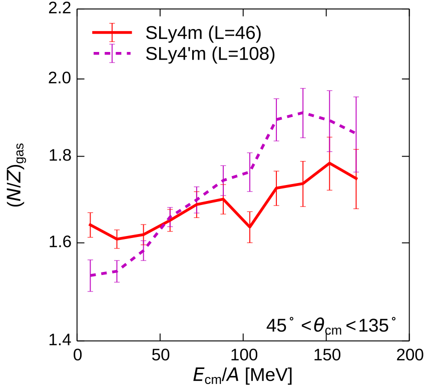
- $(m_n^* - m_p^*)/m_N = -0.05^{+0.09}_{-0.09}\,(\rho_n - \rho_p)/\rho$
- $S(\rho_s) = 16.8^{+1.2}_{-1.2}$ MeV at $\rho_s/\rho_0 = 0.43^{+0.05}_{-0.05}$

N/Z Ratio at 50 and 300 MeV/u calculated by AMD with clusters

$^{124}Sn + ^{124}Sn$, E/A = 50 MeV, $b \approx 0$



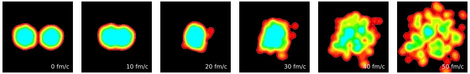
$^{132}Sn + ^{124}Sn$, E/A = 300 MeV, $b \approx 0$



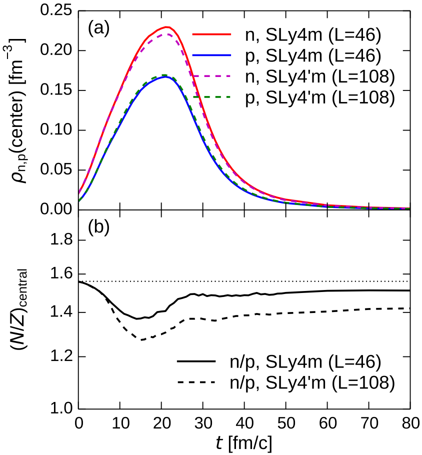
Low density effect $\Longleftrightarrow$ High density effect

$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_\alpha(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_\alpha(\epsilon)}, \qquad \epsilon = E_{\text{cm}}/A$$

For 50A MeV, the $(N/Z)_{\text{gas}}$ ratio in the AMD calculation seems to be higher than in QMD calculations.
However, the nucleon spectra (before taking ratio) need to be understood better in calculations to draw a definite conclusion.



^{132}Sn + ^{124}Sn, E/A = 300 MeV, $b \sim 0$



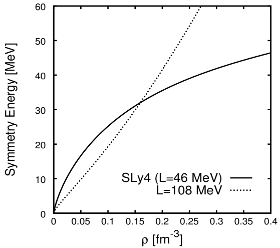
Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

Nuclear EOS (at T = 0)

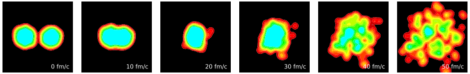
$$(E/A)(\rho_p,\rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

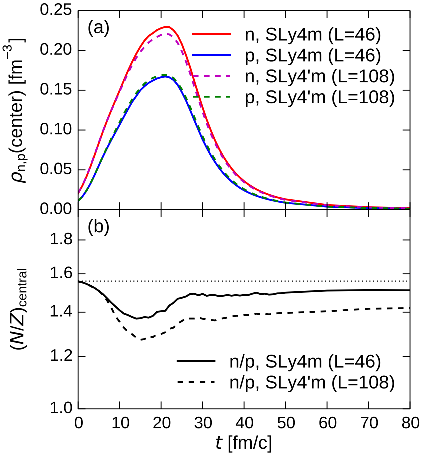
- $S_0 = S(\rho_0)$
- $L = 3\rho_0(dS/d\rho)_{\rho=\rho_0}$



Compression and expansion in collisions at 300 MeV/nucleon



^{132}Sn + ^{124}Sn, E/A = 300 MeV, b ~ 0



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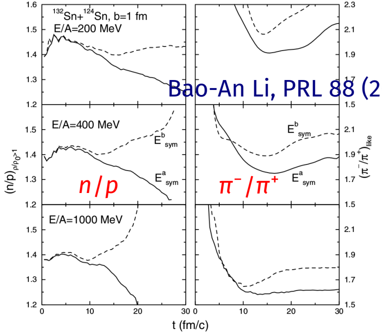
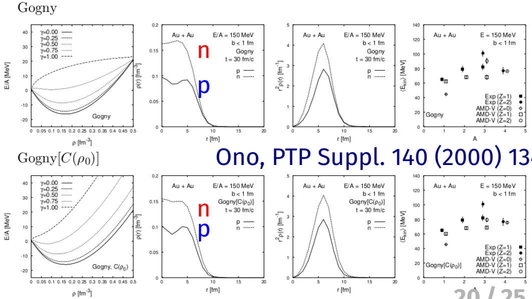
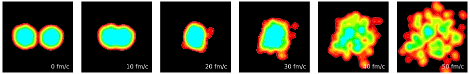


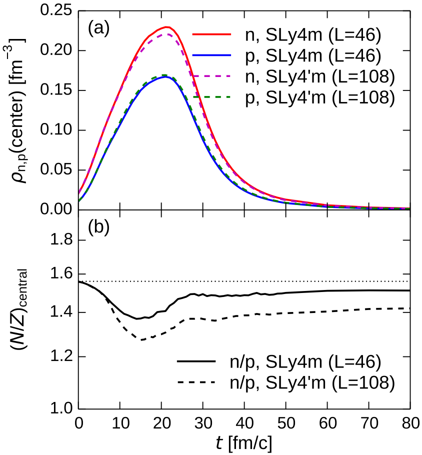
Bao-An Li, PRL 88 (2002) 192701.



Ono, PTP Suppl. 140 (2000) 134.



^{132}Sn + ^{124}Sn, E/A = 300 MeV, $b \sim 0$



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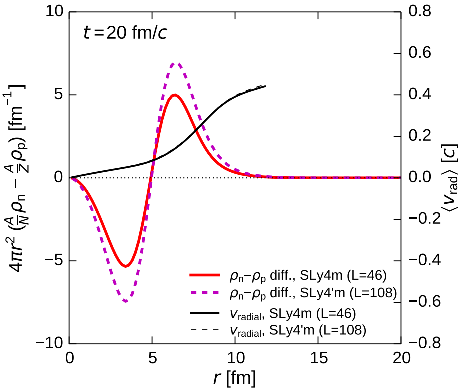
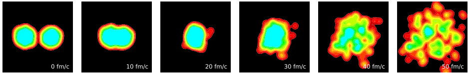
- Neutron-proton density diff. (fn of r)

$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

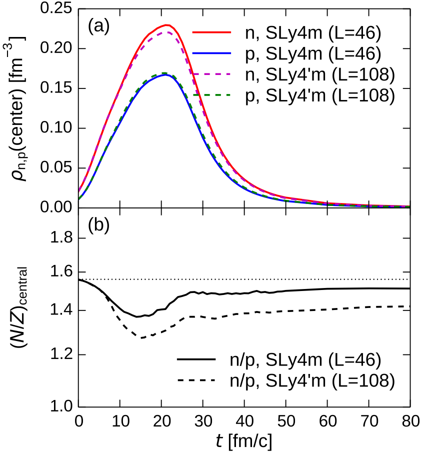
- Radial expansion velocity $v_{\mathrm{rad}}(r)$



Compression and expansion in collisions at 300 MeV/nucleon



^{132}Sn + ^{124}Sn, E/A = 300 MeV, b ~ 0



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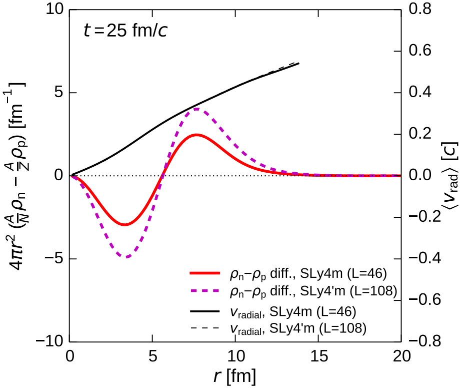
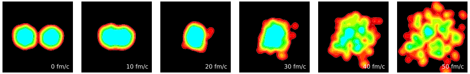
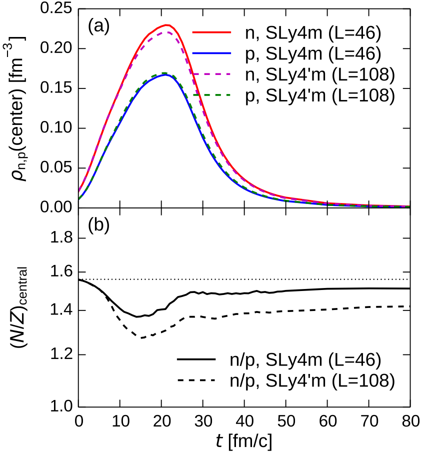
- Radial expansion velocity $v_{\rm rad}(r)$



Compression and expansion in collisions at 300 MeV/nucleon



^{132}Sn + ^{124}Sn, E/A = 300 MeV, b ~ 0



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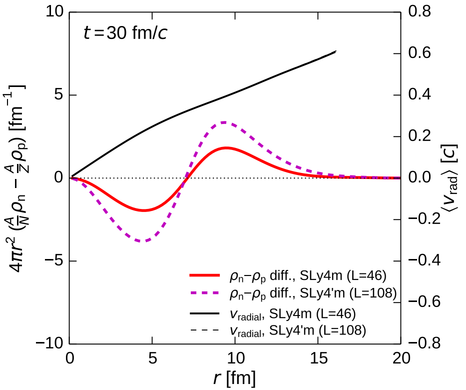
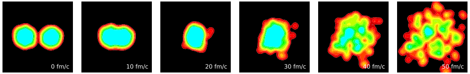
$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Radial expansion velocity $v_{\rm rad}(r)$



Compression and expansion in collisions at 300 MeV/nucleon



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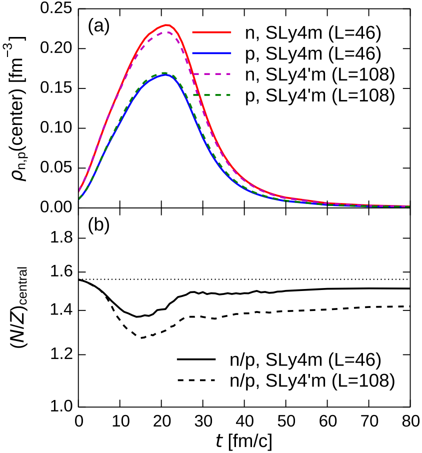
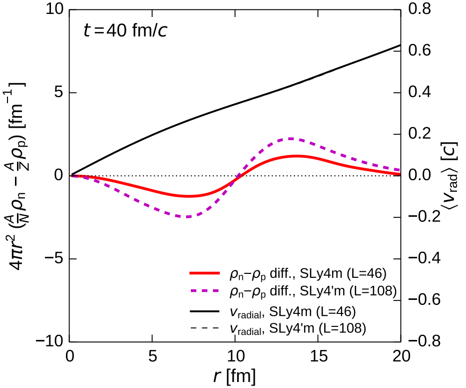


- Neutron-proton density diff. (fn of r)

$$4\pi r^2\left[\frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r)\right]$$

- Radial expansion velocity $v_{\text{rad}}(r)$



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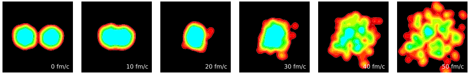

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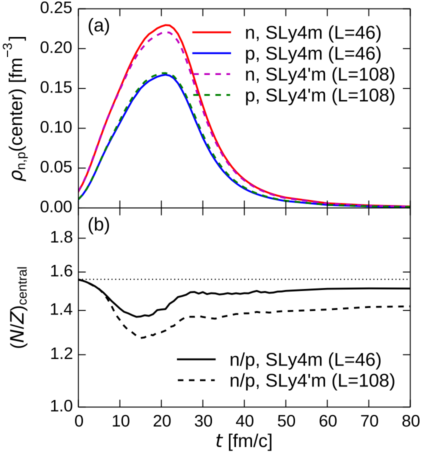

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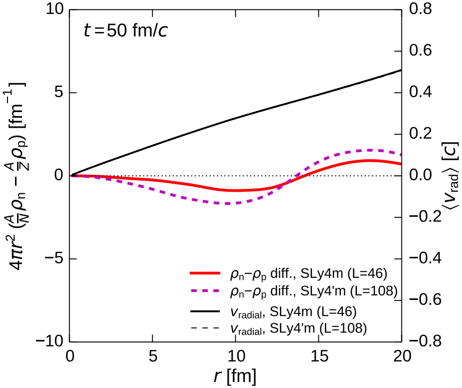
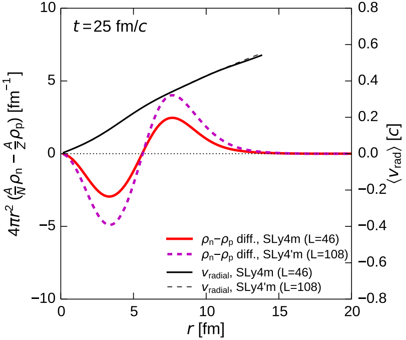
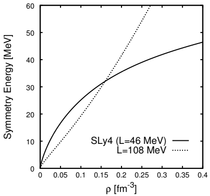


A. Ono, EPJ Web of Conferences 117 (2016) 07003.

ρ_n - ρ_p at the compression stage

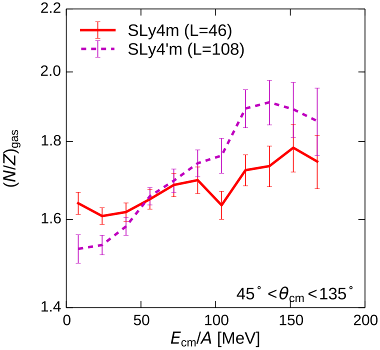




⇔

similar

N/Z of the spectrum of emitted particles



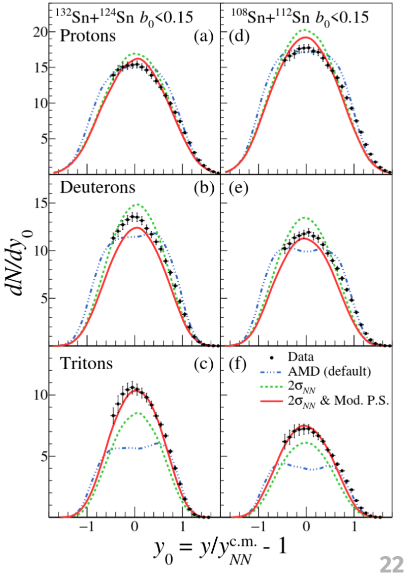
$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_\alpha(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_\alpha(\epsilon)}$$

$$\epsilon = E_{\text{cm}}/A$$

SπRIT data for *p*, *d* and *t* (Experiment at RIKEN/RIBF)

M. Kaneko, Murakami, Isobe, Kurata-Nishimura, Ono, Ikeno et al. (SπRIT), PLB 822 (2021) 136681.

neutron rich vs. neutron deficient



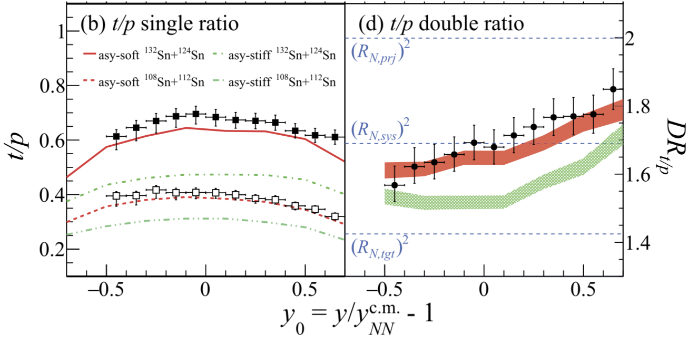
Rapidity distributions

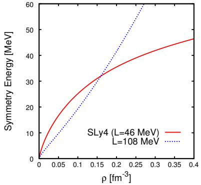
for *p*, *d* and *t*, in the central collisions of

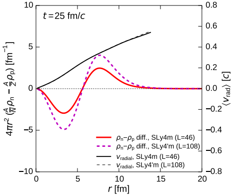
- ^{132}Sn + ^{124}Sn at 270 MeV/nucleon
- ^{108}Sn + ^{112}Sn at 270 MeV/nucleon

- Black points: SπRIT data
- Lines: AMD calculations (the asy-soft symmetry energy L = 46 MeV) with different model parameters.
  - The results in the next page don't depend much of the model parameters.

t/p ratio and its implication on the symmetry energy







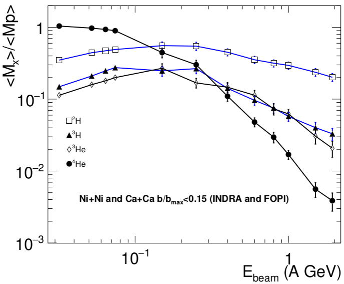
$$\frac{t/p \text{ in } {}^{132}\text{Sn} + {}^{124}\text{Sn}}{t/p \text{ in } {}^{108}\text{Sn} + {}^{112}\text{Sn}} = (t/p \text{ double ratio})$$

- The SπRIT data (black points) favor the asy-soft symmetry energy (L=46 MeV) rather than the asy-stiff symmetry energy (L=108 MeV).
- The moderate rapidity dependence of the t/p double ratio implies a partial isospin mixing.
- $t/{}^{3}$He should be a better probe.
- Stopping needs to be understood better systematically.
- Comparisons of other observables (e.g. the collective flow) are in progress.

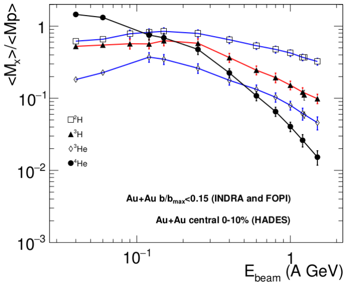
Cluster yields at various incident energies

Bougault et al., Symmetry 2021, 13, 1406. Data sets from INDRA and FOPI

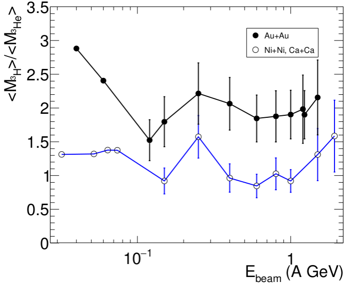
$M(x)/M(p)$ in Ni + Ni, Ca + Ca

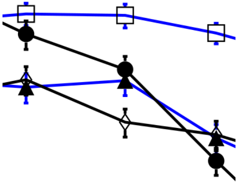


$M(x)/M(p)$ in Au + Au



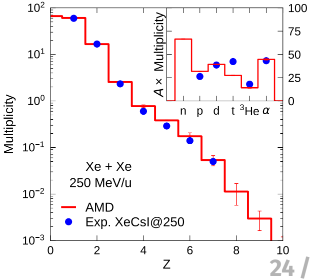
$M(^3\mathrm{H})/M(^3\mathrm{He})$





Is there any special mechanism for the triton production around 250*A* MeV (beyond the current transport model descriptions)?

Does it depend on N/Z of the system?



Summary

- Light clusters and heavier fragments cannot be ignored in understanding the global dynamics of heavy-ion collisions in the full range of energies.
- Many symmetry energy observables are related to clusters.
- In Fermi energy domain, clusters can be regarded as constituents of the gas part which has larger N/Z ratio than the liquid part in neutron-rich systems.
  - $S(\rho < \rho_0)$ from the N/Z ratio of the gas part (i.e., CI spectrum ratio).
  - Some hint on $\nabla S(\rho < \rho_0)$ from isospin drift (L = 108 MeV rather than L = 46 MeV) , from a comparison of FAZIA data and AMD calculation.
  - ...
- At higher energies (e.g., E/A = 270 MeV), clusters are mainly created in the inner part of the expanding system, and therefore they carry residual information on the N/Z of the compressed matter.
  - The t/p ratio of the rapidity distributions of the SπRIT data for Sn + Sn at 270A MeV favors L = 46 MeV rather than L = 108 MeV, in a comparison with an AMD calculation.
  - ...
- It is a highly nontrivial problem to handle cluster correlations in transport models, though there are some models which are practically working (e.g., AMD with an extension for cluster correlations).
  - How can we identify clusters in general situations?
  - How should clusters be suppressed in medium?
  - ...
- Anything special beyond the current AMD, to produce extra tritions and neutron-rich fragments?

More about cluster production in NN collisions

$$\mathrm{N}_1 + \mathrm{B}_1 + \mathrm{N}_2 + \mathrm{B}_2 \to \mathrm{C}_1 + \mathrm{C}_2$$

$$v\,d\sigma \propto |\langle \varphi_1' | \varphi_1^{+q} \rangle|^2 |\langle \varphi_2' | \varphi_2^{-q} \rangle|^2 \; |M|^2 \, \delta(E_f - E_i) \, p_{\mathrm{rel}}^2 dp_{\mathrm{rel}} d\Omega \qquad E_i,\ E_f = \frac{\langle \Phi(Z)|H|\Phi(Z)\rangle}{\langle \Phi(Z)|\Phi(Z)\rangle}$$

$$\Rightarrow P(\mathrm{C}_1, \mathrm{C}_2, p_{\mathrm{rel}}, \Omega) \times \left| M(p_{\mathrm{rel}}^{(0)}, p_{\mathrm{rel}}, \Omega) \right|^2 \times \frac{{p_{\mathrm{rel}}}^2 d\Omega}{\partial E_f / \partial p_{\mathrm{rel}}}$$

- Gaussian width $\nu_{\mathrm{cl}} = 0.24\ \mathrm{fm}^{-2}$ for the overlap factors.
- There are a huge number of final cluster configurations $(\mathrm{C}_1, \mathrm{C}_2)$.

$$\sum_{\mathrm{C}_1 \mathrm{C}_2} P(\mathrm{C}_1, \mathrm{C}_2, p_{\mathrm{rel}}, \Omega) = 1 \quad \text{for any fixed } (p_{\mathrm{rel}}, \Omega)$$

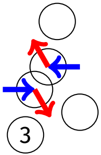
- The energy-conserving final momentum depends on the cluster configuration

$$p_{\mathrm{rel}} = p_{\mathrm{rel}}(\mathrm{C}_1, \mathrm{C}_2, \Omega)$$

When clusters are formed, p_{rel} tends to be large, and the effect of collisions will increase.

  - the phase space factor ↑
  - Pauli blocking ↓ (collision probability ↑)
  - momentum transfer ↑

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

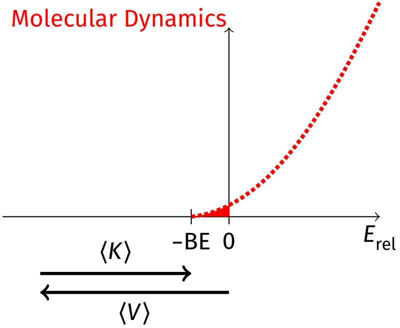


In the usual way of NN collision, only the two wave packets are changed.

$$\{|\Psi_f\rangle\} = \{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\}$$

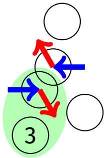
(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system



NN collisions without or with cluster correlations

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



In the usual way of NN collision, only the two wave packets are changed.

$$\{|\Psi_f\rangle\} = \{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\dots)\rangle\}$$

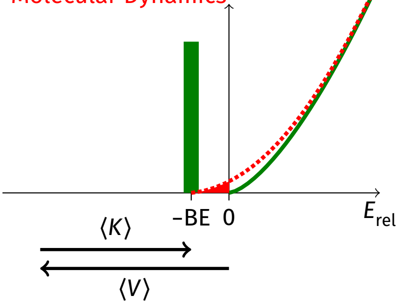
(ignoring antisymmetrization for simplicity of presentation.)

Extension for cluster correlations

Include correlated states in the set of the final states of each NN collision.

$$\{|\Psi_f\rangle\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\dots)\rangle,\ \dots$$

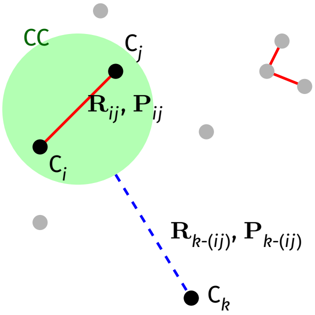
Phase space or the density of states for two nucleon system



Several clusters may form a loosely bound state.

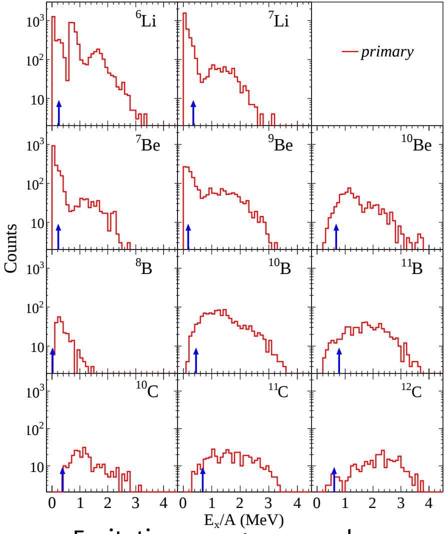
e.g., ^{7}Li = $\alpha + t - 2.5$ MeV

Need more probability of $|\alpha + t\rangle \to |^7\mathrm{Li}\rangle$ etc.



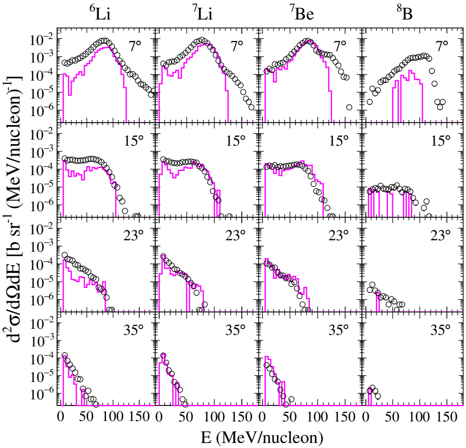
$^{12}C + ^{12}C$ at 95 MeV/nucleon

Tian et al., PRC 97 (2018) 034610.



Excitation energy per nucleon

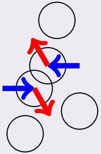
Some light nuclei are emitted at large angles (θ_{lab} > 20°) almost in their ground states, at t = 300 fm/c.



NN collisions in AMD (without/with cluster correlations)

Without cluster correlation

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



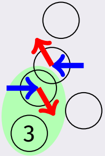
A collision of particles 1 and 2 will change only the two wave packets.

$$\{|\Psi_f\rangle\} = \{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\}$$

(ignoring antisymmetrization for simplicity of presentation.)

With cluster correlation

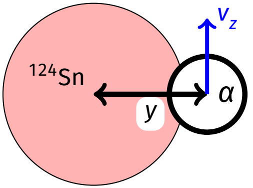
$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

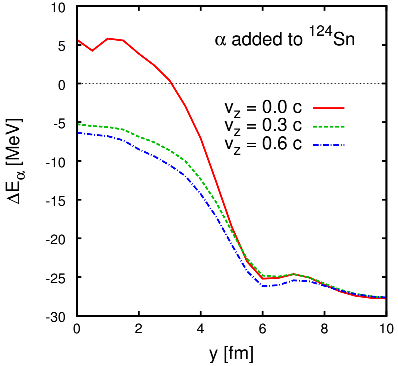


Include correlated states in the set of the final states of each NN collision.

$$\{|\Psi_f\rangle\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle, \ \ldots$$

(ignoring antisymmetrization for simplicity of presentation.)





α cluster $|\alpha, \mathbf{Z}\rangle$: Four wave packets with different spins and isospins at the same phase space point $\mathbf{Z}$.

$$E_\alpha : \quad \mathcal{A}\, |\alpha, \mathbf{Z}\rangle |^{124}\mathrm{Sn}\rangle$$

$$E_\mathrm{N} : \quad \mathcal{A}\, |\mathbf{Z}\rangle |^{124}\mathrm{Sn}\rangle \quad (\mathrm{N} = p\uparrow, p\downarrow, n\uparrow, n\downarrow)$$

$$-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

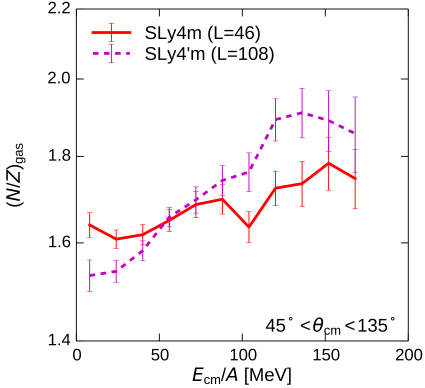
(Energies are defined relative to $|^{124}\mathrm{Sn}\rangle$.)

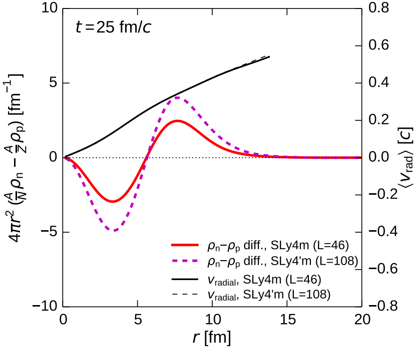
$$\frac{\mathrm{Re}\,\mathbf{Z}}{\sqrt{\nu}} = (0, y, 0), \quad \frac{2\hbar\sqrt{\nu}\,\mathrm{Im}\,\mathbf{Z}}{M} = (0, 0, v_z)$$

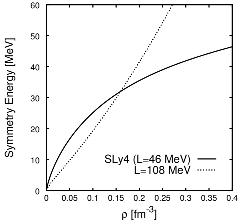
- Distance from the center: y
  ≈ Dependence on density
- Dendence on $P_\alpha = M_\alpha v_z$
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the α cluster is weakened in the nucleus.

Energy is OK, but dynamics is ...

N/Z Spectrum Ratio (AMD with clusters)



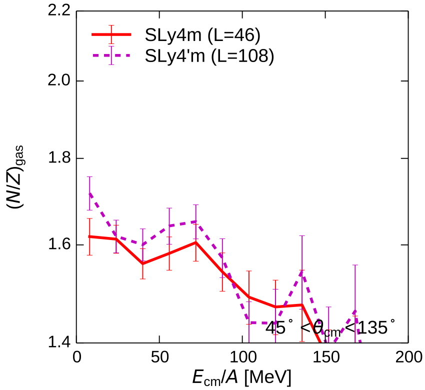


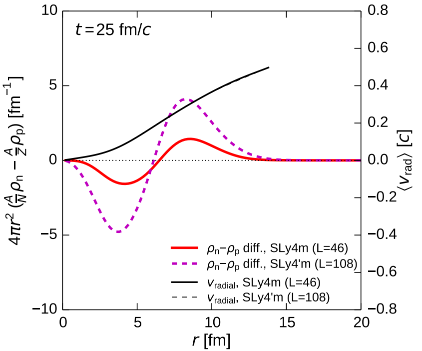


$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

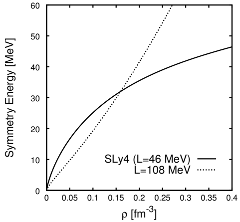
N/Z of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

N/Z Spectrum Ratio (AMD withOUT clusters)





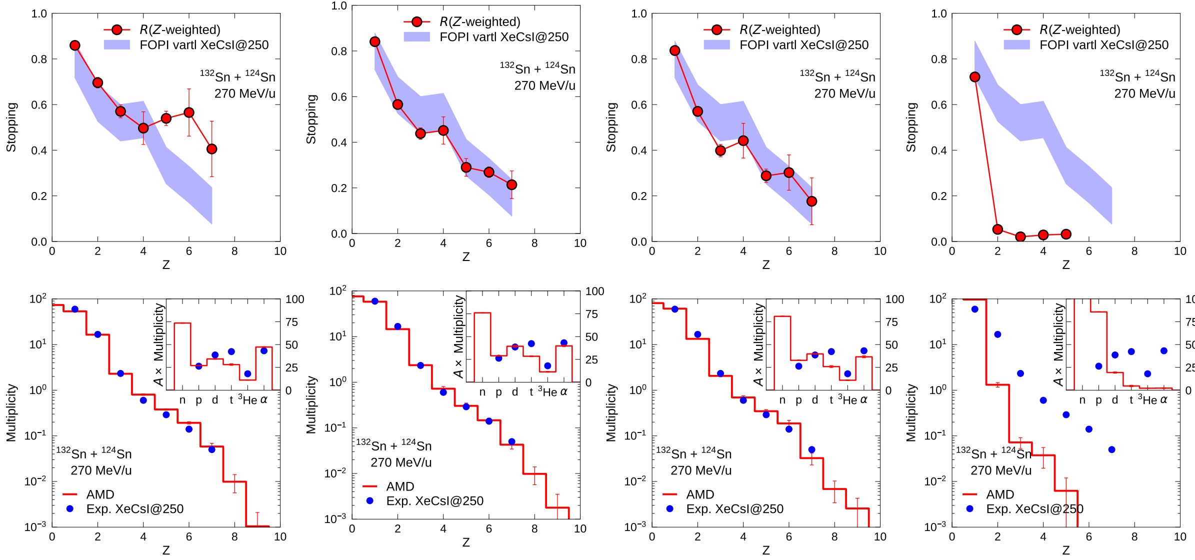
$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$



N/Z of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.

Results from different choices of cluster and σ_{NN} in medium

Cluster(full)	Cluster(0.125)	Cluster(0.060)	Cluster(0)
σ_{NN}(free)	σ_{NN}(in-medium)	σ_{NN}(free)	σ_{NN}(in-medium)



Results from different choices of cluster and σ_{NN} in medium

Cluster(full) σ_{NN}(free)	Cluster(0.125) σ_{NN}(in-medium)	Cluster(0.060) σ_{NN}(free)	Cluster(0) σ_{NN}(in-medium)