Decoding the density dependence of the symmetry energy

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What do our theorist friends often show us about the density dependence of the Symmetry Energy?

$E/A\,(\rho,\delta) = E/A\,(\rho,0) + \delta^2 \cdot S(\rho);$

$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$

$$S(\rho) = S_0 + \frac{L}{3\rho_0}(\rho - \rho_0) + \frac{K_{\rm sym}}{18\rho_0^2}(\rho - \rho_0)^2 + \cdots$$

The most popular theoretical constraint plot does not provide $S(\rho)$. Instead it provides values for parameters S_0 and L for various effective interactions that were used to describe data. These functions and their dependencies on S_0 and L differ.

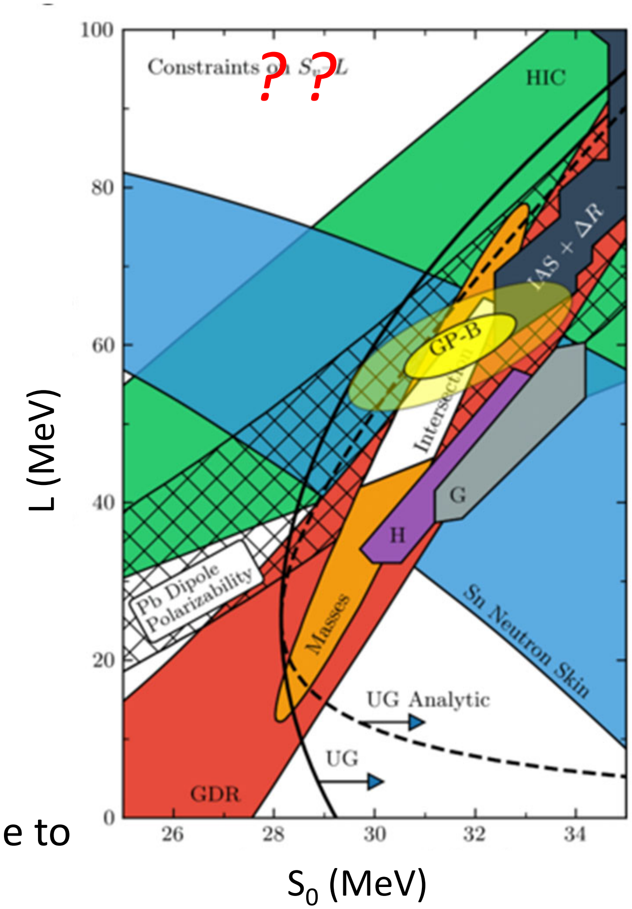
Example 5 parameter Skyrme:

$$S(\rho) = a(\rho/\rho_0) + b(\rho/\rho_0)^{1+\sigma} + c(\rho/\rho_0)^{2/3} + d(\rho/\rho_0)^{5/3}$$

$$a = S_0 - \frac{1}{3}\varepsilon_f - b - d(m_n^*, m_p^*)$$

$$b = -\left[3S_0 - L - \frac{1}{3}\varepsilon_f + 2d(m_n^*, m_p^*)\right] \Big/ (3\sigma)$$

} Relationships help one to determine $S(\rho)$



C. Drischler et al., PRL **125**, 202702 (2020); Lim & Lattimer APJ 771, 51 (2013)

Thrust of the talk:

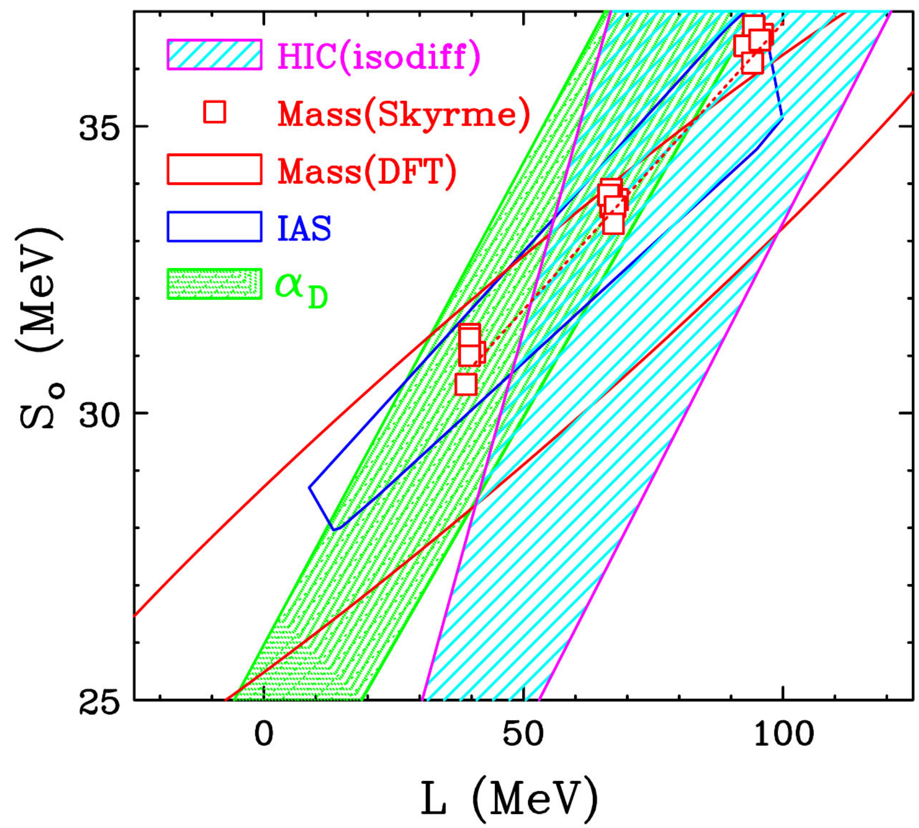
- I take seriously the published constraint curves.
- I assume that the observable is probing $S(\rho)$.
- I let the constraint curves tell me what density the measurement is probing.
- I use the authors' symmetry energy functions and ask what is the symmetry energy at the density the measurement probes. I plot the value for that symmetry energy function at that sensitive density
- I show what I get.

What do these contours really tell us?

Consider the analyses of masses by Alex Brown. The squares indicate the parameter values for his fits of doubly closed shell nuclei. Each nucleus was fit under the constraint of three possible values for neutron skin thicknesses of ΔR_{np}=0.16, 0.20 and 0.24 fm.

The dashed line and the other solid lines are lines of constant χ^2. It describes how changes in S_0 in $S(S_0,L,\rho)$ can be compensated by changes in L so that the data are reproduced with the same χ^2.

Each of these "equiprobability" contours has an inclination $\tau = \frac{\Delta S_0}{\Delta L}$. The value τ is also relates the partial derivatives of the symmetry energy at density ρ_s where the symmetry energy remains approximately constant along the straight sections of the equiprobability contour.



Horowitz et al., JPG **41**, 093001 (2014)
B.A. Brown, Phys. Rev. Lett. 111, 232502 (2013)

$$\tau = -\left(\partial S(\rho_s)/\partial L\right)/\left(\partial S(\rho_s)/\partial S_0\right);$$

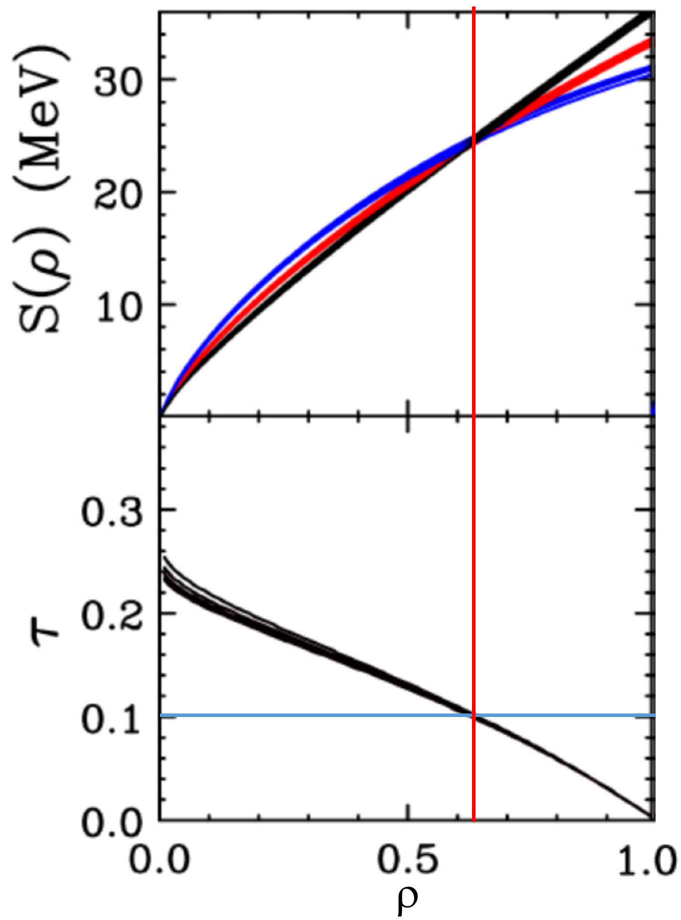
τ depends monotonically on ρ_s

What density do these fits constrain?

- Crossover technique:
  - Alex Brown fit the masses of doubly closed shell nuclei, while setting different values neutron skin thicknesses of ΔR_{np}=0.16, 0.20 and 0.24 fm. The functionals are shown in the upper panel. All Brown fits provide S(ρ)=24.8±0.7 MeV at ρ/ρ_0=0.63±.03 as shown at left.
- inclination technique
  - The inclination, τ=0.100±.006, of Brown's contour in the S_0 and L plane plus the form of the function is sufficient to determine ρ/ρ_0=0.63±.03 and S(ρ)=24.8 ±0.7 MeV.
  - The values for S_0 and L lie on a line in the S_0 and L plane along which $S(0.63\rho_0)$ remains constant. This line lies perpendicular to the gradient of $S(0.63\rho_0)$ in the S_0 and L plane.

We use the name "inclination" instead of "slope" for τ so that τ will not be confused with L.

B.A. Brown, Phys. Rev. Lett. 111, 232502 (2013)

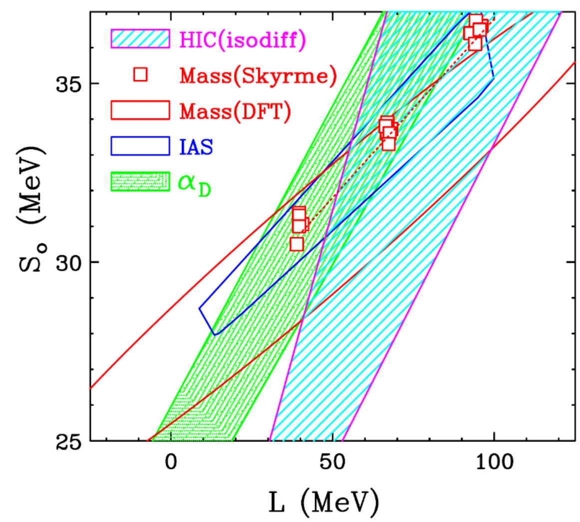


$$\tau(\rho) = -\left(\partial S(\rho)/\partial L\right)/\left(\partial S(\rho)/\partial S_0\right);$$

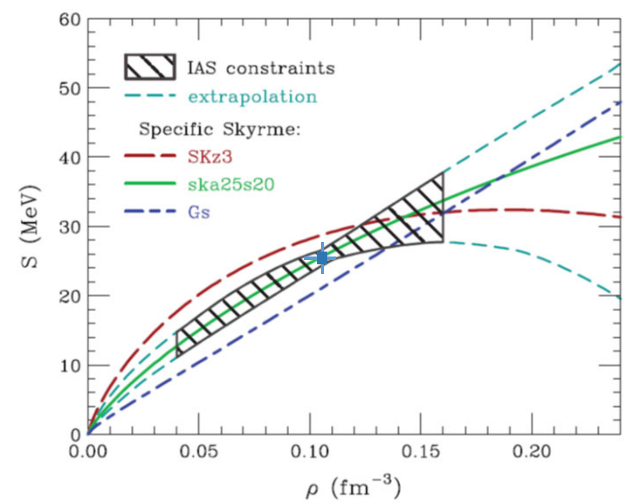
τ depends monotonically on ρ

Isobaric Analog states

- Danielewicz and Lee examined Isobaric Analog States (IAS) and obtained constrain
  - Inclination provides density.
  - Danielewicz gave us their best fit functional $S_{bf}(S_0,L,\rho)$
  - $S_{bf}(S_0,L,\rho_s)$ is plotted as the dark blue point in the lower figure.
- Pearson correlation technique:
  - Danielewicz and Lee used the Pearson correlation technique to determine what densities must be "right" to reproduce IAS?
  - Light nuclei are more sensitive to lower densities.
  - They obtained the cross-hatched constraint in the lower figure. The best fit constrained value in their Pearson correlation analysis is the same as the point obtained from the inclination analysis of the constraint contour in the upper figure. The inclination point is shown as the dark blue point in the lower figure.



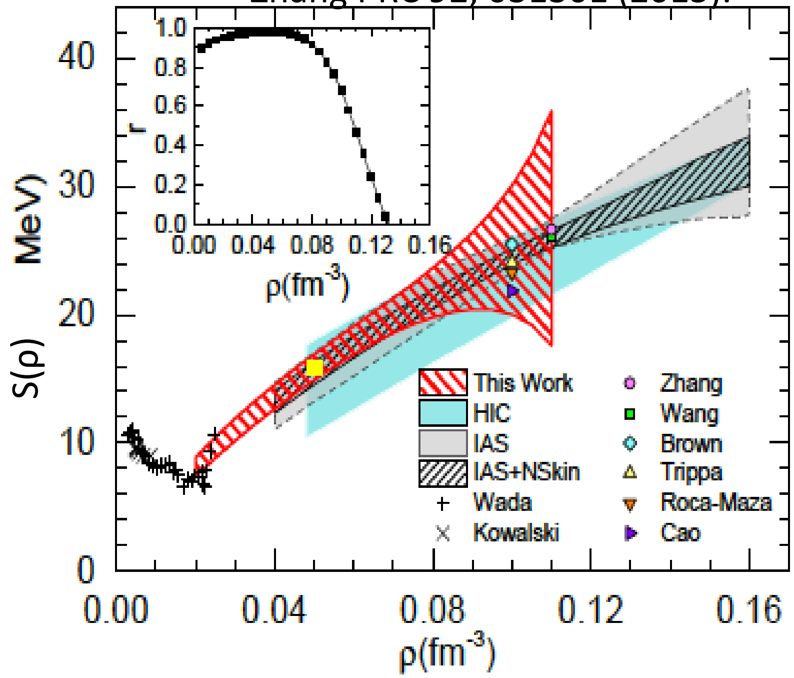
P. Danielewicz, and J. Lee, Nucl. Phys. A 922, 1 (2014).



Electrical dipole polarizability

- The dipole polarizability of a nucleus tells how the ground state energy changes in a dipole electric field. Perturbation theory tells us
  - Large contribution from GDR
  - Low energy non-collective excitations have enhanced importance.
  - Latter measured by (p,p') coulex by Tamii et al., PRL 107, 062502 (2011)..
- Analyzed by Zhang and Chen using a Pearson correlation technique.
- This involves selectively varying S(ρ) at various densities and deduce the optimal S(ρ) and the sensitivity to specific densities.
- We find that the sensitive density extracted from the inclination of the constraint contour from Roca-Maza PRC 88,024316 (2013) is also ρ=0.05 fm^{-3}

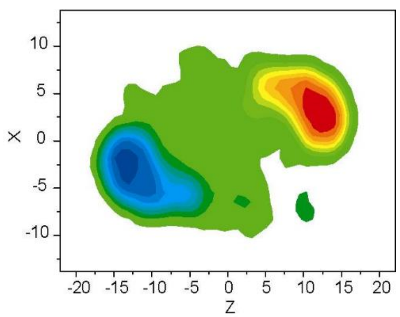
Zhang PRC 92, 031301 (2015).

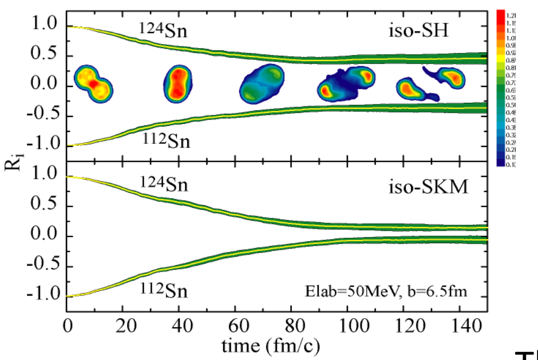


The red contour shows the constraint on S(ρ) consistent with the α_D measurement. The best determined value is S(ρ)=15.9±1.0MeV at ρ=0.05 fm^{-3}, which is marked as the yellow square point in the figure.

Isospin Diffusion

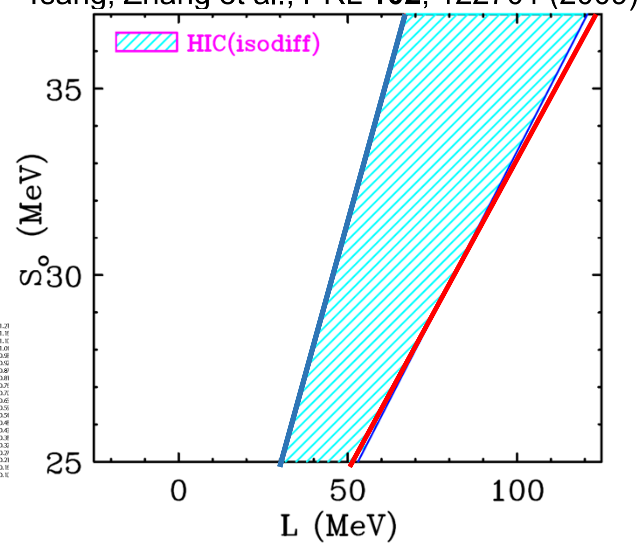
$S(\rho)=12.5(\rho/\rho_o)^{2/3}+C(\rho/\rho_o)^{\gamma}$

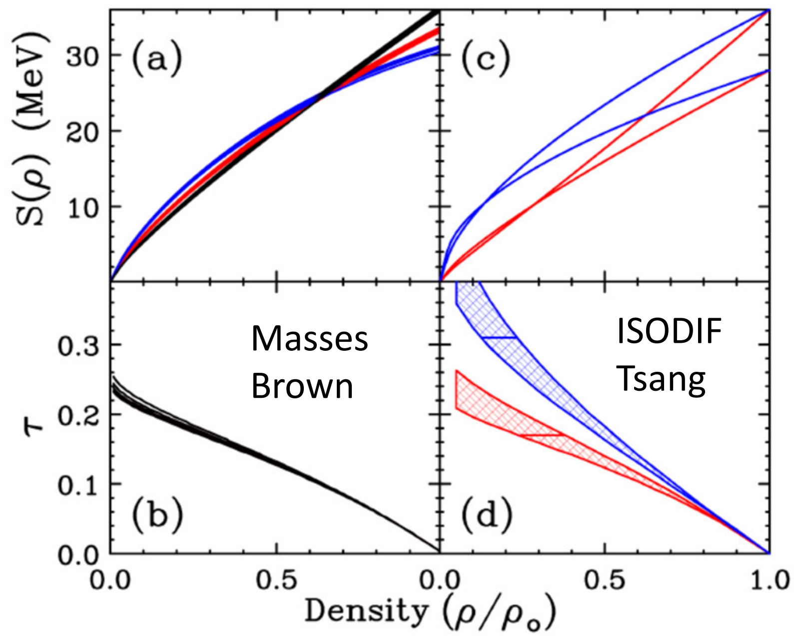




Tsang et al., PRL **92**, 062701 (2004)

Tsang, Zhang et al., PRL **102**, 122701 (2009)





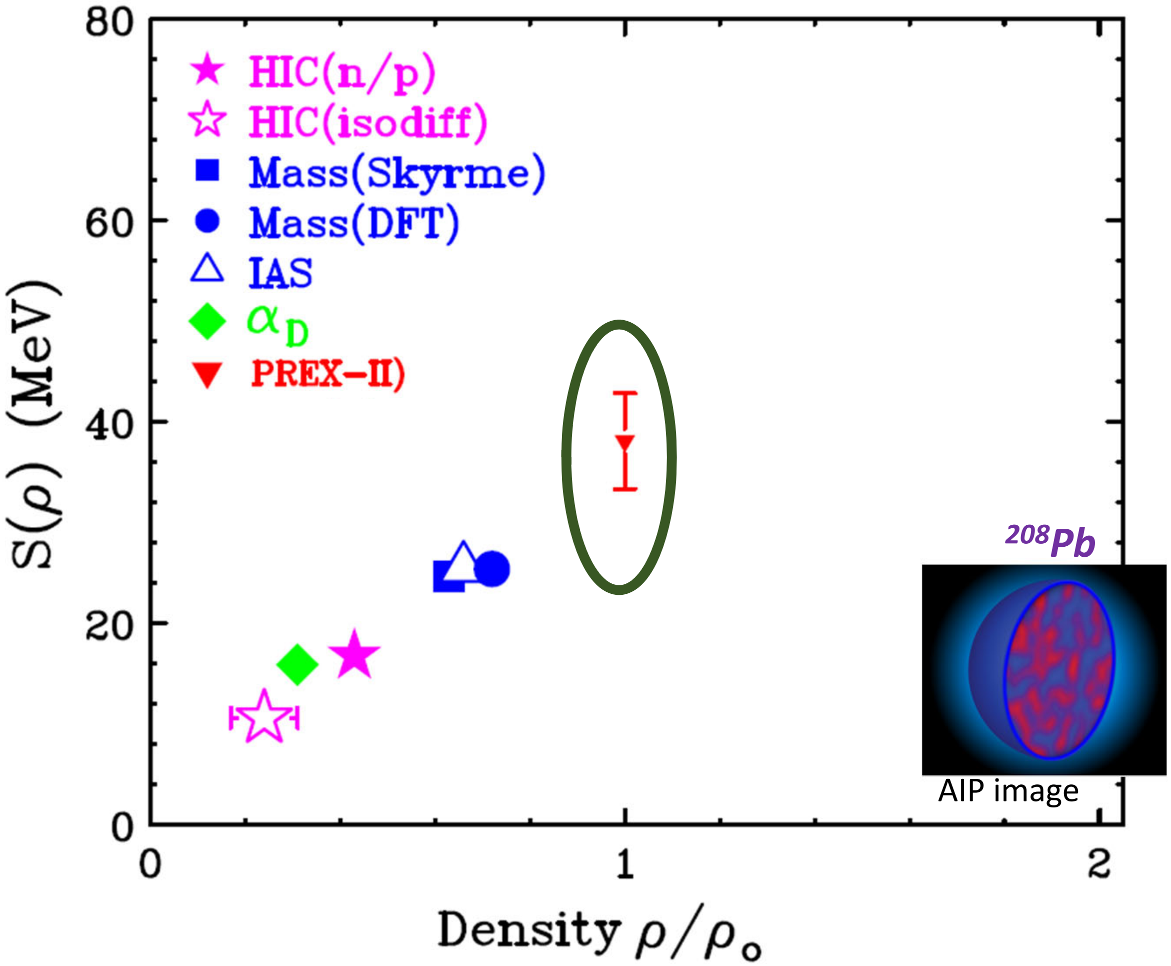
Significant uncertainty in the sensitive density.

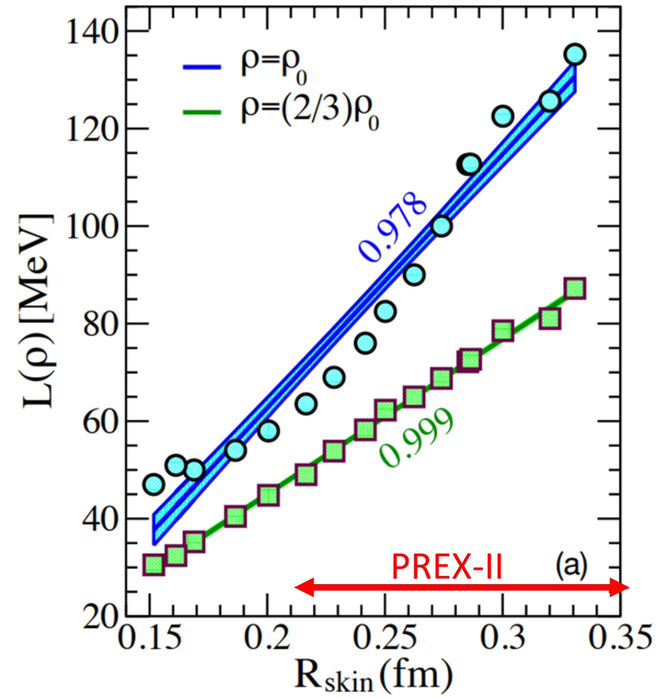
This observable reflects the exchange of neutrons and between nuclei of different asymmetries. The observable provides the density at which a given functional reaches an energy of 10.4±1 MeV and a density $\rho/\rho_0 = 0.22 \pm .07$. (Stiffer S(ρ)'s reproduce the data at higher densities.)

$$\tau = \frac{\Delta S_0}{\Delta L} = -\frac{\partial S(\rho)}{\partial L}\Big/\frac{\partial S(\rho)}{\partial S_0}$$

PREX Collaboration: R_{skin}=0.283+-0.071 fm

Parity violating elastic e scattering



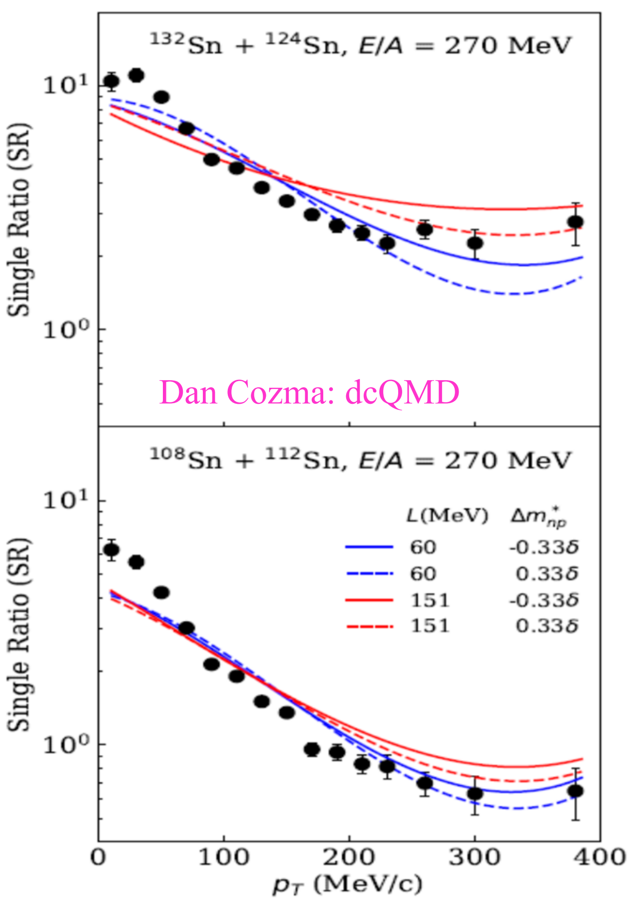


Reed et al, PRL **126**, 172503 (2021)

Adhikari et al, PRL **126**,172502 (2021)

Collaboration: Pion production in rare isotope collisions

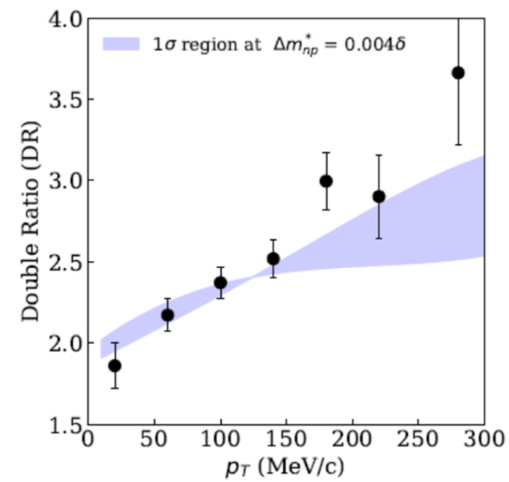
Estee et al, PRL **126**, 162701 (2021)



The pion spectra at high p_T reflect the accelerations of the nucleons due to the isovector mean field potentials that control the symmetry energy.

The discrepancy may reflect the contribution of non-resonant pion production that is neglected in the current calculations. The discrepancy is independent of the asymmetry of the system. This discrepancy is absent in the pion spectral double ratio.

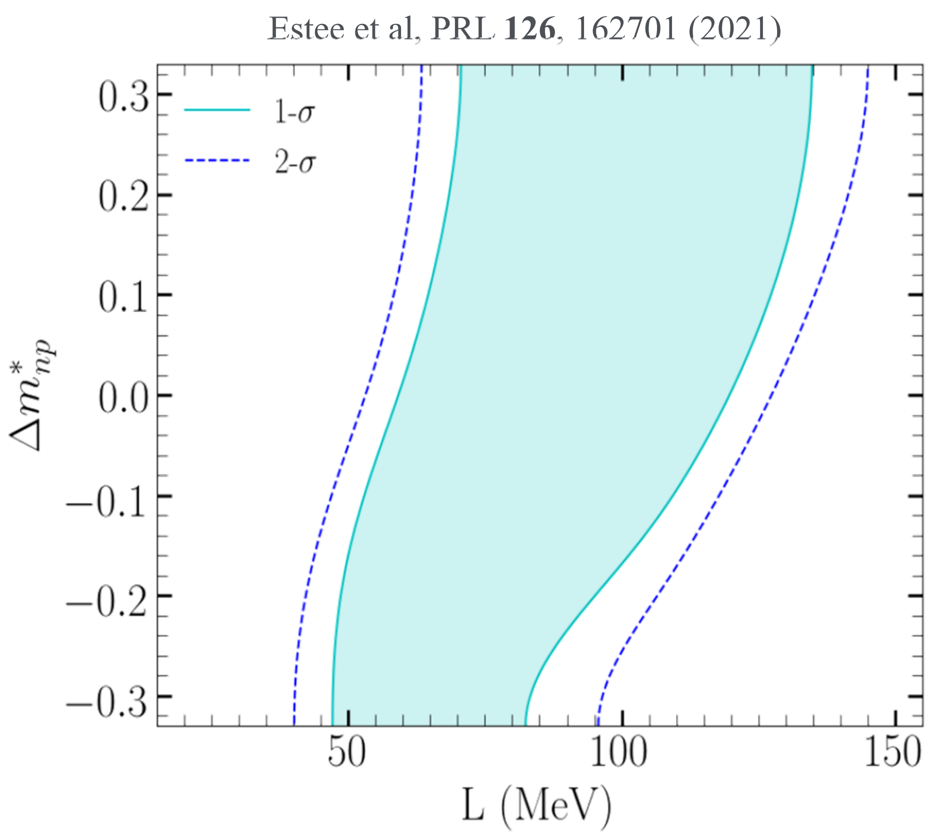
Raising the bombarding energy is an important test of this interpretation because it will decrease the relative contributions of non-resonant pion production and reduce the role of the Δ potentials.

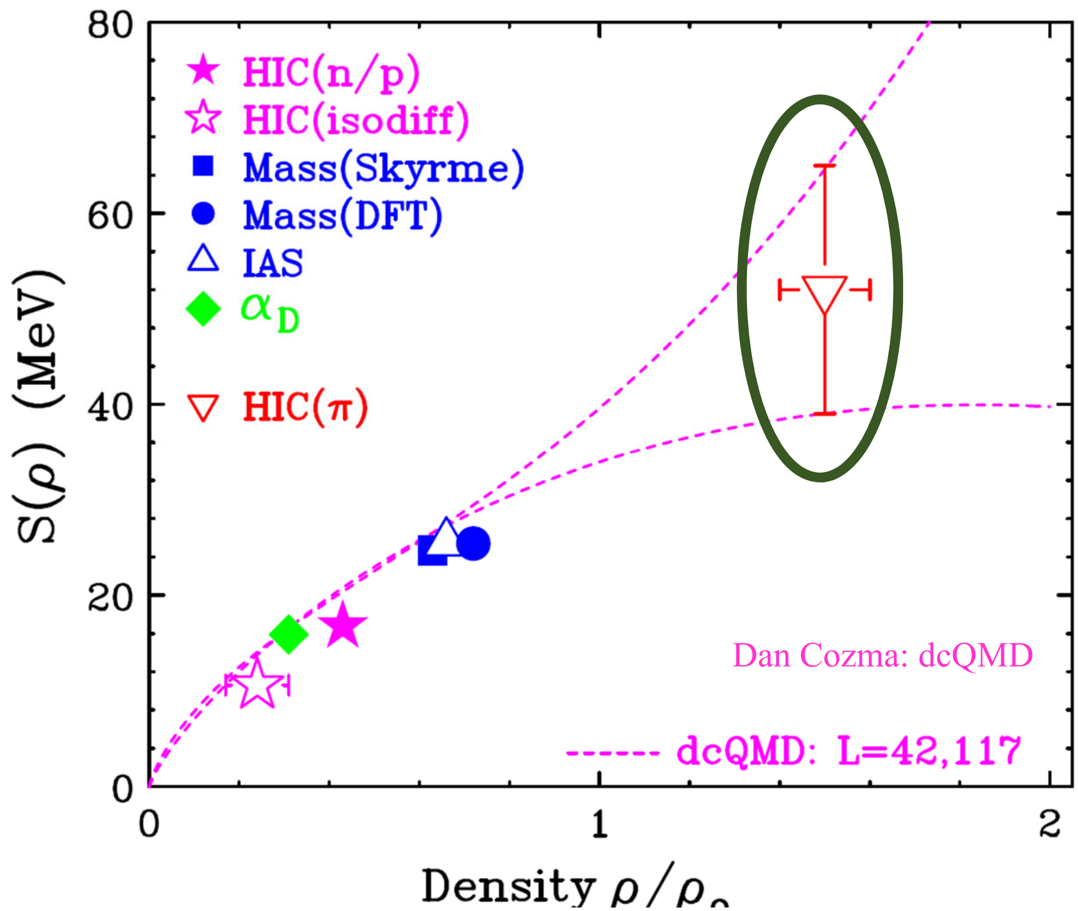


Gray region show calculated double ratio for L=77±28 MeV

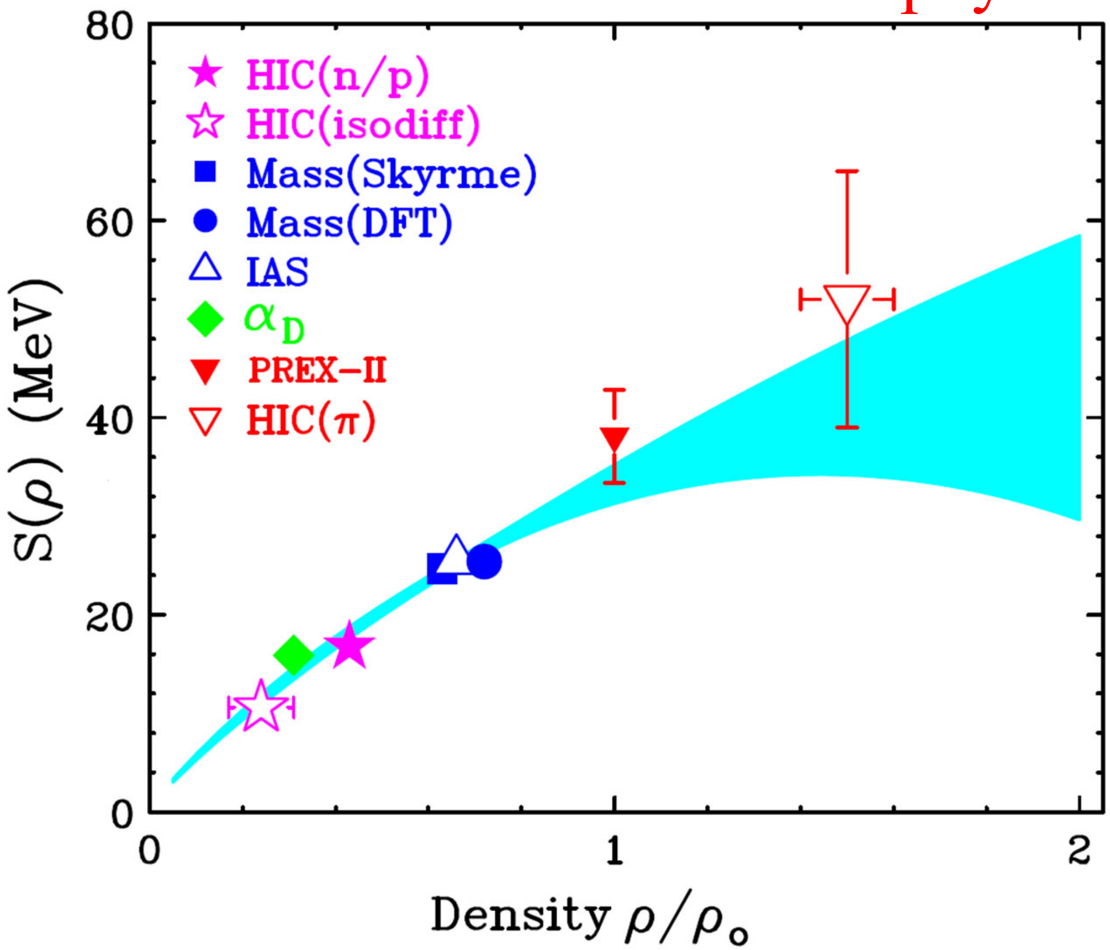
SπRIT Collaboration

The measurements provide a correlated constraint on L and Δm_{np}^*





Conclusion – Some Astrophysics & Physics implications



S_{01}=24.2+-0.5 MeV
L_{01}=53.1+-6.1 MeV
K_{01}=-79.2+-37.6 MeV

R_{np}=0.23+-0.03 fm
ρ_{cc}~$0.5\rho_0$

S_0=33.3+-1.3 MeV
L=59.6+-22.1 MeV
K_{sym}=-180+-96 MeV

P_{sym}= 3.2+-1.2 MeV
Λ(1.4)= 500-720
R(1.4) = 13.1+-0.6 km

More to come when the new density functional is included in NS calculations.

Summary

- We took every constraint contour seriously.
- We used the density function of the authors and asked what sensitive density could give rise to the observed correlation between $S_o$and L.
- We used the symmetry energy of the analysis to get the constraint.
- We conclude from the consistency of these constraints that we should take these constraints seriously.