

## **Status Report of Transport Model Evaluation Project**

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### **Content**

- 1. Nuclear matter EOS and nucleon mean-field potential**
- 2. BUU and QMD transport approaches**
- 3. Achievements from transport approaches**
- 4. Transport model evaluation project - status report**

# Nuclear matter EOS and nucleon mean-field potential

Energy per nucleon  
in asymmetric matter

Energy per nucleon  
in symmetric matter

$$\rho = \rho_n + \rho_p$$

$$\delta = (\rho_n - \rho_p) / \rho$$

Symmetry energy

$$E(\rho, \delta) \approx E_0(\rho) + E_{sym}(\rho)\delta^2$$

nucleon MF potential  
in asymmetric matter

nucleon MF potential  
in symmetric matter

Symmetry potential

$$U_{n/p}(\rho, \delta) \approx U_0(\rho) \pm U_{sym}(\rho)\delta$$

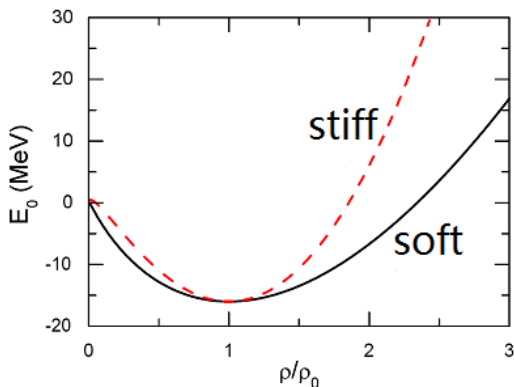
Intermediate-energy HIC

Transport  
approach

EOS of asymmetric  
nuclear matter

General  
relativity

Nstar, GW, ...

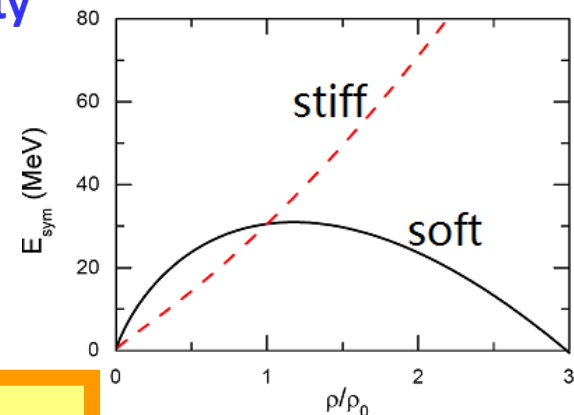


Many-body theory

Nuclear force

Many-body theory

Nuclear structure, SHE, ...



# BUU transport approach

**Boltzmann-Uehling-Uhlenbeck equation:**

$$\left( \frac{\partial}{\partial t} + \frac{\vec{p}}{m} \cdot \nabla_r - \nabla_r U \cdot \nabla_p \right) f(\vec{r}, \vec{p}; t) = I_{coll}[f; \sigma_{12}]$$

**Collision term with quantum statistics**

$$I_{coll} = \frac{1}{(2\pi)^6} \int dp_2 dp_3 d\Omega |v - v_2| \frac{d\sigma_{12}^{med}}{d\Omega} (2\pi)^3 \delta(p + p_2 - p_3 - p_4) \\ \times [f_3 f_4 (1 - f)(1 - f_2) - f f_2 (1 - f_3)(1 - f_4)]$$

**Derivation: real-time Green's function formalism; von-Neumann equation with density matrix; higher-order cutoff from TDHF; ...**

**test-particle (TP) method: parallel events**

C.Y. Wong, PRC 25, 1460 (1982); G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).

**Point particle or finite size (triangular, Gaussian)**

$$f(\vec{r}, \vec{p}; t) = \frac{1}{N_{TP}} \sum_{i=1}^{N_{TPA}} g(\vec{r} - \vec{r}_i(t)) \tilde{g}(\vec{p} - \vec{p}_i(t))$$

**Equations of motion from pseudoparticle method:**

$$d\vec{r}_i/dt = \nabla_{\vec{p}_i} H; \quad d\vec{p}_i/dt = -\nabla_{\vec{r}_i} H.$$

# QMD transport approach

single-particle wave function:

$$\phi_i(\vec{r}; t) = \frac{1}{(2\pi L)^{4/3}} \exp \left[ -\frac{(\vec{r} - \vec{r}_i(t))^2}{4L} + \frac{i\vec{p}_i(t) \cdot \vec{r}}{\hbar} \right]$$

Wigner function (phase-space distribution):

$$\begin{aligned} f_i(\vec{r}, \vec{p}) &= \frac{1}{(2\pi\hbar)^3} \int \phi_i^*(\vec{r} - \vec{s}/2) \phi_i(\vec{r} + \vec{s}/2) \exp(-i\vec{p} \cdot \vec{s}) d^3 s \\ &= \frac{1}{(\pi\hbar)^3} \exp \left[ -\frac{(\vec{r} - \vec{r}_i)^2}{2L} - \frac{2L(\vec{p} - \vec{p}_i)^2}{\hbar^2} \right], \end{aligned}$$

**Many-body Hamiltonian**

$$H = \sum_i T_i + \frac{1}{2} \sum_{i \neq j} V_{ij}$$

$\langle V_{ij} \rangle$  from  
Hartree calculation

**Equations of motion**

$$\begin{aligned} \frac{d\vec{r}_i}{dt} &= \frac{\vec{p}_i}{m} + \frac{1}{2} \sum_{j, j \neq i} \frac{\partial \langle V_{ij} \rangle}{\partial \vec{p}_i} = \frac{\partial \langle H \rangle}{\partial \vec{p}_i}, \\ \frac{d\vec{p}_i}{dt} &= -\frac{1}{2} \sum_{j, j \neq i} \frac{\partial \langle V_{ij} \rangle}{\partial \vec{r}_i} = -\frac{\partial \langle H \rangle}{\partial \vec{r}_i}. \end{aligned}$$

Ch. Hartnack et al., PRC 495, 303 (1989); J. Aichelin, Phys. Rep. 202, 233 (1988).

**AMD and FMD: wave function antisymmetrized**

Nuclear EOS,  $E_{\text{sym}}$



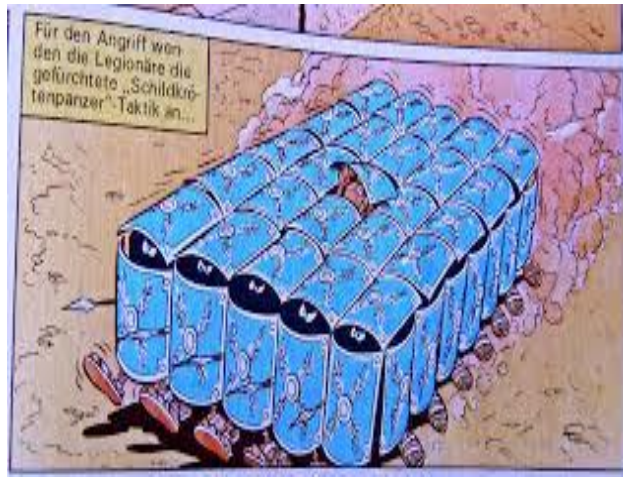
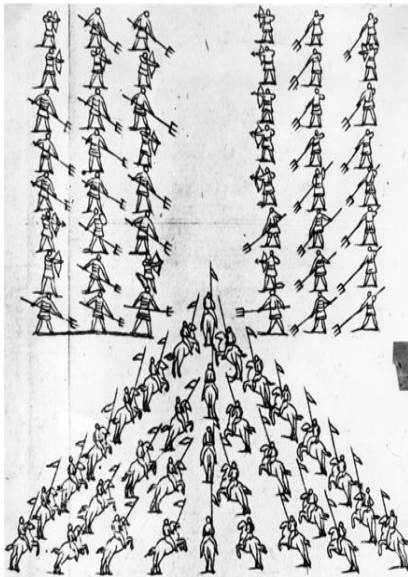
Mean-field potential

How reliable?

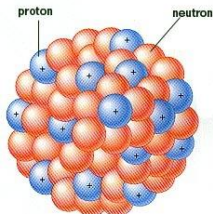


Transport simulations

Heavy-ion experiments



Initialization

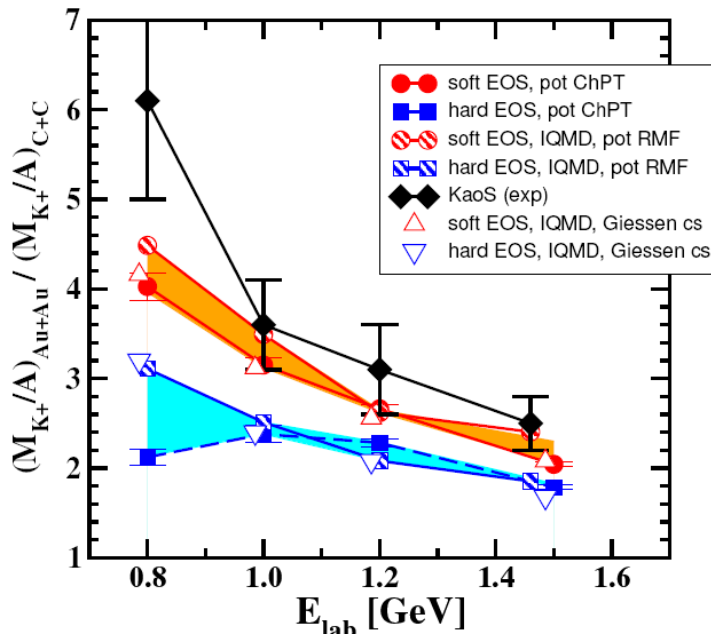
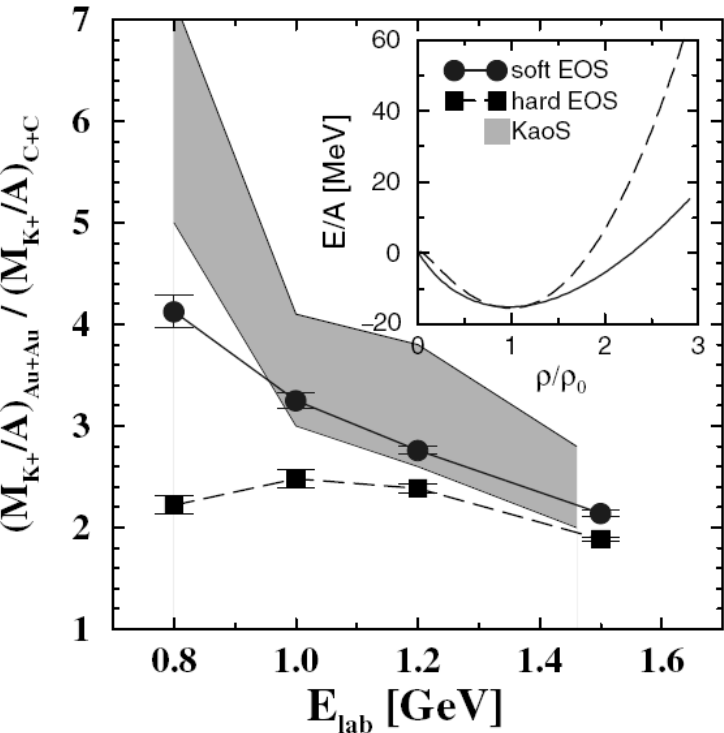


Mean Field :  
attractive  
Low Energy

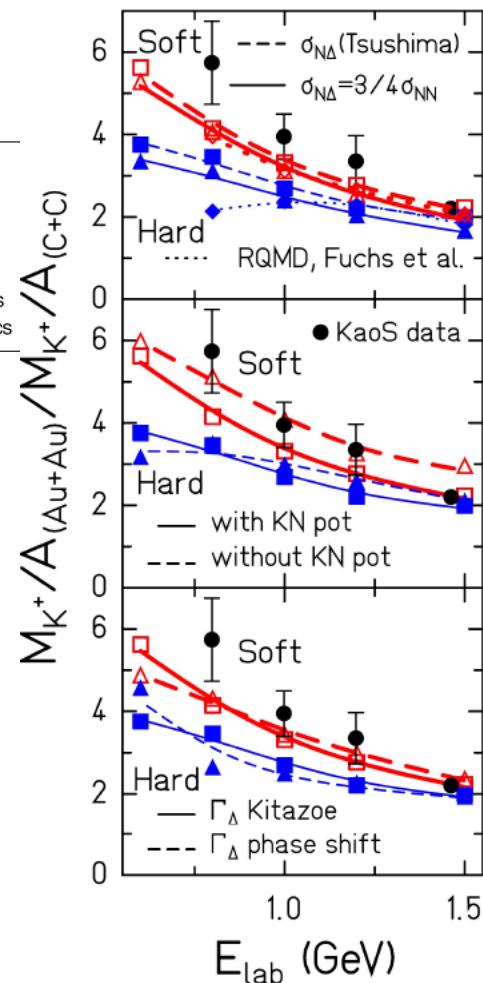
$$\vec{p}_i(t + \Delta t) = \vec{p}_i(t) - \nabla U[f(\vec{r}, \vec{p}; t)]$$

NN collisions:  
repulsive  
Pauli Blocking  
High Energy

# Probe of symmetric NM EOS: kaon production



C. Fuchs and H.H. Wolter, EPJA 30, 5 (2006)

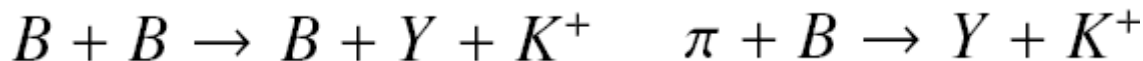


Ch. Hartnack, H. Oeschler, and J. Aichelin, PRL 96, 012302 (2006)

$$\Delta + N \rightarrow N + K^+ + \Lambda$$

J. Aichelin and C.M. Ko, PRL 55, 2661 (1985)

C. Fuchs et al., PRL 86, 1974 (2001)

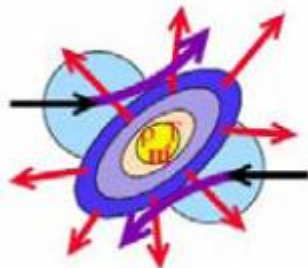
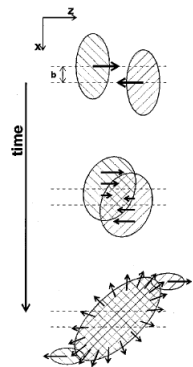


# Probes of symmetric NM EOS: collective flows

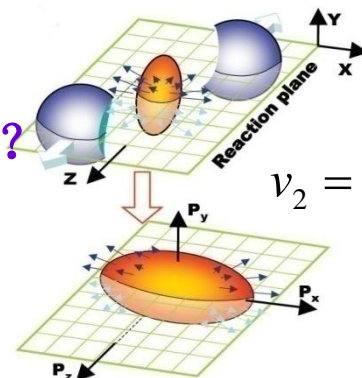
Slope of Transverse flow/directed flow ( $v_1$ )

Elliptic flow ( $v_2$ )

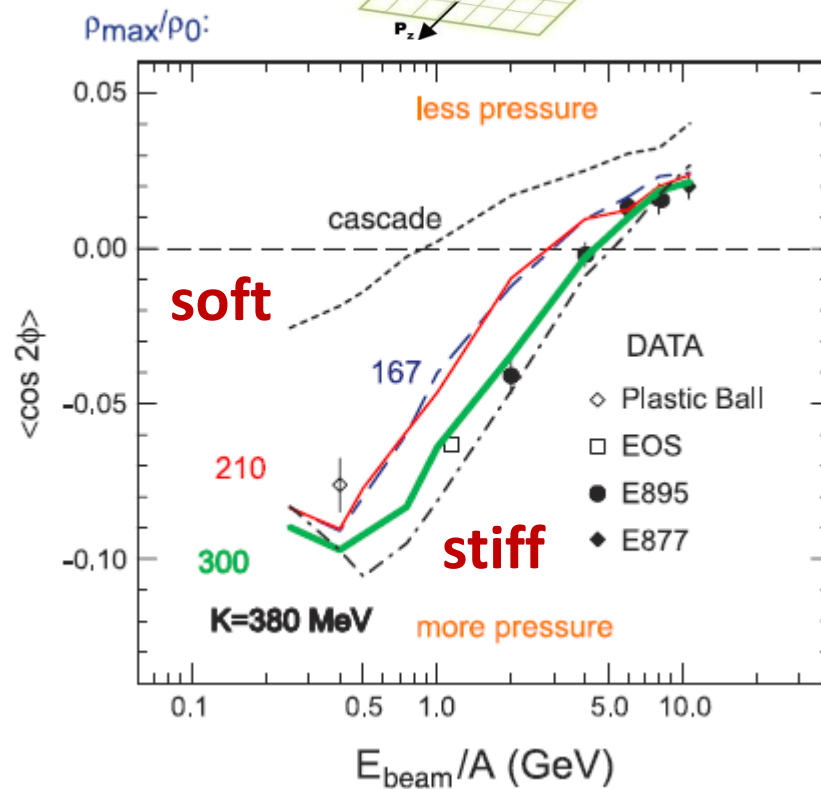
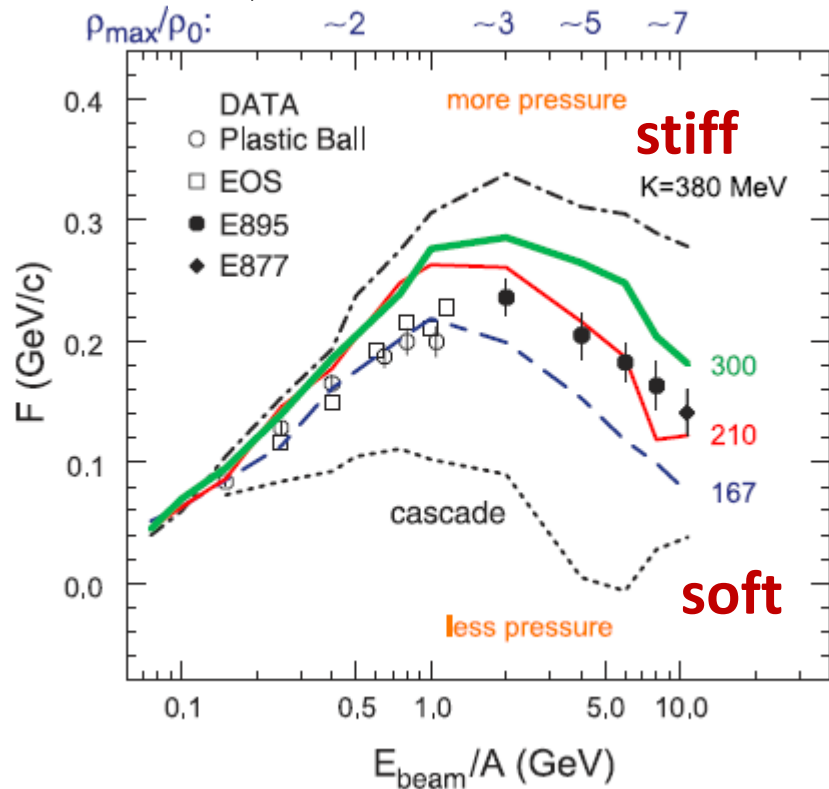
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$



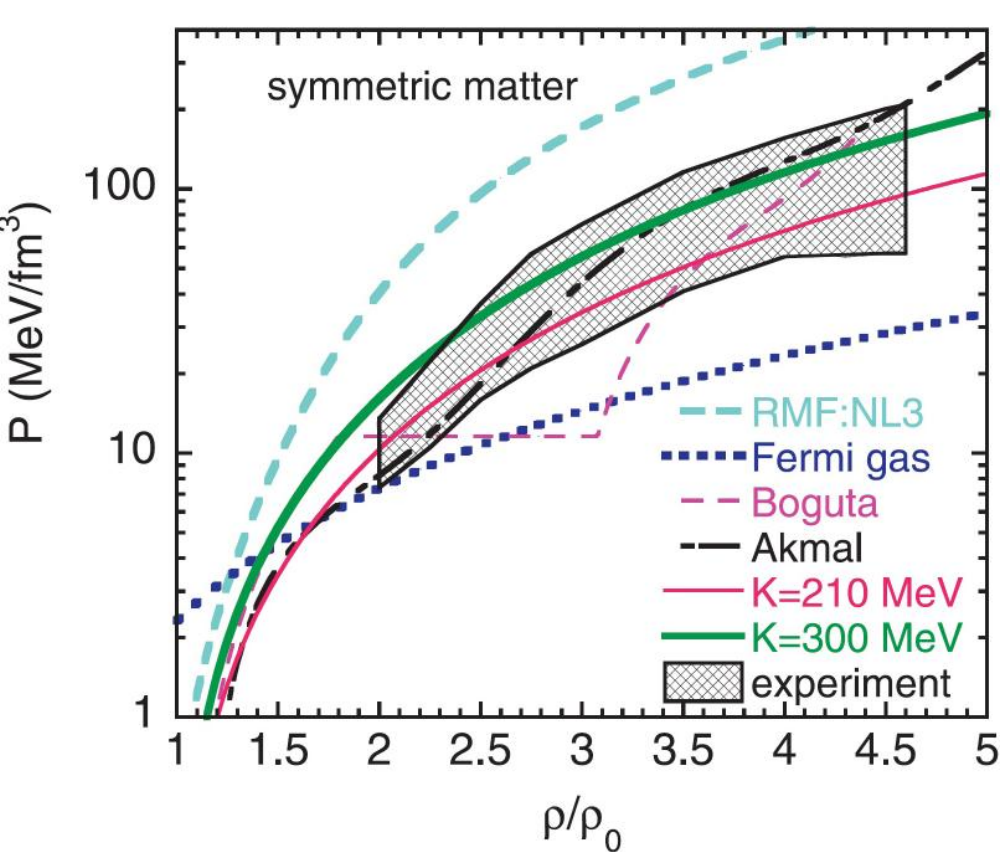
Model dependence?  
Theoretical uncertainty?



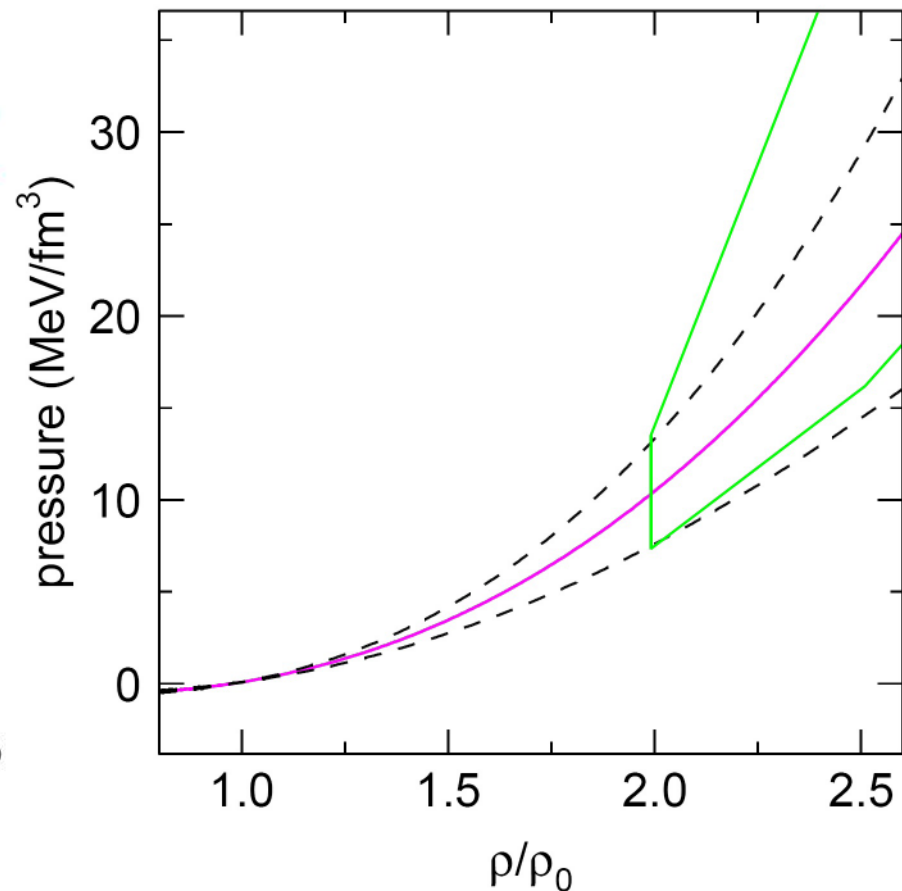
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$



# Probes of symmetric NM EOS: collective flows



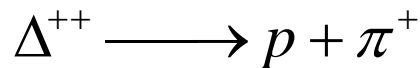
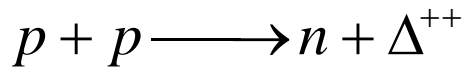
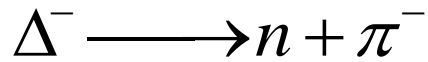
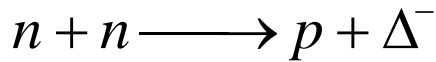
P. Danielewicz, R. Lacey,  
and W.G. Lynch, *Science* (2002)



**More latest constraint from IQMD-FOPI:**  
A. LeFèvre, Y. Leifels, W. Reisdorf, J. Aichelin,  
and Ch. Hartnack, *NPA* (2016).



# Probes of $E_{\text{sym}}$ : $\pi^-/\pi^+$ ratio

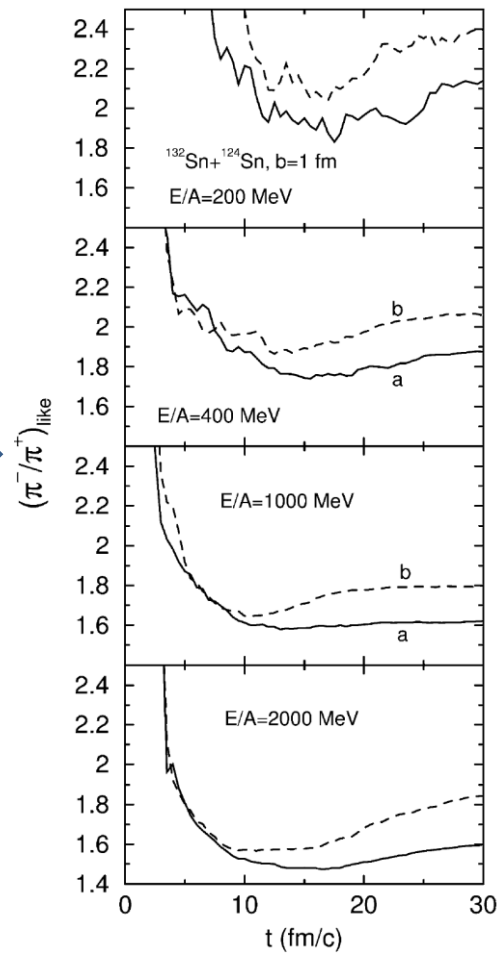
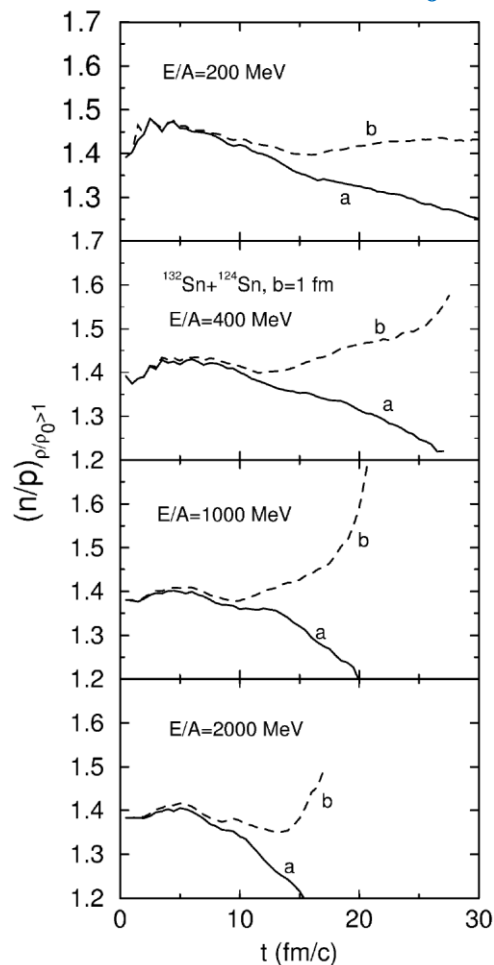
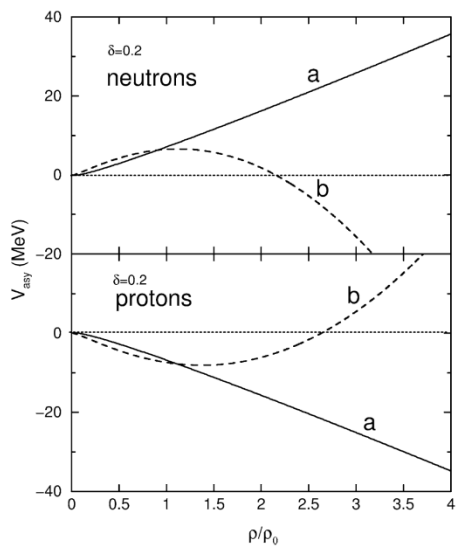
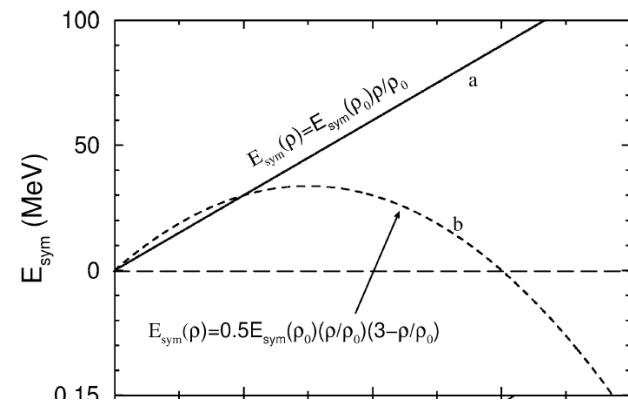


B.A. Li, NPA, (2002)

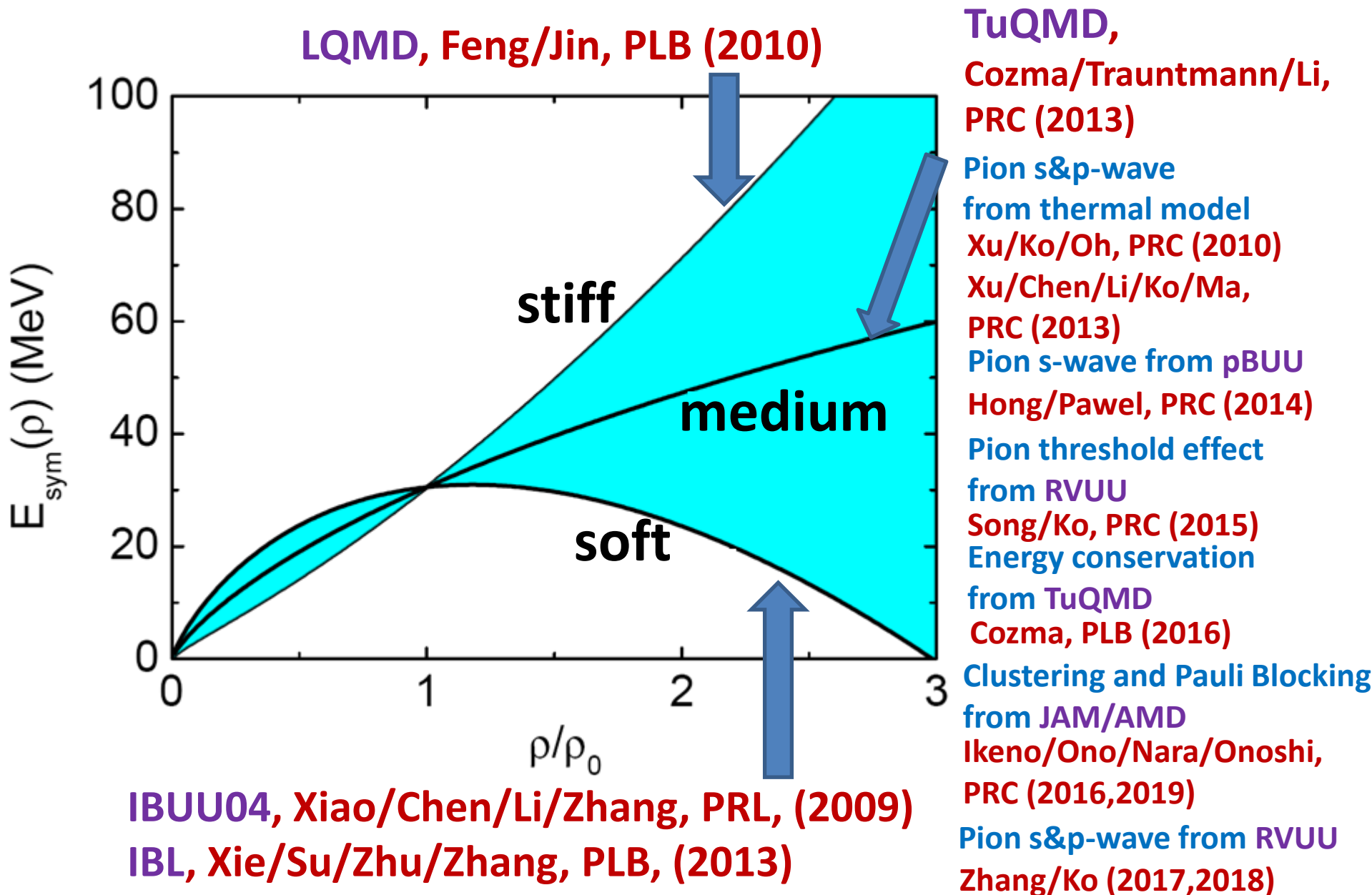
$E_{\text{sym}}$  or  $U_{\text{sym}}$  at  $\rho > \rho_0$

$n/p$  at  $\rho > \rho_0$

$\pi^-/\pi^+$  ratio



# Divergence of $E_{\text{sym}}$ from FOPI $\pi^-/\pi^+$ data



# Transport Model Evaluation Project (TMEP)

- Trento I (2004): energy 1-2 AGeV, particle production  $p$ ,  $\pi$ , K
- Trento II (2009): energy 100, 400 AMeV, not finished
- Transport2014 (2014): Mainly 100 AMeV, also 400 AMeV. stability, stopping, and flow, NN scatterings
- Transport2017 (2017): Box calculation of NN scatterings, mean-field evolution, and production of pion-like particles
- Transport2019 (2019): production of pion-like particles at 270 AMeV, ...

**Research papers  
published**

# Transport theories for heavy-ion collisions in the

**1 A GeV regime** J. Phys. G: Nucl. Part. Phys. **31** (2005) S741–S757

E E Kolomeitsev<sup>1,2</sup>, C Hartnack<sup>3</sup>, H W Barz<sup>4</sup>, M Bleicher<sup>5</sup>,  
E Bratkovskaya<sup>5</sup>, W Cassing<sup>6</sup>, L W Chen<sup>7,8</sup>, P Danielewicz<sup>9</sup>, C Fuchs<sup>10</sup>,  
T Gaitanos<sup>11</sup>, C M Ko<sup>7</sup>, A Larionov<sup>6,13</sup>, M Reiter<sup>5</sup>, Gy Wolf<sup>12</sup> and  
J Aichelin<sup>3,14</sup>

## Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions

PHYSICAL REVIEW C **93**, 044609 (2016)

Jun Xu,<sup>1,\*</sup> Lie-Wen Chen,<sup>2,†</sup> ManYee Betty Tsang,<sup>3,‡</sup> Hermann Wolter,<sup>4,§</sup> Ying-Xun Zhang,<sup>5,||</sup> Joerg Aichelin,<sup>6</sup>  
Maria Colonna,<sup>7</sup> Dan Cozma,<sup>8</sup> Pawel Danielewicz,<sup>3</sup> Zhao-Qing Feng,<sup>9</sup> Arnaud Le Fèvre,<sup>10</sup> Theodoros Gaitanos,<sup>11</sup>  
Christoph Hartnack,<sup>6</sup> Kyungil Kim,<sup>12</sup> Youngman Kim,<sup>12</sup> Che-Ming Ko,<sup>13</sup> Bao-An Li,<sup>14</sup> Qing-Feng Li,<sup>15</sup> Zhu-Xia Li,<sup>5</sup>  
Paolo Napolitani,<sup>16</sup> Akira Ono,<sup>17</sup> Massimo Papa,<sup>18</sup> Taesoo Song,<sup>19</sup> Jun Su,<sup>20</sup> Jun-Long Tian,<sup>21</sup> Ning Wang,<sup>22</sup> Yong-Jia Wang,<sup>15</sup>  
Janus Weil,<sup>19</sup> Wen-Jie Xie,<sup>23</sup> Feng-Shou Zhang,<sup>24</sup> and Guo-Qiang Zhang<sup>1</sup>

## Comparison of heavy-ion transport simulations: Collision integral in a box

PHYSICAL REVIEW C **97**, 034625 (2018)

Ying-Xun Zhang,<sup>1,2,\*</sup> Yong-Jia Wang,<sup>3,†</sup> Maria Colonna,<sup>4,‡</sup> Pawel Danielewicz,<sup>5,§</sup> Akira Ono,<sup>6,||</sup> Manyee Betty Tsang,<sup>5,¶</sup>  
Hermann Wolter,<sup>7,#</sup> Jun Xu,<sup>8,\*\*</sup> Lie-Wen Chen,<sup>9</sup> Dan Cozma,<sup>10</sup> Zhao-Qing Feng,<sup>11</sup> Subal Das Gupta,<sup>12</sup> Natsumi Ikeno,<sup>13</sup>  
Che-Ming Ko,<sup>14</sup> Bao-An Li,<sup>15</sup> Qing-Feng Li,<sup>3,11</sup> Zhu-Xia Li,<sup>1</sup> Swagata Mallik,<sup>16</sup> Yasushi Nara,<sup>17</sup> Tatsuhiko Ogawa,<sup>18</sup>  
Akira Ohnishi,<sup>19</sup> Dmytro Oliinychenko,<sup>20</sup> Massimo Papa,<sup>4</sup> Hannah Petersen,<sup>20,21,22</sup> Jun Su,<sup>23</sup> Taesoo Song,<sup>20,21</sup> Janus Weil,<sup>20</sup>  
Ning Wang,<sup>24</sup> Feng-Shou Zhang,<sup>25,26</sup> and Zhen Zhang<sup>14</sup>

## Comparison of heavy-ion transport simulations:

### Collision integral with pions and $\Delta$ resonances in a box

PHYSICAL REVIEW C **100**, 044617 (2019)

Akira Ono<sup>10</sup>,<sup>1,\*</sup> Jun Xu,<sup>2,3,†</sup> Maria Colonna,<sup>4</sup> Pawel Danielewicz,<sup>5</sup> Che Ming Ko,<sup>6</sup> Manyee Betty Tsang,<sup>5</sup> Yong-Jia Wang,<sup>7</sup>  
Hermann Wolter,<sup>8</sup> Ying-Xun Zhang,<sup>9,10</sup> Lie-Wen Chen,<sup>11</sup> Dan Cozma,<sup>12</sup> Hannah Elfner,<sup>13,14,15</sup> Zhao-Qing Feng,<sup>16</sup>  
Natsumi Ikeno,<sup>17,18</sup> Bao-An Li,<sup>19</sup> Swagata Mallik,<sup>20</sup> Yasushi Nara,<sup>21</sup> Tatsuhiko Ogawa,<sup>22</sup> Akira Ohnishi,<sup>23</sup>  
Dmytro Oliinychenko,<sup>24</sup> Jun Su,<sup>25</sup> Taesoo Song,<sup>13</sup> Feng-Shou Zhang,<sup>26,27</sup> and Zhen Zhang<sup>25</sup>

## Comparison of heavy-ion transport simulations: Mean-field dynamics in a box

PHYSICAL REVIEW C **104**, 024603 (2021)

Maria Colonna,<sup>1,\*</sup> Ying-Xun Zhang,<sup>2,3,†</sup> Yong-Jia Wang,<sup>4,‡</sup> Dan Cozma,<sup>5</sup> Pawel Danielewicz,<sup>6,§</sup> Che Ming Ko,<sup>7</sup> Akira Ono,<sup>8,||</sup>  
Manyee Betty Tsang,<sup>6,¶</sup> Rui Wang,<sup>9,10</sup> Hermann Wolter,<sup>11,#</sup> Jun Xu,<sup>12,9,\*\*</sup> Zhen Zhang,<sup>13</sup> Lie-Wen Chen,<sup>14</sup> Hui-Gan Cheng,<sup>15</sup>  
Hannah Elfner,<sup>16,17,18</sup> Zhao-Qing Feng,<sup>15</sup> Myungkuk Kim,<sup>19</sup> Youngman Kim,<sup>20</sup> Sangyong Jeon,<sup>21</sup> Chang-Hwan Lee,<sup>22</sup>  
Bao-An Li,<sup>23</sup> Qing-Feng Li,<sup>4,24</sup> Zhu-Xia Li,<sup>2</sup> Swagata Mallik,<sup>25</sup> Dmytro Oliinychenko,<sup>26,27</sup> Jun Su,<sup>13</sup> Taesoo Song,<sup>16,28</sup>  
Agnieszka Sorensen,<sup>29</sup> and Feng-Shou Zhang<sup>30,31</sup>

## Compare model calculations

### with exp data

Symmetry energy investigation with pion production from Sn+Sn systems

[Physics Letters B 813 \(2021\) 136016](#)

G. Jhang, et al. (Sπrit Collaboration and TEMP Collaboration)

## A few review papers

### related to TMEP

Dynamics of clusters and fragments in heavy-ion collisions

Akira Ono

[Progress in Particle and Nuclear Physics 105 \(2019\) 139-179](#)

Transport approaches for the description of intermediate-energy heavy-ion collisions

Jun Xu\*

[Progress in Particle and Nuclear Physics 106 \(2019\) 312–359](#)

Collision dynamics at medium and relativistic energies

M. Colonna

[Progress in Particle and Nuclear Physics 113 \(2020\) 103775](#)

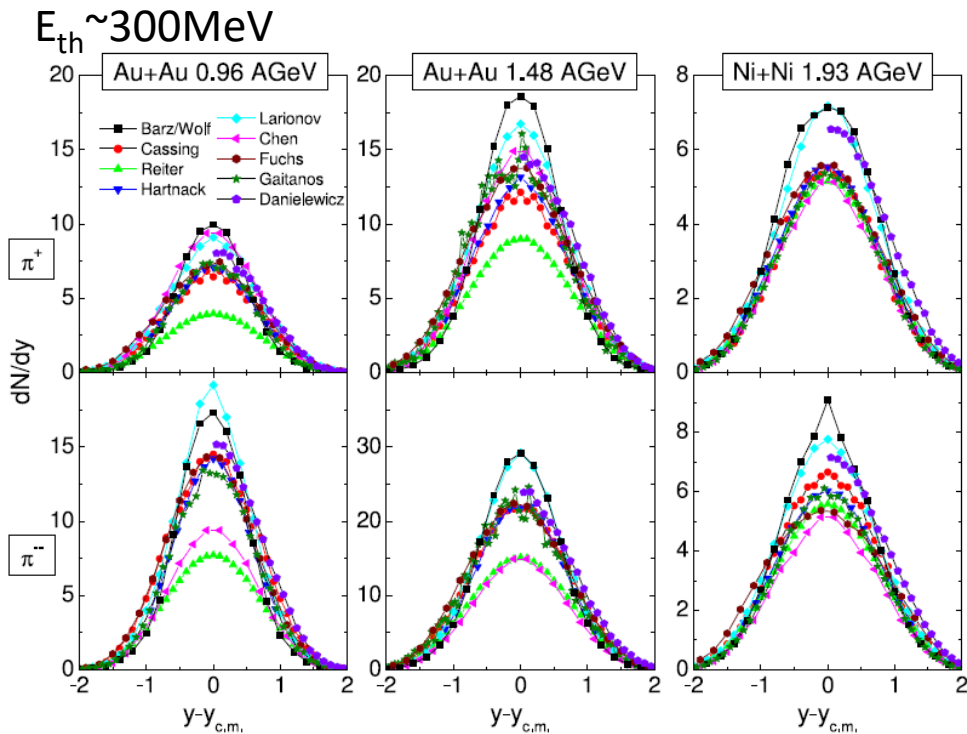
Transport models for intermediate-energy heavy-ion studies

Hermann Wolter, et al. (47 authors, >100 pages)

[In preparation](#)

# Transport2004

Also stopping from p spectrum

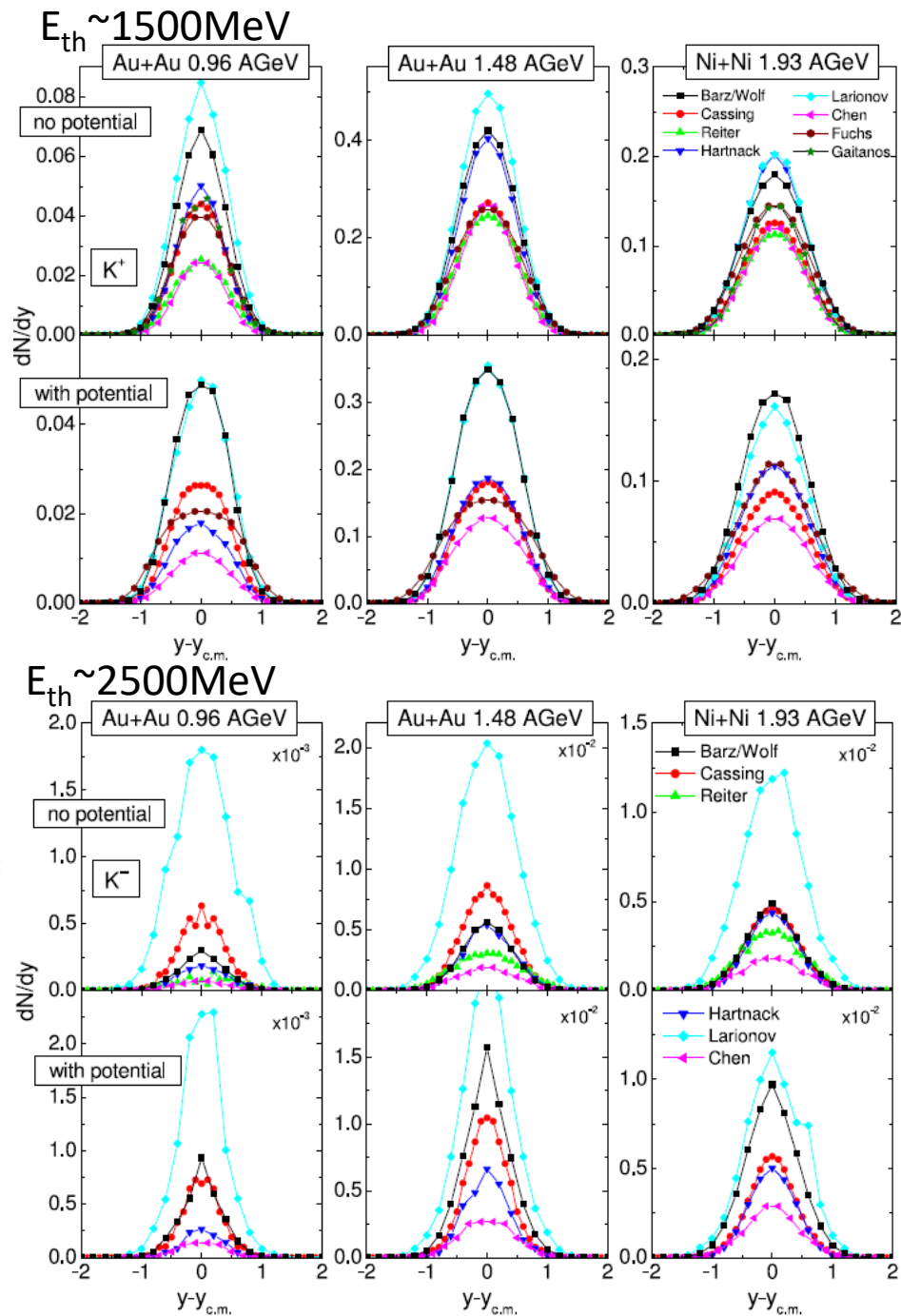


Kaon energy

$$\omega_K(\rho_B, k) = \sqrt{[m_K^*(\rho_B)]^2 + k^2}$$

$$m_K^*(\rho_B) = m_K^0 \left( 1 - \alpha \frac{\rho_B}{\rho_0} \right)$$

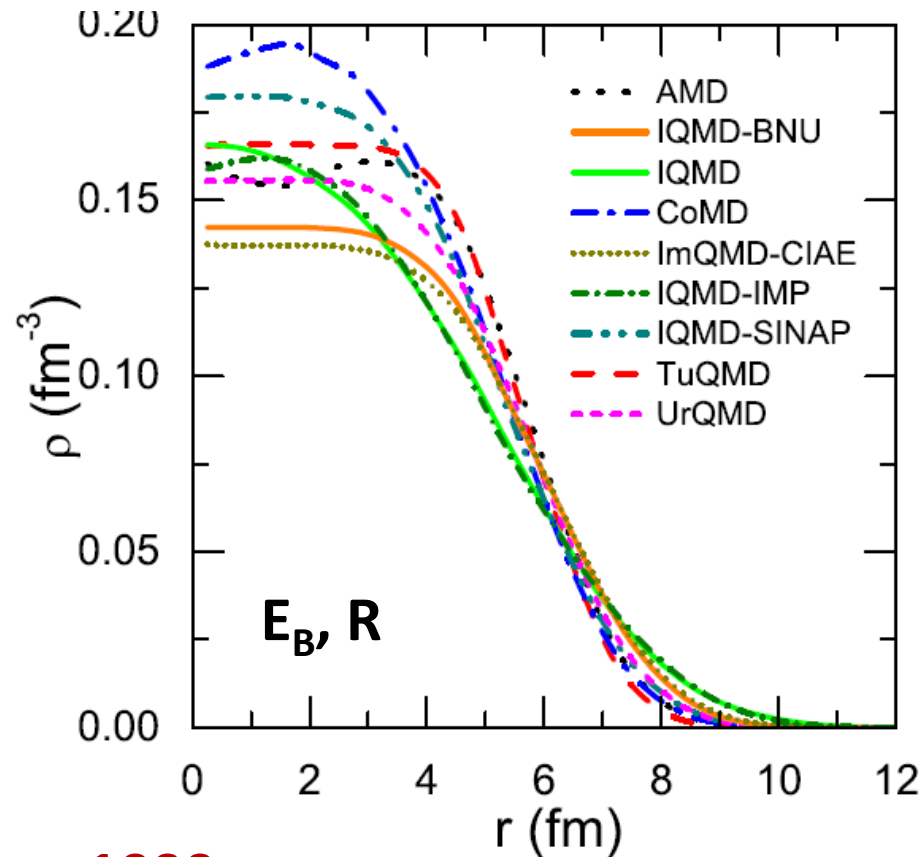
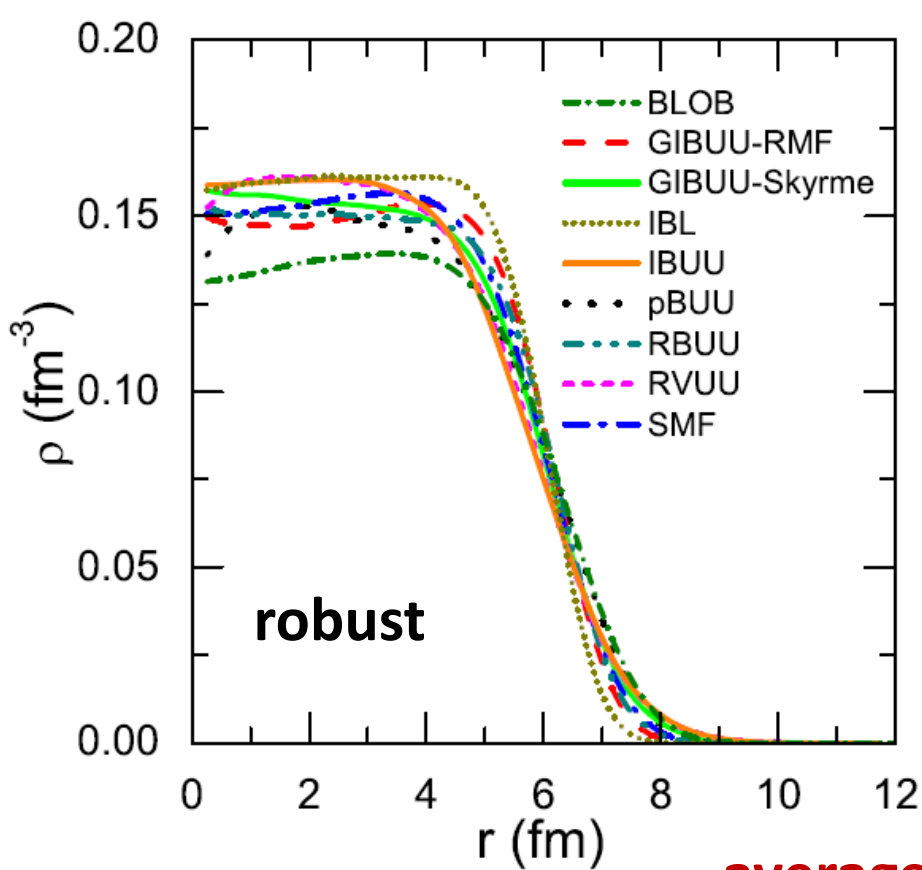
E.E. Kolomeitsev, et al., JPG (2005)



# Transport2014 at SJTU



# HIC-comparison: Initial density profile



average over 1000 events

BUU: mostly follow the suggested Woods-Saxon distribution, easily stable

QMD: mostly deviate from the suggested Woods-Saxon distribution

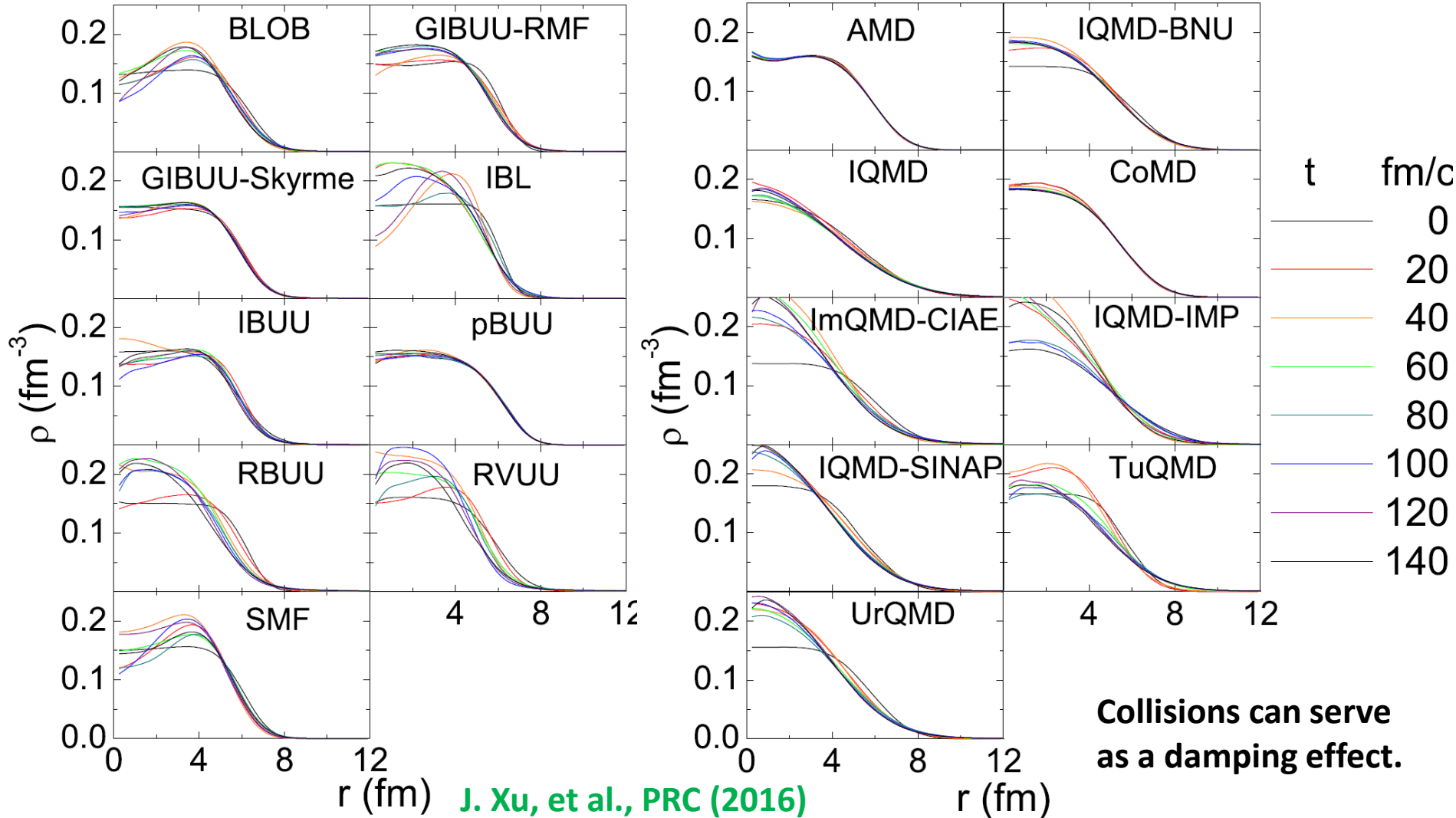
ground state? Thomas-Fermi or Hartree-Fock, frictional cooling

Difficult to get a common initialization

J. Xu, et al., PRC (2016)



# HIC-comparison: Stability (b=20 fm)



J. Xu, et al., PRC (2016)

Bubble: IBL, BLOB

Stable: GIBUU-Skyrme, pBUU, AMD, CoMD

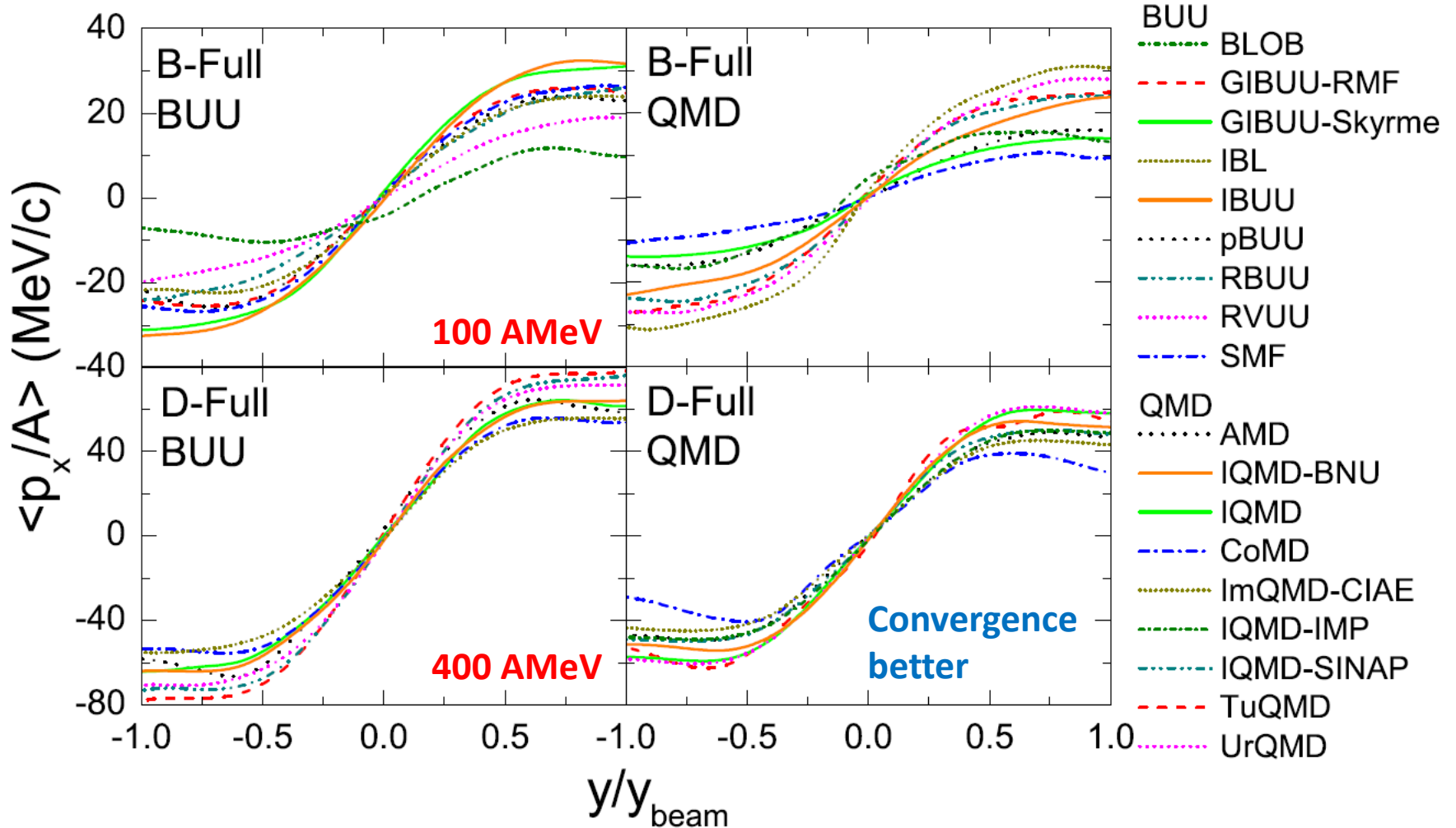
evolve to another stable configuration:

GMR: IBUU, SMF, RVUU, TuQMD, IQMD-IMP

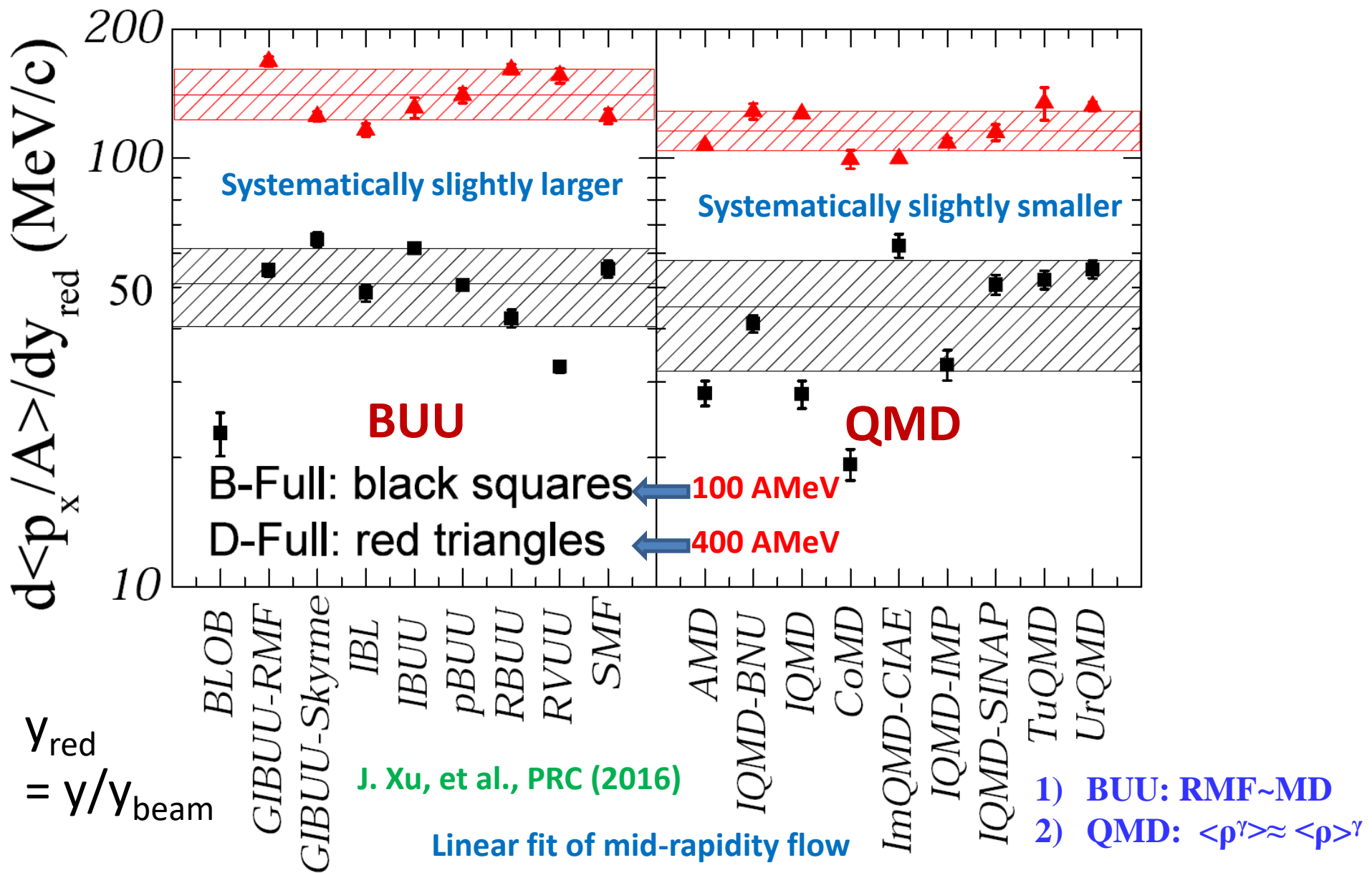
IQMD-BNU, IQMD-SINAP, UrQMD

# HIC-comparison: Transverse flow

Au+Au,  $b = 7$  fm



J. Xu, et al., PRC (2016)



**Theoretical uncertainties of flow parameter:  
 about 30% at 100 AMeV, 13% at 400 AMeV**

## Approximation used in the QMD codes:

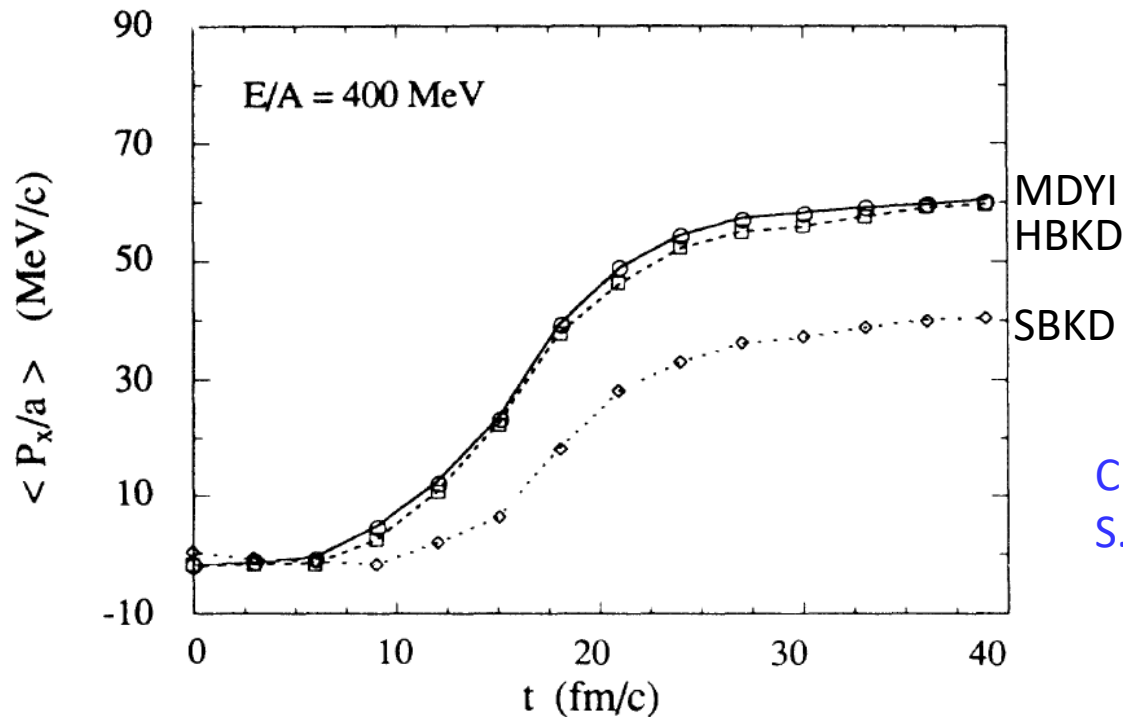
potential energy:

$$E_p = \int \epsilon \, dr = \frac{a}{2} \frac{1}{\rho_0} \sum_i \langle \rho \rangle_i + \frac{b}{\gamma + 1} \frac{1}{\rho_0^\gamma} \sum_i \langle \rho^\gamma \rangle_i$$

In most of the QMD codes, the second term is calculated as:

$$\sum_i \langle \rho^\gamma \rangle_i \approx \sum_i \langle \rho \rangle_i^\gamma$$

By Ying-Xun Zhang



**Momentum-dependent potential leads to stronger transverse flow**

C. Gale, G.M. Welke, M. Prakash, S.J. Lee, and S. Das Gupta, PRC (1990)

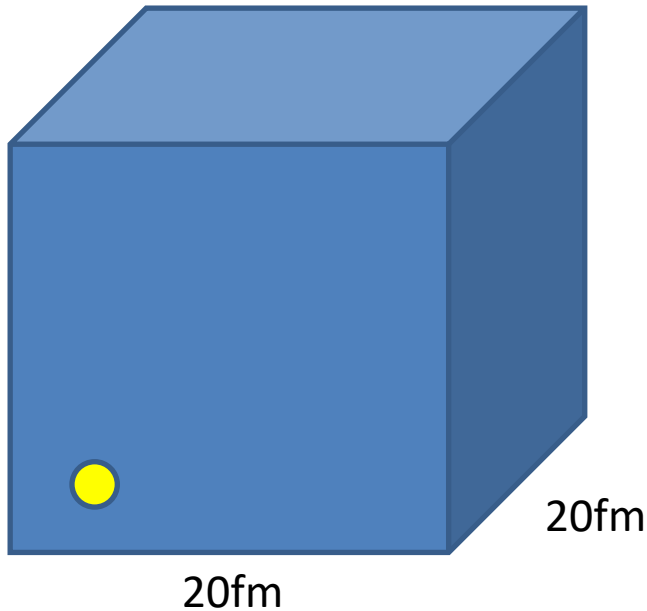
# Transport2017 at MSU



# Transport2017 at MSU



# Box calculations with periodic boundary conditions



## •Details of periodic boundary conditions

1. a box of volume  $V = L_1 * L_2 * L_3$ , where the system is confined.
2. The position of the center of box is  $(L_1/2, L_2/2, L_3/2)$ .
3. In order to keep all particles inside the box, a particle leaving the box has to enter it on the opposite side, keeping the same momentum.

## •Initialization:

**Uniform density  $\rho_0=0.16 \text{ fm}^{-3}$ , with isospin asymmetry equal to zero.** With the above size of the box this corresponds to 1280 nucleons, 640 neutrons and 640 protons. Particle positions are initialized randomly from 0 to  $L_k$ .

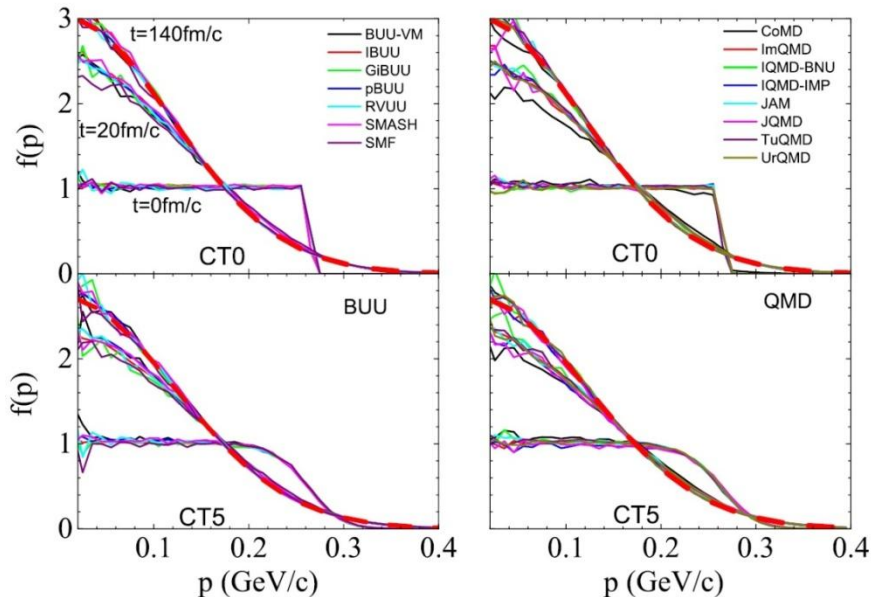
## Advantages:

1. Common initialization can be easily achieved.
2. Theoretical limits are generally available.

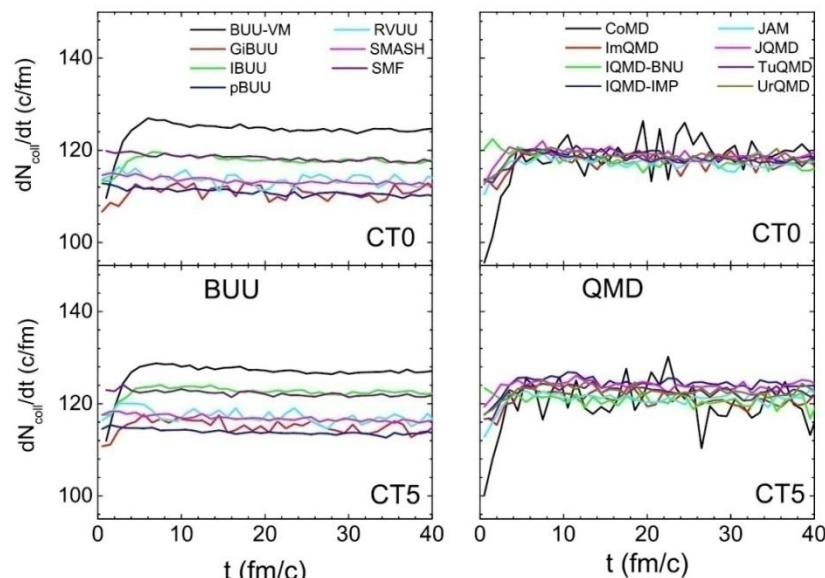
# Box-Cascade calculation

## Only NN scatterings without Pauli blocking

Time evolution of momentum distribution



Time evolution of collision rate



Y.X. Zhang, et al., PRC (2018)

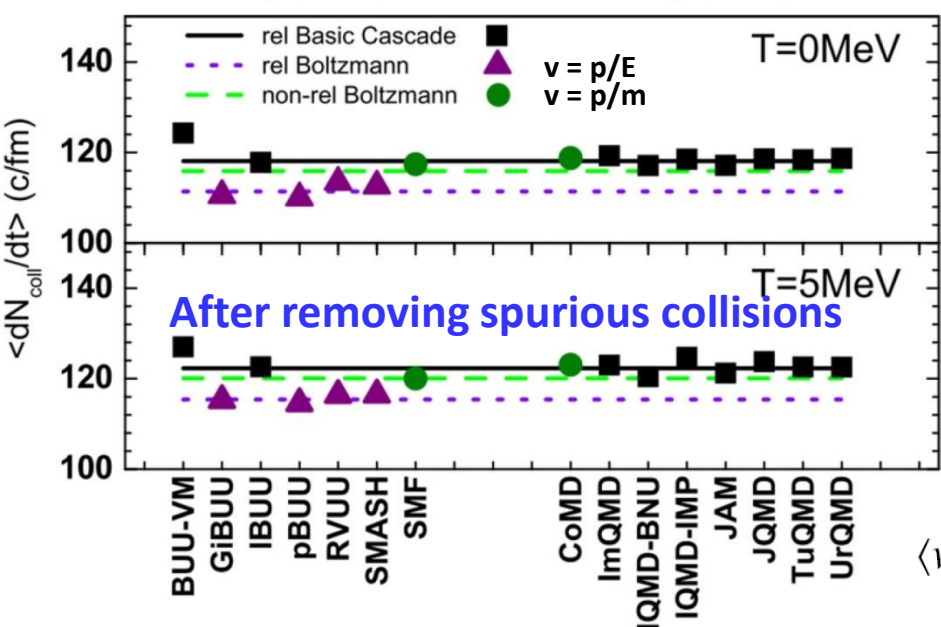
$$\frac{dN_{coll}}{dt} = \frac{1}{2} A \rho \sigma \langle v_{rel} \rangle$$

non-relativistic Boltzmann

$$\langle v_{rel} \rangle = (4 / \sqrt{5\pi}) (p_F / m)$$

Relativistic Boltzmann

$$\langle v_{rel} \rangle = \frac{1}{4m^4 T_B K_2^2(m/T_B)} \int_{2m}^{\infty} d\sqrt{s} s (s - 4m^2) K_1(\sqrt{s}/T_B)$$



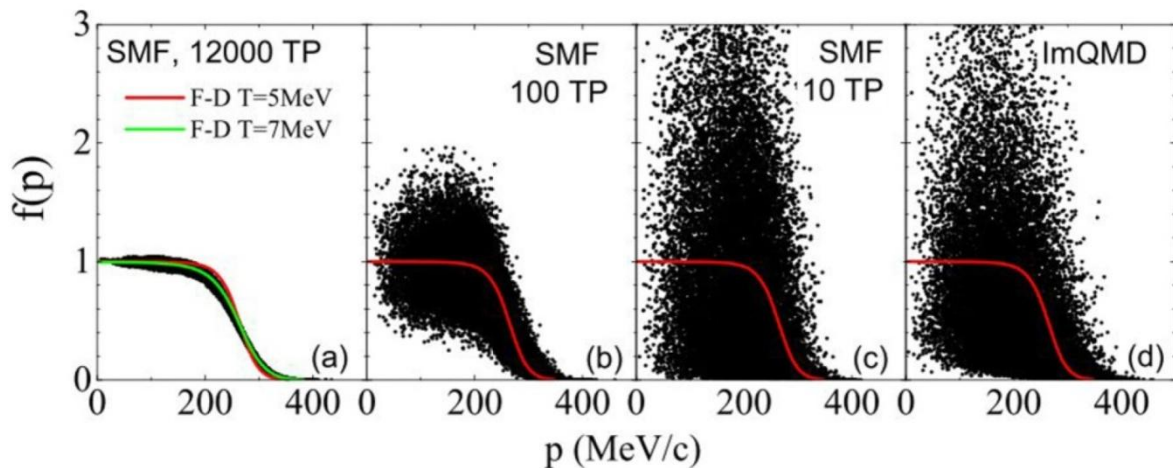
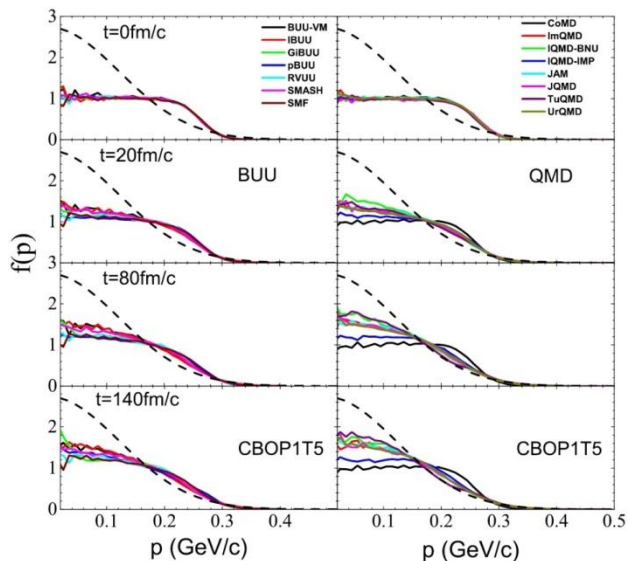


# Box-Cascade calculation

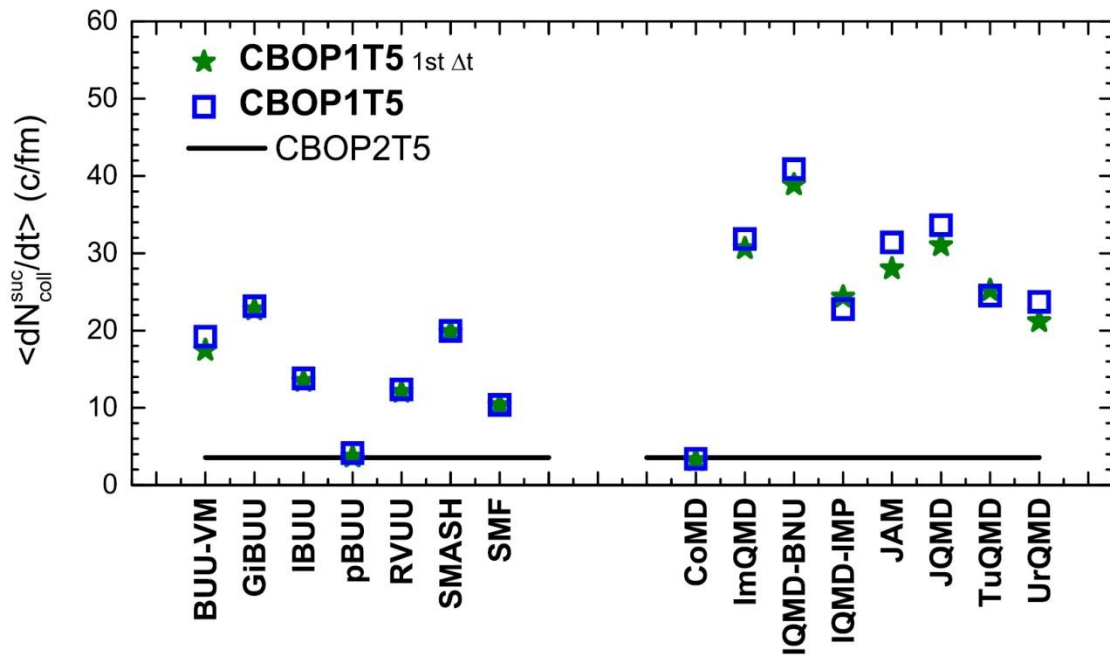
## NN scatterings with Pauli blocking

Time evolution of momentum distribution

Momentum occupation at 1<sup>st</sup> time step



Pauli blocking rate:  $1 - (1 - f_1)(1 - f_2)$

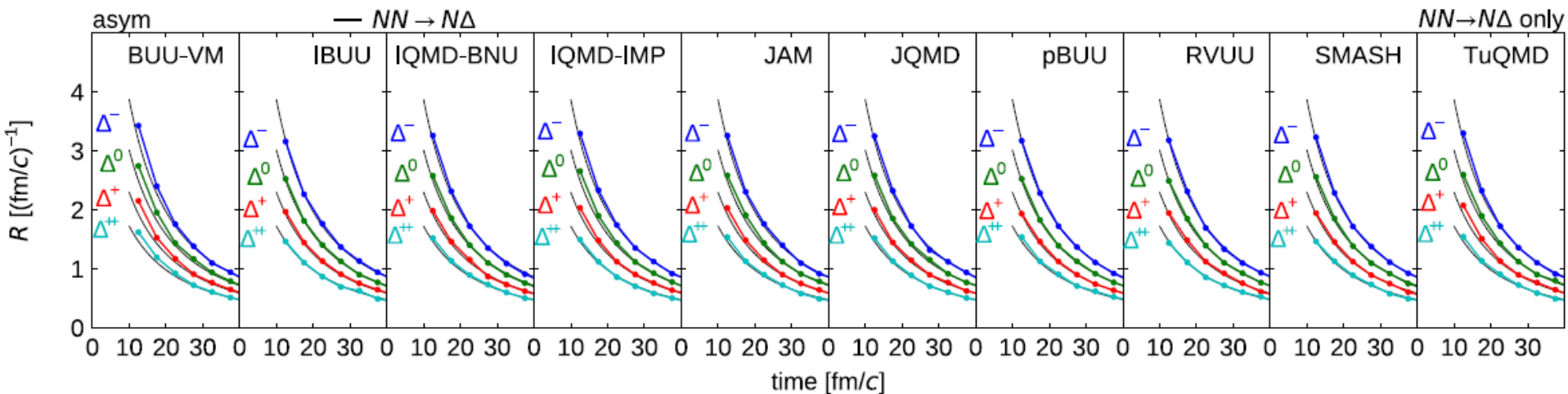


With Pauli blocking, the successful collision rates are much overestimated in QMD codes than in BUU codes, due to larger fluctuations in QMD.

Y.X. Zhang, et al., PRC (2018)

# Box-pion calculation

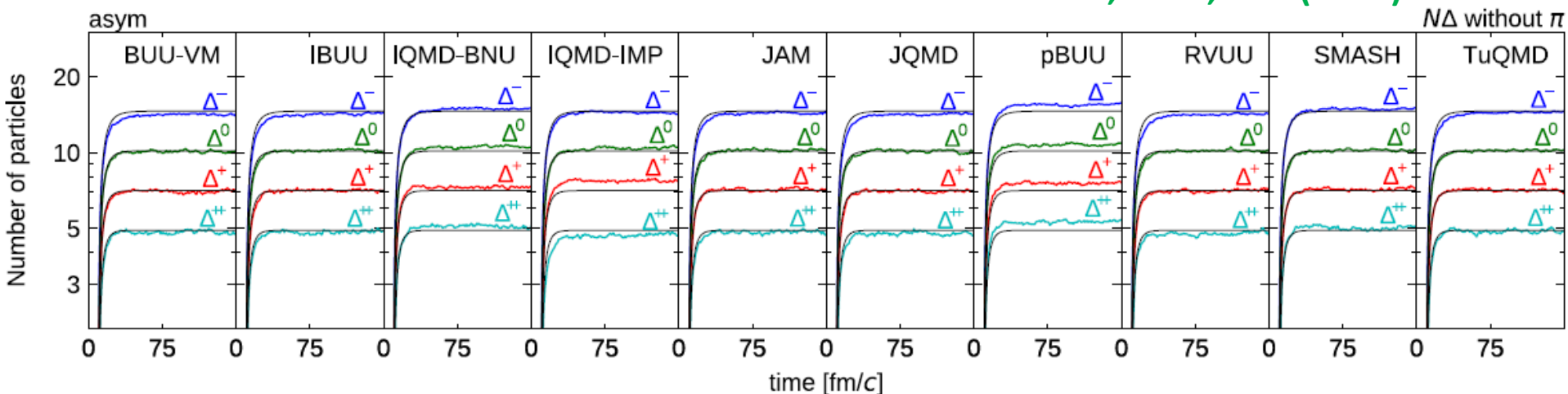
## $N+N \rightarrow N+\Delta$ and elastic $B+B \leftrightarrow B+B$



## $N+N \leftrightarrow N+\Delta$ and elastic $B+B \leftrightarrow B+B$

detailed balance

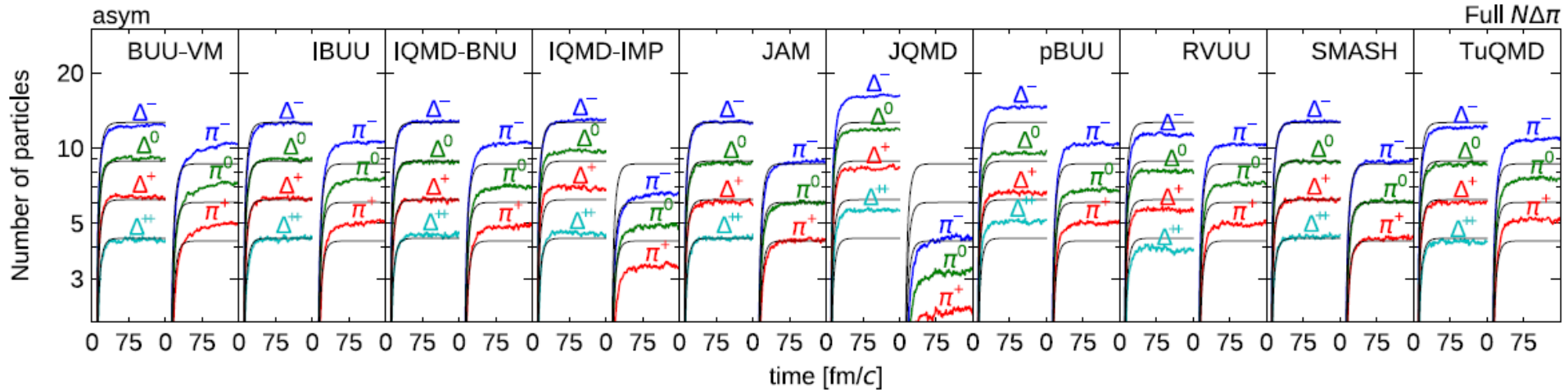
A. Ono, et al., PRC (2019)



Blacking solid lines: theoretical limits from reaction rate equations/statistical model

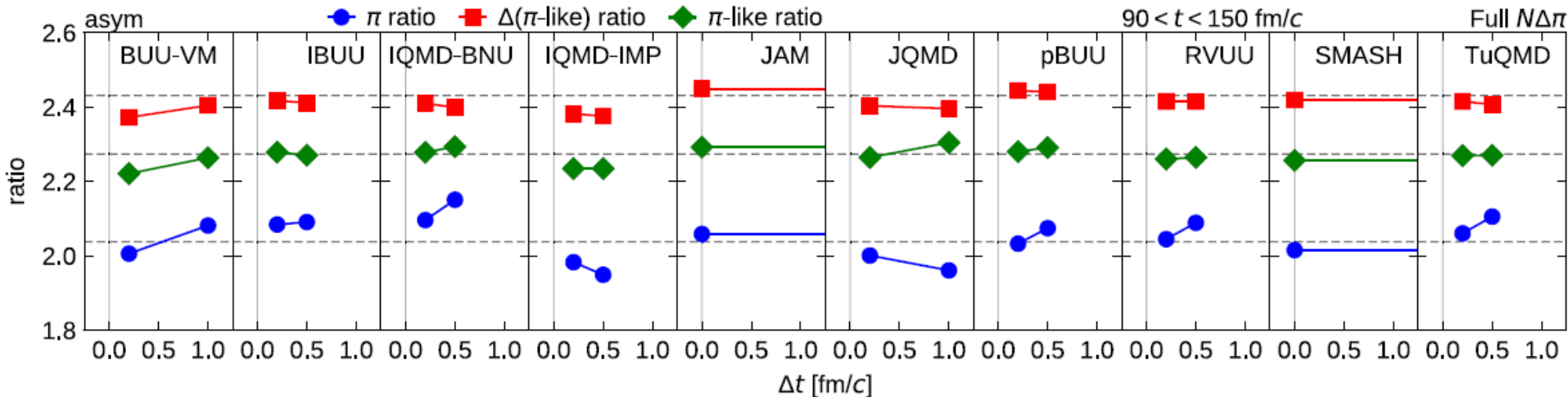
# Box-pion calculation

**$N+N \leftrightarrow N+\Delta$ , and  $\Delta \leftrightarrow N+\pi$ , and elastic  $B+B \leftrightarrow B+B$**



**Situation becomes worse with pions.**

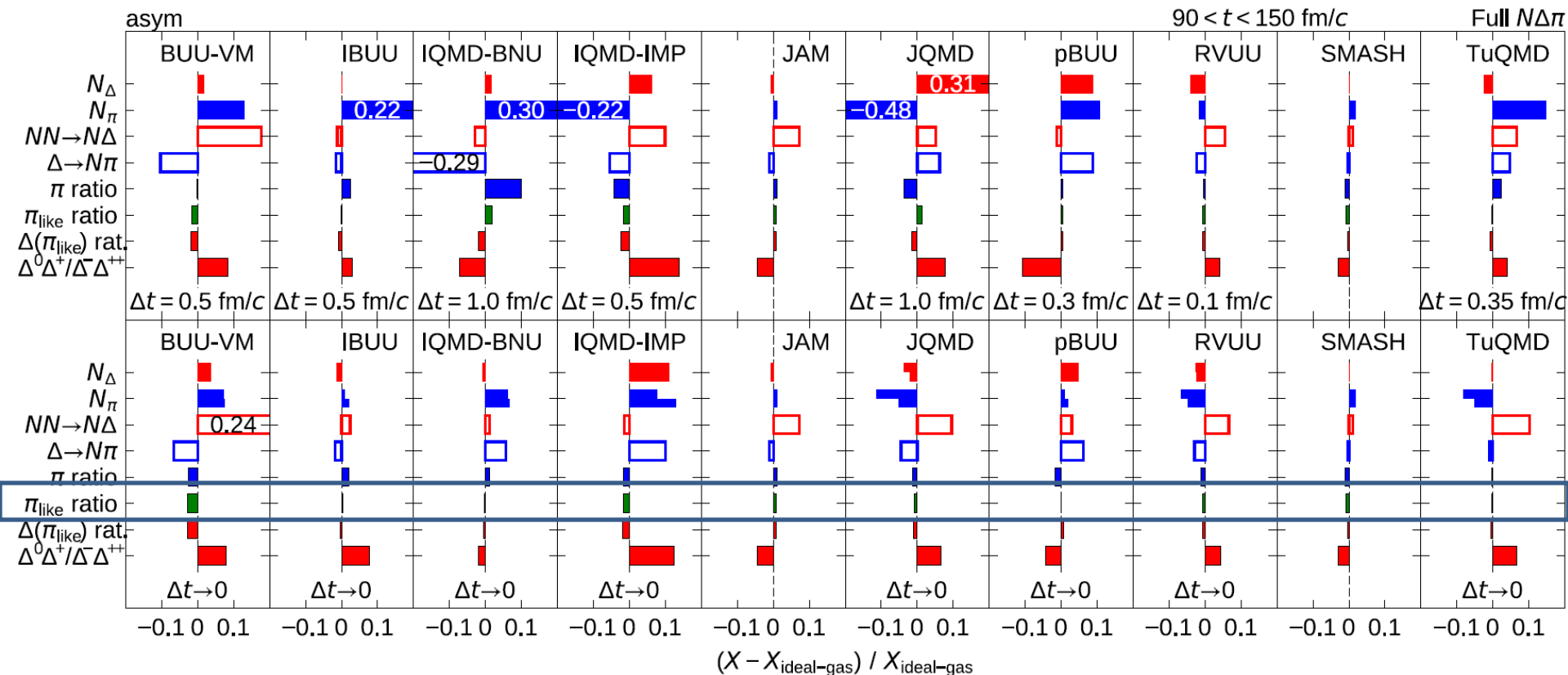
**A. Ono, et al., PRC (2019)**



**Sequence of  $N+N \leftrightarrow N+\Delta$  and  $\Delta \leftrightarrow N+\pi$  affects pion multiplicity (weak when  $\Delta t \rightarrow 0$ );  
Higher-order correlations lead to isospin violation in geometrical collision treatment (full ensemble method as a cure).**

**e.g. leading to  $\Delta^0\Delta^+/\Delta^-\Delta^{++} > 1$**

# Box-pion calculation: summary



Extrapolation to  $\Delta t \rightarrow 0$  can be helpful (or time-step free code).

$\pi_{\text{like}}$  ratio is OK due to cancellation of different effects.

A. Ono, et al., PRC (2019)

# Box-Vlasov calculation

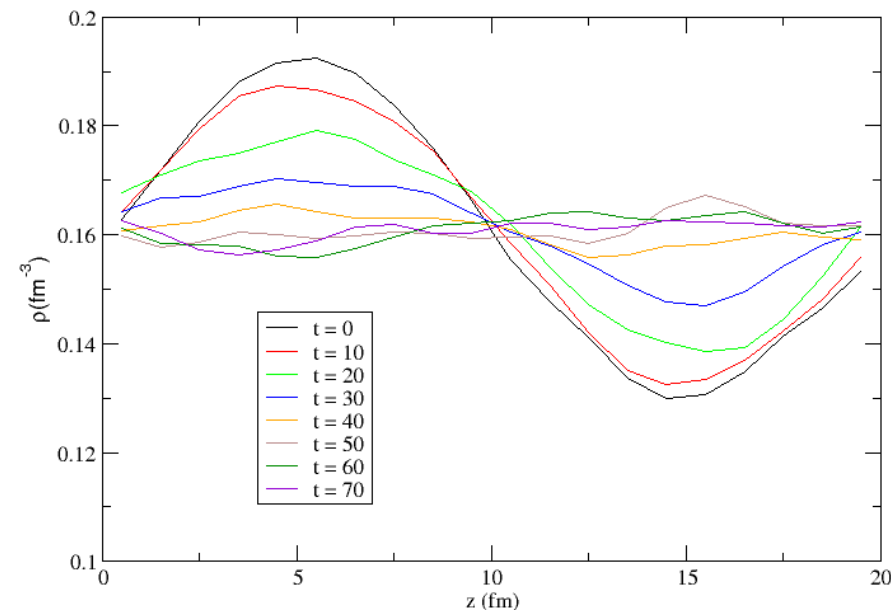
$$\rho(z, t=t_0) = \rho_0 + a_\rho \sin(k_i z)$$

$$k_i = n_i 2\pi/L$$

$$a_\rho = 0.2 \rho_0$$

Momentum sampled within local Fermi sphere determined by local density

Study the time evolution of  $\rho(z)$



Isospin symmetric nuclear matter

Only momentum-independent mean-field potential

a) Skyrme-like  
b)  $\sigma$ - $\omega$  coupling

No surface or Coulomb potential

No NN collision

due to large fluctuations and  
dissipations in QMD models

Incompressibility  $K_0=240$  MeV  $\Rightarrow$   $K_0=500$  MeV

## Damping sources:

- 1) Landau damping: mixing modes
- 2) Numerical damping: fluctuations
  - 1) Decreases with increasing TP numbers
  - 2) Decreases with increasing particle size

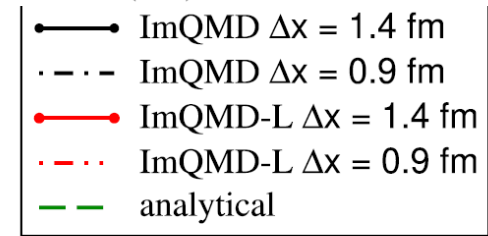
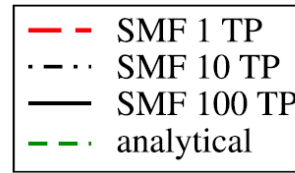
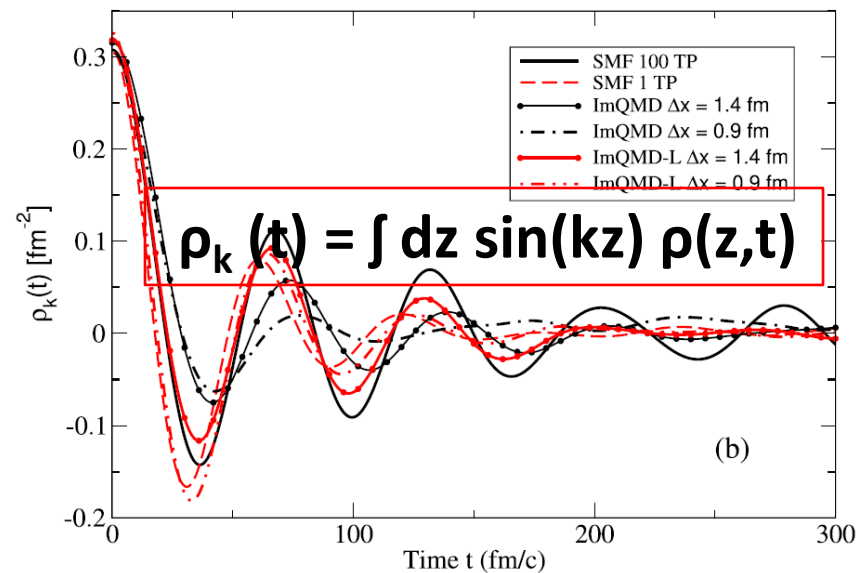
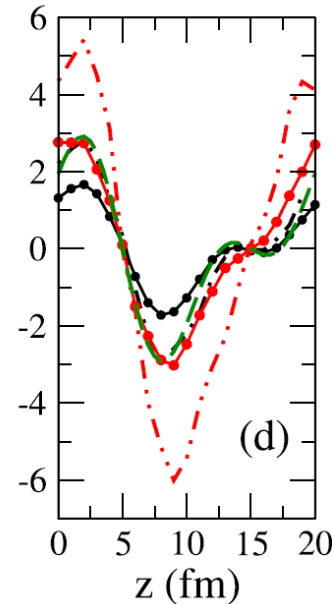
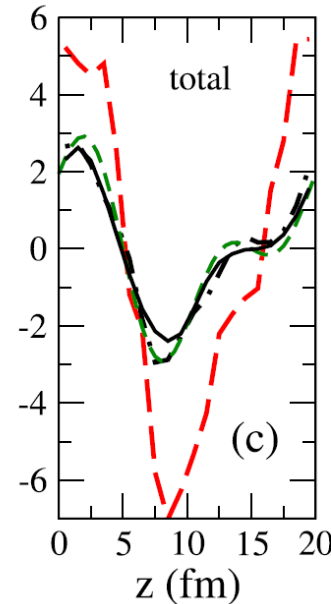
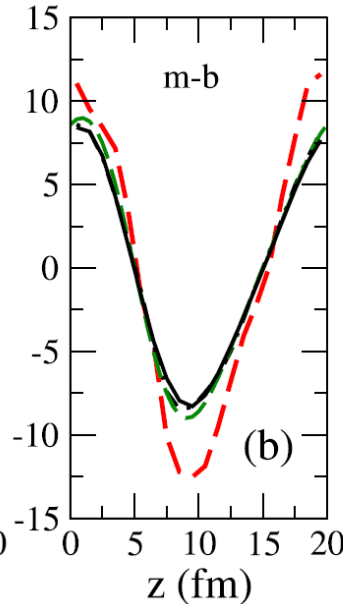
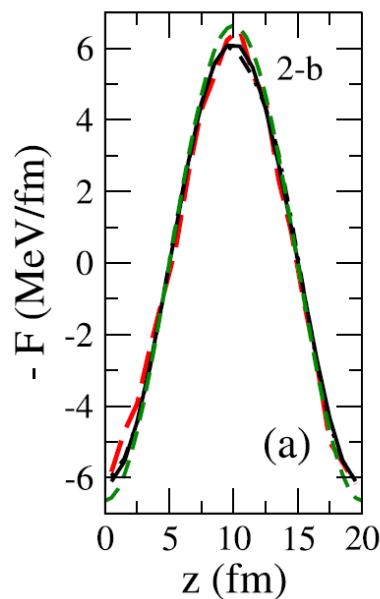
$$-F = \frac{\partial U}{\partial Z_i} \approx \int d^3r U(\rho) \frac{\partial G(\vec{r} - \vec{R}_i)}{\partial Z_i} = \frac{\partial H_{\text{pot}}}{\partial Z_i}$$

$$H_{\text{pot}} = \int d^3r \left[ \frac{a}{2} (\rho^2 / \rho_0) + \frac{b}{\sigma + 1} (\rho^{\sigma+1} / \rho_0^\sigma) \right]$$

$$H_{\text{pot}}^{2\text{body, QMD}} = \frac{a}{2\rho_0} \sum_i \tilde{\rho}_i$$

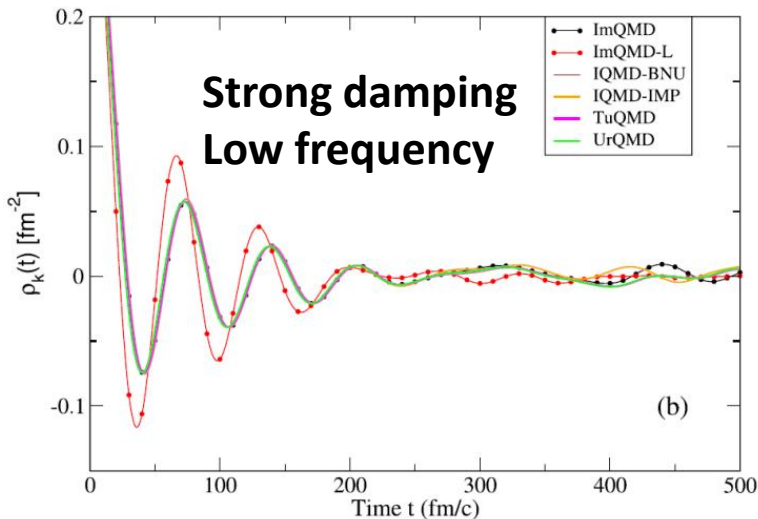
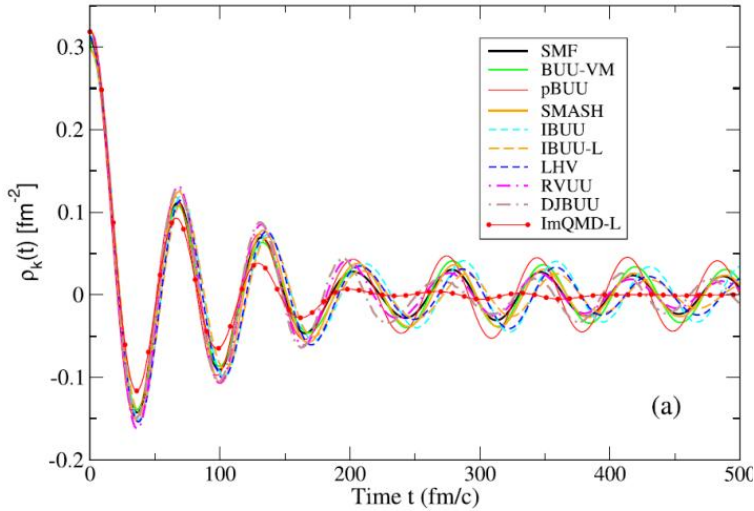
$$H_{\text{pot}}^{3\text{body, QMD}} = \frac{b}{(\sigma + 1)\rho_0^\sigma} \sum_i \tilde{\rho}_i^\sigma$$

**Inaccurate 3-body force calculation with few TPs (or in QMD);**  
**Suitable size of particle**



# Time Fourier transformation

$$\rho_k(t) = \int dz \sin(kz) \rho(z,t)$$

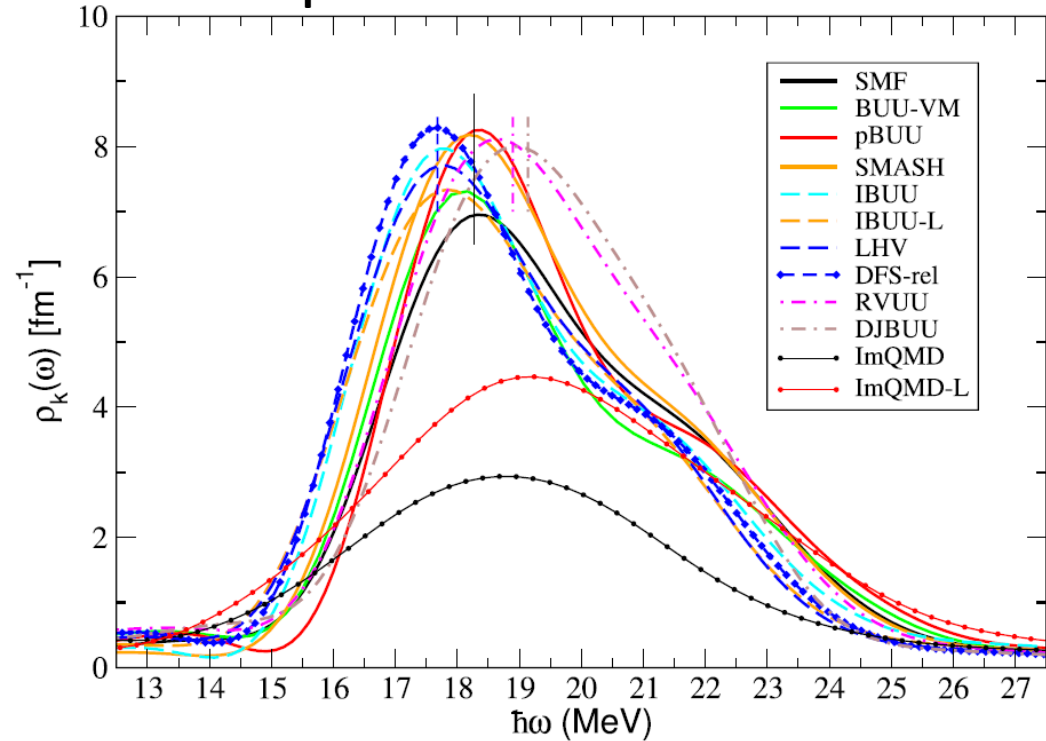


$$\rho_k(\omega) = \int dt \cos(\omega t) a_k(t)$$

Nonrel:  $v = p/m$

Rel:  $v = p/E$

Cov: depends on  $m^*$



Peak of  $\rho_k(\omega) \Leftrightarrow$  zero-sound

Landau parameter

M. Colonna, et al., PRC (2021)

# Transport2019 at ECT\*



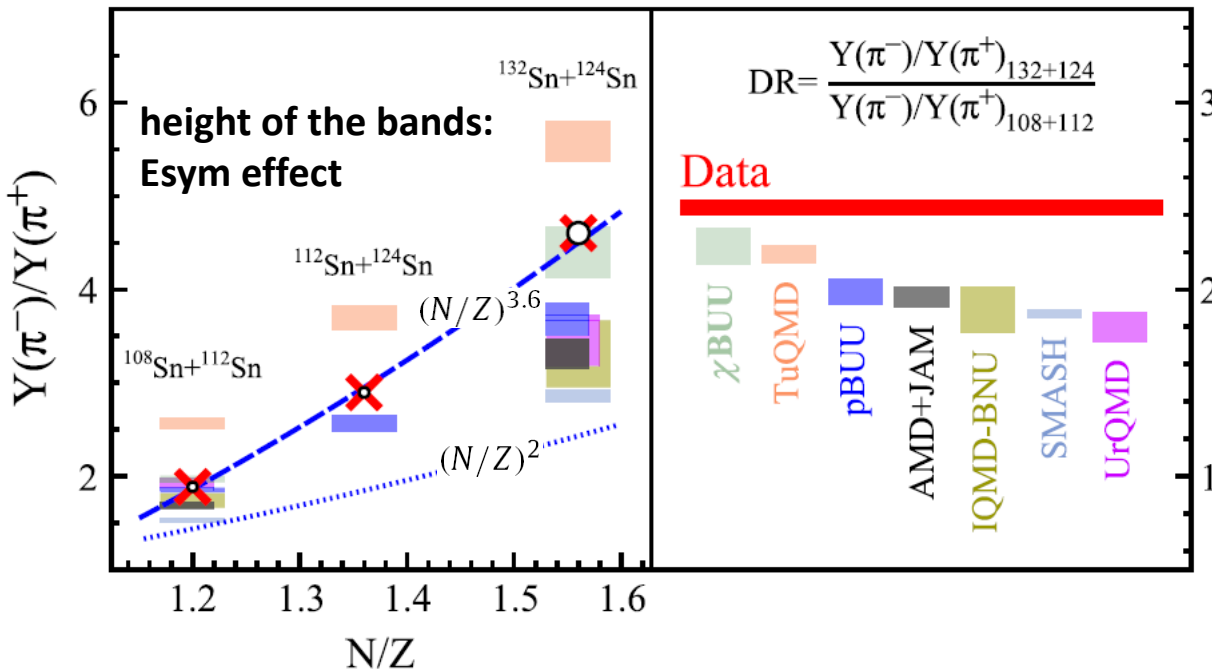
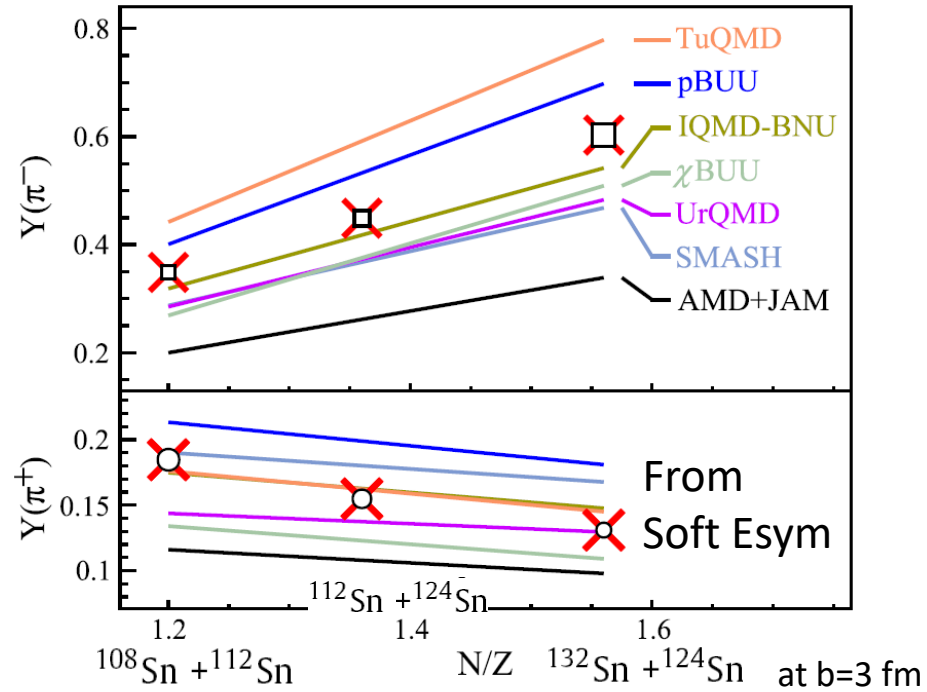
Workshop „Challenges to Transport Theory for Heavy-Ion Collisions“, ECT\*, Trento, May 20-24, 2019



# HIC-pion calculation with realistic setups by code authors

Compared with  $S_{\pi rit}$  data without first knowing the data

Deviations among code predictions on  $\pi^-/\pi^+$  are larger than the Esym effect.



Uncertainties of different physics + Uncertainties of their different incorporations

# HIC-pion calculation (in progress)

Setup:  $^{112}\text{Sn}+^{108}\text{Sn}@270\text{ AMeV}$ ,  $^{132}\text{Sn}+^{124}\text{Sn}@270\text{ AMeV}$ ,  $b = 4\text{ fm}$

Same nucleon setup as HIC comparison

Same pion related inelastic channels as Box-pion calculation

$$N + N \leftrightarrow N + \Delta$$

$$\Delta \leftrightarrow N + \pi$$

Common initialization by using the same initial phase-space data

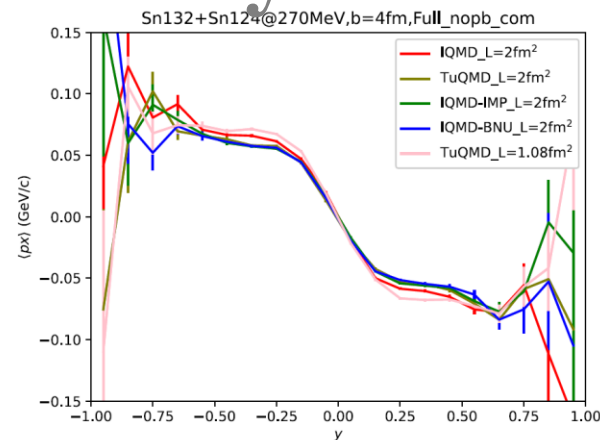
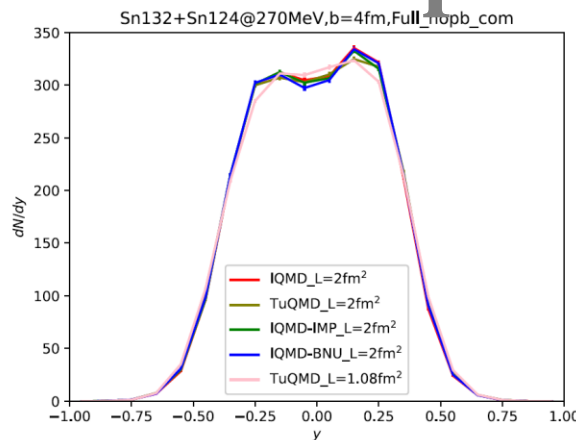
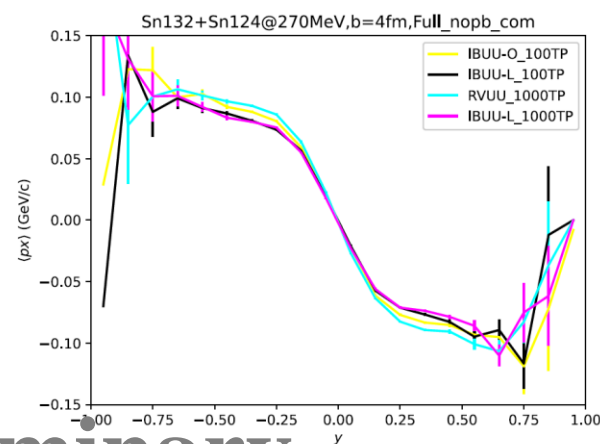
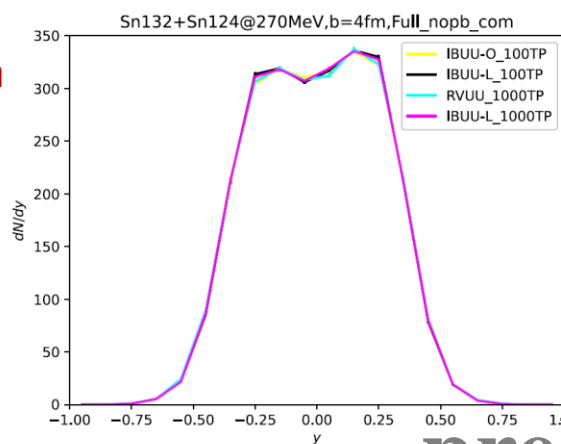
Investigate nucleon dynamics by revisiting stopping and flow

Effect of test-particle size

Effect of  $\langle p^y \rangle \approx \langle \rho \rangle^y$

Stronger flow in BUU than QMD

Higher central density in QMD than BUU



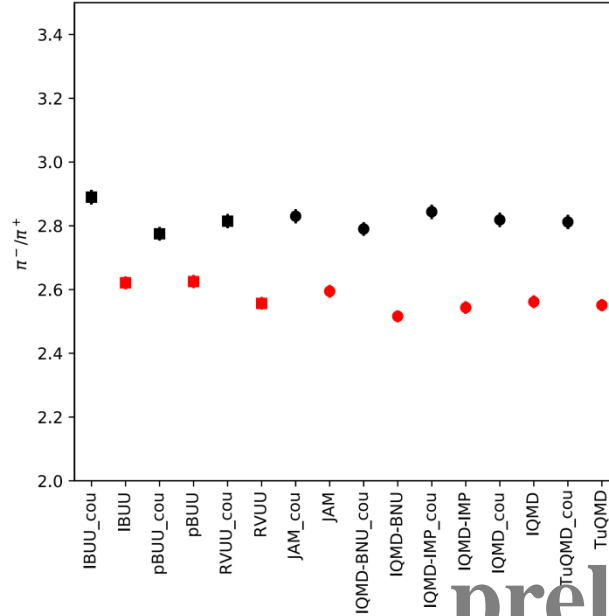
preliminary

# Effects of Pauli blocking and Coulomb I

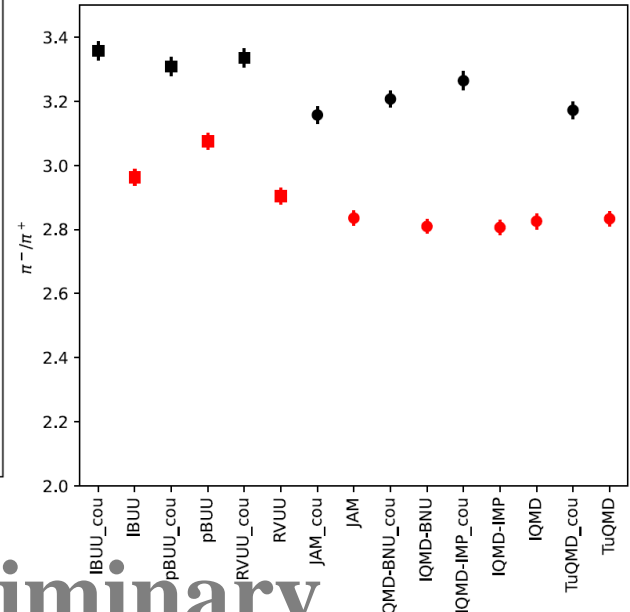
## Pauli blocking:

- 1) Suppress more NN $\rightarrow$ N $\Delta$  than N $\Delta\rightarrow$ NN, suppress  $\Delta(\pi)$  production
- 2) Suppress more pp $\rightarrow$ n $\Delta^{++}$  than nn $\rightarrow$ p $\Delta^-$ , enhance  $\pi^-/\pi^+$  yield ratio
- 3) Effect 2) remain when taking double ratio

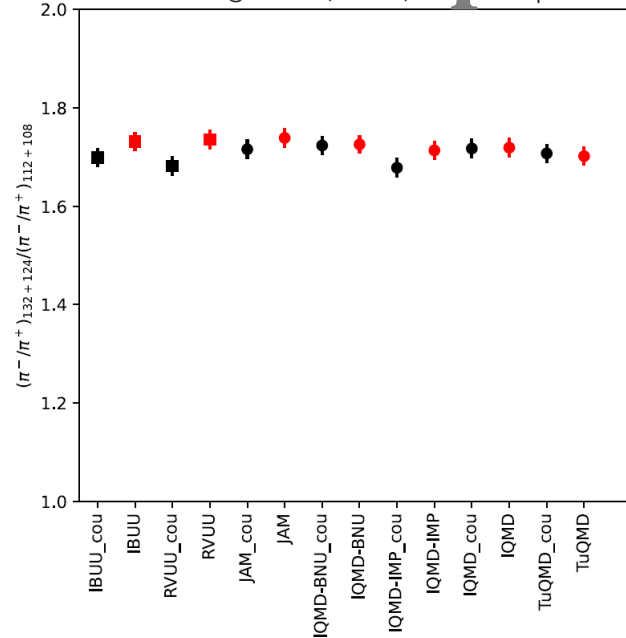
Sn132+Sn124@270MeV,b=4fm,Cascade&nopb



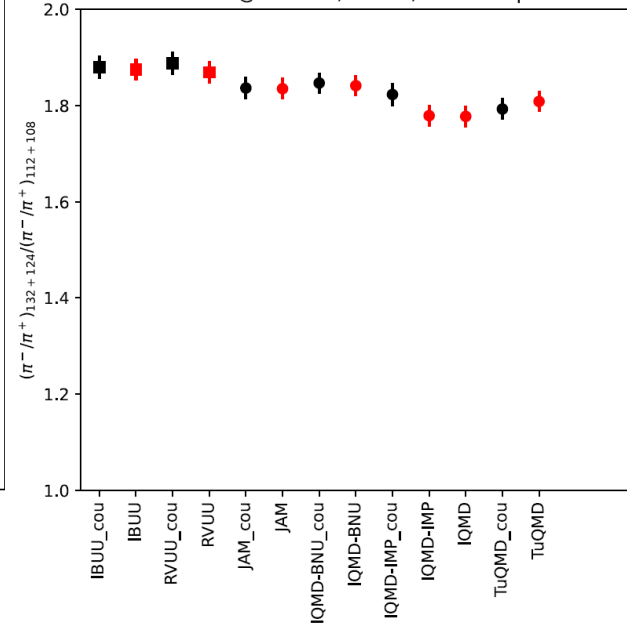
Sn132+Sn124@270MeV,b=4fm,Cascade&pb



Sn+Sn@270MeV,b=4fm,Cascade&nopb



Sn+Sn@270MeV,b=4fm,Cascade&pb



## Coulomb potential:

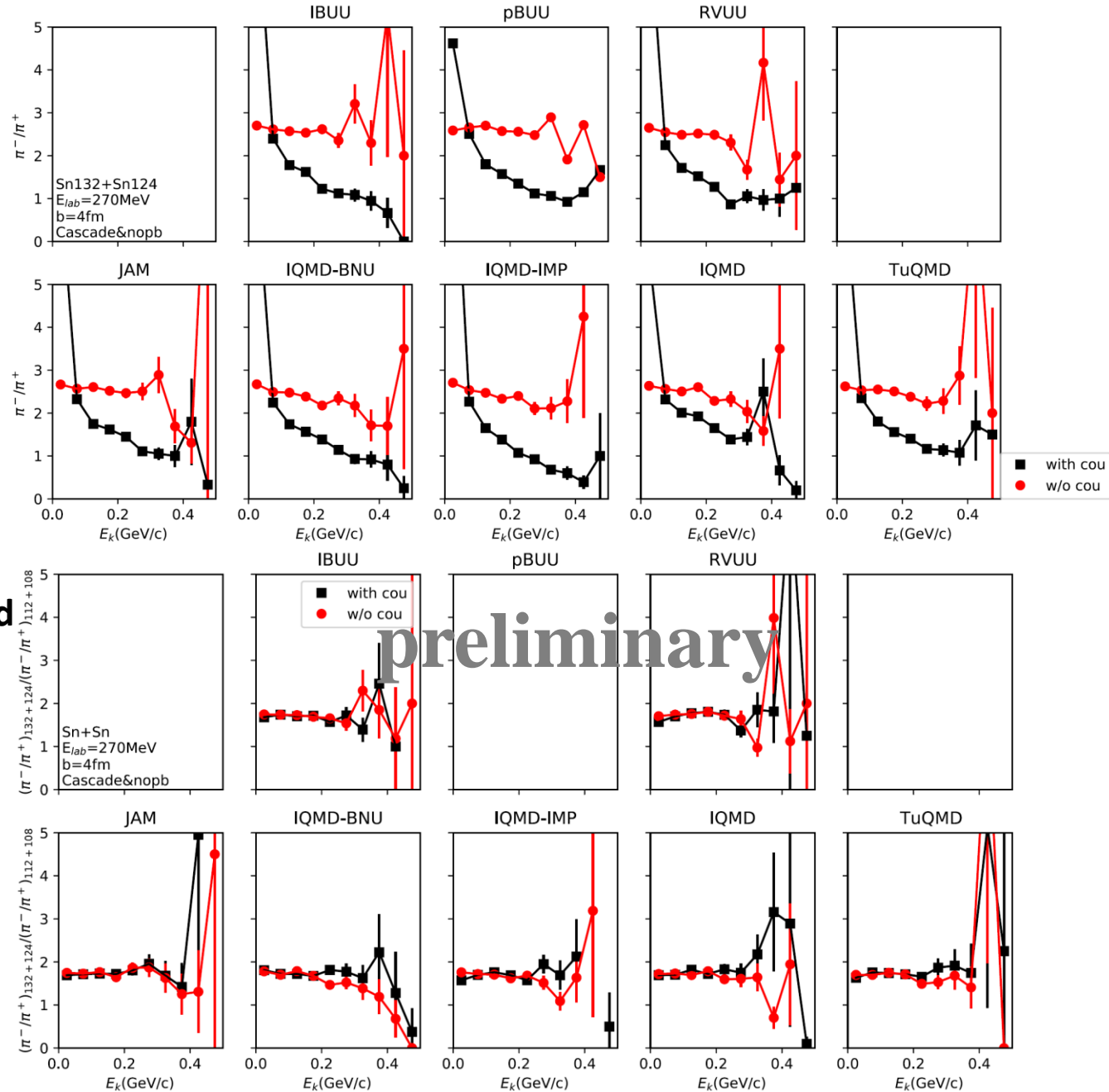
- 1) Suppress slightly the total  $\Delta(\pi)$  production (not shown)
- 2) Affect isospin asymmetry in the high-density phase and thus  $\pi^-/\pi^+$  yield ratio
- 3) Effect 2) largely cancelled by taking double ratio

preliminary

# Effects of Pauli blocking and Coulomb II

## Coulomb potential:

- 1) Repulsive for  $\pi^+$  and attractive for  $\pi^-$ , affect the energy spectrum of  $\pi^-/\pi^+$
- 2) Effect 1) largely cancelled when taking double ratio



# Concluding remarks

Accurate knowledge of nuclear force/EOS extracted from intermediate-energy HIC needs well calibrated transport approaches.

**Strategies** {  
HIC comparison among models, simple physics input  
Box comparison among models, theoretical limits available  
HIC comparison with exp data, realistic physics input

Transport models that (partially) participated in transport model evaluation project

Boltzmann-Uehling-Uhlenbeck approach	Quantum Molecular Dynamics approach
Boltzmann-Langevin One Body (BLOB)	Antisymmetrized Molecular Dynamics (AMD)
BUU by Budapest/Rosendorf group (BUU-BR)	Constrained Molecular Dynamics (CoMD)
BUU by VECC and McGill University (BUU-VM)	Improved QMD at CIAE (ImQMD-CIAE)
Daejeon BUU (DJBUU)	Improved QMD at GXNU (ImQMD-GXNU)
BUU by Giessen group (GiBUU)	Isospin-dependent QMD (IQMD)
Hadron String Dynamics (HSD)	Isospin-dependent QMD at BNU (IQMD-BNU)
Isospin-dependent Boltzmann-Langevin (IBL)	Isospin-dependent QMD at IMP (IQMD-IMP)
Isospin-dependent BUU (IBUU)	Isospin-dependent QMD at SINAP (IQMD-SINAP)
Lattice BUU (LBUU or LHV)	jet AA microscopic (JAM) & sJAM
Pawel's BUU (pBUU)	QMD at Japan Atomic Energy Research Institute (JQMD)
Relativistic BUU (RBUU)	Tübingen QMD(TuQMD)
Relativistic Vlasov-Uehling-Uhlenbeck (RVUU)	Ultra-relativistic QMD (UrQMD)
Simulating Many Accelerated Strongly-interacting Hadron (SMASH)	<b>In progress:</b>
Stochastic Mean-Field (SMF)	<b>Energy conservation with momentum-</b>
BUU based on MF from $\chi$ EFT ( $\chi$ BUU)	<b>dependent potential in a box</b>

# What we learned

- **Initialization:** ground-state distribution - stable
- **Mean-field potential:** size of particles
  - **BUU:** Lattice Hamiltonian method
  - **QMD:** accurate calculation of  $\langle \rho^\gamma \rangle$
- **NN collisions:**
  - **Attempted:**  $\gamma$  factor, remove spurious collisions, reduce higher-order correlations, prefer full-ensemble method, prefer stochastic method
  - **Pauli blocking:**
    - **BUU:** more test particles, effective temperature fit
    - **QMD:** antisymmetrized wave function?
  - **Inelastic:** prefer time-step free or full-ensemble method, reduce higher-order correlations, randomize collision order list
  - **Coulomb(?):** Poisson equation, independent of distance cut, TP size, cell size, boundary condition, ...

$\Delta t \rightarrow 0$

# Acknowledgement

Lie-Wen Chen, Maria Colonna, Dan Cozma,  
Pawel Danielewicz, Che Ming Ko, Akira Ono, Betty Tsang,  
Yong-Jia Wang, Hermann Wolter, and Ying-Xun Zhang  
and all code correspondents

TMEP Collaboration

**Thank you!**

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