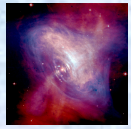


Modelling inhomogeneous matter (at finite temperature) in compact stars

Anthea F. Fantina

**in collaboration with: F. Gulminelli, H. Dinh Thi, T. Carreau (LPC Caen)
N. Chamel, S. De Ridder (IAA, ULB, Belgium)
J. M. Pearson (Univ. Montreal, Canada)**

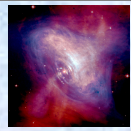


Outline

- ❖ Introduction and motivation
 - Inhomogeneous matter in astrophysical (compact-star) context
 - One-component (OCP) vs multi-component (MCP) plasma

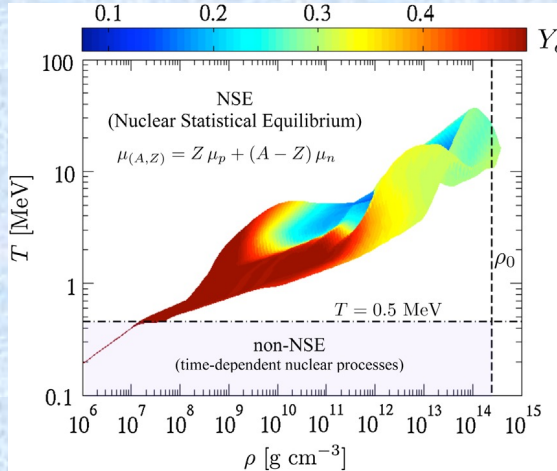
- ❖ Some (selected) examples : recent results
 - Impurities in (proto-)neutron stars ((P)NS)
 - Pasta phase (at $T=0$ and finite T) and impact of empirical parameters

- ❖ Conclusions & outlooks



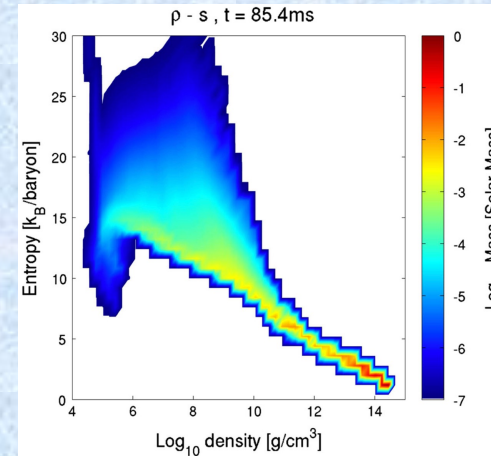
Inhomogeneous matter (at finite T)

Core-collapse supernovae (SNe)



Simulation T. Fischer

Binary neutron-star (NS) mergers

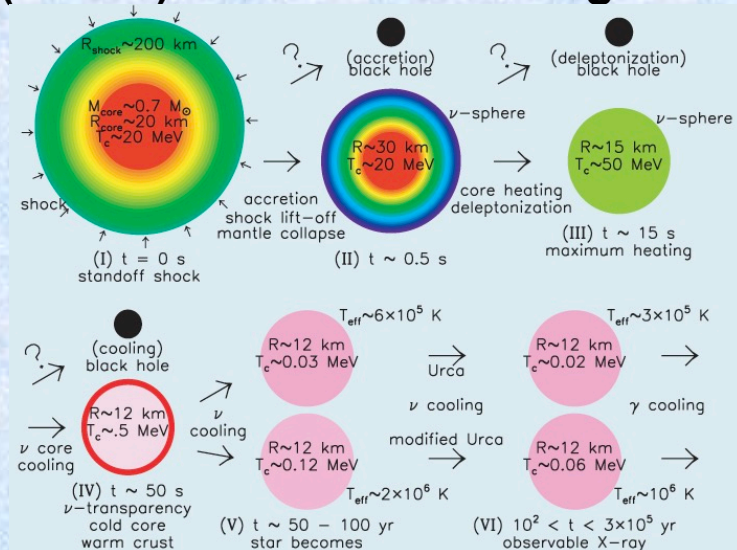


Simulation A. Perego

Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

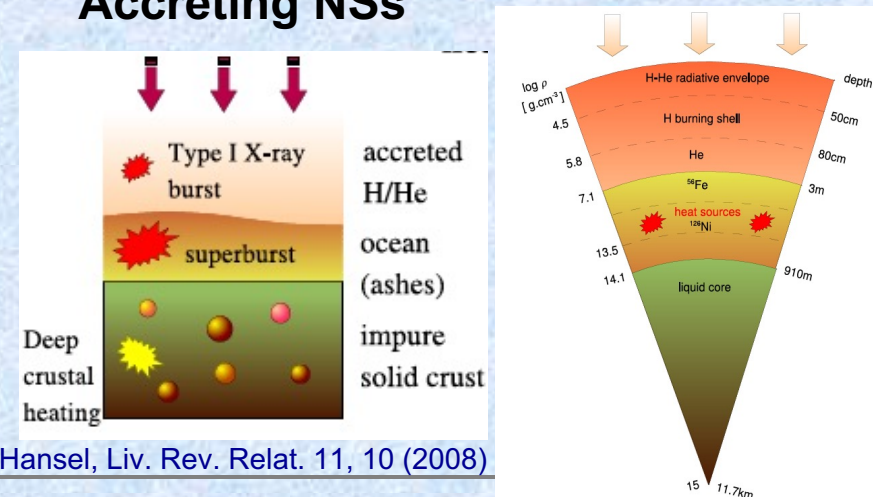
Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

(Proto-) NS crust & cooling



Lattimer & Prakash, Science 304, 536 (2004)

Accreting NSs



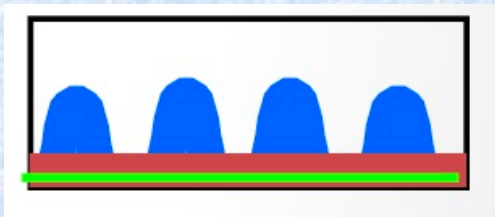
Chamel & Hansel, Liv. Rev. Relat. 11, 10 (2008)

N.B.: also off-(beta-) equilibrium !

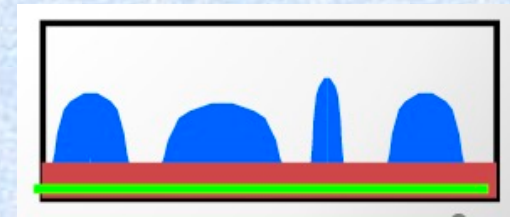


Which approach ?

- Which theoretical framework for clustered matter at finite T ?
 - *ab-initio methods*, but: not affordable for inhomogeneous matter in all range
 - *phenomenological methods* → **Nuclear energy-density functional theory**
- One-component (OCP) single-nucleus (SNA) vs Multi-component (MCP)



→ energy minimisation gives favoured *cluster* (A, Z)



→ energy minimisation gives favoured *cluster distribution*

$$p(A, Z) \propto \exp \left[-(k_B T)^{-1} (F_N + \delta\Omega - \mu_n N - \mu_p Z) \right]$$

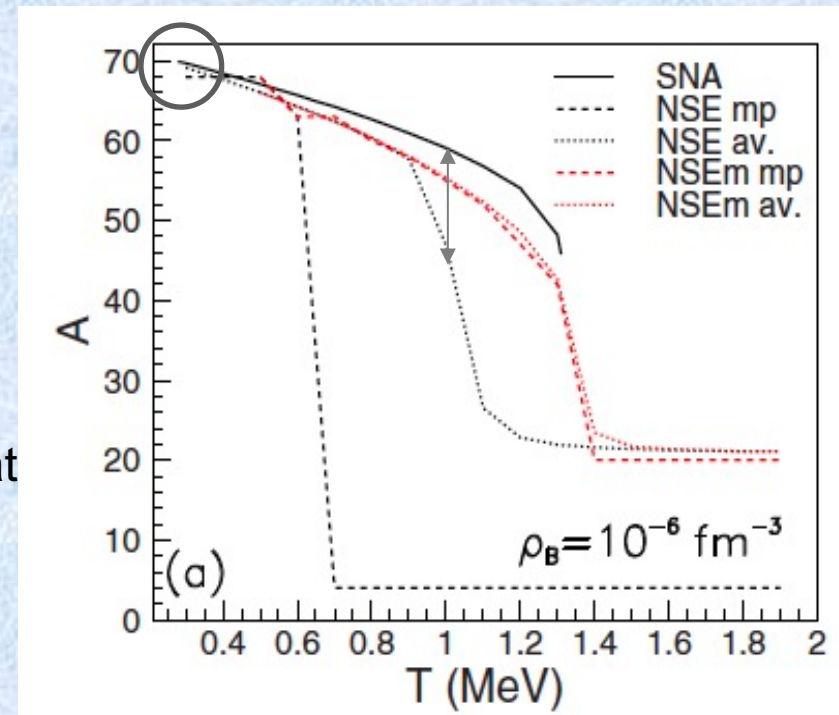
- ✓ OCP OK for thermodynamic quantities and faster computationally
- ✗ but: at finite T more microstates populated → nuclear distributions
- ✗ MCP needed to compute reaction (electron-capture) rates (CCSN, cooling, ...)



Treatment of clusters

→ Need to go beyond the standard Wigner-Seitz (one-component) approximation

- Possible strategies:
 1. Micro calculations (MD, TDHF)
 - computationally expensive (e.g. Seville et al., 2009, 2011; Nandi & Schramm 2018, ...)
 2. **Statistical (NSE) models**
 - **cluster degrees of freedom**
 - more flexible but more difficult to treat beyond mean-field effects (e.g. Gulminelli & Raduta 2015, Grams et al. 2018, Pais et al. 2020, ...)



Gulminelli & Raduta, PRC 92, 055803 (2015)

for a review: Oertel et al., Rev. Mod. Phys. 89, 015007 (2017);
Burgio & Fantina, Astrophys. Space Sci. Lib. 457, 255 (2018)

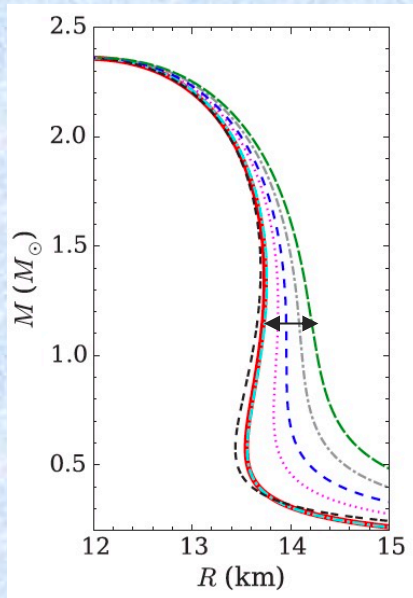


Why a unified treatment ?

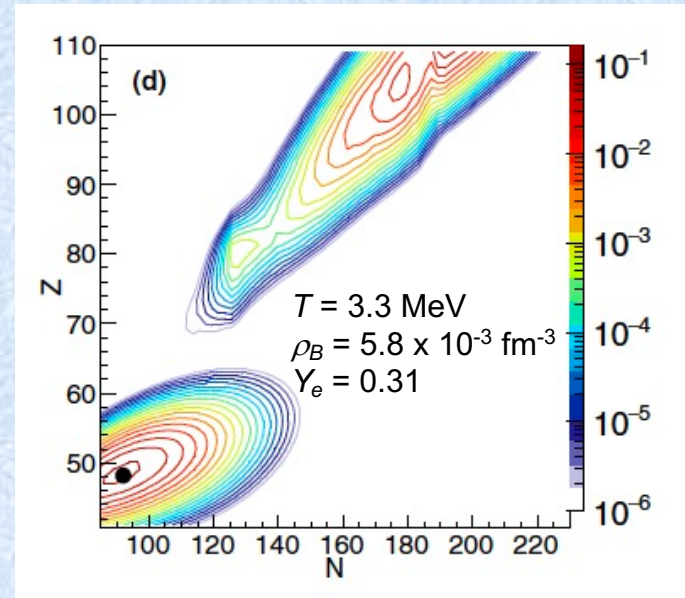
Unified treatment of finite nuclei & infinite matter

→ same nuclear model employed in different regions of star

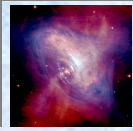
- Challenging because of wide range of density, temperature, isospin asymmetry
- Challenging because different states of matter (cluster, “pasta”, homogeneous matter)
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016); $T = 0$

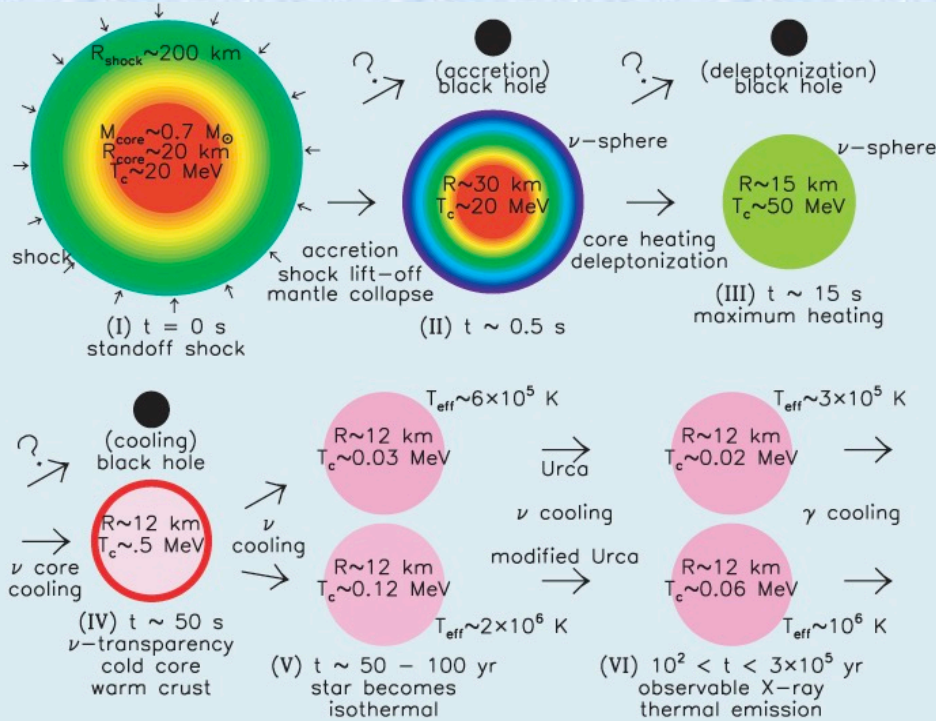


Grams et al., PRC 97, 035807 (2018); CCSN, $T \neq 0$

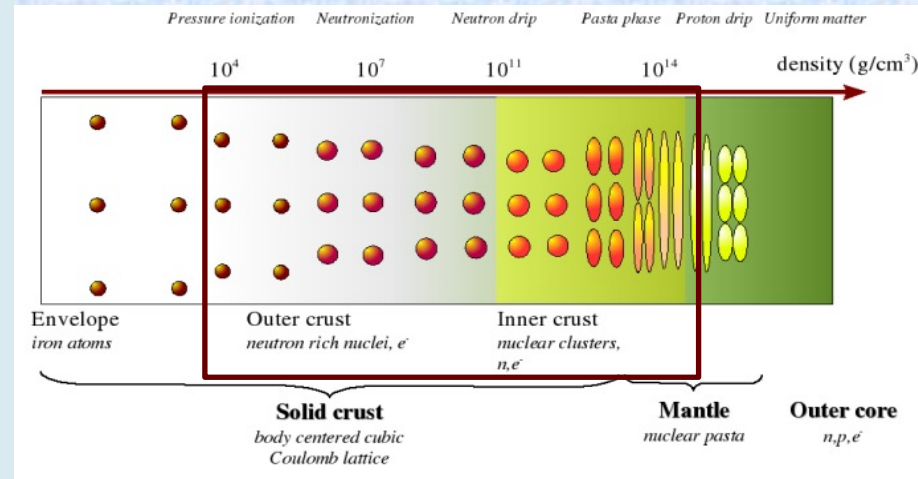


(Proto-)NS crust (isolated NSs)

NS formation from CCSN (here from shock stall)



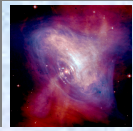
“Cold” (catalysed) NS
($T = 0$ approx. \rightarrow OCP)



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008)

Lattimer & Prakash, Science 304, 536 (2004)

- \rightarrow NS are born hot from core-collapse SN ($T > 1$ MeV)
- \rightarrow ensemble of nuclei expected at formation
- \rightarrow possible existence of “pasta” layer in the crust



Impurities in the (P)NS crust

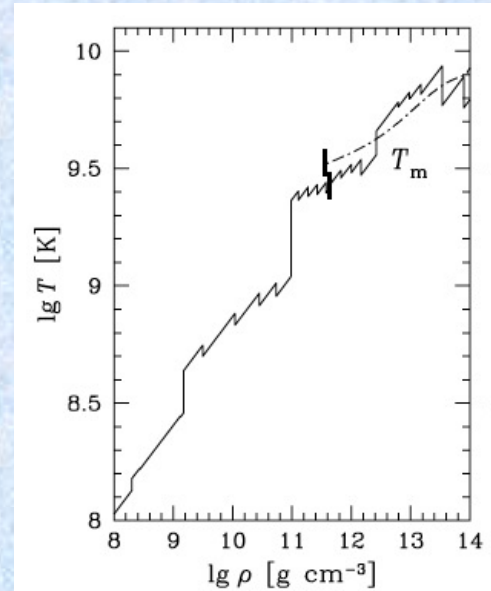
- Crystallization temperature in crust :
 $10^9 < T_m < 10^{10}$ K (~ few 100 keV)

- Static properties (e.g. EoS)
 similar to ground-state cold catalysed matter ($T=0$)
- Dynamic, magneto-rotational, and transport
 properties affected by impurities

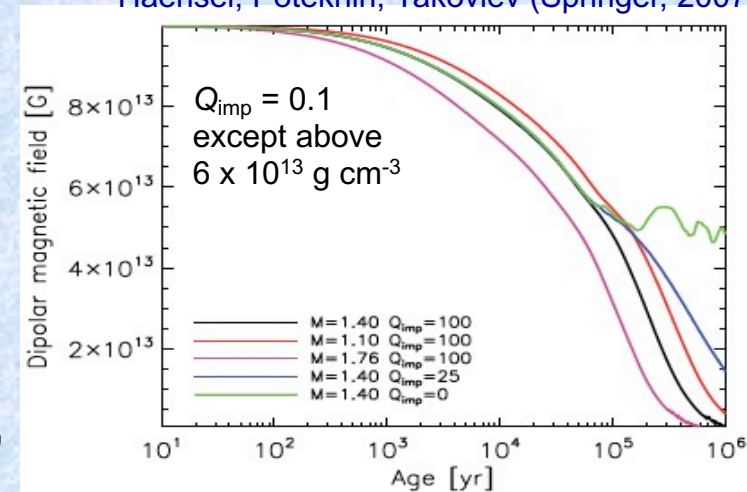
- ❖ Cooling simulations use ground-state EoS and composition
- ❖ “Impurity factor” (free parameter adjusted on observational data)

$$Q_{\text{imp}} = \sum_j p(Z^{(j)}) (Z^{(j)} - \langle Z \rangle)^2$$

- **consistent calculation of nuclear distributions, Q_{imp} (and transport properties)**



Haensel, Potekhin, Yakovlev (Springer, 2007)

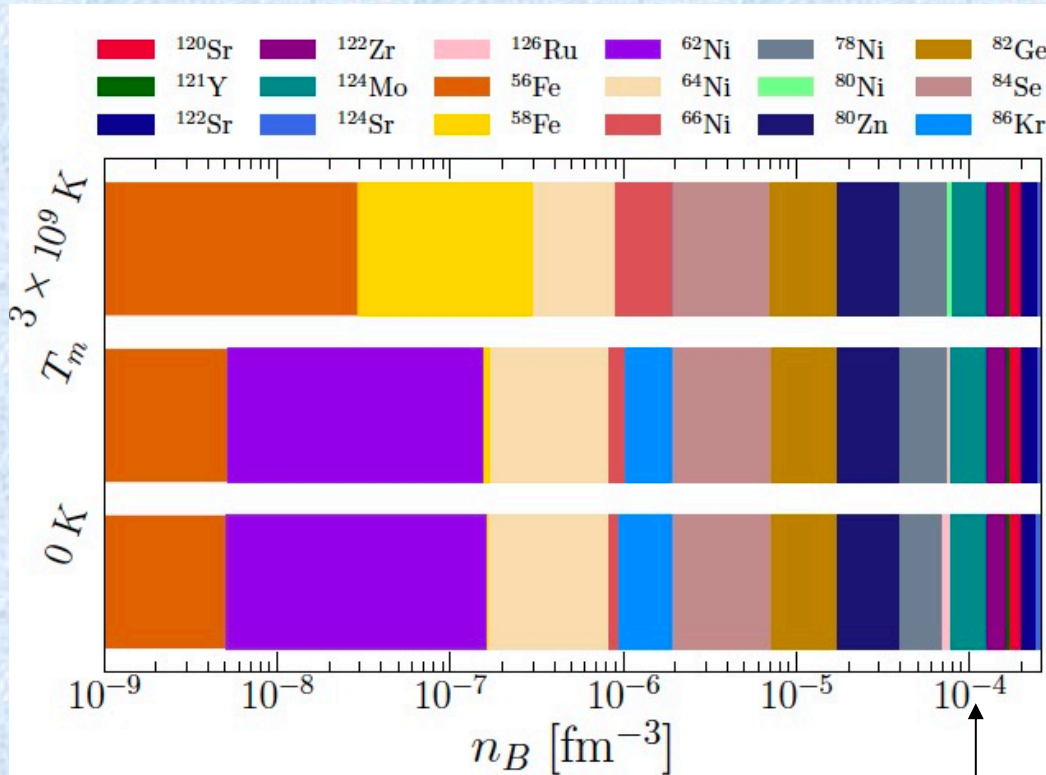


Viganò et al., MNRAS 434, 123 (2013)



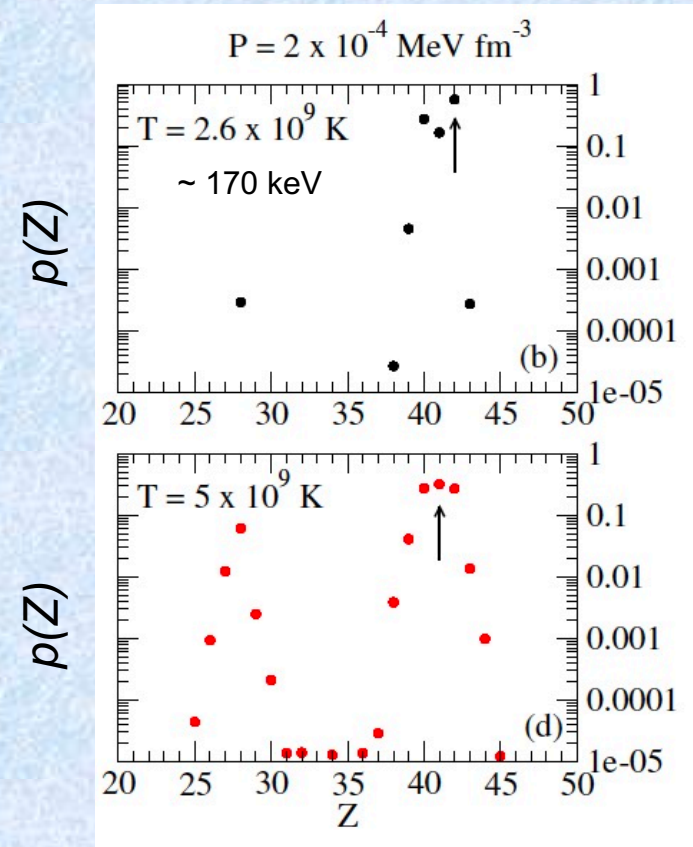
Impurities in the (P)NS crust

Composition of outer crust



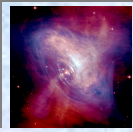
T. Carreau, PhD thesis (2020)

- if composition frozen at $T > T_m$
 - different from ground state !
- OCP approx. very good at lower P (or n_B), T
- larger distribution at higher T → higher Q_{imp}



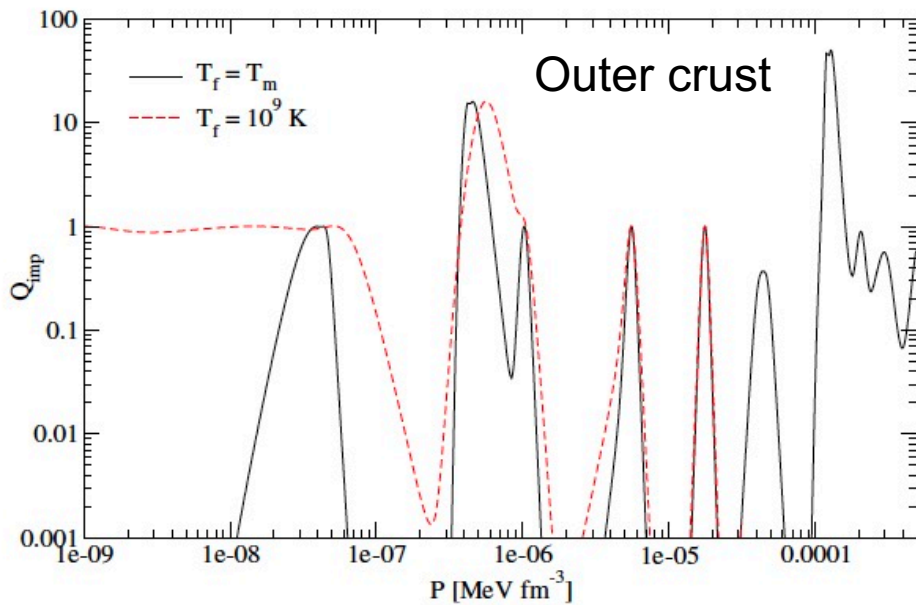
Fantina et al., A&A 633, A149 (2020)

*Exp. masses (AME2016)
+ HFB-24*

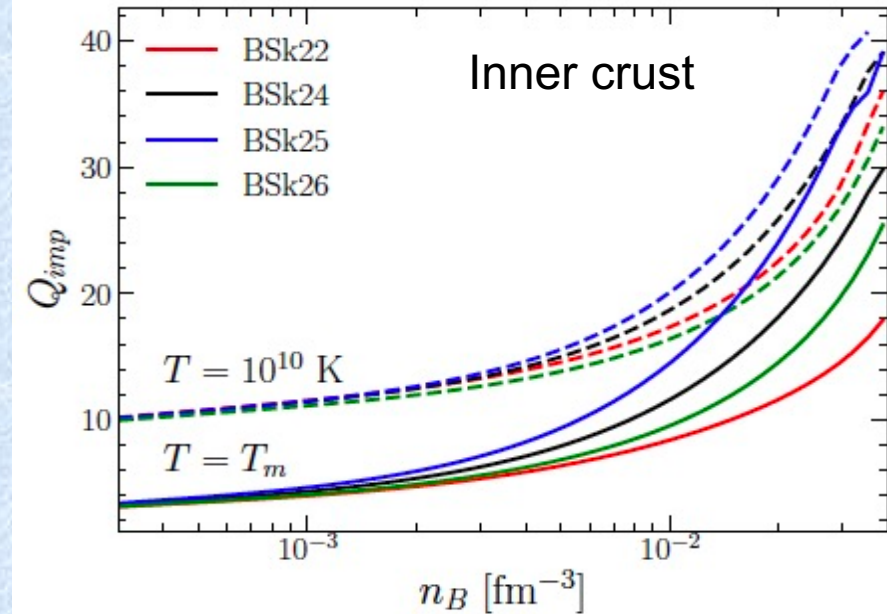


Impurities in the (P)NS crust

- ✓ First self-consistent (microscopic) calculation of $Q_{\text{imp}} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z \rangle)^2$



Fantina et al., A&A 633, A149 (2020)



Carreau et al., A&A 640, A77 (2020)

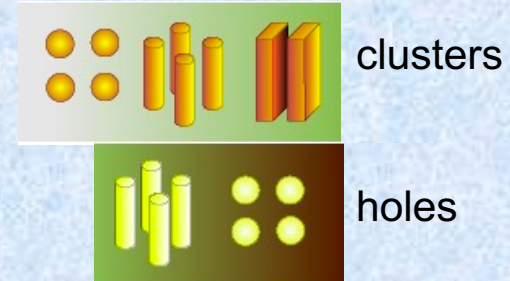
- variation of Q_{imp} → alternation of pure (conductive) and impure (resistive) layers
- larger Q_{imp} at higher pressure/density and T
- consistent calculations of transport coefficients/properties needed
- implementation in numerical simulations of (P)NS cooling



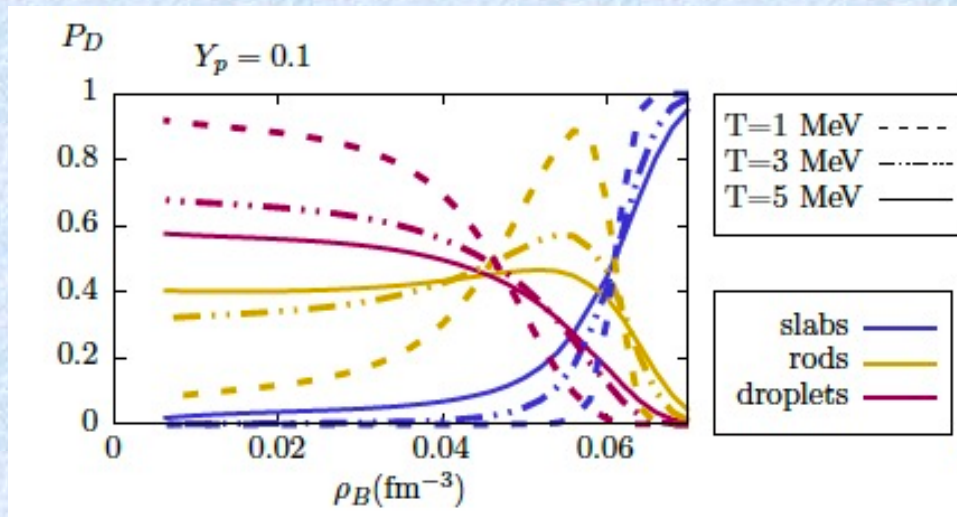
Impurities in the (P)NS crust & pasta

What about non-spherical nuclei/clusters (“pasta” phases) ?

$$p(A, Z) \propto \exp \left[-(k_B T)^{-1} (F_{N,d} + \delta\Omega_d - \mu_n N - \mu_p Z) \right]$$

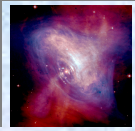


✓ (very recent) calculations at finite T



Pelicer et al., PRC 104, L022801 (2021) - RMF model

- different geometries can co-exist in large fraction of pasta phase
- dependence on spatial direction
- can affect transport properties



Pasta-phase properties ($T=0$): model

CLD approach :

- **Meta-model** approach for nucleons (nucleons only considered): flexible functional
 → expansion in density and asymmetry around n_{sat} and $\delta = 0$

$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left(\left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m$$

$x = (n - n_{\text{sat}})/3n_{\text{sat}}$
 $\delta = (n_n - n_p)/n$

Empirical parameters $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$ } ~ 18 param.

- + surface and Coulomb term → surface parameters ($\sigma_0, \sigma_{0,c}, \dots$)
 → dimensionality of pasta only enters here

Bayesian analysis :

$$p_{\text{post}}(\vec{X}) = \mathcal{N} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X}) e^{-\chi^2(\vec{X})/2} p_{\text{prior}}(\vec{X})$$

Low-Density filters
 → ab-initio (EFT)

(Drischler et al, PRC 93,
 054316 (2016))

High-Density filters
 → causality, stability,
 $M_{\text{NS,max}}, e_{\text{sym}} > 0$

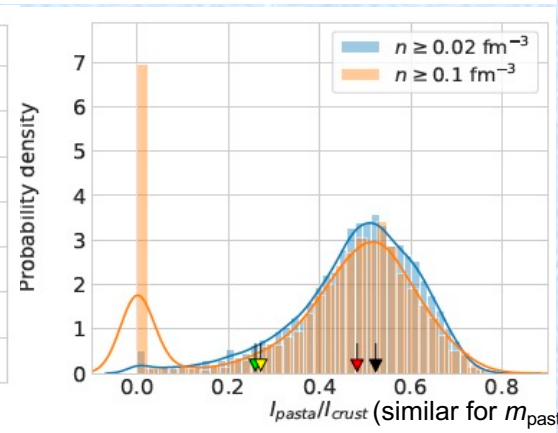
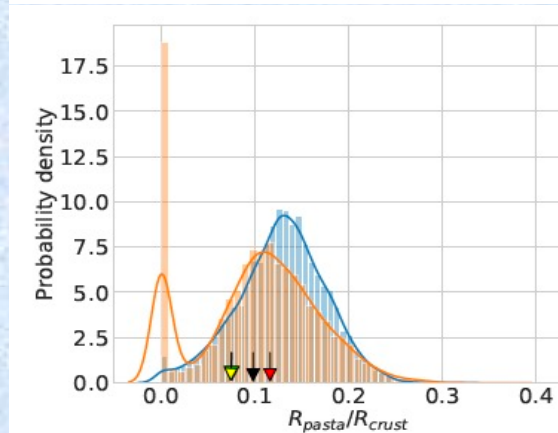
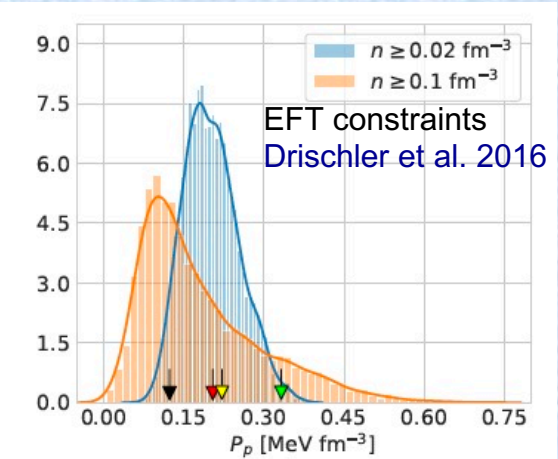
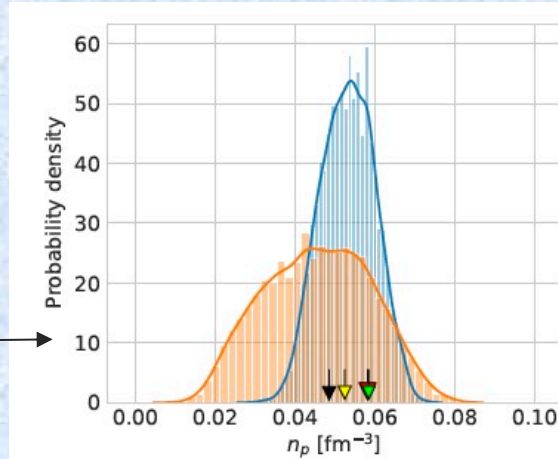
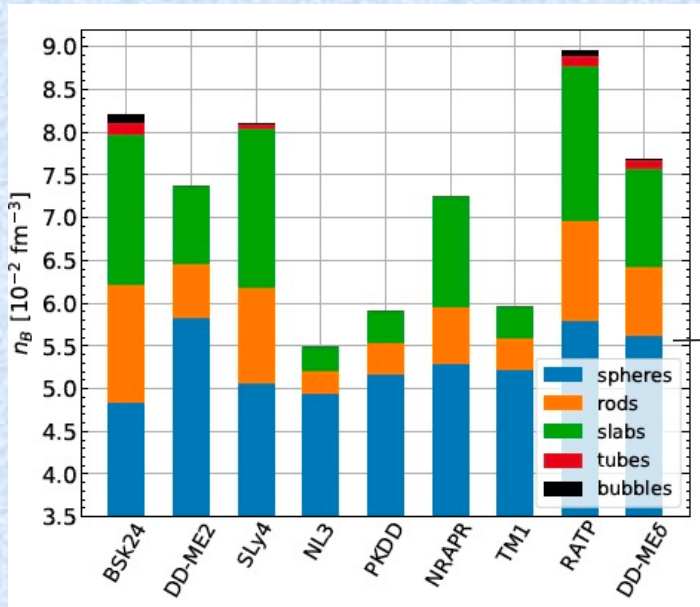
nuclear masses
 (AME2016)
 → surf. param.

flat non-informative prior
 → span large parameter space

see also Margueron et al., PRC 97, 025805 (2018), Carreau et al, EPJA 55, 188 (2019),
 Dinh Thi et al., A&A accepted (2021)



Pasta-phase properties ($T = 0$)

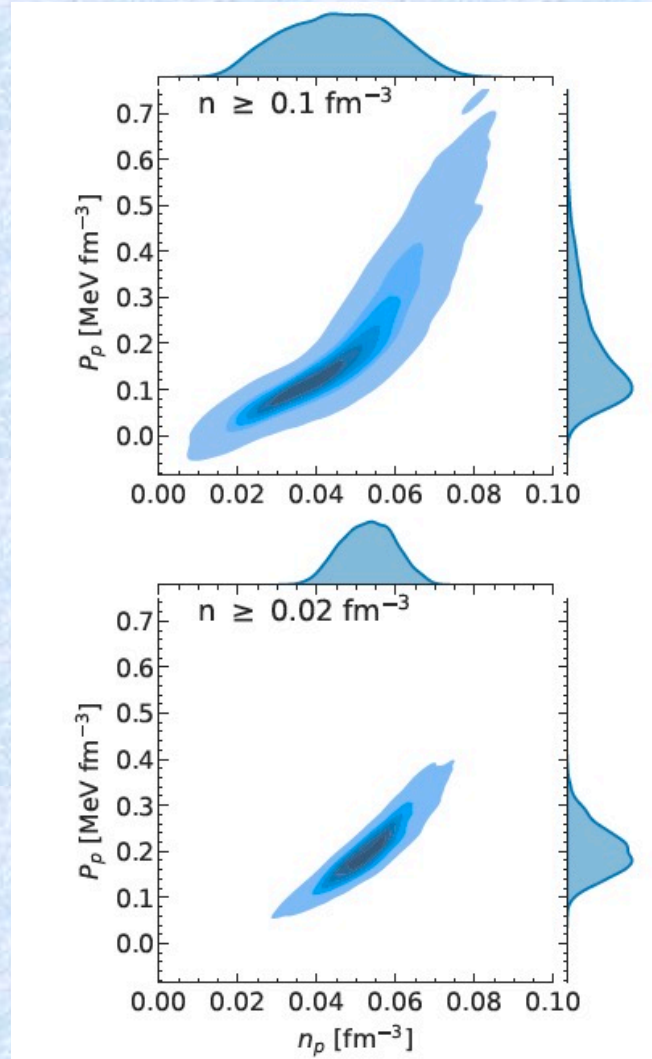
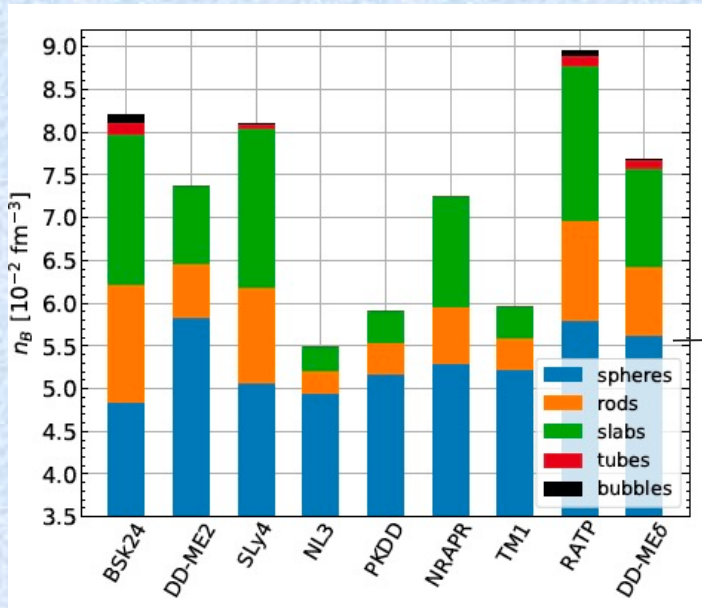


- Pasta predicted (robustly) but model dependences
- $I_{\text{pasta}} \sim 0.5 I_{\text{crust}}$; $m_{\text{pasta}} \sim 0.5 m_{\text{crust}}$
 $R_{\text{pasta}} \sim 0.1 R_{\text{crust}}$
- Importance of (low-density) nuclear physics constraints

Dinh Thi et al., A&A accepted (2021)
<https://doi.org/10.1051/0004-6361/202141192>



Pasta-phase properties ($T = 0$)



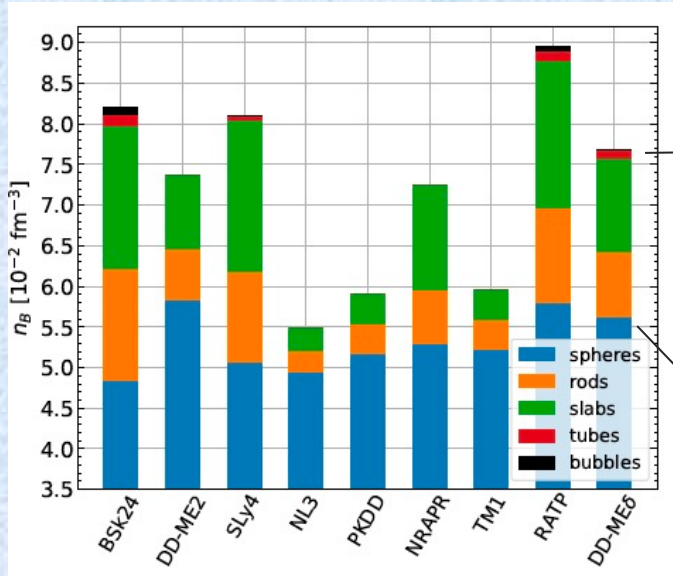
EFT constraints
 Drischler et al. 2016

Dinh Thi et al.,
 EPJA submitted (2021)

- Pasta predicted (robustly) but model dependences
- $I_{\text{pasta}} \sim 0.5 I_{\text{crust}}$; $m_{\text{pasta}} \sim 0.5 m_{\text{crust}}$
 $R_{\text{pasta}} \sim 0.1 R_{\text{crust}}$
- Importance of (low-density) nuclear physics constraints



Pasta-phase properties: correlations



		n_{cc}														
LD+HD	$n \geq 0.02 \text{ fm}^{-3}$	-0.04	-0.07	0.11	-0.05	-0.02	-0.30	-0.57	-0.15	0.45	-0.15	0.05	0.52	-0.15	-0.04	0.51
LD+HD	$n \geq 0.1 \text{ fm}^{-3}$	-0.06	-0.06	0.33	-0.46	0.17	-0.15	-0.29	-0.10	0.39	-0.16	0.06	0.34	-0.11	-0.08	0.33
Prior		0.14	0.09	0.13	-0.18	0.02	0.08	-0.56	0.11	0.20	-0.05	-0.17	0.07	0.29	0.18	0.18
		E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p

		n_p														
LD+HD	$n \geq 0.02 \text{ fm}^{-3}$	-0.86	0.20	0.32	-0.27	0.02	0.32	-0.20	-0.25	0.22	-0.00	0.87	0.08	-0.73	-0.84	-0.09
LD+HD	$n \geq 0.1 \text{ fm}^{-3}$	-0.44	0.03	0.39	-0.42	0.14	0.18	-0.19	-0.40	0.45	-0.14	0.44	0.02	-0.36	-0.47	-0.04
Prior		-0.28	0.01	0.09	-0.11	0.03	-0.04	0.06	-0.49	0.44	-0.06	0.29	0.06	-0.26	-0.22	-0.04
		E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p

		$I_{\text{pasta}}/I_{\text{crust}}$														
LD+HD	$n \geq 0.02 \text{ fm}^{-3}$	0.59	-0.21	-0.17	0.12	-0.02	-0.43	-0.22	0.09	0.15	-0.09	0.59	0.48	0.42	0.59	0.66
LD+HD	$n \geq 0.1 \text{ fm}^{-3}$	0.27	-0.10	0.09	-0.30	0.15	-0.26	-0.05	0.19	0.07	-0.06	0.28	0.42	0.15	0.26	0.51
Prior		0.37	0.01	0.04	0.06	0.07	0.20	-0.05	0.35	0.29	0.08	0.40	-0.01	0.45	0.36	0.24
		E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p



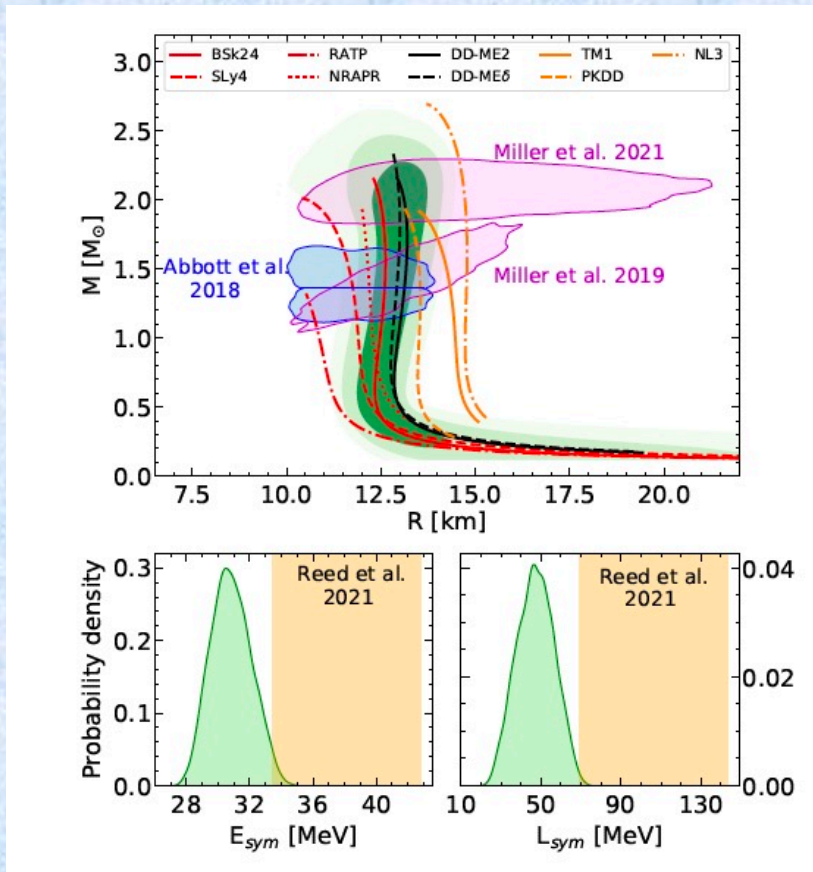
Dinh Thi et al., A&A accepted (2021);
<https://doi.org/10.1051/0004-6361/202141192>

Dinh Thi et al., EPJA submitted (2021)

→ Importance of nuclear physics constraints (*LD*) and parameters (both *bulk* and *surface*) on pasta observables

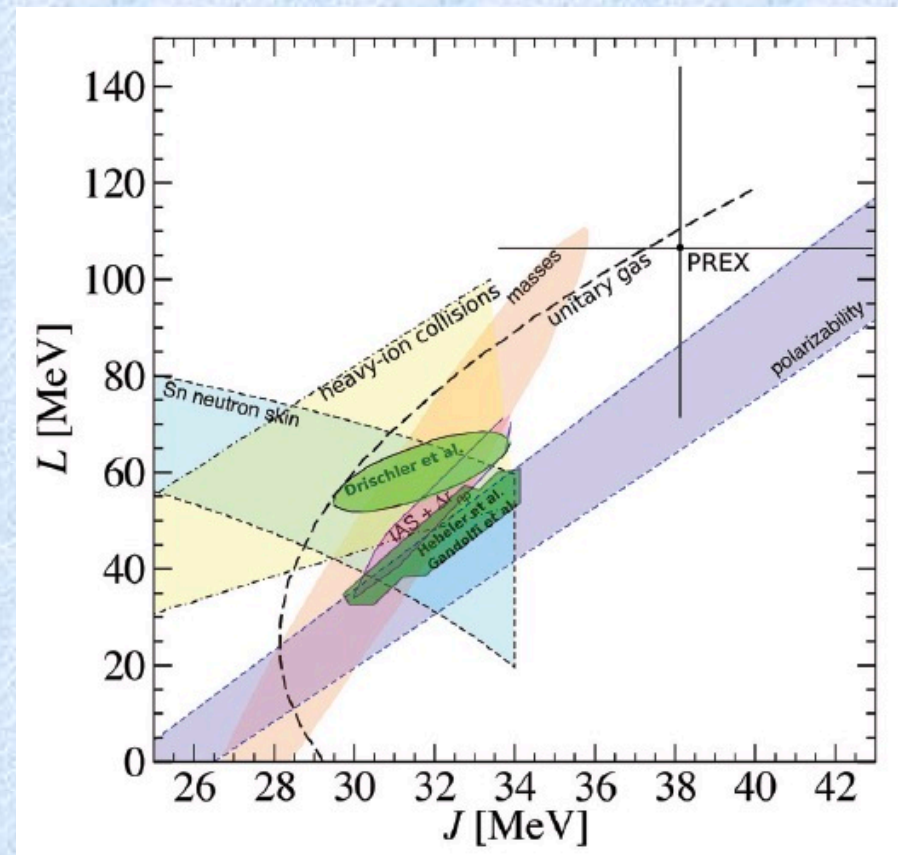


Observations and symmetry parameters



Dinh Thi et al., A&A accepted (2021);
<https://doi.org/10.1051/0004-6361/202141192>

→ posterior compatible with observations
but: some popular models are not !



Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021)
 (see also Burgio & Fantina, ASSL 457, 255 (2018);
 Reed et al., PRL 126, 172503 (2021) and refs. therein)

→ “tension” in parameter estimation



Conclusions & outlooks

- ❖ Modelling of clustered matter (at finite temperature) challenging but important for description of different compact-object properties / observables
 - ❖ Importance of *unified* and *consistent* treatment
 - ❖ Importance nuclear parameters (both bulk & surface)
 - *symmetry parameters for crust-core transition (and pasta observables)*
 - low-density part of the EoS
-
- Need of constraints on nuclear parameters (both isoscalar and isovector) and proper treatment of low-density EoS to reliably model the crust
 - Calculations of transport properties
 - Need of *consistent* implementation of **microphysics inputs** in **macroscopic simulations** and/or **analysis of observational data**



Thank you