Equation of state constraints from chiral effective field theory and astrophysics

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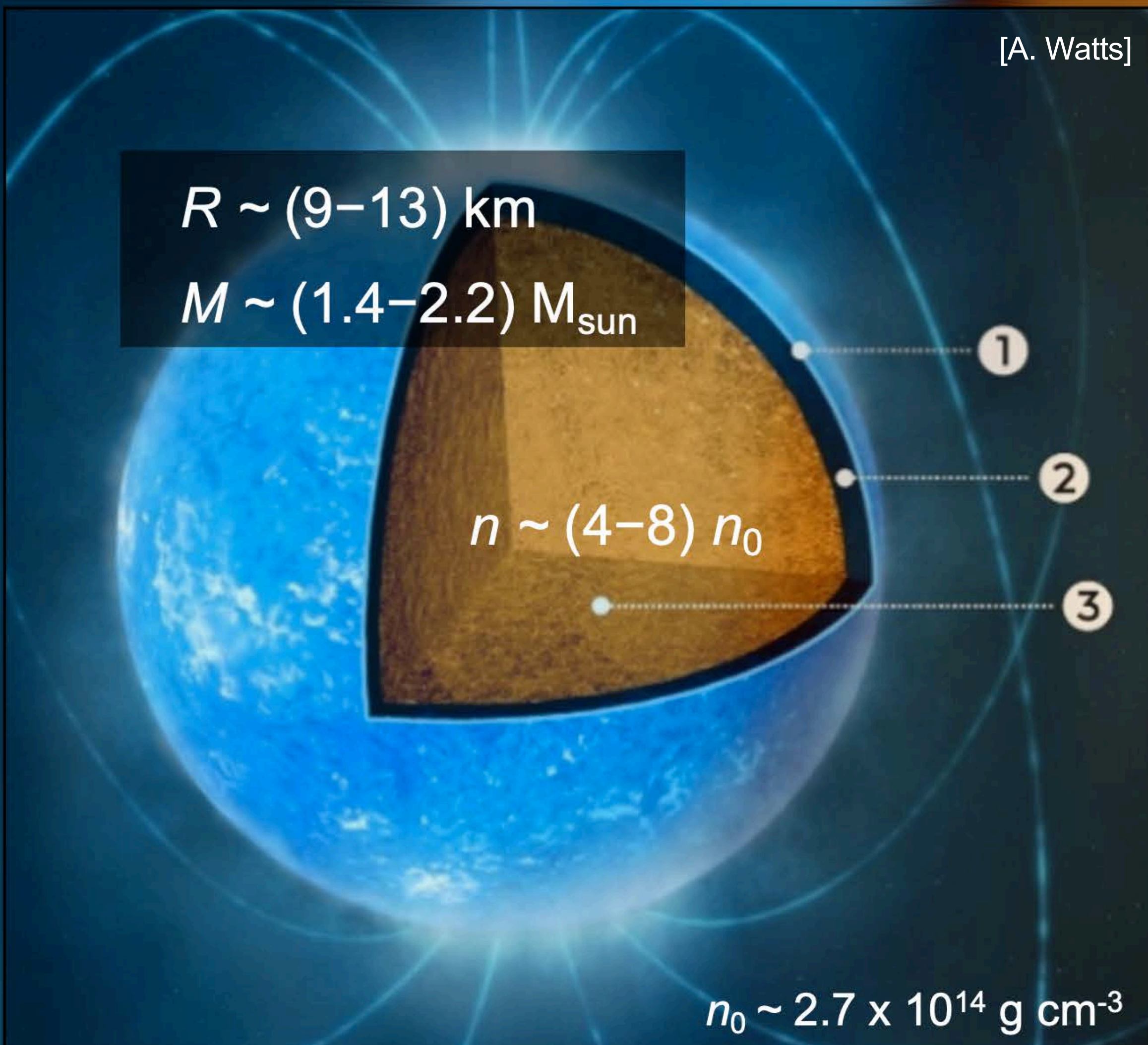
Christian Drischler

September 23, 2021 | International Symposium on Nuclear Symmetry Energy



Keywords:

+ Chiral EFT
+ infinite nuclear matter
+ Bayesian UQ
+ symmetry energy
+ N^3LO NN + 3N forces
+ nuclear saturation
+ …

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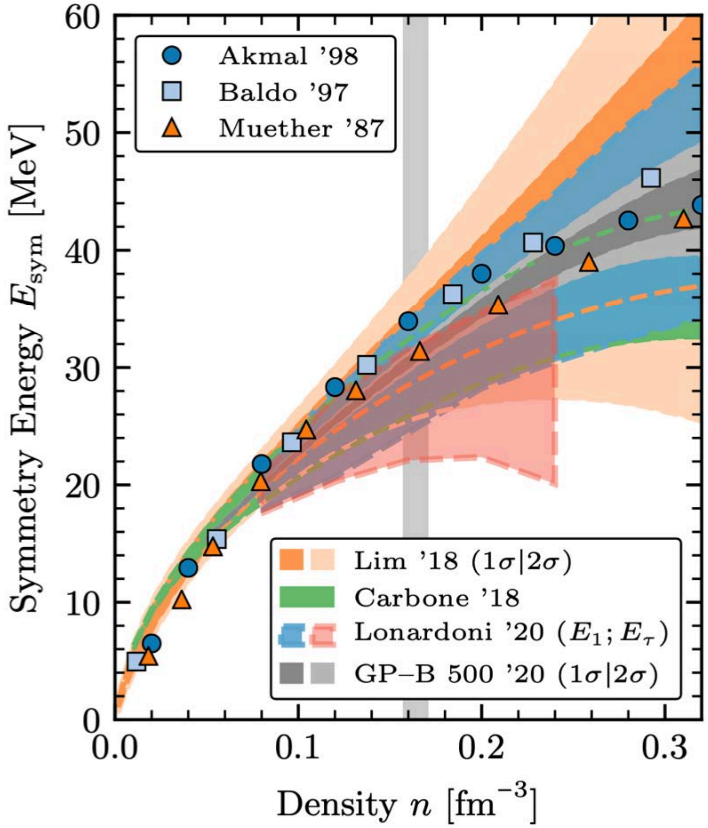
Recent review article

CD, Holt, and Wellenhofer, ARNPS **71**, 403



Chiral Effective Field Theory and the High-Density Nuclear Equation of State

C. Drischler,[1,2,3] J. W. Holt,[4] and C. Wellenhofer,[5,6]

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invited contribution to Annu. Rev. Nucl. Part. Sci. **71**, 403
see also: Lattimer, Annu. Rev. Nucl. Part. Sci. **71**, 433

See also:
Burgio, Schulze, Vidaña & Wei,
PPNP **120**, 103879

Annu. Rev. Nucl. Part. Sci. 2021. 71:1–30

This article's doi:
10.1146/annurev-nucl-102419-041903

Annu. Rev. Nucl. Part. Sci. in press.

Keywords

chiral effective field theory, nuclear matter, neutron stars, many-body perturbation theory, bayesian uncertainty quantification

Abstract

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Multi-messenger astronomy

ligo.caltech.edu

+ Virgo
+ GEO600
+ KAGRA
+ …

Binary neutron star merger GW170817

$$R_{1.4} \lesssim 13.6\,\mathrm{km}$$
$$M_{\max} \lesssim 2.3\,\mathrm{M}_\odot$$

e.g., see:
Margalit, Metzger, APJ **850**, 19
Rezzolla *et. al.*, APJ **852**, L25
De *et al.*, PRL **121**, 091102
Lim and Holt, EPJ A **55**, 209
Capano *et al.*, NA **4**, 625
Al-Mamun *et al.*, PRL **126**, 061101
…

What is the secondary object in GW190425 and GW190814?

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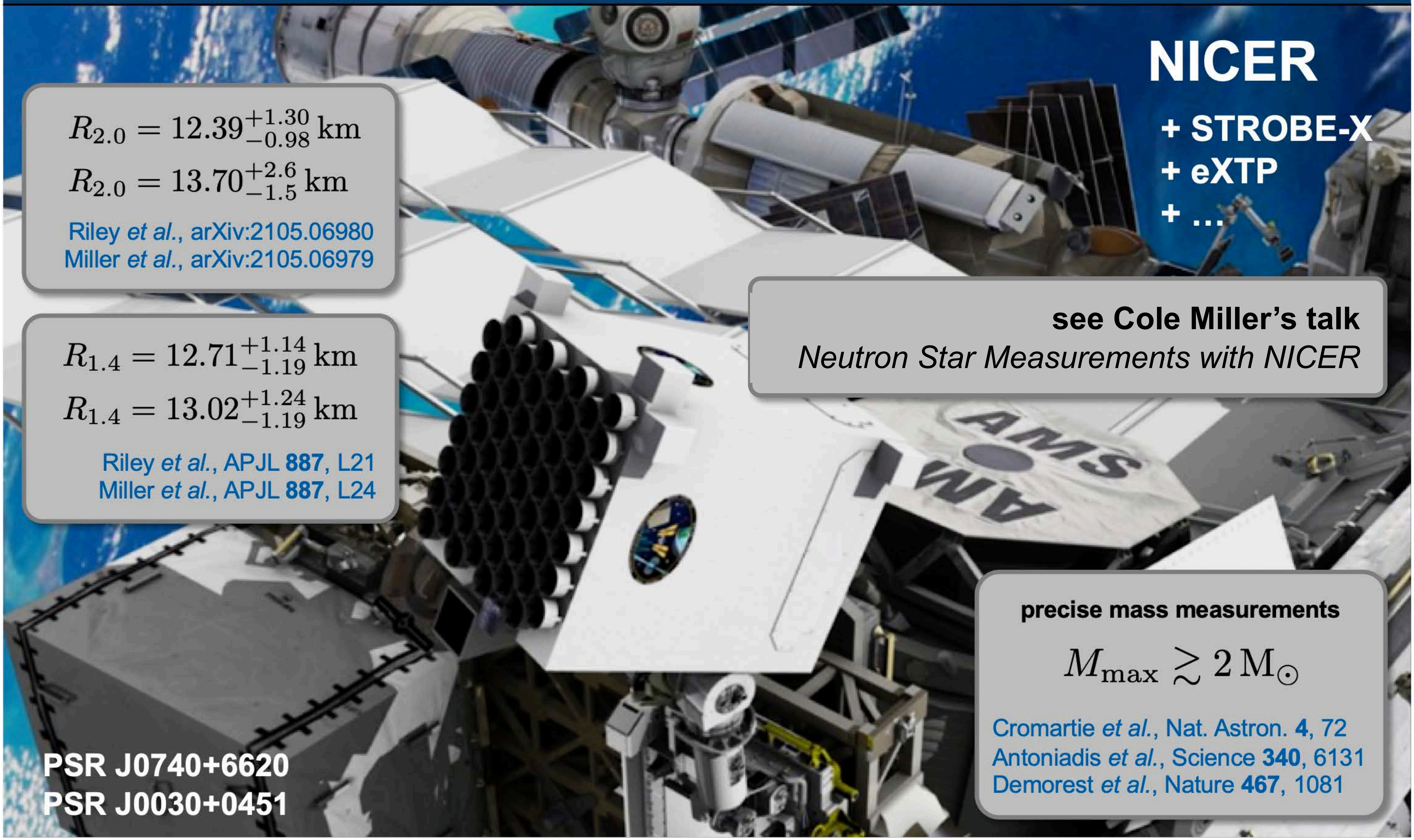
Recent neutron star observations

NASA



NICER

+ STROBE-X
+ eXTP
+ ...

$$R_{2.0} = 12.39^{+1.30}_{-0.98}\,\mathrm{km}$$

$$R_{2.0} = 13.70^{+2.6}_{-1.5}\,\mathrm{km}$$

Riley *et al.*, arXiv:2105.06980
Miller *et al.*, arXiv:2105.06979

$$R_{1.4} = 12.71^{+1.14}_{-1.19}\,\mathrm{km}$$

$$R_{1.4} = 13.02^{+1.24}_{-1.19}\,\mathrm{km}$$

Riley *et al.*, APJL **887**, L21
Miller *et al.*, APJL **887**, L24

see Cole Miller's talk
Neutron Star Measurements with NICER

precise mass measurements

$$M_{\max} \gtrsim 2\,\mathrm{M}_{\odot}$$

Cromartie *et al.*, Nat. Astron. **4**, 72
Antoniadis *et al.*, Science **340**, 6131
Demorest *et al.*, Nature **467**, 1081

PSR J0740+6620
PSR J0030+0451

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Nuclear experiments: neutron-rich nuclei, collisions…

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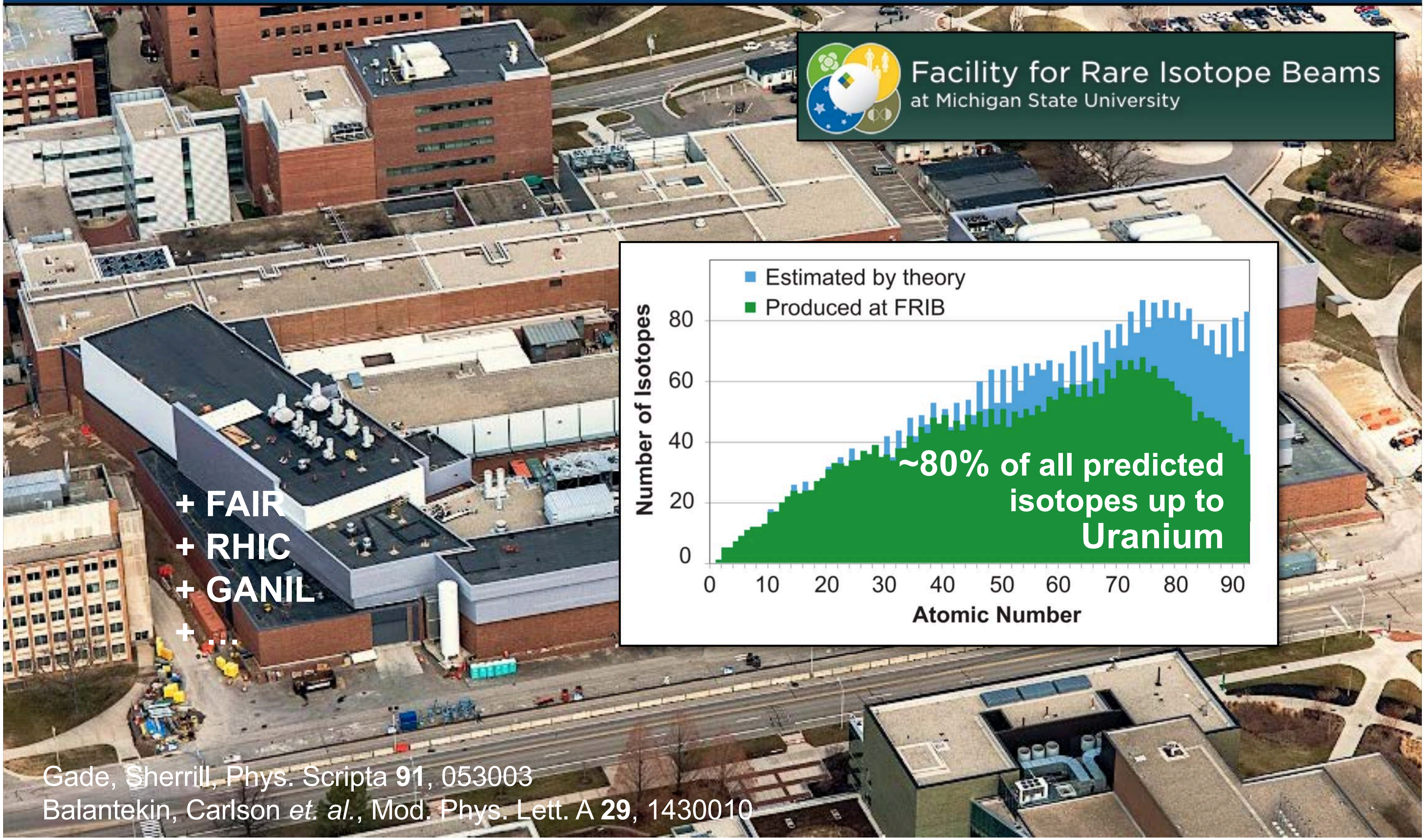
frib.msu.edu



Gade, Sherrill, Phys. Scripta **91**, 053003

Balantekin, Carlson *et. al.*, Mod. Phys. Lett. A **29**, 1430010

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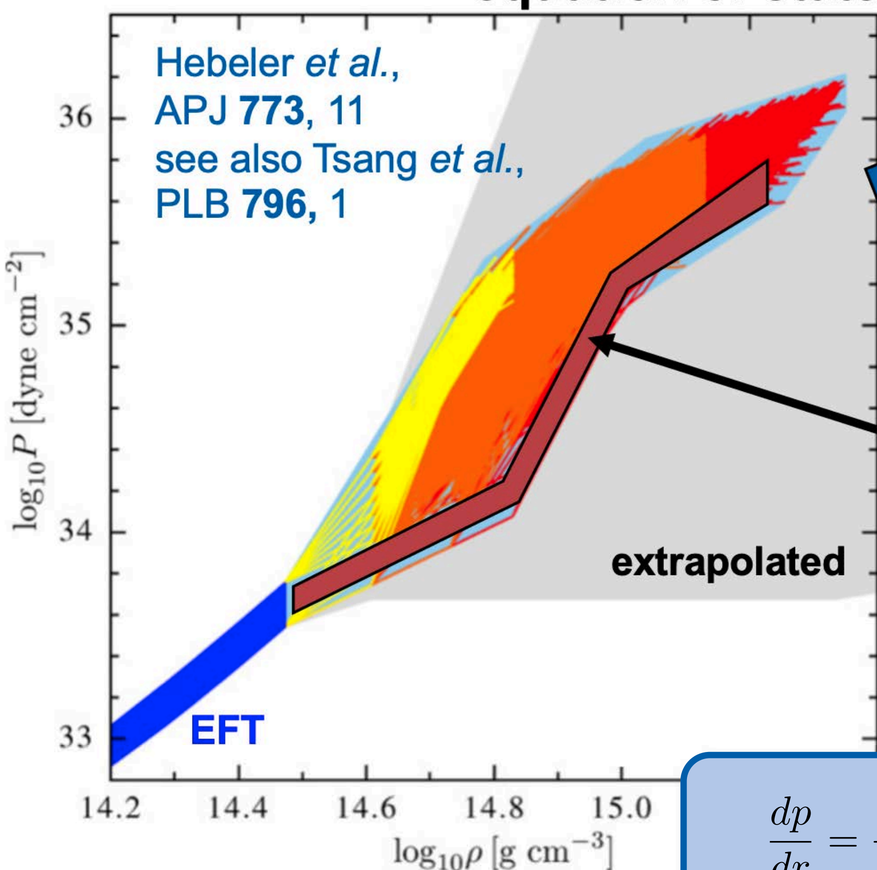
Direct correspondence: *M*–*R* relation and EOS

equation of state

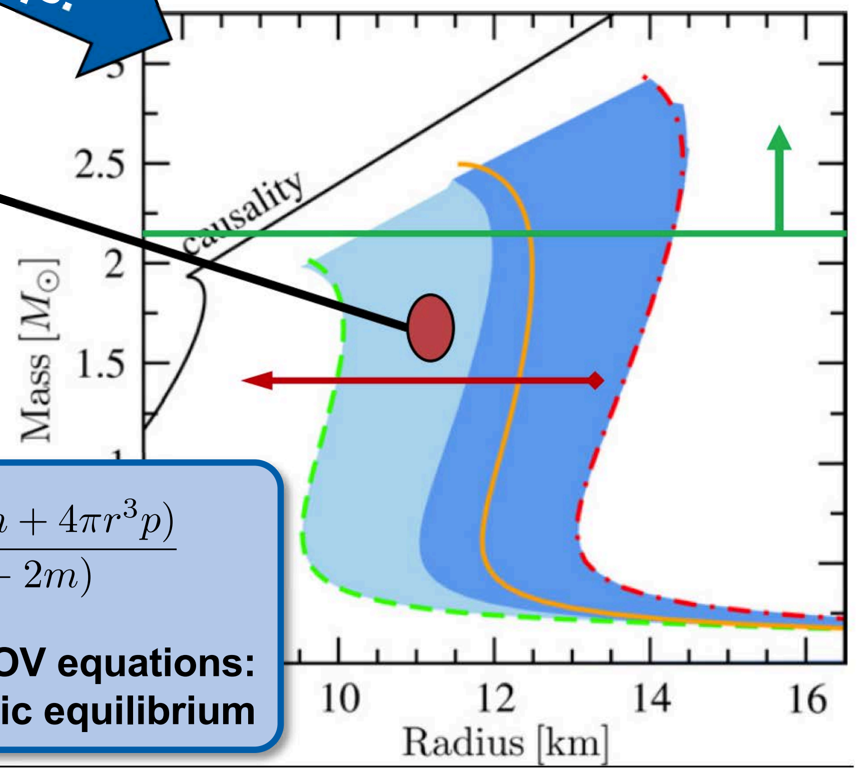


TOV eqs.

constraints from

- \+ **mass measurements**
- \+ radius measurements
- \+ **GW170817**, …
- \+ experiment (*e.g.*, skins)

mass–radius relation



$$\frac{dp}{dr} = -\frac{(\varepsilon + p)(m + 4\pi r^3 p)}{r(r - 2m)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

TOV equations: hydrostatic equilibrium

for recent reviews, see:
Hebeler, Holt *et al.*, ARNPS **65**, 457
Tews, Front. in Phys. **8**,153
CD, Holt, and Wellenhofer, ARNPS **71**, 403

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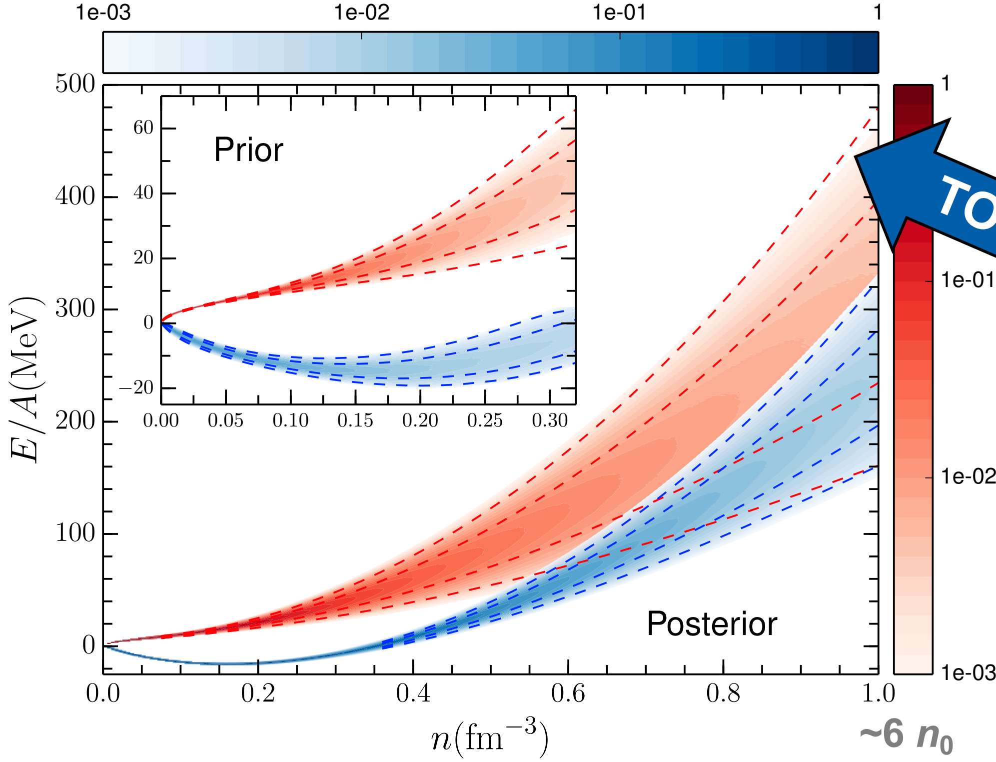
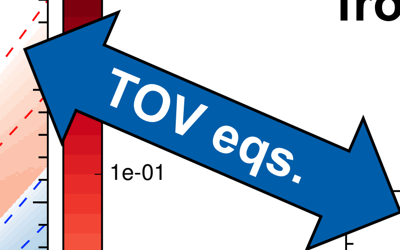
Bayesian modeling of the EOS

Lim & Holt, PRL **121**, 062701

equation of state





constraints from
- \+ mass measurements
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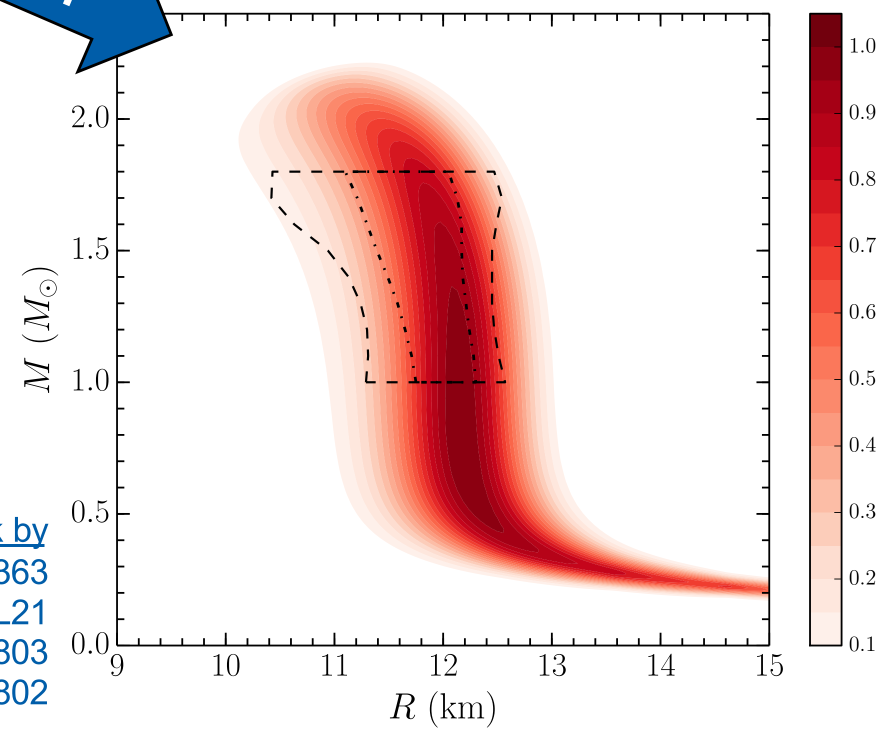
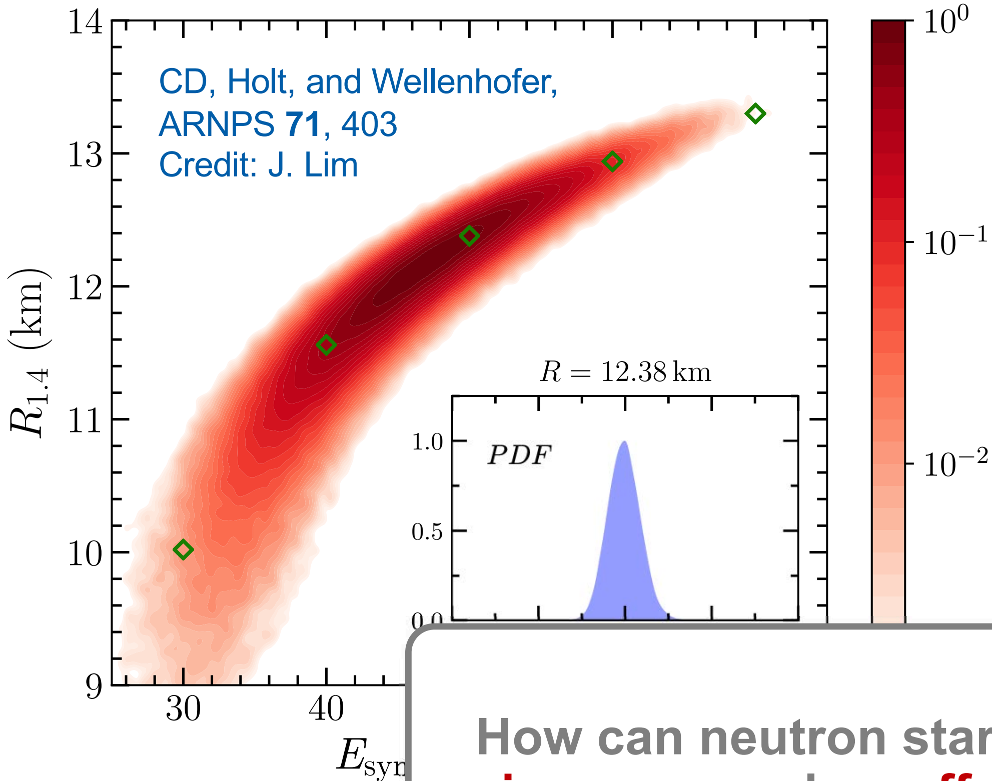
mass–radius relation

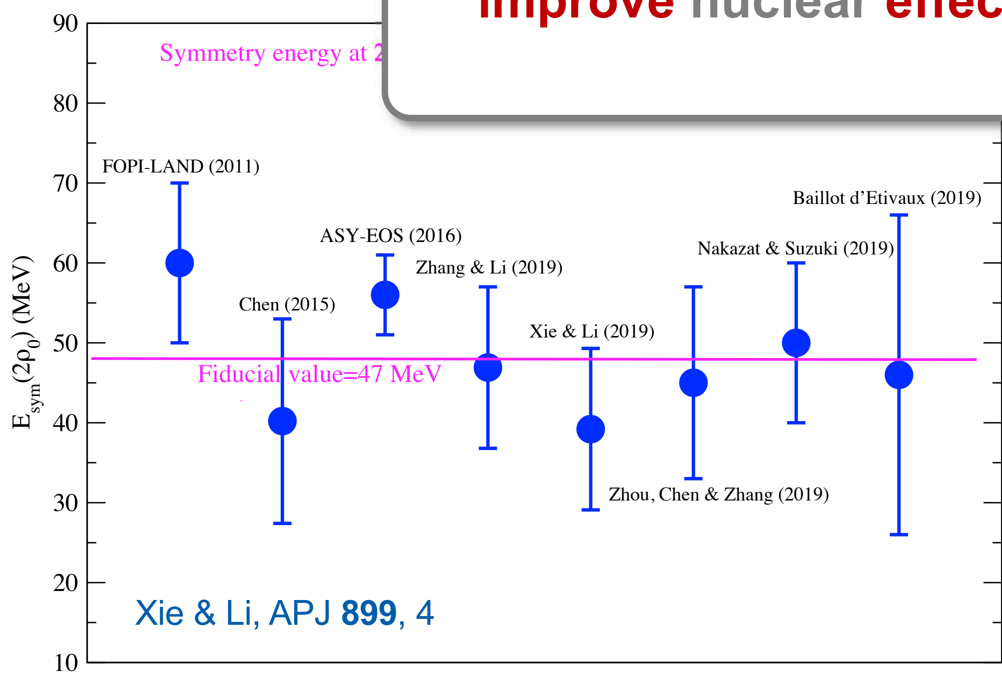


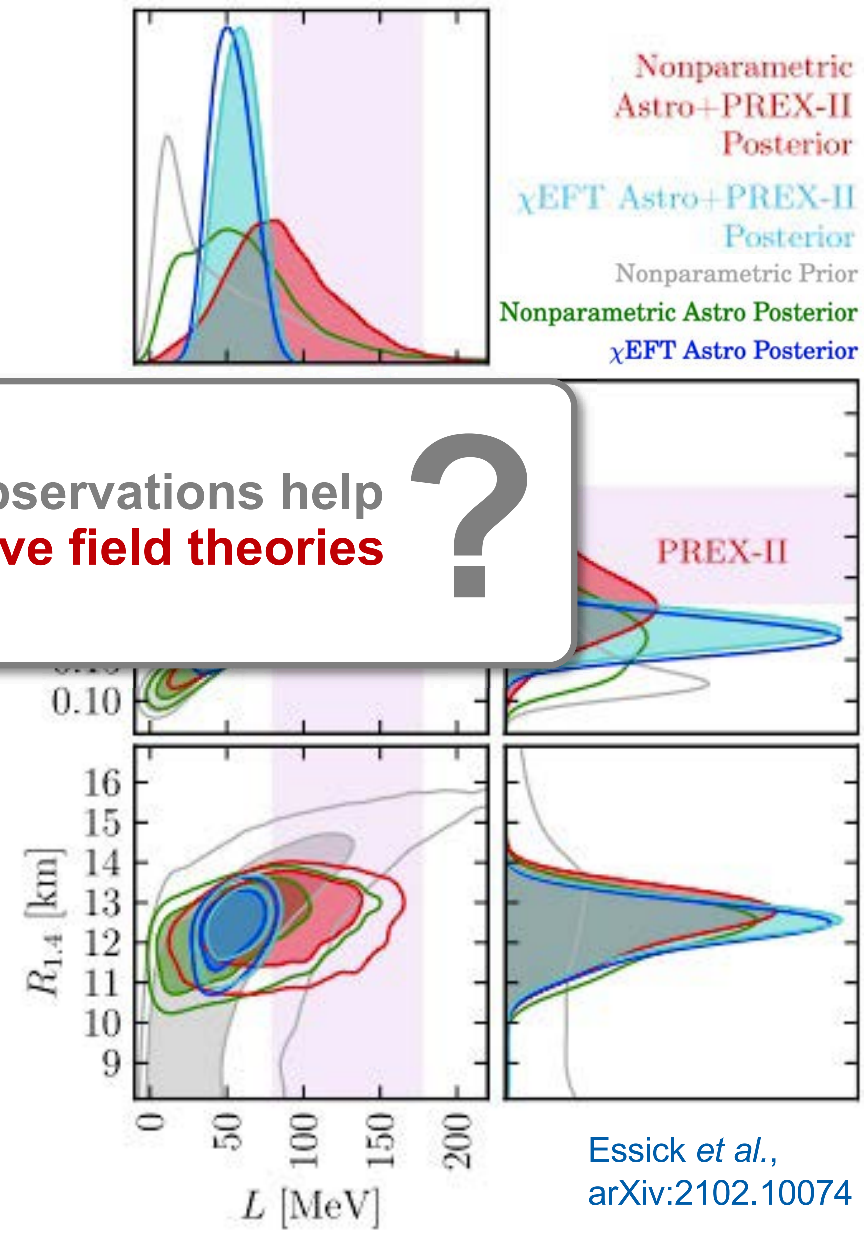
see also work by
Greif *et al.*, MNRAS **485**, 45363
Raaijmakers *et al.*, APJ **893**, L21
Essick *et al.*, PRC **102**, 055803
Xie and Li, PRC **103**, 035802
…

for recent reviews, see:
Hebeler, Holt *et al.*, ARNPS **65**, 457
Tews, Front. in Phys. **8**,153
CD, Holt, and Wellenhofer, ARNPS **71**, 403

Recent constraints on the nuclear symmetry energy:
a few examples







How can neutron star observations help improve nuclear effective field theories?

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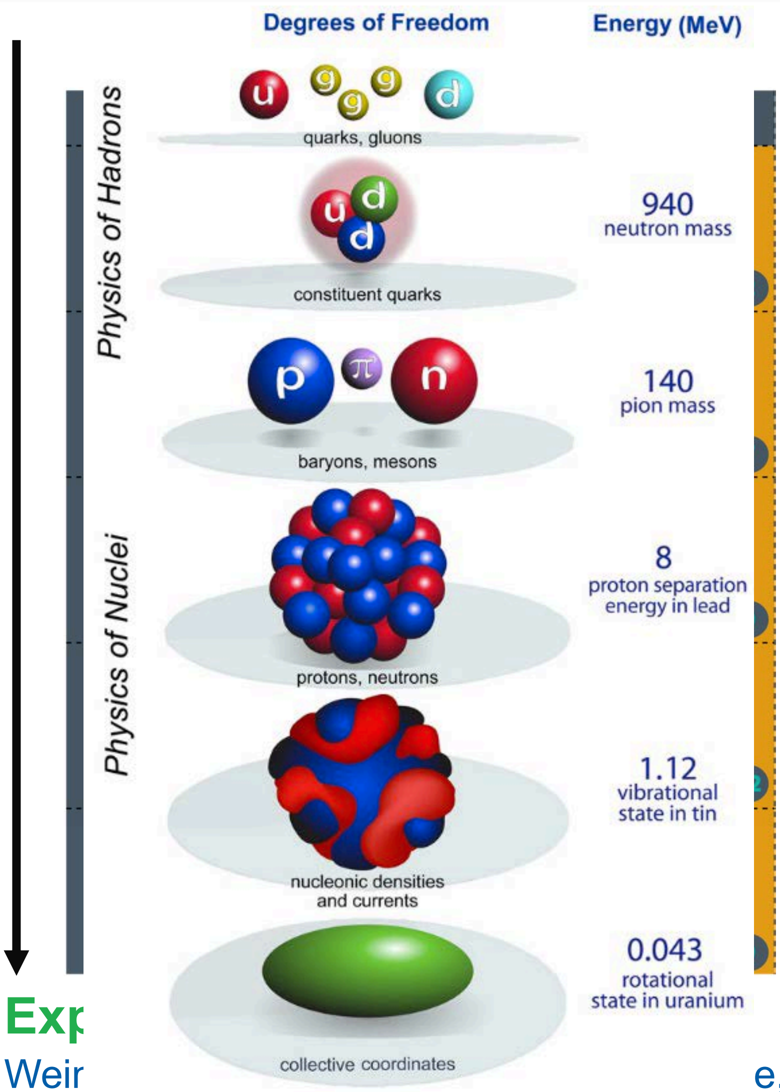
Microscopic nuclear forces

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



Chiral EFT: modern approach to deriving *microscopic* nuclear forces consistent with the symmetries of low-energy QCD

- use relevant instead of the fundamental degrees of freedom: *e.g.*, **nucleons** and **pions**
- **pion exchanges** and short-range **contact interactions** (∝ LECs)
- **systematic expansion** enables improvable **uncertainty estimates**

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \geq \frac{1}{3}$$

Exp

Weir… e, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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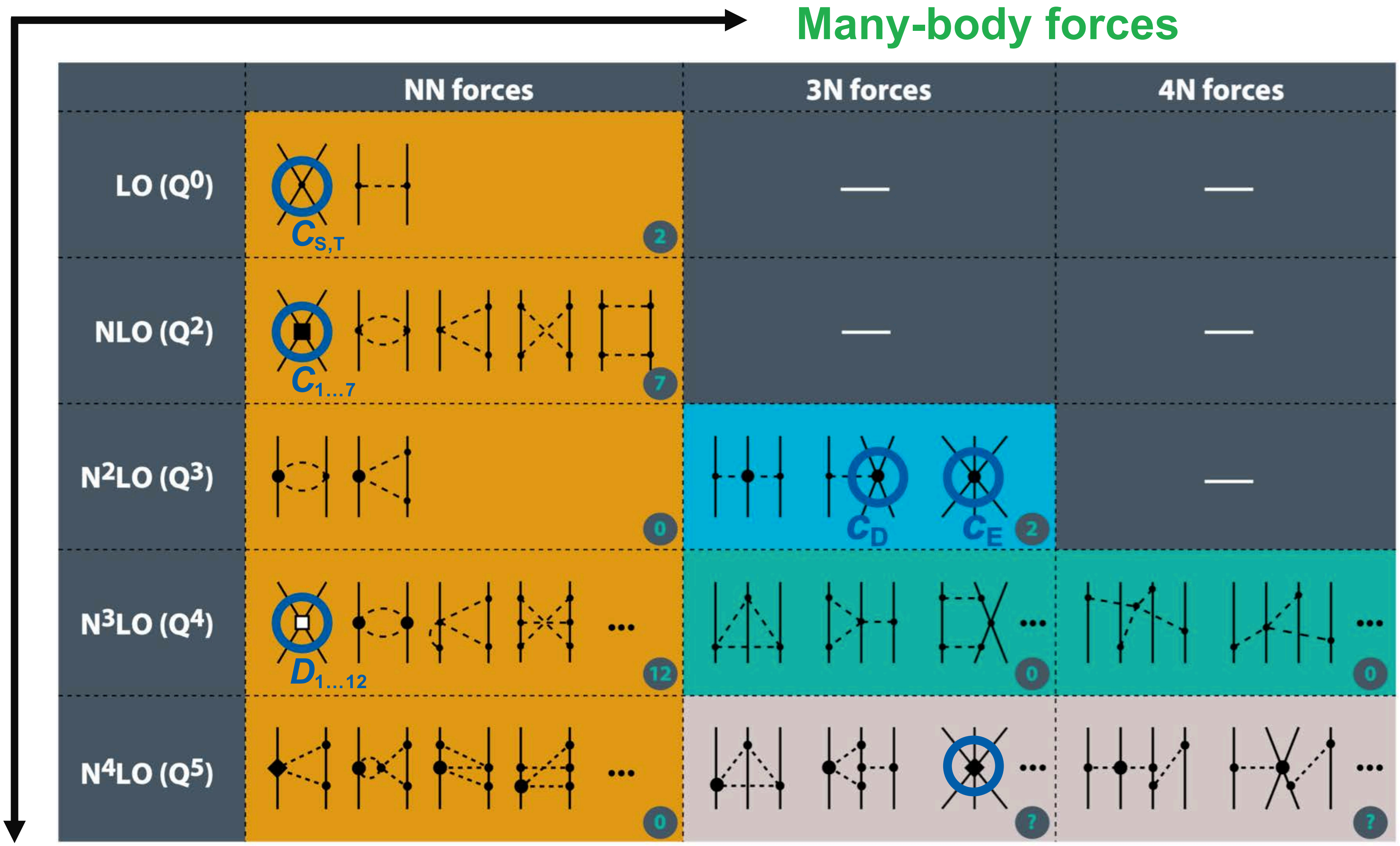
Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1

Many-body forces



S. Weinberg

no unknown parameters

not (completely) worked out

Expansion

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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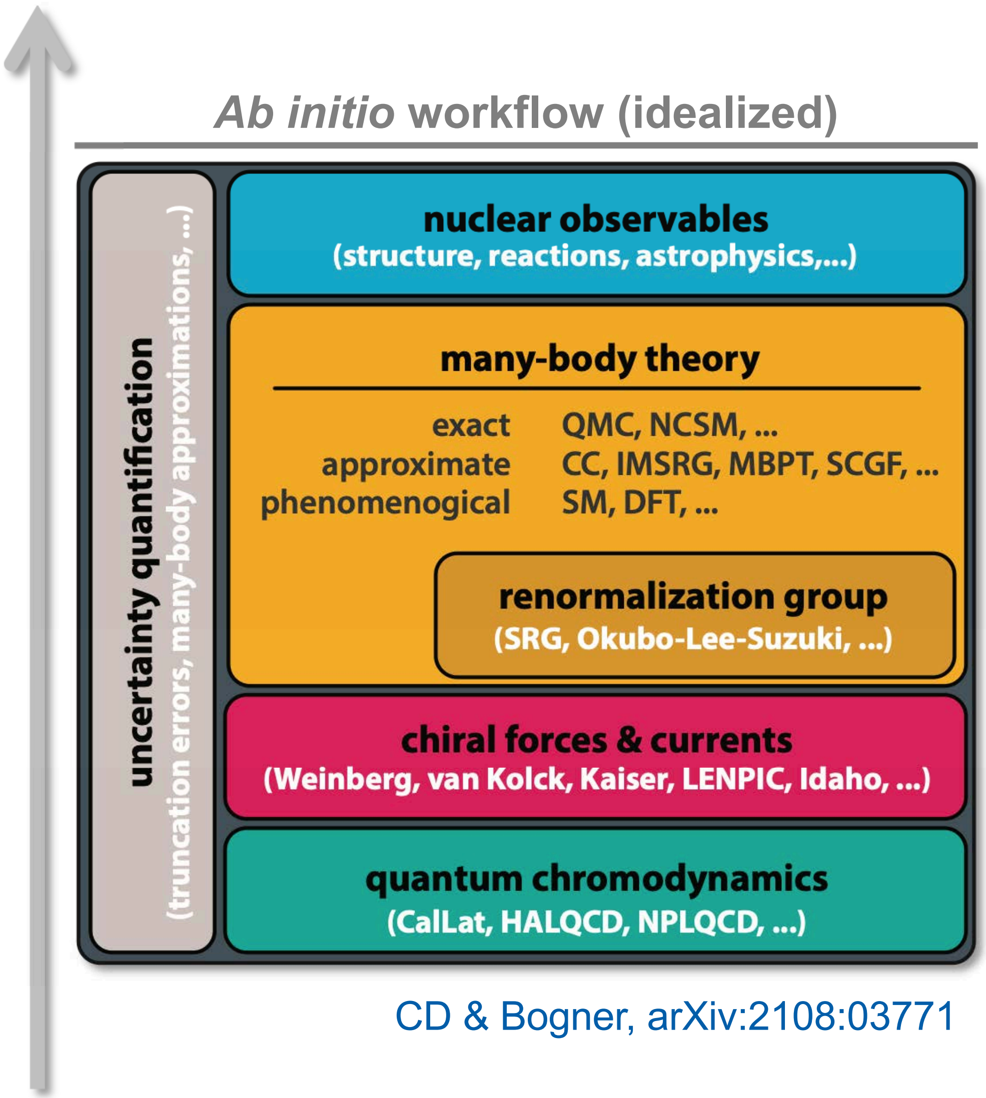
Microscopic calculations of the nuclear EOS

e.g., Hebeler, Holt *et al.*, ARNP **65**, 457



CD & Bogner, arXiv:2108:03771

great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\lesssim 2n_0$

1 Many-body frameworks
- treatment of 3N forces
- improved order-by-order calculations

2 Uncertainty quantification
- development of Bayesian methods
- quantification of EFT truncation errors

Hebeler, Lattimer *et al.*, APJ **773**, 11

see also Anthea Fantina's talk
Modelling inhomogeneous matter at finite temperature in compact stars

Lonardoni, Tews *et al.*, PRR **2**, 022033(R)
Piarulli, Bombaci *et al.*, PRC **101**, 045801

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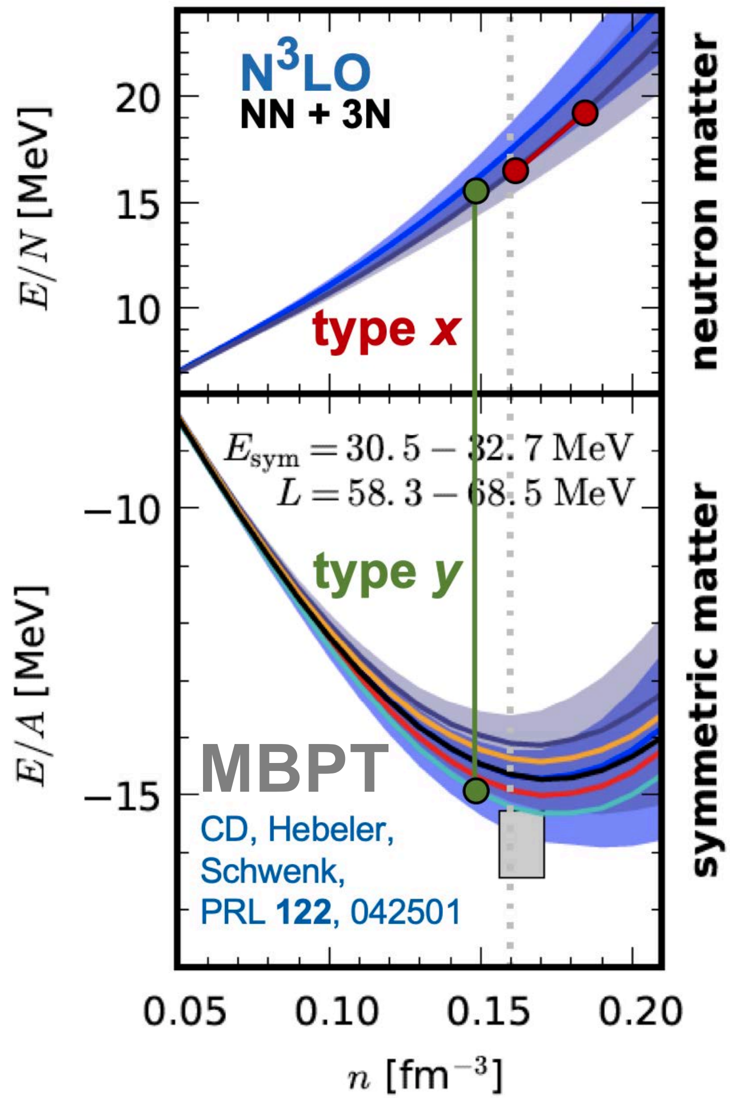
Improved many-body calculations



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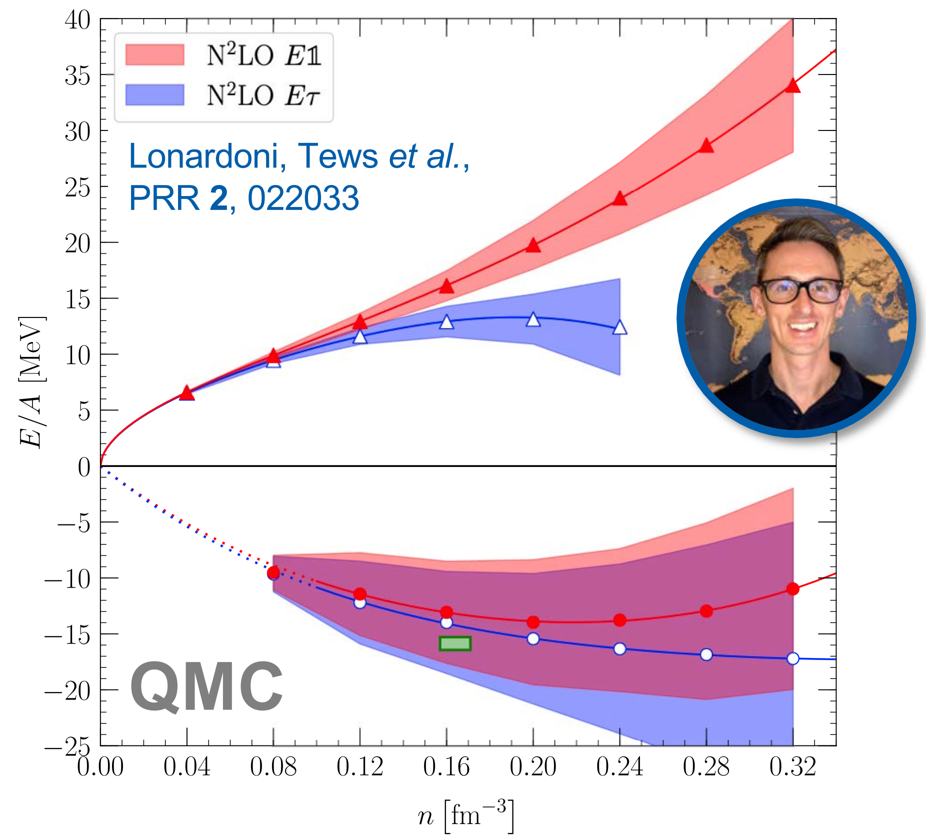
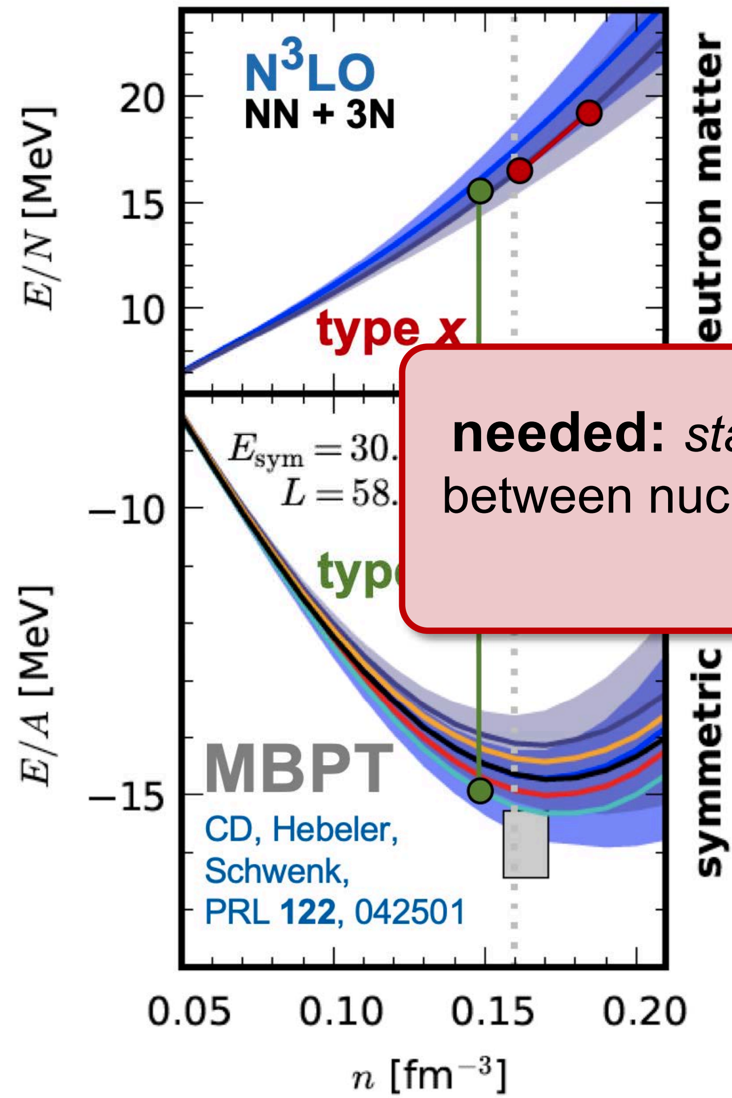


Equation of state constraints from chiral effective field theory and astrophysics

Improved many-body calculations



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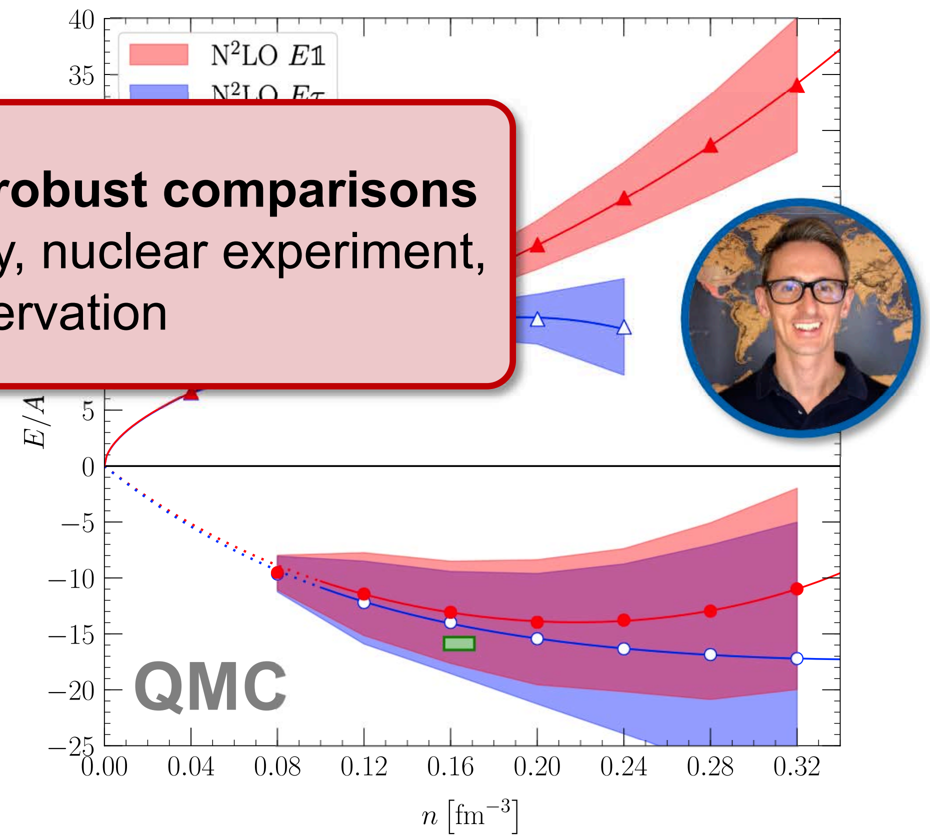

needed: *statistically* **robust comparisons** between nuclear theory, nuclear experiment, and observation

Equation of state constraints from chiral effective field theory and astrophysics

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Efficient Monte Carlo framework

CD, Hebeler, Schwenk, PRL **122**, 042501

efficient evaluation of **MBPT diagrams**
with **NN**, **3N**, and **4N forces** (single-particle basis)

- **implementing diagrams** has become **straightforward** (incl. particle-hole terms)
- multi-dimensional momentum integrals: (improved) VEGAS algorithm
- acceleration: openMP, MPI, and CUDA
- **controlled computation** of arbitrary interaction and many-body diagrams

EOS up to high orders

automatic code generation

analytic form of diagrams/forces

The number of diagrams increases rapidly!

1, 3, 39, 840, 27 300, 1 232 280, …

n =	2	3	4	5	6	7
	1	3	39	840	27 300	1 232 280

Integer sequence A064732:
Number of labeled Hugenholtz diagrams with n nodes.

ADG: Automated generation and evaluation of many-body diagrams I. Bogoliubov many-body perturbation theory

Pierre Arthuis, Thomas Duguet, Alexander Tichai, Raphaël-David Lasseri, Jean-Paul Ebran

Comput. Phys. **240**, 202

fully automated approach to MBPT

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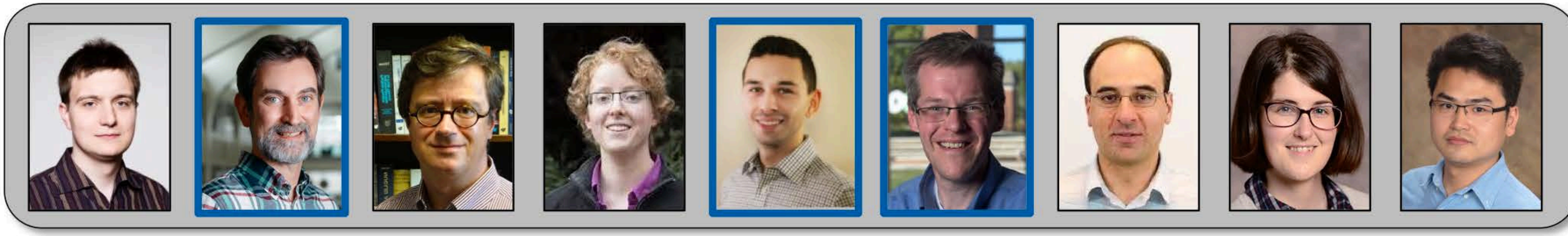
New framework for UQ of EFT calculations

buqeye.github.io



CD, Furnstahl, Melendez, and Phillips

How well do we know the neutron-matter equa… stars … unce…

…nty …ation:

CD, …

BUQEYE Collaboration

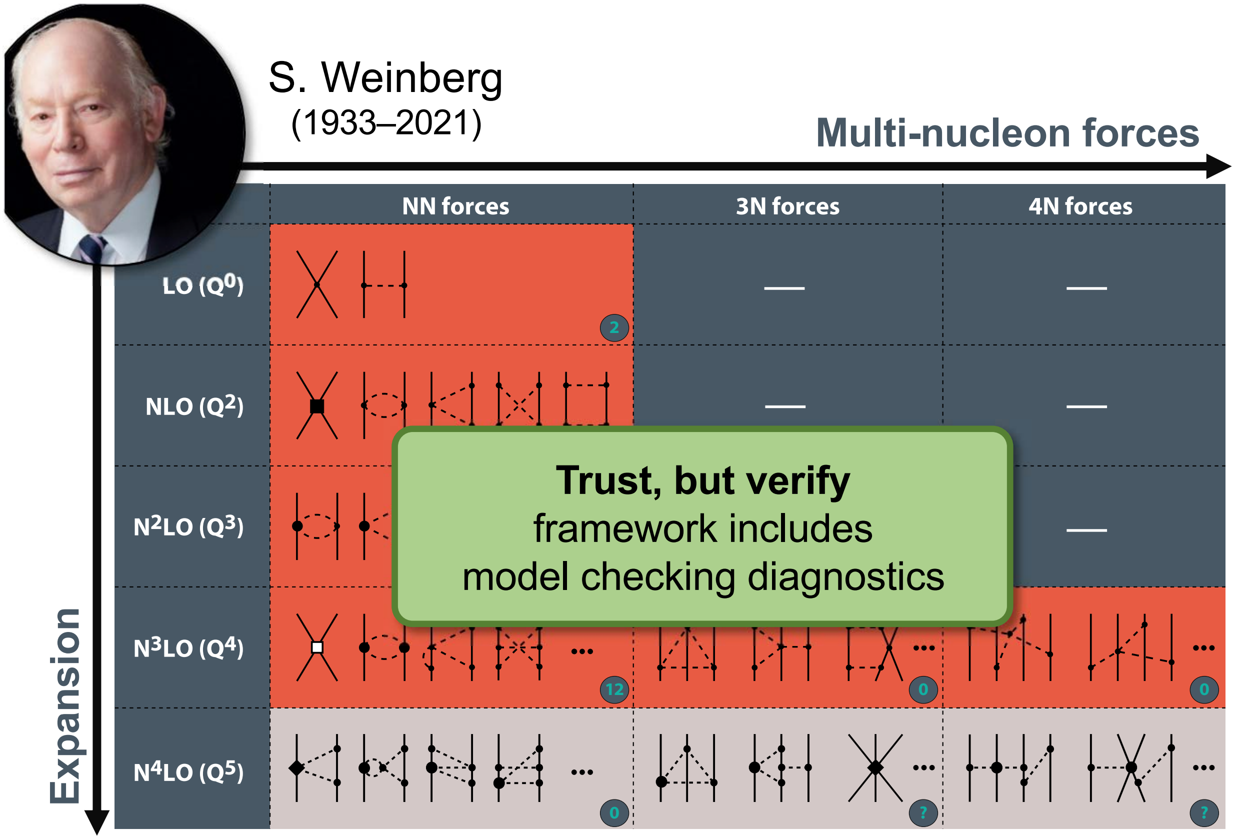
Correlated EFT truncation errors are important!

statistically **robust uncertainty estimates**
for key quantities of **neutron stars**

Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, PRC **102**, 054315

See also: Melendez *et al.*, PRC **100**, 044001
Wesolowski *et al.*, JPG **43**, 074001

UQ framework available at
https://buqeye.github.io



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum,
Kaiser, Krebs, Machleidt, Meißner, ...

**predict observable y_k
order by order in EFT**

$$y_k = y_{\mathrm{ref}} \sum_{n=0}^{k} c_n Q^n$$

c_n are *not* the EFT's LEC

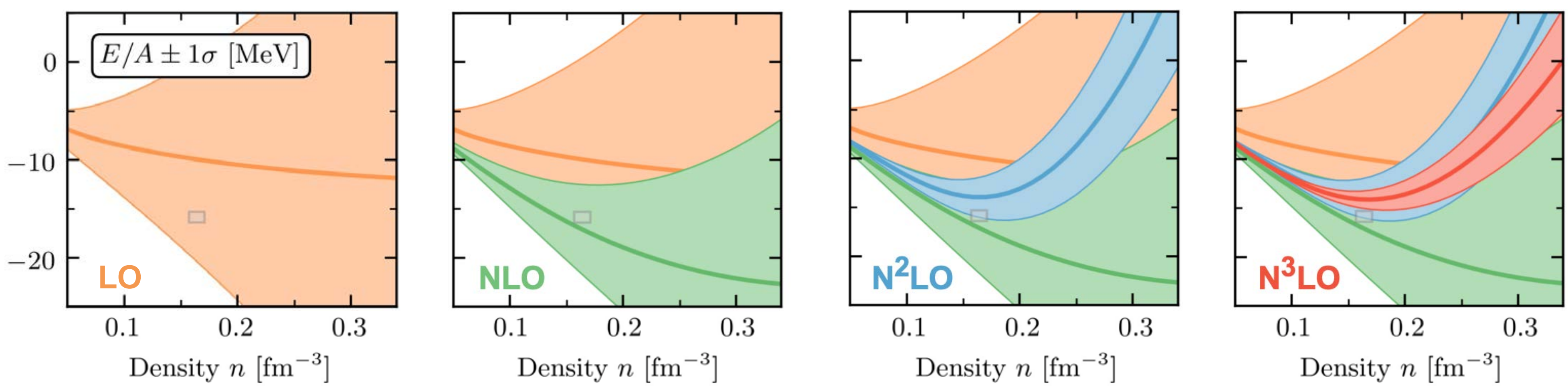
**treat all c_n as
independent draws from
a Gaussian Process**

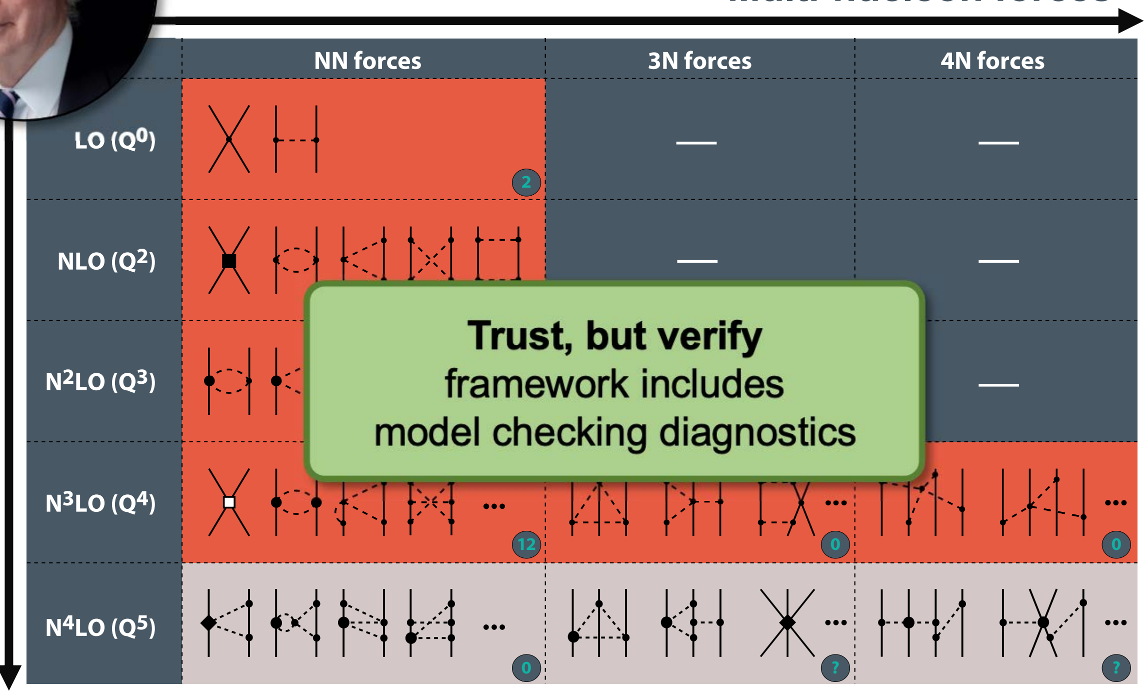
**learn GP's hyperparameters
& infer EFT truncation error**

$$\delta y_k = y_{\mathrm{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

geometric sum

For example: y_k = E/A in SNM at chiral order k





Trust, but verify
framework includes
model checking diagnostics

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum,
Kaiser, Krebs, Machleidt, Meißner, ...

predict observable y_k
order by order in EFT

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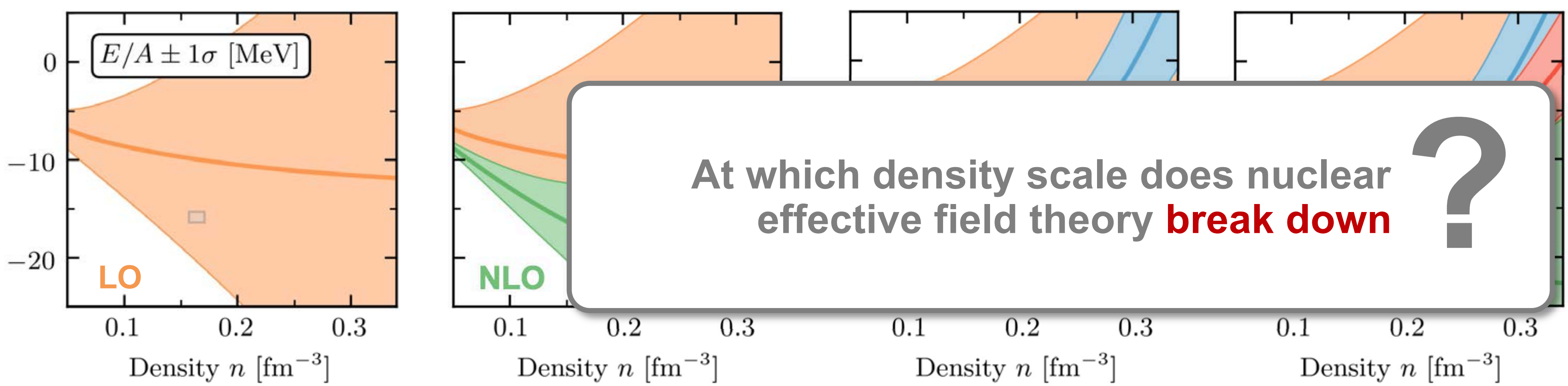

At which density scale does nuclear effective field theory **break down** ?

Equation of state constraints from chiral effective field theory and astrophysics

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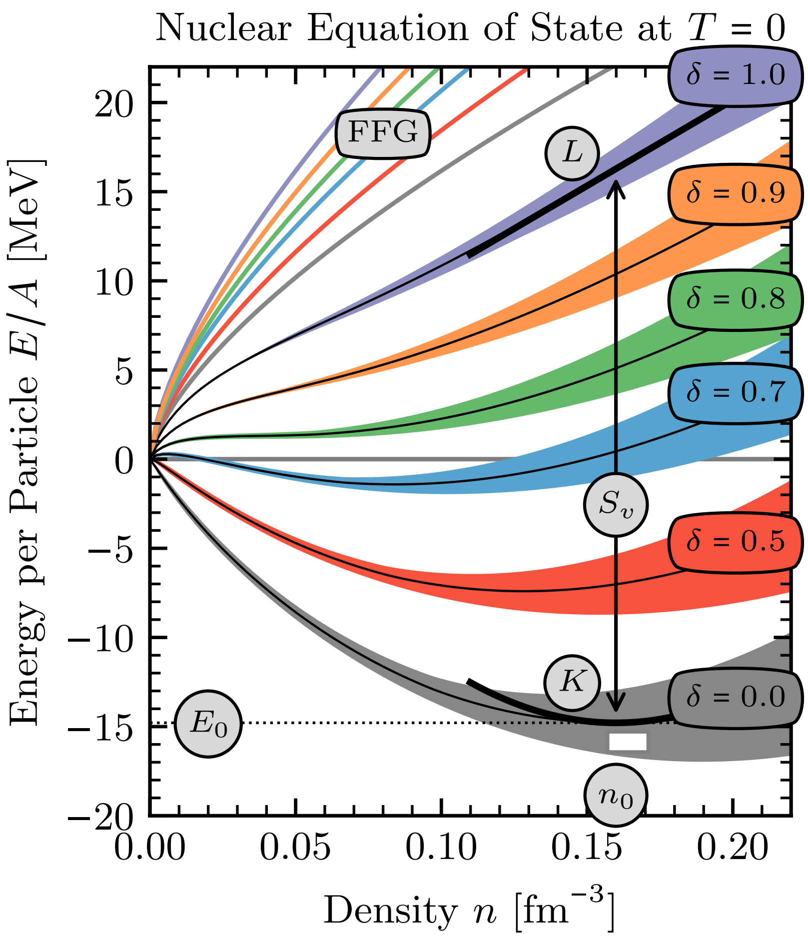
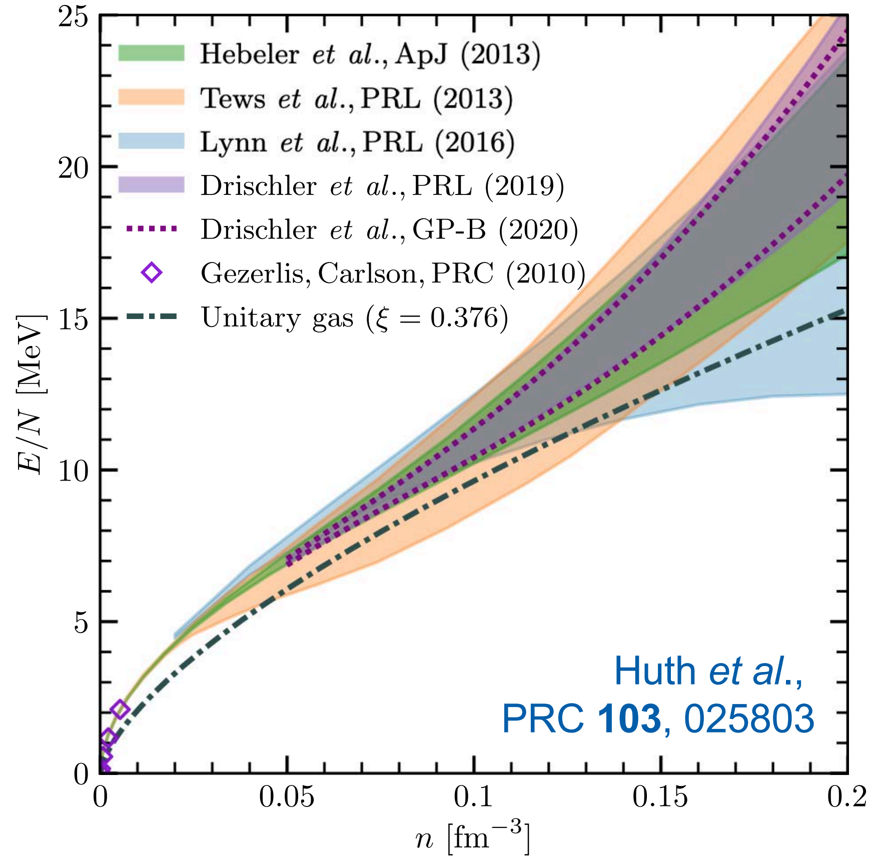
Parameters of the low-density EOS

CD, Holt, and Wellenhofer, ARNPS. **71**, 403





FFG: free Fermi gas; ***δ*** = $(n_n - n_p)/n$: isospin asymmetry Annotations: **(λ / Λ_{3N})** in fm^{-1} or **(Λ)** in MeV

for nuclear saturation, see also Atkinson *et al.*, PRC **102**, 044333; Dewulf *et al*, PRL **90**, 152501

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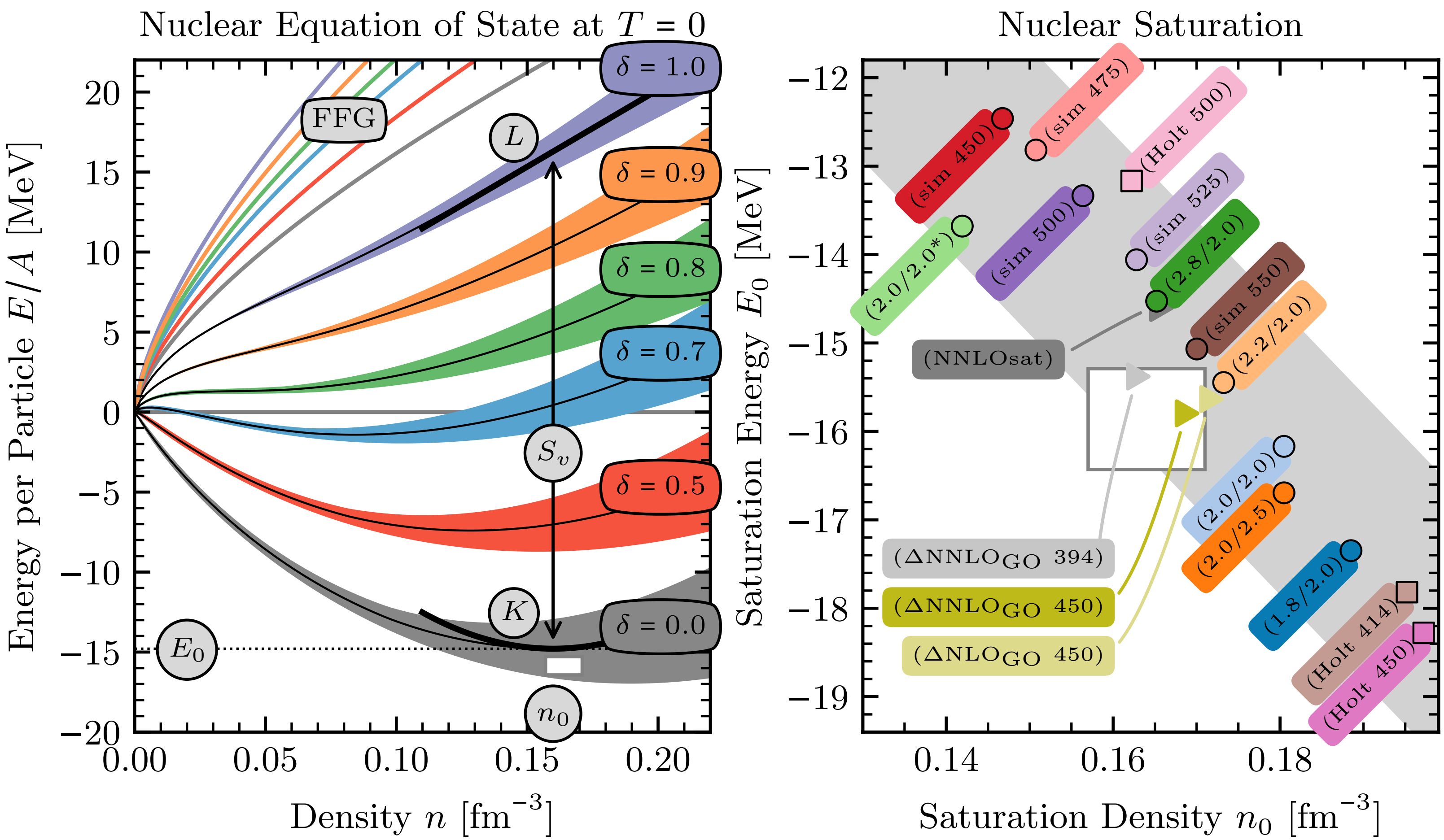

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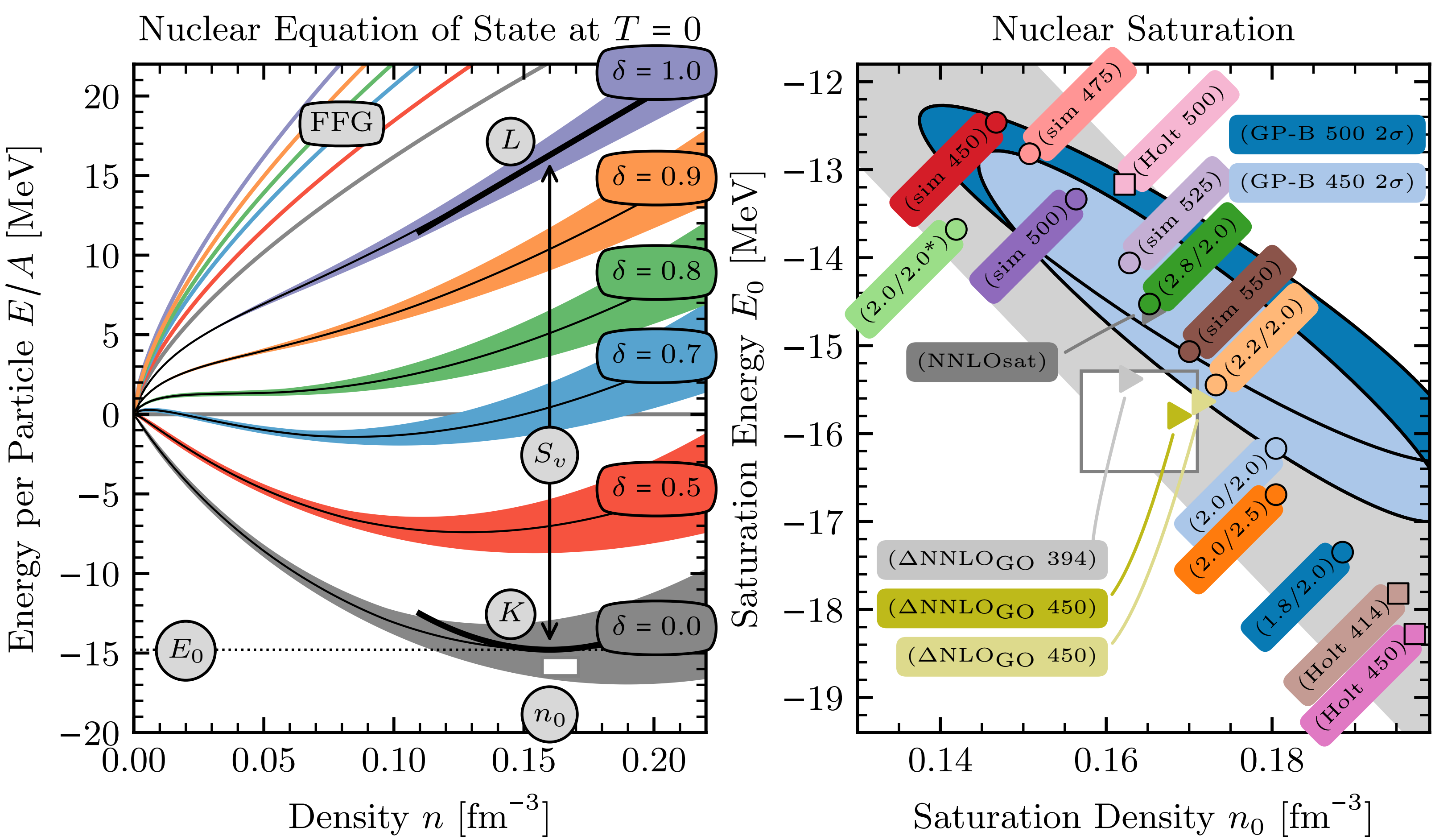

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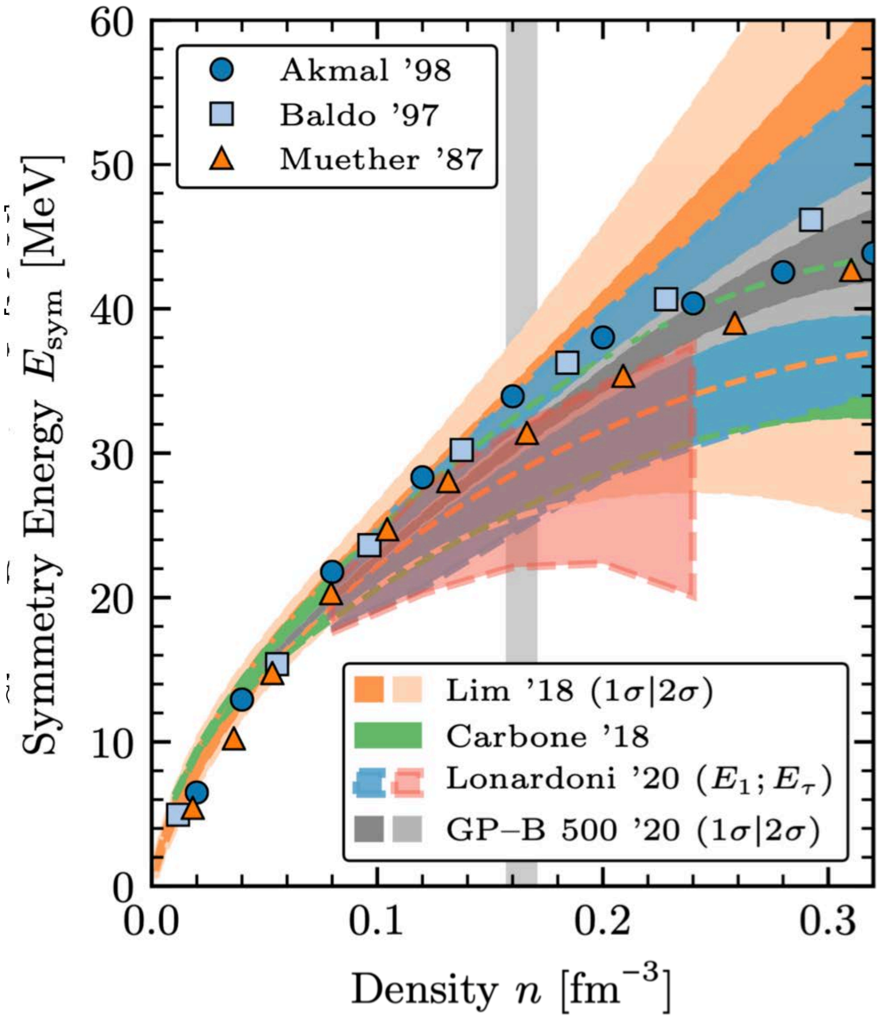
Confronting chiral EFT with empirical constraints

CD, Furnstahl *et al.*, PRL **125**, 202702



$S_2(n) \equiv$

$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$

!

Excellent agreement with experiment

Lattimer and Lim, APJ **771**, 51

$$\mathrm{pr}(S_v, L \,|\, \mathcal{D}) = \int \mathrm{d}n_0 \, \mathrm{pr}(S_2, L \,|\, n_0, \mathcal{D}) \, \mathrm{pr}(n_0 \,|\, \mathcal{D})$$

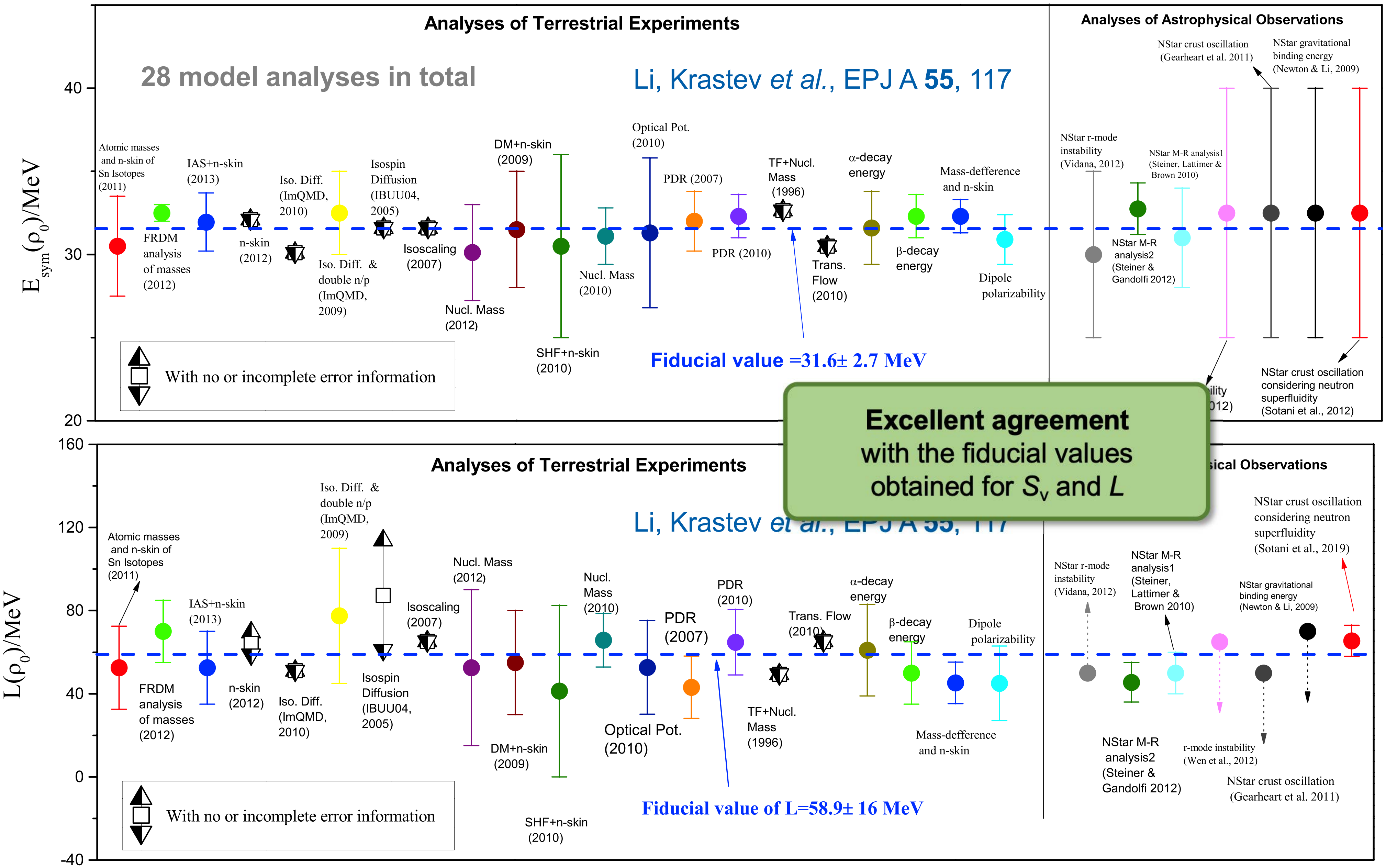
$$\mathrm{pr}(n_0 \,|\, \mathcal{D}) \approx 0.17 \pm 0.01 \, \mathrm{fm}^{-3}$$

2σ ellipse (light yellow) is completely within the *conjectured* unitary gas limit

predicted range in ***S_v* agrees** with other **theoretical constraints**; but ~15 MeV stronger density-dependence of $S_2(n_0)$

GP–B (500): two-dimensional Gaussian

$$\begin{bmatrix} \mu_{S_v} \\ \mu_L \end{bmatrix} = \begin{bmatrix} 31.7 \\ 59.8 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 1.11^2 & 3.27 \\ 3.27 & 4.12^2 \end{bmatrix}$$



Excellent agreement
with the fiducial values
obtained for S_v and L

GP–B (500): two-dimensional Gaussian

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Compilation of recent terrestrial and astrophysical **constraints on S_v and L**

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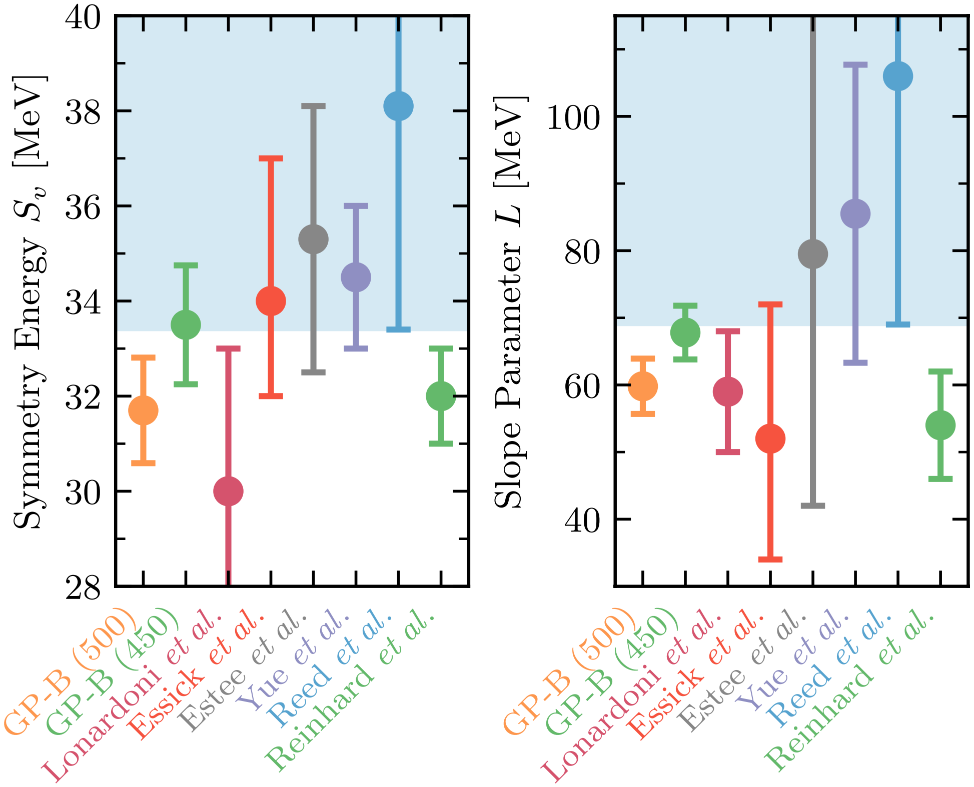
PREX–II vs theory and observation

see also Yue *et al.*, arXiv:2102.05267



Parity violating elastic e scattering

$$R_{\text{skin}}\left(^{208}\text{Pb}\right) = 0.283 \pm 0.071\,\text{fm}$$

PREX collaboration, PRL **126**, 172502

Exploiting strong correlations (EDFs)

$$S_v = 38.1 \pm 4.7\,\text{MeV}$$

$$L = 105.9 \pm 36.9\,\text{MeV}$$

Reed *et al.*, PRL **126**, 172503

Astron. data + chiral EFT only (incl. GP-B)

$$R\left(^{208}\text{Pb}\right) = 0.18^{+0.04}_{-0.04}\,\text{fm}$$

$$S_v = 34^{+3}_{-2}\,\text{MeV} \quad L = 52^{+20}_{-18}\,\text{MeV}$$

Essick *et al.*, arXiv:2102.10074

Different set of modern EDFs

$$R\left(^{208}\text{Pb}\right) = 0.19 \pm 0.02\,\text{fm}$$

$$S_v = 32 \pm 1\,\text{MeV} \quad L = 54 \pm 8\,\text{MeV}$$

Reinhard, Roca-Maza *et al.*, arXiv:2105.15050

Take away from PREX-II-informed results:

- **uncertainties are still large**
- allows for stiffer EOS at ~n_0, but within the large uncertainties consistent with chiral EFT
- **tension between A_{PV} and α_{D}**

see also: Piekarewicz, PRC **104**, 024329

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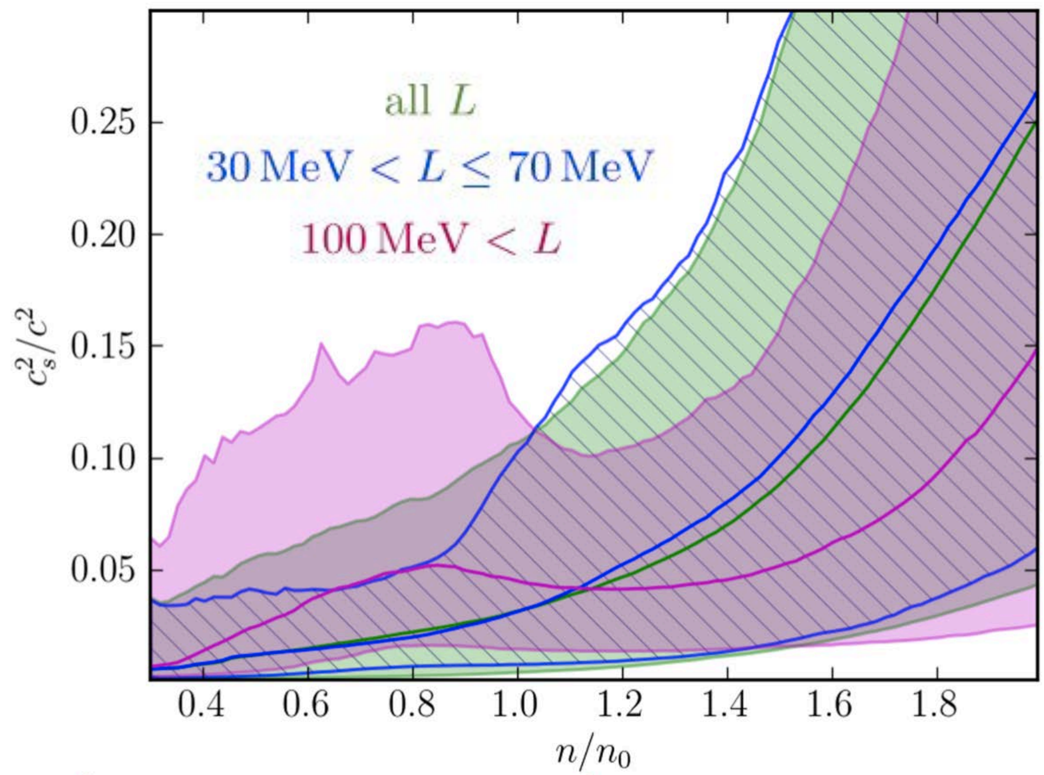

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PREX collaboration, PRL **126**, 172502

Exploiting strong correlations (EDFs)

$$S_v = 38.1 \pm 4.7\,\mathrm{MeV}$$

$$L = 105.9 \pm 36.9\,\mathrm{MeV}$$

Reed *et al.*, PRL **126**, 172503

Astron. data + chiral EFT only (incl. GP-B)

$$R\left(^{208}\mathrm{Pb}\right) = 0.18^{+0.04}_{-0.04}\,\mathrm{fm}$$

$$S_v = 34^{+3}_{-2}\,\mathrm{MeV} \quad L = 52^{+20}_{-18}\,\mathrm{MeV}$$

Essick *et al.*, arXiv:2102.10074

Different set of modern EDFs

$$R\left(^{208}\mathrm{Pb}\right) = 0.19 \pm 0.02\,\mathrm{fm}$$

$$S_v = 32 \pm 1\,\mathrm{MeV} \quad L = 54 \pm 8\,\mathrm{MeV}$$

Reinhard, Roca-Maza *et al.*, arXiv:2105.15050

Take away from PREX-II-informed results:

- **uncertainties are still large**
- allows for stiffer EOS at ~n_0, but within the large uncertainties consistent with chiral EFT
- **tension between A_{PV} and α_{D}**

see also: Piekarewicz, PRC **104**, 024329

Equation of state constraints from chiral effective field theory and astrophysics

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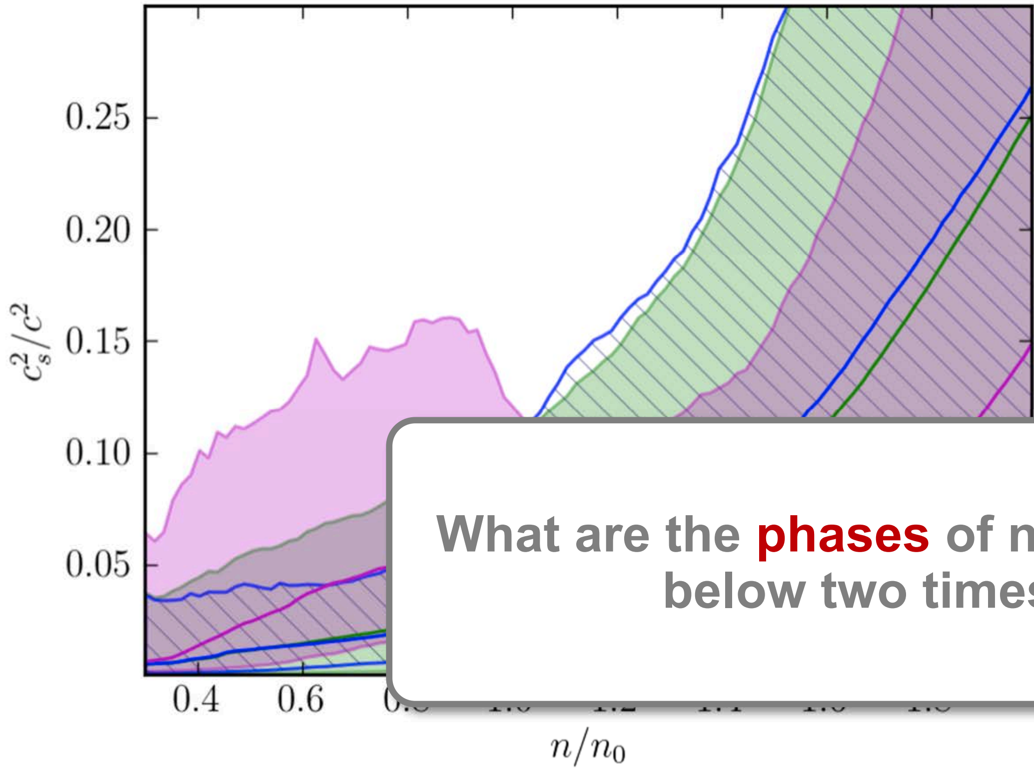
PREX–II vs theory and observation

see also Yue *et al.*, arXiv:2102.05267



Parity violating elastic e scattering

$$R_{\text{skin}}\left({}^{208}\text{Pb}\right) = 0.283 \pm 0.071\,\text{fm}$$

PREX collaboration, PRL **126**, 172502

Exploiting strong correlations (EDFs)

$$S_v = 38.1 \pm 4.7\,\text{MeV}$$

$$L = 105.9 \pm 36.9\,\text{MeV}$$

…RL **126**, 172503

…ly (incl. GP-B)

What are the phases of neutron star matter below two times normal densities ?

$$S_v = 34^{+\ldots}_{-2}\,\text{MeV} \qquad L = 52^{+20}_{-18}\,\text{MeV}$$

Essick *et al.*, arXiv:2102.10074

Different set of modern EDFs

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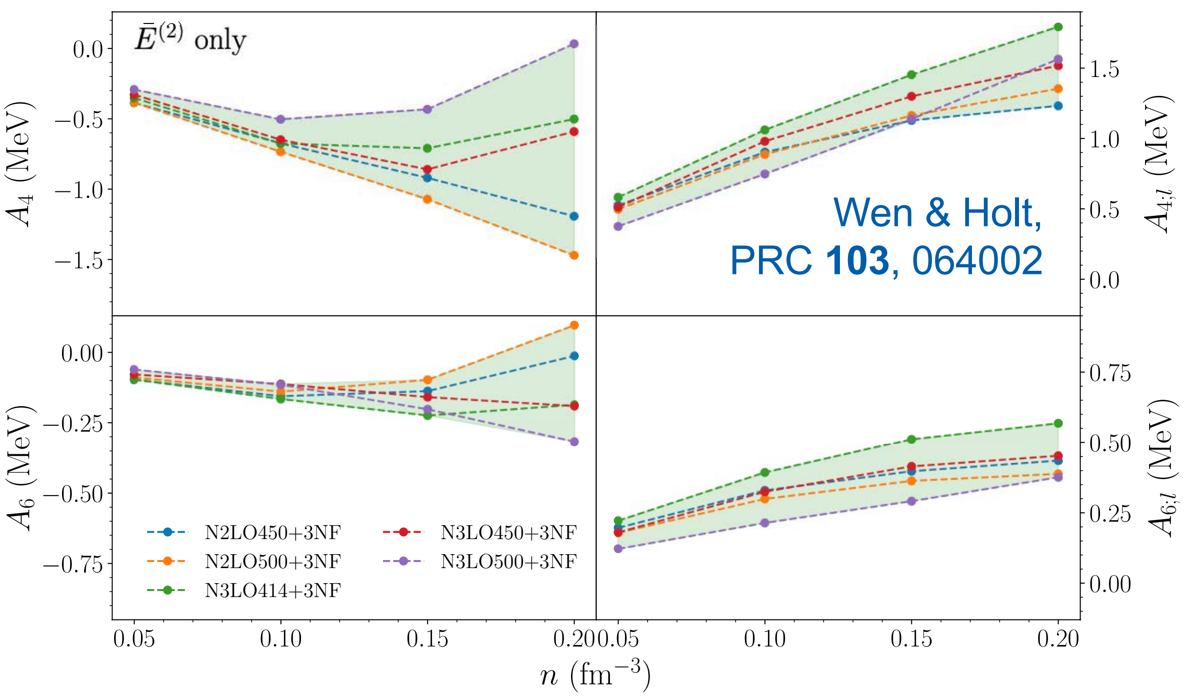
Nonquadratic contributions

to the nuclear symmetry energy

Kaiser, PRC **91**, 065201
Wellenhofer, Holt, and Kaiser, PRC **93**, 055802
Somasundaram, CD, Tews *et al.*, PRC **103**, 045803

$$\frac{E}{A}(n,\delta) = \frac{E}{A}(n,\delta=0) + S_2(n)\delta^2 + \sum_{i>1}\left(A_{2i}(n) + A_{2i,l}(n)\log|\delta|\right)\delta^{2i}$$

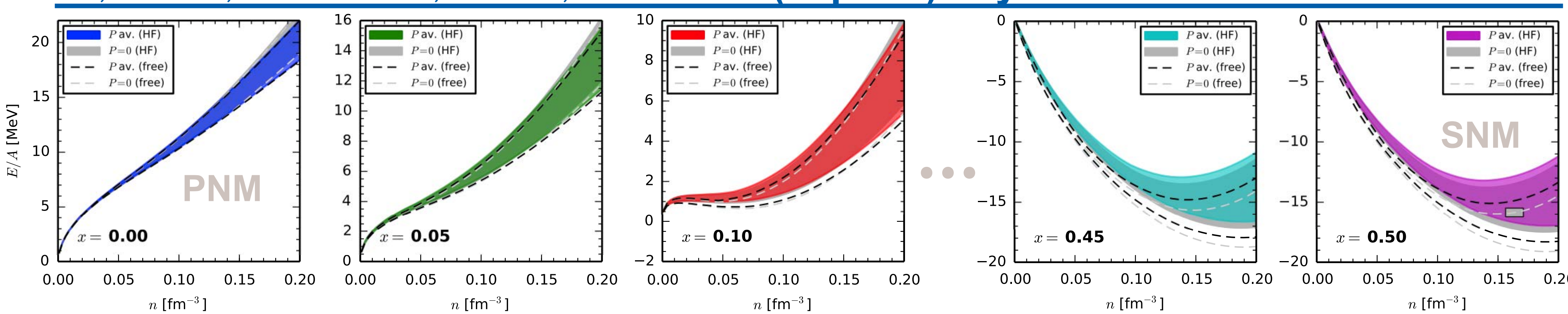


MBPT beyond Hartree-Fock gives rise to (nonanalytic) **logarithmic contributions**

Precision MBPT calculations can **extract** high-order symmetry energy **coefficients**

Overall small contribution from nonquadratic terms
(but can impact β-equilibrium)

CD, Hebeler, and Schwenk, PRC **93**, 054314 **(explicit) asymmetric matter calculations**



Equation of state constraints from chiral effective field theory and astrophysics

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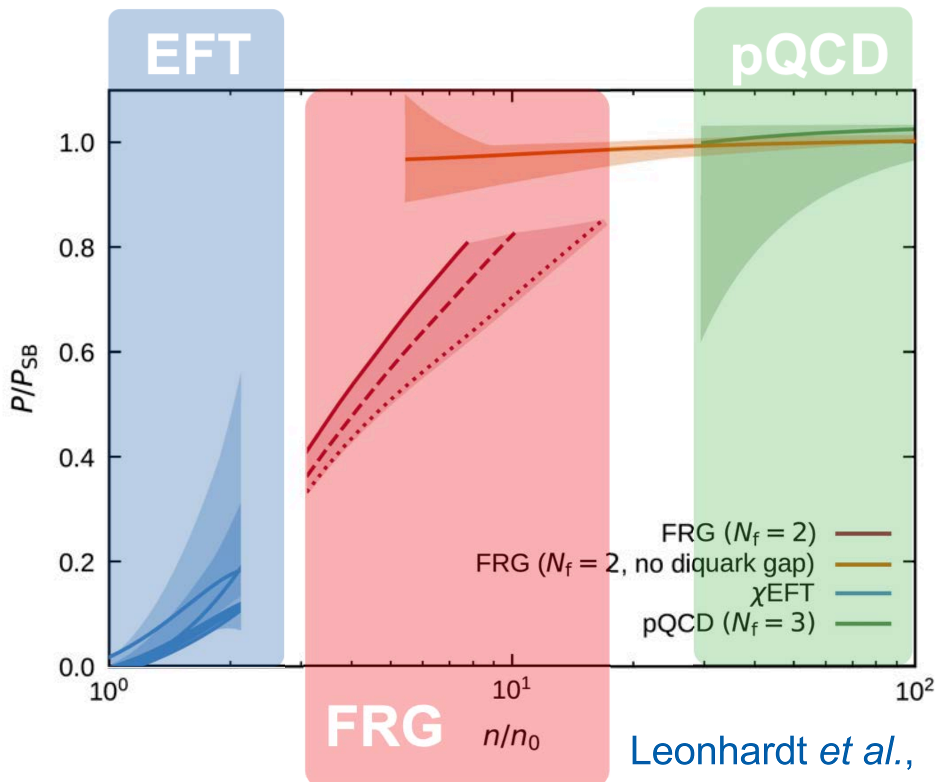
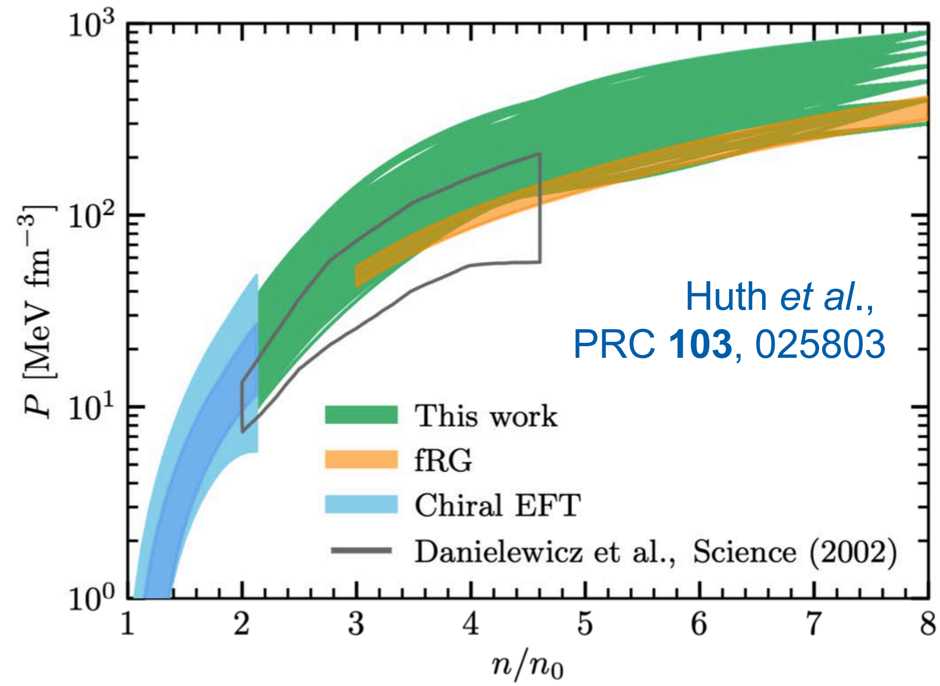
New developments: symmetric nuclear matter



Leonhardt *et al.*, PRL **125**, 142502



Huth *et al.*, PRC **103**, 025803

Functional Renormalization Group: ***complementary* constraints at > 3n_0** (beyond the range of chiral EFT) from the QCD action

New insights into the high-density EOS: **remarkable consistency** between the constraints, which suggests that they can be combined via **simple extrapolations**

Equation of state constraints from chiral effective field theory and astrophysics

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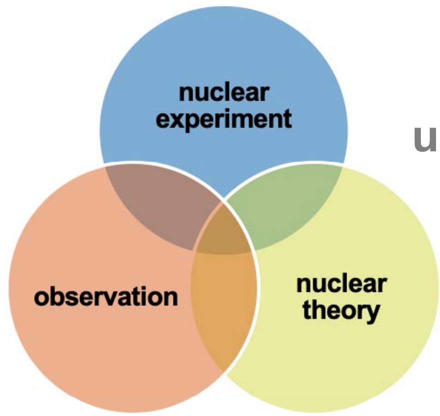
Conclusion

multi-messenger
nuclear precision
FRIB
} **era**



unique opportunity to **constrain the nuclear EOS**, with great **potential for discoveries**

1 Microscopic EOS constraints *statistically* robust uncertainties

- excellent agreement of predicted S_v–L correlation with experiment
- PNM and SNM show a regular EFT convergence pattern with increasing order
- extracted Λ_b is consistent with NN scattering • N^2LO coefficient may be an outlier

2 full Bayesian UQ: sample over LECs & hyperparameters

- in future: consistently include uncertainties in the LECs of chiral interactions
- promising: new potentials up to N^2LO by Wesolowski *et al.*, arXiv:2104.04441

thanks to my collaborators:

R. Furnstahl **J. Melendez** **K. McElvain** D. Phillips
S. Han J. Lattimer M. Prakash S. Reddy **T. Zhao**