

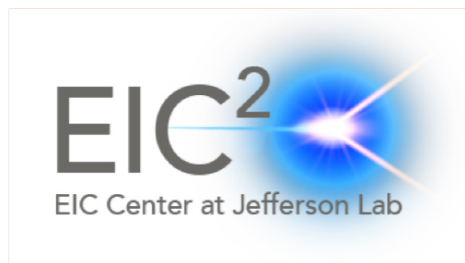
SarWorS 2021  
7 September 2021

# TMDs & EIC impact studies

**Chiara Bissoletti**  
**Università degli Studi di Pavia**

**MAP Collaboration**

(Multi-dimensional Analyses of Partonic distributions)



# Unpolarized quark TMDs

## TMD PDF

$$\text{scales } \mu = Q$$
$$\zeta = Q^2$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left( C_{q/j} \otimes f_1^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu)} e^{g_K(b) \ln \frac{\mu}{\mu_0}} f_{\text{NP}}^q(x, b, \zeta)$$

# Unpolarized quark TMDs

## TMD PDF

scales  $\mu_b = \frac{2e^{-\gamma_E}}{b_*}$   
 $\mu = Q$   
 $\zeta = Q^2$

collinear PDFs

Sudakov factor  
 perturbative evolution

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left( C_{q/j} \otimes f_1^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu)} e^{g_K(b) \ln \frac{\mu}{\mu_0}} f_{\text{NP}}^q(x, b, \zeta)$$

matching coefficients

non perturbative  
 evolution

intrinsic non perturbative  
 transverse content

# Unpolarized quark TMDs

## TMD PDF

collinear PDFs

Sudakov factor  
perturbative evolution

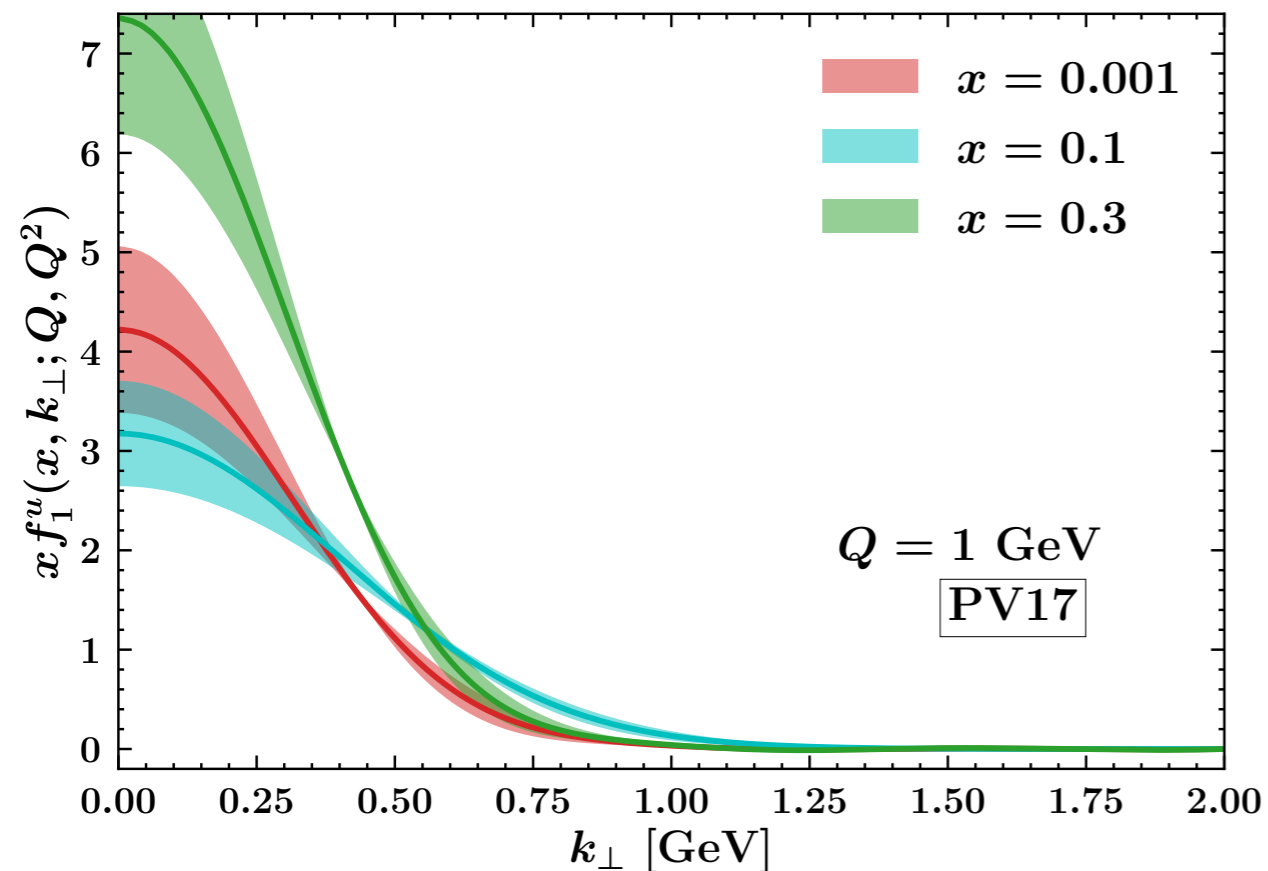
$$f_1^q(x, b; \mu, \zeta) = \sum_j \left( C_{q/j} \otimes f_1^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu)} e^{g_K(b) \ln \frac{\mu}{\mu_0}} f_{\text{NP}}^q(x, b, \zeta)$$

matching coefficients

non perturbative  
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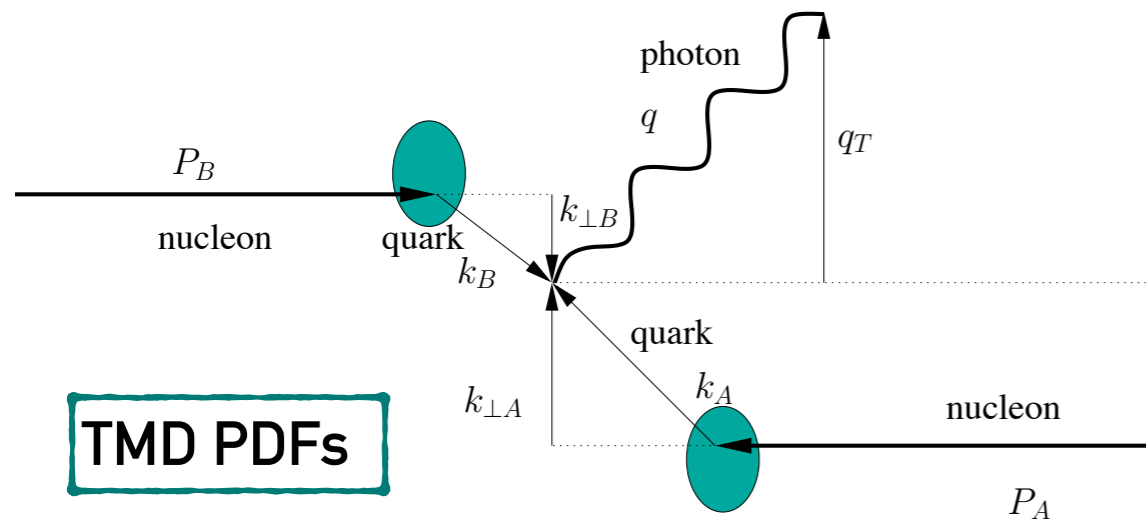
**non perturbative**  
parametrized and fitted to data



A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori  
 JHEP06 (2017) 081, arXiv:1703.10157

# Drell-Yan and SIDIS

$$N(P_A) + N(P_B) \rightarrow \gamma^*/Z \rightarrow l^+l^-$$



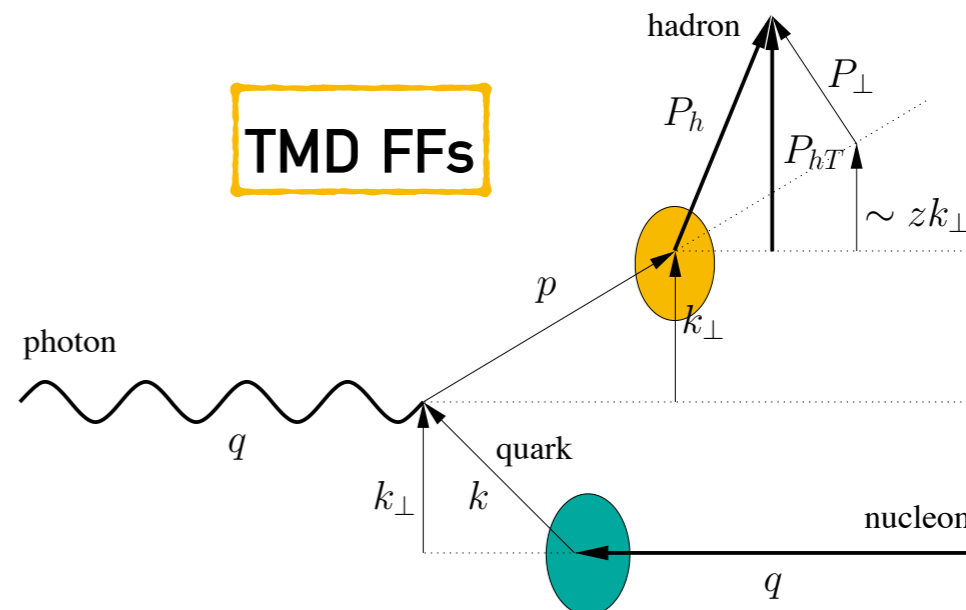
$$q_T \ll Q$$

**TMD factorization**

$$\left( \frac{d\sigma}{dq_T} \right) \propto$$

$$\int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 \boxed{f_1^q(x_1, \mathbf{b})} x_2 \boxed{f_1^{\bar{q}}(x_2, \mathbf{b})}$$

$$\ell(l) + N(p) \rightarrow \ell(l') + h(P_h) + X$$



$$M^2 \ll Q^2 \quad P_{hT}^2 \ll Q^2$$

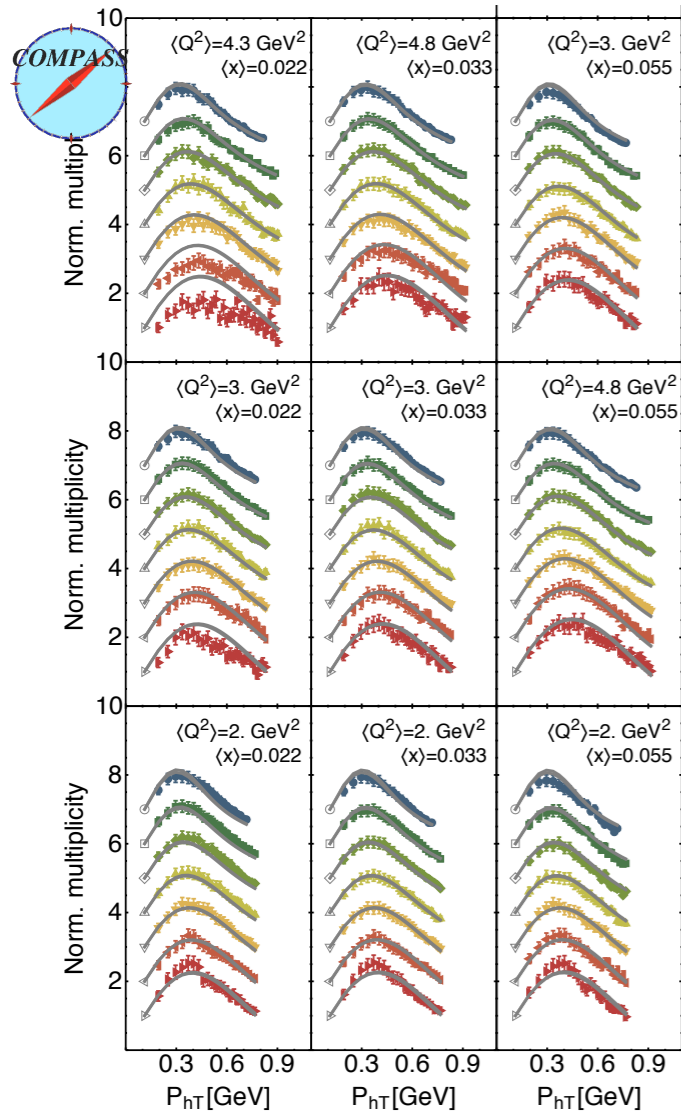
$$\left( \frac{d\sigma}{dq_T} \right) \propto$$

$$\int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \boxed{f_1^q(x, \mathbf{b})} \boxed{D_1^{q \rightarrow h}(z, \mathbf{b})}$$

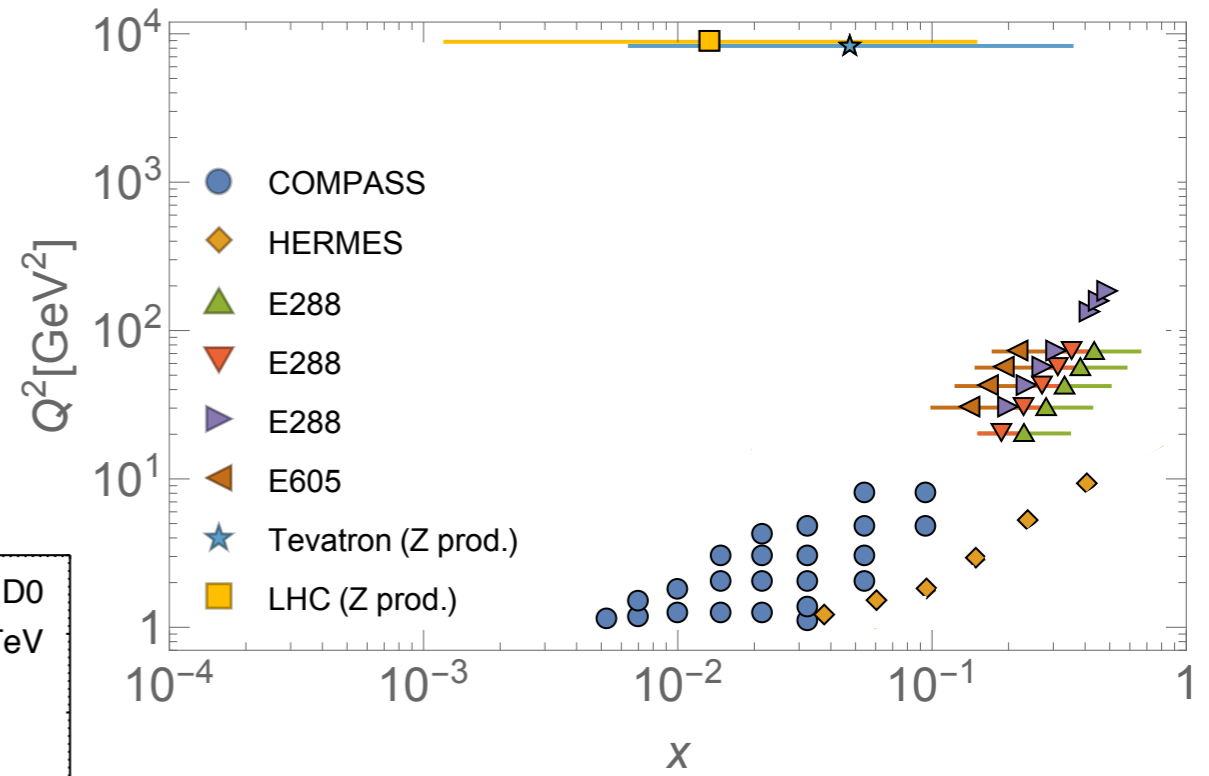
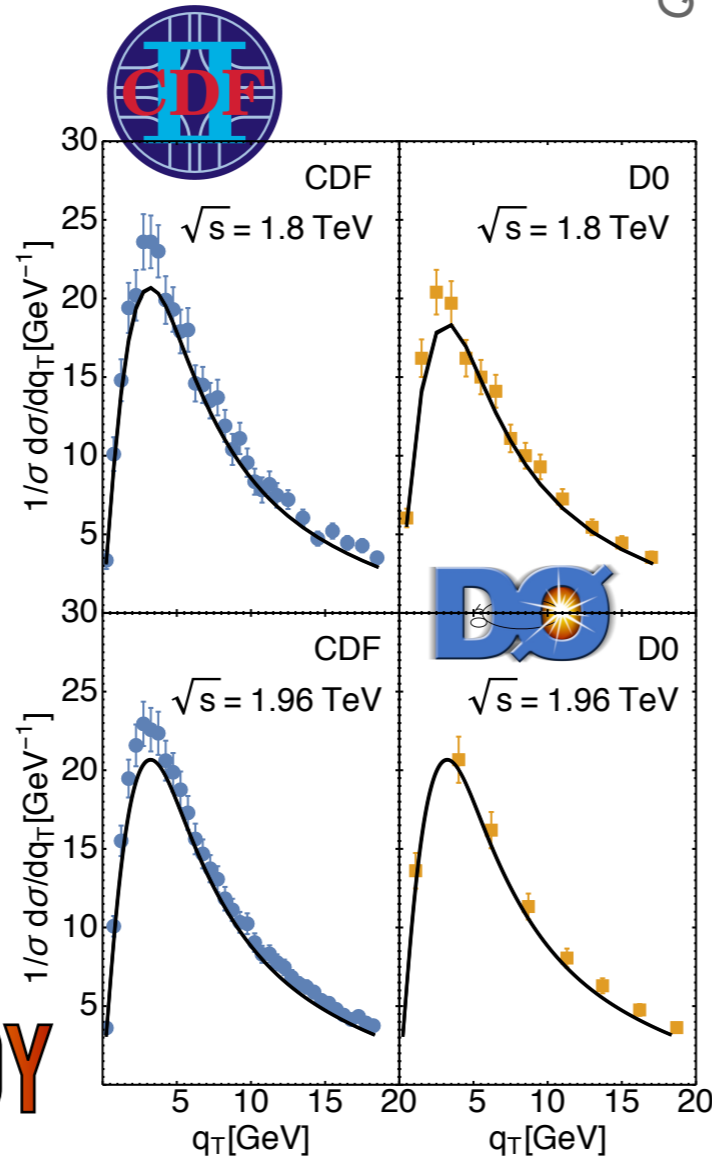
# PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori  
 JHEP06 (2017) 081, arXiv:1703.10157

## SIDIS



**8059**  
 data points



with  
 normalization  
 coefficients

**NLL**

global

$\chi^2 = 1.55$

# PV17 non perturbative functions

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori

JHEP06 (2017) 081, arXiv:1703.10157

$$f_{1\text{NP}}^a(x, \mathbf{k}_\perp^2) = \frac{1}{\pi} \frac{(1 + \lambda \mathbf{k}_\perp^2)}{g_{1a} + \lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

$$D_{1\text{NP}}^{a \rightarrow h}(z, \mathbf{P}_\perp^2) = \frac{1}{\pi} \frac{1}{g_{3a \rightarrow h} + (\lambda_F/z^2) g_{4a \rightarrow h}^2} \left( e^{-\frac{\mathbf{P}_\perp^2}{g_{3a \rightarrow h}}} + \lambda_F \frac{\mathbf{P}_\perp^2}{z^2} e^{-\frac{\mathbf{P}_\perp^2}{g_{4a \rightarrow h}}} \right)$$

x-dependence

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$g_{3,4}(z) = N_{3,4} \frac{(z^\beta + \delta) (1-z)^\gamma}{(\hat{z}^\beta + \delta) (1-\hat{z})^\gamma}$$

non-perturbative Sudakov factor

$$g_K(b_T) = -g_2 b_T^2 / 2$$

total of **11 parameters**

# PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori  
JHEP06 (2017) 081, arXiv:1703.10157

8059 points  
11 parameters  
SIDIS + DY  
NLL

$$\chi_{\text{d.o.f.}}^2 = 1.55 \pm 0.05$$

all standard deviations at 68% c.l.

$g_2$	$N_1$ [GeV <sup>2</sup> ]	$\beta$	$\alpha$	$\lambda_F$ [GeV <sup>-2</sup> ]
0.13 ± 0.01	0.28 ± 0.06	1.65 ± 0.49	2.95 ± 0.05	5.50 ± 1.23

$$\langle x \rangle \pm \Delta x$$

*fairly well determined*

*not very constrained*

$g_2 \rightarrow$  non-perturbative evolution

$\beta \rightarrow$  low-z width of TMD FF

$N_1 \rightarrow$  mid-x width of TMD PDF

$\alpha \rightarrow$  high-x width of TMD PDF

$\lambda_F \rightarrow$  weight of second Gaussian in TMD FF



# PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori  
JHEP06 (2017) 081, arXiv:1703.10157

8059 points  
11 parameters  
SIDIS + DY  
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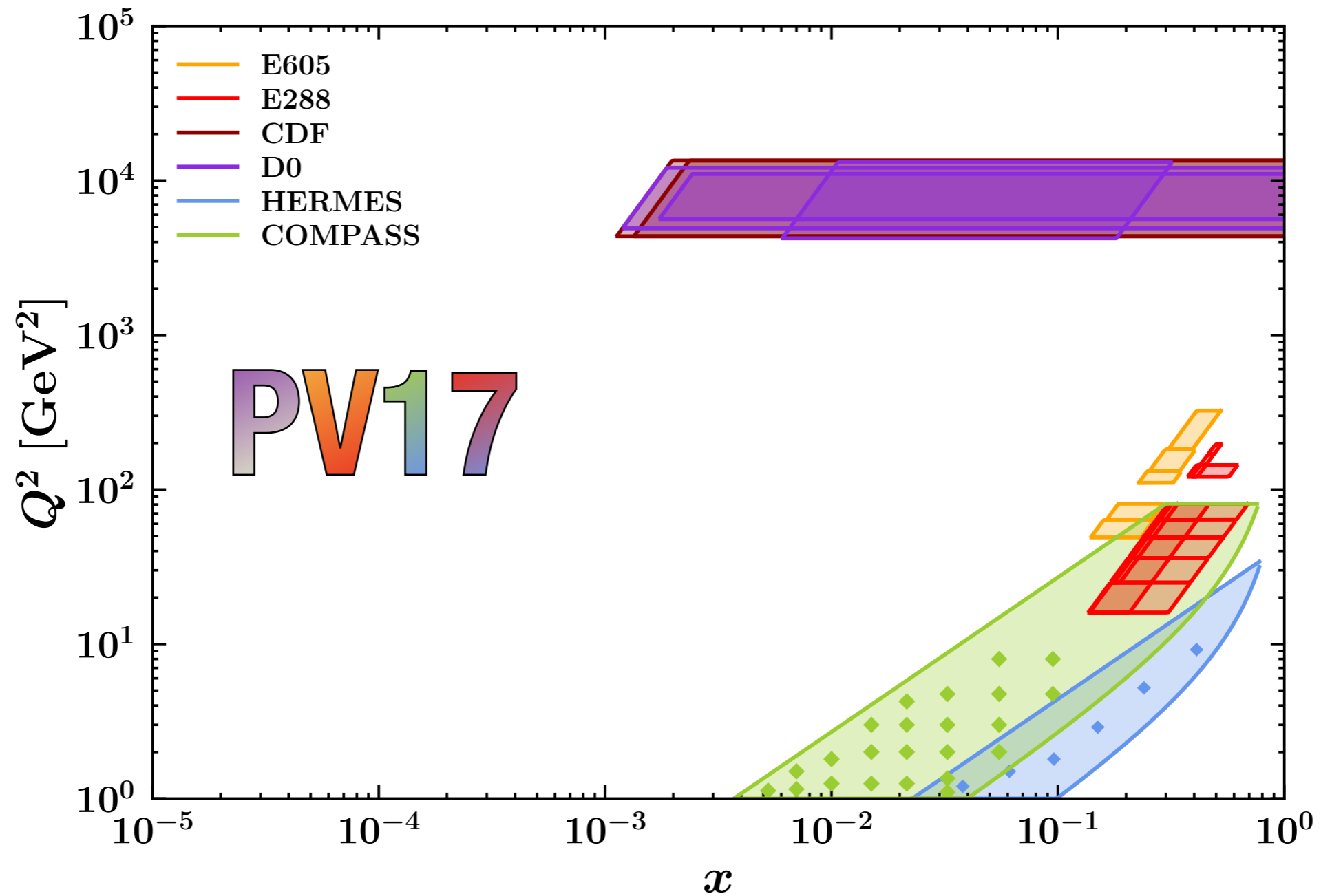
$$\langle x \rangle \pm \Delta x$$

*fairly well determined*

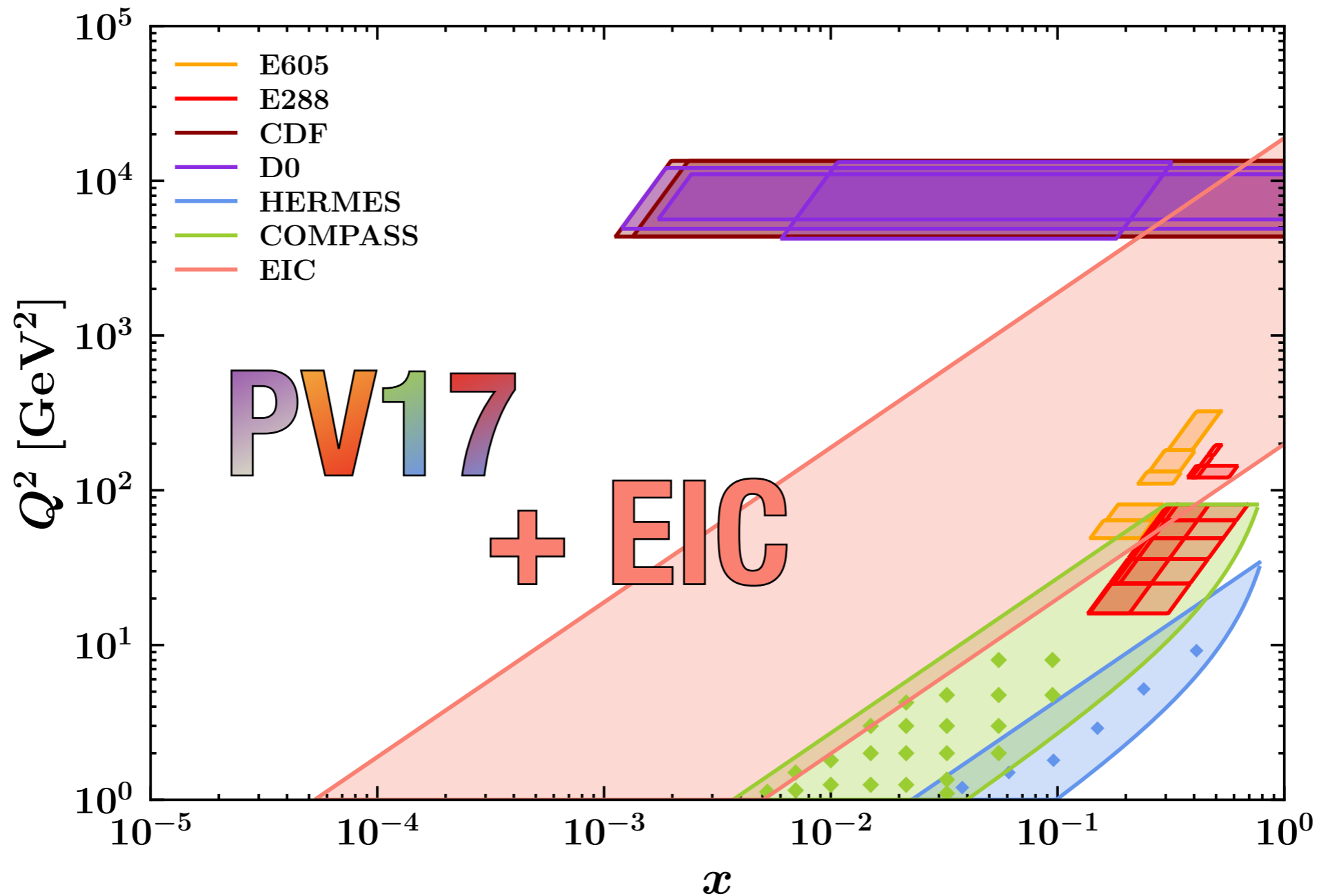
*not very constrained*

**what impact will the EIC have on those uncertainties?**

# What happens to PV17 TMDs ...

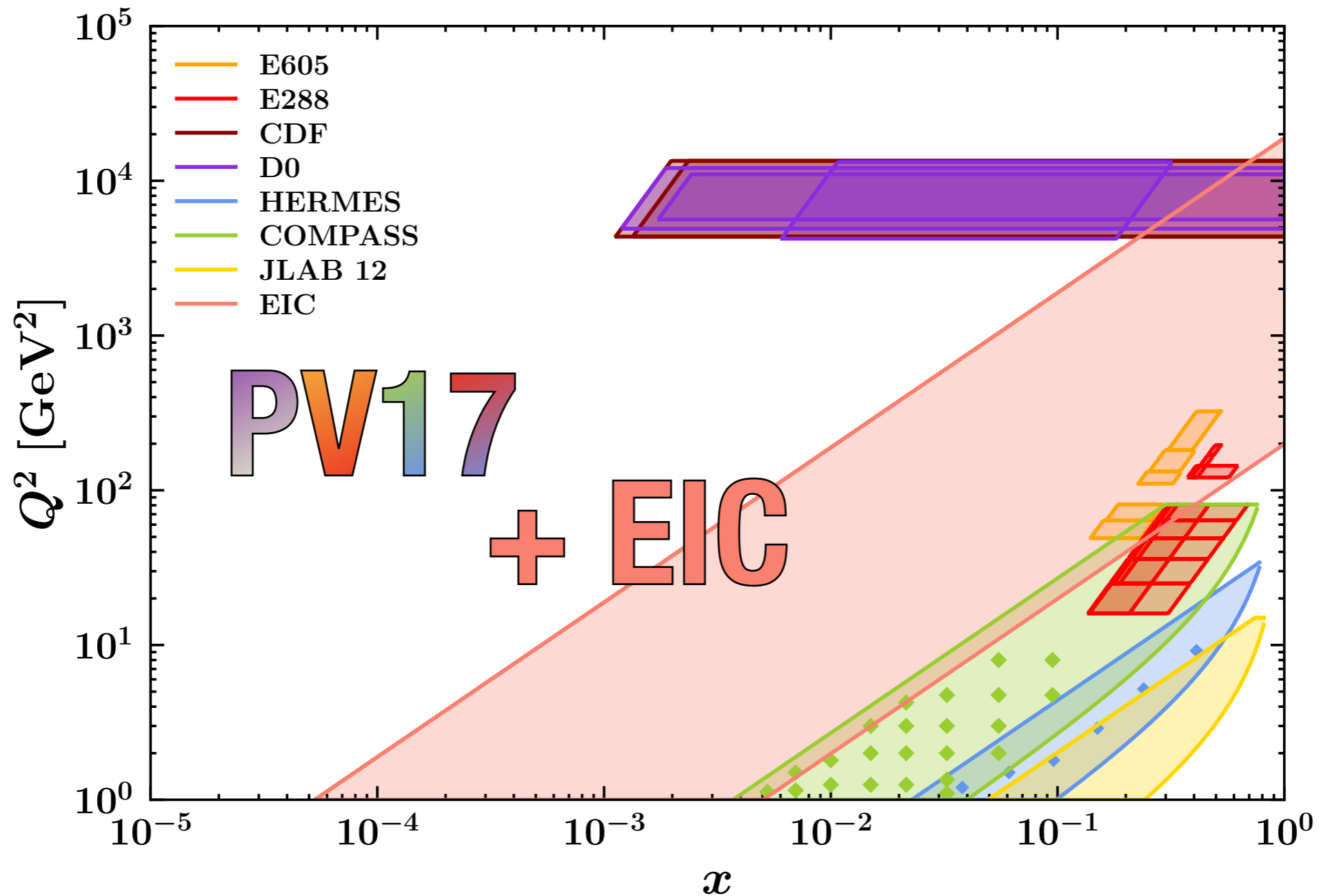


# What happens to PV17 TMDs ...



... once we include EIC data?

# Electron-Ion Collider



*little side-note:*

impact studies can be also done for **JLAB 12**

# EIC pseudodata

generated by Ralf Seidl and available on [https://github.com/VladimirovAlexey/EIC\\_YR\\_TMD](https://github.com/VladimirovAlexey/EIC_YR_TMD)

$\pi^+$   
 $\pi^-$  final state hadrons  
 $K^+$   
 $K^-$

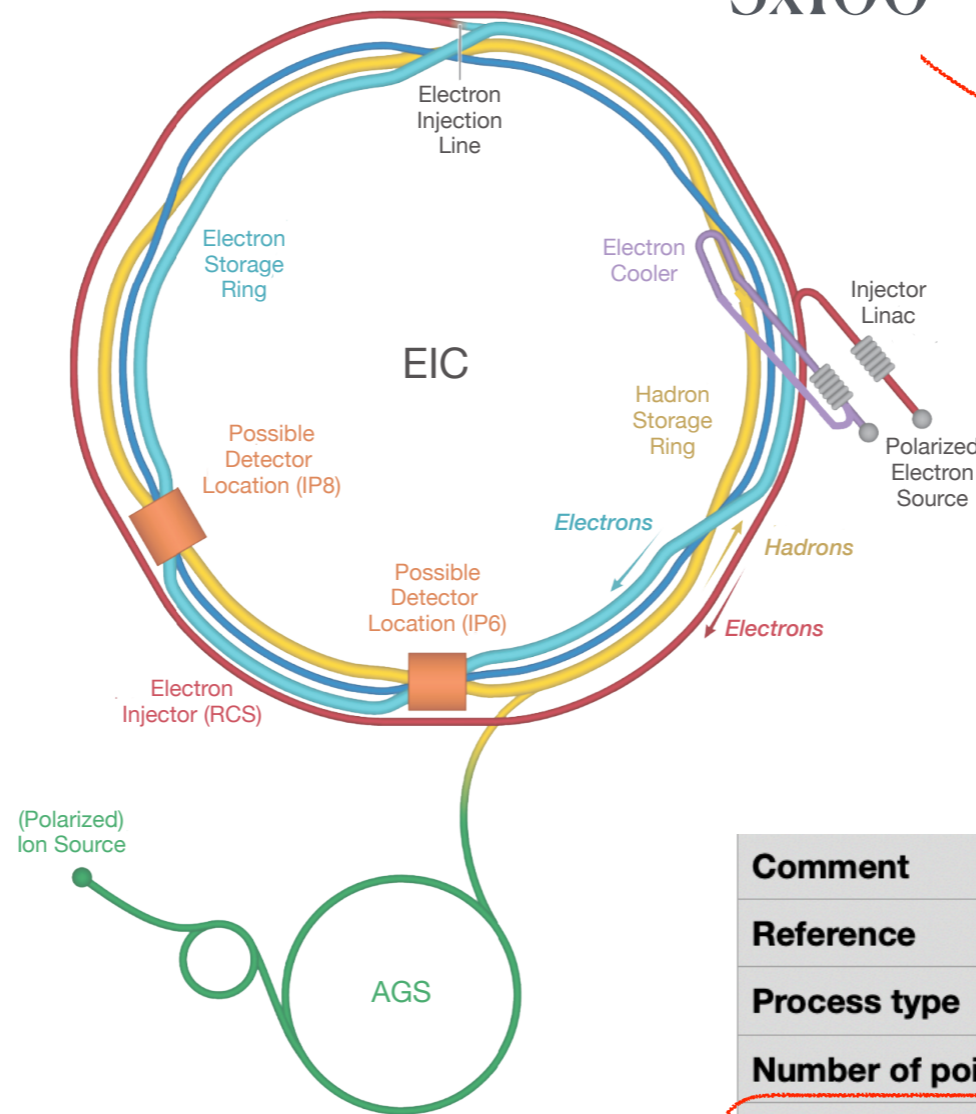


image from EIC YR  
arXiv:2103.05419

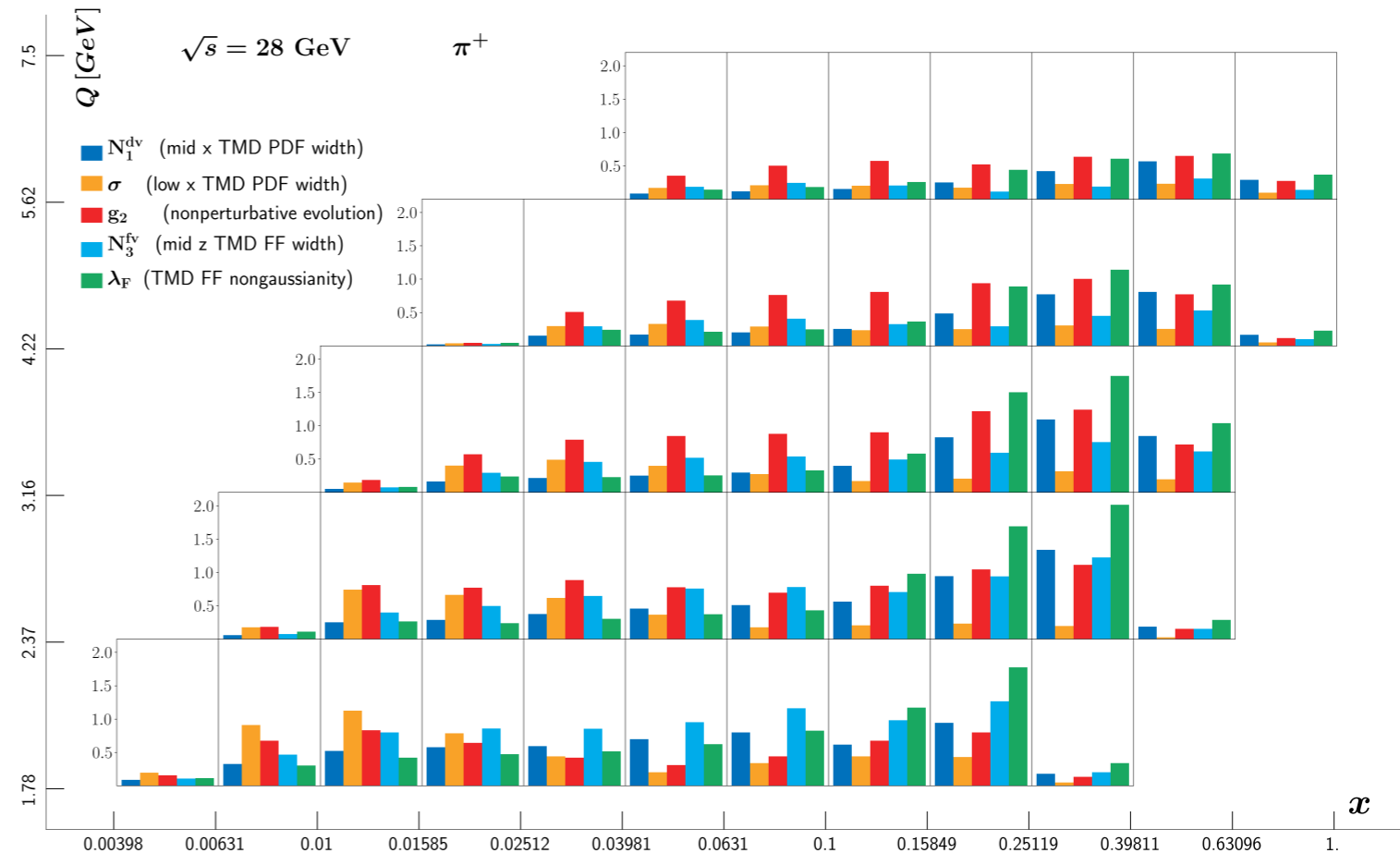
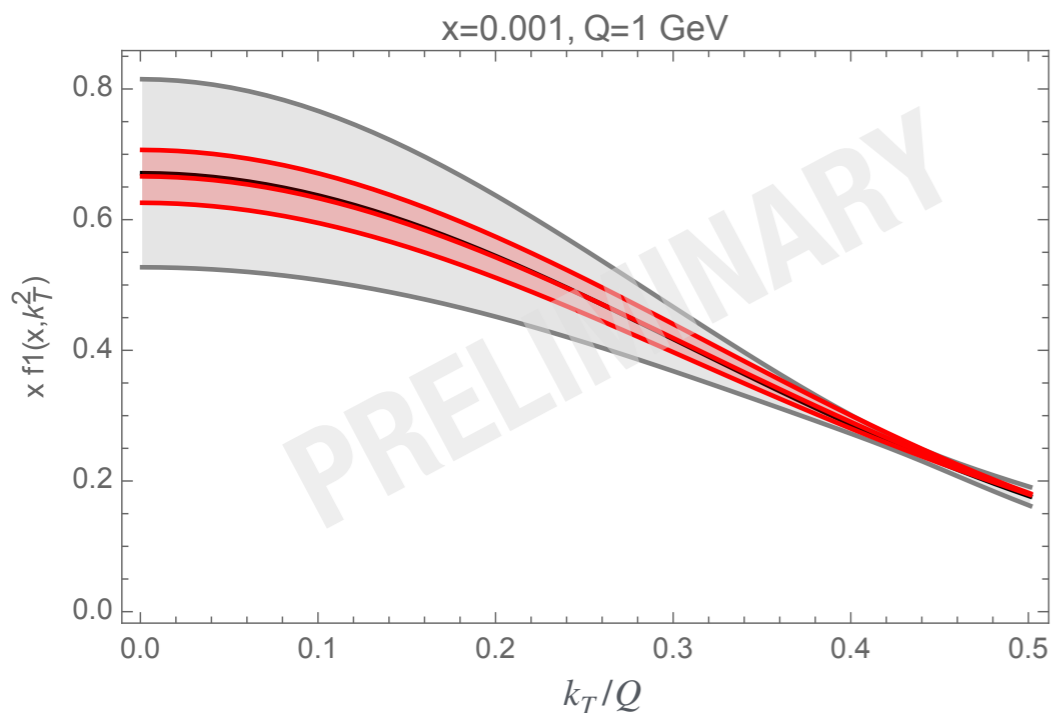
5x41 ————— 18x275  
 configurations  
 5x100 ————— 18x100  
 10x100

Comment	Ralf's pseudo data for EIC.
Reference	Ralf
Process type	SIDIS
Number of points	2958
Number of uncorr.errors	2
Number of corr.errors	<b>uncertainties</b> 0
Number of norm.errors	1
List of norm.errors (relative)	0.03
Total cross-section normalized	False

eight options for EIC settings:  
we choose **option 8**

# A few tools to estimate EIC impact

## reweighting



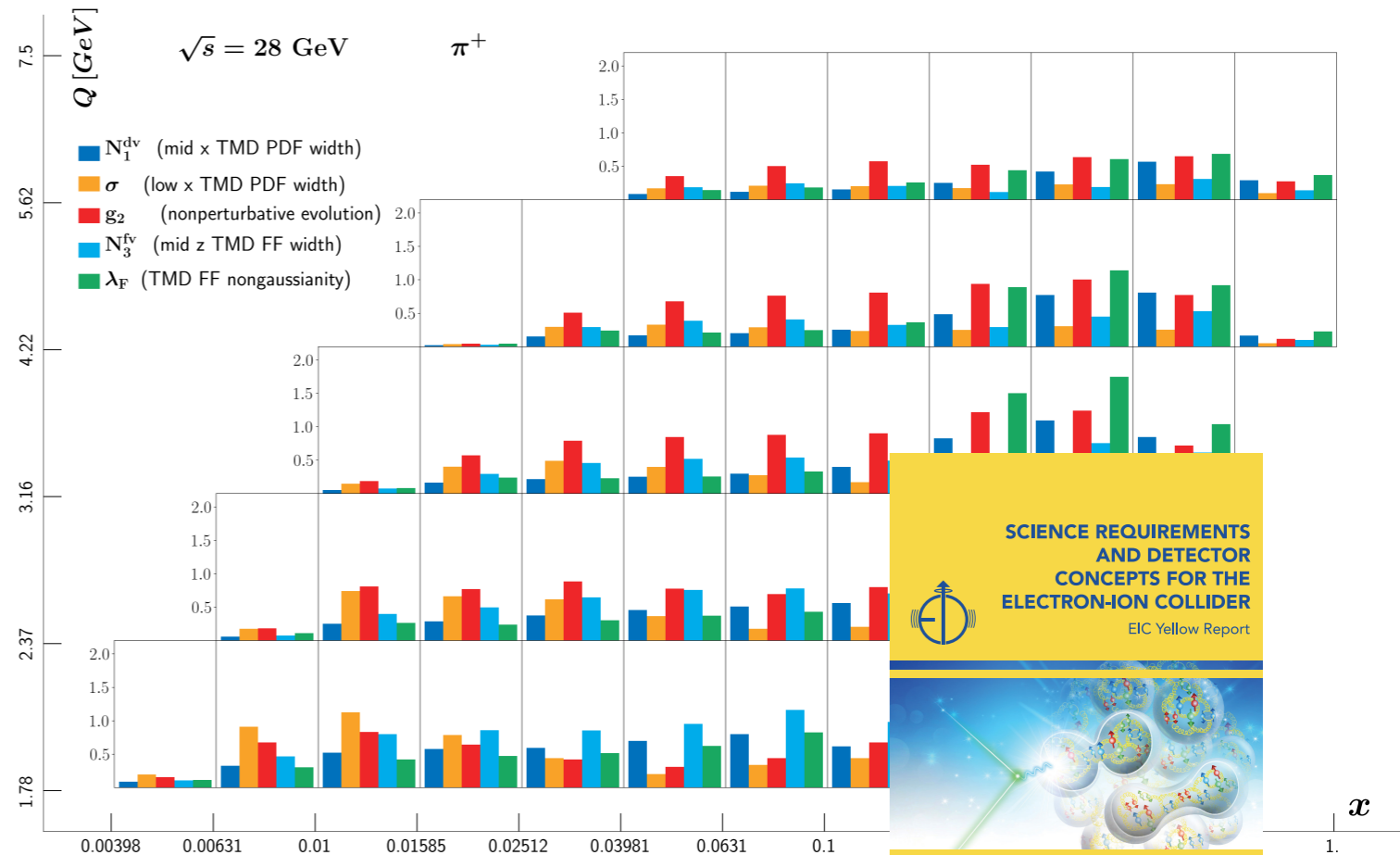
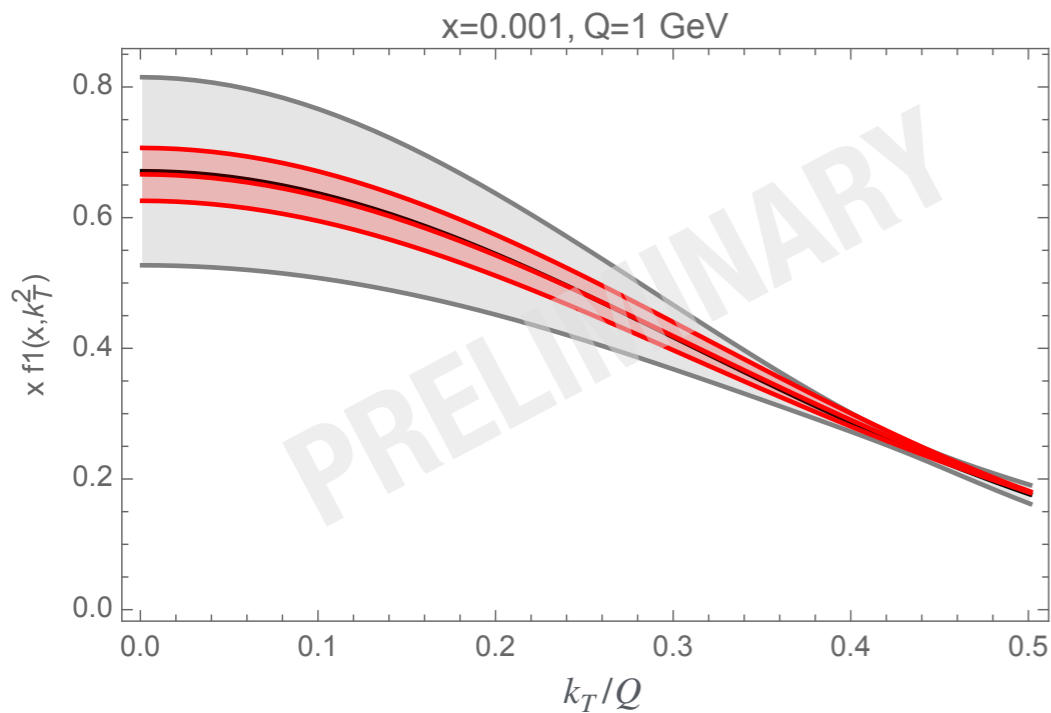
## sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$

# A few tools to estimate EIC impact

reweighing

not satisfactory results



sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$

**NECESSARY**

new fit that includes EIC pseudodata

# Baseline fit

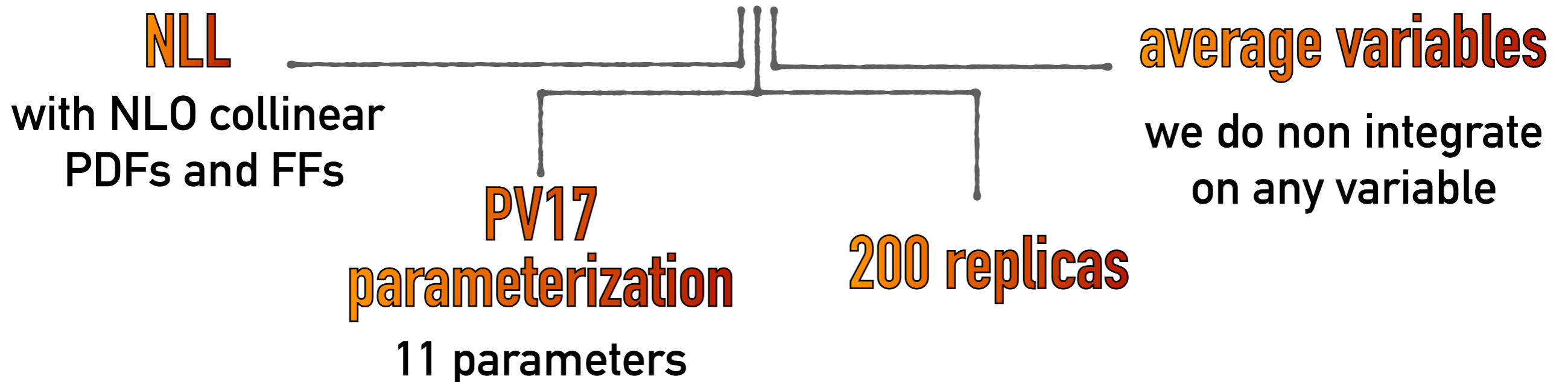
<https://github.com/MapCollaboration/NangaParbat>



with **NangaParbat**  
is not possible to replicate  
*exactly* **PV17 results**

**APFEL++**

**NEW FIT**  
to use as baseline





# Baseline fit - datasets

## NEW FIT

(almost) same datasets as in PV17

global fit



SIDIS



Drell-Yan



2017 COMPASS



Fermilab  
low energy

### remarks:

- ☼ normalization coeff. for Tevatron
- ☼ NO LHC data
- ☼ **uncertainties correlations taken into account**

cut in  $q_T/Q$

**more restrictive than PV17**

$$q_T/Q < \min \left[ \min[0.2/z, 0.5] + 0.3/zQ, 1 \right]$$

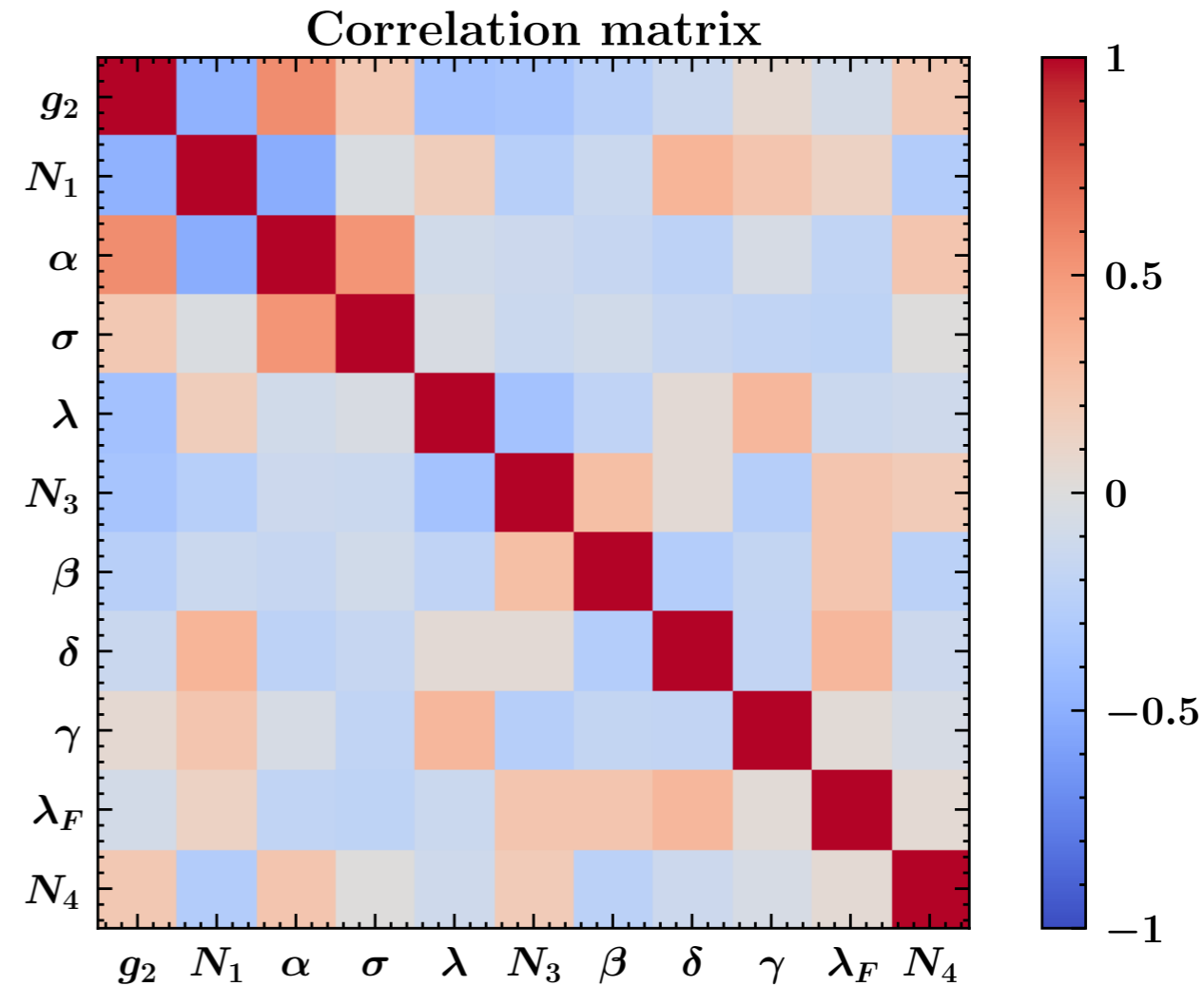
**2516**

data points

# Baseline fit - results

## parameters

Parameter	Average over replicas à la PV17	
$g_2$	0.118	$\pm 0.013$
$N_1$	0.288	$\pm 0.030$
$\alpha$	2.473	$\pm 1.538$
$\sigma$	-0.142	$\pm 0.110$
$\lambda$	0.069	$\pm 0.285$
$N_3$	0.220	$\pm 0.021$
$\beta$	2.948	$\pm 0.966$
$\delta$	0.115	$\pm 0.0316$
$\gamma$	2.454	$\pm 0.151$
$\lambda_F$	7.022	$\pm 3.298$
$N_4$	0.0317	$\pm 0.006$



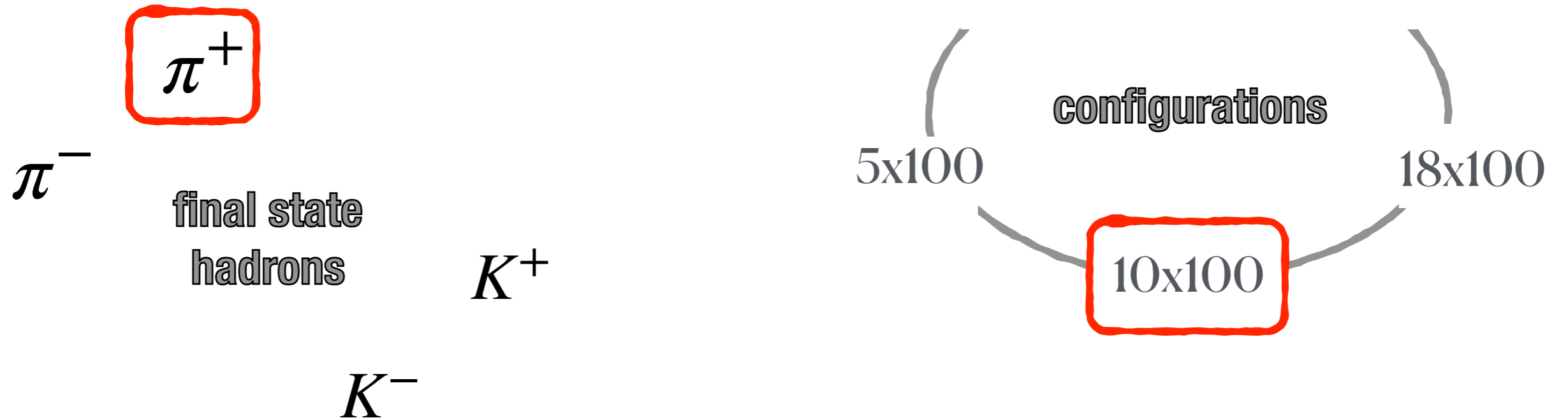
with **NangaParbat**

a **new fit** with uncertainties  
*similar* to **PV17**

$$\chi_{\text{d.o.f.}}^2 = 1.14 \pm 0.06$$

# Step towards PV17 + EIC fit

generation of pseudo data



**~ 2500**  
pseudodata points

**central value** of pseudo data obtained  
using average parameters of the PV17 baseline fit

**uncertainties** of pseudo data  
are given by simulations

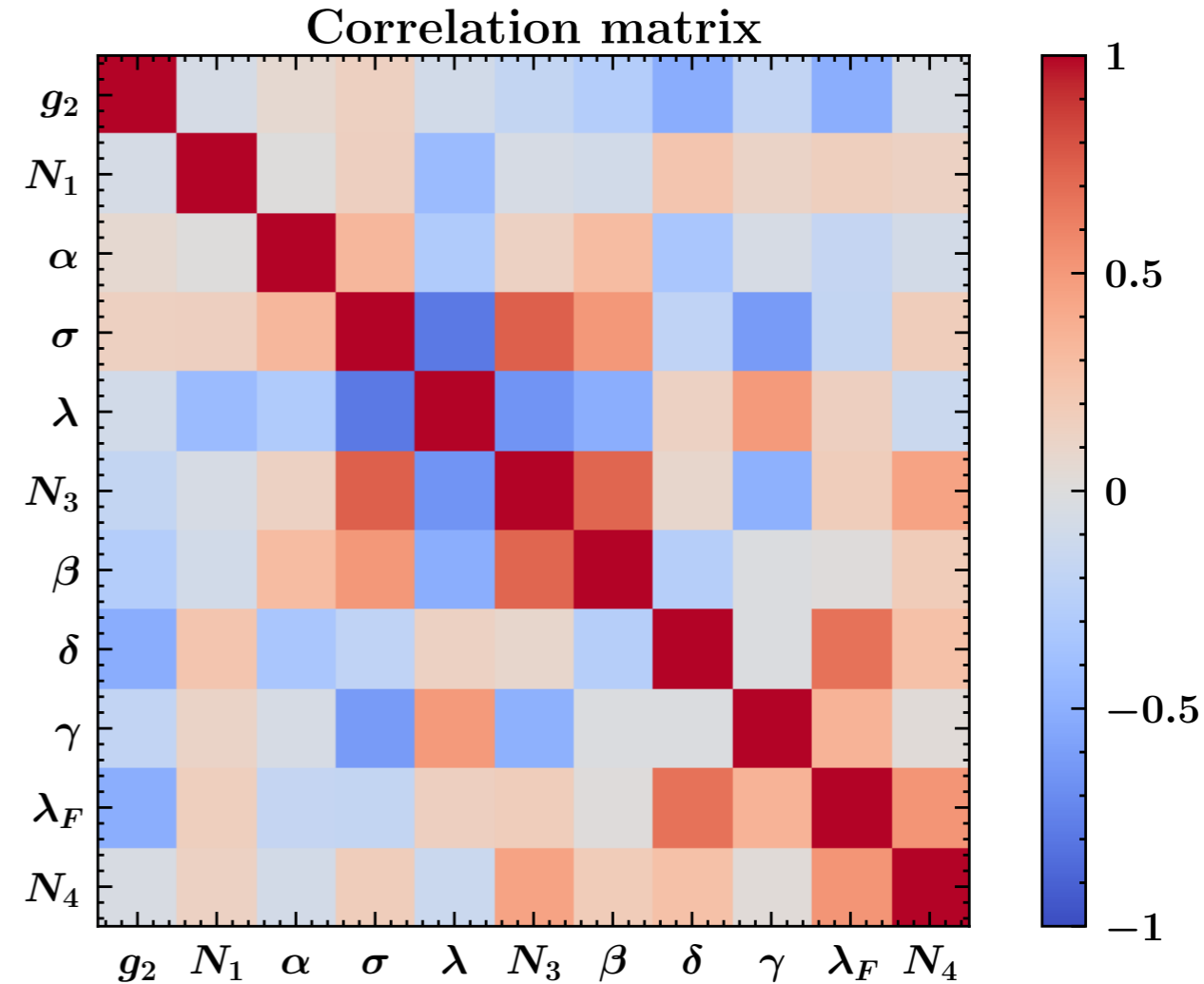
# PV17 baseline + EIC fit

## impact on parameters

$$g_K(b_T) = -g_2 b_T^2 / 2$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$g_{3,4}(z) = N_{3,4} \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$



$$f_{1\text{NP}}^a(x, \mathbf{k}_\perp^2) = \frac{1}{\pi} \frac{(1 + \lambda \mathbf{k}_\perp^2)}{g_{1a} + \lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

$$\chi_{\text{d.o.f.}}^2 = 0.88 \pm 0.32$$

$$D_{1\text{NP}}^{a \rightarrow h}(z, \mathbf{P}_\perp^2) = \frac{1}{\pi} \frac{1}{g_{3a \rightarrow h} + (\lambda_F / z^2) g_{4a \rightarrow h}^2} \left( e^{-\frac{\mathbf{P}_\perp^2}{g_{3a \rightarrow h}}} + \lambda_F \frac{\mathbf{P}_\perp^2}{z^2} e^{-\frac{\mathbf{P}_\perp^2}{g_{4a \rightarrow h}}} \right)$$

# Impact of EIC

PV17 baseline

reduction by factor 10

PV17 baseline + EIC

Average over replicas à la PV17		Parameter	Average over replicas à la PV17	
0.118	$\pm 0.013$	$g_2$	0.129	$\pm 0.002$
0.288	$\pm 0.030$		0.300	$\pm 0.036$
2.473	$\pm 1.538$		3.409	$\pm 1.079$
-0.142	$\pm 0.110$		0.2147	$\pm 0.073$
0.069	$\pm 0.285$	$\lambda$	0.136	$\pm 0.551$
0.220	$\pm 0.021$	$N_3$	0.213	$\pm 0.0149$
2.948	$\pm 0.966$	$\beta$	2.039	$\pm 0.238$
0.115	$\pm 0.0316$	$\delta$	0.090	$\pm 0.0181$
2.454	$\pm 0.151$	$\gamma$	2.439	$\pm 0.1114$
7.022	$\pm 3.298$	$\lambda_F$	4.497	$\pm 1.0423$
0.0317	$\pm 0.006$	$N_4$	0.0331	$\pm 0.002$

non - perturbative evolution

PRELIMINARY

# Impact of EIC

PV17 baseline

Average over replicas à la PV17	
0.118	$\pm 0.013$
0.288	$\pm 0.030$
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7.022	$\pm 3.298$
0.0317	$\pm 0.006$

reduction by  
factor 10

Parameter
$g_2$
$N_1$
$\alpha$
$\sigma$
$\lambda$
$N_3$
$\beta$
$\delta$
$\gamma$
$\lambda_F$
$N_4$

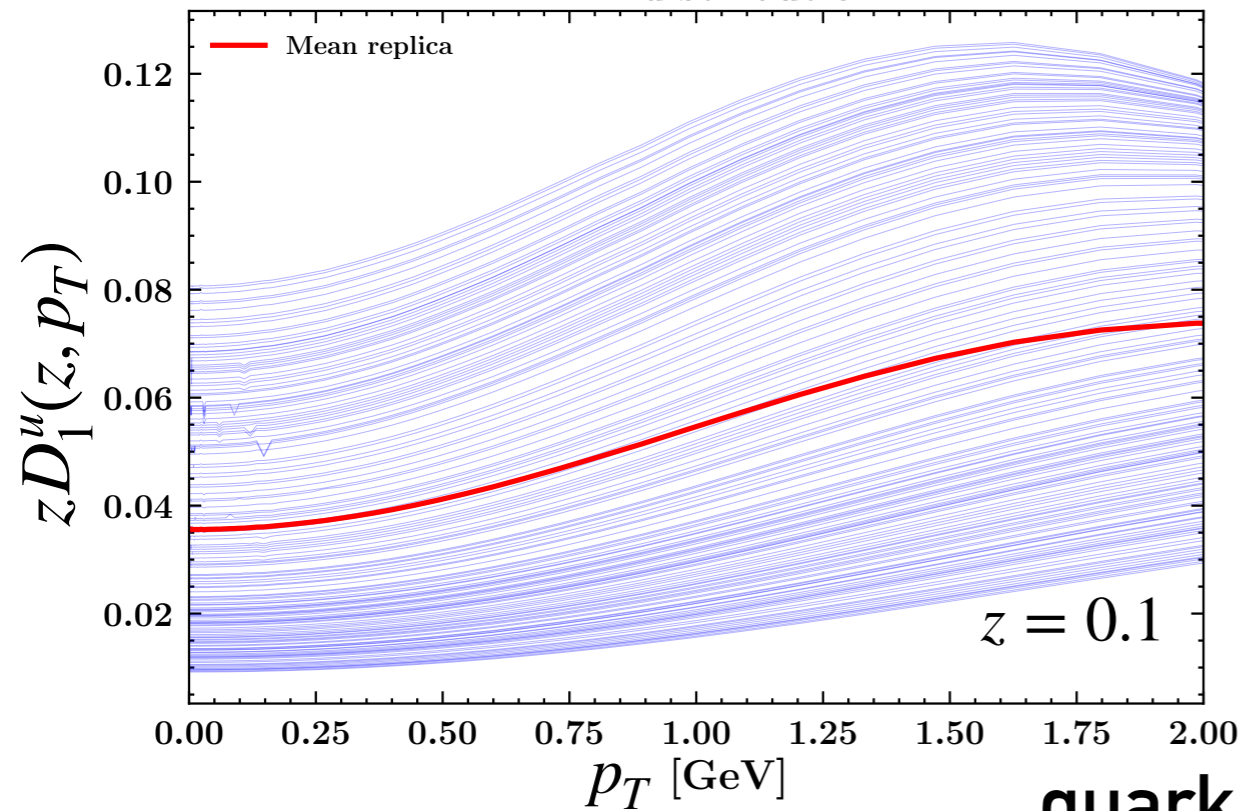
PV17 baseline  
+ EIC

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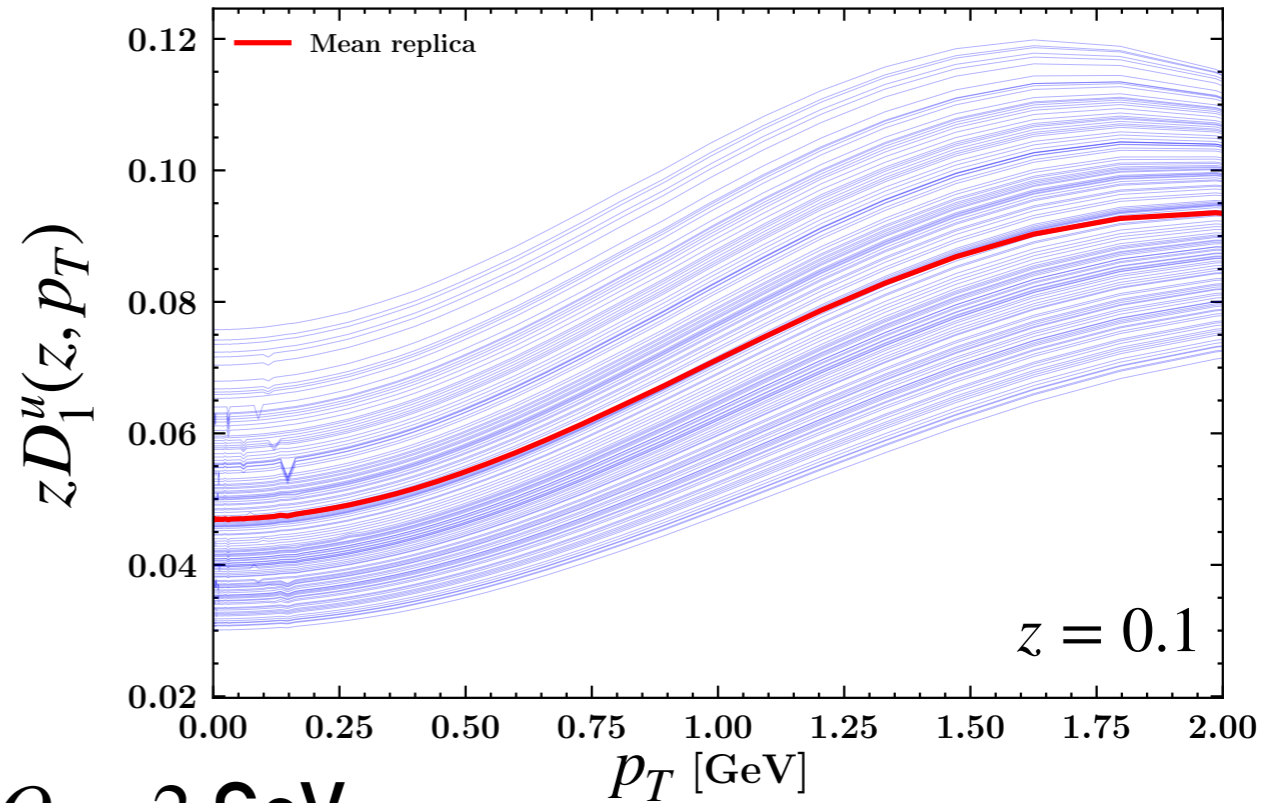
general improvement of uncertainties

# Impact on unpolarized TMD FFs

TMD distribution

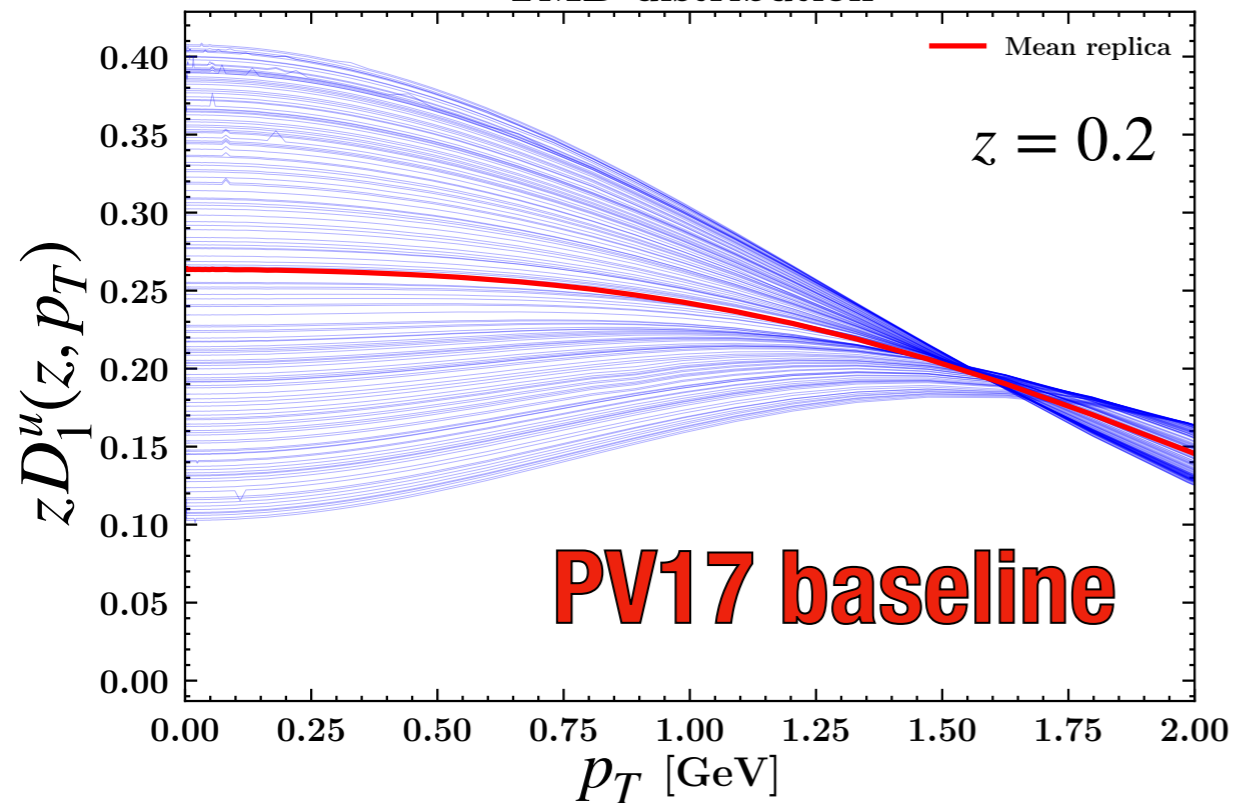


TMD distribution

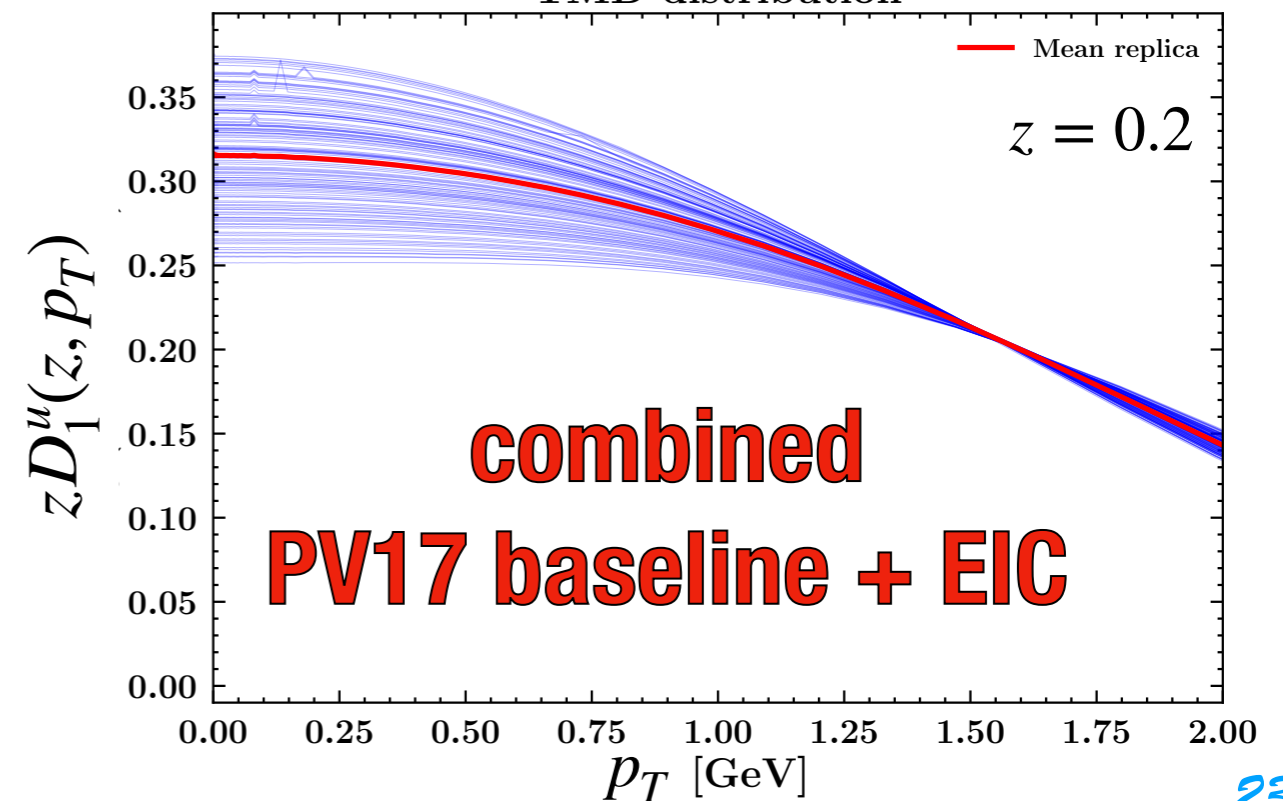


quark up,  $Q = 2$  GeV

TMD distribution



TMD distribution



# Conclusions

**EIC** will have a very big **impact** on **TMDs**

will cover a large region  
not covered by present data

from **impact studies**,

and in particular from **EIC PSEUDO DATA FITS**

we have **encouraging results** on **uncertainties reduction**

## **CAVEAT**

results depend on  
the chosen parameterization

**preliminary work**

need to include also  
other EIC energy configurations



Backup

# EIC impact studies

## sensitivity coefficients

E. Aschenauer, I. Borsa, G. Lucero, A. S. Nunes, R. Sassot

arXiv:2007.08300

$F_{UU,T}(x, z, q_T; Q^2)$  — **observable** — **distribution** — **TMD parameters**

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \Delta \mathcal{O} \Delta f_i}$$

experimental uncertainty  
(from pseudodata)

$$\xi \equiv \frac{\delta \mathcal{O}}{\Delta \mathcal{O}}$$

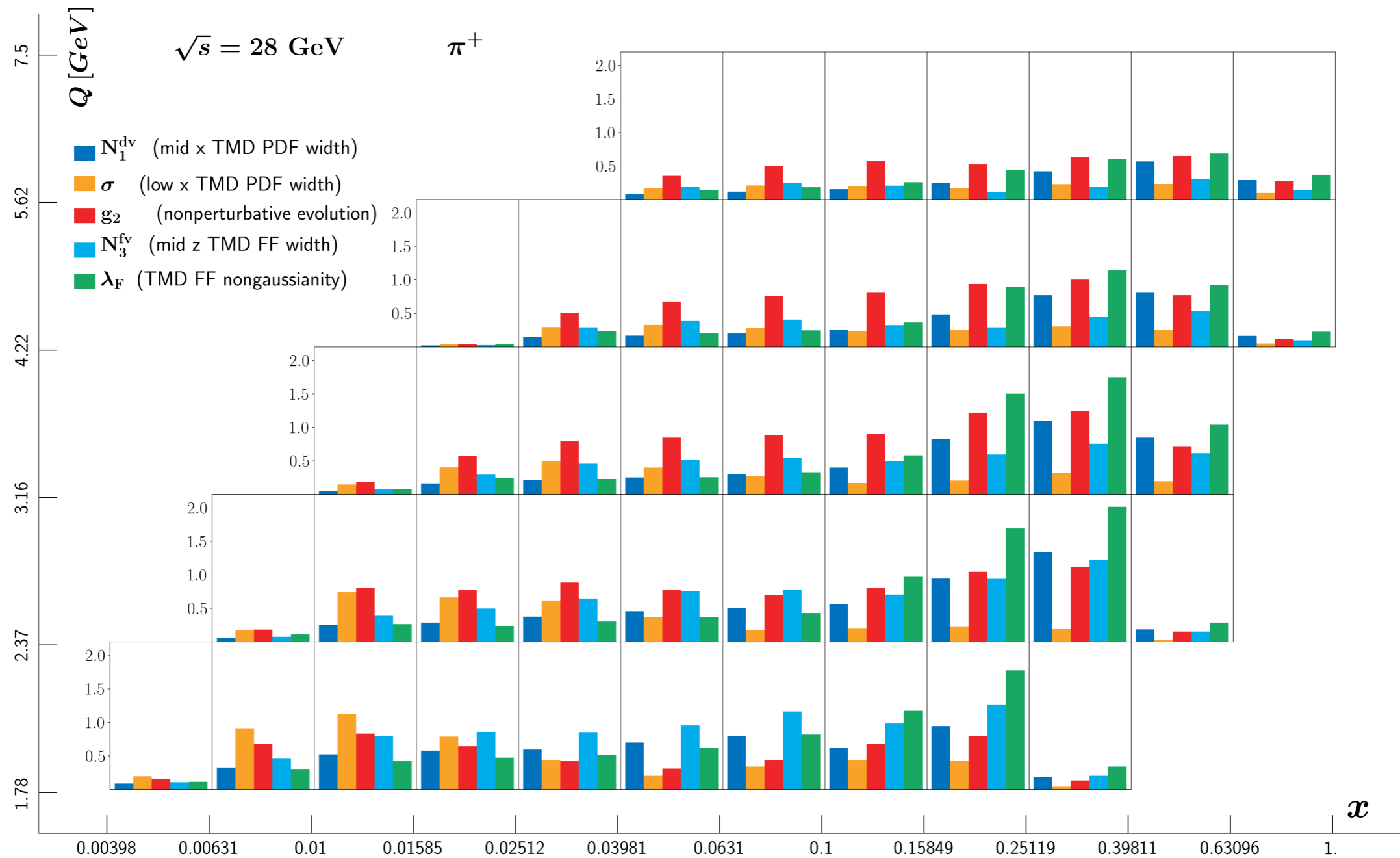
~~theoretical uncertainty~~

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{k=1}^N \mathcal{O}[f_i^{(k)}]$$

# ELC impact studies

## sensitivity coefficients

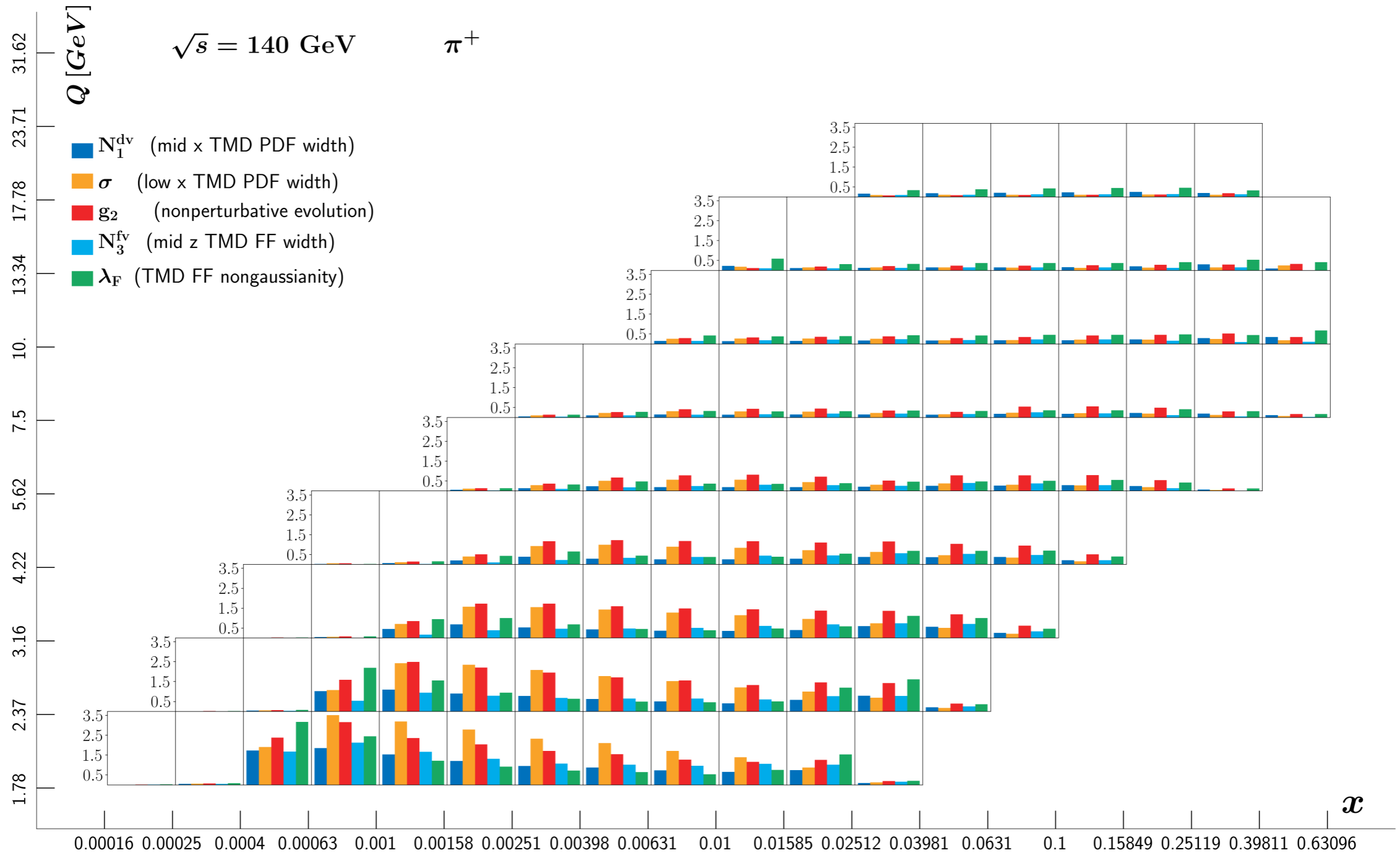
$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$



# ELC impact studies

## sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$



# Sensitivity coefficient and standard deviation

## how to estimate uncertainties reduction?

$$S[\mathcal{O}, f] = \frac{\langle \mathcal{O} f \rangle - \langle \mathcal{O} \rangle \langle f \rangle}{\delta \mathcal{O} \Delta f}$$

from **PV17**

we know a parameter A with error  $\Delta A$

☀ if we perform a new measurement that produces on A an error equal to its initial standard deviation,  $\delta A = \Delta A$

→ this corresponds to  $S(A) = 1$



in fact, if A can be ideally considered as parameter and observable, then

☀ the error on A scales as  
 $1/\sqrt{2} = 1/\sqrt{1 + (S = 1)}$

$$S(A, A) = \frac{\langle A A \rangle - \langle A \rangle \langle A \rangle}{\delta A \Delta A} = \frac{(\Delta A)^2}{\Delta A \Delta A} = 1$$

if the new measurement is more precise, then  $S > 1$  and the error is further reduced

for n measurements, the error on A should scale as

$$1/\sqrt{1 + S_1 + \dots + S_n}$$

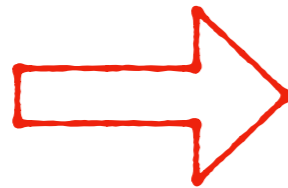
# Non perturbative evolution $g_2$

## which energy configuration has the highest impact?

summing over all  $(x, Q^2)$  bins

**PV17 fit**

$$\Delta g_2 = 0.01$$



$$R(A) = \frac{(\Delta A)_{\text{prev}}}{(\delta A)_{\text{EIC}}}$$

run at $\sqrt{s} = 28$ GeV $\pi^+$	→	0.00155
run at $\sqrt{s} = 44$ GeV $\pi^+$	→	0.00120
run at $\sqrt{s} = 63$ GeV $\pi^+$	→	0.00108
run at $\sqrt{s} = 84$ GeV $\pi^+$	→	0.00105
run at $\sqrt{s} = 140$ GeV $\pi^+$	→	0.00096

$$R(g_2) = 6.45$$

$$R(g_2) = 8.33$$

$$R(g_2) = 9.26$$

$$R(g_2) = 9.52$$

$$R(g_2) = 10.36$$

(no correlation between measurements in different bins)

consistent trend:  
 evolution parameter better constrained by larger covered  $(x, Q^2)$   
 → larger  $\sqrt{s}$  configuration

# Attempts at reweighting

200 replicas are compared  
with pseudodata

$$\chi_k^2 = \chi_{k,\text{EIC}}^2 + \chi_{k,\text{PV17}}^2$$

'original'  $\chi^2$   
with respect to PV17 data

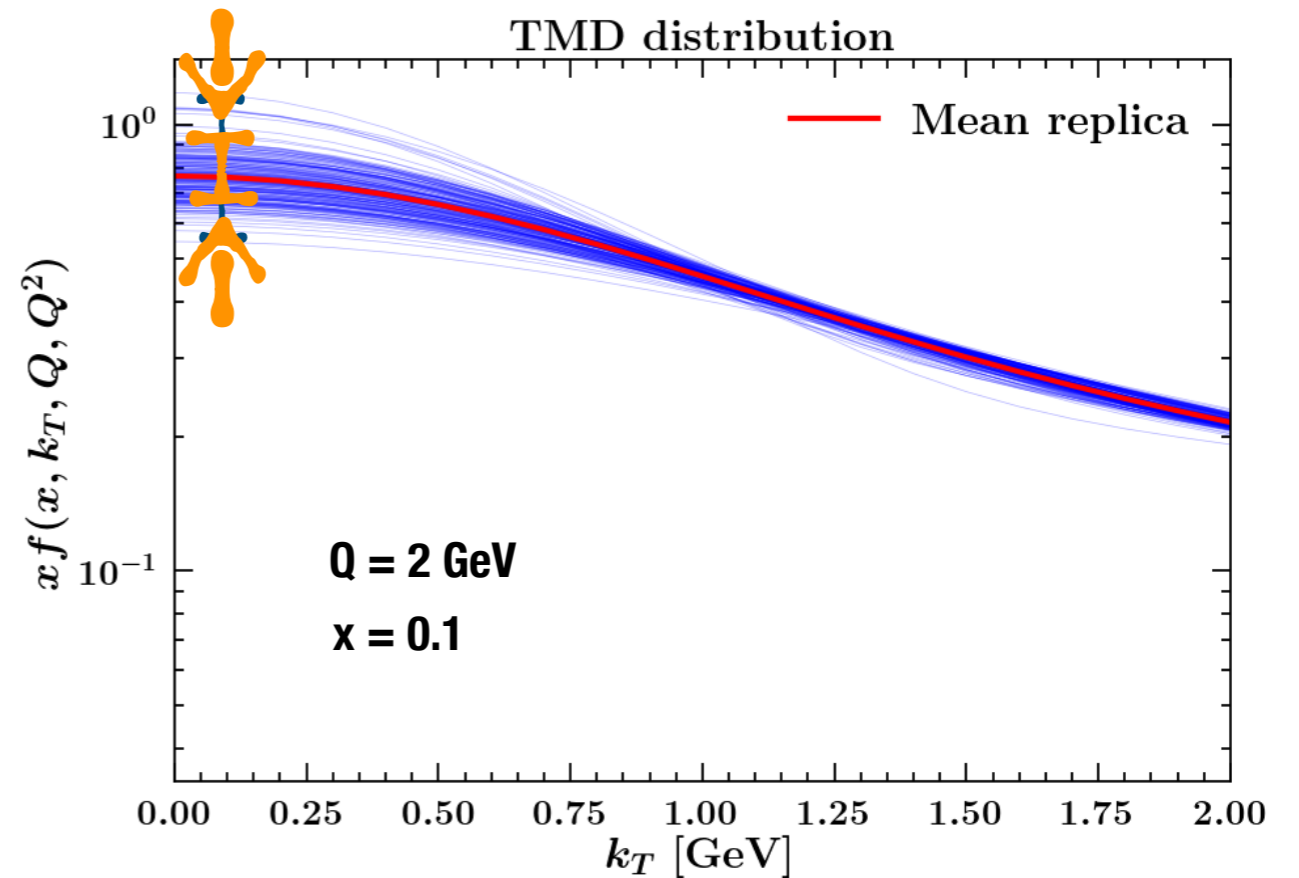
**weights**

$$w_k \propto \mathcal{P}(f_k | \chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

**used to select replicas**

from NNPDF Collaboration  
[arXiv:1108.1758](https://arxiv.org/abs/1108.1758)

reflect the impact of EIC data on extracted TMDs



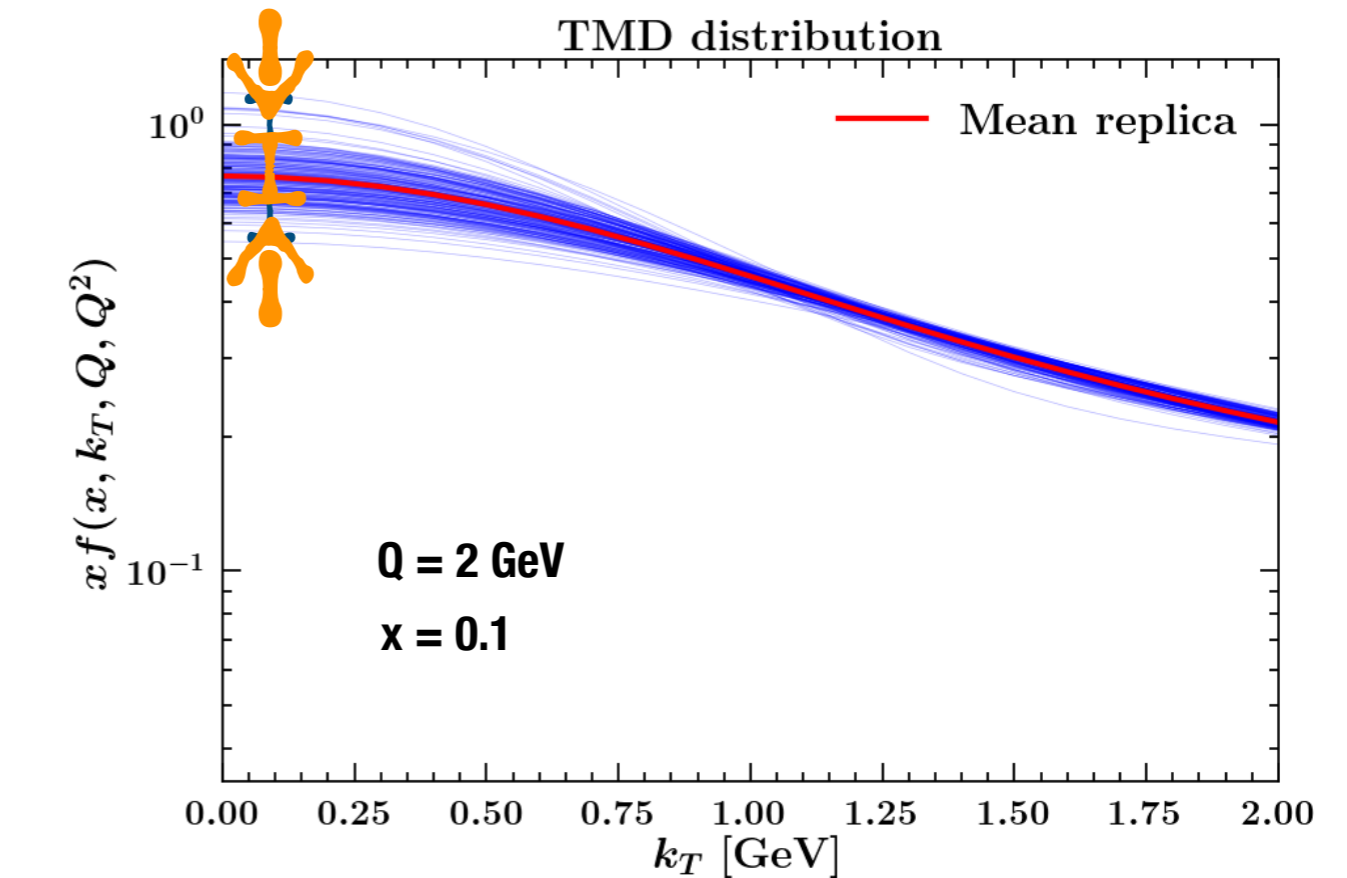
# Attempts at reweighting

different mathematical formulas  
to compute the weights

N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020;  
H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

$$w_k \propto \mathcal{P}(f_k | \chi_k) \propto e^{-\frac{1}{2} \chi_k^2}$$

selects replicas with very low  $\chi^2$



NNPDF Collaboration  
[arXiv:1108.1758](https://arxiv.org/abs/1108.1758)

$$w_k \propto \mathcal{P}(f_k | \chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2} \chi_k^2}$$

suppresses replicas with very  
high AND very low  $\chi^2$

total number  
of points