

# Higher twist fragmentation and mass generation

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University of Pavia and INFN

Sardinian Workshop on Spin (SarWorS) 2021

September 6, 2021



# Outline

Quark hadronization, propagation, mass generation

Inclusive jets

Semi-inclusive processes



# Some references:

A [selection of references](#) related to the topics discussed in this talk:

- ▶ Collinear factorization for deep inelastic scattering structure functions at large Bjorken  $x_B$

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- ▶ Quark fragmentation as a probe of dynamical mass generation

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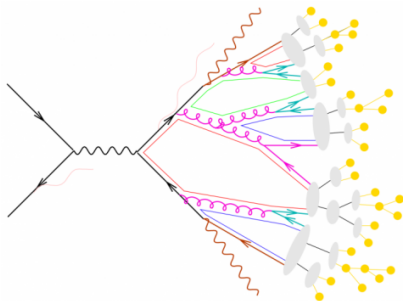
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- ▶ Pion parton distribution and fragmentation functions beyond the leading twist in a confining Nambu–Jona-Lasinio model  
I. Cloet, [A. Signori](#) - in preparation

# Fragmentation functions

**Hadronization:** dynamical generation of hadronic properties from quarks/gluons  
→ fundamental topic

It follows any QCD hard scattering event and populates the final states with hadrons.

Maps of hadronization in momentum space: fragmentation functions (FFs)

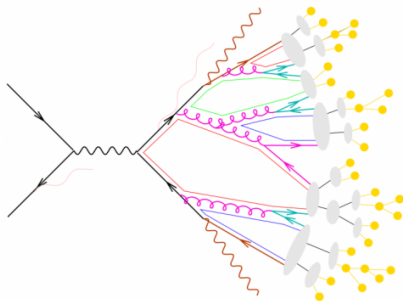


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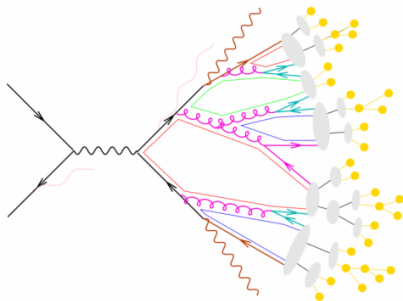


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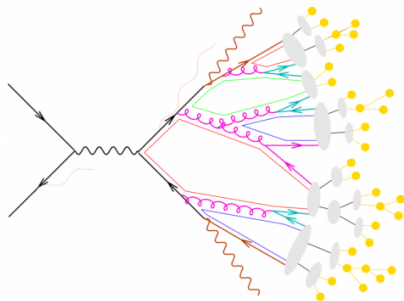
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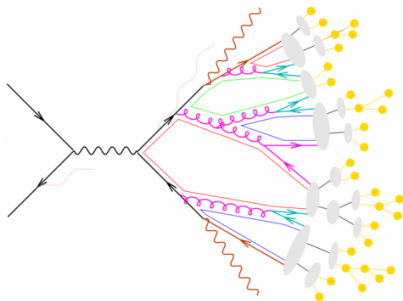
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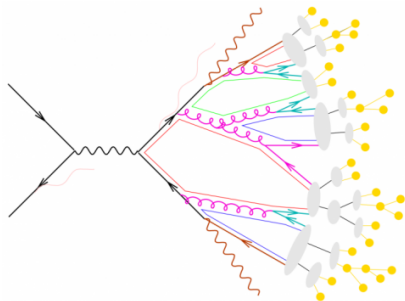
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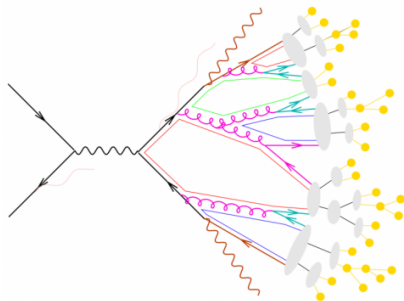
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What generates the masses of partons and hadrons?

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The dynamical generation of mass in QCD can be addressed in different ways:

- ▶ gap equation  
e.g. in the NJL model of QCD:  $M_q = m_q - 4G_\pi \langle \bar{q}q \rangle \gg m_q$
- ▶ Energy Momentum Tensor  $\rightarrow$  hadron mass decomposition

$$\left( \text{---} \text{---} \text{---} \right)^{-1} = \left( \text{---} \text{---} \text{---} \right)^{-1} + \text{---} \text{---} \text{---}$$

The diagram illustrates the gap equation for the quark propagator. On the left, a horizontal line with a grey circle in the middle represents the inverse propagator with a dynamical mass  $M_q$ . The momentum is  $p$ . This is equal to the sum of two terms. The first term is a horizontal line with a grey circle in the middle, representing the inverse propagator with a bare mass  $m_q$ , also with momentum  $p$ . The second term is a loop diagram. It consists of a horizontal line with a grey circle in the middle, representing a quark with mass  $\nu$  and momentum  $k$ . A wavy line, representing a pion with mass  $\mu$ , forms a loop with the quark line. The pion has momentum  $q = p - k$ . The loop is attached to the quark line at two points, with the external momentum  $p$  entering from the left and exiting to the right.

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- ▶ Energy Momentum Tensor  $\rightarrow$  hadron mass decomposition
- ▶ “mass sum rule” for **fragmentation functions** - **new and observable!**

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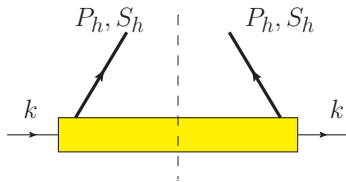
Inclusive jets

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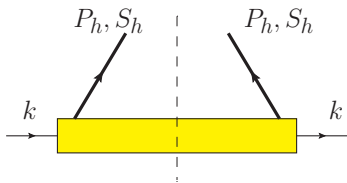
# Quark 1h-FFs

$$\Delta_{ij}(k, P_h, S_h) = \int \frac{d^4\xi}{(2\pi)^4} e^{ikx} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{T} W_1(\infty, \xi) \psi_i(\xi) a^\dagger a \bar{\psi}_j(0) W_2(0, \infty) | \Omega \rangle$$



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		U	L	T
hadron pol.	U	$D_1$		$H_1^\perp$
	L		$G_{1L}$	$H_{1L}^\perp$
	T	$D_{1T}^\perp$	$G_{1T}$	$H_1, H_{1T}^\perp$

8 (TMD) fragmentation functions at leading twist

# Quark higher twist 1h-FFs

Twist 3 transverse momentum dependent FFs  $\mathcal{D}_{\dots}^{a \rightarrow h}(z, P_{h\perp}^2)$   
for a quark hadronizing into a spin 1/2 hadron

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hadron pol.

Black and magenta: survive transverse momentum integration

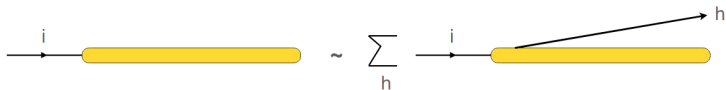
Red and magenta: T-odd

Blue: T-even, w/o collinear counterpart

# Inclusive jets and 1h-FFs

Accardi, Signori 1903.04458 - PLB  
Accardi, Signori 2005.11310 - EPJC

“Fully” inclusive jet *correlator* (quark propagator)  
 $\equiv$  inclusive limit of 1h-fragmentation *correlator*

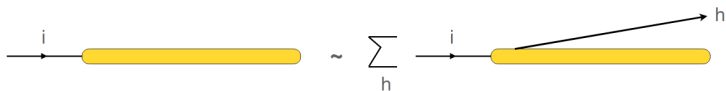




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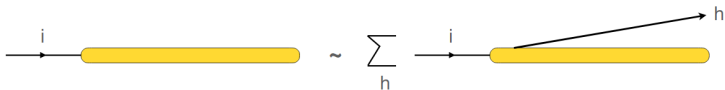


$$k^\mu \Xi^i(k) = \sum_{h, S_h} \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2) P_h^\mu \Delta^{i \rightarrow h}(k, P_h, S_h)$$

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Dirac projections:

**momentum sum rules** for FFs in terms of quark propagator

# Quark propagator

Källén-Lehman representation in terms of **spectral functions**  $\rho_{1,3}$ :

$$\Xi(k) \rightarrow S_F(k) = \int \frac{d\mu^2}{(2\pi)^4} \{ \not{k} \rho_3(\mu^2) + \sqrt{\mu^2} \rho_1(\mu^2) \mathbb{I} \} \frac{\theta(\mu^2)}{k^2 - \mu^2 + i\epsilon}$$

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The **non-perturbative** structure of the **jet** is **trivial** at **twist 2**, **but not at twist 3**

# Quark/jet mass

“Mass sum rule” for twist 3  $E$  fragmentation function:

$$\sum_h \int dz M_h E^h(z) = M_j$$

quark/jet dynamical mass  $M_j$  as the  
“average” of produced hadron masses  
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Full QCD:  $M_j = m_q + m_q^{corr}$  (current and dynamical components), where

$$\sum_h \int dz M_h \tilde{E}^h(z) = M_j - m_q = m_q^{corr}$$

$\tilde{E}$  and  $m_q^{corr}$  probe quark-gluon-quark  $\sim \langle 0 | \bar{\psi} A \psi | 0 \rangle$  dynamical correlations

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estimate of  $M_j$  in **NJL model**

evolution: large- $N_c$  and LO in  $\alpha_s$

(A. Belitsky - hep-ph/9703432)

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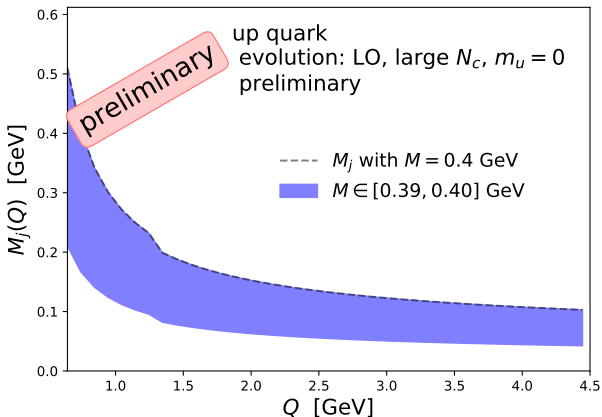
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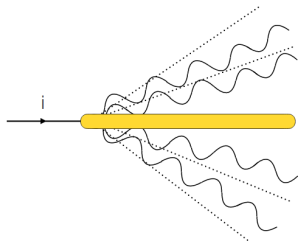
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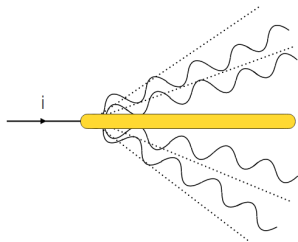


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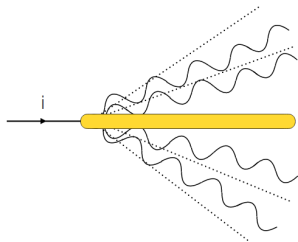
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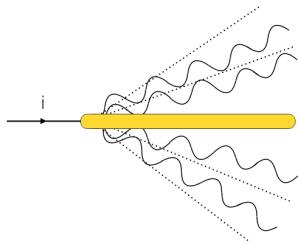


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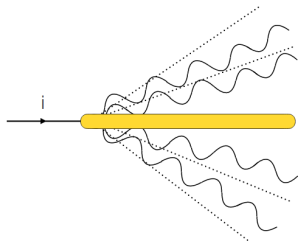
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- ▶ non-perturbative quark propagation (yellow blob)

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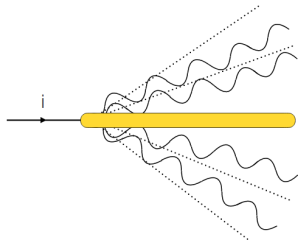
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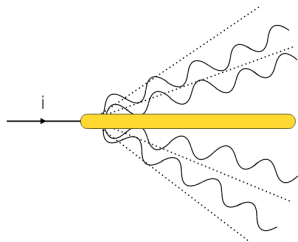
Procura, Stewart 0911.4980 - PRD  
Jain, Procura, Waalewijn 1101.4953 - JHEP



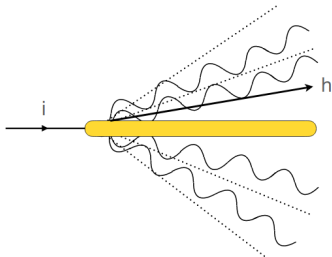
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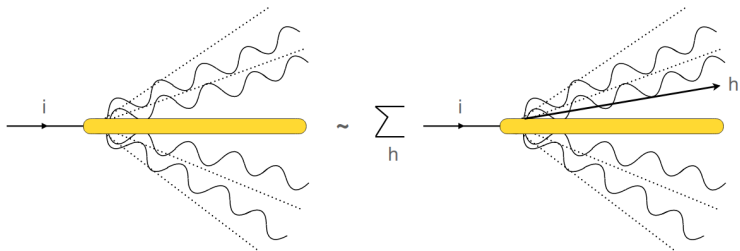
Inclusive jet function  $J_i(s)$ :  
sensitive to the jet virtuality  $s$   
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Fragmenting jet function (FJF)  $\mathcal{G}^{i \rightarrow h}(s, z)$ :  
sensitive to jet virtuality  $s$   
and hadron momentum fraction  $z$   
(less inclusive)

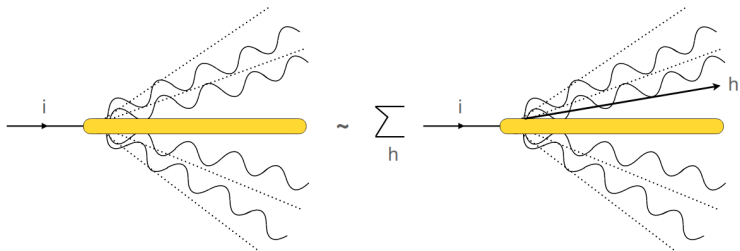
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Jain, Procura, Waalewijn 1101.4953 - JHEP

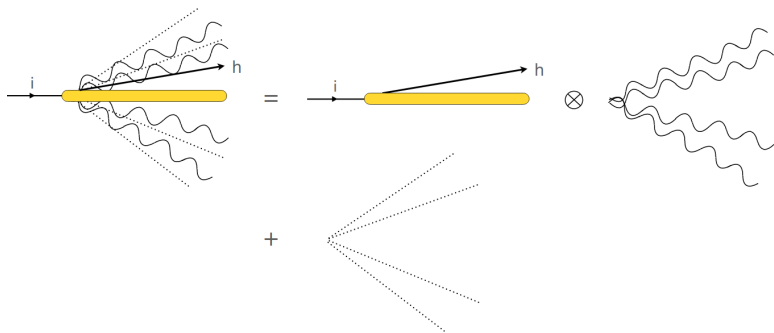


$$J_i(s) = \frac{1}{2(2\pi)^3} \sum_h \int dz z \mathcal{G}^{i \rightarrow h}(s, z)$$

Connection between the unpolarized (twist 2) jet function and FJFs :  
jet as the “inclusive” limit of the in-jet fragmentation

# FJFs and 1h-FFs

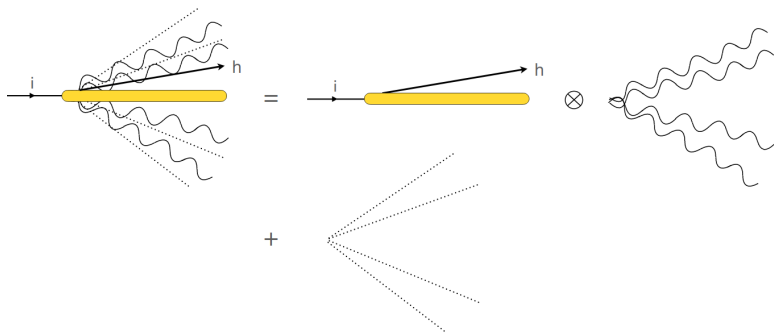
Procura, Stewart 0911.4980 - PRD  
Jain, Procura, Waalewijn 1101.4953 - JHEP





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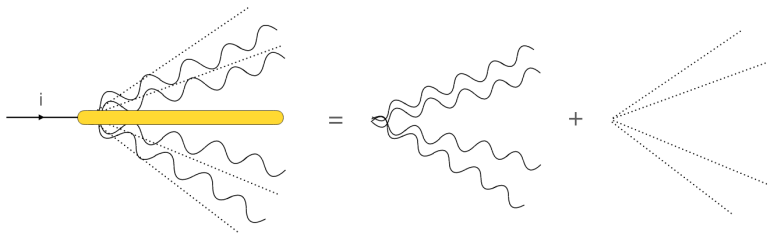


$$\mathcal{G}^{i \rightarrow h}(s, z) = \sum_j \mathcal{J}_{ij}(s, z) \otimes D_1^{j \rightarrow h}(z) + \mathcal{O}(\Lambda_{qcd}^2 s^{-1})$$

Large- $s$  expansion of the unpolarized **FJF**  $\mathcal{G}$   
 on the single-hadron collinear **FF**  $D_1$

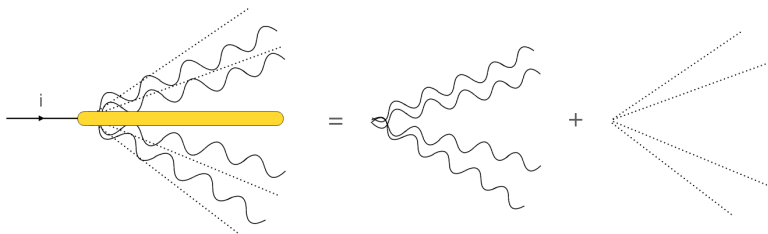
# Twist two jets

Procura, Stewart 0911.4980 - PRD  
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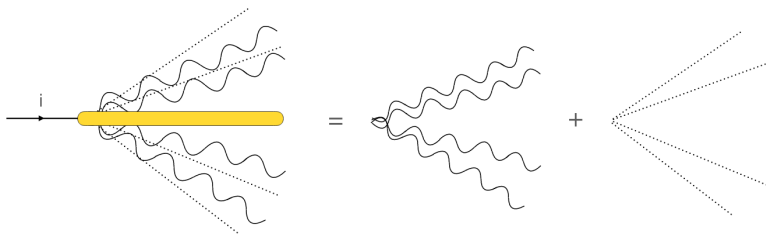
Procura, Stewart 0911.4980 - PRD  
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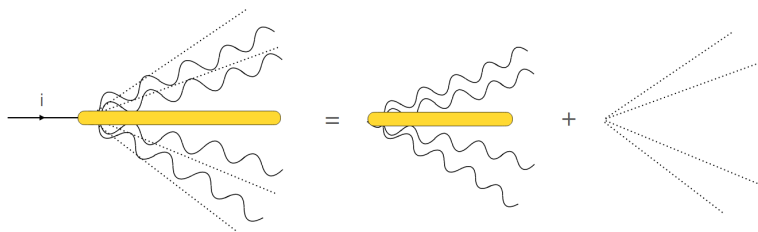
Procura, Stewart 0911.4980 - PRD  
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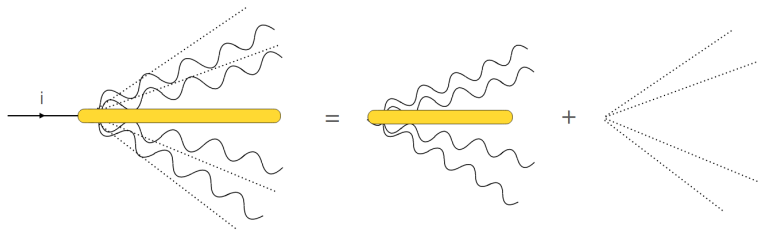
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At **twist 2** the jet function  $J_i(s)$  “**decouples**” from the 1h-FF  $D_1(z)$   
and the **non-perturbative structure** gets **simplified**

# Twist three jets



# Twist three jets



$$\tilde{J}_i(s) \sim M_j \otimes \tilde{J} + \mathcal{O}(\Lambda_{qcd}^2 s^{-1}) \quad (\text{"mass sum rule" for } E)$$

More complex non-perturbative structure:  
normalization of the associated quark spectral function ( $\rho_1$  in this case)

Just a speculation?



# Just a speculation? **NO**

Accardi, Signori 1903.04458 - PLB  
Accardi, Signori 2005.11310 - EPJC

The quark/jet mass can have a sizeable impact on physical observables:



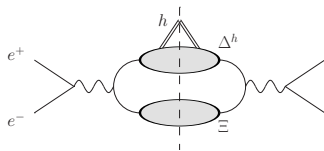
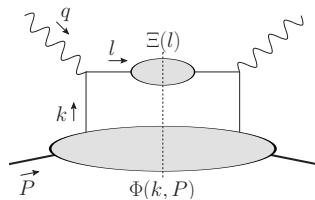


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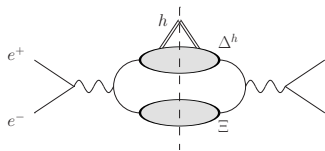
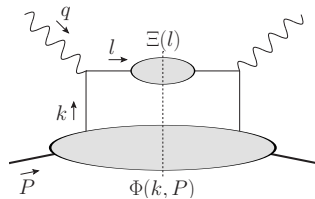


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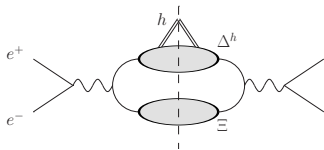
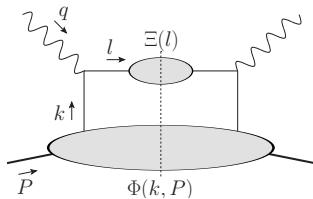


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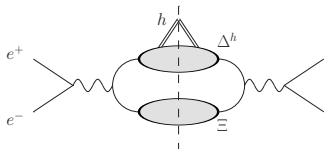
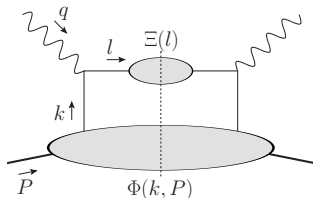


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- ▶ **calculable**: quark  $E$  FFs [ $M_j = \sum_h \int dz M_h E^h(z)$ ]  $\leftarrow$  **this work!**



# Outline

Quark hadronization, propagation, mass generation

Inclusive jets

Semi-inclusive processes



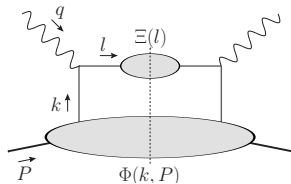
# Semi-inclusive processes

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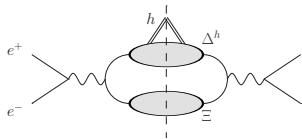
dynamical mass coupled to the transversity PDF

A. Accardi, A. Bacchetta - 1706.02000 - PLB

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(Accardi, Signori et al. - in progress)

$$\frac{d\sigma^L(e^+e^- \rightarrow h^\uparrow X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_{1L}(z) \right. \\ \left. + D(y) |\mathbf{S}_T| \cos(\phi_S) \frac{2M_h}{Q} \left( \frac{G_T(z)}{z} + \frac{m_q^{corr}}{M_h} H_1(z) \right) \right\}$$



# Conclusions

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- ▶ this mass is gauge-invariant, and the dynamical component **can be measured** at twist three in scattering experiments

INT Electronic Workshop 21-80W

Fragmentation Functions 2021

November 1 - 5, 2021

**Organizers:**

- Marco Radici, INFN Pavia (Italy), [marco.radici@pv.infn.it](mailto:marco.radici@pv.infn.it)
- Ralf Seidl, RIKEN (Japan), Riken BNL Research Center (NY, USA), [rseidl@riken.jp](mailto:rseidl@riken.jp)
- Andrea Signori, University of Pavia and INFN Pavia (IT), Jefferson Lab (VA, USA), [andrea.signori@unipv.it](mailto:andrea.signori@unipv.it)

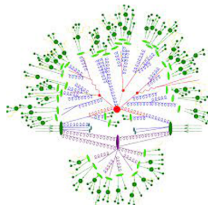
**Diversity Coordinator:**

- Ralf Seidl, RIKEN (Japan), Riken BNL Research Center (NY, USA), [rseidl@riken.jp](mailto:rseidl@riken.jp)

**Program Coordinator:**

- Alesha Vertrees, [aleshav@uw.edu](mailto:aleshav@uw.edu), (206) 221-8914

Application Form



<https://sites.google.com/uw.edu/int/programs/21-80w>

# Backup



# Useful references/1:

A selection of useful references related to **inclusive jets** and **dynamical mass effects**:

- ▶ Fully unintegrated parton correlation functions and factorization in lowest order hard scattering  
J.C. Collins, T.C. Rogers, A.M. Stasto - 0708.2833
- ▶ Collinear factorization for deep inelastic scattering structure functions at large Bjorken  $x_B$   
A. Accardi, J.W. Qiu - 0805.1496
- ▶ Quark fragmentation as a probe of dynamical mass generation  
A. Accardi, A. Signori - 1903.04458
- ▶ On the connection between quark propagation and hadronization  
A. Accardi, A. Signori - 2005.11310
- ▶ Accessing the nucleon transverse structure in deep-inelastic scattering  
A. Accardi, A. Bacchetta - 1706.02000



# Useful references/2:

A selection of useful references dealing with fragmentation functions, inclusive jets in pQCD,  $e^+e^-$  annihilation:

- ▶ Parton fragmentation functions (review)

A. Metz, A. Vossen - 1607.02521

- ▶ Quark fragmentation within an identified jet

M. Procura, I. Stewart - 0911.4980

- ▶ Parton fragmentation within an identified jet at NNLL

A. Jain, M. Procura, W. Waalewijn - 1101.4953

- ▶ Asymmetries in polarized hadron production in  $e^+e^-$  annihilation up to order  $1/Q$

D. Boer, R. Jakob, P.J. Mulders - hep-ph/9702281

- ▶ Angular dependences in inclusive two-hadron production at Belle

D. Boer - 0804.2408

# The NJL model of QCD

The Nambu–Jona-Lasinio (NJL) model of QCD is a **chiral effective theory** which is useful to help understand **non-perturbative** phenomena in low energy QCD. In particular:

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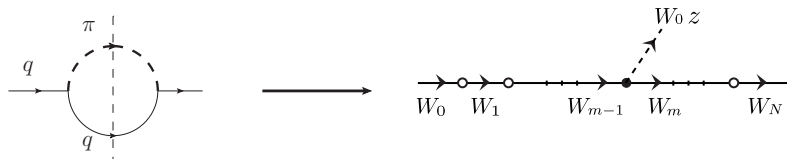
*Proper-time* regularization scheme: it can incorporate aspects of confinement

The NJL model has been used to describe:

- ▶ hadrons as bound states of quarks
- ▶ nuclear matter and nuclei in terms of quarks (medium modifications)
- ▶ phases of strongly interacting matter at high densities (e.g. neutron stars, etc.)

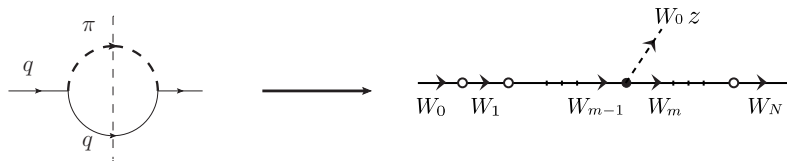
(Klevansky - Rev.Mod.Phys. 64 (1992) 649-708)

# The NJL-jet model for FFs



- ▶ Within the NJL it is possible to calculate PDFs and FFs by calculating and regularizing the associated Feynman diagrams
- ▶ A more realistic model of FFs: take into account that the fragmentation process occurs as a *cascade*: the **NJL-jet** (Ito et al. - 0906.5362)

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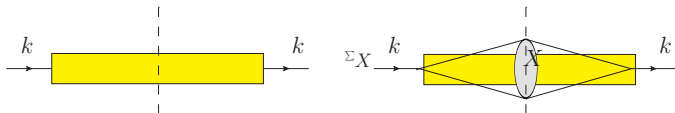
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$$D_q^\pi(z) = \sum_{m=1}^N \int_0^1 d\eta_1 \cdots \int_0^1 d\eta_N 6^N \sum_{Q_N} d_q^{Q_1}(\eta_1) \cdots d_{Q_{m-1}}^\pi(z) \cdots d_{Q_{N-1}}^{Q_N}(\eta_N)$$

The physical FF  $D_q^\pi$  can be calculated from the *elementary*  $d_q^\pi$  solving two integral Volterra equations

# The cut quark propagator

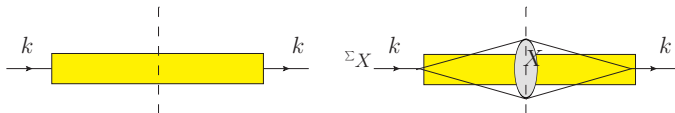
$$\Xi_{ij}(k; v) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ikx} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{T} W_1(\infty, \xi; v) \psi_i(\xi) \bar{\psi}_j(0) W_2(0, \infty; v) | \Omega \rangle$$



- ▶ **Partonic picture:** gauge invariant dressed quark correlator
  - ▶ only the discontinuity is considered  $\rightarrow$  on-shellness
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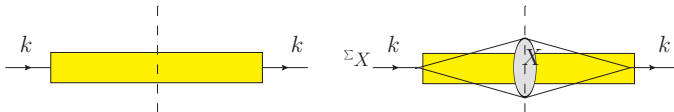


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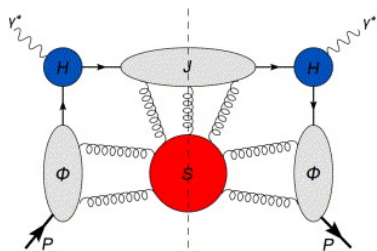
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- ▶ insights into **dynamical generation** of mass and momentum and **chiral symmetry** breaking

# The cut quark propagator

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See Serman NPB 281 ('87) 310, Chen et al. NPB 763 ('07) 183, Accardi et al. - 0805.1496, Collins et al. - 0708.2833 (and refs. therein)

(figure from Chen et al.)

- ▶  $\Xi$  emerges in the factorization theorem for DIS at large  $x$ , where a new semi-hard scale appears
- ▶  $\Xi$  captures the physics at  $Q^2(1-x) \sim Q\Lambda_{QCD}$ , which becomes increasingly non-perturbative at low energy and large  $x$
- ▶ the end-point factorization should be extended to different processes (e.g.  $e^+e^-$ )
- ▶ here we study the properties of  $\Xi$  and  $\Delta$  regardless of processes

# The quark/jet mass

$$M_j(k^-) \sim \int dk^+ \text{Tr}_D [\Xi \mathbb{I}]$$



Mass associated with the scalar term (**chiral-odd**) of the cut quark propagator:

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In the light-cone gauge we can relate it to the chiral-odd spectral function for the quark propagator:

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This mass term:

- ▶ gauge-invariant
- ▶ renormalization scale dependent
- ▶ calculable via the spectral functions of the cut quark propagator
- ▶ **accessible via momentum sum rules for twist-3 FFs**

# Semi-inclusive processes

- ▶ We can study the phenomenology of the dynamical mass in (semi-) inclusive hard processes
- ▶ interesting but challenging: **chiral-odd** sector at least at **twist-3**
- ▶ working in collinear factorization :
  - ▶ (?)  $pp^\uparrow \rightarrow h_1 h_2 j X \xrightarrow[\text{sum rule}]{\text{mass}} f_1(x_1) \otimes h_1(x_2) \otimes D_1(z) \otimes m_q^{corr}$   
(fixed-target configuration at LHC)
- ▶ (?) potentially also TMD factorization
- ▶ in order to make quantitative predictions and extractions the (“end-point”) factorization of these processes has to be addressed