Generalised Partons Distributions: recent developments and perspectives

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Introduction

- Generalised Parton Distributions (GPDs):

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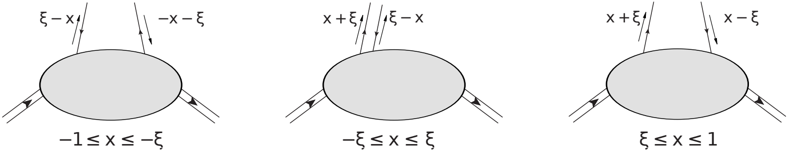
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,



  - ★ x: average momentum fraction carried by the active parton
  - ★ ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - ★ t: the Mandelstam variable

Generalised Parton Distributions

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

- Generalised Parton Distributions (GPDs):
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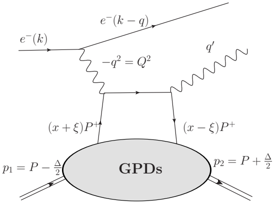
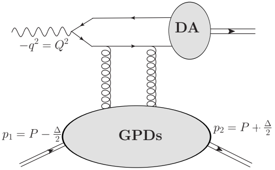
  - can be split into quark flavour and gluon contributions,
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t;\mu) + mod(m,2)\xi^{m+1} C^q_{m+1}(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\; H^q(x,\xi,t;\mu) = F_1^q(t)$$

Lorentz Covariance

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines
  $\rightarrow$ GPDs are continuous albeit non analytical at $x = \pm\xi$

J. Collins and A. Freund, PRD 59 074009 (1999)

Factorisation theorem

Properties

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Scale evolution property
  $\rightarrow$ generalization of DGLAP and ERBL evolution equations

  D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

  Renormalization

Properties

- Polynomiality Property:

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- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

Factorisation theorem

- Scale evolution property

Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

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- Possibility to extract density from experimental data

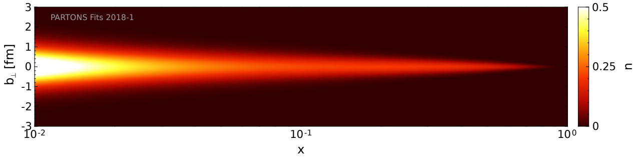

figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

Interpretation of GPDs I

2+1D structure of the nucleon

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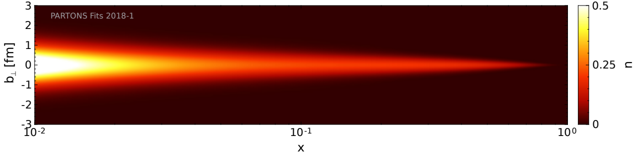

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- Correlation between x and $b_\perp$ $\to$ going beyond PDF and FF.

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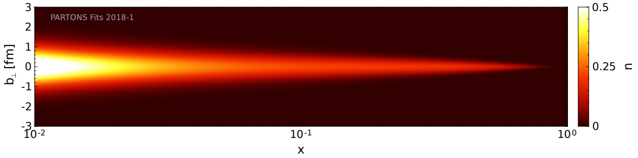

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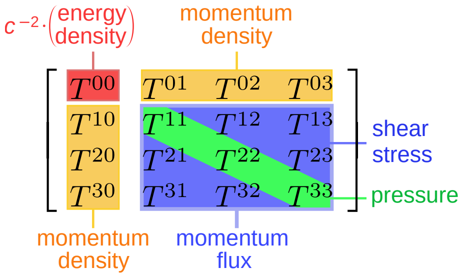
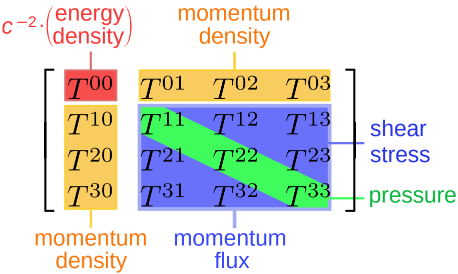
- Correlation between x and $b_\perp$ $\to$ going beyond PDF and FF.
- Caveat: no experimental data at $\xi = 0$
  $\to$ extrapolations (and thus model-dependence) are necessary



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89



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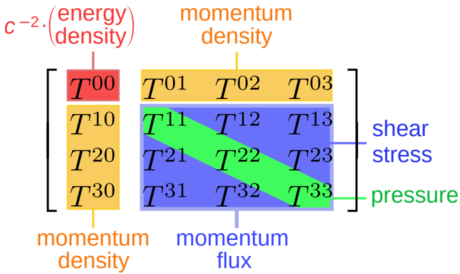
$$\langle p', s' | T^{\mu\nu}_{q,g} | p, s\rangle = \bar{u}\Bigg[P^{\{\mu}\gamma^{\nu\}} A_{q,g}(t;\mu) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_{q,g}(t;\mu) + M g^{\mu\nu}\bar{C}_{q,g}(t;\mu) + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{2M} B_{q,g}(t;\mu) + \frac{P^{[\mu}i\sigma^{\nu]\Delta}}{2M} D_{q,g}(t;\mu)\Bigg] u$$



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$$\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,t;\mu) = A_q(t;\mu) + (2\xi)^2 C_q(t;\mu)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,t;\mu) = B_q(t;\mu) - (2\xi)^2 C_q(t;\mu)$$

- Ji sum rule
- Fluid mechanics analogy

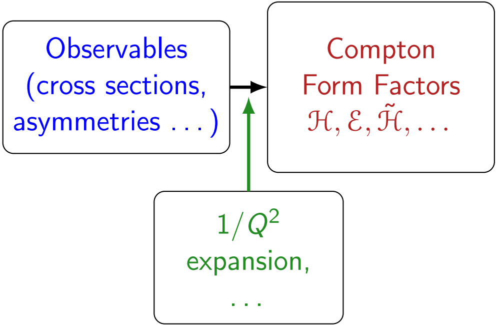
X. Ji, PRL 78, 610-613 (1997)
M.V. Polyakov PLB 555, 57-62 (2003)

Phenomenology of GPDs

Experimental connection to GPDs

Observables
(cross sections,
asymmetries . . .)

Experimental connection to GPDs



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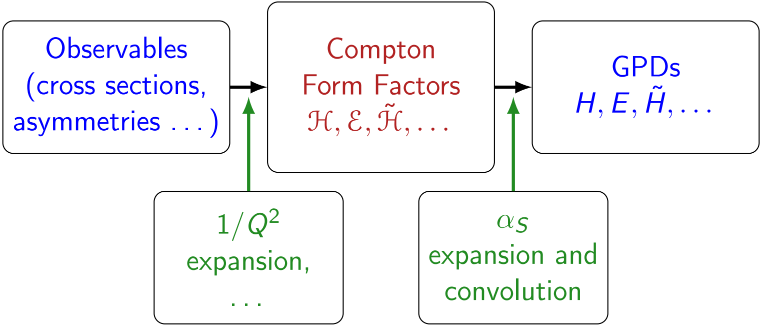

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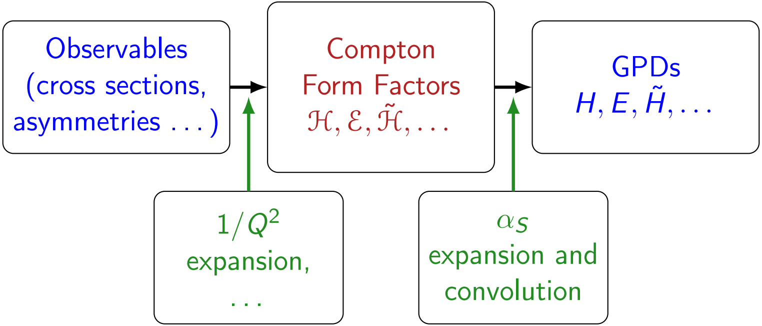
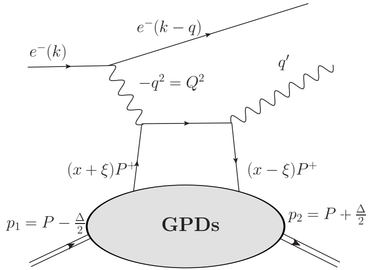
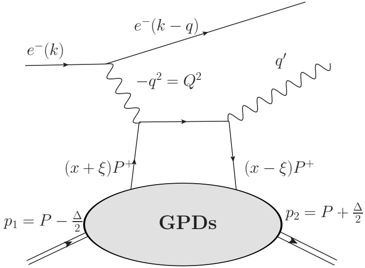


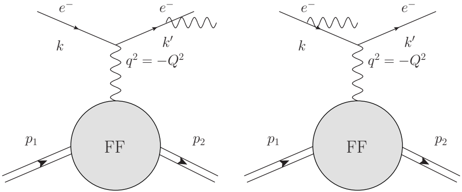
- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs



- Best studied experimental process connected to GPDs
  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12

Deep Virtual Compton Scattering

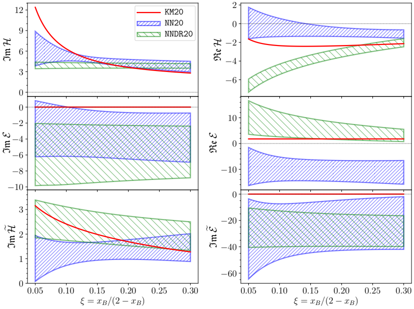




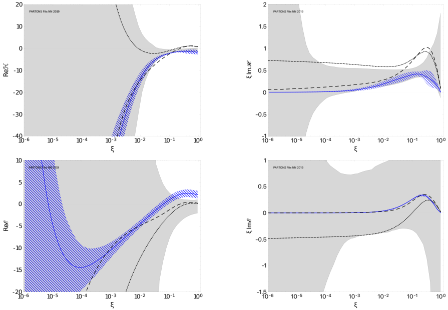
- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

Recent CFF extractions



M. Cuič *et al.*, PRL 125, (2020), 232005



H. Moutarde *et al.*, EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)

  additional ongoing studies, J. Grigsby *et al.*, PRD 104 (2021) 016001
- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,. . .)
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)

  see *e.g.* H. Dutrieux *et al.*, EPJA 57 8 250 (2021)

DVCS on Nuclei

- DVCS off the deuteron

F. Cano *et al.*, EPJA 19 (2004) 423
M. Benali *et al.*, Nature Phys. 16 (2020) 2, 191-198

  - Incoherent scattering : DVCS off the quasi-free neutron
    $\rightarrow$ significant step toward flavour separation

    M. Cuic *et al.*, PRL 125 (2020) 23, 232005

  - Coherent scattering : probing partons inside a deuteron
    $\rightarrow$ Spin 1 target: richer spin structure $\rightarrow$ more GPDs
    $\rightarrow$ Extraction more complicated

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- DVCS off He^4

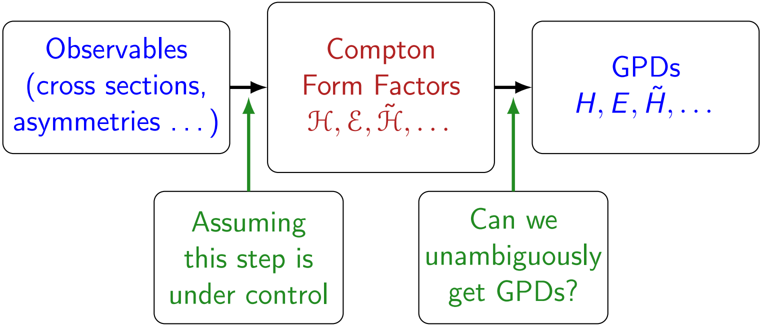
M. Hattawy *et al.*, PRL 119 (2017) 20, 202004

  - Coherent scattering on a scalar target
    $\rightarrow$ Less spin structure $\rightarrow$ less GPDs
  - Incoherent scattering: information on the structure of a bound nucleon

S. Fucini *et al.*, Phys.Rev.C 102 (2020) 065205

The DVCS deconvolution problem I

From CFF to GPDs



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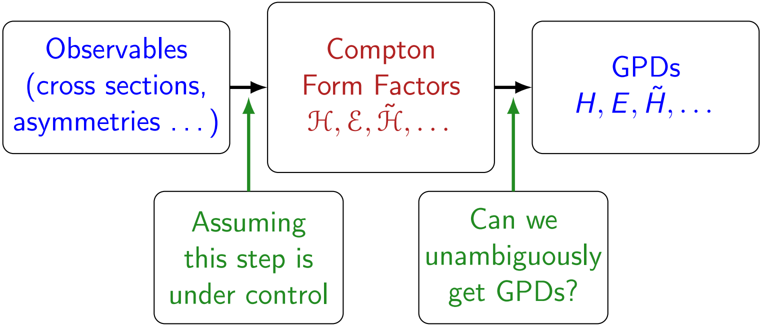

- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).

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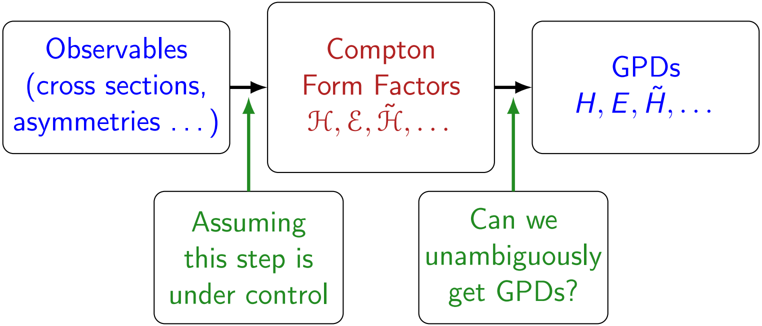

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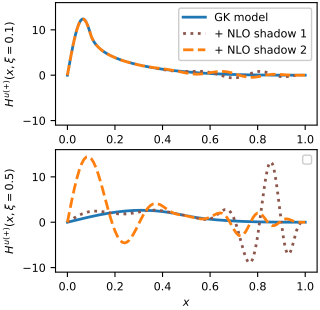
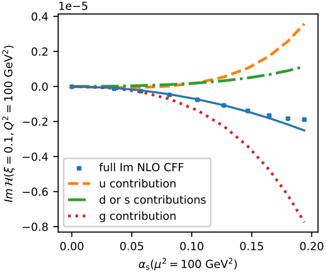
- Are QCD corrections improving the situation?

The DVCS deconvolution problem II





- NLO analysis of shadow GPDs:
  - Cancelling the line $x = \xi$ is necessary but **no longer** sufficient
  - Additional conditions brought by NLO corrections reduce the size of the “shadow space”...
  - ... but do not reduce it to 0
    $\rightarrow$ NLO shadow GPDs

H. Dutrieux *et al.*, PRD 103 114019 (2021)

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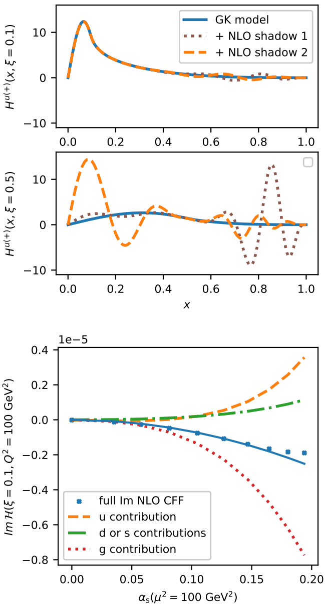

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- Evolution
  - it was argued that evolution would solve this issue

    A. Freund PLB 472, 412 (2000)
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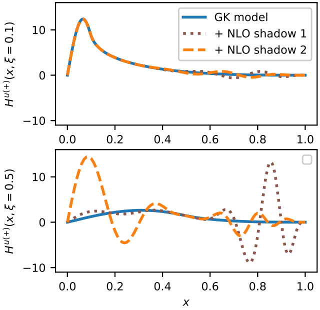

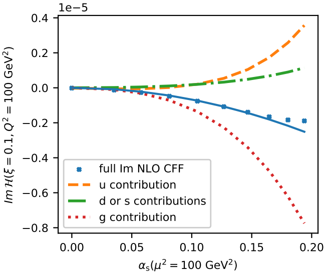

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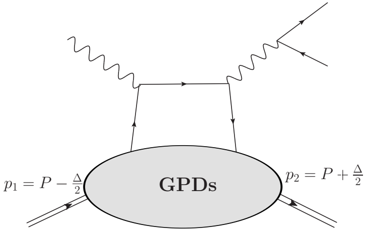
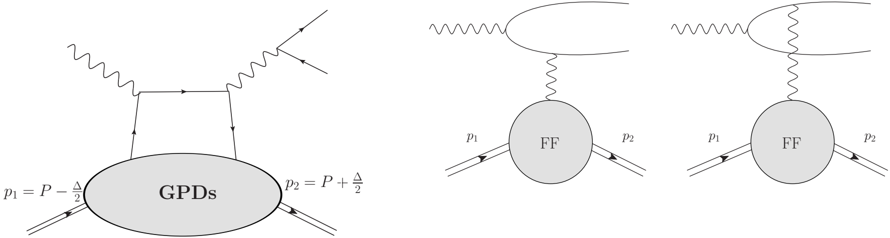
Multichannel Analysis required to fully determine GPDs



- Amplitude related to the DVCS one ($Q^2 \to -Q^2, \ldots$)
  $\to$ theoretical development for DVCS can be extended to TCS

  E. Berger *et al.*, EPJC 23 (2002) 675
- Excellent test of GPD universality but not the best option to solve the deconvolution problem

Timelike Compton Scattering



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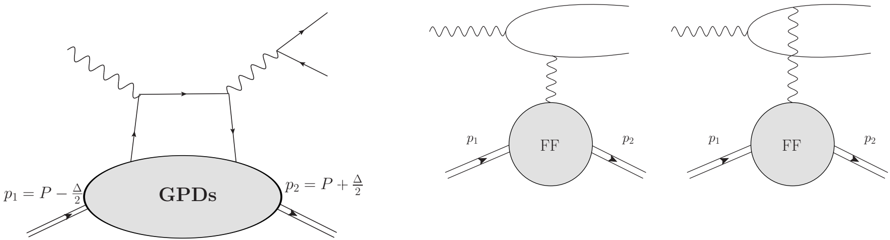

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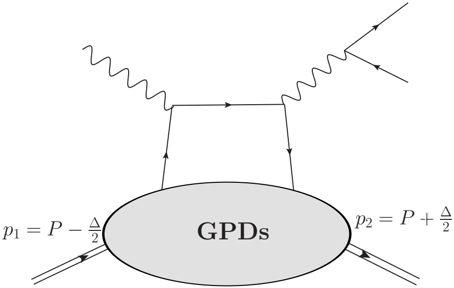
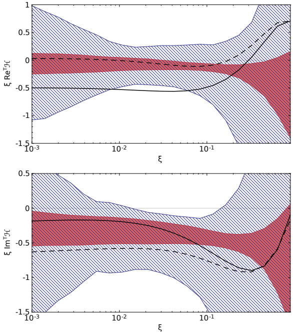
- Same type of final states as exclusive quarkonium production





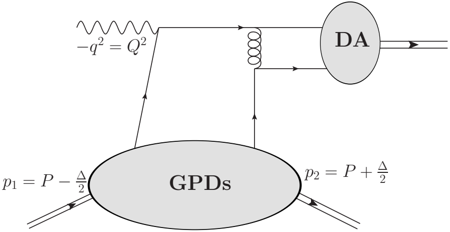
O. Grocholski *et al.*, EPJC 80, (2020) 61

- DVCS Data-driven prediction for TCS at LO and NLO
- First experimental measurement at JLab through forward-backward asymmetry (interference term)

P. Chatagnon *et al.*, arXiv:2108.11746

- Measurable at the LHC in UPC ?

Deep Virtual Meson Production



- Factorization proven for γ_L^*

  J. Collins *et al.*, PRD 56 (1997) 2982-3006
- Same GPDs than previously
- Depends on the meson DA
- Formalism available at NLO

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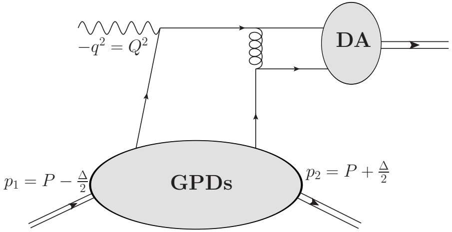

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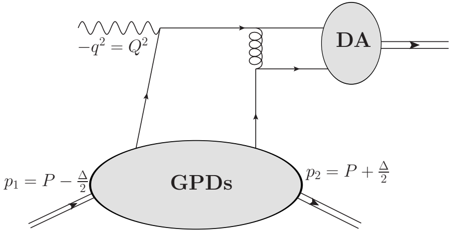

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  - Help flavour separation
  - Leading-order access to gluon GPDs
- Factorisation proven $\neq$ factorisation visible at achievable Q^2
  - Leading-twist dominance at a given Q^2 is process-dependent
    $\rightarrow$ for DVMP it can change between mesons.
  - At JLab kinematics, higher-twist contributions are very strong
    $\rightarrow$ hide factorisation of σ_L

Status of DVMP

- π^0 electroproduction
  - $\sigma_T > \sigma_L$ at JLab 6 and likely at JLab 12 kinematics ($Q^2 = 8.3 GeV^2$)

    M. Dlamini *et al.*, arXiv:2011.11125
  - No extraction of σ_L at JLab 12 yet
  - Model-dependent treatment of σ_T using higher-twist contributions

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- ρ^0 electroproduction
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    see *e.g.* L. Favart, EPJA 52 (2016) 6, 158
  - $\sigma_T \neq 0$ though $\rho_{0;T}$ production vanishes at leading twist
    $\rightarrow$ No LT access to chiral-odd GPDs.
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DVMP is as interesting as challenging
Additional data would be more than welcome

PARTONS	Gepard
partons.cea.fr	calculon.phy.hr/gpd/server/index.html

B. Berthou *et al.*, EPJC 78 (2018) 478

K. Kumericki, EPJ Web Conf. 112 (2016) 01012

- Similarities : NLO computations, BM formalism, ANN, ...
- Differences : models, evolution, ...

Physics impact

These integrated softwares are the mandatory path toward reliable multichannel analyses.

Evolution of GPDs

- No publicly available and maintained evolution code for GPDs even at LO
  (Vinnikov code is not maintained anymore but available in PARTONS)

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- PARTONS $\rightarrow$ adapt Apfel++ to get a flexible and standardised evolution code in x space for GPDs.
  - LO splitting functions have been implemented and validated
  - Additional polishing before public release of the code
  - NLO splitting functions are the next target

V. Bertone *et al.*, *in preparation*

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V. Bertone *et al.*, *in preparation*

Benchmarking

Benchmarking GPDs evolution code is a real topic today between PARTONS and GeParD groups. But some difficulties need to be overcome (among them: choice of the model for benchmarking)

Dispersion relation and the D-term

- At all orders in α_S, dispersion relations relate the real and imaginary parts of the CFF.

I. Anikin and O. Teryaev, PRD 76 056007
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- For instance at LO:

$$Re(\mathcal{H}(\xi,t)) = \frac{1}{\pi}\int_{-1}^{1}\mathrm{d}x\ Im(\mathcal{H}(x,t))\left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] + \underbrace{2\int_{-1}^{1}\mathrm{d}\alpha \frac{D(\alpha,t)}{1-\alpha}}_{\text{Independent of } \xi}$$

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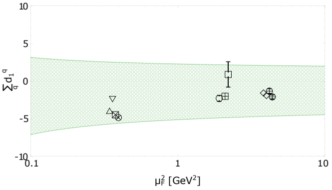

figure from H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- First attempt from JLab 6 *GeV* data

Burkert *et al.*, Nature 557 (2018) 7705, 396-399

- Tensions with other studies
  → uncontroled model-dependence

K. Kumericki, Nature 570 (2019) 7759, E1-E2
H. Moutarde *et al.*, Eur.Phys.J.C 79 (2019) 7, 614
H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- Scheme/scale dependence

Modelling GPDs

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

Factorisation theorem

- Scale evolution property

Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- GPDs are related to Double Distributions (DDs) through:

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta\left(x - \beta - \xi\alpha\right)$$

The Dirac δ insures that the polynomiality is fulfilled, independently of our choice of F and G

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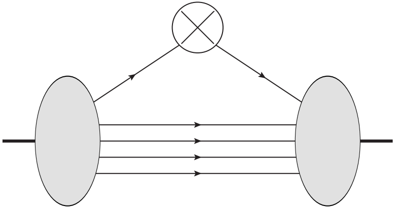
- DDs have been widely used for phenomenological purposes (VGG, GK...)
- They also appear naturally in covariant modelling attempts

Positivity property is not guaranteed, and may be violated.

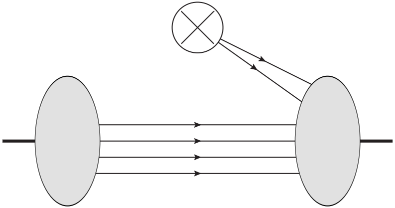
- On the light front, hadronic states can be expanded on a Fock basis

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- No ambiguity

ERBL: $|x| < |\xi|$



- N and $N+2$ partons LFWFs
- Ambiguity

M. Diehl *et al.*, Nucl.Phys. B596 (2001) 33-65

Fulfilling positivity

LFWFs approach to GPDs

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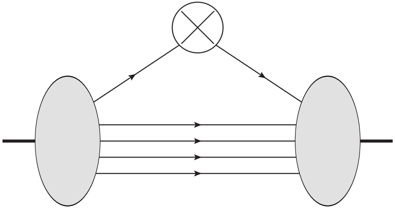

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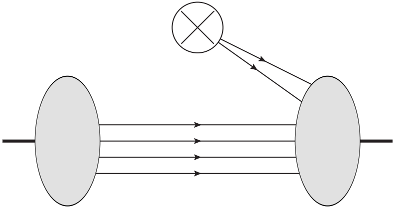

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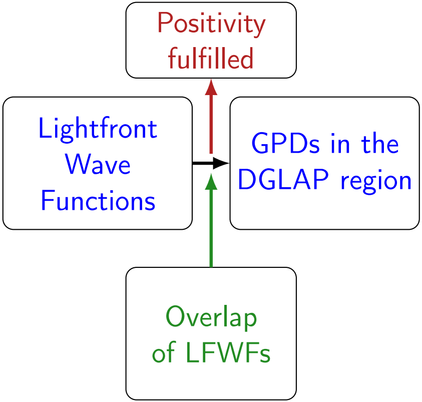
LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

A new modelling procedure

Lightfront
Wave
Functions

A new modelling procedure



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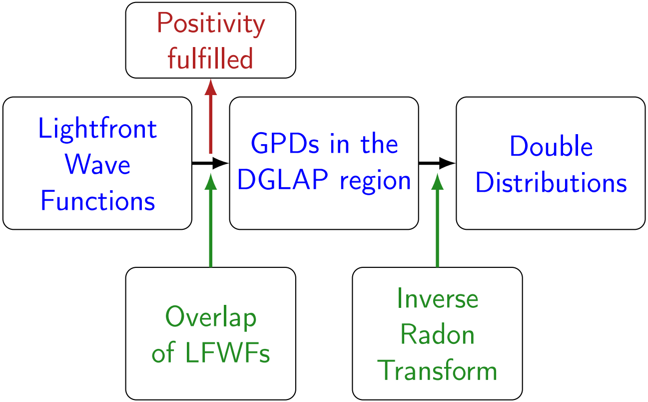

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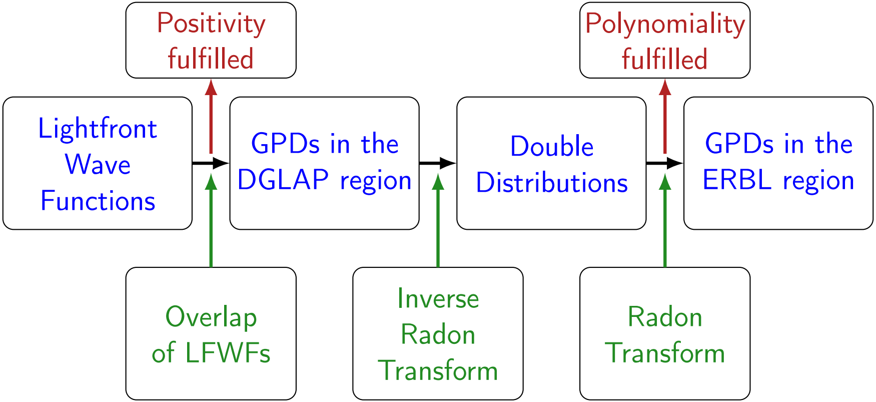

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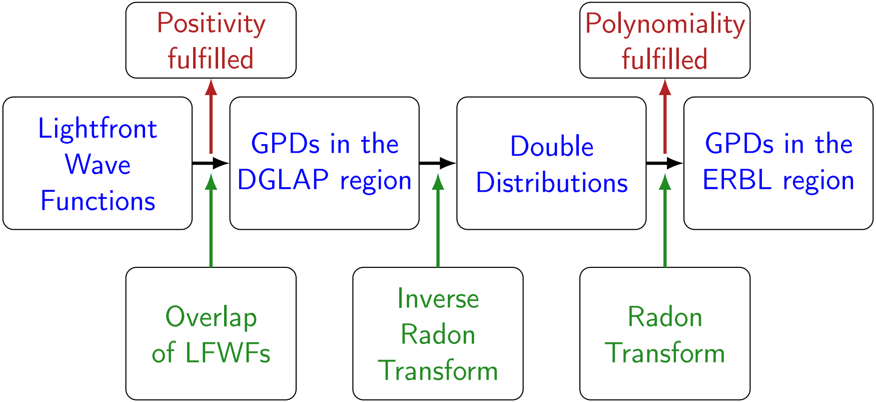
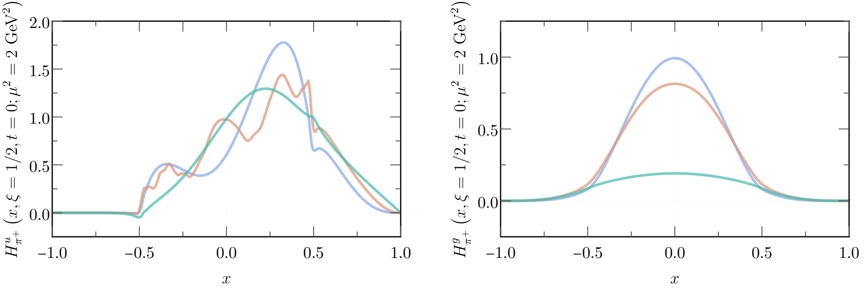


Not necessary to start from LFWFs
$\rightarrow$ Fulfilling the positivity and forward limit properties is enough

An example on the pion

Morgado *et al.*, in preparation



- Blue: GPD based on algebraic PDFs model
- Orange: GPD based on refine numerical PDF model
- Green: GPD based on standard Ansatz (RDDA)

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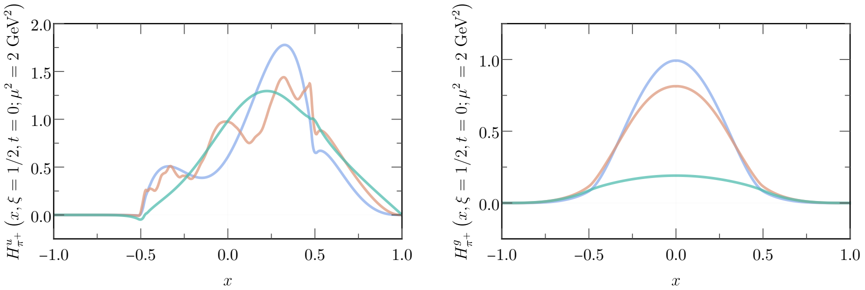

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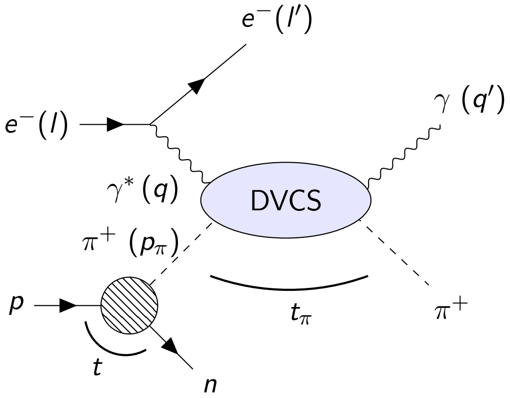
All theoretical constraints are fulfilled by construction !

Morgado *et al.*, in preparation

Can we measure DVCS on a virtual pion ?

D. Amrath *et al.*, EPJC 58 (2008) 179-192

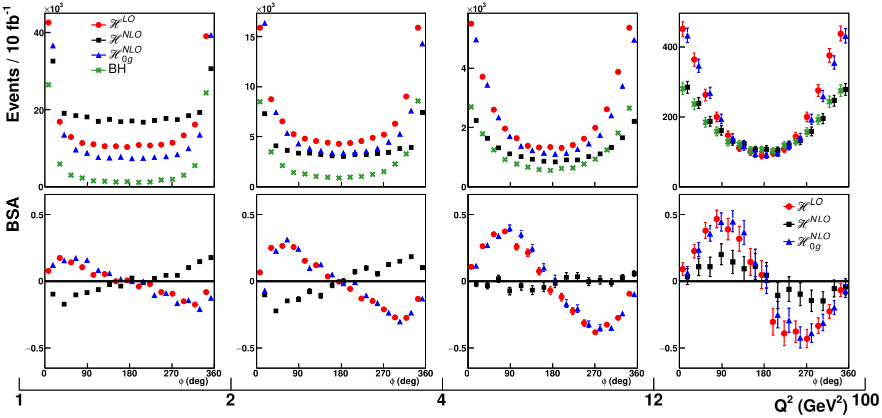


- $e^- p \to e^- \gamma \pi^+ n$
- kinematical cuts to avoid N^* resonances
- Already used to extract pion EFF at JLab
- Considered for pion structure function at EIC and EicC

EIC Yellow report, arXiv:2103.05419
EicC white paper, arXiv:2102.09222

An example on the pion

Morgado *et al.*, in preparation



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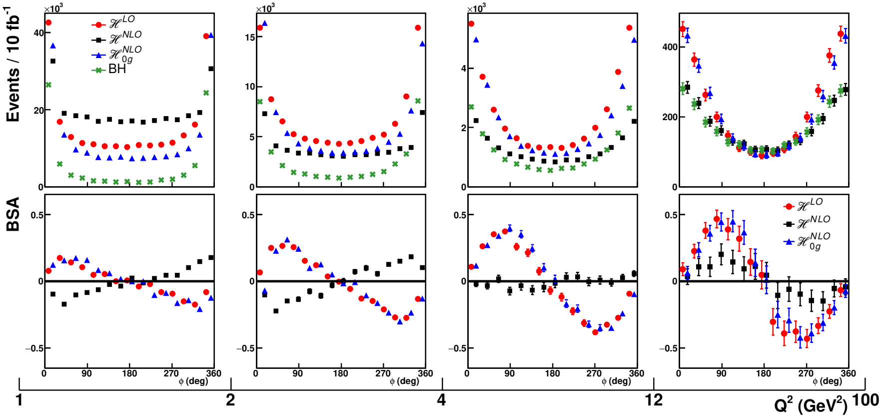
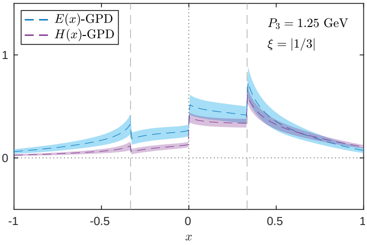


DVCS off virtual pion measurable at EIC and EicC

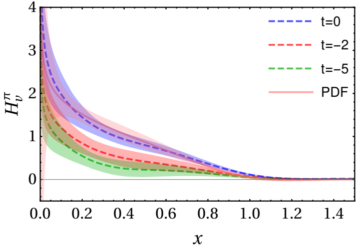
Lattice-QCD computations

- Lattice practicionner used to compute matrix element of local operators
  $\rightarrow$ Mellin moments of GPDs
- new techniques allow to extrapolate euclidean matrix element on the lightcone

X. Ji Phys. Rev. Lett., 2013, 110, 262002
Y.-Q. Ma and J.-W. Qiu Phys. Rev. D, 2018, 98, 074021
A. Radyushkin, Phys. Rev. D, 2017, 96, 034025



C. Alexandrou *et al.*, PRL 125 (2020) 26, 262001



J.-W. Chen Nucl.Phys.B 952 (2020) 114940

Unmentioned topics

- Phenomenological parametrisations (KM, GK, VGG, MSW, ...)
- Extension of the relation between CFF and observables

  B. Kriesten *et al.* PRD 101 (2020) 054021
- Continuum Schwinger Method computations

  see e.g. Jin-Lin Zhang *et al.*, arXiv:2009.11384
  A. Freese *et al.*, Phys.Rev.C 101 (2020) 3, 035203
- Pseudo-distribution on the lattice
- Exclusive charmonium production (EIC and LHC)
- Transition GPDs
- ...

Conclusions

Summary

- After 25 years, GPDs formalism is well established . . .
- . . . but the GPDs themselves remain poorly known
- the situation may change with JLAB 12, EIC and EicC

Perspectives

- Significant efforts in phenomenology remain be done (CFF and GPD)
- Multichannel analysis could help solving the deconvolution problem
- Ab-initio computations may provide insights in the next decade

In the perspective of EIC and EicC, a lot of work remains to be done to exploit the forthcoming data.

Thank you for your attention

Back up slides

Lattice and CSM distributions

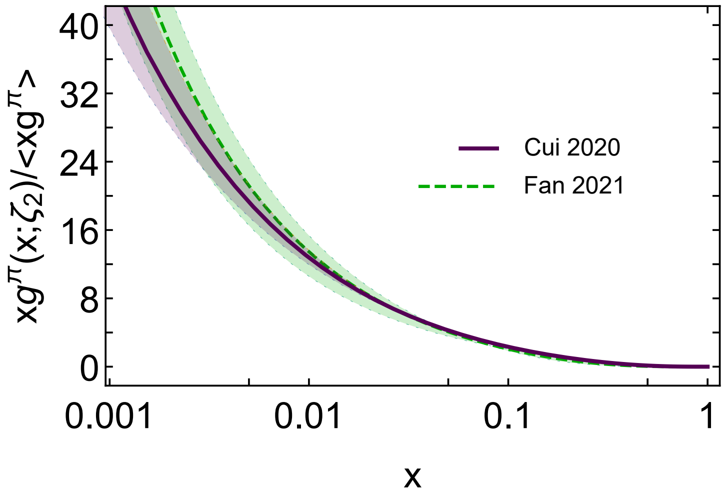
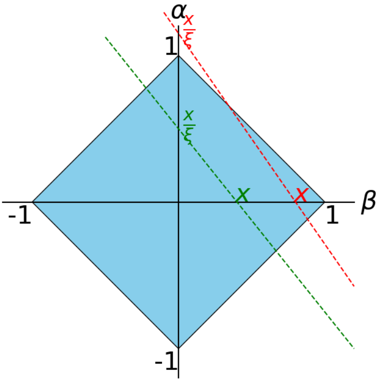


figure from L. Chang and C.D. Roberts, Chin.Phys.Lett. 38 (2021) 8, 081101

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha)+\xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

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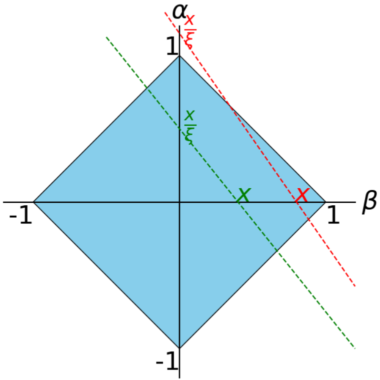

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Is it possible to recover the DDs from the DGLAP region only?

GPD properties from Radon Transform

- Double Distribution representation:

$$H(x,\xi) = D(x/\xi) + \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) F_D(\beta,\alpha)$$

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$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \mathrm{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

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New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**