Overview on gluon TMDs

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Outline

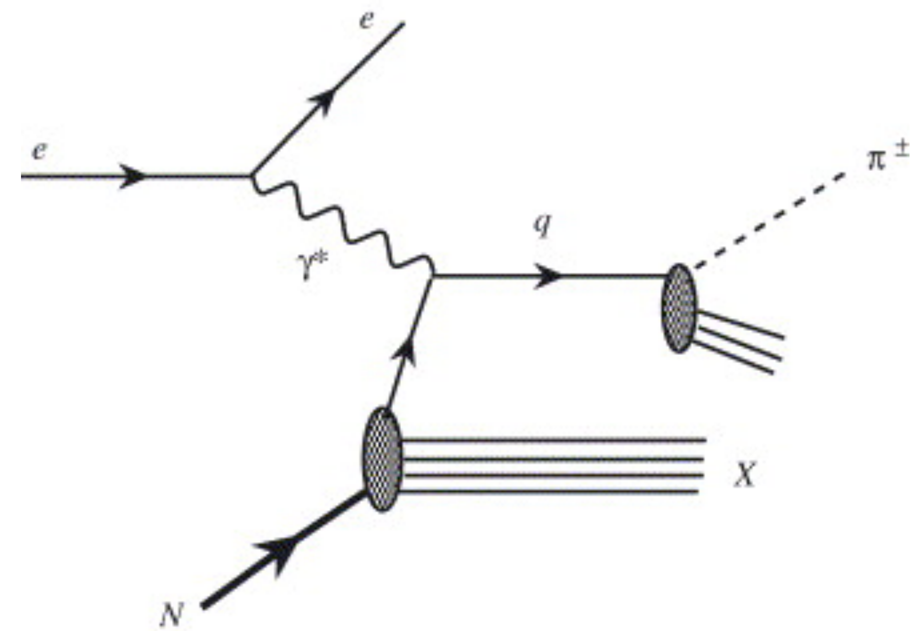
- Gluon TMDs & the parallels with quark TMDs
- Process dependence of gluon TMDs
  - sign change relation for gluon Sivers TMDs
  - small-x limit & Wilson loop matrix elements
- Probes of unpolarized gluon TMDs
- Probes of linearly polarized gluon TMDs

Gluon TMDs & the parallels with quark TMDs

Typical TMD processes

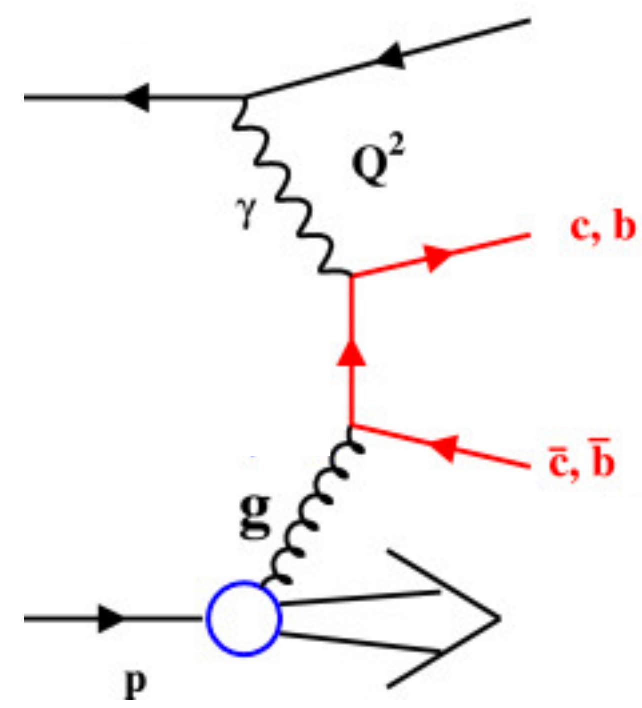
Semi-inclusive DIS is a process sensitive to the transverse momentum of quarks

$$e\,p \to e'\,h\,X$$



D-meson pair production is sensitive to transverse momentum of gluons

$$e\,p \to e'\,D\,\bar{D}\,X$$



Gluons TMDs

The transverse momentum dependent gluon correlator:

$$\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P | F^{+\nu}(0)\,\mathcal{U}\,F^{+\mu}(\xi^-, \xi_T)\,\mathcal{U}' | P \rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu} f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

[Mulders, Rodrigues, 2001]

For transversely polarized protons:

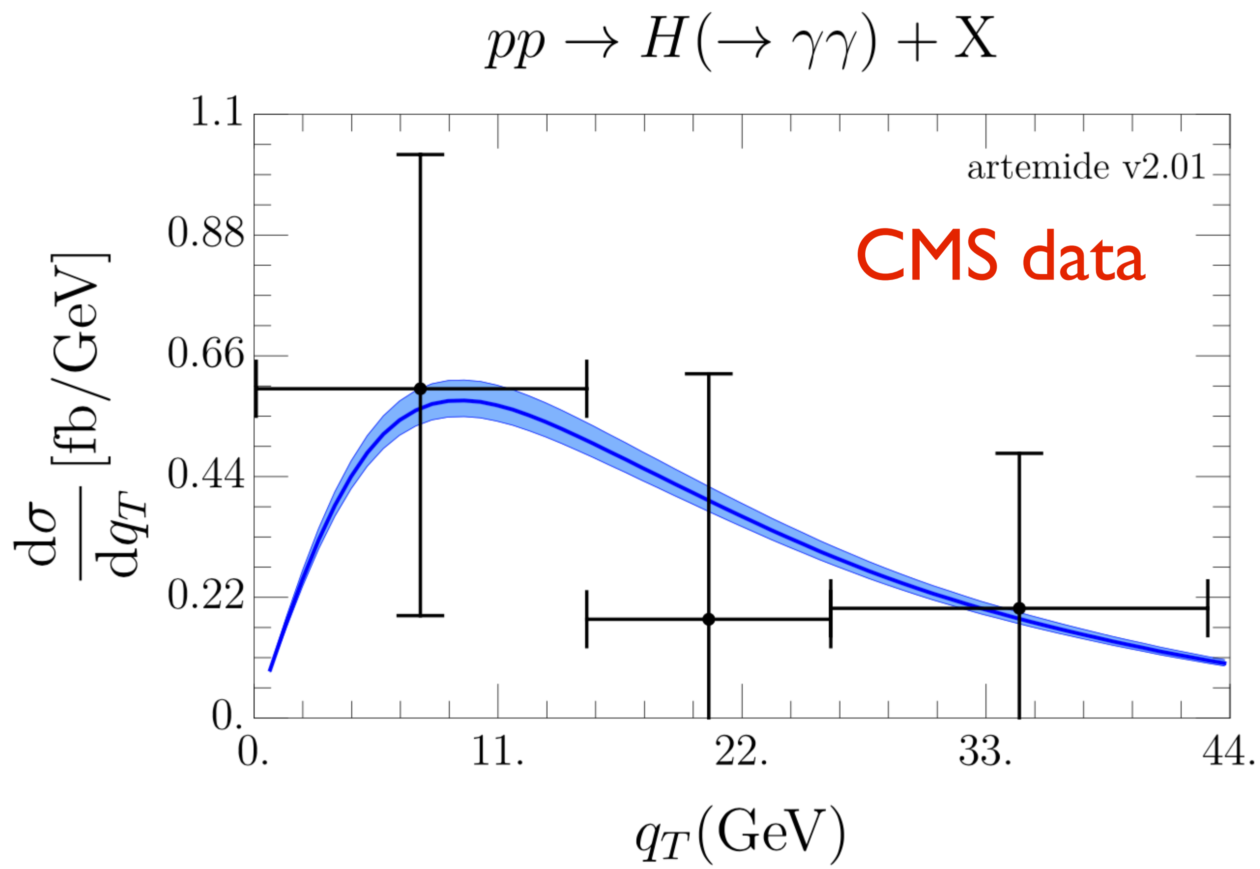
gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp\, g}(x, \boldsymbol{p}_T^2) + \ldots \right\}$$

Perhaps surprisingly, these TMDs have not been extracted from experiments yet

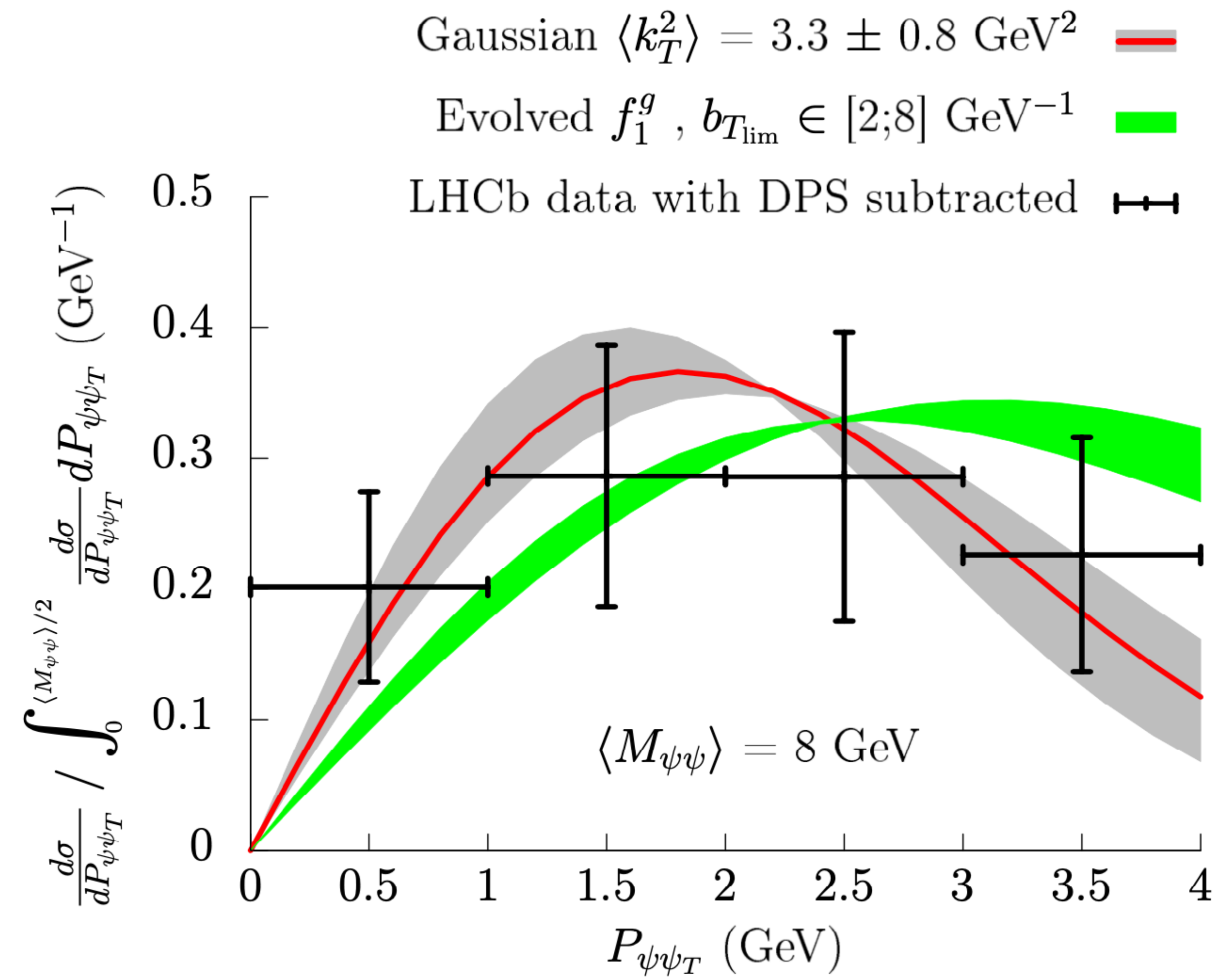
Entering the gluon TMD era

Higgs p_T distribution



[Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, 2019]

J/ψ pair production



[Scarpa et al., 2020]

Sivers asymmetry in high-p_T hadron pair production

$A^{Siv} = -0.23 \pm 0.08$ (stat) ± 0.05 (syst) at $\langle x_g \rangle = 0.15$

[COMPASS Collab., 2017]

GPM studies of $A_N^{\pi,D}$

[D'Alesio, Murgia, Pisano, 2015; & Taels, 2017]

Parallels between quarks and gluons

$$\Phi_U(x,\boldsymbol{k}) = \frac{1}{2}\left[\not{n}\, f_1(x,\boldsymbol{k}^2) + \frac{\sigma_{\mu\nu}k_T^\mu \bar{n}^\nu}{M}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_L(x,\boldsymbol{k}) = \frac{1}{2}\left[\gamma^5\not{n}\, S_L\, g_1(x,\boldsymbol{k}^2) + \frac{i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu k_T^\nu S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\not{n}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5\not{n}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) + i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5\bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

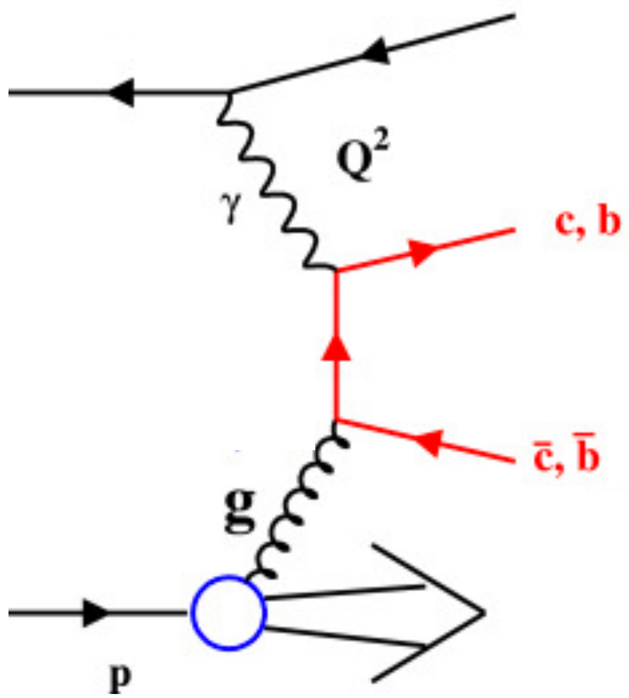
$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For gluons $h_1^\perp$ is T-even and h_1 is k_T-odd, T-odd and unrelated to transversity

Parallels between SIDIS and HQ pair production



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_\perp = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_\perp|$$

$\phi_T, \phi_\perp$ are the angles of q_T, $K_\perp$

Linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_\perp)$ distribution

Despite the differences in properties of some of the quark and gluon TMDs, the asymmetries they lead to are analogous for SIDIS and HQ pair production

There is a "Collins" asymmetry without a Collins function, but it does probe h_1^g which is not transversity however

Parallels between SIDIS and HQ pair production

LO asymmetries in HQ pair production:

[Boer, Pisano, Mulders, Zhou, 2016]

$$|\langle\cos 2\phi_T\rangle| = \left|\frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, \cos 2\phi_T\, \mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, \mathrm{d}\sigma}\right| = \frac{\boldsymbol{q}_T^2\, |B_0^U|}{2\, A_0^U} = \frac{\boldsymbol{q}_T^2}{2M^2} \frac{\left|h_1^{\perp\, g}(x, \boldsymbol{p}_T^2)\right|}{f_1^g(x, \boldsymbol{p}_T^2)} \frac{\left|\mathcal{B}_0^{eg\to eQ\overline{Q}}\right|}{\mathcal{A}_0^{eg\to eQ\overline{Q}}}$$

$$A_N^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{A_0^T}{A_0^U} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{f_{1T}^{\perp\, g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

$$A_N^{\sin(\phi_S+\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{B_0'^T}{A_0^U} = \frac{2(1-y)\,\mathcal{B}_{0T}^{\gamma^* g\to Q\overline{Q}}}{[1+(1-y)^2]\,\mathcal{A}_{U+L}^{\gamma^* g\to Q\overline{Q}} - y^2 \mathcal{A}_L^{\gamma^* g\to Q\overline{Q}}}\frac{|\boldsymbol{q}_T|}{M_p}\frac{h_1^g(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

$$A_N^{\sin(\phi_S-3\phi_T)} = -\frac{|\boldsymbol{q}_T|^3}{M_p^3}\frac{B_0^T}{2A_0^U} = -\frac{2(1-y)\,\mathcal{B}_{0T}^{\gamma^* g\to Q\overline{Q}}}{[1+(1-y)^2]\,\mathcal{A}_{U+L}^{\gamma^* g\to Q\overline{Q}} - y^2 \mathcal{A}_L^{\gamma^* g\to Q\overline{Q}}}\frac{|\boldsymbol{q}_T|^3}{2M_p^3}\frac{h_{1T}^{\perp\, g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

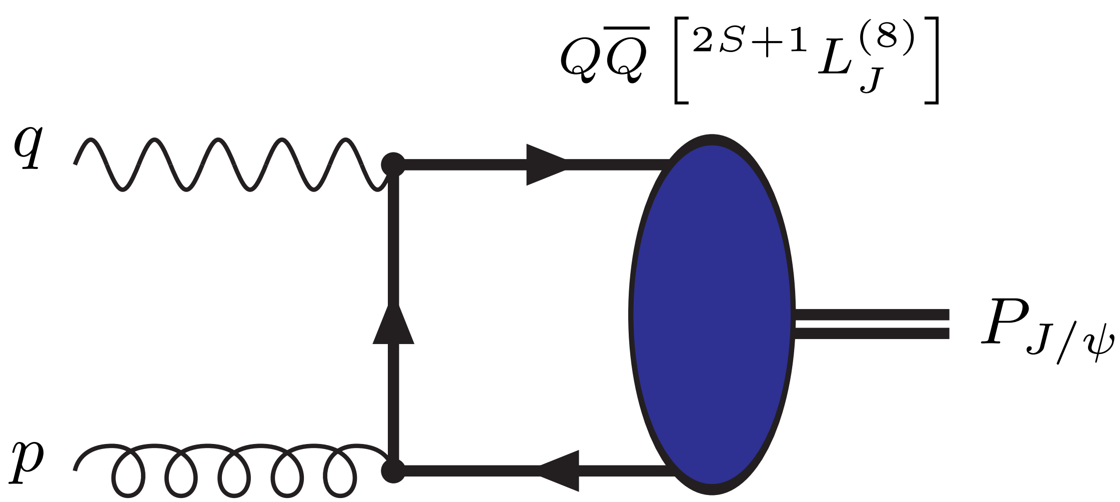
SIDIS - Fragmentation functions

HQ pairs - calculable amplitudes

Quarkonium production

$e\,p \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018]



A $\cos(2\phi_T)$ asymmetry probes $h_1^{\perp\,g}$

$$\langle \cos 2\phi_T \rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g \to \mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g \to \mathcal{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to \mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}.$$

In LO NRQCD the prefactor of the asymmetry depends on y, Q, M_Q and on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

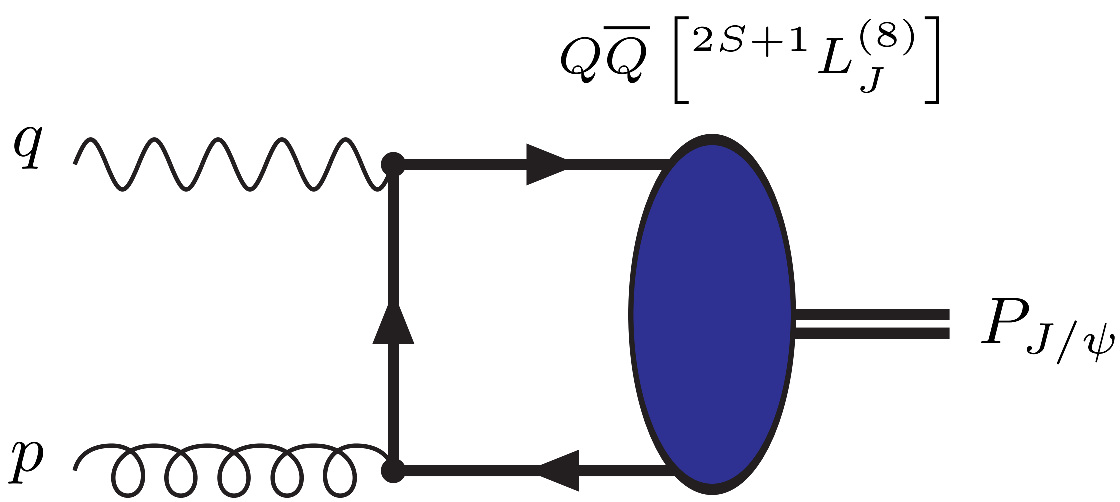
One can cancel out the CO LDMEs by considering ratios with spin asymmetries

[Bacchetta, Boer, Pisano, Taels, 2018]

Quarkonium production in ep

$$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X \text{ with } \mathcal{Q} \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson}$$

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018]



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

Other asymmetries depend on the quite uncertain CO NRQCD LDMEs, but one can consider ratios of asymmetries to cancel them out

[Bacchetta, Boer, Pisano, Taels, 2018]

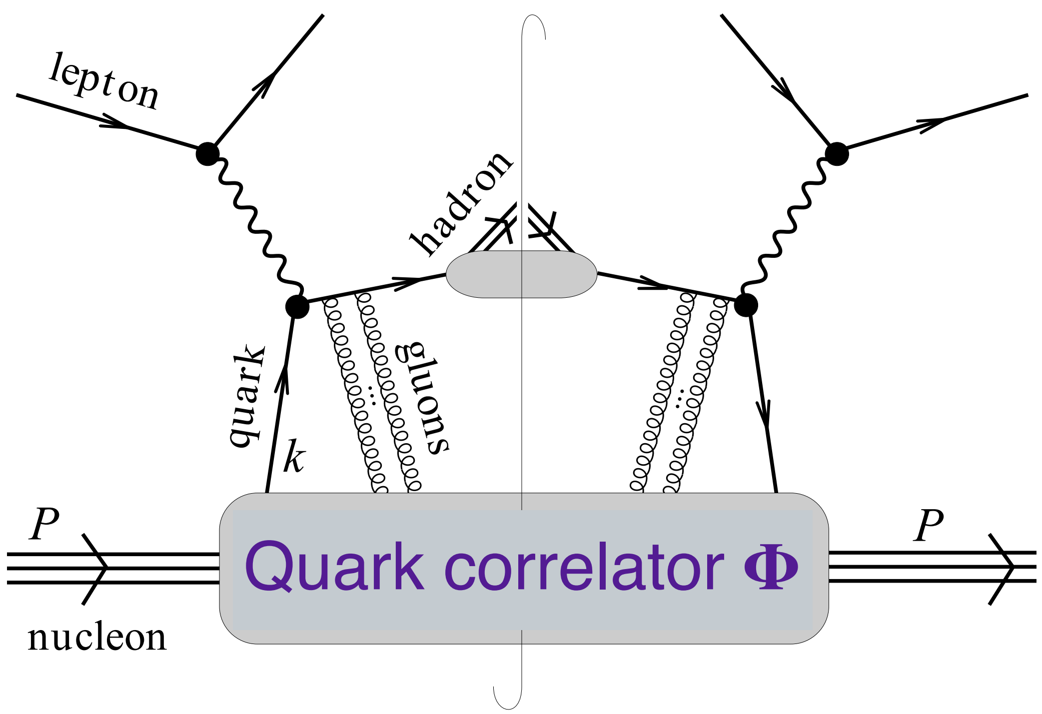
$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

Process dependence of gluon TMDs

Gauge invariance of correlators



summation of all gluon exchanges leads to *path-ordered exponentials* in the correlators

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{U}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003; Boer, Mulders & Pijlman, 2003]

These *gauge links* may or may not affect observables and it turns out that they do in certain cases sensitive to the transverse momentum

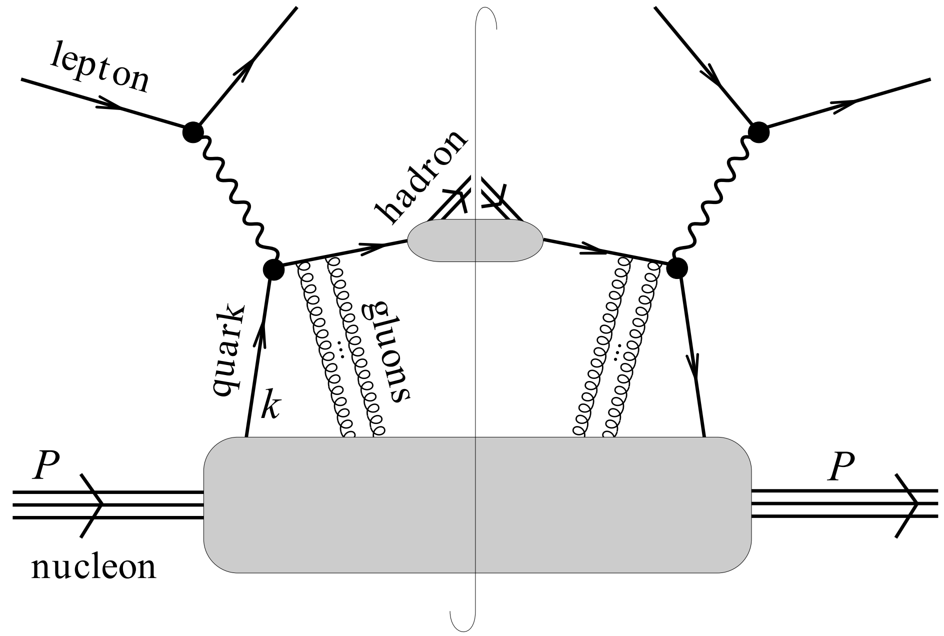
Gauge links in quark TMDs

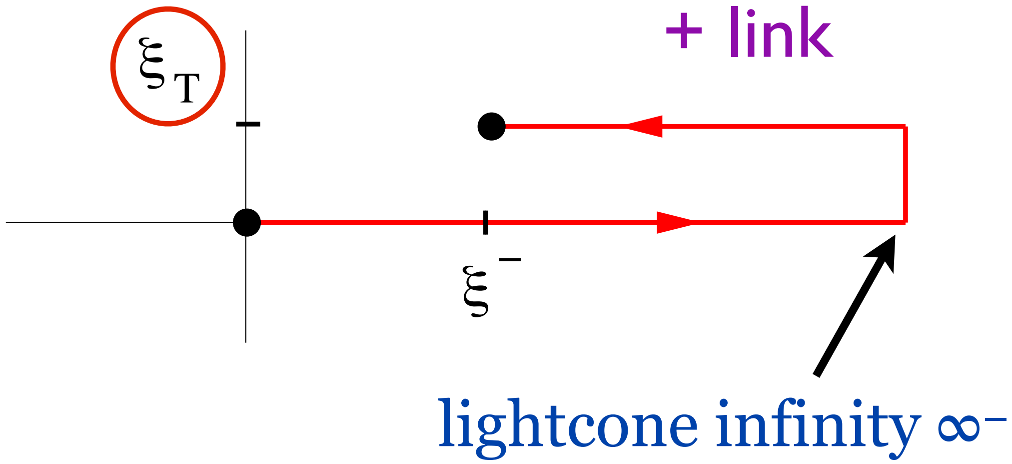
semi-inclusive DIS

$e\,p \to e'\,h\,X$

$k \approx xP + k_T$

$P^\mu \approx P^+$

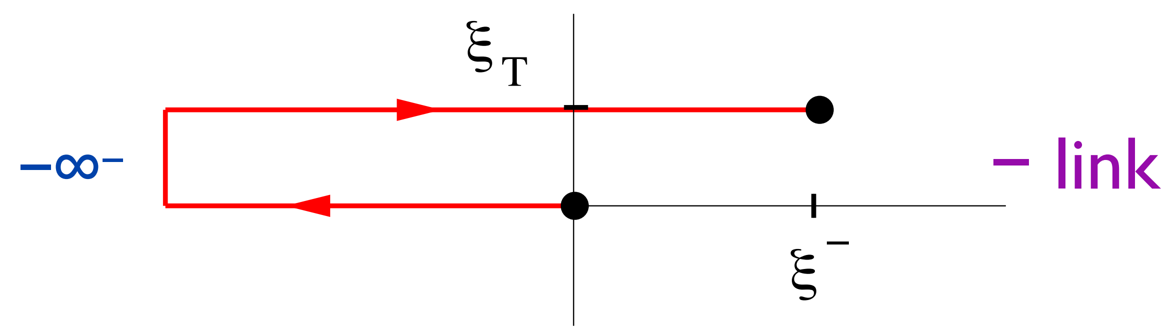




$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

$$\xi = [0^+, \xi^-, \boldsymbol{\xi}_T]$$

$pp \to \gamma^* X$ Drell-Yan:



This has observable effects, as was first noted for quark Sivers asymmetries

[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

$$f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\mathrm{DY}]}(x, k_T^2)$$

[Collins '02]

Process dependence of gluon TMDs

A similar sign change relation for gluon Sivers functions holds, but due to the appearance of two gauge links, there are more possibilities

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For most gluon TMDs there are only 2 link combinations of interest: [+,+] & [+,−]

[−,−] & [−,+] are related to them by parity and time reversal

More complicated links arise in processes where TMD factorization is questionable

The gauge link dependence even affects unpolarized gluon TMDs

[Dominguez, Marquet, Xiao, Yuan, 2011]

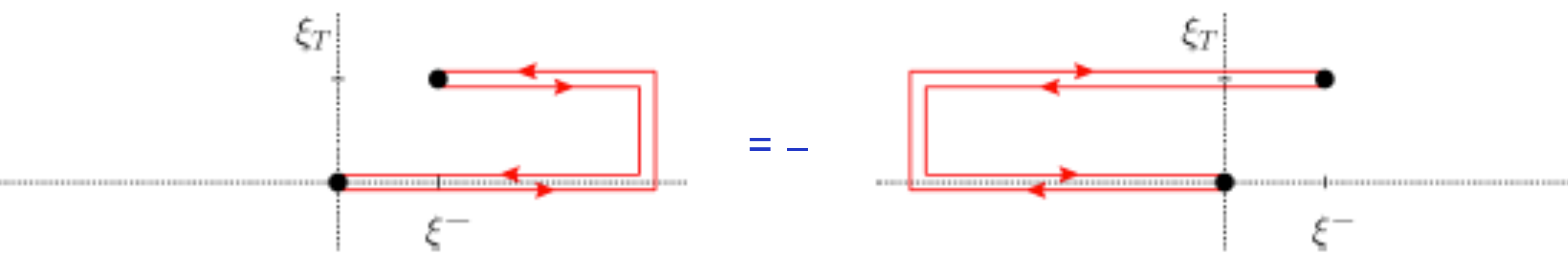
Sign change relation for gluon Sivers TMD

$e\, p^\uparrow \to e'\, Q\bar{Q}\, X$ $\gamma^*\, g \to Q\bar{Q}$ probes [+,+]

$p^\uparrow\, p \to \gamma\, \gamma\, X$ Qiu, Schlegel, Vogelsang, 2011

In the kinematic regime where pair rapidity is central, one effectively selects the subprocess:

$g\, g \to \gamma\, \gamma$ probes [–,–]



$$f_{1T}^{\perp\, g\, [e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X]}(x, p_T^2) = -f_{1T}^{\perp\, g\, [p^\uparrow\, p \to \gamma\, \gamma\, X]}(x, p_T^2)$$

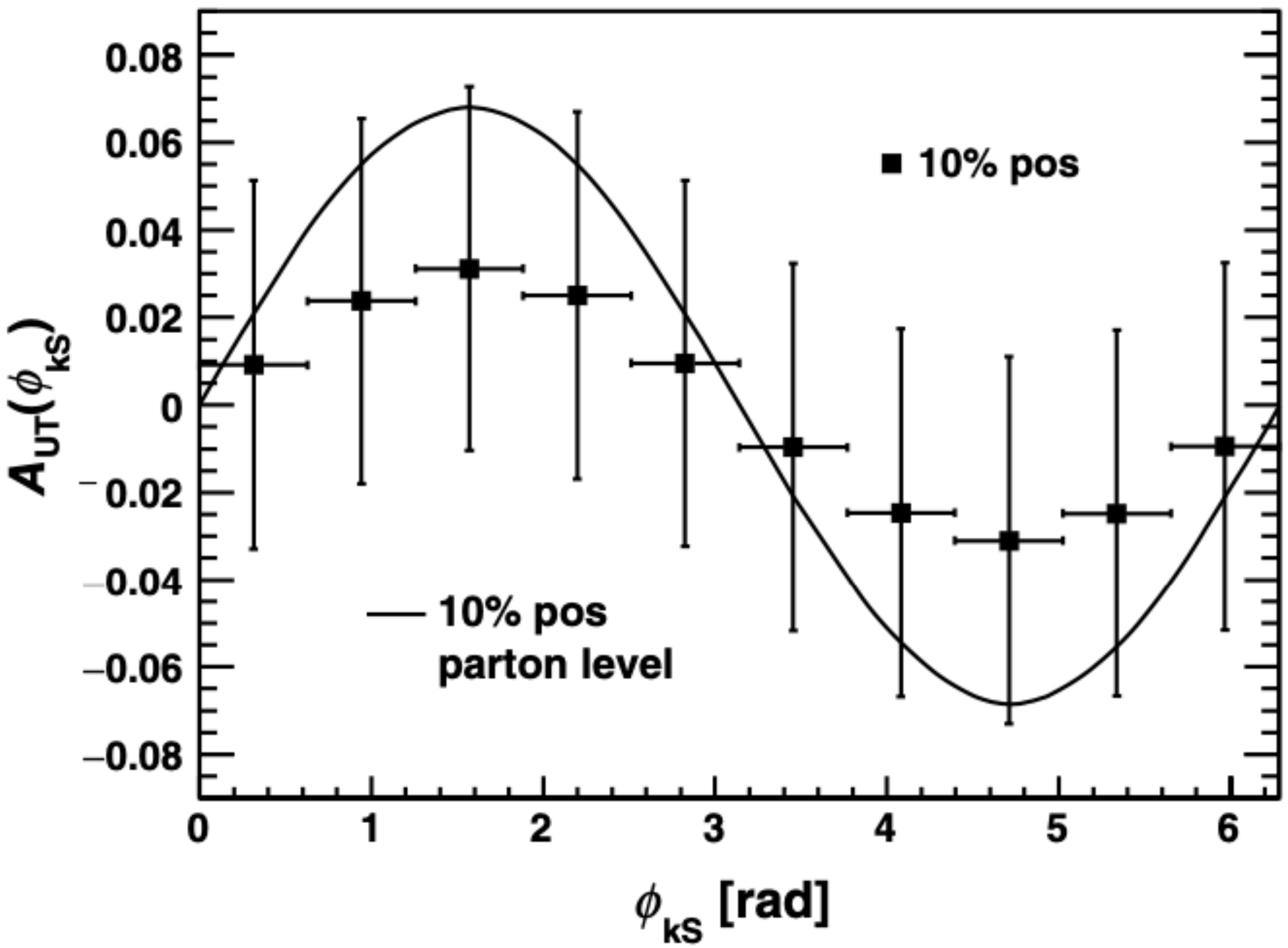
EIC RHIC

Boer, Mulders, Pisano, J. Zhou, 2016

Asymmetries in heavy quark pair production

Sivers asymmetry in D-meson pair production at EIC:

$$f_{1T}^{\perp\, g\,[+,+]}$$



Assumes gluon Sivers TMD that is 10% of the positivity bound

Luminosity of 10 fb^{-1}

Zheng, Aschenauer, Lee, Xiao, Jin, 2018

$\langle x_B \rangle = 0.0012$

The [+,+] Sivers TMD lacks the 1/x growth of the unpolarized gluon TMD (at least in the perturbative k_T regime), hence 10% at x=0.001 may be too optimistic

Boer, Echevarria, Mulders, J. Zhou, PRL 2016

Jet pair production more promising, but receives contributions from quark TMDs

High-p_T hadron pairs

COMPASS measured the gluon Sivers asymmetry in high-p_T hadron pair production in muon-deuteron and muon-proton scattering

The combined result for PGF:

$$f_{1T}^{\perp g\,[+,+]}$$

$A^{Siv} = -0.23 \pm 0.08$ (stat) ± 0.05 (syst) at $\langle x_g \rangle = 0.15$

[C. Adolph et al., PLB 2017]

Imposed requirement: $p_{1T} > 0.7$ GeV/c and $p_{2T} > 0.4$ GeV/c
The bulk of the data was for $p_{1T} < 1.7$ GeV/c and $p_{2T} < 1.1$ GeV/c

The gluon contribution has been estimated using MC
Significant contribution to the asymmetry from QCD Compton (quark TMDs)

EIC projections are promising for gluon Sivers TMDs 5% of the positivity bound

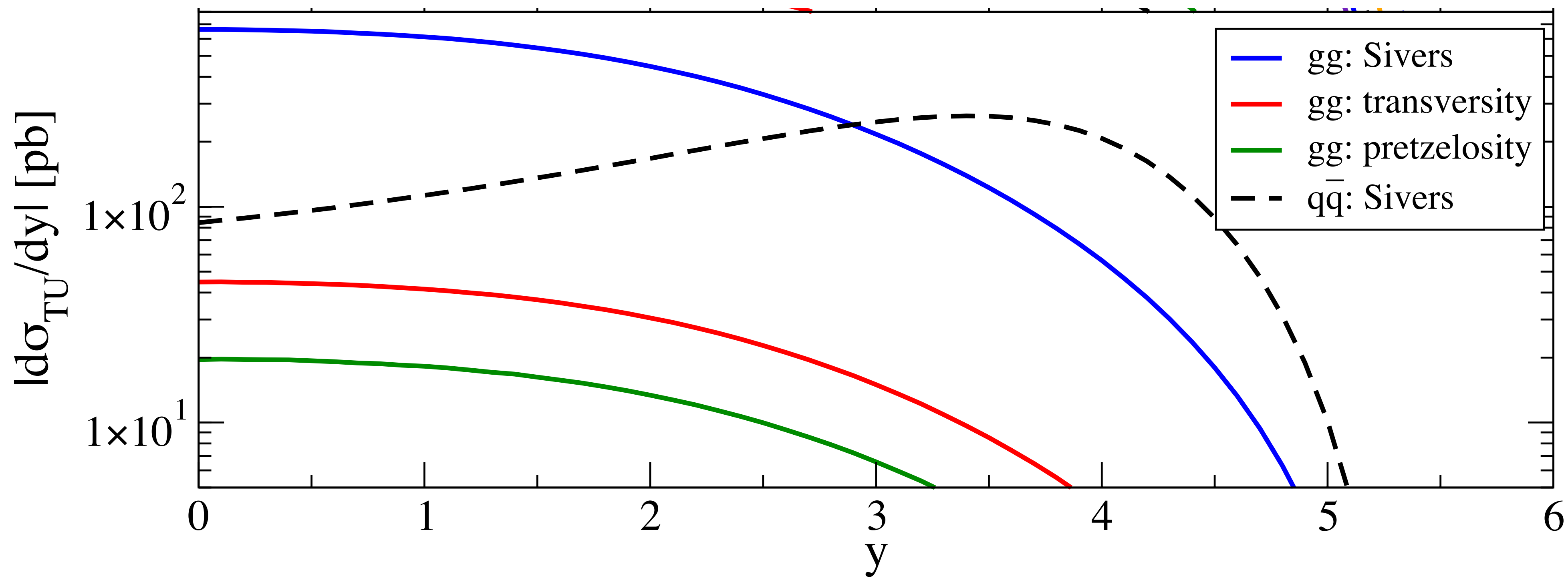
Zheng, Aschenauer, Lee, Xiao, Jin, 2018

Gluon Sivers effect in double γ production

$$f_{1T}^{\perp\, g\, [-,-]}$$

[Qiu, Schlegel, Vogelsang, 2011]

$p^{\uparrow}p \rightarrow \gamma\gamma X$



$\sqrt{s}$=500 GeV, $p_T^{\gamma} \geq 1$ GeV, integrated over $4 < Q^2 < 30$ GeV^2, $0 \leq q_T \leq 1$ GeV

At photon pair rapidity y < 3 gluon Sivers dominates and max($d\sigma_{TU}/d\sigma_{UU}$) ~ 30-50%

Asymmetry may be large but di-photon rate is not at RHIC, so also very challenging

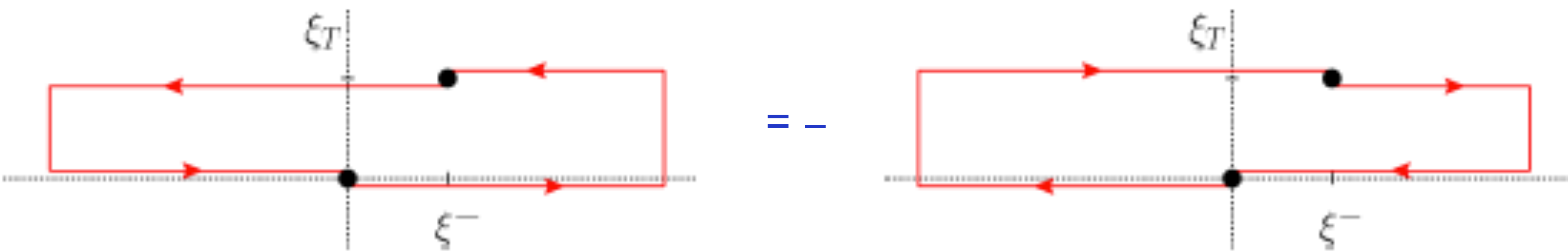
Same may apply to J/ψ γ and J/ψ J/ψ pair production in $p^{\uparrow}p$ collisions

f and d type gluon Sivers TMD

$e\, p^{\uparrow} \to e'\, Q\bar{Q}\, X$ $\gamma^*\, g \to Q\bar{Q}$ probes [+,+]

$p^{\uparrow}\, p \to \gamma\, \mathrm{jet}\, X$

In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess: $g\, q \to \gamma\, q$ probes [+,−]



These processes probe 2 distinct, *independent* gluon Sivers functions

Related to the antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Conclusion: gluon Sivers TMD studies using different processes can be related or they can be complementary

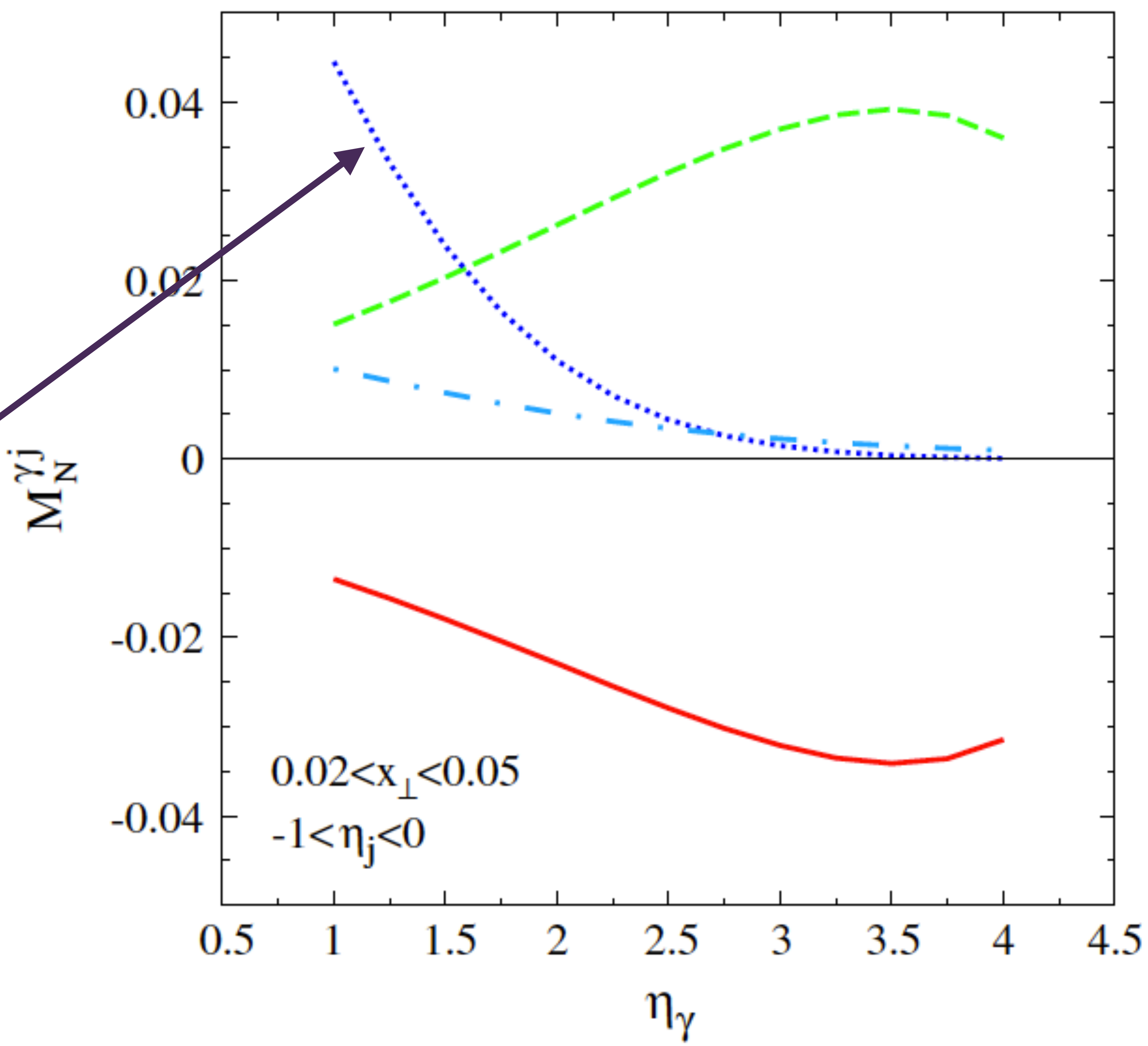
Gluon Sivers effect in γ jet production

[Bacchetta, Bomhof, D'Alesio, Mulders, Murgia, 2007]

$$f_{1T}^{\perp\, g\, [+,-]}$$

$p^{\uparrow}p \rightarrow \gamma$ jet X

maximum contribution from the gluon Sivers function (absolute value)



Prediction for the azimuthal moment at $\sqrt{s}$=200 GeV and $p_T^\gamma \geq 1$ GeV

$$M_N^{\gamma j}(\eta_\gamma, \eta_j, x_\perp) = \frac{\int d\phi_j\, d\phi_\gamma \frac{2|\boldsymbol{K}_{\gamma\perp}|}{M} \sin(\delta\phi) \cos(\phi_\gamma)\, \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}{\int d\phi_j\, d\phi_\gamma\, \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}$$

Gluon Sivers effect at small x

Selection of processes that probe the f-type or d-type Sivers gluon TMD:

	$e\,p^\uparrow \to e'\,Q\,\overline{Q}\,X$ $e\,p^\uparrow \to e'\,j_1\,j_2\,X$	$p^\uparrow\,A \to h\,X$ $(x_F < 0)$	$p^\uparrow A \to \gamma^{(*)}\,\text{jet}\,X$	$p^\uparrow p \to \gamma\,\gamma\,X$ $p^\uparrow p \to J/\psi\,\gamma\,X$ $p^\uparrow p \to J/\psi\,J/\psi\,X$
$f_{1T}^{\perp\,g\,[+,+]}$	$\surd$	$\times$	$\times$	$\surd$
$f_{1T}^{\perp\,g\,[+,-]}$	$\times$	$\surd$	$\surd$	$\times$

The d-type Sivers gluon TMD can be probed in $p^\uparrow A \to h\,X$ in the backward region

→ small x in the polarized proton and large A will enhance gluon-gluon scattering

The *leading twist* [+,−] correlator becomes in the small-x limit:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$

Boer, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, JHEP 2016

d-type gluon Sivers effect

The d-type gluon Sivers function $f_{1T}^{\perp\, g\, [+,-]}$ at small x is part of:

$$\Gamma^{(T-\mathrm{odd})}_{(d)} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\,\langle P, S_T|\mathrm{Tr}\left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T)\right]|P, S_T\rangle$$

Boer, Echevarria, Mulders, J. Zhou, PRL 2016

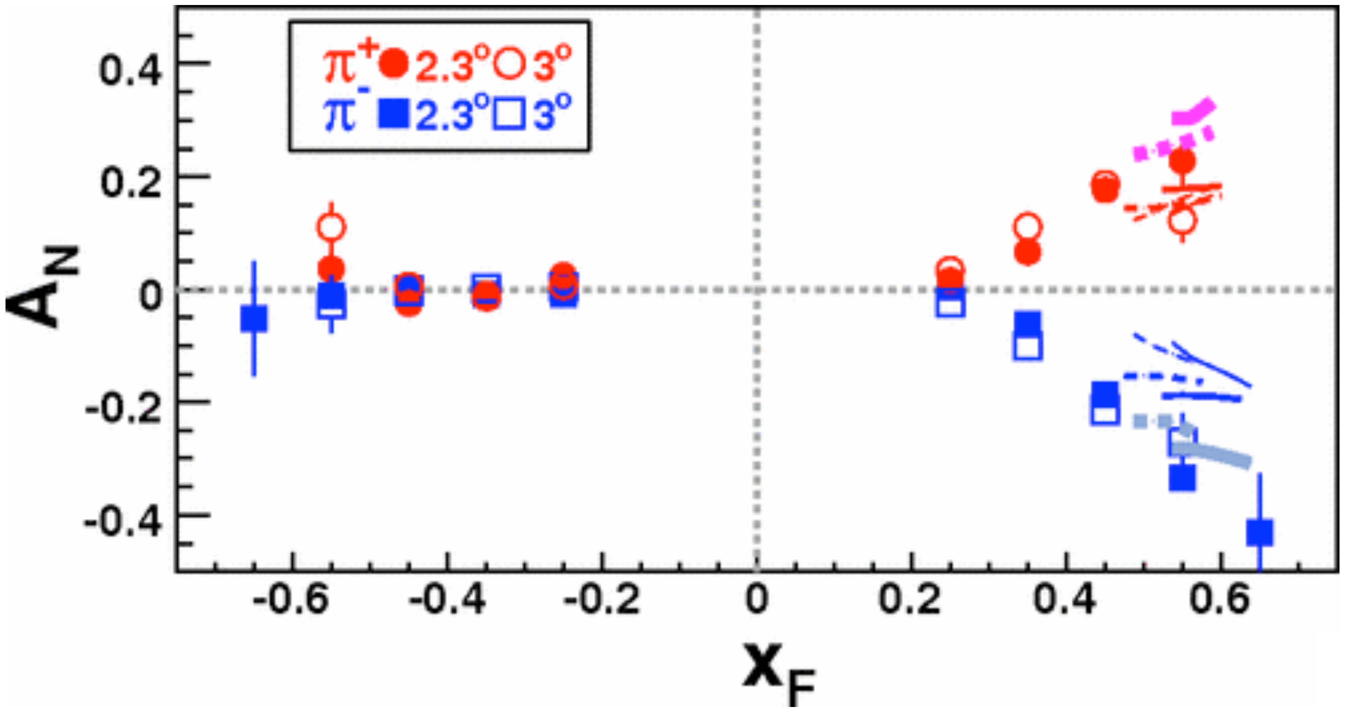
At small x it can be identified with the *spin-dependent odderon* [J. Zhou, 2013]

It is the only relevant contribution to A_N in backward ($x_F < 0$) charged hadron production in $p^\uparrow A$

A_N is not a TMD factorizing process, but at small x one can apply a hybrid factorization (at least at one-loop order) [Chirilli, Xiao, Yuan, 2012]

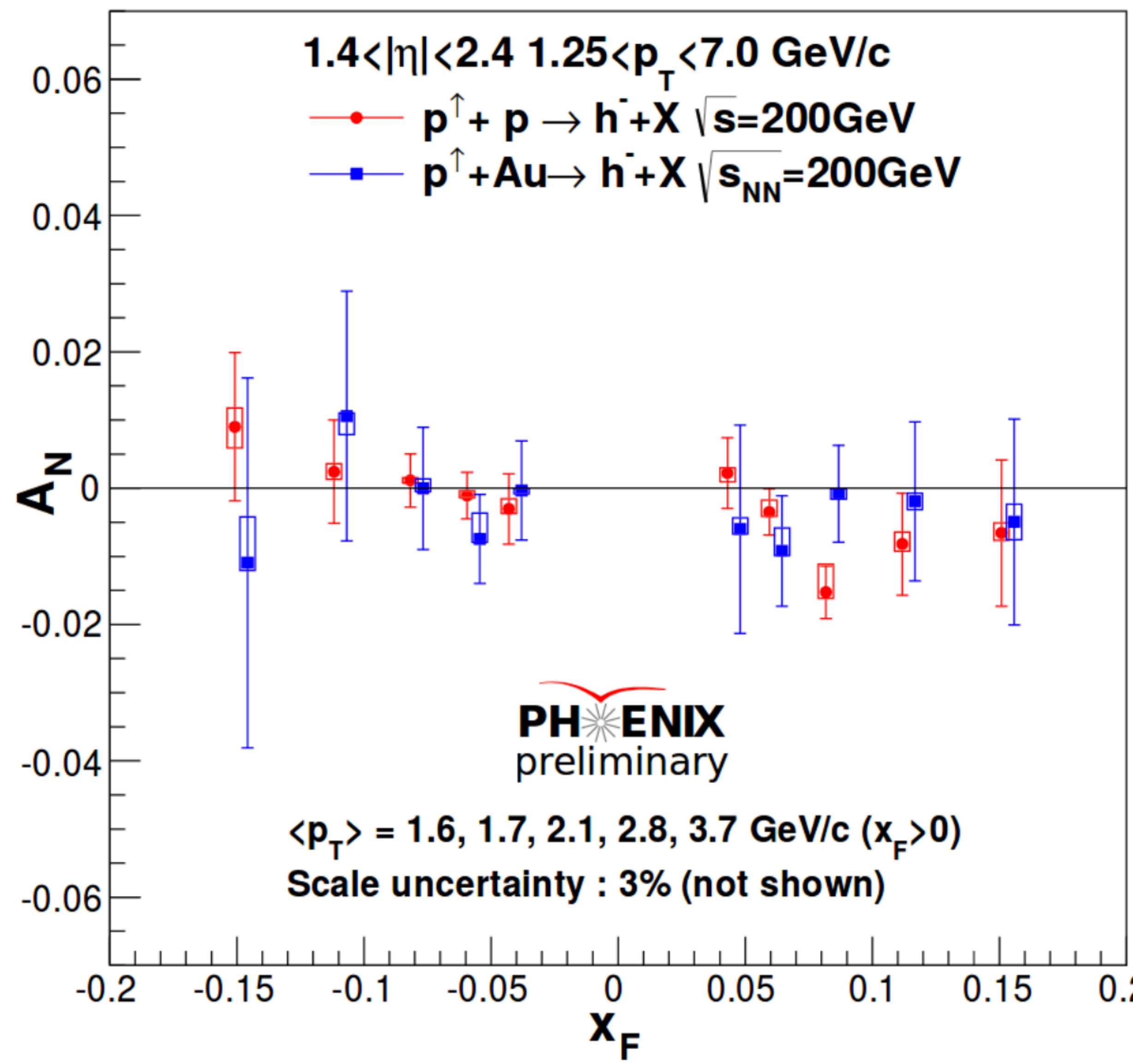
As the odderon is C-odd, for gg-dominated scattering one should select final states that are not C-even, hence charged hadron production (as opposed to jets or π^0)

$p^{\uparrow}p \to h^{\pm} X$ at $x_F < 0$



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

PHENIX, 2017
$\sqrt{s}$ = 200 GeV
p_T between 1.25 and 7 GeV



Interesting process to study e.g. at NICA

Probes of unpolarized gluon TMDs

WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,+]$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,-]$$

For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \quad \text{WW}$$

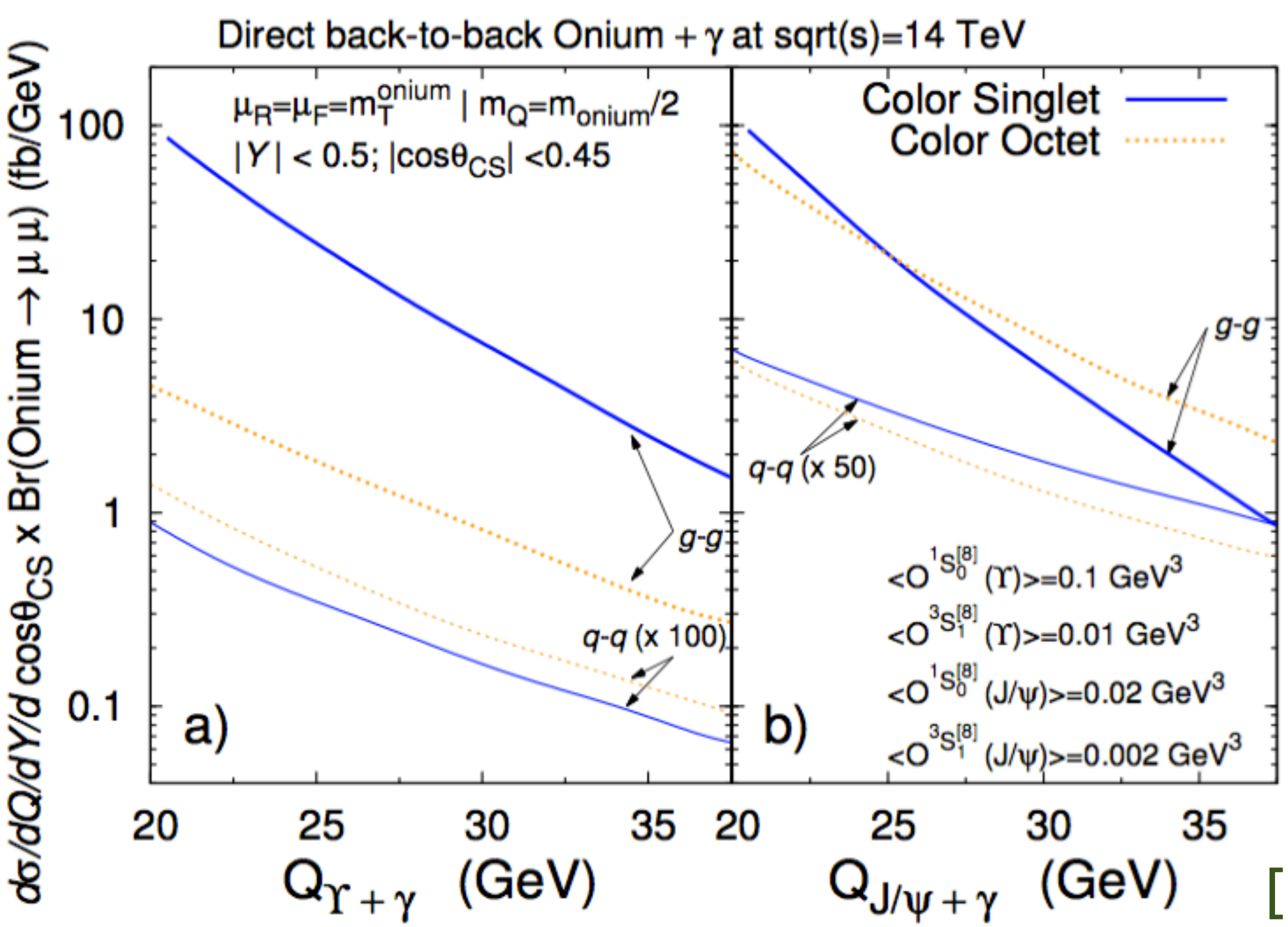
$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \quad \text{DP}$$

Different processes probe one or the other or a mixture, so this can be tested

Processes for unpolarized gluon TMDs

Selection of processes that probe the WW or DP unpolarized gluon TMD:

	$pA \to \gamma\,\text{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$f_1^{g\,[+,+]}$ (WW)	$\times$	$\surd$	$\surd$	$\surd$
$f_1^{g\,[+,-]}$ (DP)	$\surd$	$\times$	$\times$	$\times$



$p\,p \to \mathcal{Q}\,\gamma\,X$

a good process at LHC to extract

$$f_1^{g\,[+,+]}$$

[Den Dunnen, Lansberg, Pisano, Schlegel, 2014]

Processes for unpolarized gluon TMDs

Heavy quarks are generally promising to exploit, but not in all processes:

$pp \to J/\psi\, X$ or $\Upsilon\, X$ Color Singlet (CS) vector quarkonium production from 2 gluons is forbidden by Landau-Yang theorem, while Color Octet (CO) production involves a more complicated link structure

For C-even (pseudo-)scalar quarkonium production gg→CS is leading contribution

$pp \to \eta_c X$ or $pp \to \chi_c X$ could be studied at NICA

In LO NRQCD the differential cross sections in pp and pA are:

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\frac{\pi^3\alpha_s^2}{M^3\, s}\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\, \mathcal{C}\left[f_1^g\, f_1^g\right]\left[1 - R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q0})}{dy\, d^2\boldsymbol{q}_T} = \frac{8}{3}\frac{\pi^3\alpha_s^2}{M^5\, s}\langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\, \mathcal{C}\left[f_1^g\, f_1^g\right]\left[1 + R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q2})}{dy\, d^2\boldsymbol{q}_T} = \frac{32}{9}\frac{\pi^3\alpha_s^2}{M^5\, s}\langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\, \mathcal{C}\left[f_1^g\, f_1^g\right]$$

[Boer, Pisano, 2012]

$R(q_T^2)$ is the contribution from the linearly polarized gluon TMD $h_1^{\perp g}$

Probes of linearly polarized gluons TMDs

Probes of linear gluon polarization

For linearly polarized gluons [+,+] = [−,−] and [+,−] = [−,+]

Processes that probe the linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\text{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	√	×	√	√	√
$h_1^{\perp\,g\,[+,-]}$ (DP)	×	√	×	×	×

1% level at RHIC
Qiu, Schlegel, Vogelsang, 2011

5% level at RHIC
Boer, Mulders, J. Zhou, Y. Zhou, 2017

10% level at EIC
Boer, Brodsky, Pisano, Mulders, 2011;
Dumitru, Lappi, Skokov, 2015;
Boer, Pisano, Mulders, J. Zhou, 2016;
Efremov, Ivanov, Teryaev, 2018

10% level for η_Q and
1% level for Higgs at LHC
Boer & den Dunnen, 2014;
Echevarria, Kasemets, Mulders, Pisano, 2015

Higgs and $0^{\pm+}$ quarkonium production uses the angular *independent* p_T distribution

All other suggestions use angular modulations

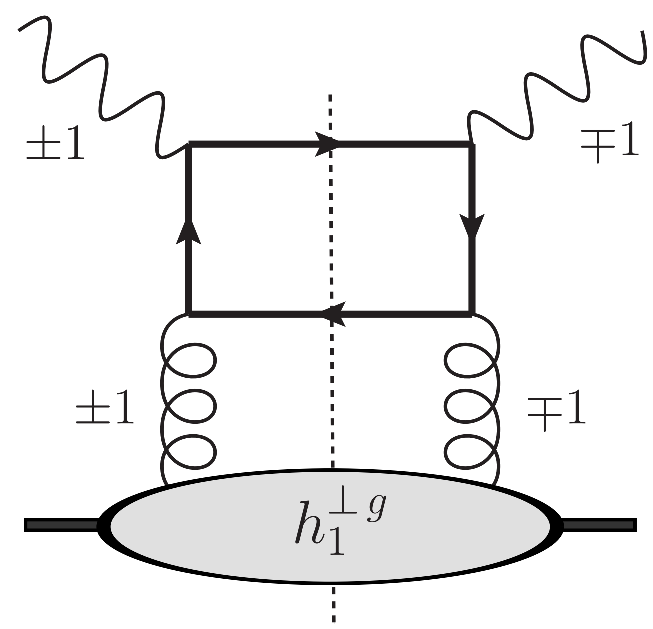
$pp \to Q\bar{Q}\,X$ [Akcakaya, Schäfer, Zhou, 2013; Pisano, Boer, Brodsky, Buffing, Mulders, 2013]

TMD factorization is a concern here [Catani, Grazzini, Torre, 2015]

Open heavy quark electro-production

Unpolarized open heavy quark production at EIC allows to probe $h_1^{\perp\, g}(x, p_T^2)$

$$ep \to e' Q\bar{Q} X$$



no convolution!

[Boer, Brodsky, Mulders & Pisano, 2010]

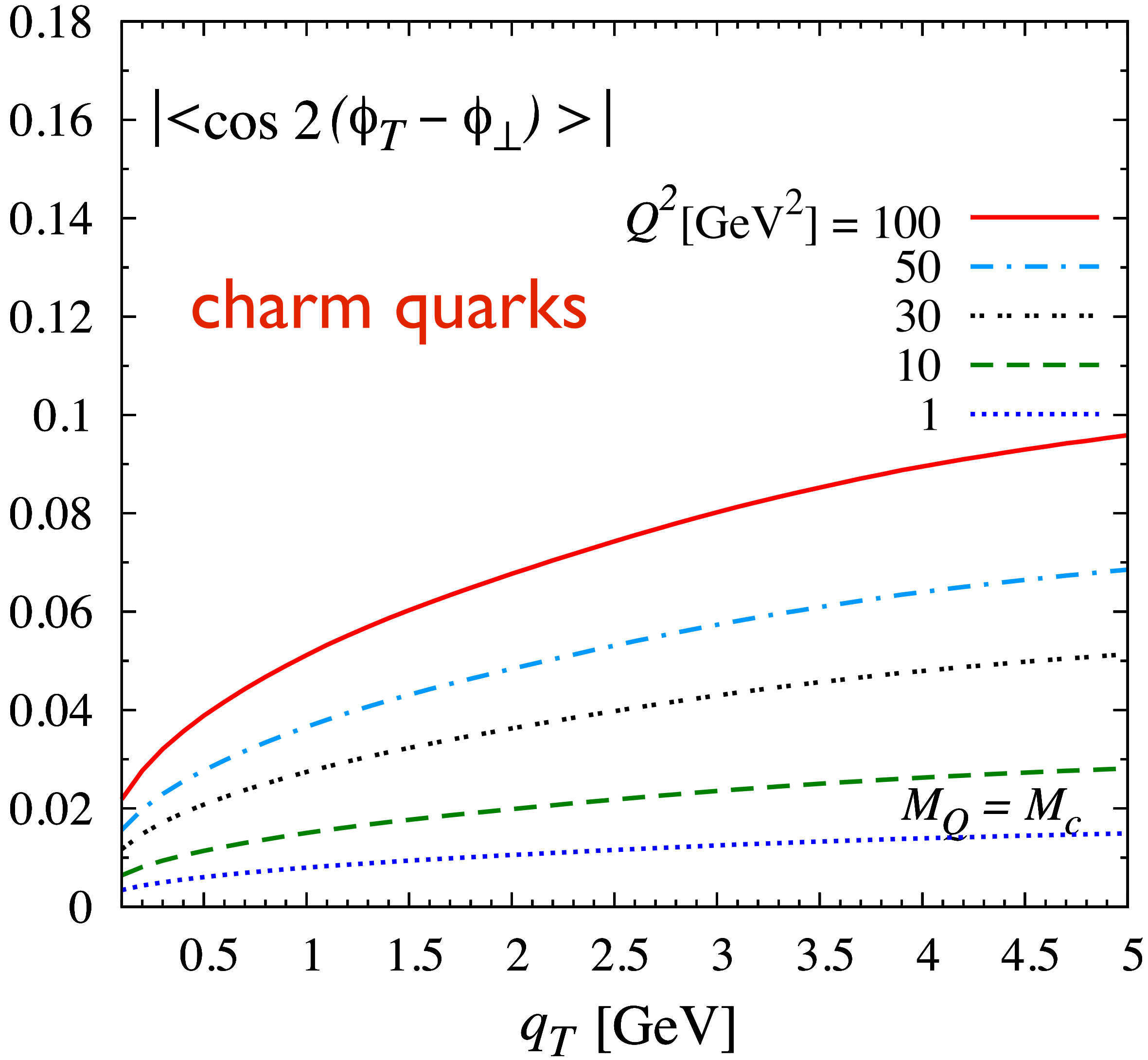
The individual transverse momenta have to be large but their sum has to be small

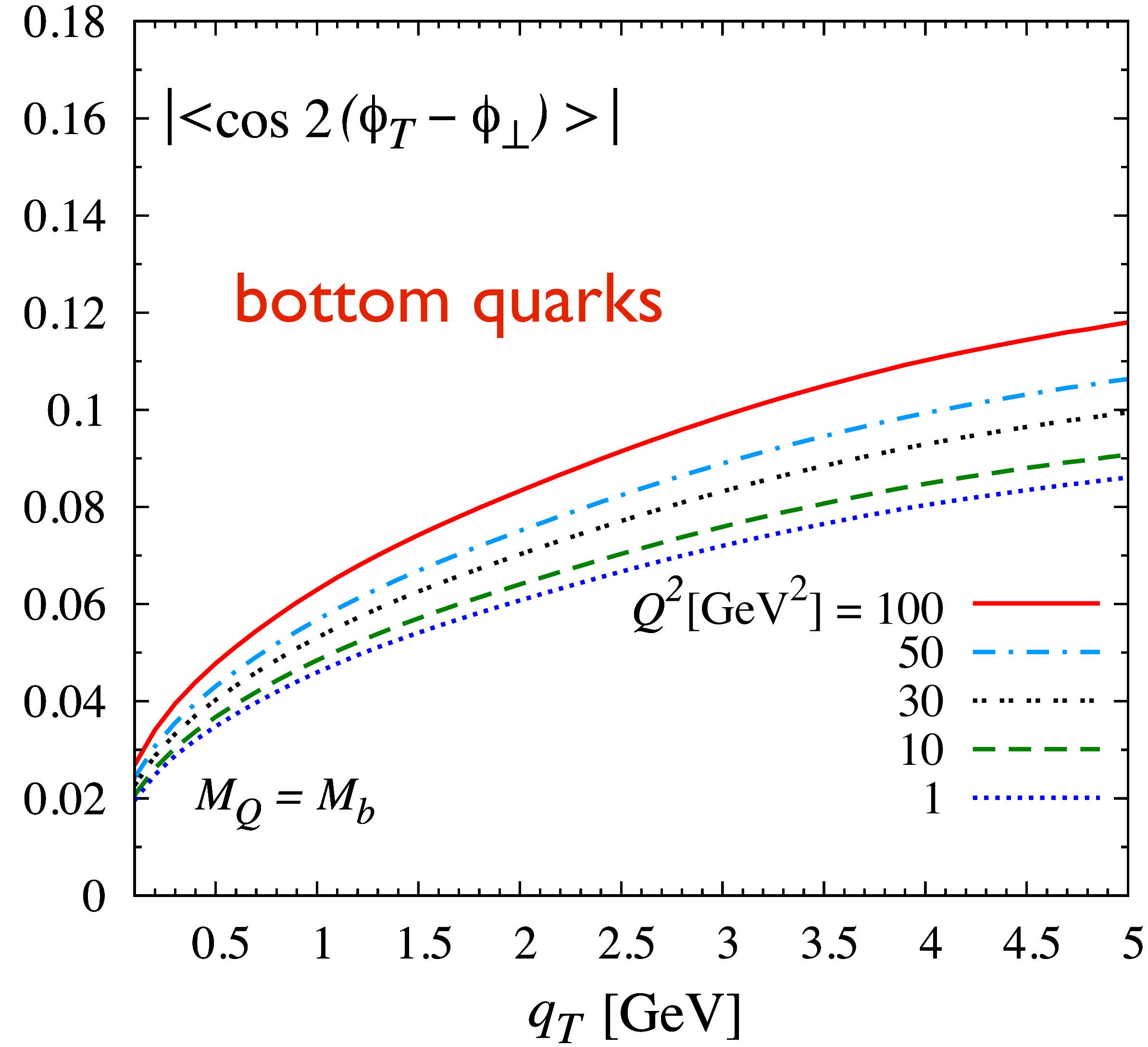
Linearly polarized gluons lead to $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_\perp)$ angular modulation

$h_1^{\perp g}$ appears by itself, so its sign can be determined and the effects could be significant, especially towards smaller x as it follows the fast growth of f_1^g

Asymmetries in heavy quark pair production

$h_1^{\perp\, g}$ expected to keep up with growth of the unpolarized gluons TMD as x → 0





small x
MV model

$|\boldsymbol{K}_\perp| = 10\,\text{GeV}$

$z = 0.5$

$y = 0.3$

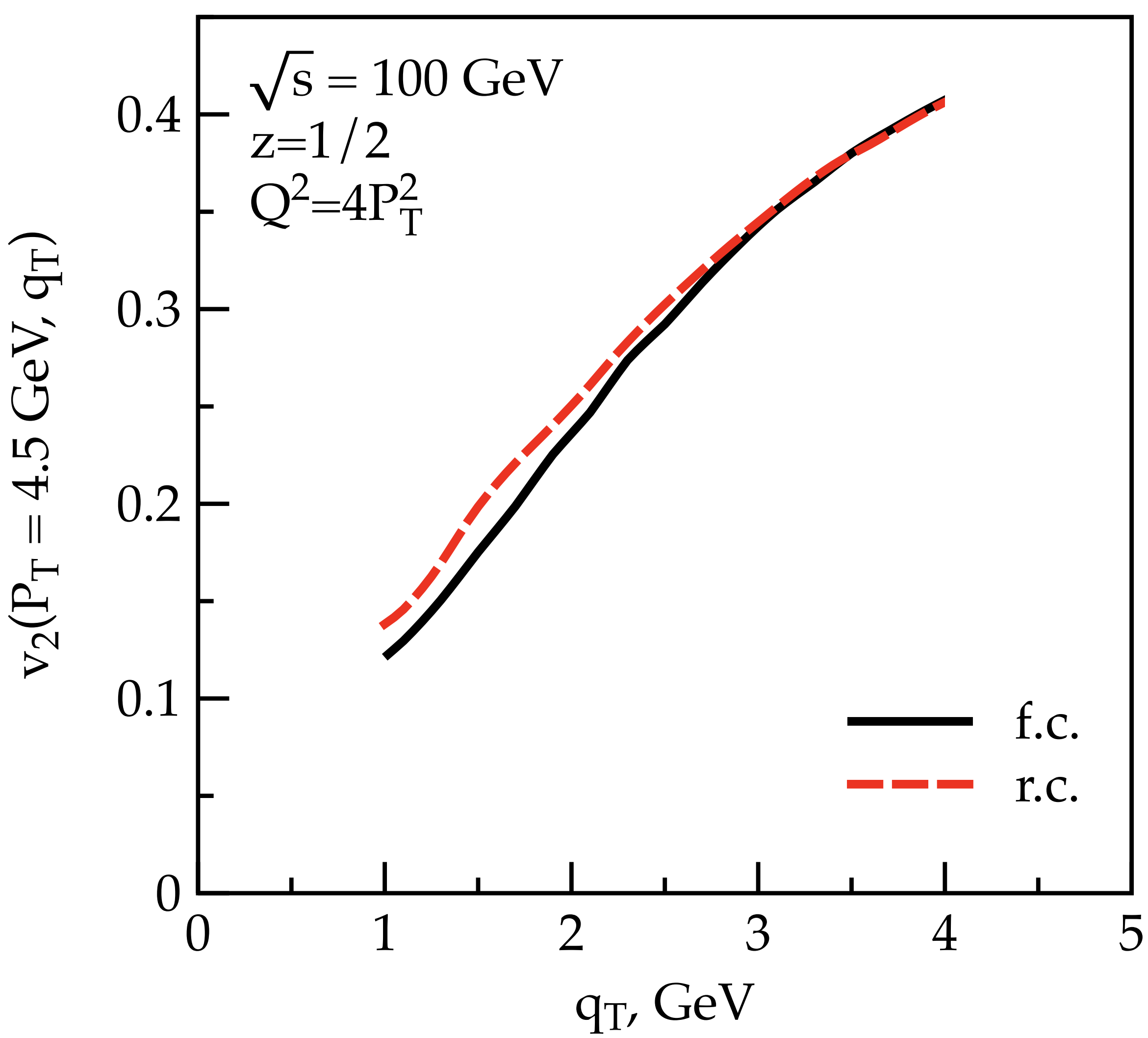
Sizable asymmetries at EIC

[Boer, Pisano, Mulders, Zhou, 2016]

Dijet production at EIC

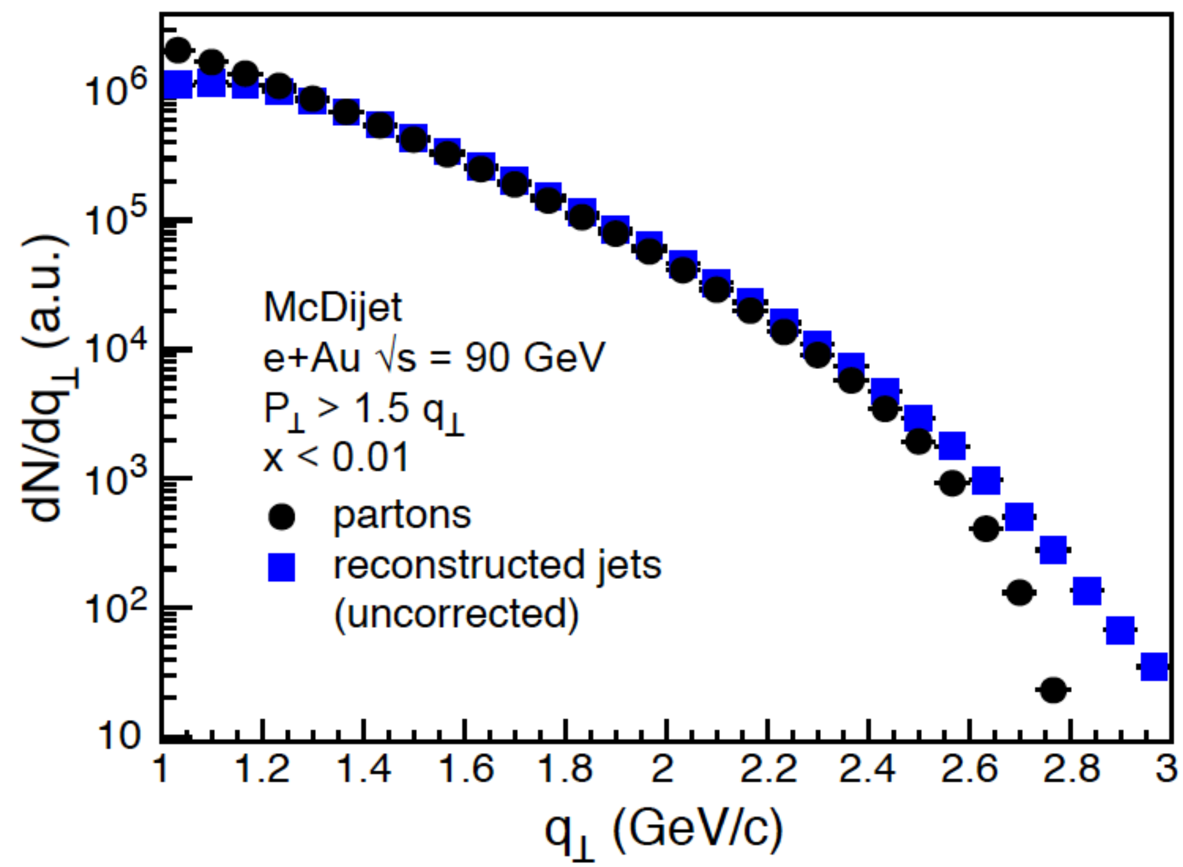
$h_1^{\perp g}$ (WW) is also accessible in dijet production at EIC
[Metz, Zhou 2011; Pisano, Boer, Brodsky, Buffing, Mulders, 2013; Boer, Pisano, Mulders, Zhou, 2016]

Linear gluon polarization shows itself through a cos2ϕ distribution ("v_2")



Large effects are found

Dumitru, Lappi, Skokov, 2015



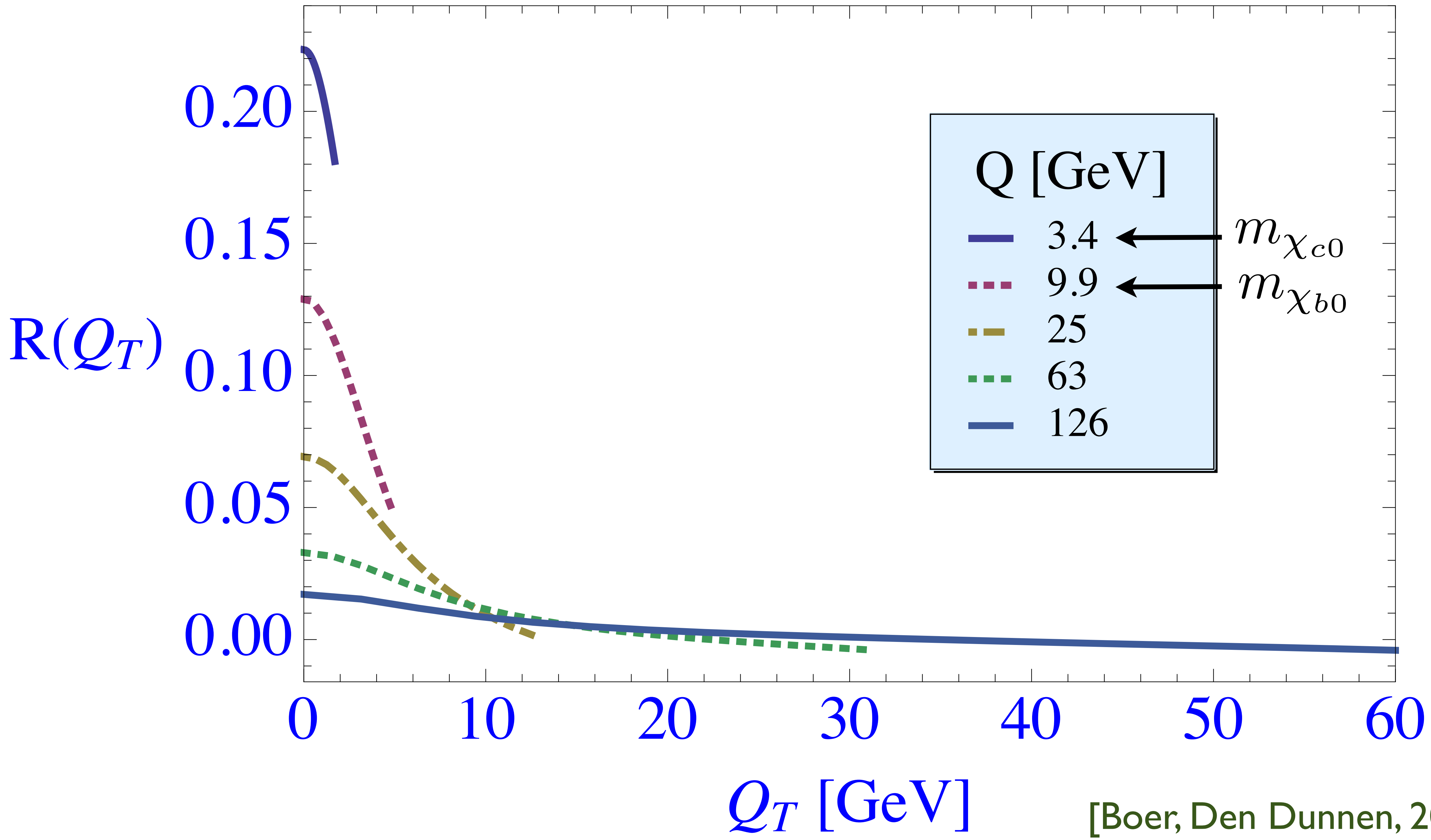
Jets are reasonable proxies for outgoing quarks concerning the q_T distribution

Dumitru, Skokov, Ullrich, 2018

Quarkonium production in pp

pp→ η_c X or pp→ χ_c X allow to probe the linearly polarized gluon TMD

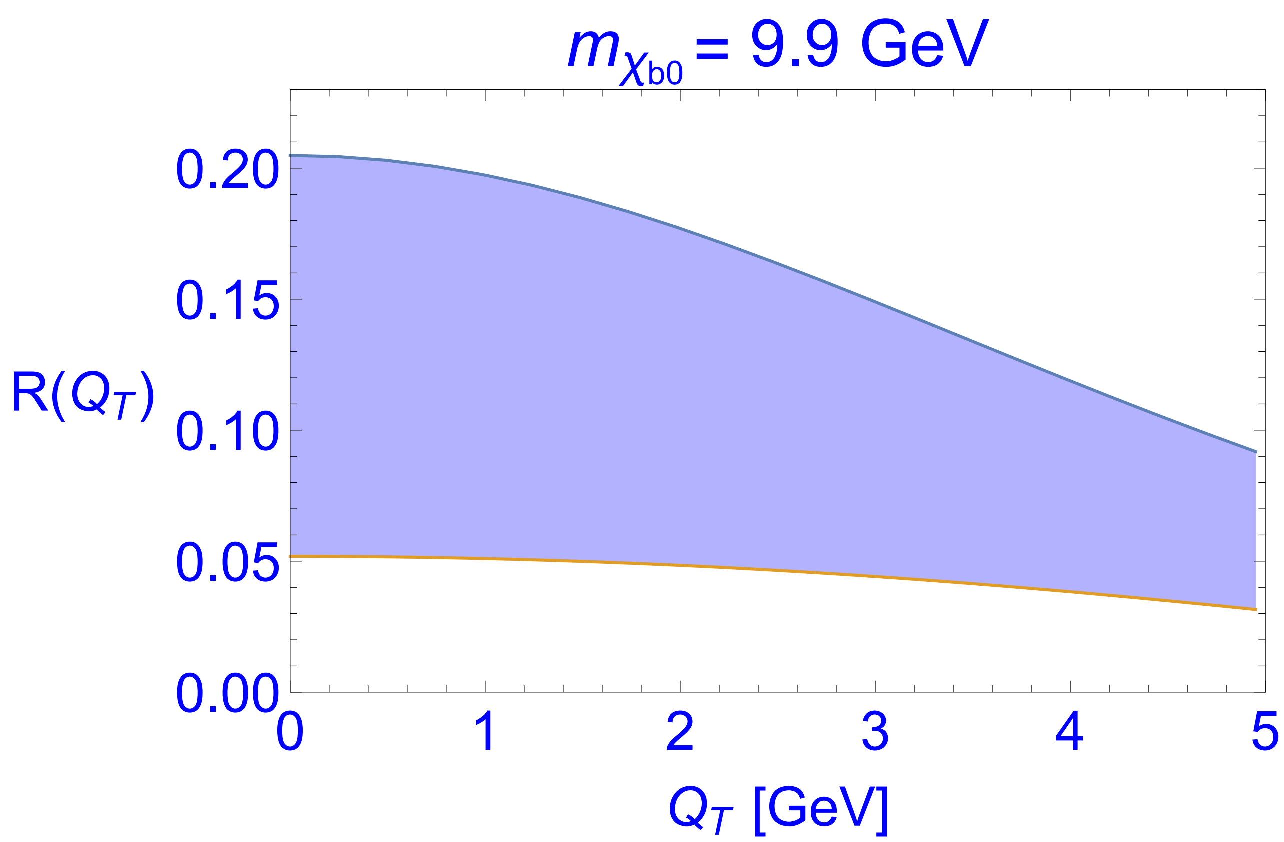
The relative contribution to the diff. cross section w.r.t. the unpolarized gluons:



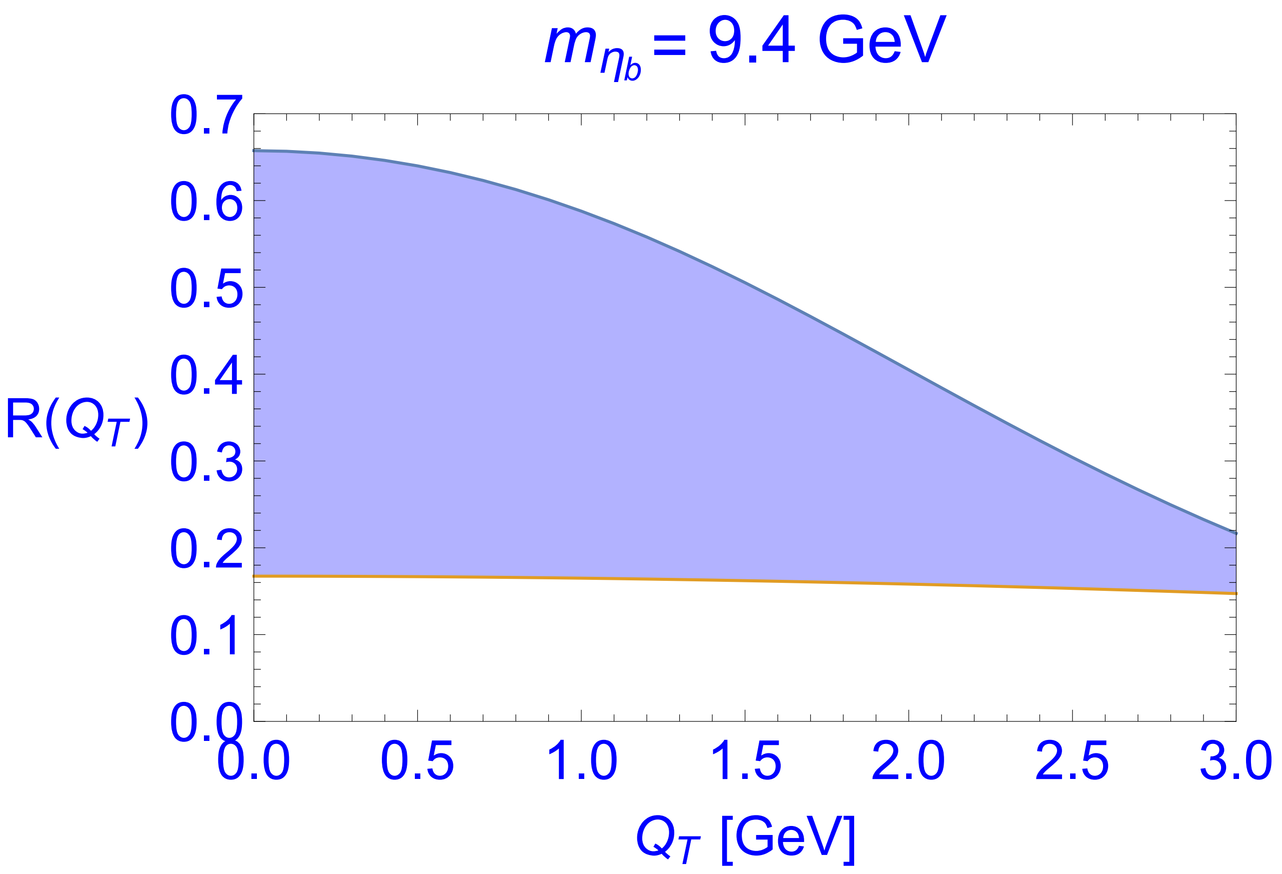
[Boer, Den Dunnen, 2014]

Bottomonium production in pp

The range of predictions for C-even bottomonium production:



Boer & den Dunnen, 2014



Echevarria, Kasemets, Mulders, Pisano, 2015

Very large theoretical uncertainties (from the nonperturbative part of the TMDs), even more for charmonium production, but contribution of 20% or more is expected

γ^*-jet production

$h_1^{\perp g}$ is power suppressed in *pp*→ *γ jet X* [Boer, Mulders, Pisano, 2008]

It is not power suppressed in *pp*→ *γ* jet X if* $Q^2 \sim P_{\perp,jet}^2$ [Boer, Mulders, Zhou & Zhou, 2017]

Consider $Q^2 \sim P_{\perp,jet}^2$ also to avoid a three-scale problem

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\text{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\surd$	$\times$	$\surd$	$\surd$	$\surd$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\surd$	$\times$	$\times$	$\times$

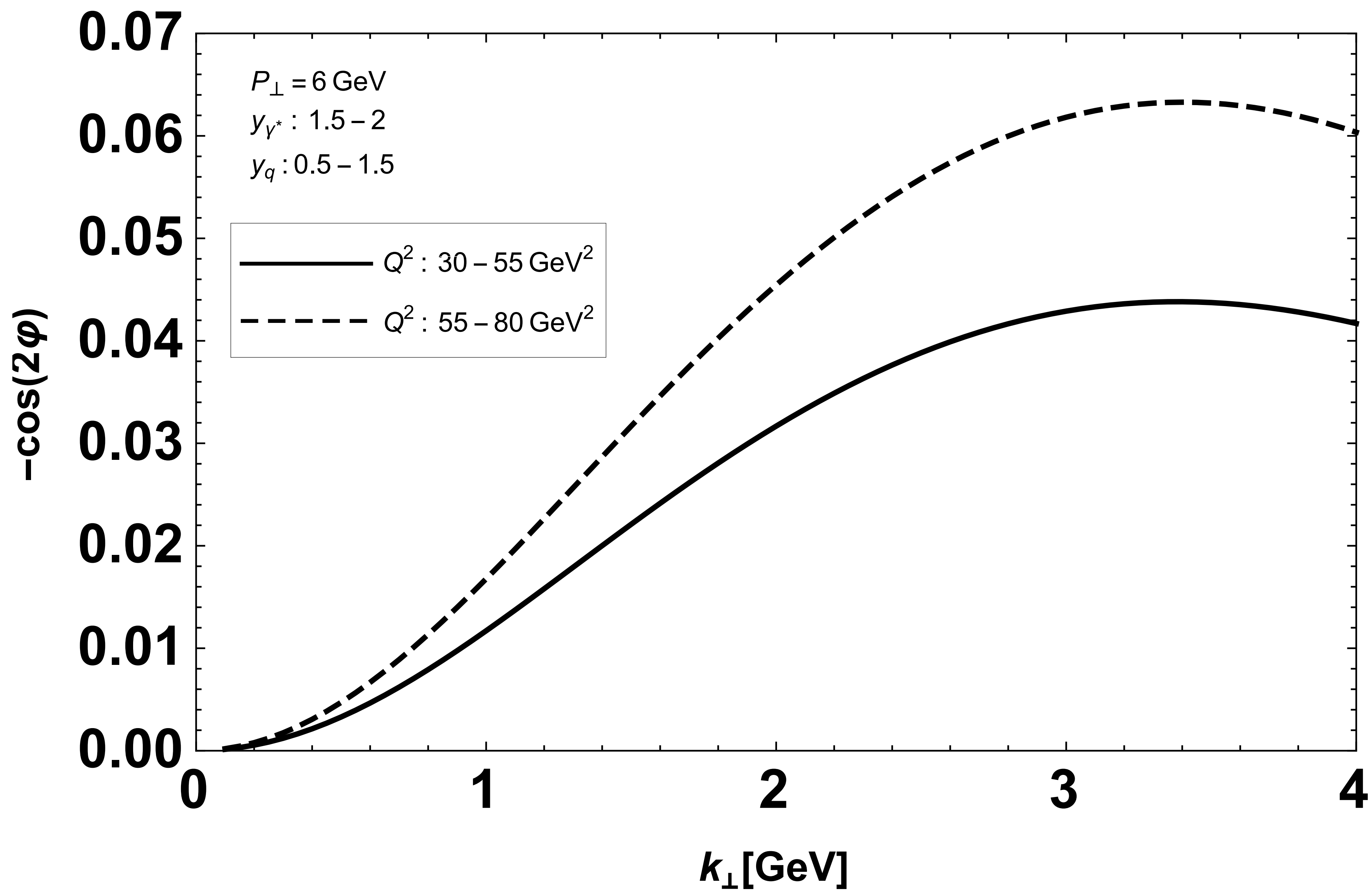
pp→ *γ* jet X* offers a *unique* opportunity to study the DP linear gluon polarization

At high gluon density (large A and/or small x) the DP linear gluon polarization is expected to become maximal, as was first shown in the MV model for the CGC

$$xh_{1,DP}^{\perp g}(x,k_\perp) = 2xf_{1,DP}^{g}(x,k_\perp)$$

[Metz & Jian Zhou, 2011; Boer, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016]

Sudakov suppression of linear gluon polarization

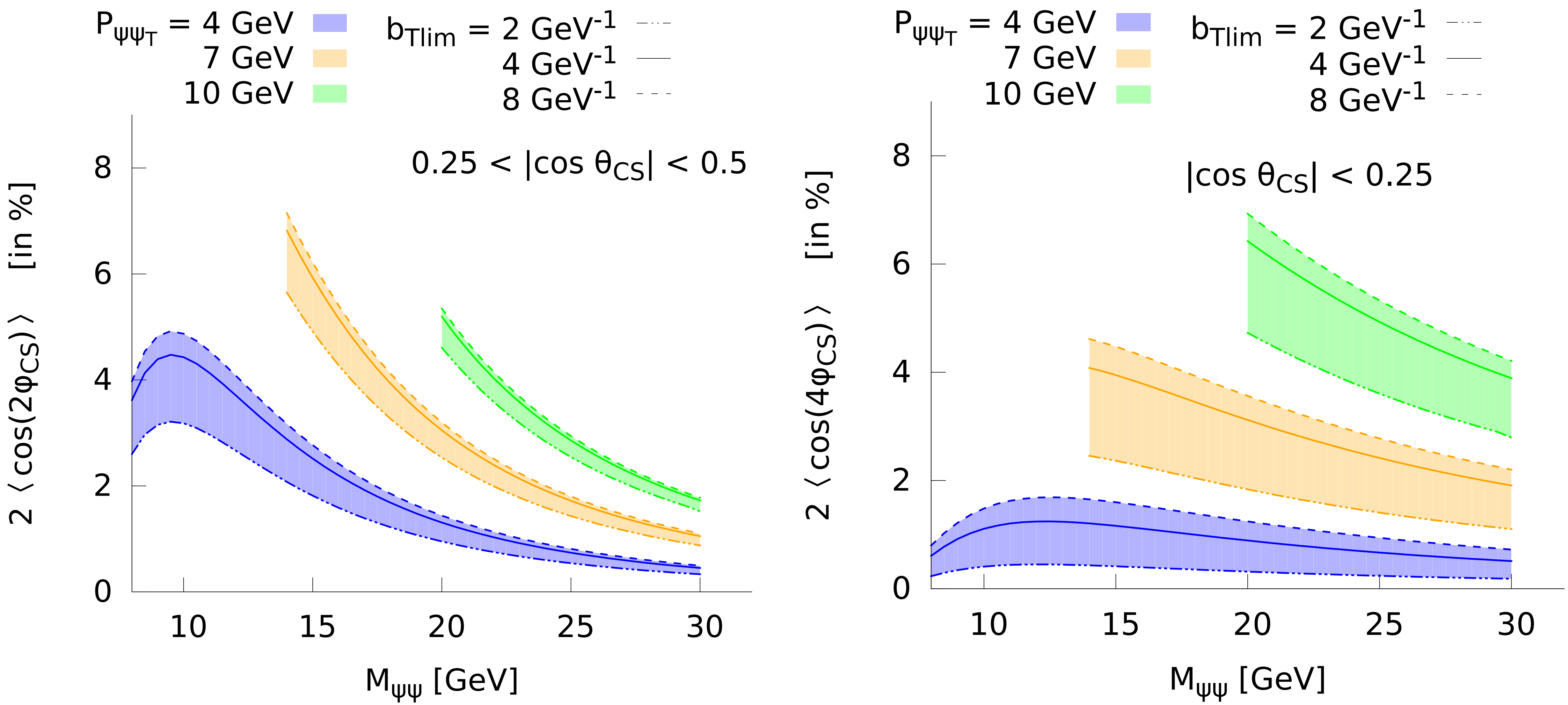


Despite the DP linear gluon polarization becoming maximal at small x, there is amplitude and Sudakov suppression of the cos(2ϕ) asymmetry in pA→γ^* jet X:

~5% asymmetry at RHIC   [Boer, Mulders, Jian Zhou & Ya-Jin Zhou, 2017]

It becomes effectively power suppressed as Q~P⊥ increases from 6 to 90 GeV

Linear gluon polarization in di-J/Ψ production



At LHC $h_1^{\perp g}$ can be probed in $p\,p \to J/\psi\; J/\psi\; X$

Estimated to lead to 1-5% level azimuthal modulations

[Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, 2019]

Conclusions

Conclusions

- Gluon TMDs measurements generally require higher energy collisions, less inclusive observables (particle pair correlations) and several processes (process dependence)

- Even the unpolarized gluon TMDs have not been extracted yet, but there are plenty of future opportunities

- For the linearly polarized gluon TMDs small x will be beneficial, for the f-type gluon Sivers TMD small x is not favorable

- The f-type [+,+] gluon Sivers TMD enters di-hadron production in SIDIS and satisfies the sign-change relation

- The d-type [+,−] gluon Sivers TMD at small x corresponds to the spin-dependent odderon, a C-odd Wilson loop matrix element that fully determines A_N at negative x_F

Main opportunities

$f_1^{g\,[+,+]}$	$pp \to \gamma\, J/\psi\, X$	LHC
	$pp \to \gamma\, \Upsilon\, X$	LHC
$f_1^{g\,[+,-]}$	$pp \to \gamma\, \text{jet}\, X$	LHC & RHIC
$h_1^{\perp\, g\,[+,+]}$	$e\, p \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p \to e'\, \text{jet}\, \text{jet}\, X$	EIC
	$pp \to \eta_{c,b}\, X$	LHC & NICA
	$pp \to H\, X$	LHC
$h_1^{\perp\, g\,[+,-]}$	$pp \to \gamma^*\, \text{jet}\, X$	LHC & RHIC
$f_{1T}^{\perp\, g\,[+,+]}$	$e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p^\uparrow \to e'\, \text{jet}\, \text{jet}\, X$	EIC
$f_{1T}^{\perp\, g\,[-,-]}$	$p^\uparrow p \to \gamma\, \gamma\, X$	RHIC
$f_{1T}^{\perp\, g\,[+,-]}$	$p^\uparrow A \to \gamma^{(*)}\, \text{jet}\, X$	RHIC
	$p^\uparrow A \to h\, X\ \ (x_F < 0)$	RHIC & NICA