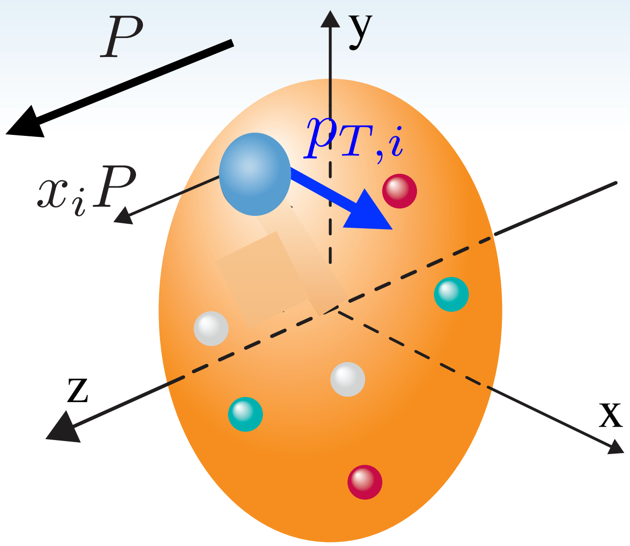
Probing TMDs with jets

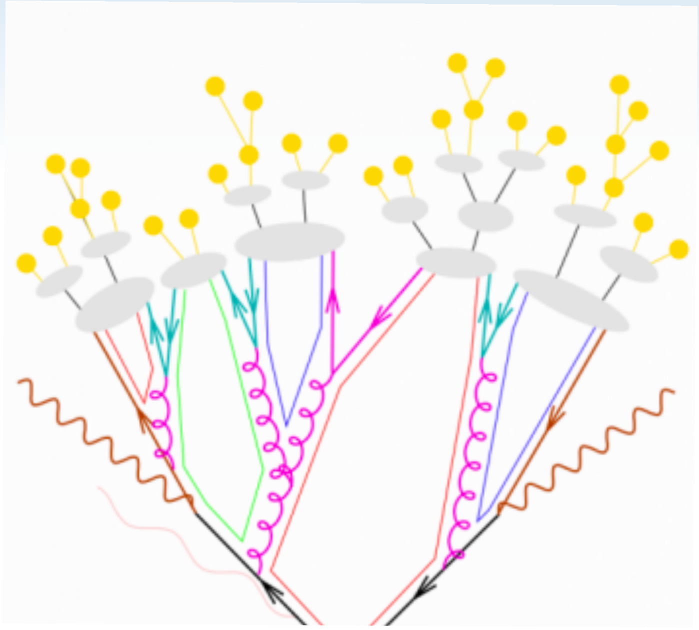
Kyle Lee
LBNL

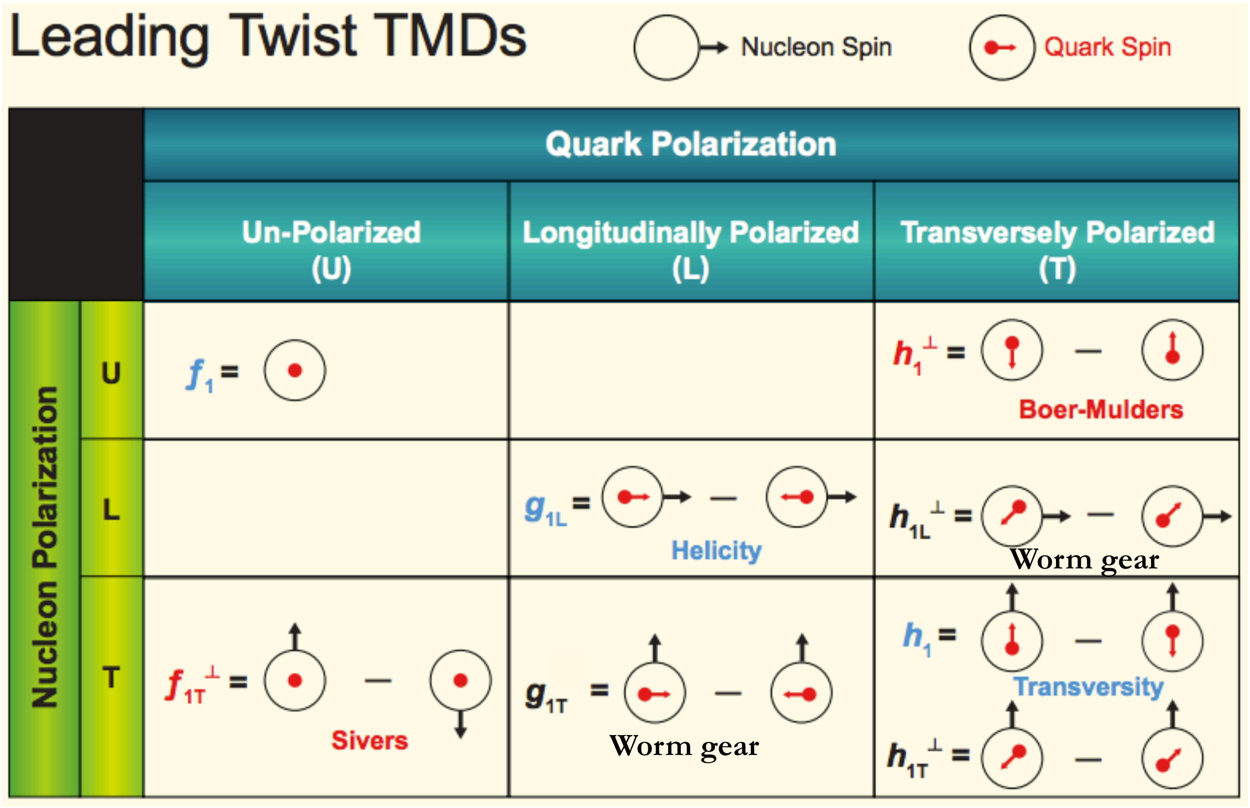
Sar Wors

6 - 8 September 2021

TMD structure







Leading Twist TMDs

Nucleon Spin Quark Spin

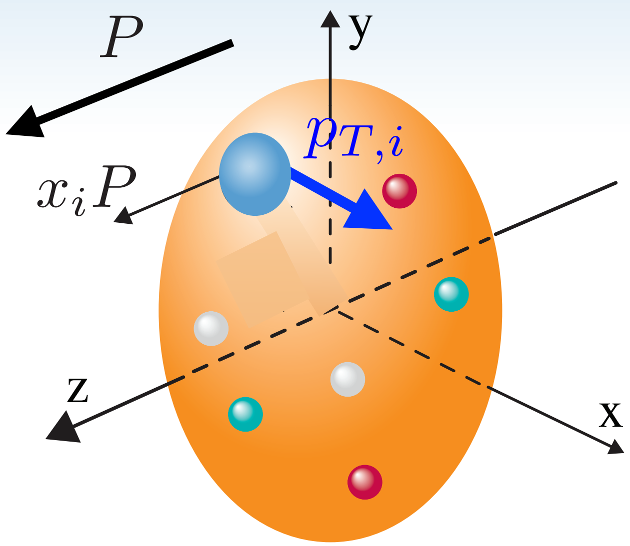
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = ⊙		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
	L		g_{1L} = ⊙ − ⊙ Helicity	$h_{1L}^\perp$ = ⊙ − ⊙ Worm gear
	T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	g_{1T} = ⊙ − ⊙ Worm gear	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙

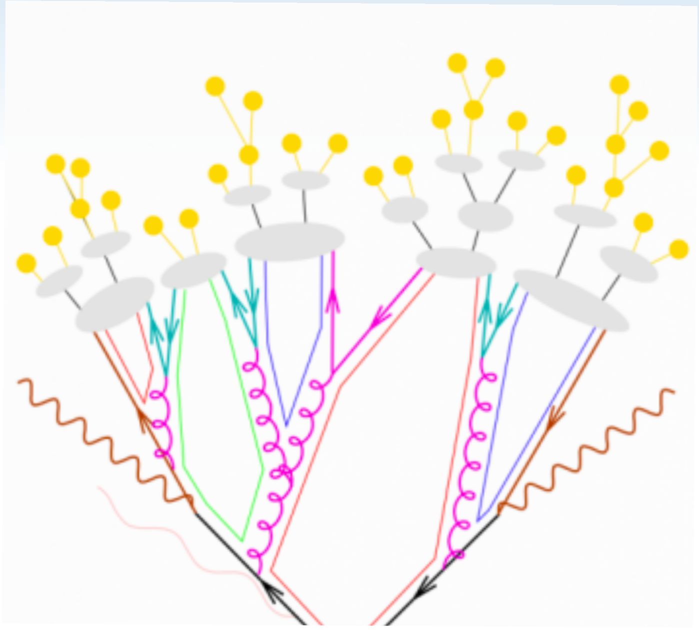
		Quark polarization		
		U	L	T
Hadron polarization	U	$D^{h/q}$		$H^{\perp h/q}$
	L		$G^{h/q}$	$H_L^{\perp h/q}$
	T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

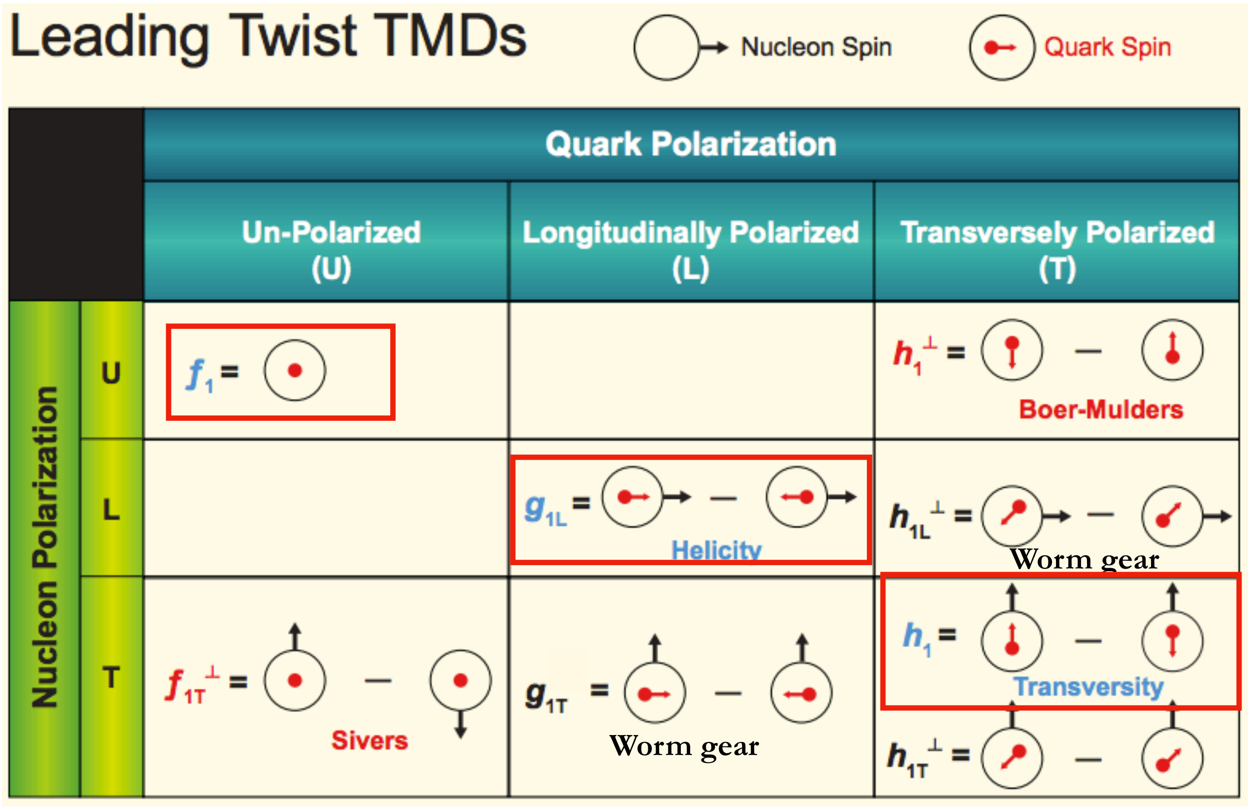
Quark **TMDPDF** inside spin-$\frac{1}{2}$ hadron

Quark **TMDFF** inside spin-$\frac{1}{2}$ hadron

TMD structure







Leading Twist TMDs

○→ Nucleon Spin ⊖ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^{\perp}$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^{\perp}$ = − Worm gear
	T	$f_{1T}^{\perp}$ = − Sivers	g_{1T} = − Worm gear	h_1 = − Transversity $h_{1T}^{\perp}$ = −

Collinear analogs!

Quark polarization

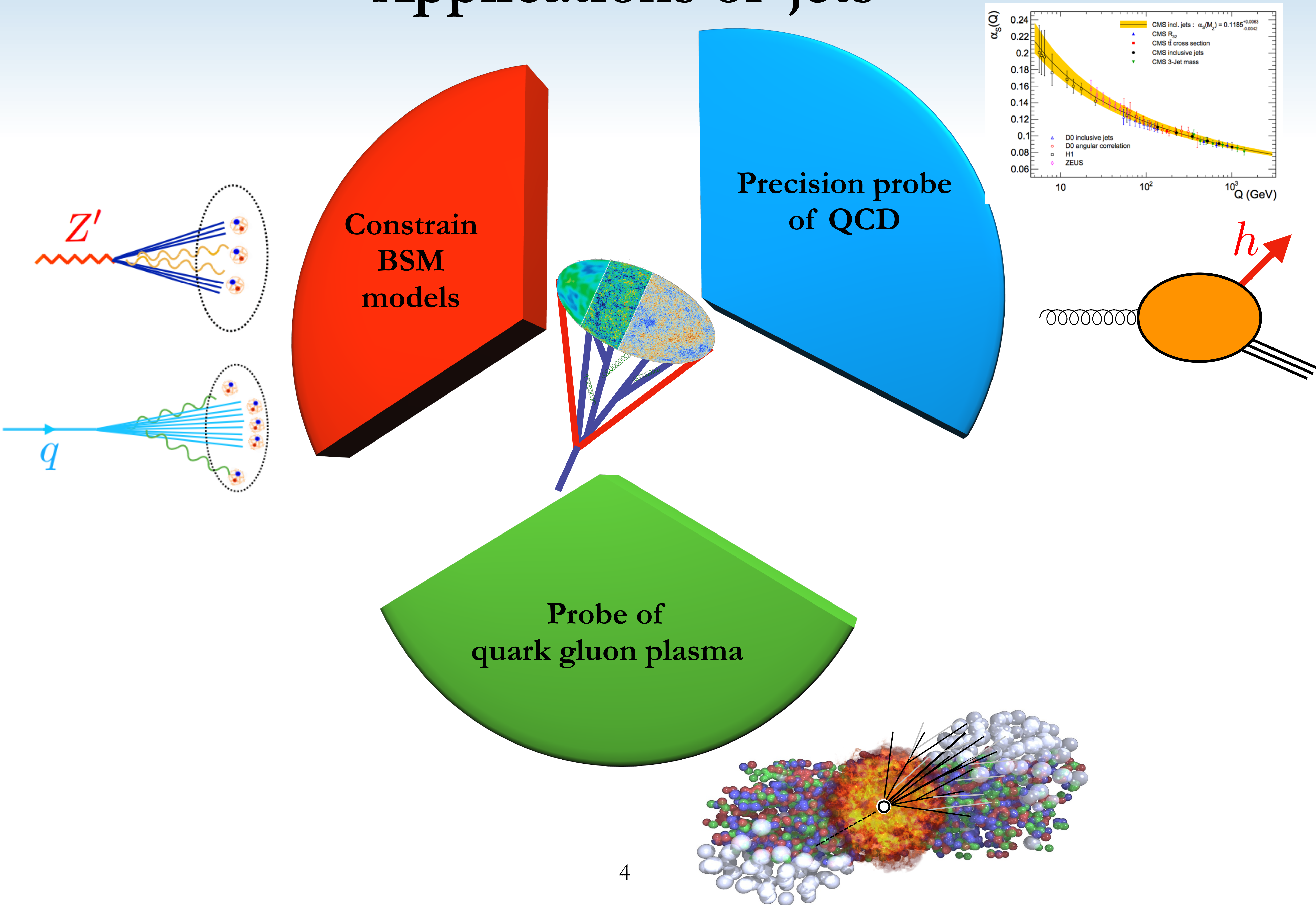
Hadron polarization

	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

Quark TMDPDF inside spin-$\frac{1}{2}$ hadron

Quark TMDFF inside spin-$\frac{1}{2}$ hadron

Applications of jets

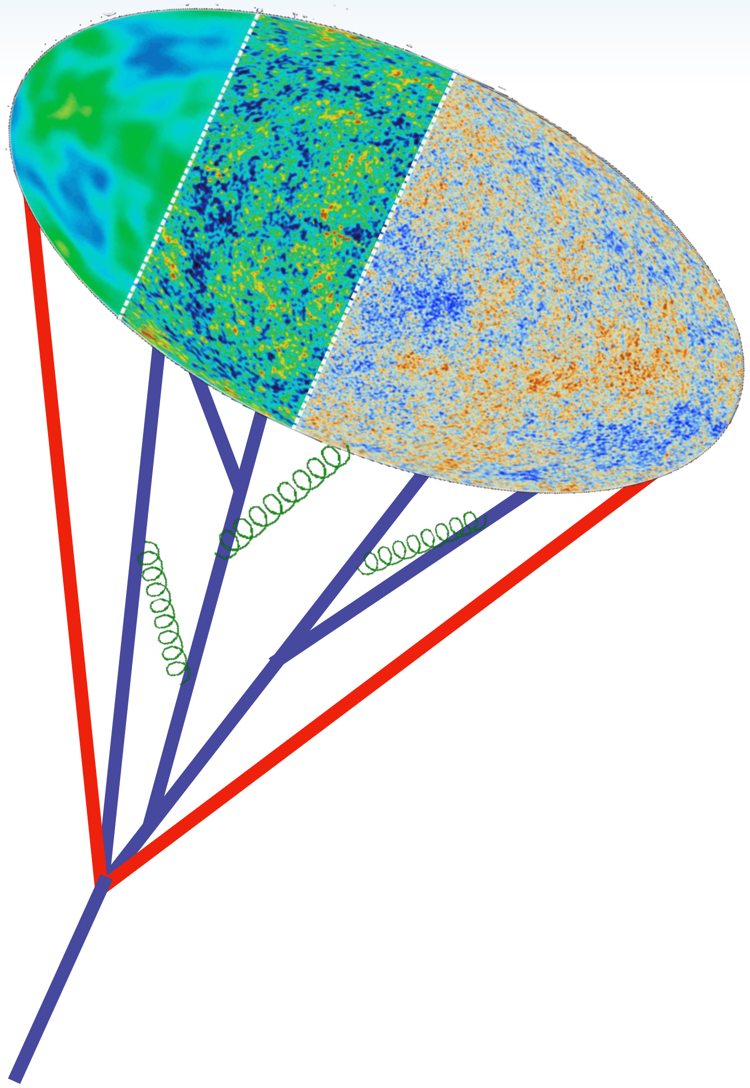


Constrain
BSM
models

Precision probe
of QCD

Probe of
quark gluon plasma

Study of hadron structures

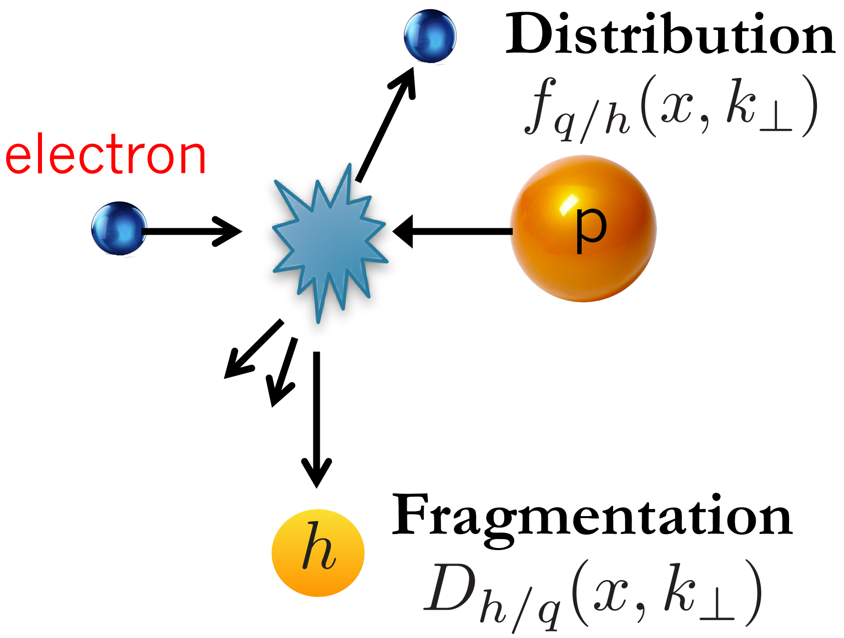


“Can we use jets to probe these TMD structure?”

Standard processes to study TMD structure

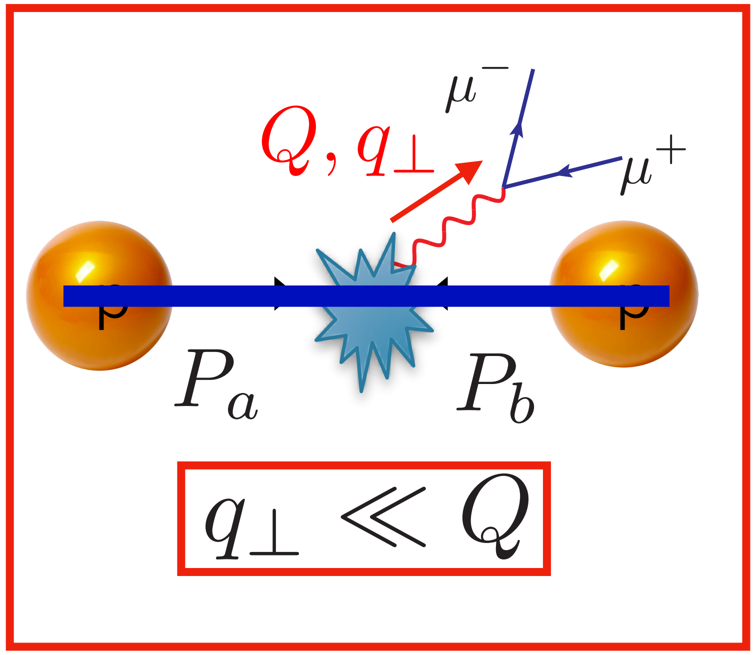
Semi-Inclusive DIS (SIDIS)

$$\sigma \sim f_{q/P}(x,k_\perp)D_{h/q}(x,k_\perp)$$



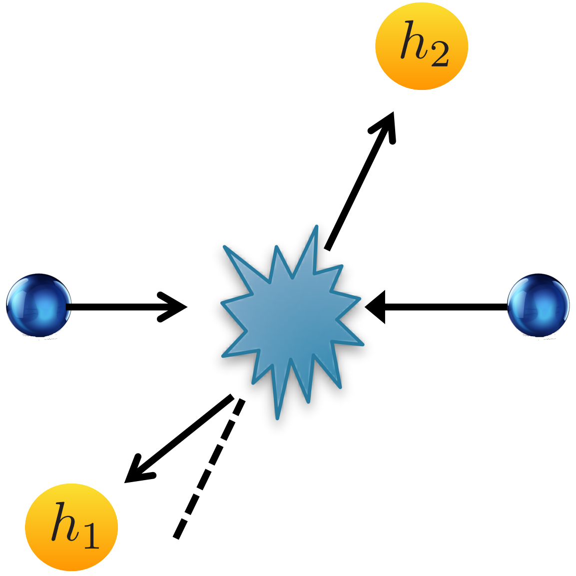
Drell-Yan

$$\sigma \sim f_{q/P}(x,k_\perp)f_{\bar{q}/P}(x,k_\perp)$$



Dihadrons in e^+e^-

$$\sigma \sim D_{h_1/q}(x,k_\perp)D_{h_2/q}(x,k_\perp)$$



- They have a well-established factorization formalism
- Small transverse momentum measured with respect to an axis

(CSS) Collin, Soper, Sterman `81-`85
Ji, Ma, Yuan ` 04
Becher, Neubert, Wilhelm`11-`13
Echevarria, Idilbi, Scimemi `11-`14
…

Beyond the standard processes

- Many other imaginable processes with sensitivity to the TMD structure

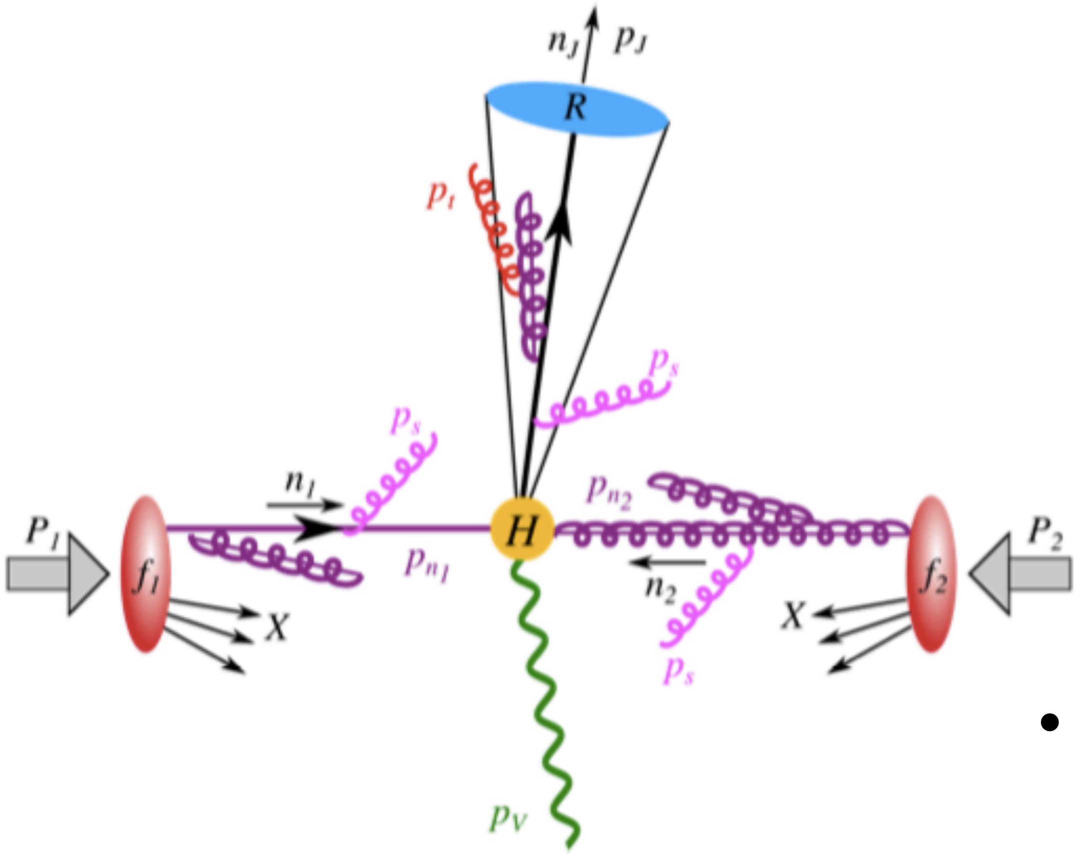


Fig. from Chien, Shao, Wu `19

$$PP \to J_1 + J_2 + X,$$
$$PP \to J + V + X,$$
$$PP \to J(h) + X, \ldots$$

LHC / RHIC

$$e\,P \to e + J + X$$
$$e\,P \to Q + \bar{Q} + X,$$
$$e\,P \to J(h) + X, \ldots$$

EIC

- Many experiments sensitive to such processes
- Standard processes have low sensitivity to gluon TMDs.
- Standard processes sensitive to two TMDs simultaneously; many involving jets will only be sensitive to a single TMD.

1) Inclusive jet production

TMDFFs $PP\,/\,e\,P \to J(h) + X$

2) Lepton + Jet imbalance

TMDPDFs $e\,P \to e + J + X$

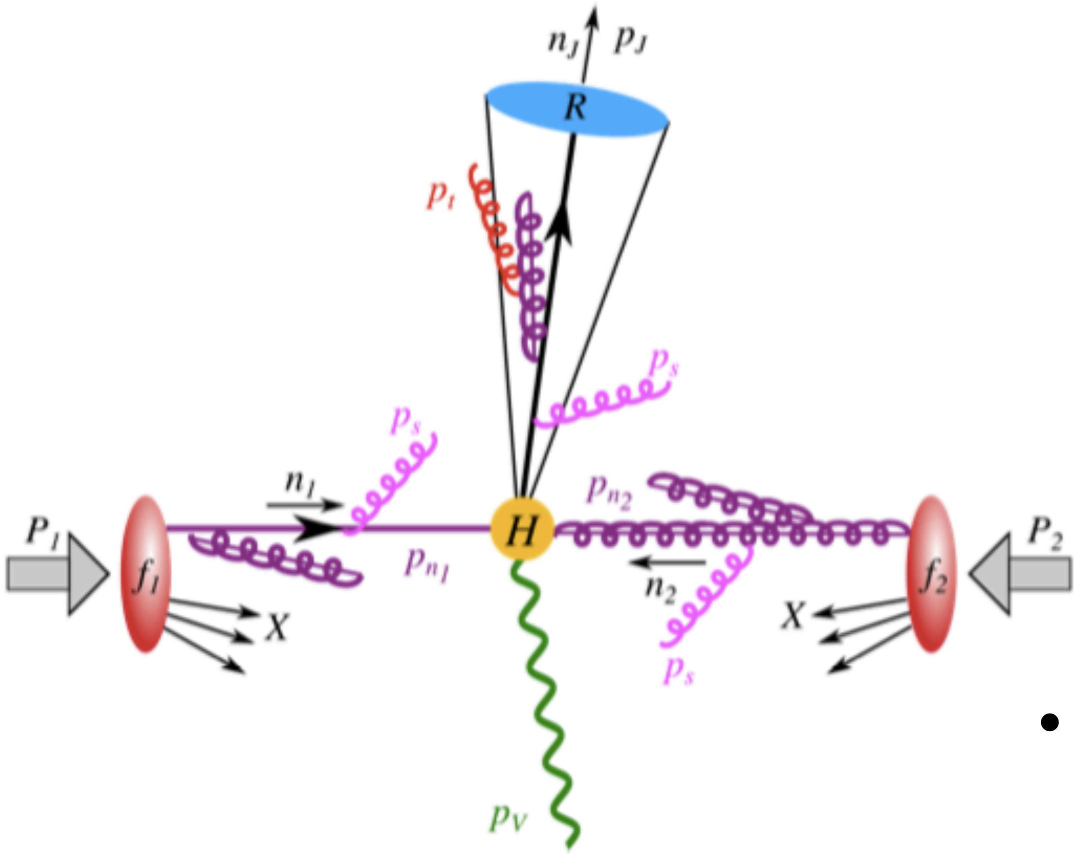
3) Lepton + Jet imbalance with hadron in jet

$$e\,P \to e + J(h) + X$$

TMDFFs / TMDPDFs

Beyond the standard processes

- **Many other imaginable processes with sensitivity to the TMD structure**



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- **Many experiments sensitive to such processes**
- **Standard processes have low sensitivity to gluon TMDs.**
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Fig. from Chien, Shao, Wu `19

collinear structure

1) Inclusive jet production

TMDFFs $PP\,/\,e\,P \to J(h) + X$

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3) Lepton + Jet imbalance with hadron in jet

$e\,P \to e + J(h) + X$

TMDFFs / TMDPDFs

Jet Fragmentation Functions (JFFs)

Unpolarized case:

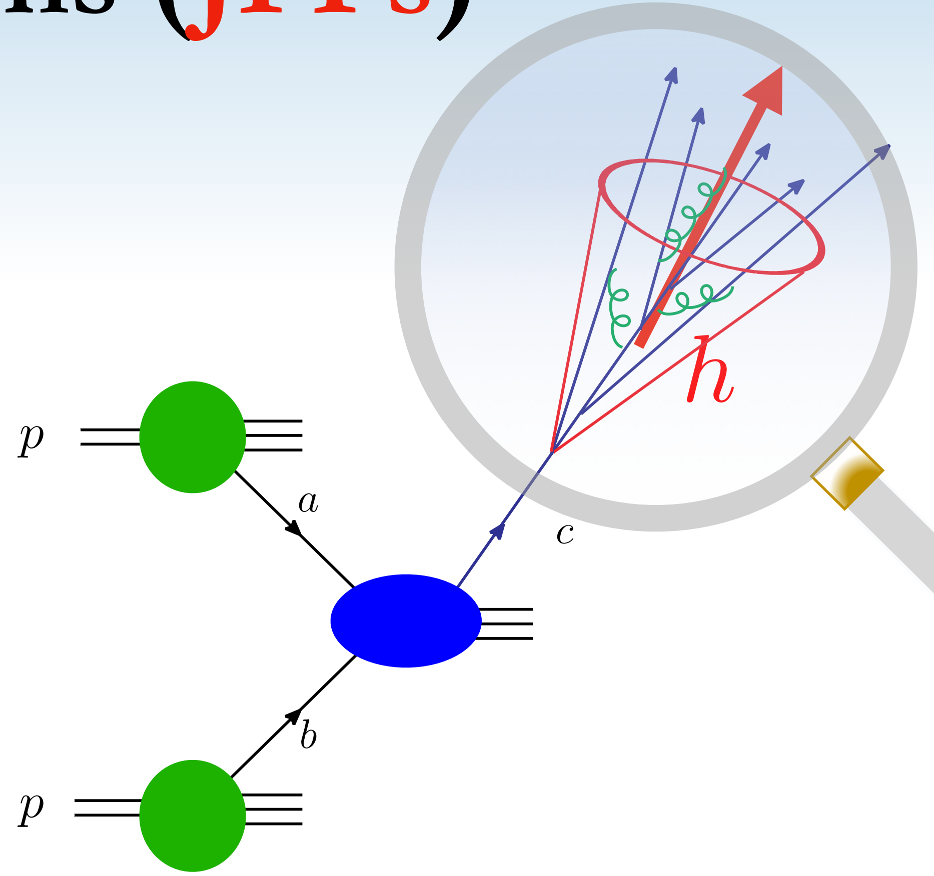
(replace pp with ep for EIC)

$$\frac{d\sigma^{pp\to\text{jet}(h)X}}{dp_T\,d\eta\,dz_h} = \sum_{a,b,c} \underset{\Lambda_{\text{QCD}}}{f_{a/A} \otimes f_{b/B}} \otimes \underset{p_T}{H^c_{ab}} \otimes \underset{\substack{p_T R\\ \Lambda_{\text{QCD}}}}{\mathcal{G}^h_c(z_h)}$$

where

$$z = p_T^J/p_T^c$$

$$z_h = p_T^h/p_T^J$$



$$\boxed{\frac{d\sigma^{pp\to hX}}{dp_T\,d\eta} = \sum_{a,b,c} \underset{\Lambda_{\text{QCD}}}{f_{a/A} \otimes f_{b/B}} \otimes \underset{p_T}{H^c_{ab}} \otimes \underset{\Lambda_{\text{QCD}}}{D^h_c}}$$

where $z = p_T^h/p_T^c$

1) Inclusive jet production

TMDFFs

collinear PDFs/FFs

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

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collinear PDFs

where
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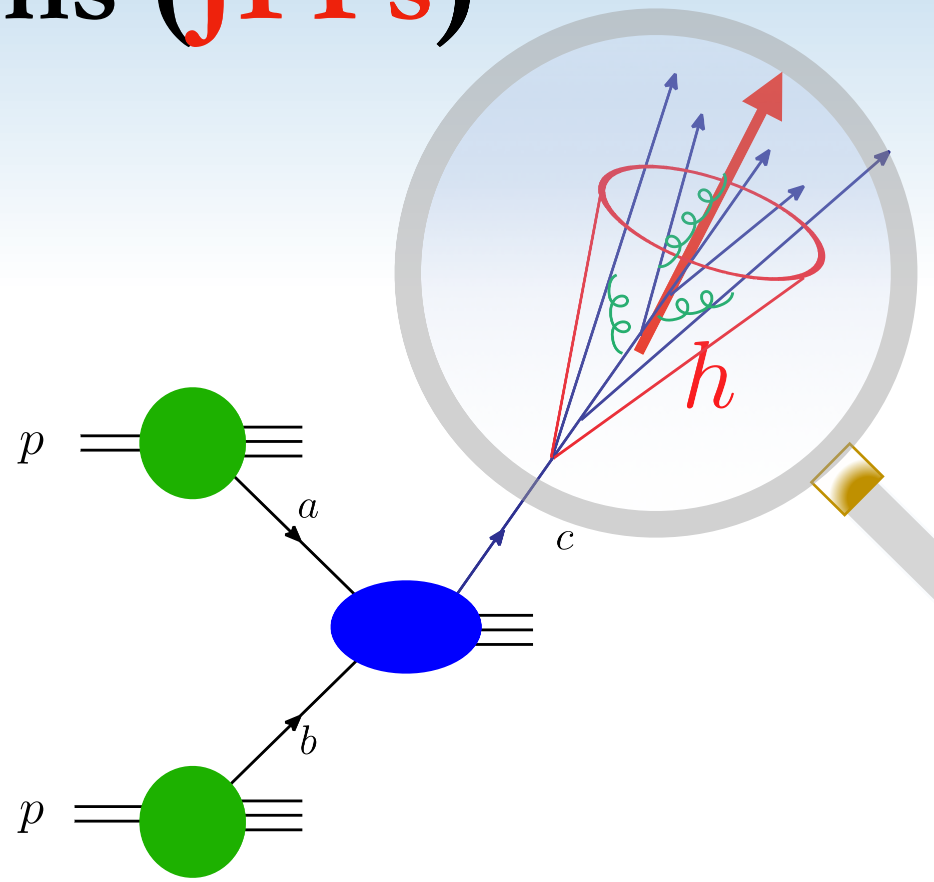

1) Inclusive jet production
TMDFFs
collinear structure

$$\boxed{\frac{d\sigma^{pp\to hX}}{dp_T\, d\eta} = \sum_{a,b,c} \underset{\Lambda_{\rm QCD}}{f_{a/A} \otimes f_{b/B}} \otimes \underset{p_T}{H_{ab}^c} \otimes \underset{\Lambda_{\rm QCD}}{\boxed{D_c^h}}}$$

where $z = p_T^h / p_T^c$

collinear FFs

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
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Jet Fragmentation Functions (JFFs)

Unpolarized case:

(replace pp with ep for EIC)

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^{c} \otimes \mathcal{G}_c^h(z_h)$$

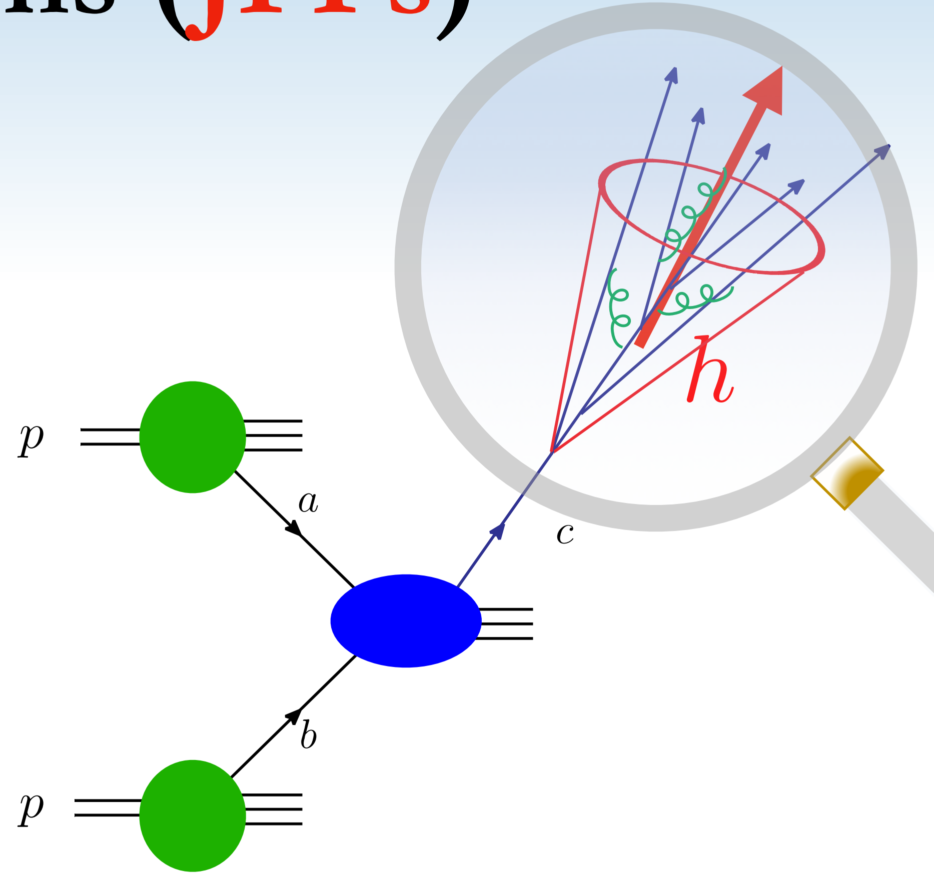
Λ_{QCD} p_T $p_T R$ Λ_{QCD}

collinear PDFs

where

$$z = p_T^J / p_T^c$$

$$z_h = p_T^h / p_T^J$$



IR sensitivity and require matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D_j^h(z_h, \mu)$$

$p_T R$ Λ_{QCD}

matching coefficients

collinear FFs

1) Inclusive jet production
TMDFFs
collinear structure

- **Collinear JFFs can be related to collinear FFs**

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
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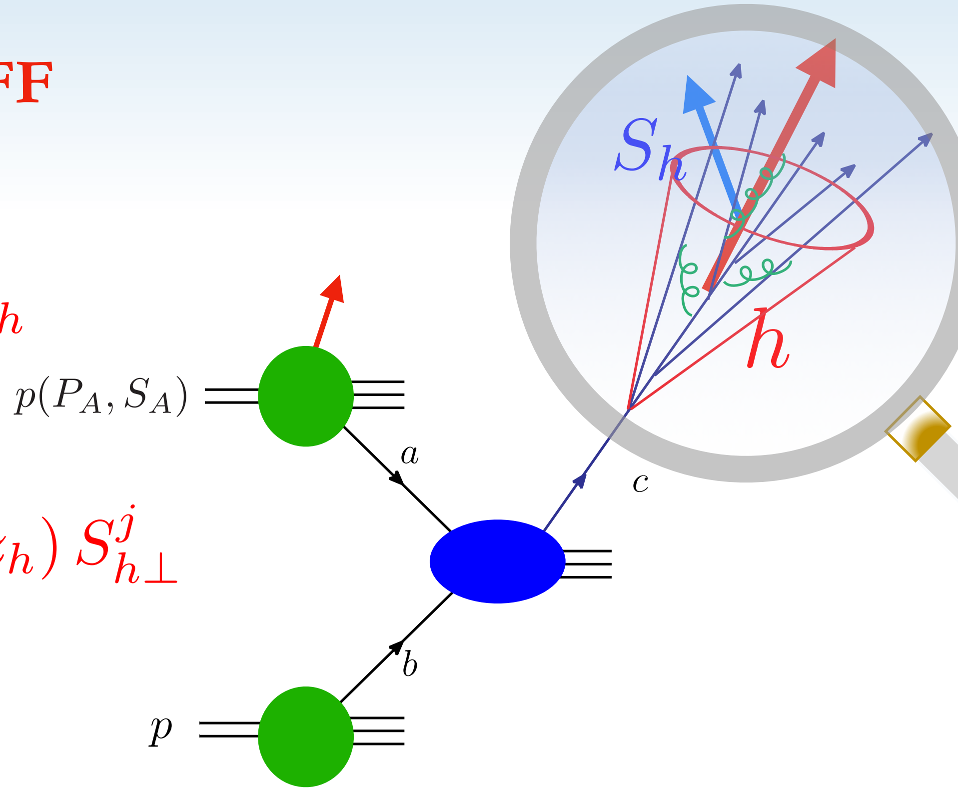
Polarized Jet Fragmentation Functions (JFFs)



Polarized case:

Helicity PDF

Helicity JFF

$$\frac{d\sigma_{LL}^{\bar{p}p\to \text{jet}(\bar{h})X}}{dp_T\, d\eta\, dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h$$

Transversity PDF

Transversity JFF

$$\frac{d\sigma_{TT}^{\bar{p}p\to \text{jet}(\bar{h})X}}{dp_T\, d\eta\, dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT}(H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j$$

where

$$z = p_T^J / p_T^c$$

$$z_h = p_T^h / p_T^J$$

$$d\sigma_{LL} \equiv \frac{d\sigma(\Lambda_A, \Lambda_h) - d\sigma(\Lambda_A, -\Lambda_h)}{2}$$

$$d\sigma_{TT} \equiv \frac{d\sigma(\vec{S}_{A\perp}, \vec{S}_{h\perp}) - d\sigma(\vec{S}_{A\perp}, -\vec{S}_{h\perp})}{2}$$

Similar definitions for $\Delta_{LL} H_{\bar{a}b}^{\bar{c}}$ **and** $\Delta_{TT}(H_{ij})$

Kang, KL, Zhao, `20

Polarized Jet Fragmentation Functions (JFFs)

Polarized case:

Helicity PDF **Helicity JFF**

$$\frac{d\sigma_{LL}^{\bar{p}p\to\text{jet}(\bar{h})X}}{dp_T\,d\eta\,dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A}\otimes f_{b/B}\otimes \Delta_{LL}H_{\bar{a}b}^{\bar{c}}\otimes \Delta_L\mathcal{G}_c^h(z_h)\,\Lambda_h$$

Transversity PDF **Transversity JFF**

$$\frac{d\sigma_{TT}^{\bar{p}p\to\text{jet}(\bar{h})X}}{dp_T\,d\eta\,dz_h} = \sum_{a,b,c} S_{A\perp}^{i}\, h_{a/A}\otimes f_{b/B}\otimes \Delta_{TT}(H_{ij})_{\bar{a}b}^{\bar{c}}\otimes \Delta_T\mathcal{G}_c^h(z_h)\,S_{h\perp}^{j}$$

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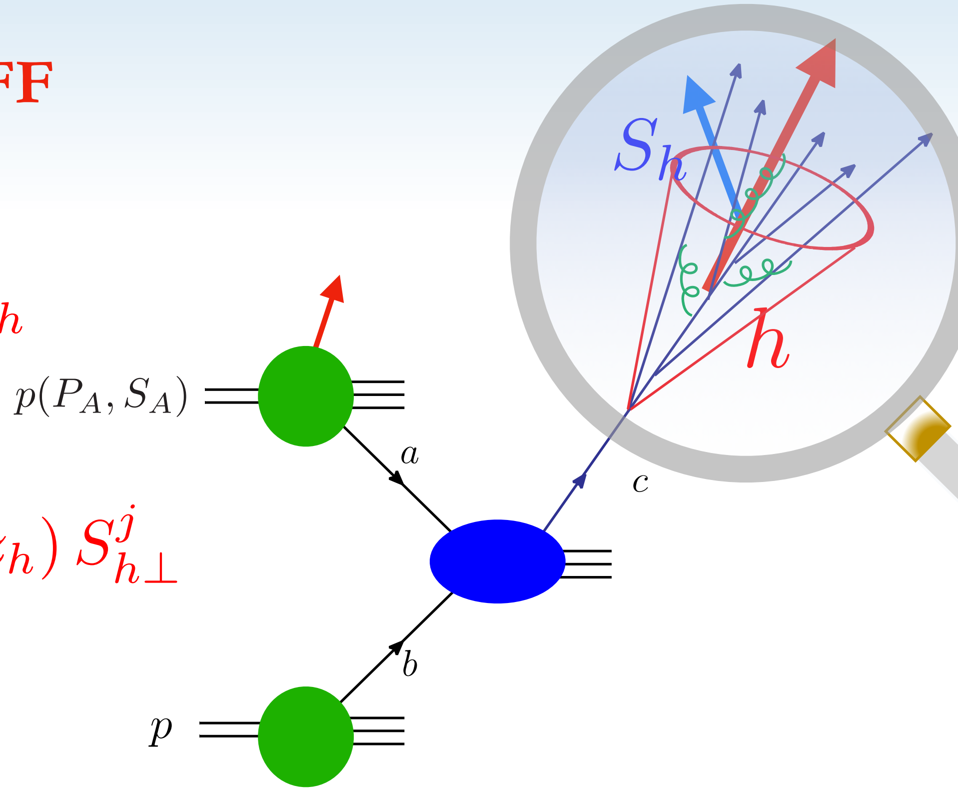


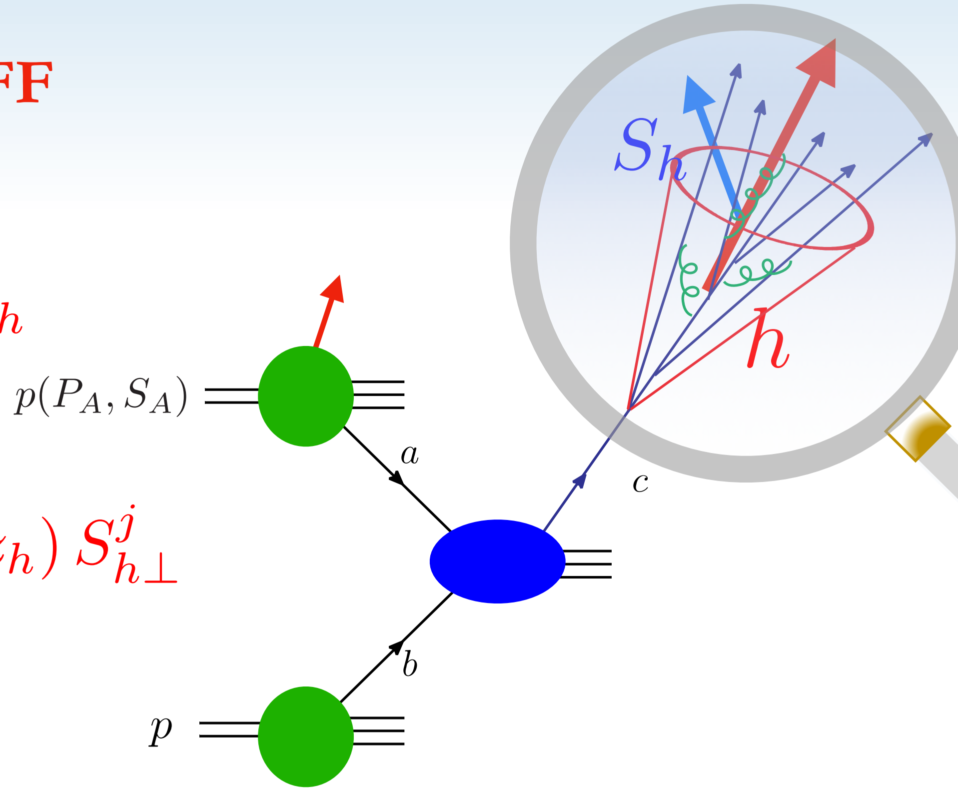
$$\frac{d\sigma_{LL}^{\bar{p}p\to\bar{h}X}}{dp_T\,d\eta} = \sum_{a,b,c} \Lambda_A\, g_{a/A}\otimes f_{b/B}\otimes \Delta_{LL}H_{\bar{a}b}^{\bar{c}}\otimes G_c^h$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to\bar{h}X}}{dp_T\,d\eta} = \sum_{a,b,c} S_{A\perp}^{i}\, h_{a/A}\otimes f_{b/B}\otimes \Delta_{TT}(H_{ij})_{\bar{a}b}^{\bar{c}}\otimes H_c^h\, S_{h\perp}^{j}$$

		Quark polarization		
		U	**L**	**T**
Hadron polarization	**U**	$D^{h/q}$		
	L		$G^{h/q}$	
	T			$H^{h/q}$

Kang, KL, Zhao, `20

Polarized Jet Fragmentation Functions (JFFs)



Polarized case:

Helicity PDF **Helicity JFF**

$$\frac{d\sigma_{LL}^{\bar{p}p\to \text{jet}(\bar{h})X}}{dp_T\, d\eta\, dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h$$

Transversity PDF **Transversity JFF**

$$\frac{d\sigma_{TT}^{\bar{p}p\to \text{jet}(\bar{h})X}}{dp_T\, d\eta\, dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j$$

where $$z = p_T^J / p_T^c$$

$$z_h = p_T^h / p_T^J$$

IR sensitivity and require matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D_j^h(z_h, \mu)$$

$p_T R$ Λ_{QCD}

$$\Delta_L \mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \Delta_L \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes G_j^h(z_h, \mu)$$

$$\Delta_T \mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \Delta_T \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes H_j^h(z_h, \mu)$$

		Quark polarization		
		U	**L**	**T**
Hadron polarization	**U**	$D^{h/q}$		
	L		$G^{h/q}$	
	T			$H^{h/q}$

Kang, KL, Zhao, `20

Jet Fragmentation Functions (JFFs)

- **Light charged hadrons**

  Arleo, Fontannaz, Guillet, Nguyen `14
  Kaufmann, Mukherjee, Vogelsang `15
  Kang, Ringer, Vitev `16
  Neill, Scimemi, Waalewijn `16

- **Photons**

  Kaufmann, Mukherjee, Vogelsang `16

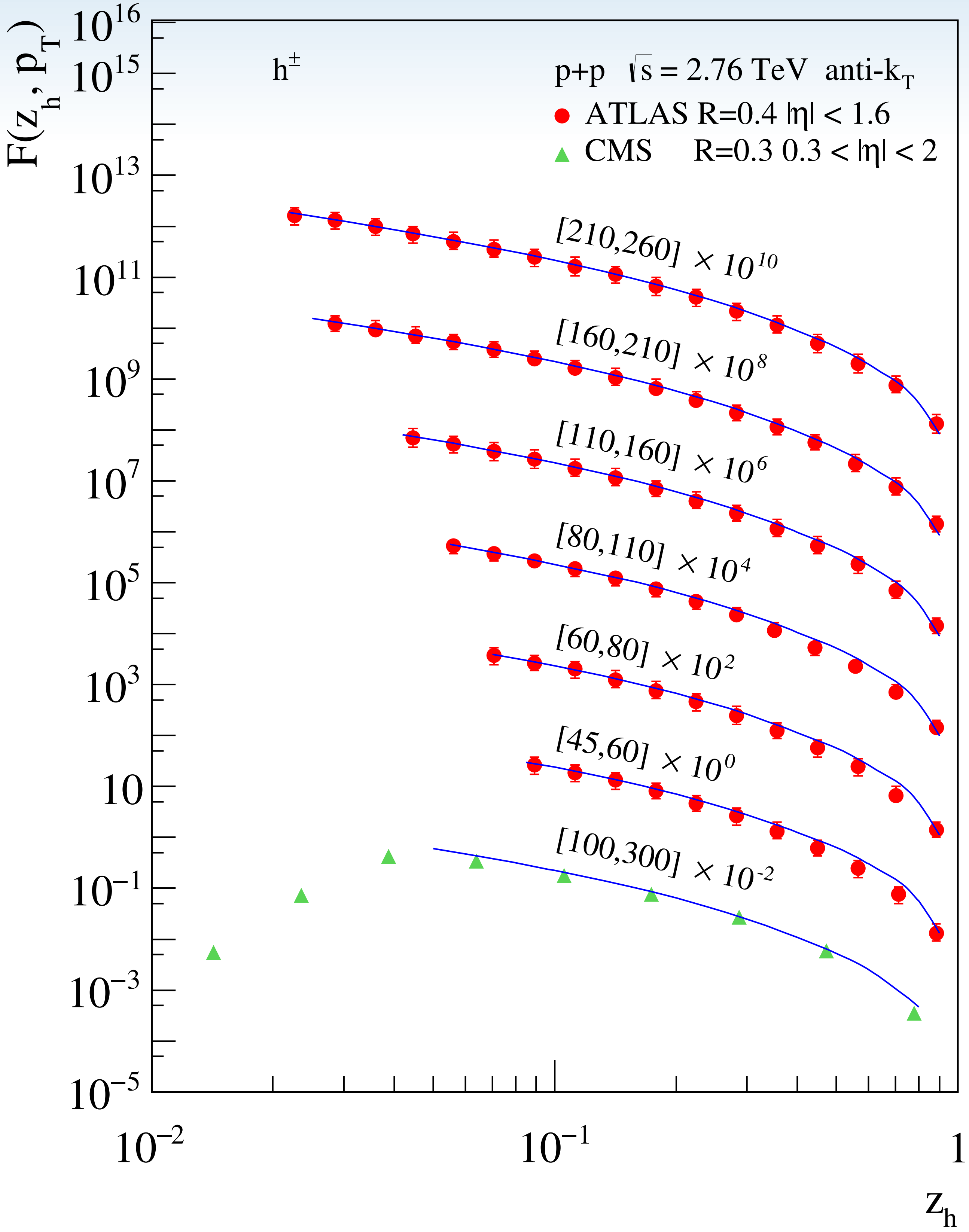
- **Heavy flavor mesons**

  Chien, Kang, Ringer, Vitev, Xing `15
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Anderle, Kaufmann, Stratmann, Ringer, Vitev `17

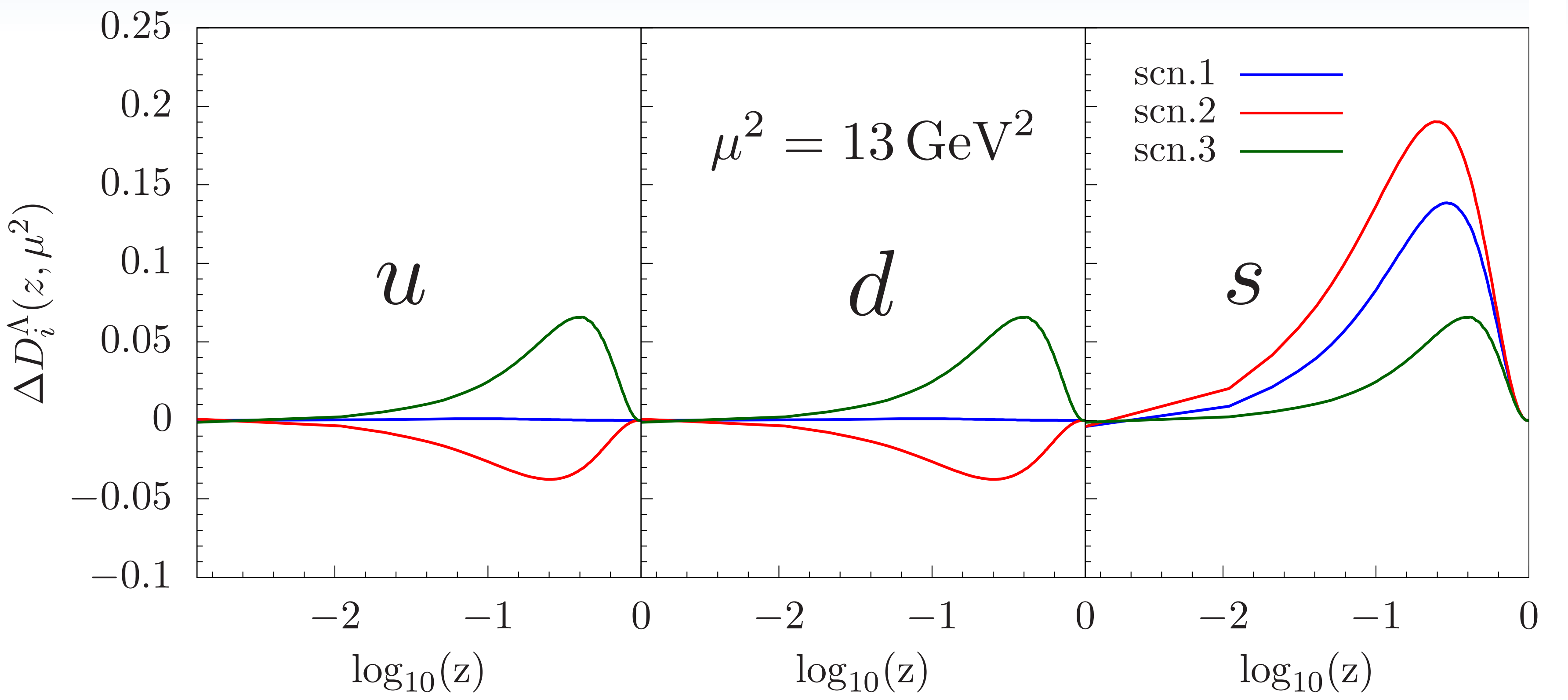
- **Quarkonia**

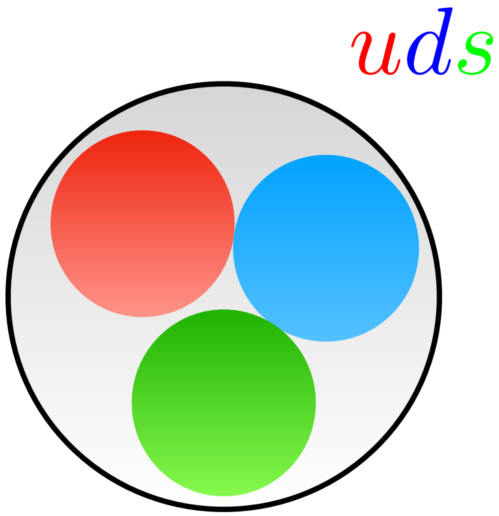
  Baumgart, Leibovich, Mehen, Rothstein `14
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Kang, Qiu, Ringer, Xing, Zhang `17
  Bain, Dai, Leibovich, Makris, Mehen `17

$$F(z_h, p_T) = \frac{d\sigma^{pp\to(\text{jet}h)X}}{dp_T d\eta dz_h} \Big/ \frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta}$$



Longitudinally polarized Λ



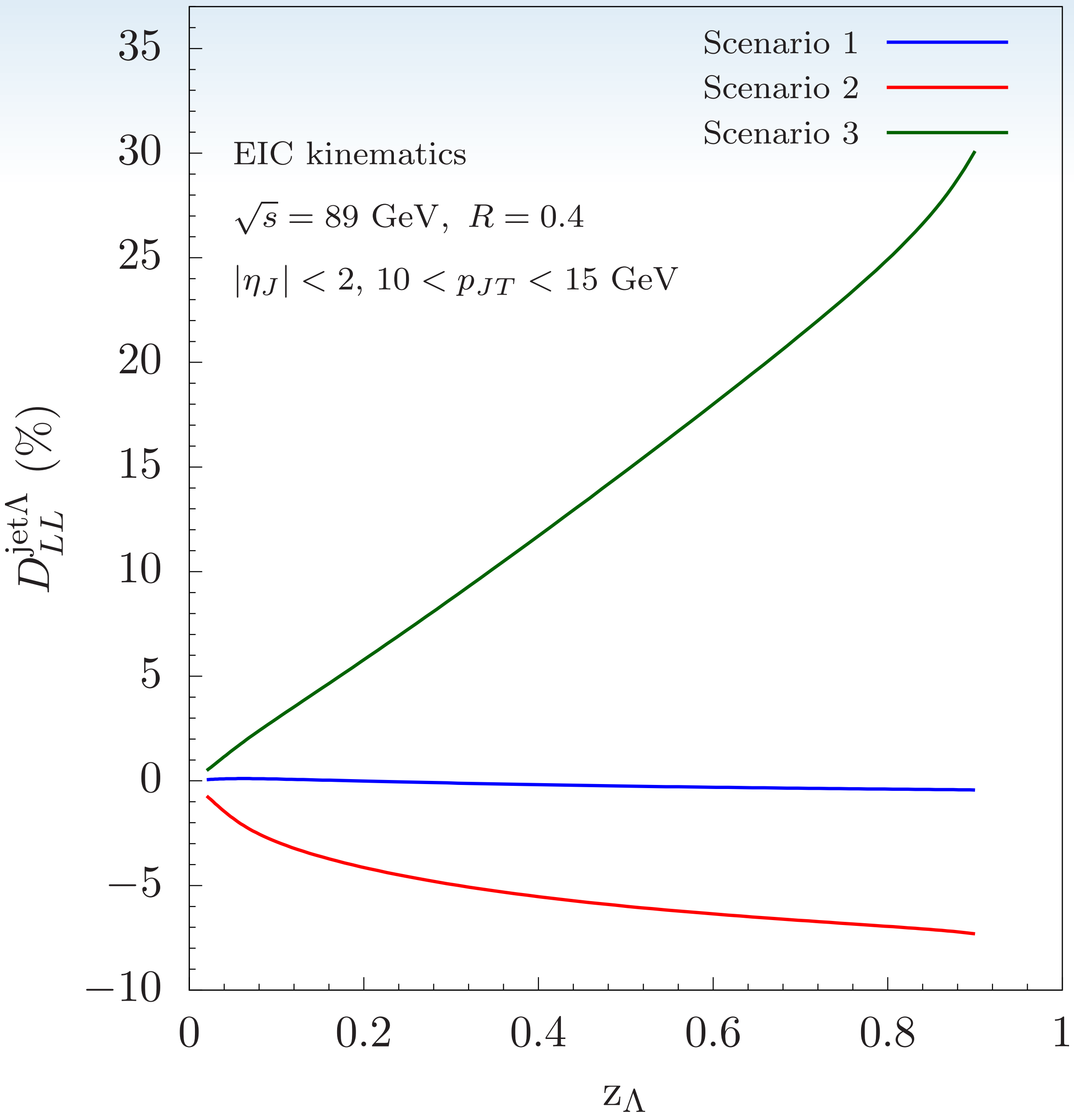


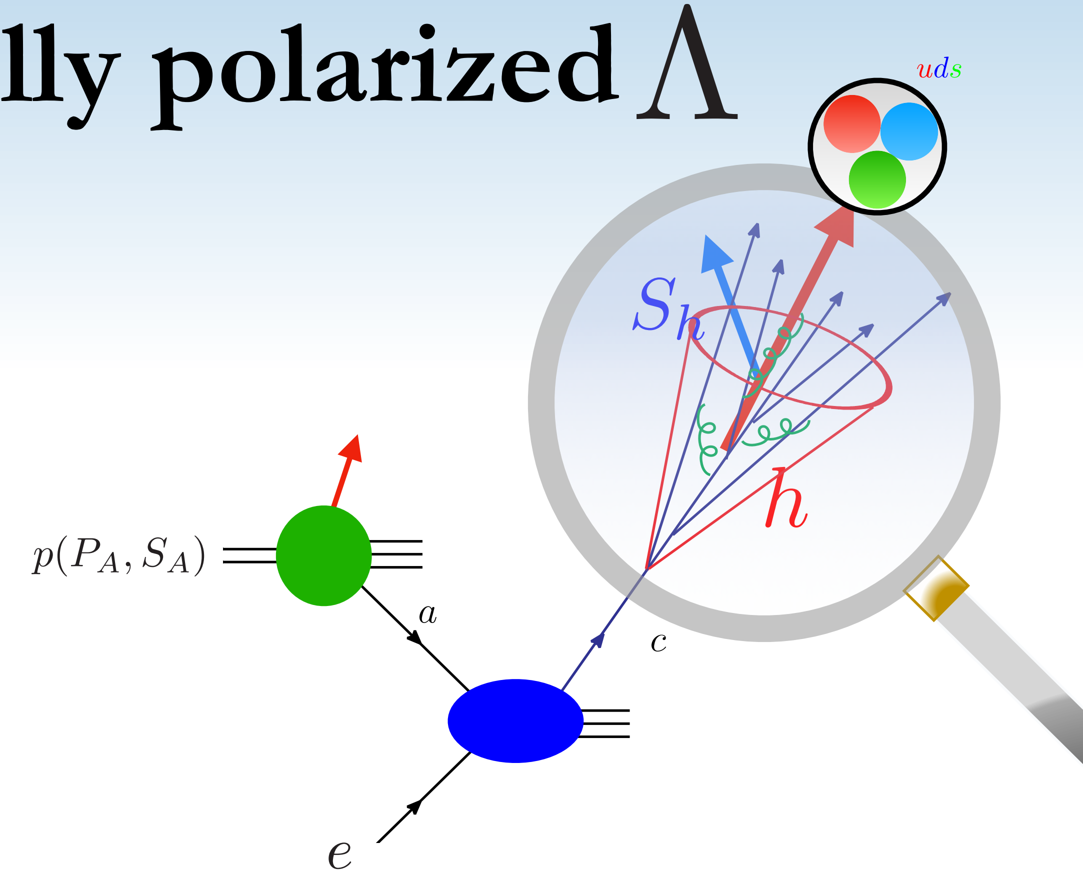
Three different scenarios can explain LEP data equally well

- **Scenario 1:** Only strange quarks have contribution to the fragmentation.
- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

Burkardt, Jaffe, `93
De Florian, Stratmann, Vogelsang, `98

JFFs to study longitudinally polarized Λ





Gives shape expected from the scenarios

$$D_{LL}^{\text{jet}\Lambda} = \frac{d\Delta_{LL}\sigma}{d\sigma}$$

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- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

Kang, KL, Zhao, `20

TMD hadron in jet (TMDJFFs)

Unpolarized case:

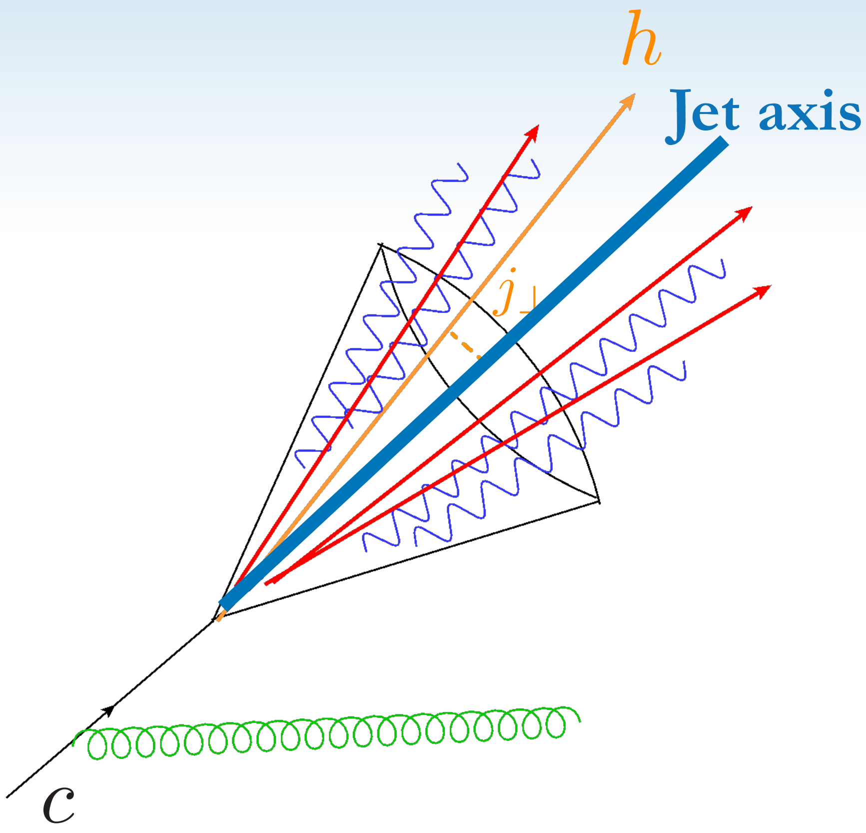
(replace pp with ep for EIC)

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h, \boldsymbol{j}_\perp)$$

Λ_{QCD} p_T $p_T R$ Λ_{QCD}

where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$



(including polarized jet fragmentation functions)

TMD Fragmentation Functions (TMDFFs)

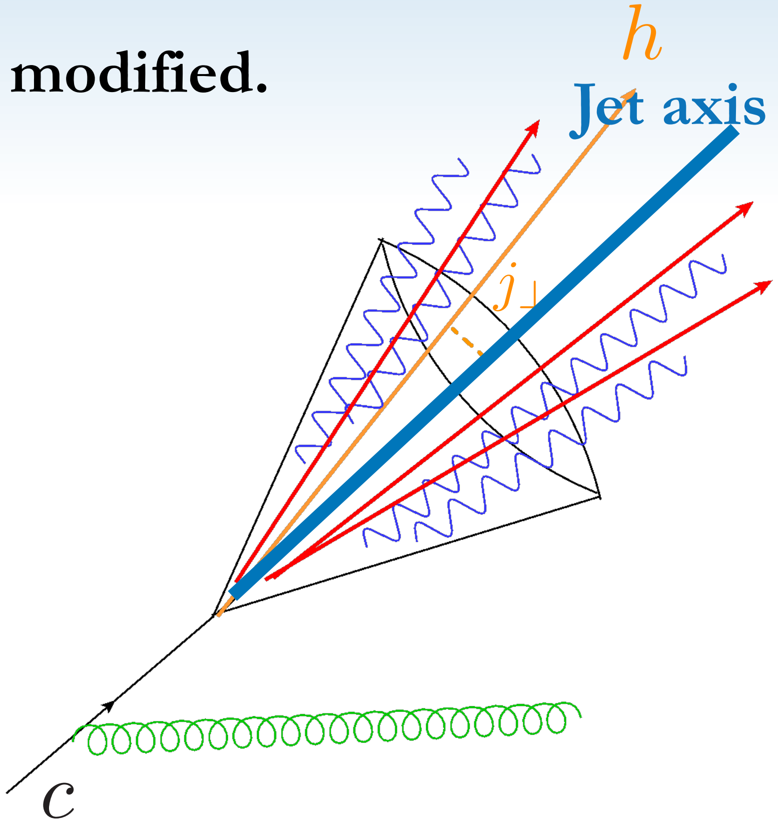
		Quark polarization		
		U	L	T
Hadron polarization	U	$D^{h/q}$		$H^{\perp h/q}$
	L		$G^{h/q}$	$H_L^{\perp h/q}$
	T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

TMD Jet Fragmentation Functions (TMDJFFs)

		Quark polarization		
		U	L	T
Hadron polarization	U	$\mathcal{D}^{h/q}$		$\mathcal{H}^{\perp h/q}$
	L		$\mathcal{G}^{h/q}$	$\mathcal{H}_L^{\perp h/q}$
	T	$\mathcal{D}_T^{\perp h/q}$	$\mathcal{G}_T^{h/q}$	$\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$

TMD hadron in jet (TMDJFFs)

- **Still hard-collinear factorization structure other than jet function modified.**



$$\frac{d\sigma^{pp\to\text{jet}(h)X}}{dp_T\,d\eta\,dz_h\,d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A}\otimes f_{b/B}\otimes H^c_{ab}\otimes\mathcal{G}^h_c(z_h,\boldsymbol{j}_\perp)$$

When $\Lambda_{\text{QCD}}\lesssim j_\perp\ll p_T R,\qquad \lambda\sim j_\perp/p_T$

collinear $k_c\sim p_T(\lambda^2,1,\lambda)$

soft $k_s\sim p_T(\lambda R,\lambda/R,\lambda)$

Unpolarized TMDJFF

$$\mathcal{D}_1^{h/q}(z,z_h,j_\perp^2,\mu,\zeta_J) = \mathcal{H}_{c\to i}(z,p_T R,\mu)\int_{\boldsymbol{k}_\perp,\boldsymbol{\lambda}_\perp} D_1^{h/q,\,\text{unsub}}(z_h,k_\perp^2,\mu,\zeta'/\nu^2)S_q(\lambda_\perp^2,\mu,\nu\mathcal{R})$$

$$= \mathcal{H}_{c\to i}(z,p_T R,\mu)\int\frac{b\,db}{2\pi}J_0\left(\frac{j_\perp b}{z_h}\right)\tilde{D}_1^{h/q,\,\text{unsub}}(z_h,b^2,\mu,\zeta'/\nu^2)S_q(b^2,\mu,\nu\mathcal{R})$$

$$= \mathcal{H}_{c\to i}(z,p_T R,\mu)\underbrace{D_1^{h/q}(z_h,j_\perp^2,\mu,\zeta_J)}$$

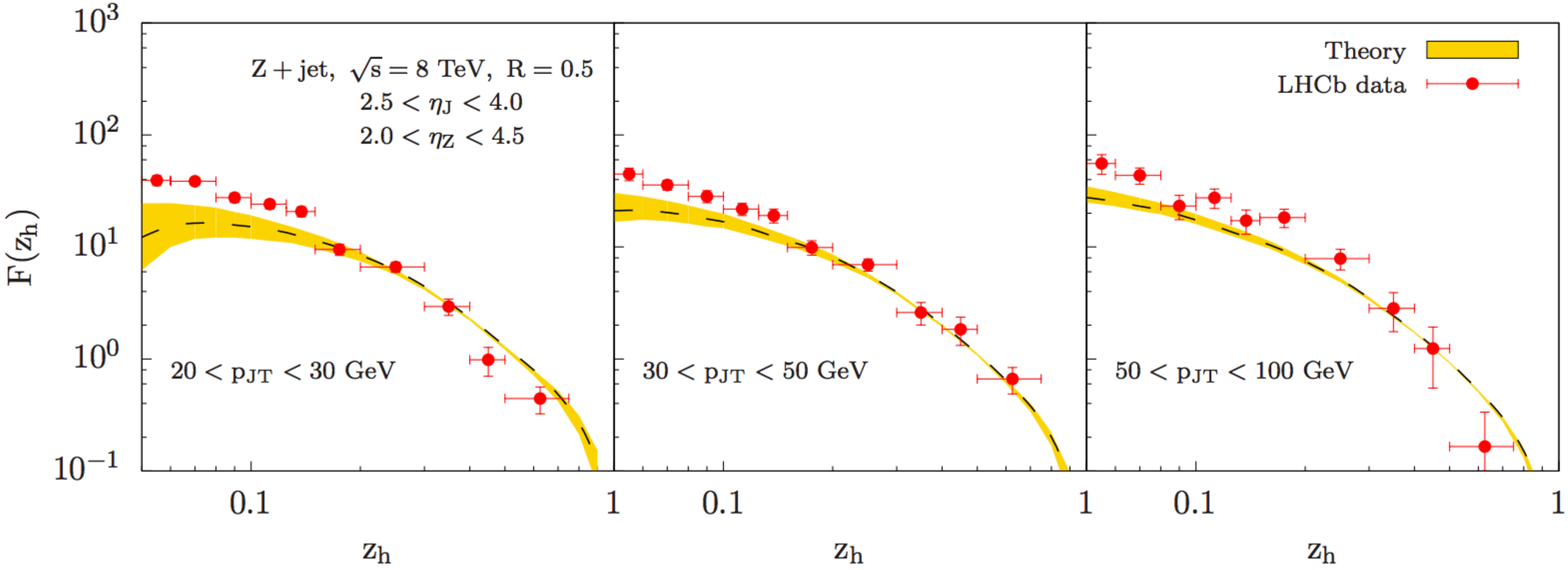
Standard subtracted TMDFFs, say in SIDIS

Relation also holds for other TMDJFFs.

- **TMDJFFs can be related to TMDFFs**

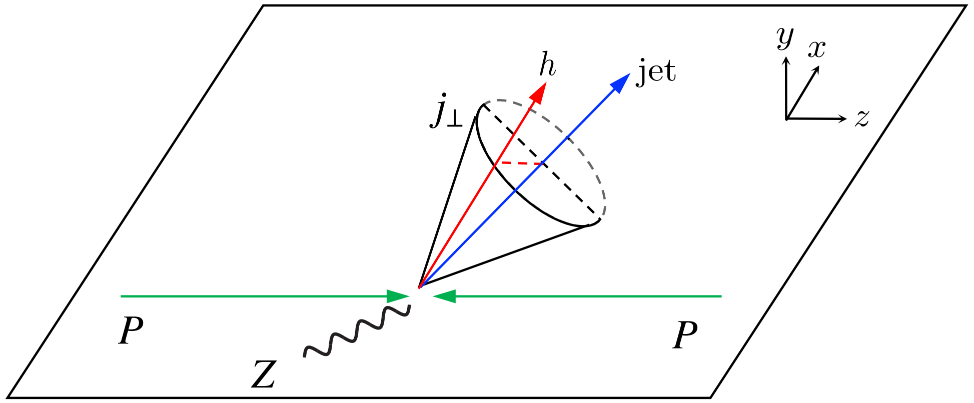
Kang, Liu, Ringer, Xing, `17
Kang, KL, Shao, Zhao `21

Z-tagged jet



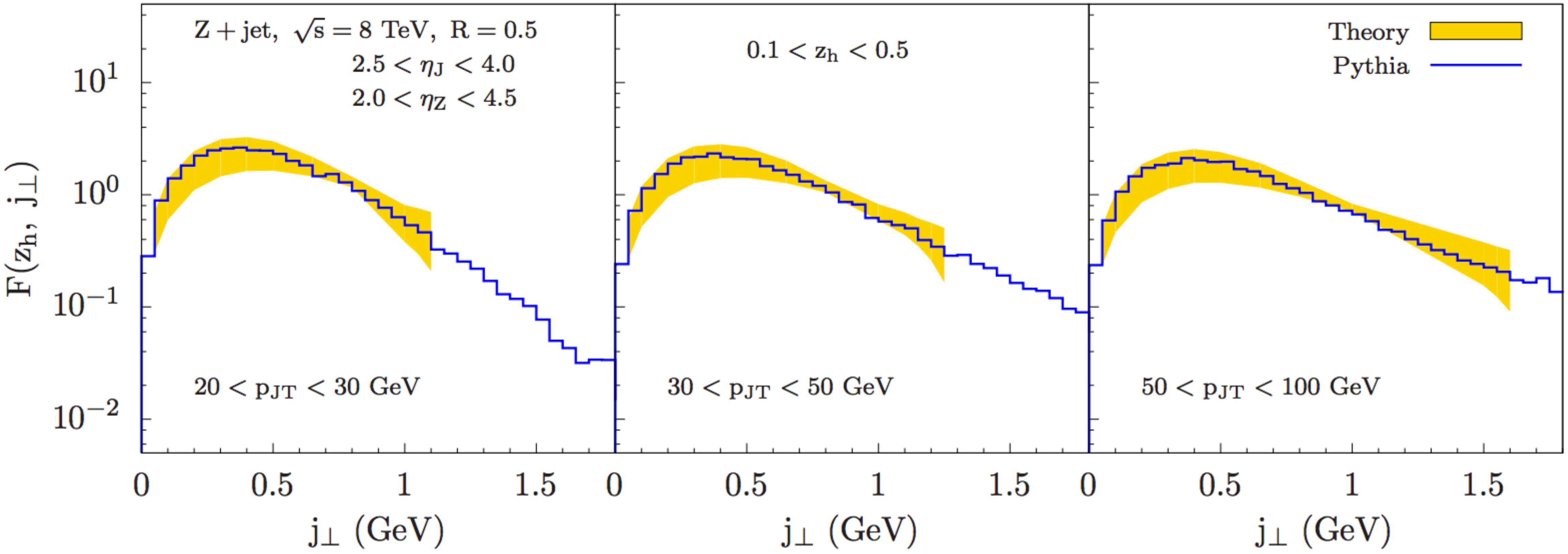
- **LHCb collaboration measured collinear FFs and** $j_\perp$

  Agrees well in the $0.1 < z_h < 0.5$ **region**



Kang, KL, Terry, Xing `19

Z-tagged jet

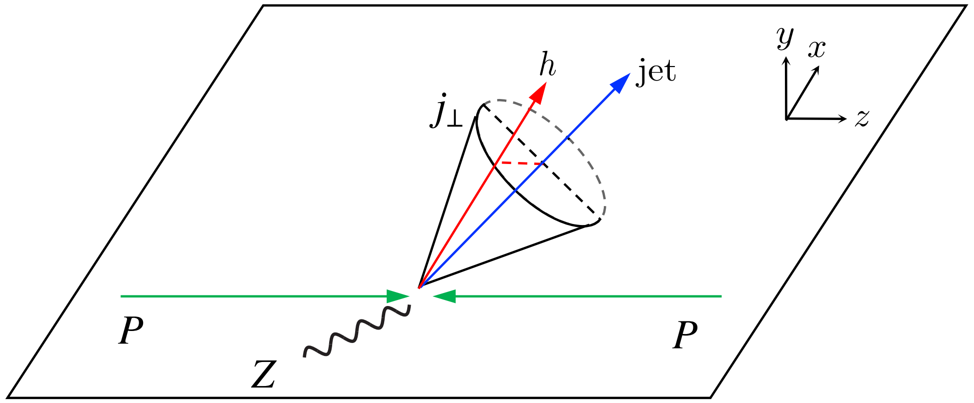


- **LHCb collaboration measured collinear FFs and $j_\perp$**

  Agrees well in the $0.1 < z_h < 0.5$ region

- **LHCb collaboration measured $j_\perp$ recently.**

  but with entire $0 < z_h < 1$



Kang, KL, Terry, Xing `19

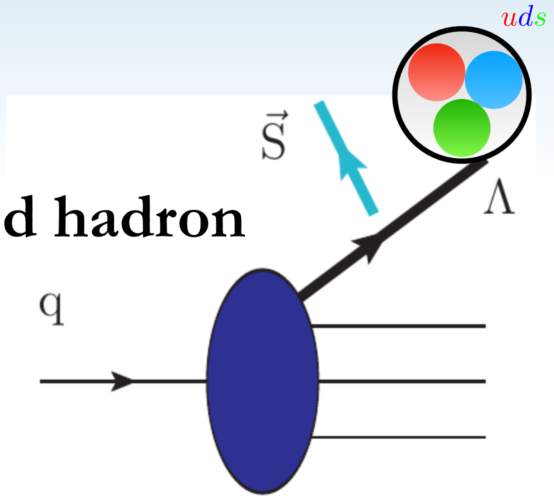
Polarizing FF

TMD Fragmentation Functions

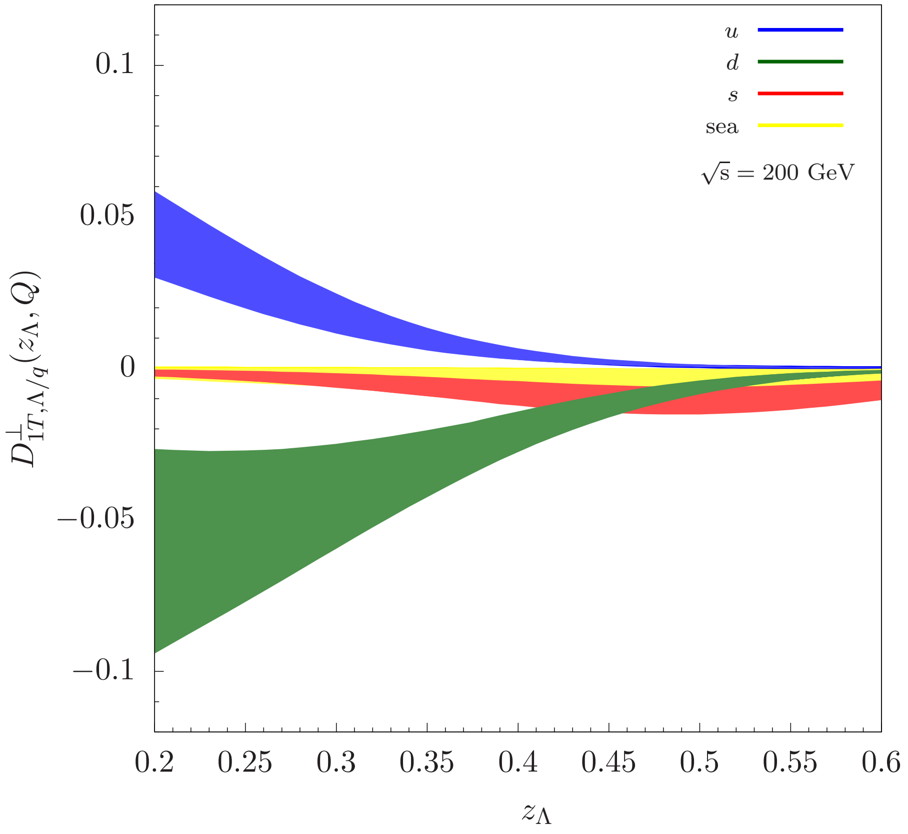
Quark polarization

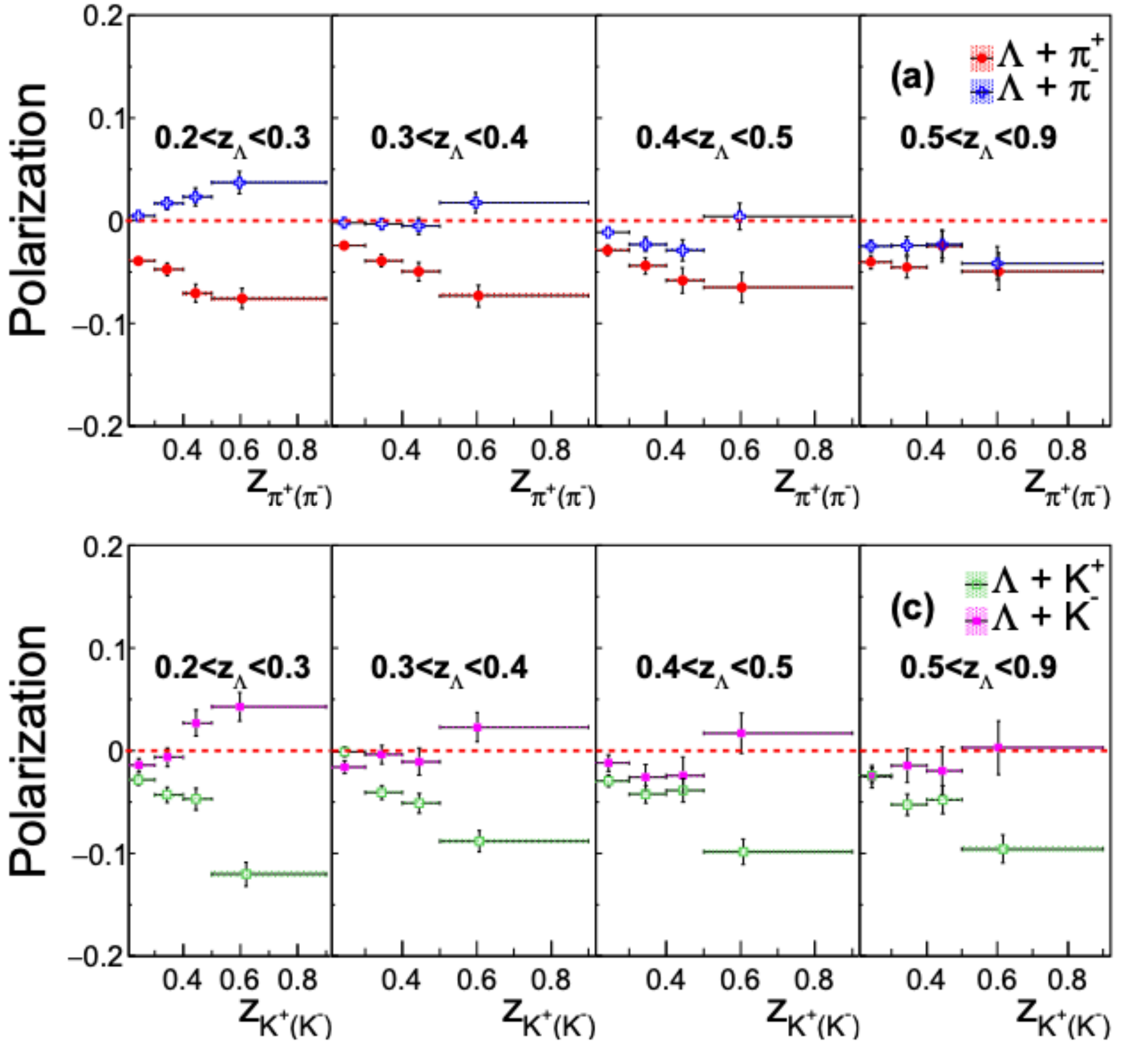
Hadron polarization

	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$



- Describes transversely polarized hadron inside unpolarized parton.





- Used PFF fits from Belle data

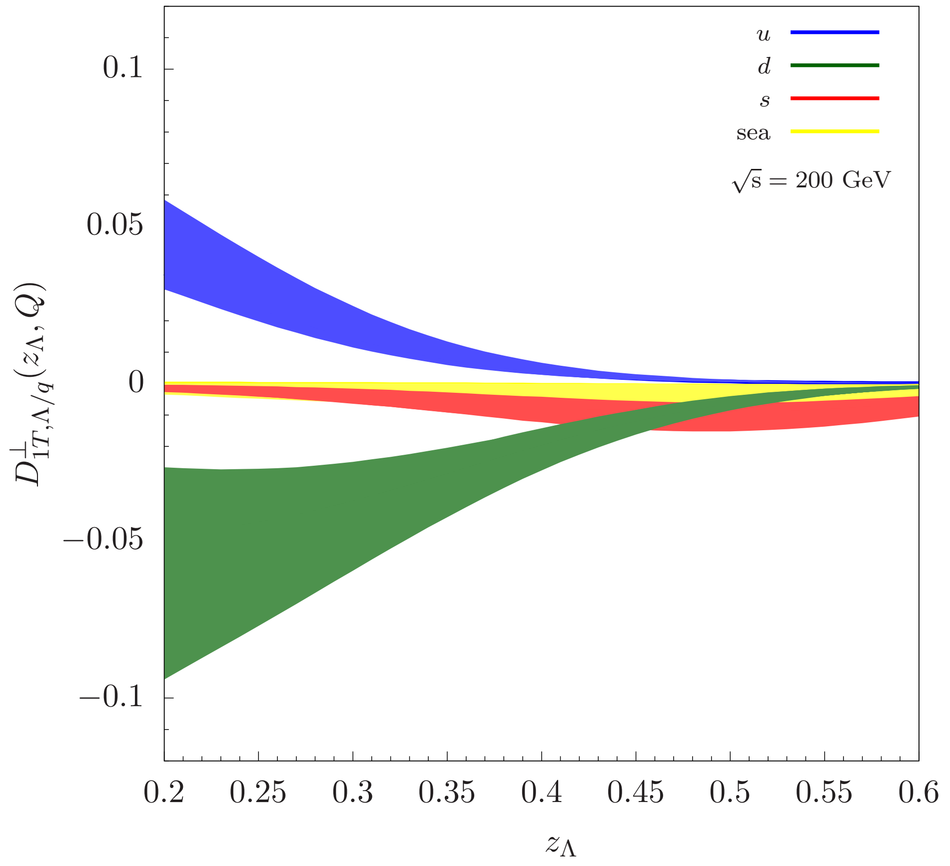
Callos, Kang, Terry, `20

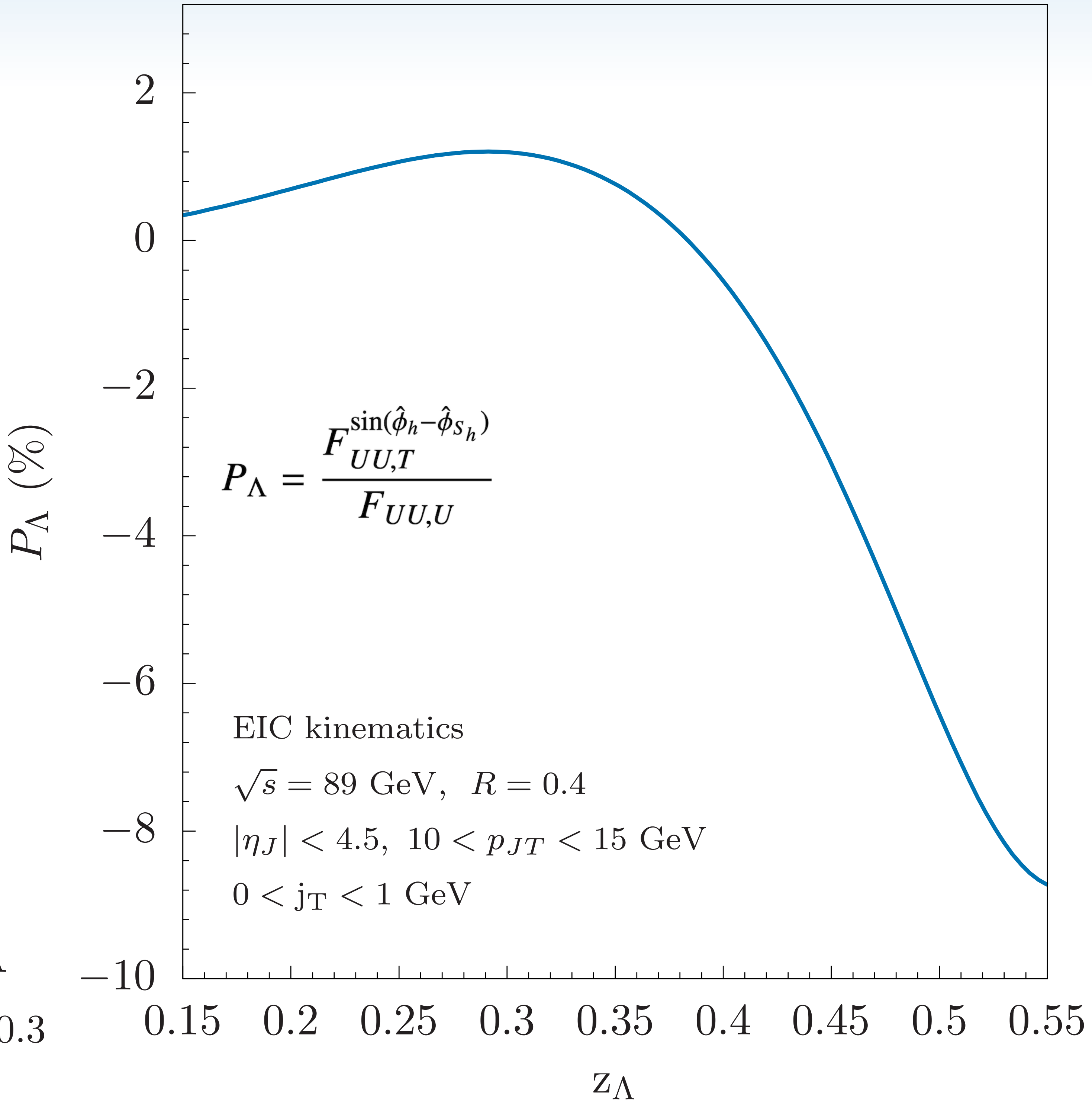
Belle Collaboration `19

Polarizing JFF

- **Used PFF fits from Belle data**

Callos, Kang, Terry, `20





- **Predictions at the LHC kinematics**
- **Positive from up quark PFF at small z_Λ**
- **Negative from down quark PFF at $z_\Lambda \gtrsim 0.3$**

Kang, KL, Zhao, `20

Azimuthal angular dependence

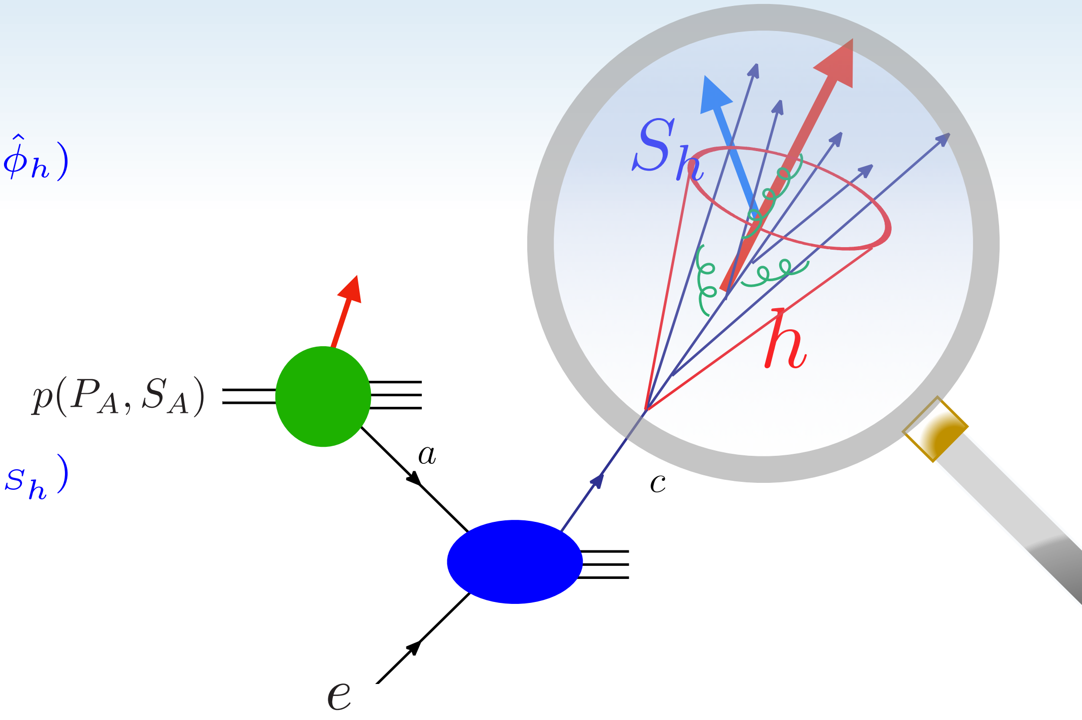
$$\frac{d\sigma^{p(S_A)+p/e\to(\text{jet}\,h(S_h))X}}{dp_{JT}d\eta_J dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + |\boldsymbol{S}_T|\sin(\phi_{S_A}-\hat{\phi}_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)}$$

$$+\Lambda_h\left[\lambda F_{LU,L} + |\boldsymbol{S}_T|\cos(\phi_{S_A}-\hat{\phi}_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)}\right]$$

$$+|\boldsymbol{S}_{h\perp}|\Big\{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda\cos(\hat{\phi}_h-\hat{\phi}_{S_h})F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})}$$

$$+|\boldsymbol{S}_T|\Big(\cos(\phi_{S_A}-\hat{\phi}_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})}$$

$$+\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})}\Big)\Big\},$$



$$\underset{\uparrow}{F_{S_A S_B,S_h}}$$

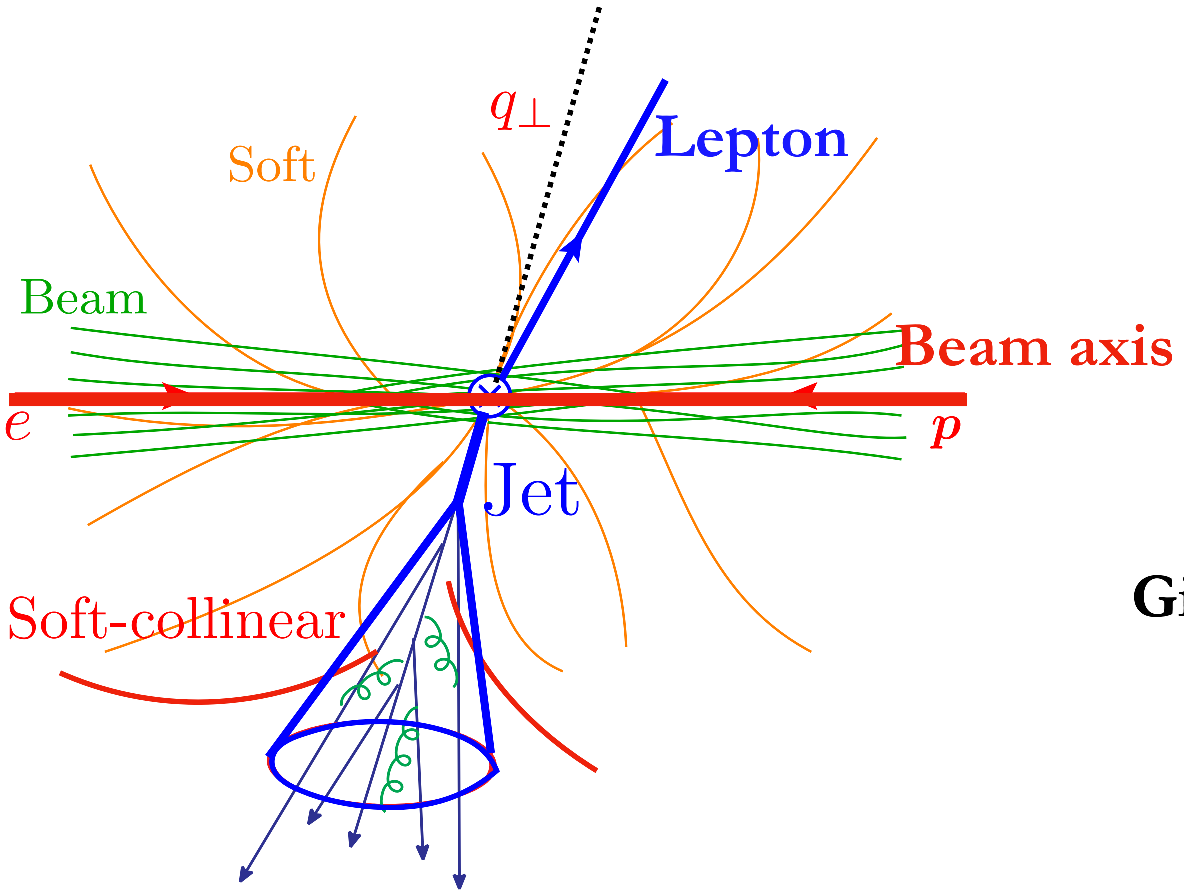
Polarization of A, B, h

- **Different structures come with different characteristic angular dependence.**

Kang, KL, Zhao, `20

Lepton + Jet imbalance

- **One of the simplest process** $e + P \to e + \text{Jet} + X$



$$q_\perp \equiv |\vec{p}_{e\perp} + \vec{p}_{J\perp}|, \qquad p_\perp \equiv |\vec{p}_{e\perp} - \vec{p}_{J\perp}|/2$$

$q_\perp \ll p_\perp$, **sensitive to the large logs of** $\ln(q_\perp/p_\perp)$ **and TMD structures of the hadrons.**

$$q_\perp = p_{X,\perp} = |\vec{k}_{c,\perp} + \vec{k}_{gs,\perp} + \vec{k}_{sc,\perp}|$$

Giving relevant modes : $(+,-,\perp)$ $\qquad \lambda = q_\perp/p_\perp$

n-collinear $\qquad k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$

global soft $\qquad k_{gs} \sim p_\perp(\lambda, \lambda, \lambda)$

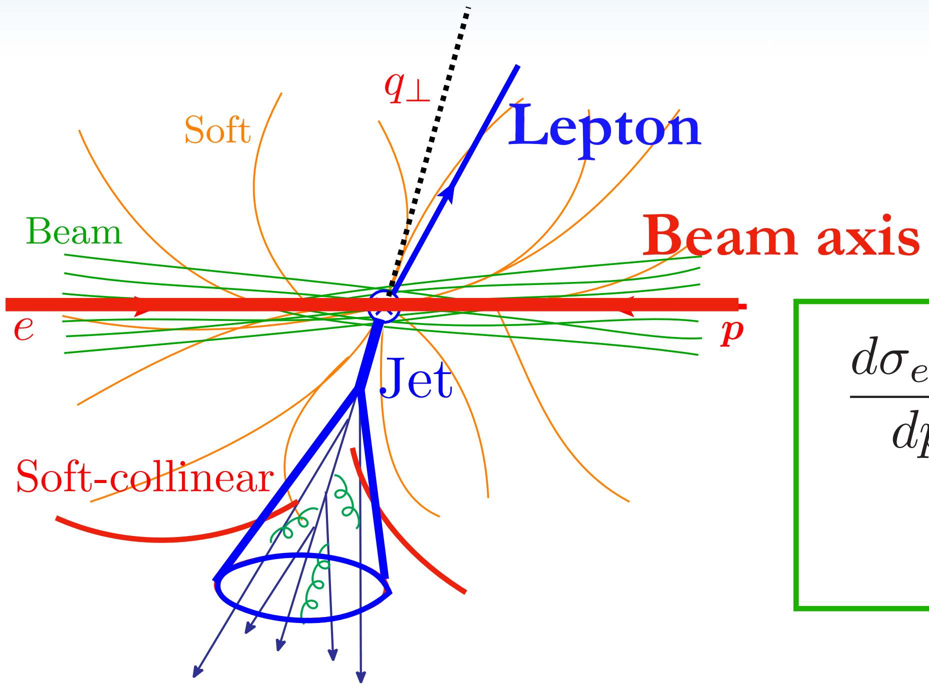
soft-collinear $\qquad k_{sc} \sim p_\perp R(\lambda R, \lambda/R, \lambda)_{n_J,\bar{n}_J}$

n_J-collinear $\qquad k_J \sim p_\perp(R^2, 1, R)_{n_J,\bar{n}_J}$

2) Lepton + Jet imbalance
TMDPDFs

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20

Lepton + Jet imbalance



$$\boxed{\begin{aligned}\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \int \prod_i^3 d^2k_{i\perp} H(Q)\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp) \\ \times f_a(x,\vec{k}_{1\perp}) S^{\text{global}}(\vec{k}_{2\perp}) S_{J_c}(\vec{k}_{3\perp}) J_c(p_\perp R)\end{aligned}}$$

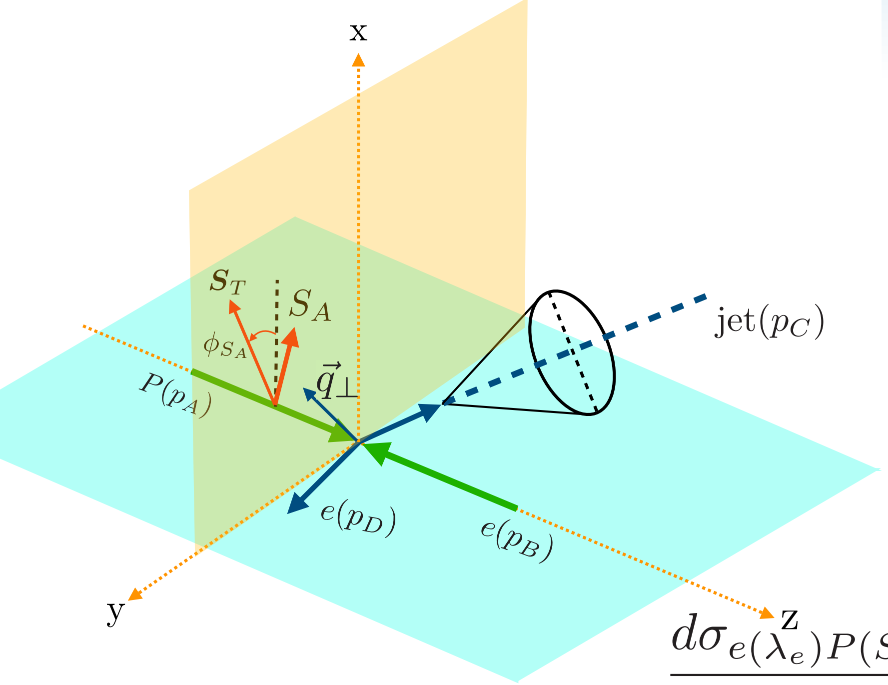
We arrive at factorization using SCET

n-collinear	$k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$	**TMDPDFs**
global soft	$k_{gs} \sim p_\perp(\lambda, \lambda, \lambda)$	**Soft functions**
soft-collinear	$k_{sc} \sim p_\perp R(\lambda R, \lambda/R, \lambda)_{n_J,\bar{n}_J}$	
n_J-collinear	$k_J \sim p_\perp(R^2, 1, R)_{n_J,\bar{n}_J}$	**Jet function**

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20

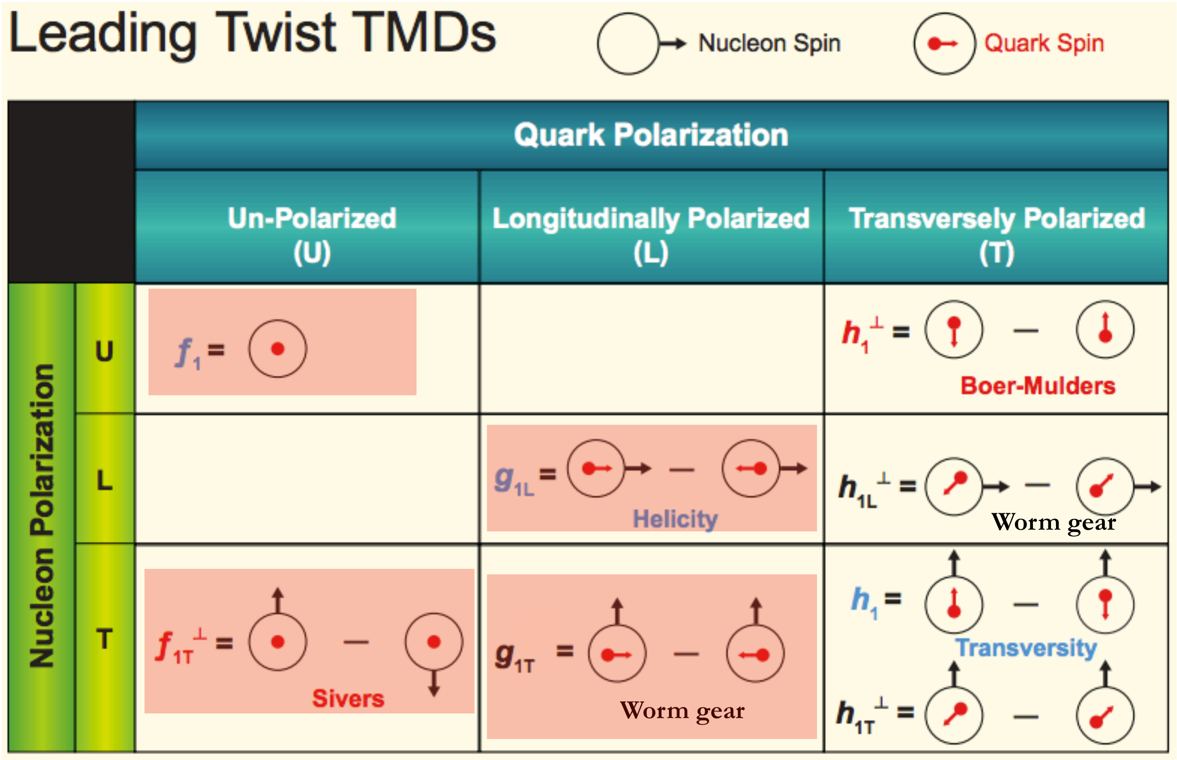
Also, Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi `19, `20

Lepton + Jet imbalance



$$\boxed{\begin{aligned}\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} &= \int \prod_i^3 d^2k_{i\perp} H(Q)\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp)\\ &\quad\times f_a(x,\vec{k}_{1\perp})S^{\text{global}}(\vec{k}_{2\perp})S_{J_c}(\vec{k}_{3\perp})J_c(p_\perp R)\end{aligned}}$$

$$\begin{aligned}\frac{d\sigma_{e(\lambda_e)P(S)\to e+\text{jet}}}{dp_\perp dq_\perp} &= \overset{\sim f_1}{\boxed{F_{UU}}}+\lambda_p\lambda_e \overset{\sim g_{1L}}{F_{LL}}\\ &\quad+S_T\left\{\underset{\sim f_{1T}^\perp}{\sin(\phi_{S_A}-\phi_q)F_{TU}^{\sin(\phi_{S_A}-\phi_q)}}+\underset{\sim g_{1T}}{\lambda_e\cos(\phi_{S_A}-\phi_q)F_{TL}^{\cos(\phi_{S_A}-\phi_q)}}\right\},\end{aligned}$$



Leading Twist TMDs

→ Nucleon Spin
→ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = ⊙		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
	L		g_{1L} = ⊙ − ⊙ Helicity	$h_{1L}^\perp$ = ⊙ − ⊙ Worm gear
	T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	g_{1T} = ⊙ − ⊙ Worm gear	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙

- **With jet, only sensitive to single TMDs (compared to standard processes)**
- **We do not get sensitivity to all TMDPDFs (only to chiral-even TMDPDFs)**

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20
Kang, KL, Shao, Zhao `21

Sivers asymmetry

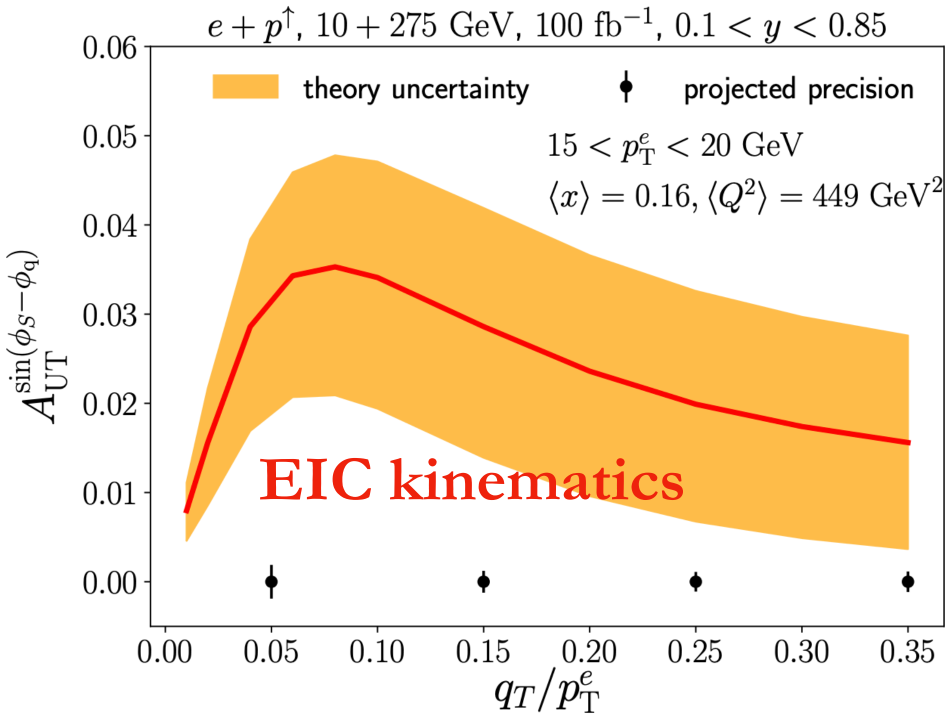
$$\frac{d\sigma_{e(\lambda_e)P(S)\to e+\text{jet}}}{dp_\perp dq_\perp} = \overset{\sim f_1}{F_{UU}} + \lambda_p\lambda_e \overset{\sim g_{1L}}{F_{LL}}$$

$$+S_T\left\{\sin(\phi_{S_A}-\phi_q)\underset{\sim f_{1T}^\perp}{F_{TU}^{\sin(\phi_{S_A}-\phi_q)}} + \lambda_e\cos(\phi_{S_A}-\phi_q)\underset{\sim g_{1T}}{F_{TL}^{\cos(\phi_{S_A}-\phi_q)}}\right\},$$

$$f_a(x_a,k_\perp) \to \frac{\epsilon_\perp^{\rho\sigma}S_{\perp\rho}k_{\perp\sigma}}{M}f_{aT}^\perp(x_a,k_\perp)$$

$$\boxed{\begin{aligned}\frac{d\Delta\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \frac{\epsilon_\perp^{\rho\sigma}S_{\perp\rho}}{M}\int\prod_i^3 d^2k_{i\perp}H(Q)\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp)\\ \times k_{1\perp\sigma}\, f_{aT}^\perp(x_a,k_{1\perp})S^{\text{global}}(\vec{k}_{2\perp})S_{J_c}(\vec{k}_{3\perp})J_c(p_\perp R)\end{aligned}}$$

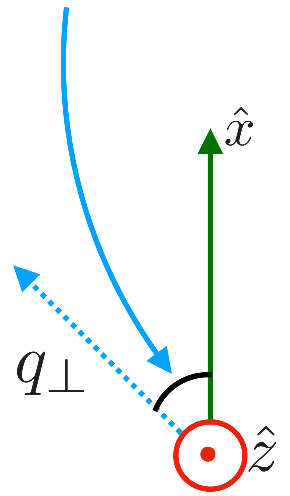
$$\propto \sin\boxed{(\phi_{S_A}-\phi_q)}$$



$$A_{UT}^{\sin(\phi_{S_A}-\phi_q)} = \frac{F_{UT}^{\sin(\phi_{S_A}-\phi_q)}}{F_{UU}}$$

- **Positive $\Delta\sigma$ $\Longrightarrow$ a preference of imbalance to be on left**

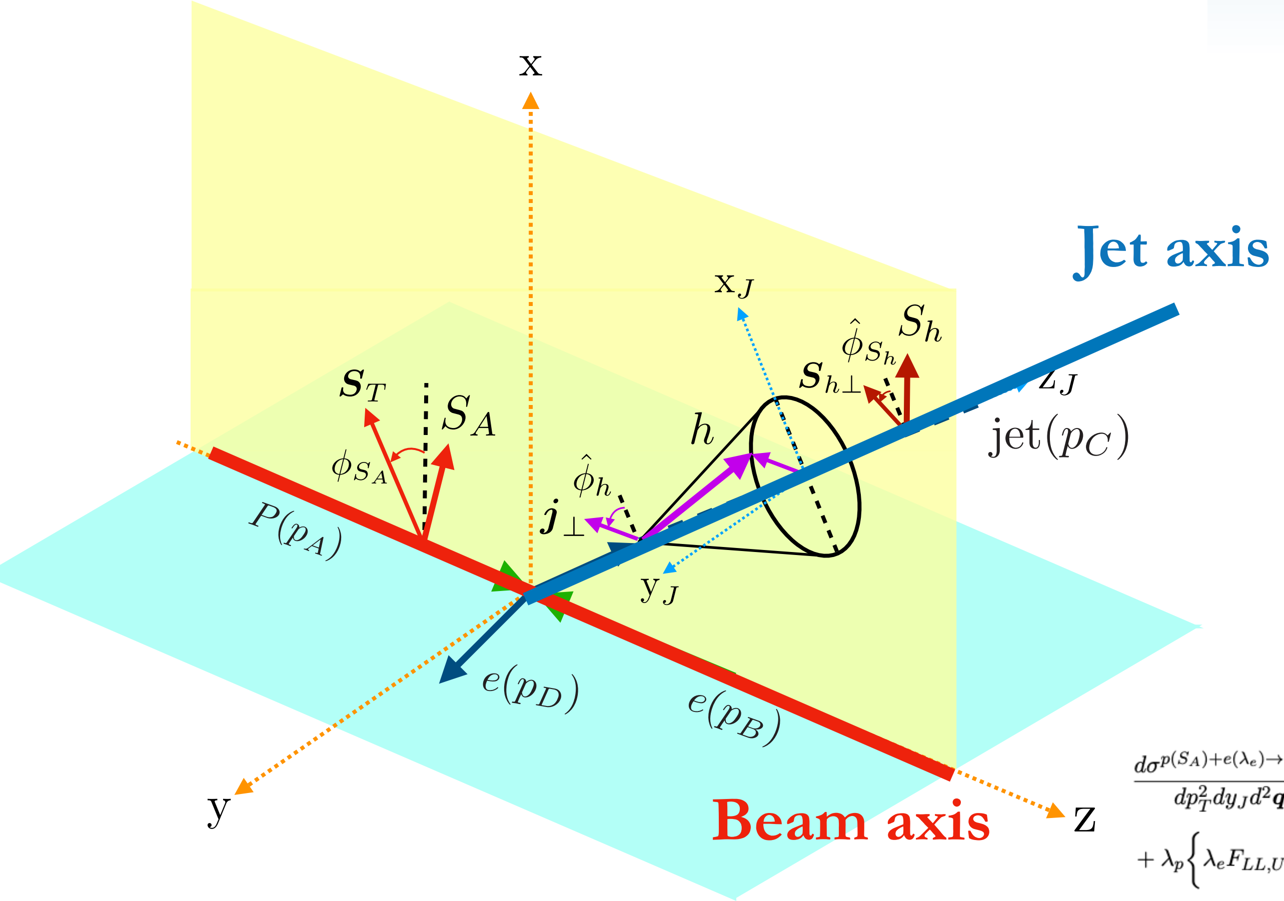
(When polarized proton moving towards us and transverse spin pointing up.)



Sivers from SIDIS extraction
Echevarria, Idilbi, Kang, Vitev, `14

Arratia, Kang, Prokudin, Ringer `20
Kang, KL, Shao, Zhao `21

Polarized Jet Fragmentation Functions and lepton + jet imbalance



3) Lepton + Jet imbalance with hadron in jet TMDFFs / TMDPDFs

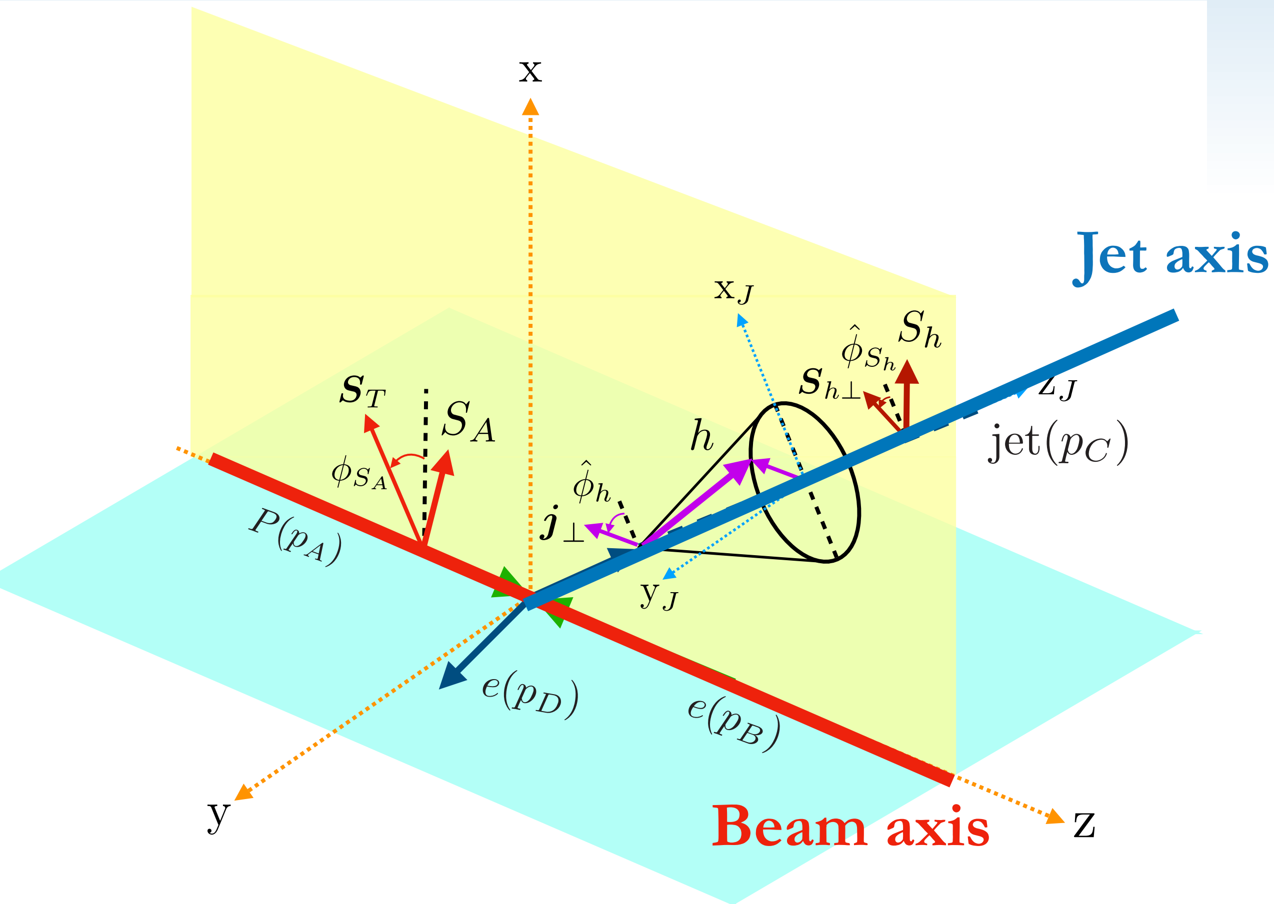
- **Observation of polarized hadron inside jet gives sensitivity to all TMDPDFs and TMDFFs. (analogous correlations to standard SIDIS)**
- **Sensitivity to two TMDs, but sensitive to $\vec{q}_\perp$ and $\vec{j}_\perp$ separately (advantage of two axes)**

Many characteristic correlations

$$
\begin{aligned}
&\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q-\hat{\phi}_h)F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+\lambda_p\Big\{\lambda_e F_{LL,U} + \sin(\phi_q-\hat{\phi}_h)F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)}\Big\} \\
&+S_T\Big\{\sin(\phi_q-\phi_{S_A})F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e\cos(\phi_q-\phi_{S_A})F_{TL,U}^{\cos(\phi_q-\phi_{S_A})} \\
&\qquad +\sin(\phi_{S_A}-\hat{\phi}_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})}\Big\} \\
&+\lambda_h\Big\{\lambda_e F_{UL,L} + \sin(\hat{\phi}_h-\phi_q)F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \lambda_p\Big[F_{LU,L} + \cos(\hat{\phi}_h-\phi_q)F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)}\Big] \\
&\qquad +S_T\Big[\cos(\phi_q-\phi_{S_A})F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e\sin(\phi_q-\phi_{S_A})F_{TL,L}^{\sin(\phi_q-\phi_{S_A})} \\
&\qquad\qquad +\cos(\phi_{S_A}-\hat{\phi}_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)}\Big]\Big\} \\
&+S_{h\perp}\Big\{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e\cos(\hat{\phi}_h-\hat{\phi}_{S_h})F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&\qquad +\sin(\hat{\phi}_{S_h}-\phi_q)F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} \\
&\qquad +\lambda_p\Big[\cos(\hat{\phi}_h-\hat{\phi}_{S_h})F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q-\hat{\phi}_{S_h})F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad\qquad +\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e\sin(\hat{\phi}_h-\hat{\phi}_{S_h})F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})}\Big] \\
&\qquad +S_T\Big[\cos(\phi_{S_A}-\hat{\phi}_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})} \\
&\qquad\qquad +\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})} \\
&\qquad\qquad +\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A}))F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A})} \\
&\qquad\qquad +\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad\qquad +\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\qquad\qquad +\lambda_e\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)} \\
&\qquad\qquad +\lambda_e\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q))F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)}\Big]\Big\},
\end{aligned}
$$

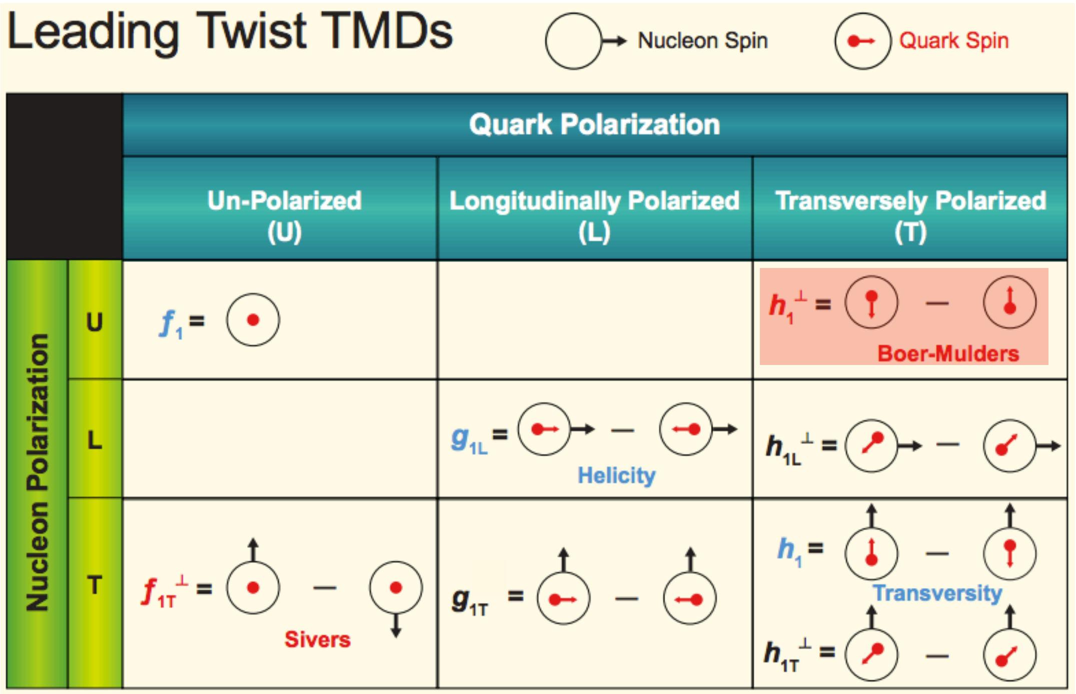
Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}$



$$A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}(q_\perp, j_\perp)}{F_{UU,U}(q_\perp, j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

- Boer-Mulders and Collins functions sensitive to transverse momentum measured with respect to different axes.
- "Separation" of the incoming and outgoing dynamics.



Leading Twist TMDs ○→ Nucleon Spin ⊙→ Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = ⊙		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
L		g_{1L} = ⊙→ − ⊙→ Helicity	$h_{1L}^\perp$ = ⊙→ − ⊙→
T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	g_{1T} = ⊙ − ⊙	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙

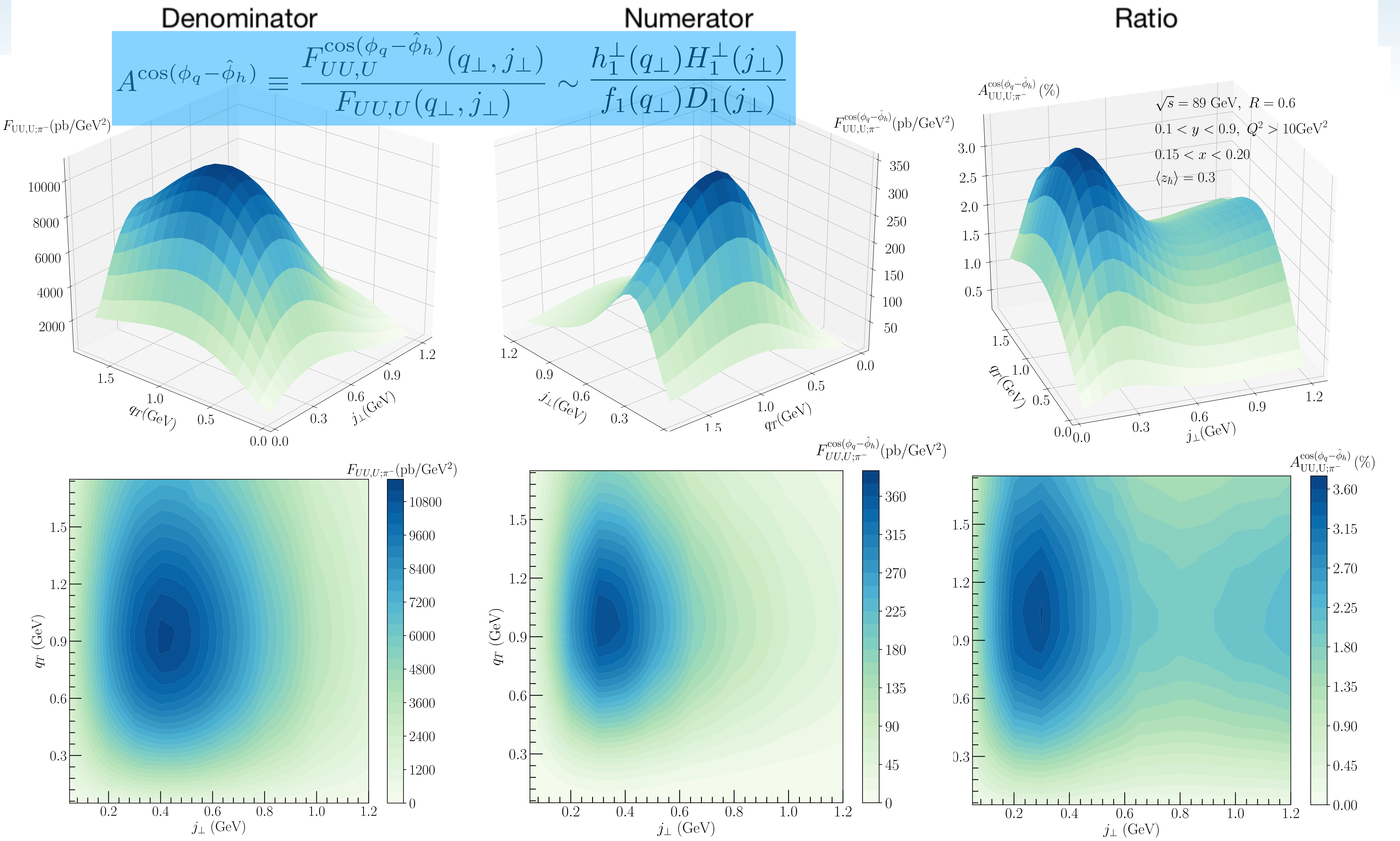
Quark polarization

Hadron polarization	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$ Collins
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

Kang, KL, Shao, Zhao `21

Unpolarized π in jet (Boer-Mulders, Collins)

π⁻ q_T [0,1.8], j_T [0,1.2]

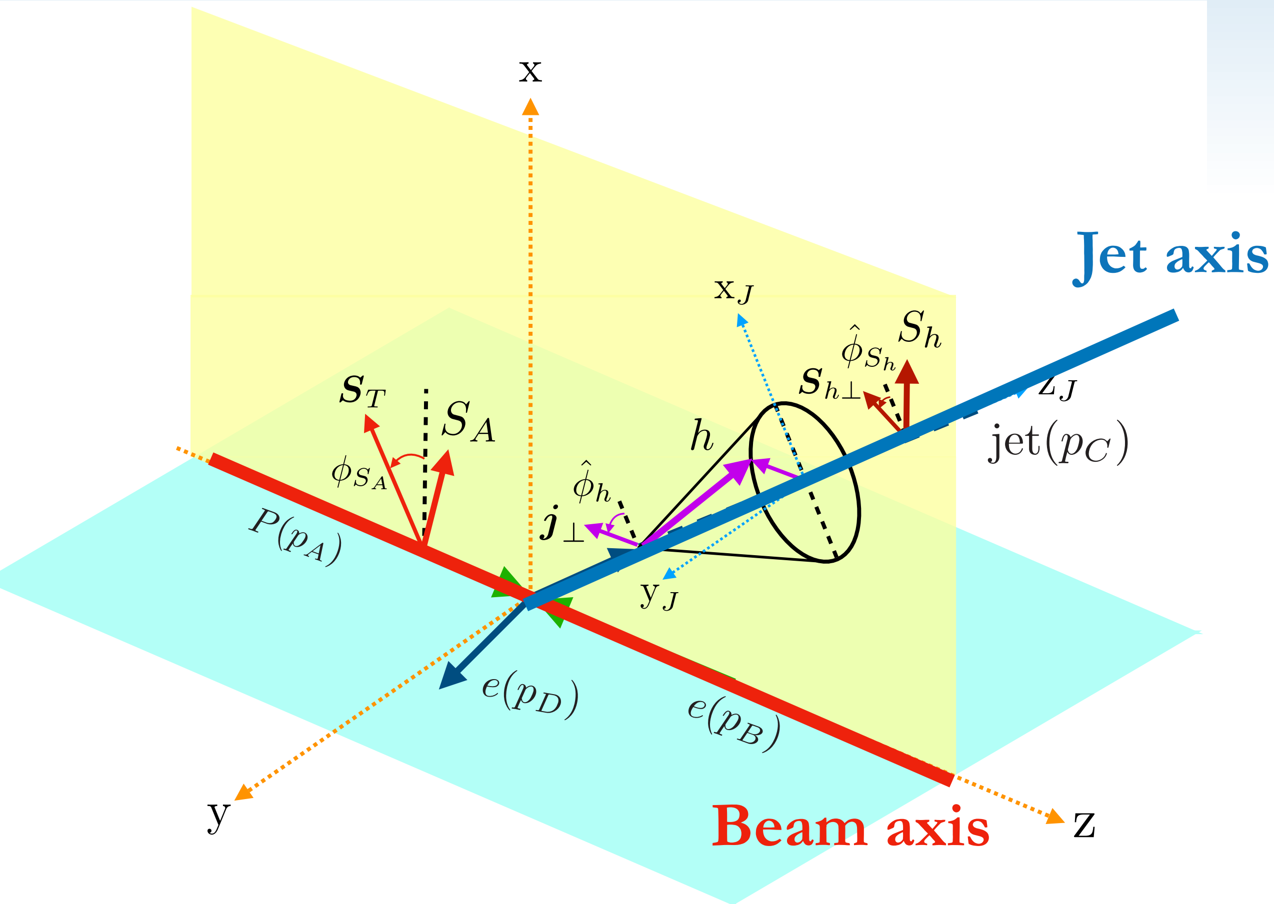


$$A^{\cos(\phi_q-\hat\phi_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat\phi_h)}(q_\perp,j_\perp)}{F_{UU,U}(q_\perp,j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

Parametrization from *Barone, Melis, Prokudin `10* (Boer-Mulders)
Kang, Prokudin, Sun, Yuan `15 (Collins)

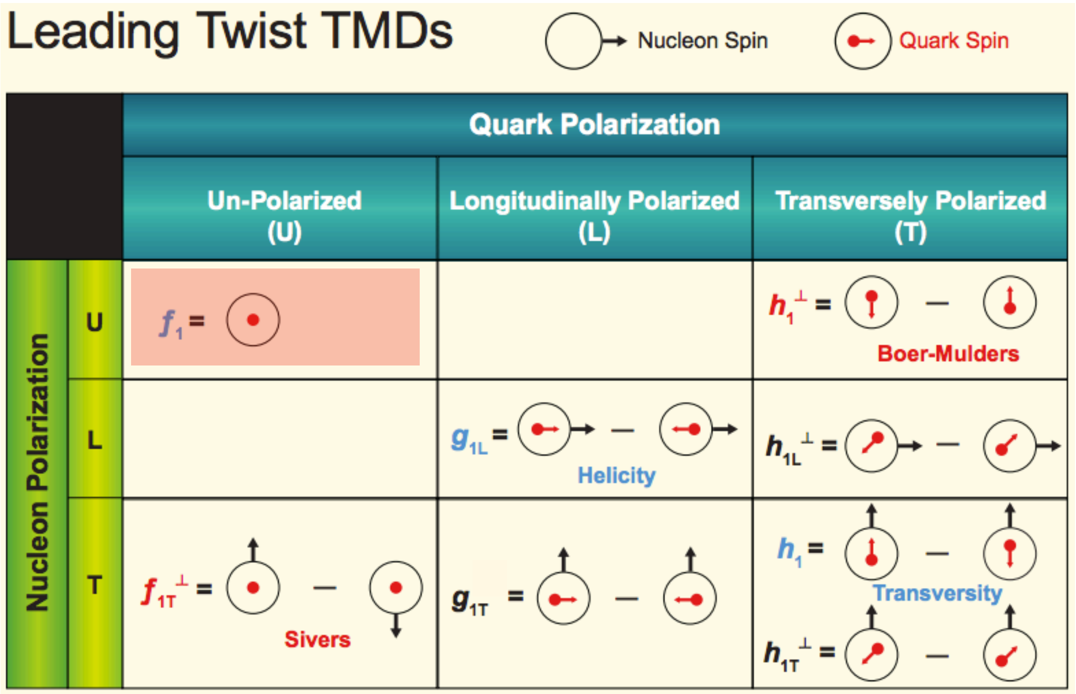
Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda-\hat{\phi}_{S_\Lambda})}$



$$A_{UU,T}^{\sin(\hat{\phi}_\Lambda-\hat{\phi}_{S_\Lambda})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_\Lambda-\hat{\phi}_{S_\Lambda})}}{F_{UU,U}} \sim \frac{f_1(q_\perp)D_{1T}^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

- "**Separation**" of the incoming and outgoing dynamics cancel the q_T dependence for this case.



Leading Twist TMDs ○→ Nucleon Spin ⊙ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = ⊙		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
	L		g_{1L} = ⊙→ − ⊙→ Helicity	$h_{1L}^\perp$ = ⊙→ − ⊙→
	T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	g_{1T} = ⊙ − ⊙	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙

Quark polarization

Hadron polarization

	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

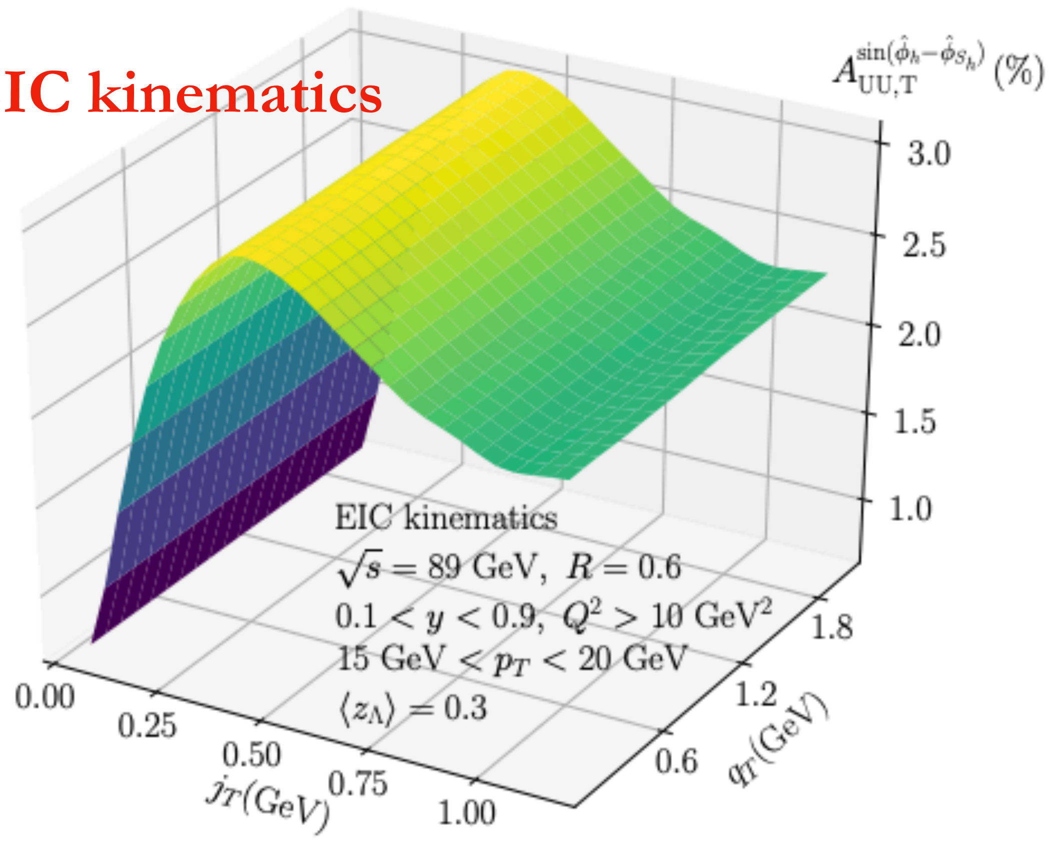
Polarizing FF

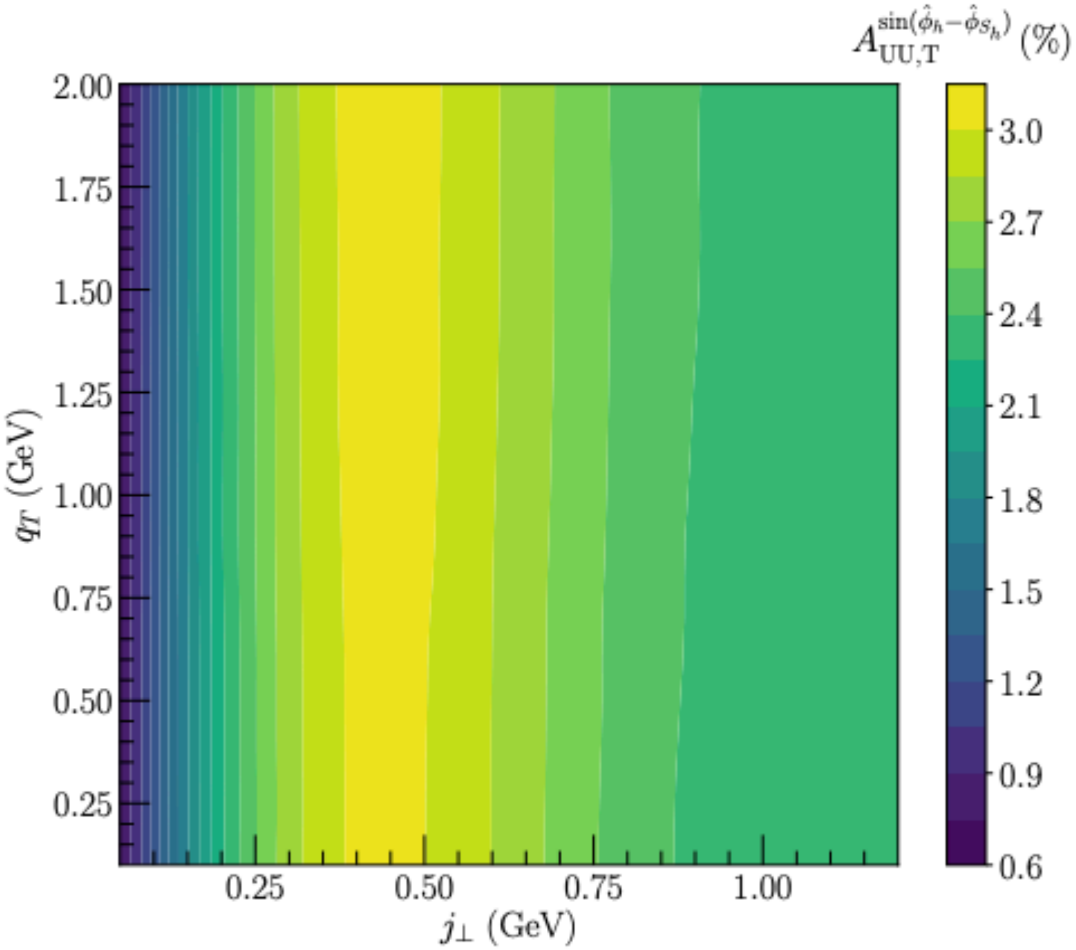
Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}$

$$A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}}{F_{UU,U}} \sim \frac{f_1(q_\perp) D_{1T}^\perp(j_\perp)}{f_1(q_\perp) D_1(j_\perp)}$$

EIC kinematics

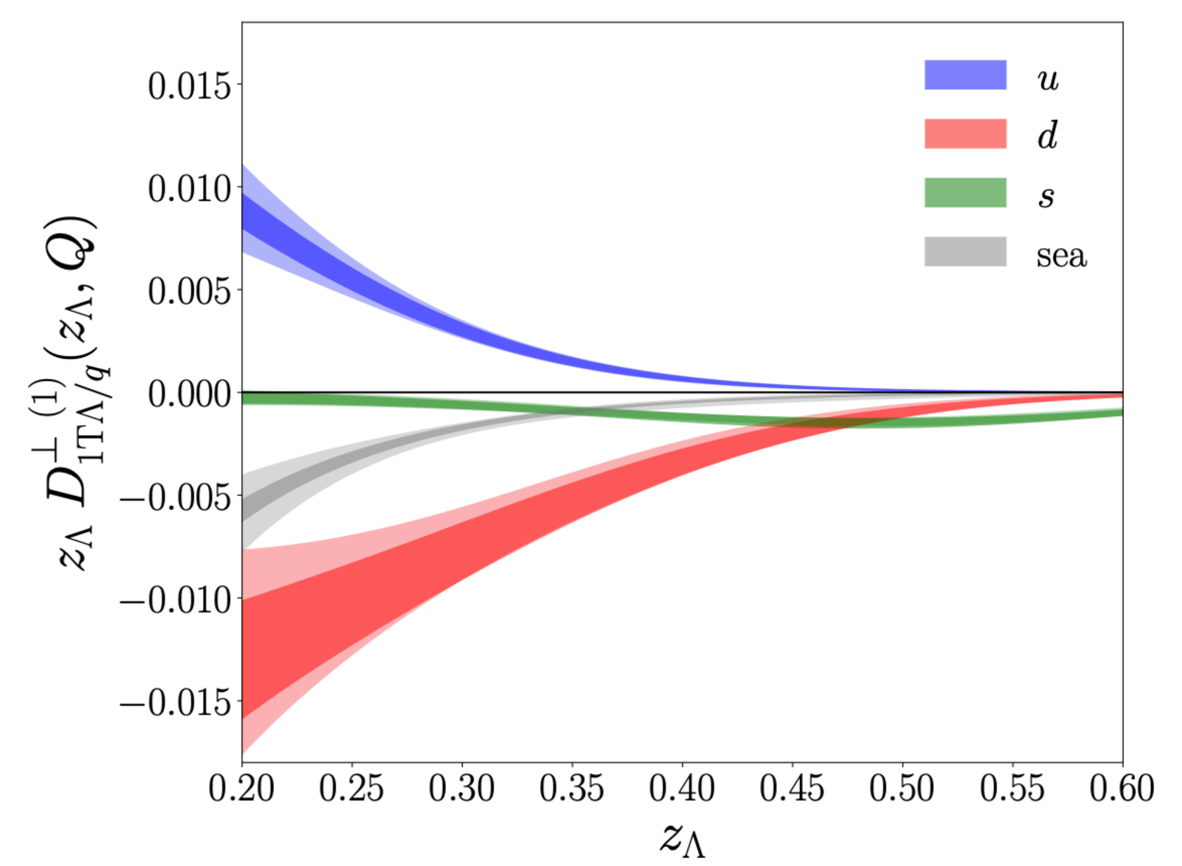




- q_T **dependence indeed cancels and is only sensitive to TMDFFs.**

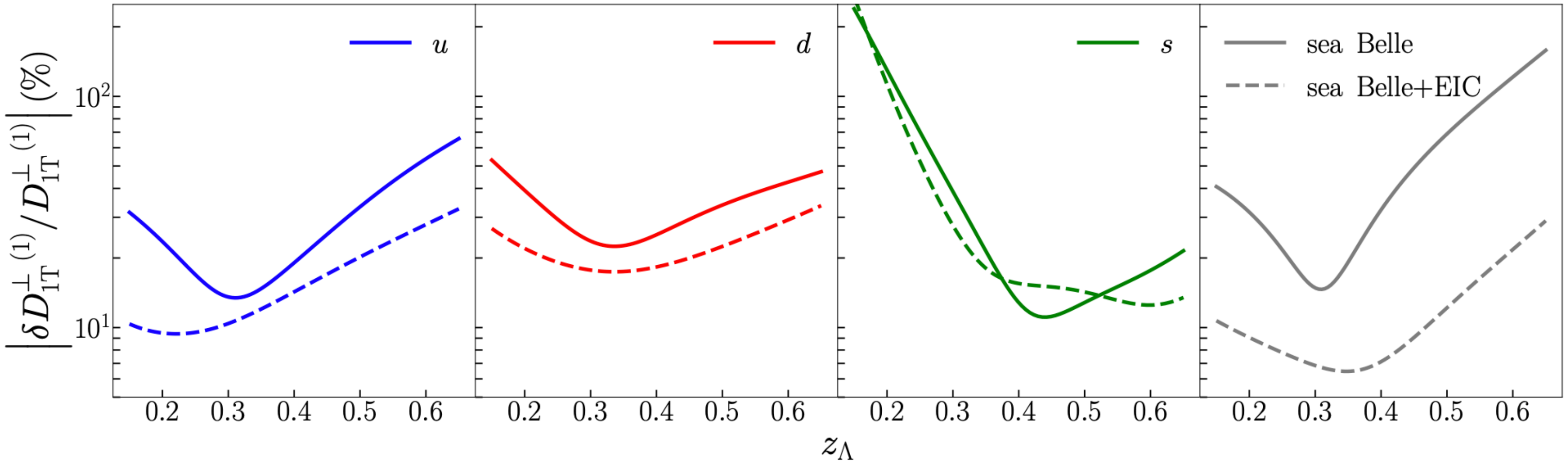
Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}$



- **EIC pseudo-data significantly decreases the uncertainties in the determination of TMDPFFs.**

Kang, Terry, Vossen, Xu, Zhang `21



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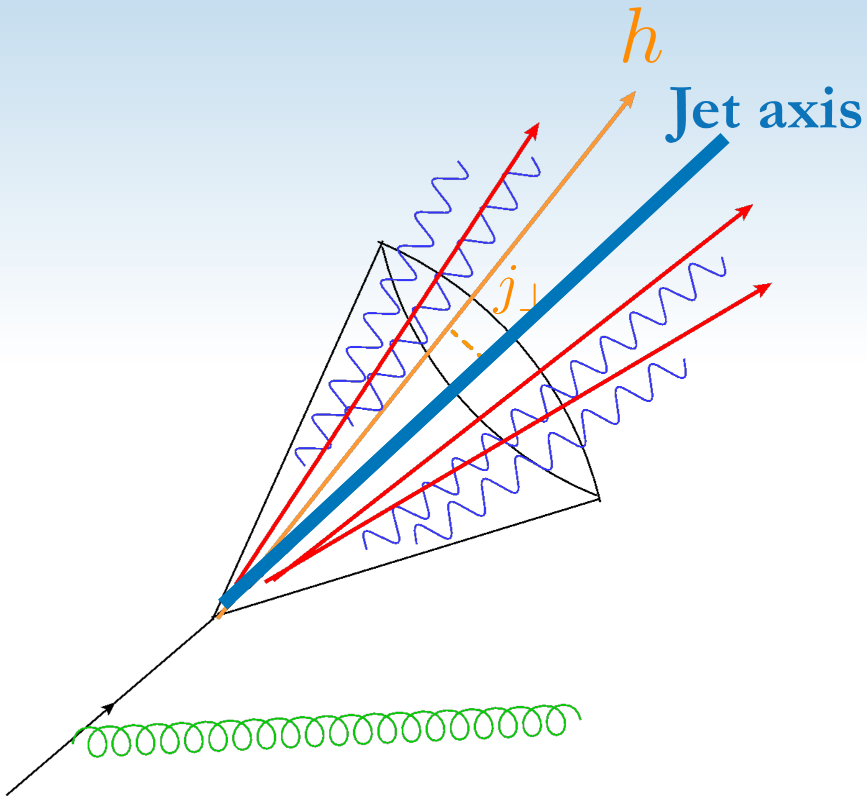
Conclusion



- New processes involving jets to non-perturbative structure

$$PP/eP \to J(h) + X, \quad eP \to e + J + X, \quad eP \to e + J(h) + X \;, \quad \cdots$$

- Jet substructure techniques can be used to access information about FFs
  - Information differential in z_h allow more direct access to the FFs
  - Jet axis provides us a mean to access TMDFF structure

- Jet processes at the EIC can deconvolve the dependence between the TMDPDF and TMDFF.
  - Its high luminosity, wide energy range, and polarized beams will illuminate our understanding of the hadron structure and process of hadronization.
    - **Jets are great way to '*isolate*' and obtain '*differential information*' of the non-perturbative structure of interest!**

Thank you!