Reweighting the Sivers function with jet data

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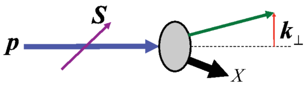
M. Boglione, U. D'Alesio, CF,
J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin,
Phys. Lett. B 815 (2021) 136135

Introduction - the Sivers function



D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$f_{a/p^\uparrow}(x,\boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x,-\boldsymbol{k}_\perp)$$

$$= \Delta^N f_{a/p^\uparrow}(x,k_\perp^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (TO-CA)$$

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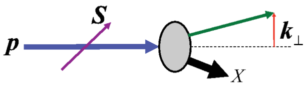

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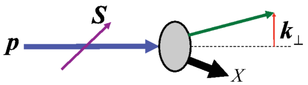

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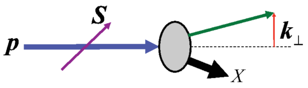

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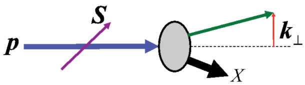

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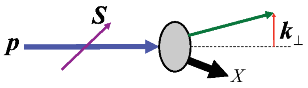

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- in the GPM, Sivers effect generates single spin asymmetries (SSAs) in $p^\uparrow p \to$ jet X

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020;
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- a color gauge invariant formulation of GPM (CGI-GPM) was developed, with inclusion of initial and final state interaction; process dependence is recovered
- A_N in $p^\uparrow p \to \text{jet}\, X$:

$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \frac{d\Delta\sigma}{2d\sigma}\,,$$

with

$$d\Delta\sigma^{\text{CGI-GPM}} = \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a\, dx_b}{x_a\, x_b}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b} \left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp a}(x_a, k_{\perp a}) \cos\varphi_a \times f_{b/p}(x_b, k_{\perp b})\, H^{\text{Inc}}_{ab\to cd}\, \delta(\hat{s}+\hat{t}+\hat{u})$$

and

$$d\sigma = \frac{\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a\, dx_b}{x_a\, x_b}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, f_{a/p}(x_a, k_{\perp a}) f_{b/p}(x_b, k_{\perp b})\, H^U_{ab\to cd}\, \delta(\hat{s}+\hat{t}+\hat{u})\,.$$

[note: $d\Delta\sigma^{\text{GPM}} = d\Delta\sigma^{\text{CGI-GPM}}\left[H^{\text{Inc}}_{ab\to cd} \to H^U_{ab\to cd}\right]$]

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

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  Never applied to a fit with actual data...

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...up to now!

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING:

- consider a model for a TMD depending on $\boldsymbol{a} = \{a_1, \cdots, a_n\}$ parameters with prior probability distribution $\pi(\boldsymbol{a})$

[if no prior knowledge exists, $\pi(\boldsymbol{a})$ is flat]

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- for a set of data $\boldsymbol{y} = y_1, \cdots, y_{N_{\rm dat}}$, χ^2 is defined as

$$\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}] - y_j)$$

[if only uncorrelated uncertainties: $\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)^2 / \sigma_i^2$]

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- parameter sets $\boldsymbol{a}_k$ produced using Monte Carlo (MC) procedures

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BAYESIAN REWEIGHTING (continued):

- Posterior density through Bayes theorem:

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Let's reweight the Sivers function with STAR data!

Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148

- quark Sivers function parametrization:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f^{(1)}_{q/p^\uparrow}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f^{(1)}_{q/p^\uparrow}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
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- 220 data points from SIDIS data, up to $x \lesssim 0.3$

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$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f^{(1)}_{q/p^\uparrow}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f^{(1)}_{q/p^\uparrow}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
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- 220 data points from SIDIS data, up to $x \lesssim 0.3$
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Reweighting the Sivers function (I)

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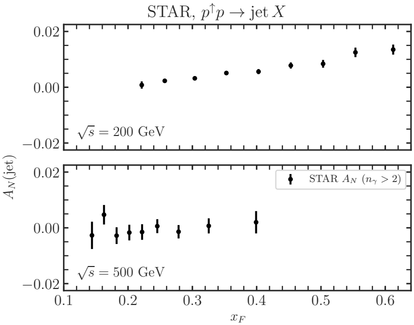
Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135; STAR Coll., arXiv:2012.11428



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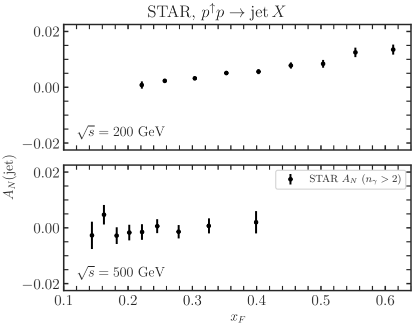

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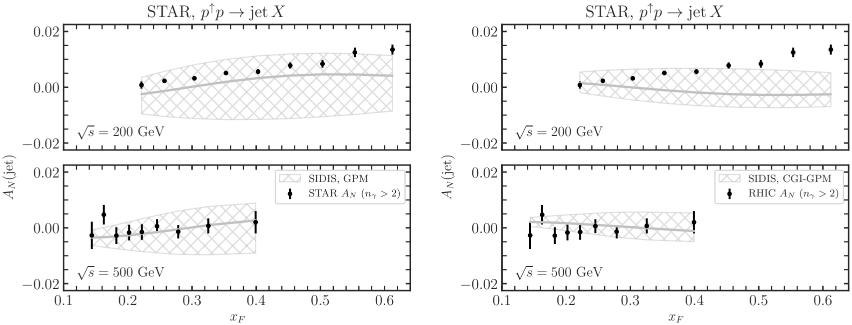

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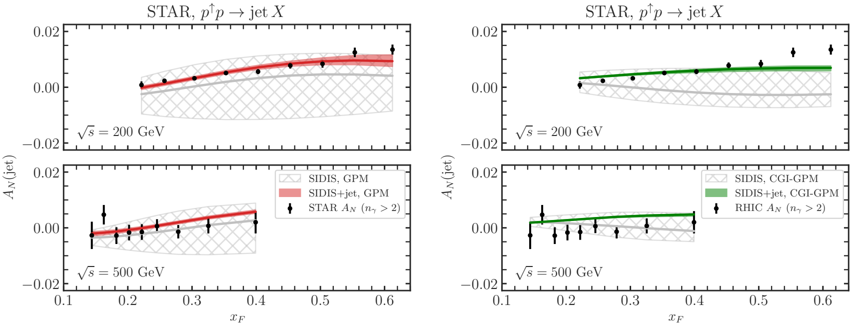

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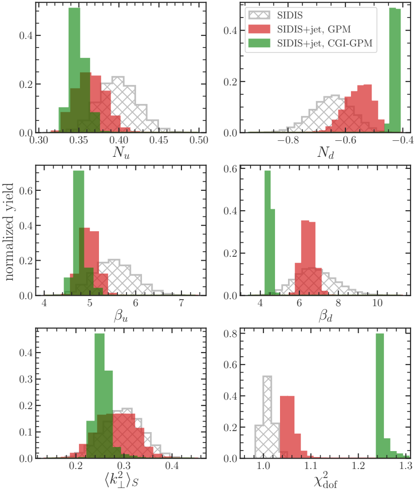
- reweighted curves show reduced uncertainties

Reweighting the Sivers function (II)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



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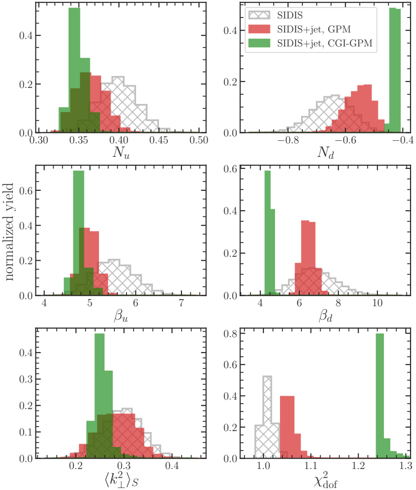

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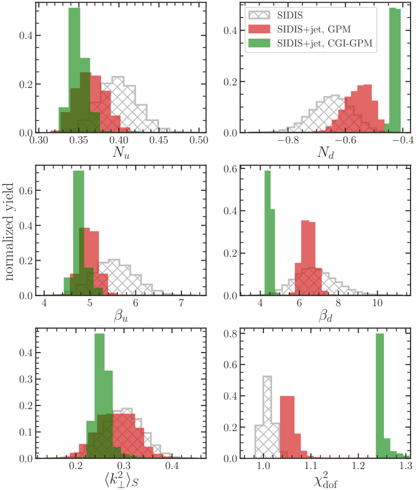

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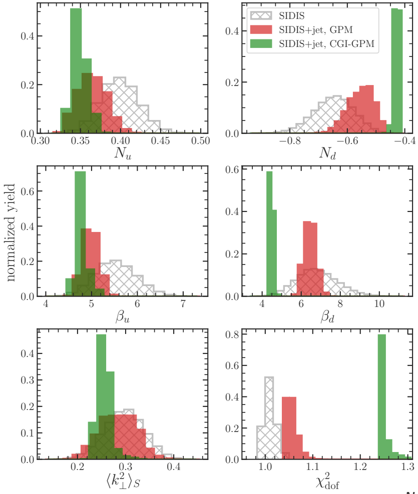

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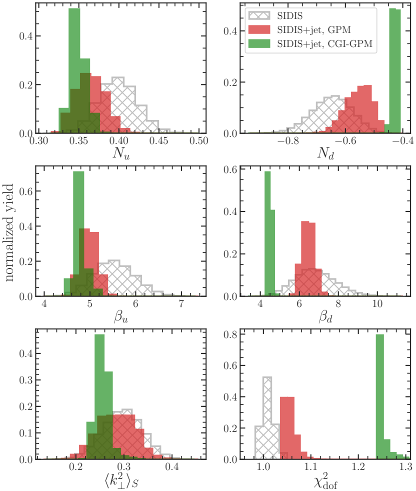

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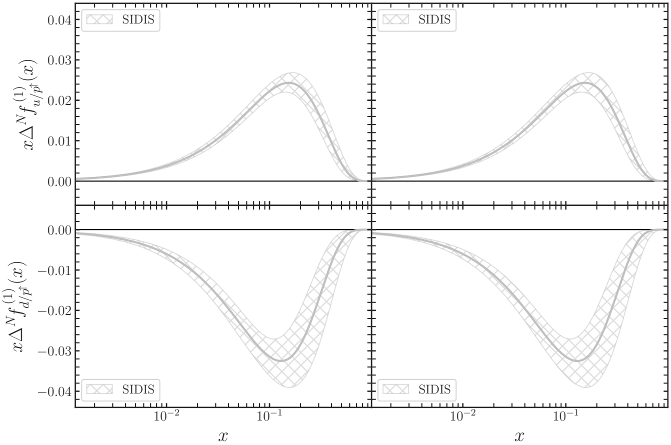

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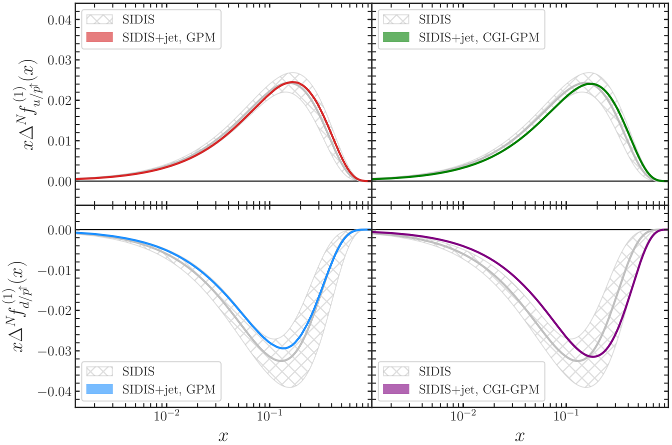

drastic reduction of uncertainties

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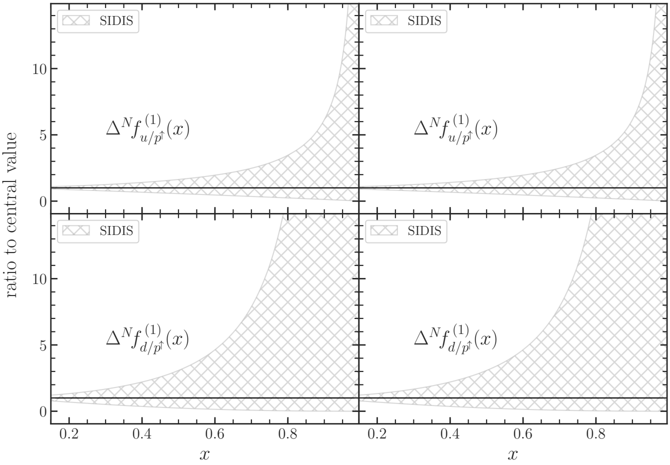

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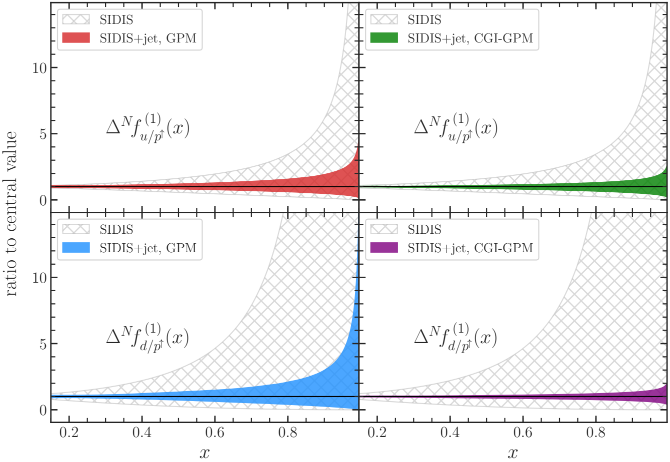

STAR data allows to constrain the Sivers function at large *x*!

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  Bayesian reweighting extended to the case of asymmetric uncertainties

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- Forthcoming measurements from COMPASS, JLab and the EIC will play a crucial role in unravelling the nucleon structure in its full complexity

Thank you

Backup

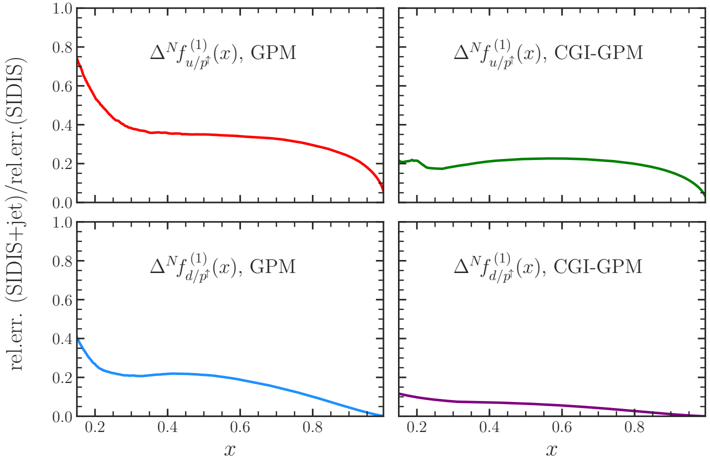
Reweighting the Sivers function - parameters

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

	SIDIS	SIDIS+jet, GPM	SIDIS+jet, CGI-GPM
N_u	$0.40^{+0.05}_{-0.04}$	$0.36^{+0.04}_{-0.03}$	$0.35^{+0.02}_{-0.01}$
N_d	$-0.65^{+0.13}_{-0.15}$	$-0.55^{+0.07}_{-0.10}$	$-0.43^{+0.01}_{-0.02}$
β_u	$5.52^{+0.93}_{-0.83}$	$4.98^{+0.34}_{-0.30}$	$4.79^{+0.28}_{-0.19}$
β_d	$6.77^{+2.29}_{-1.85}$	$6.45^{+0.63}_{-0.52}$	$4.48^{+0.17}_{-0.13}$
$\langle k_\perp^2 \rangle_S$	$0.30^{+0.08}_{-0.08}$	$0.28^{+0.07}_{-0.07}$	$0.26^{+0.03}_{-0.02}$
χ^2_{dof}	$1.01^{+0.03}_{-0.02}$	$1.05^{+0.03}_{-0.01}$	$1.25^{+0.04}_{-0.01}$

Reweighting the Sivers function - relative errors

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



Ratio of relative errors