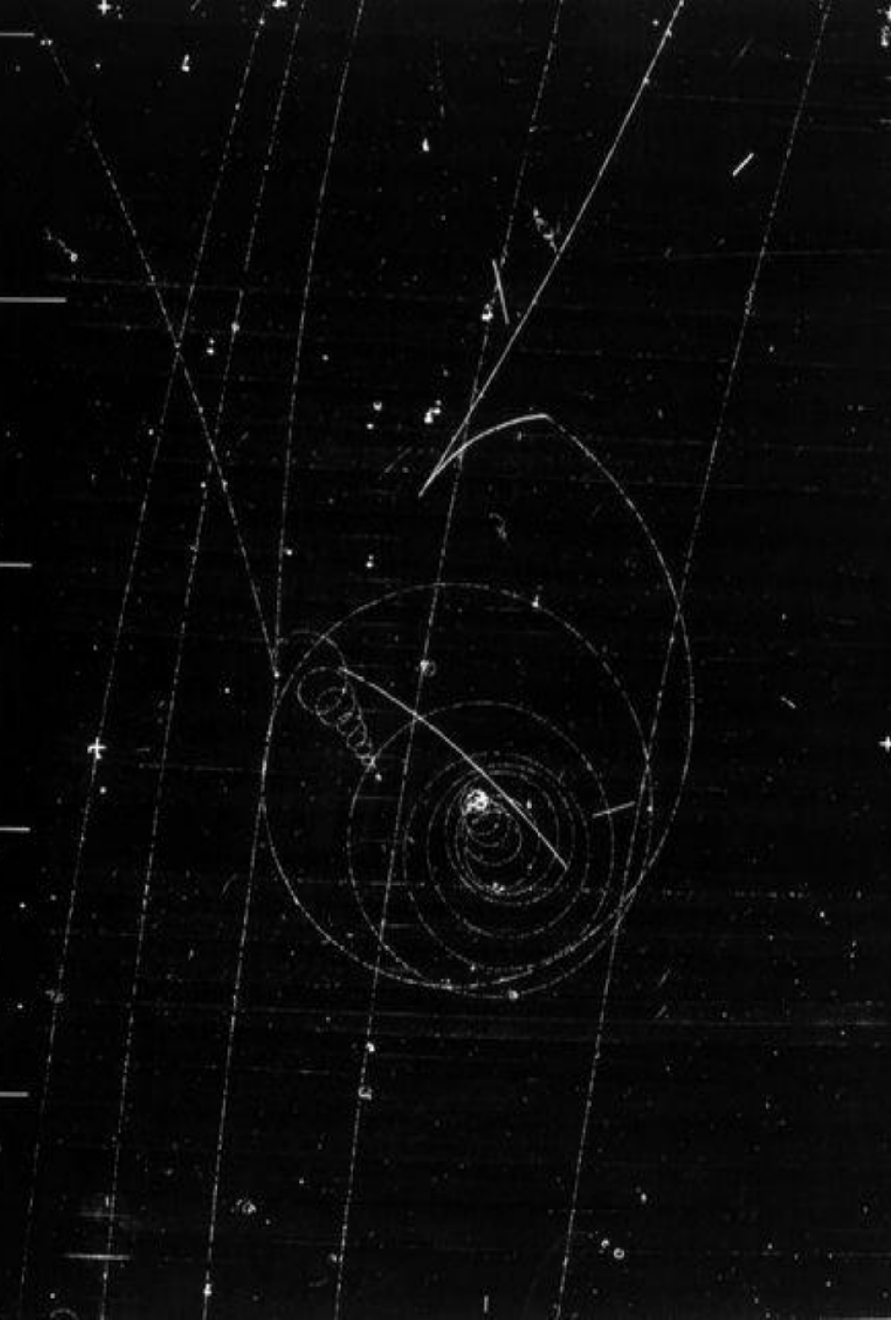

Transverse Lambda Polarization in e^+e^- processes

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Istituto Nazionale di Fisica Nucleare

Contents

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- TMDs Evolution and preliminary results
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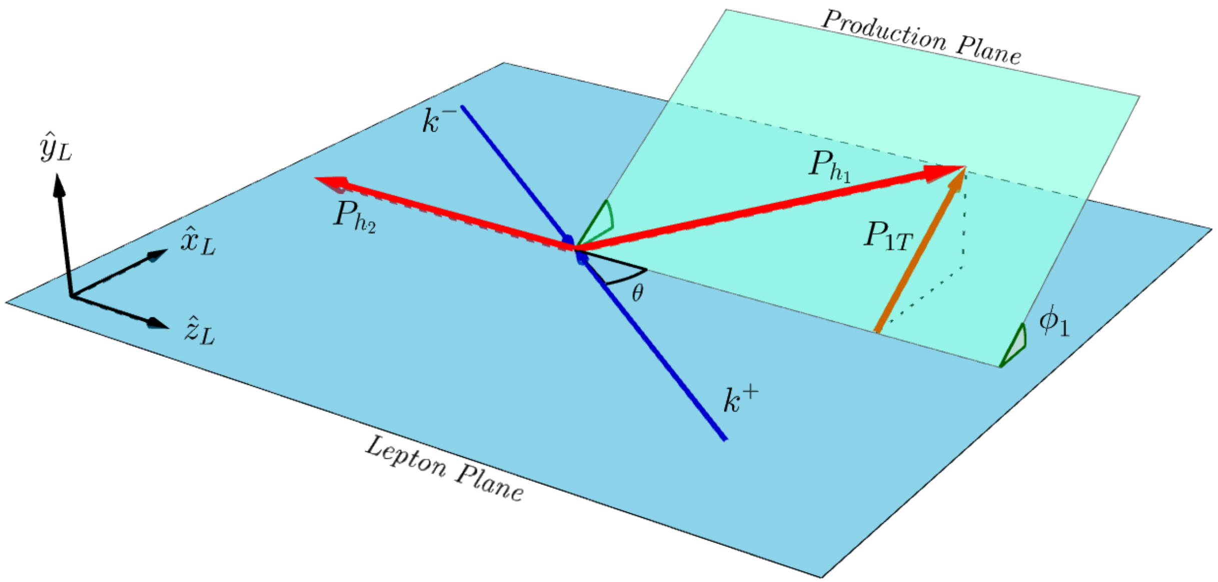
$e^+e^- \to h_1 h_2 X$: Helicity Formalism



		Hadron		
		U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta\hat{D}^{h}_{S_Y/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

Derived general structure for double hadron production in terms of convolutions:

$$\mathcal{C}[w\Delta D^{h_1}\Delta D^{h_2}] = \sum_q e_q^2 \int d^2\boldsymbol{p}_{\perp 1} d^2\boldsymbol{p}_{\perp 2}\, \delta^{(2)}(\boldsymbol{p}_{\perp 1} - \boldsymbol{P}_{1T} + \boldsymbol{p}_{\perp 2}\, z_{p_1}/z_{p_2})\, w(\boldsymbol{p}_{\perp 2}, \boldsymbol{P}_{1T}) \Delta D_{h_1/q}(z_1, p_{\perp 1}) \Delta D_{h_2/\bar{q}}(z_2, p_{\perp 2})$$

Full results in: U. D'Alesio, F. Murgia, M.Z., arXiv 2108.05632 [hep-ph]

Results consistent with [D. Boer, R. Jakob, and P.J. Mulders. Nucl. Phys. B 504 (1997)]

The polarization is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

The polarization projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos\widetilde{\phi} + P_y^{h_1} \sin\widetilde{\phi}$$

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$$P_n^{h_1}(z_1, z_2) = -\frac{\int d^2 P_{1T}\, F_{TU}^{\sin(\phi_1 - \phi_{S_1}^L)}}{\int d^2 P_{1T}\, F_{UU}}$$

$$F_{UU} = \mathcal{C}[D_1 \bar{D}_1]$$

$$F_{TU}^{\sin(\phi_1 - \phi_{S_1}^L)} = \mathcal{C}\left[\left(\frac{z_{p_1}}{z_{p_2}} \frac{p_\perp}{p_\perp} \hat{\boldsymbol{p}}_{\perp 2} \cdot \hat{\boldsymbol{P}}_{1T} - \frac{P_{1T}}{p_{\perp 1}}\right) \Delta^N D_{h_1^\uparrow/q} D_{h_2/\bar{q}}\right]$$

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$$\Delta D^h_{S_Y/q}(z, p_\perp) = \Delta D^h_{S_Y/q}(z) \sqrt{2e}\, \frac{p_\perp}{M_{pol}}\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi \langle p_\perp^2 \rangle_h}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi \langle p_\perp^2 \rangle_h}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
- $\langle p_\perp^2 \rangle_h = 0.2\ GeV^2$ width of unp. FF

The polarization is measured along:

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The polarization projection along $\hat{n}$:

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$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

$$\mathcal{P}_n(z_1, z_2) = \sqrt{\frac{e\pi}{2}}\frac{1}{M_{\rm pol}}\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_{\perp 1}^2\rangle}\frac{z_2}{\left\{[z_1(1 - m_{h_1}^2/(z_1^2 s))]^2\langle p_{\perp 2}^2\rangle + z_2^2\langle p_\perp^2\rangle_{\rm pol}\right\}^{1/2}} \times \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1) D_{h_2/\bar{q}}(z_2)}{\sum_q e_q^2\, D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
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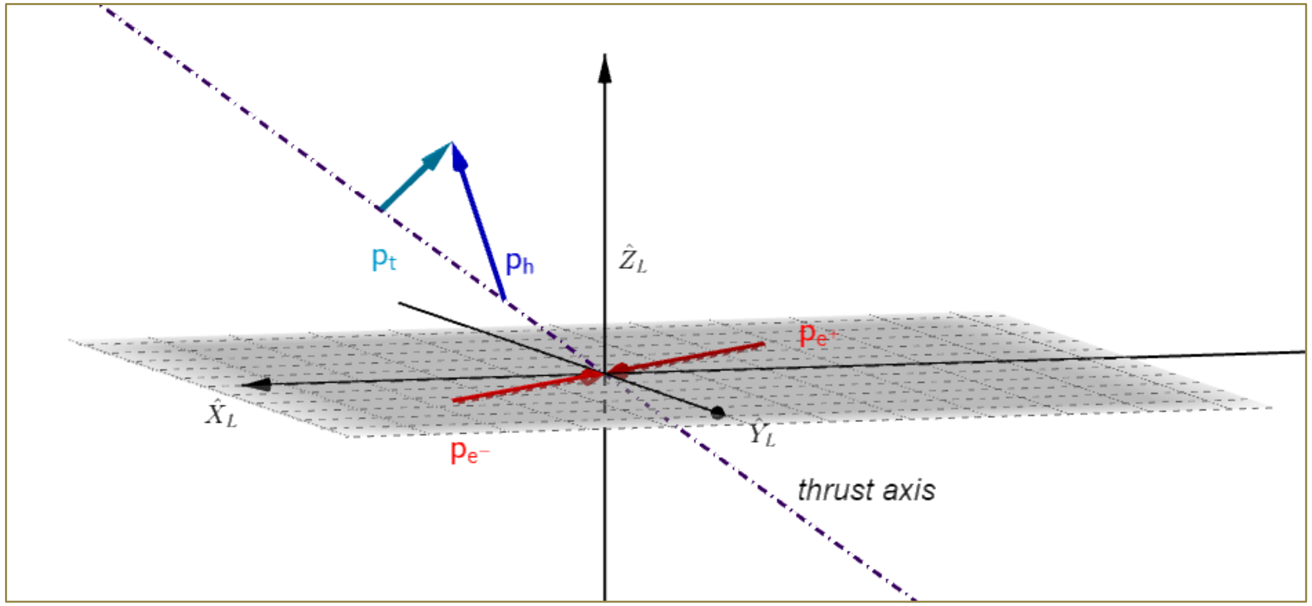
$e^+e^- \to h_1 + (\text{jet})\, X$: Thrust Frame

The polarization is measured along:

$$\hat{n} = \hat{T} \times \hat{P}_{h_1}$$

- Data depend only on z_Λ - $p_\perp$
- Direct access to $p_\perp$ dependence



Within a phenomenological approach

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2\, D_{h_1/q}(z_1, p_{\perp 1})}$$

Simple gaussian model

Fit and Results

Belle data: $\sqrt{s} = 10.58$ GeV

- 128 points $\Lambda + h$, in bins of the energy fractions z_Λ - $z_{\pi,K}$
- 32 points $\Lambda(jet)$, in bins of z_Λ - $p_\perp$

Fit and Results

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5\ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.5\ - 0.9]$ bin excluded → 24 data points

Fit and Results

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- $\Lambda(jet)$: $z_{\Lambda} = [0.5 \ - 0.9]$ bin excluded →   24 data points

$$\Delta D^{h}_{S_Y/q}(z) = \mathcal{N}^{p}_{q}(z) D_{h/q}(z)$$

$$\mathcal{N}^{p}_{q}(z) = \mathcal{N}^{p}_{q} z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

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Fitted Parameters Value	
Nu	$0.47\ ^{+0.32}_{-0.20}$
Nd	$-0.32\ ^{+0.13}_{-0.13}$
Ns	$-0.57\ ^{+0.29}_{-0.43}$
Nsea	$-0.27\ ^{+0.12}_{-0.20}$
α_s	$2.30\ ^{+1.08}_{-0.91}$
β_{sea}	$2.60\ ^{+2.60}_{-1.74}$
β_u	$3.50\ ^{+2.33}_{-1.82}$
$< p_\perp^2 >_{pol}$	$0.10\ ^{+0.02}_{-0.02}$

$$\Delta D^h_{S_Y/q}(z) = \mathcal{N}^p_q(z) D_{h/q}(z)$$

$$\mathcal{N}^p_q(z) = N^p_q z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

Full data Fit : $\chi^2_{dof} = 1.94$

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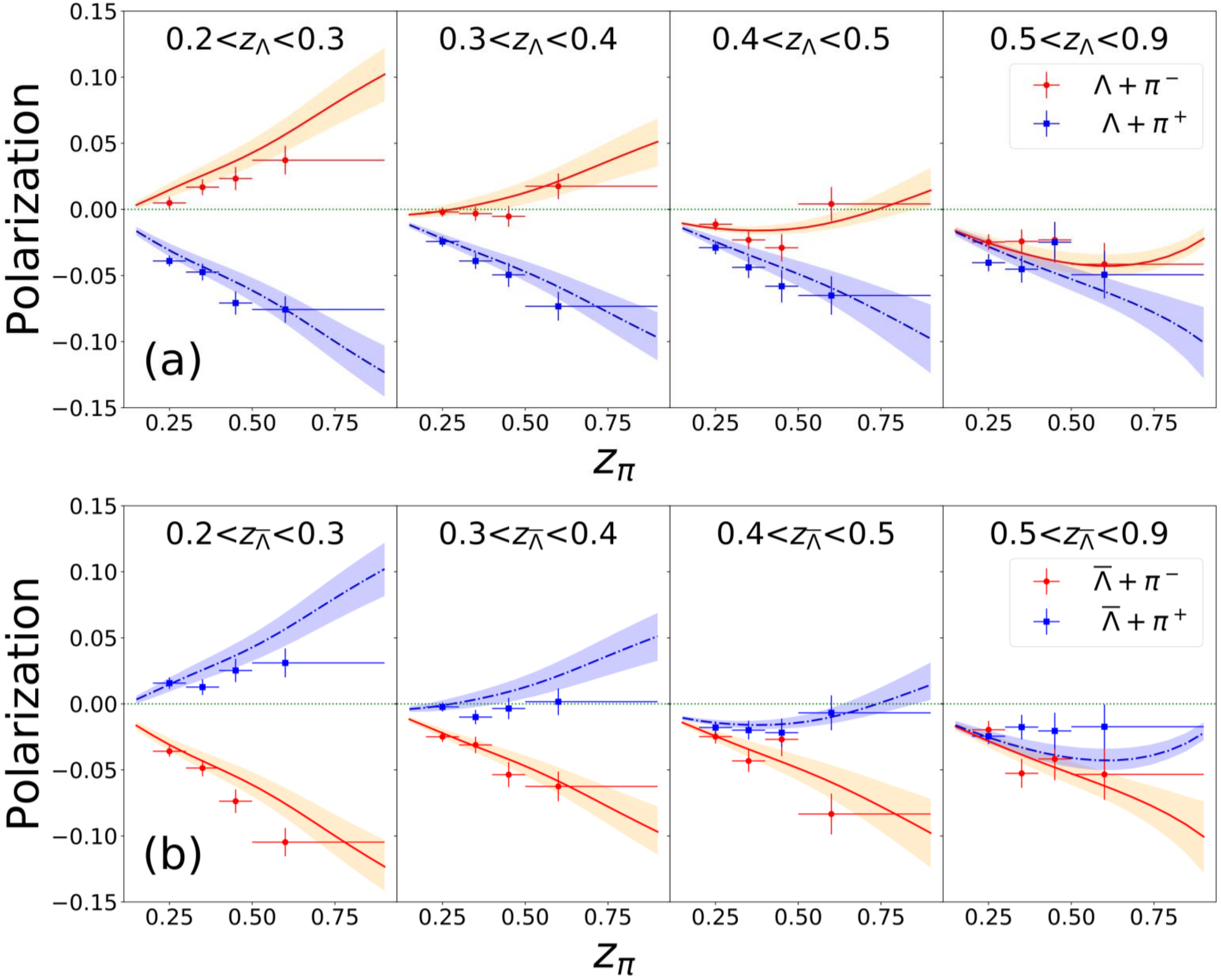
Full data Fit : $\chi^2_{dof} = 1.94$

$\Lambda + \pi/K$ data Fit : $\chi^2_{dof} = 1.26$

Published on [Phys. Rev. D 102, 054001 (2020)]

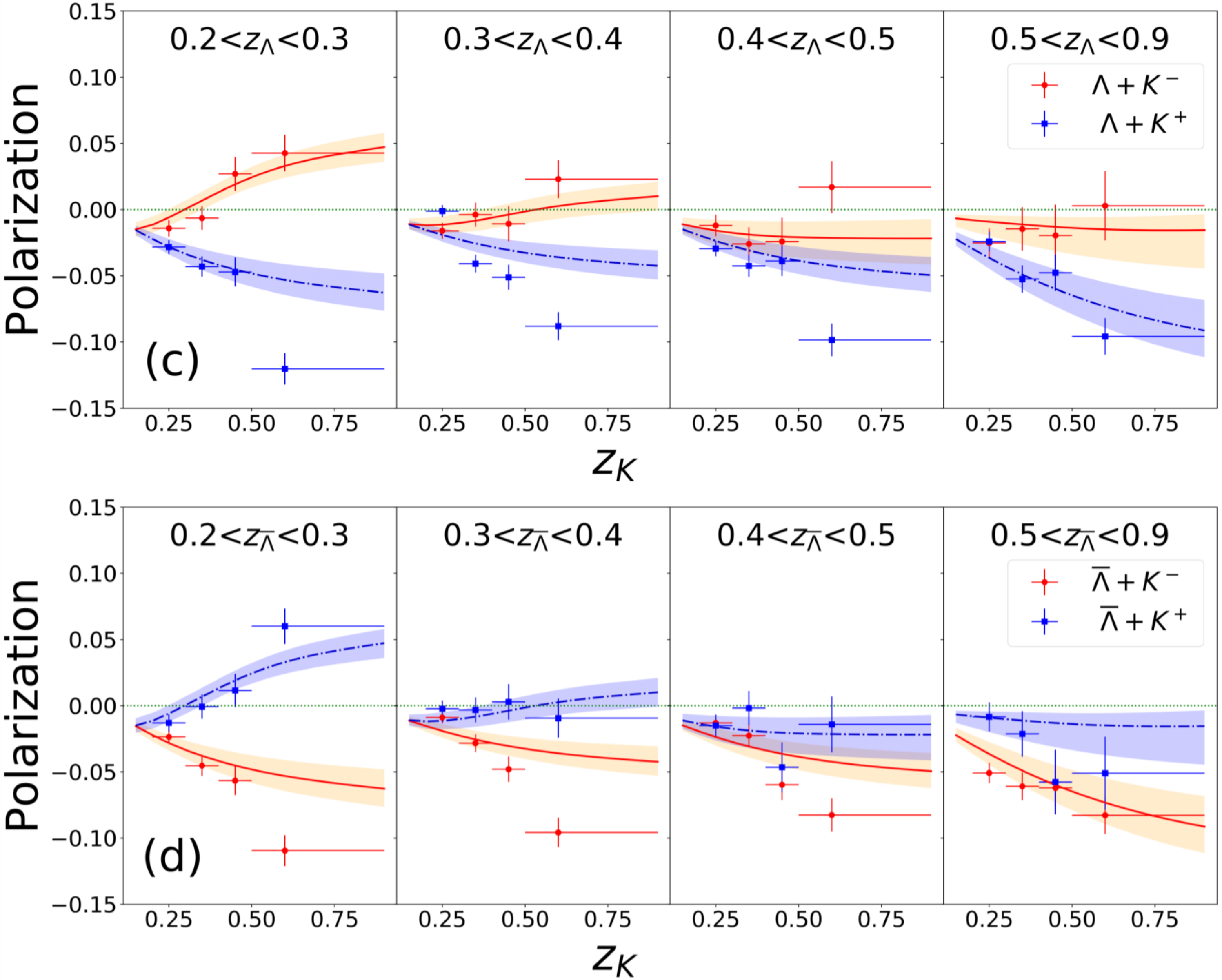
Lambda-pion



Bin excluded
$z_\pi = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

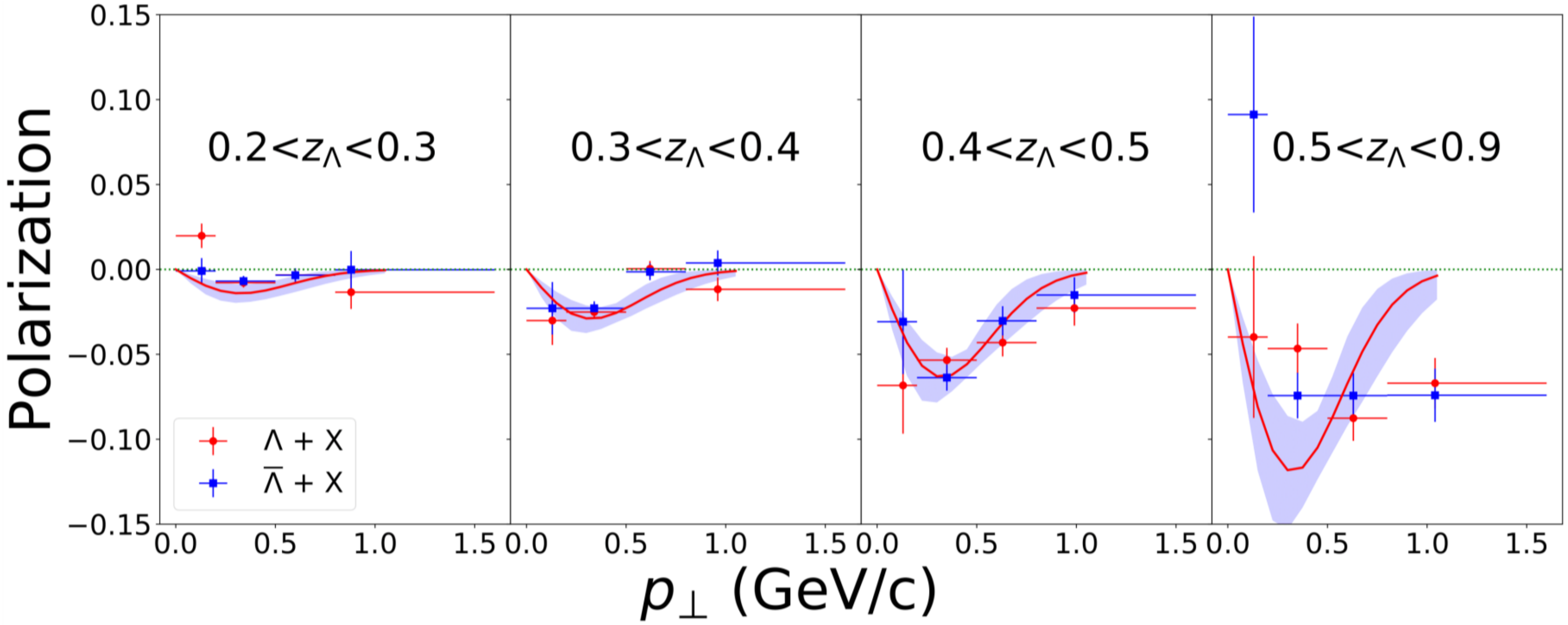
Lambda-kaon



Bin excluded
$z_K = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

Lambda-jet



Bin excluded
$z_\Lambda = [0.5 \ - 0.9]$

$\chi^2_{dof} = 1.94$

Expected features:

- $P_T = 0$ when $p_\perp = 0$;
- $P_T(\Lambda) = P_T(\bar{\Lambda})$.

Fit with TMD evolution

PRELIMINARY WORK

Polarization 2-h: double hadron production

$$P_T^h(z_1, z_2) = \frac{\int d^2\mathbf{q}_T \, F_{TU}^{\sin(\phi_1 - \phi_S)}}{\int d^2\mathbf{q}_T \, F_{UU}} = \frac{M_{h_1} \int d^2\mathbf{q}_T \, \mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)} \, \widetilde{\widetilde{D}}\right]}{\int d^2\mathbf{q}_T \, \mathcal{B}_0\left[\widetilde{D}\widetilde{\widetilde{D}}\right]}$$

Polarization 2-h: double hadron production

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$$\begin{aligned}\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] &= \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T^2 J_1(q_T\,b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)\\ &\times\exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*})+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu'))-\ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}\\ &\times M_{D_1}^{\perp}(b_T)\,M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] &= \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T J_0(b_T\,q_T)\widetilde{D}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)\\ &\times\exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*})+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu'))-\ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}\\ &\times M_{D_1}(b_T)\,M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}\end{aligned}$$

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Polarization 2-h: double hadron production

$$P_T^h(z_1, z_2) = \frac{\int d^2\mathbf{q}_T\, F_{TU}^{\sin(\phi_1-\phi_S)}}{\int d^2\mathbf{q}_T\, F_{UU}} = \frac{M_{h_1}\int d^2\mathbf{q}_T\, \mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}\right]}{\int d^2\mathbf{q}_T\, \mathcal{B}_0\left[\widetilde{D}\,\widetilde{\bar{D}}\right]}$$

$$\mathcal{N}_q^p(z) = N_q^p\, z^{\alpha_q}(1-z)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

- Times unpolarized Lambda FF

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T^2 J_1(q_T\,b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times\exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
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$$\mathcal{N}_q^p(z) = \mathcal{N}_q^p z^{\alpha_q}(1-z)^{\beta_q}\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

- Times unpolarized Lambda FF

- Gauss model
- Power Law model

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
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$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right]=\mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T^2 J_1(q_T b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times\exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*})+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu'))-\ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\,M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

DSS FF for π/K
AKK FF for Λ

- Gauss model
- Power Law model

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right]=\mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T J_0(b_T q_T)\widetilde{D}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times\exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*})+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu'))-\ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}(b_T)\,M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

Polarization 2-h: double hadron production

$$P_T^h(z_1,z_2) = \frac{\int d^2\mathbf{q}_T\, F_{TU}^{\sin(\phi_1-\phi_S)}}{\int d^2\mathbf{q}_T\, F_{UU}} = \frac{M_{h_1}\int d^2\mathbf{q}_T\, \mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}\right]}{\int d^2\mathbf{q}_T\, \mathcal{B}_0\left[\widetilde{D}\,\widetilde{\bar{D}}\right]}$$

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T) \exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

Gaussian model

$$\mathcal{B}_0\left[\widetilde{D}\,\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}(b_T)\, M_{D_2}(b_T) \exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{N}_q^p(z) = N_q^p\, z^{\alpha_q}(1-z)^{\beta_q}\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

- Times unpolarized Lambda FF

DSS FF for π/K
AKK FF for Λ

- Gauss model
- Power Law model

Polarization 2-h: double hadron production

$$P_T^h(z_1,z_2) = \frac{\int d^2\mathbf{q}_T\, F_{TU}^{\sin(\phi_1-\phi_S)}}{\int d^2\mathbf{q}_T\, F_{UU}} = \frac{M_{h_1}\int d^2\mathbf{q}_T\,\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right]}{\int d^2\mathbf{q}_T\,\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right]}$$

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1,b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2,b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{N}_q^p(z) = \mathcal{N}_q^p z^{\alpha_q}(1-z)^{\beta_q}\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

- Times unpolarized Lambda FF

DSS FF for π/K
AKK FF for Λ

Gaussian model

- Gauss model
- Power Law model

Different $g_K(b_T)$ functions

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\overline{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2\, J_1(q_T\, b_T)\,\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\,\widetilde{\overline{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\,\widetilde{\overline{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T\, J_0(b_T\, q_T)\,\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\,\widetilde{\overline{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_0(b_T\, q_T) = \frac{q_{T_{max}}}{b_T} J_1(b_T\, q_{T_{max}})$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_0(b_T\, q_T) = \frac{q_{T_{max}}}{b_T} J_1(b_T\, q_{T_{max}})$$

$$q_{Tmax} = Q * \eta$$

η = [0,2 – 0,3]

Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2p_\perp}{d\sigma/dz_1 d^2p_\perp}$$

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$$\frac{d\Delta\sigma}{dz_1 d^2p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T\, q_T) \widetilde{D}_{1T}^{\perp(1)}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

$$\frac{d\sigma}{dz_1 d^2p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T\, q_T) \widetilde{D}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}(b_T)\, \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2 p_\perp}{d\sigma/dz_1 d^2 p_\perp}$$

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$$\frac{d\Delta\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T\, q_T) \widetilde{D}_{1T}^{\perp(1)}(z_1, \mu_b) U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}^{\perp}(b_T) \exp\left\{ -g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

$$\frac{d\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T\, q_T) \widetilde{D}(z_1, \mu_b) U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}(b_T) \exp\left\{ -g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2p_\perp}{d\sigma/dz_1 d^2 p_\perp}$$

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$$\frac{d\Delta\sigma}{dz_1 d^2p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(b_T\, q_T)\, \widetilde{D}_{1T}^{\perp(1)}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}} \right) \right\}$$

$$U_{NG}(\mu_{b_*}, Q) = \exp\left[- C_A C_F \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu^c)} \right]$$

M. Dasgupta, G.P. Salam, Phys. Lett. B 512 (2001) 323

$$\frac{d\sigma}{dz_1 d^2p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\, \widetilde{D}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}(b_T)\, \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}} \right) \right\}$$

$$u = \frac{1}{\beta_0} \ln\left[\frac{\alpha_s(\mu_b)}{\alpha_s(Q)} \right]$$

$g_K(b_T)$: 2 hadrons

$$\exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}} \right) \right\}$$

$g_K(b_T)$: 2 hadrons

$$\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$g_2 = 0{,}84$
$Q_0^2 = 2{,}4\ GeV^2$

$$g_2 \ln\left(\frac{b_T}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)$$ [1]

$$\frac{\alpha_s(C_1/b_*)C_F}{\pi}\ln(1+b_T^2/b_{max}^2)\ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

$$g_0(b_{max})\left(1-\exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{max})b_{max}^2}\right]\right)\ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

$g_0 = 0{,}55$
$b_{max} = 0{,}8$

$$\frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*})\ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

[1] C.A. Aidala, B. Field, L.P. Gamberg, T.C. Rogers, Phys. Rev. D 89 (2014) 094002
P. Sun, J. Isaacson, C.P. Yuan, F. Yuan, Int. J. Mod. Phys. A 33 (2018) 1841006

[2] J. Collins, T. Rogers,Phys.Rev.D 91 (2015) 7, 074020

$g_K(b_T)$: 2 hadrons

$$g_2 = 0{,}84$$
$$Q_0^2 = 2{,}4\ GeV^2$$

$$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$$ [1]

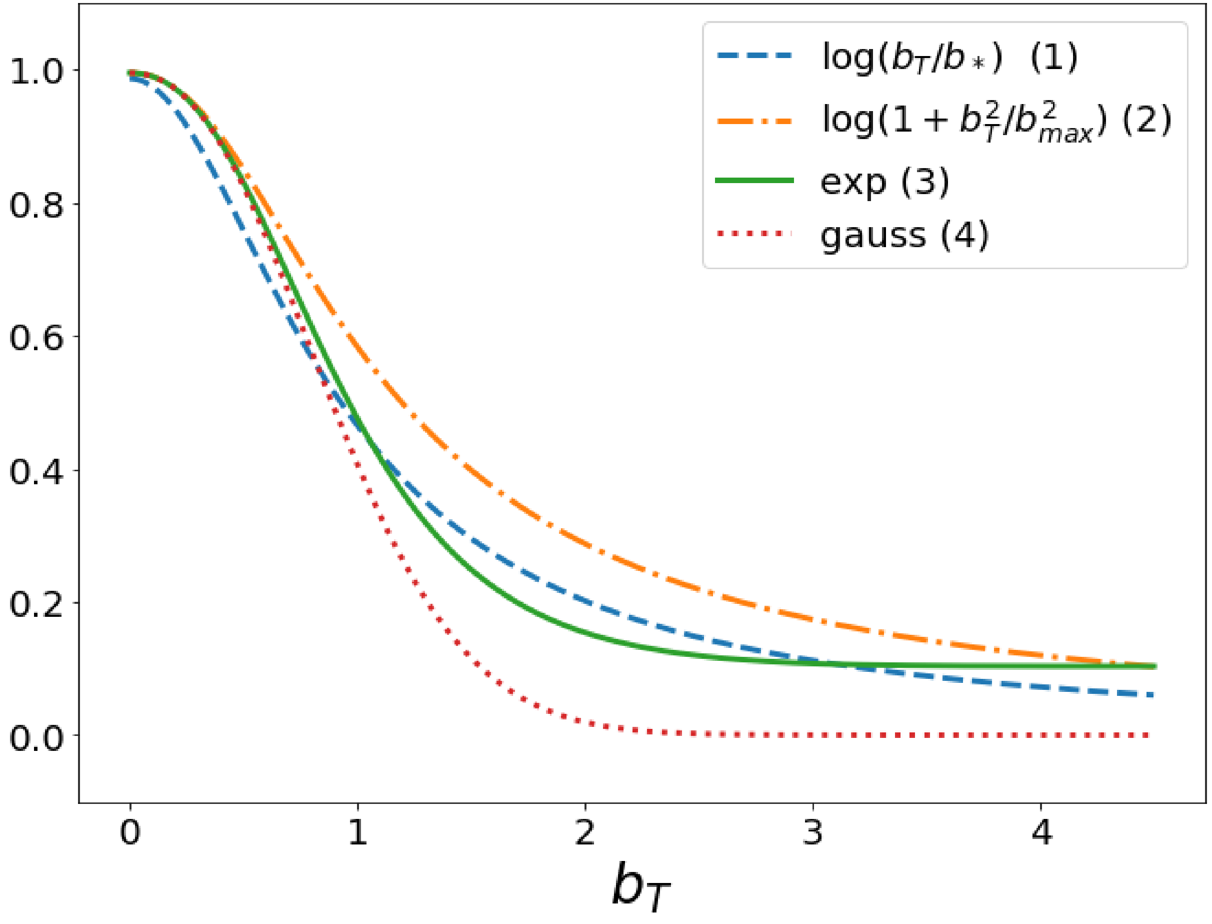
$$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

$$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{max})b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

$$g_0 = 0{,}55$$
$$b_{max} = 0{,}8$$

$$\frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1}M_{h2}}\right)$$ [2]

$$\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$



[1] C.A. Aidala, B. Field, L.P. Gamberg, T.C. Rogers, Phys. Rev. D 89 (2014) 094002
P. Sun, J. Isaacson, C.P. Yuan, F. Yuan, Int. J. Mod. Phys. A 33 (2018) 1841006

[2] J. Collins, T. Rogers, Phys.Rev.D 91 (2015) 7, 074020

Preliminary results: 2-h data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points

Fitted 8 parameters

$b_{max} = 0{,}8$

Preliminary results: 2-h data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points

Fitted 8 parameters

$b_{max} = 0{,}8$

$$M_D(b_T) = \exp\left(-\frac{\langle p_\perp^2 \rangle b_T^2}{4 z_p^2}\right)$$

Gaussian model		
$g_K(b_T, b_{max}) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$	1,286	0,27
$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,363	0,27
$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,286	0,27
$\frac{C_F}{\pi} \frac{b_T^2}{b_{max}^2} \alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,264	0,27

Preliminary results: 2-h data Fit

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \; - 0.9]$ bin excluded → 96 data points

Fitted 8 parameters

$b_{max} = 0{,}8$

$$M_D(b_T) = \exp\left(- \frac{\langle p_\perp^2 \rangle b_T^2}{4 z_p^2} \right)$$

$$M_D^\perp(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m)$$

Gaussian model		
$g_K(b_T, b_{max}) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$	1,286	0,27
$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,363	0,27
$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,286	0,27
$\frac{C_F}{\pi} \frac{b_T^2}{b_{max}^2} \alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,264	0,27

Power Law model		
$g_K(b_T, b_{max}) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$	1,293	0,27
$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,368	0,27
$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,323	0,27
$\frac{C_F}{\pi} \frac{b_T^2}{b_{max}^2} \alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,283	0,27

Preliminary results: both data set Fit

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5\ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.2\ - 0.3]$ bin excluded → 23 data points

Preliminary results: both data set Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \; - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.2 \; - 0.3]$ bin excluded → 23 data points

Power Law Model

$$M_D^{\perp}(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m)$$

m = 1 (fixed)
p to be fitted

Preliminary results: both data set Fit

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- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \; - 0.9]$ bin excluded → 96 data points
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Power Law Model

$$M_D^{\perp}(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m)$$

m = 1 (fixed)
p to be fitted

$$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$$

$g_2 = 0{,}84$
$Q_0^2 = 2{,}4 \; GeV^2$
Q=10,58 GeV

Fitted 8 parameters

Preliminary results: both data set Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.2 \ - 0.3]$ bin excluded → 23 data points

Power Law Model

$$M_D^{\perp}(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m)$$

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$$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$$

$g_2 = 0{,}84$

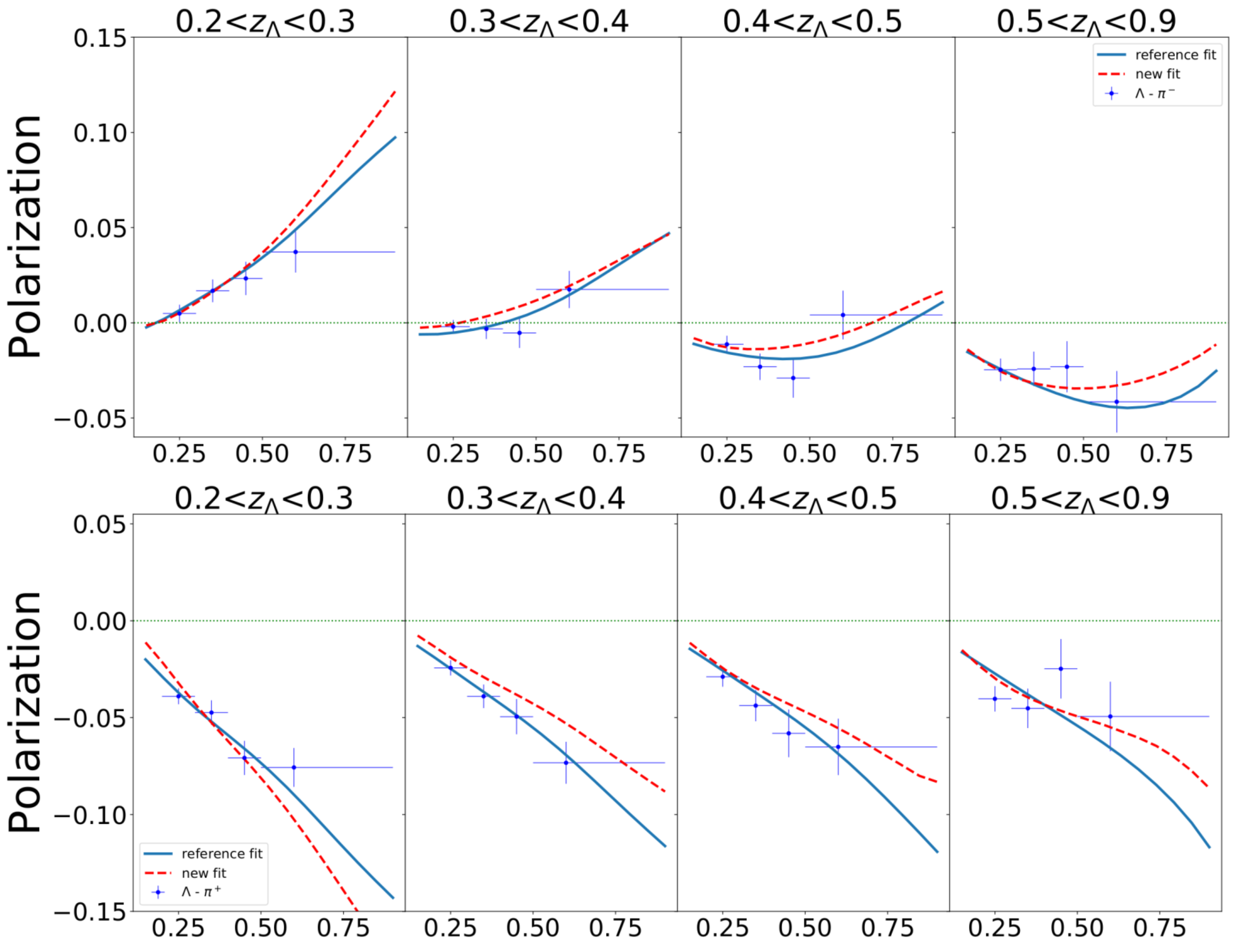
$Q_0^2 = 2{,}4\ GeV^2$

Q=10,58 GeV

Fitted 8 parameters

$\chi^2_{dof} = 1.58$

Lambda – pion: fit comparison



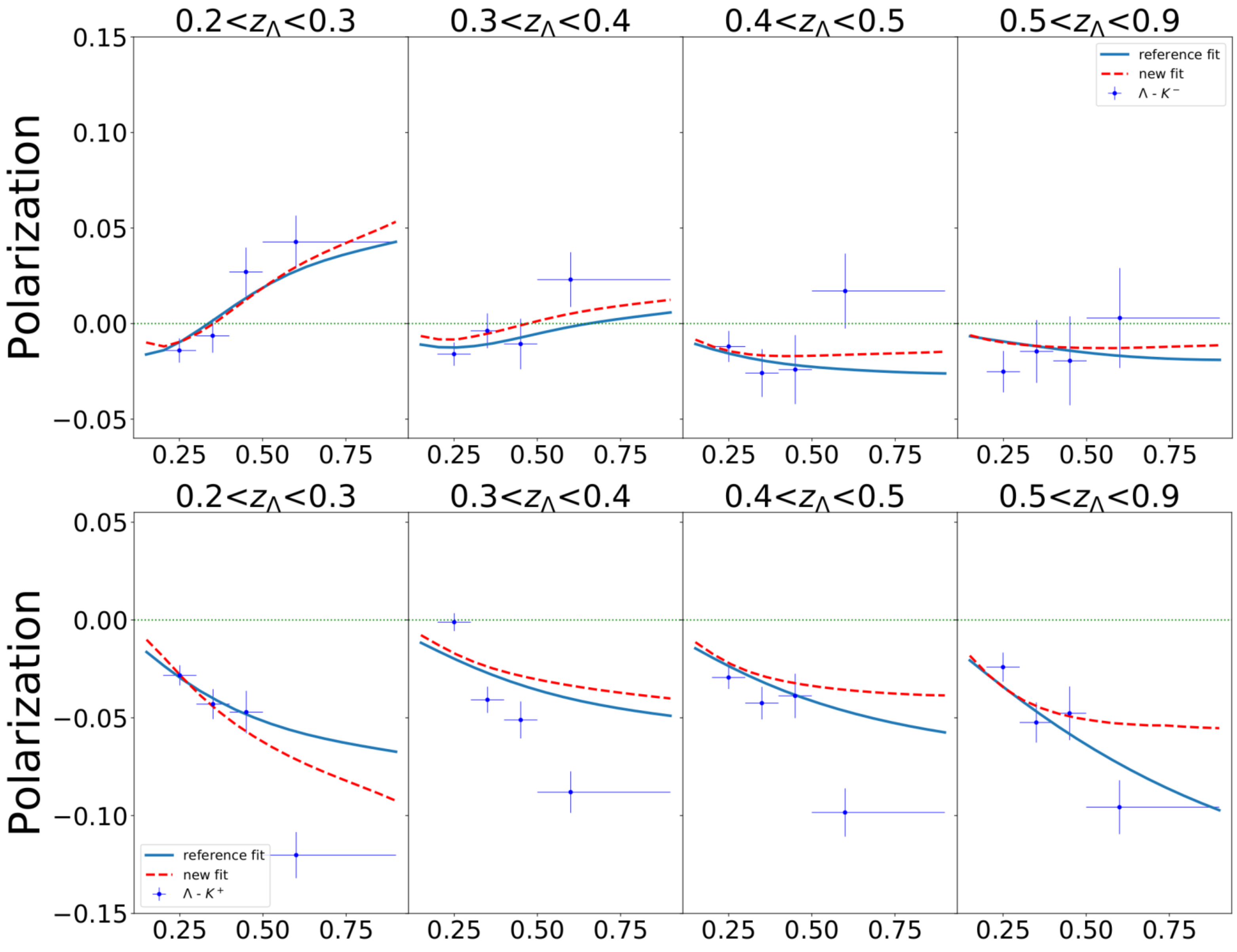
Bin excluded

$z_{\pi} = [0.5 \ - 0.9]$

Power Law Model

$\chi^2_{dof} = 1.58$

Lambda – kaon: fit comparison



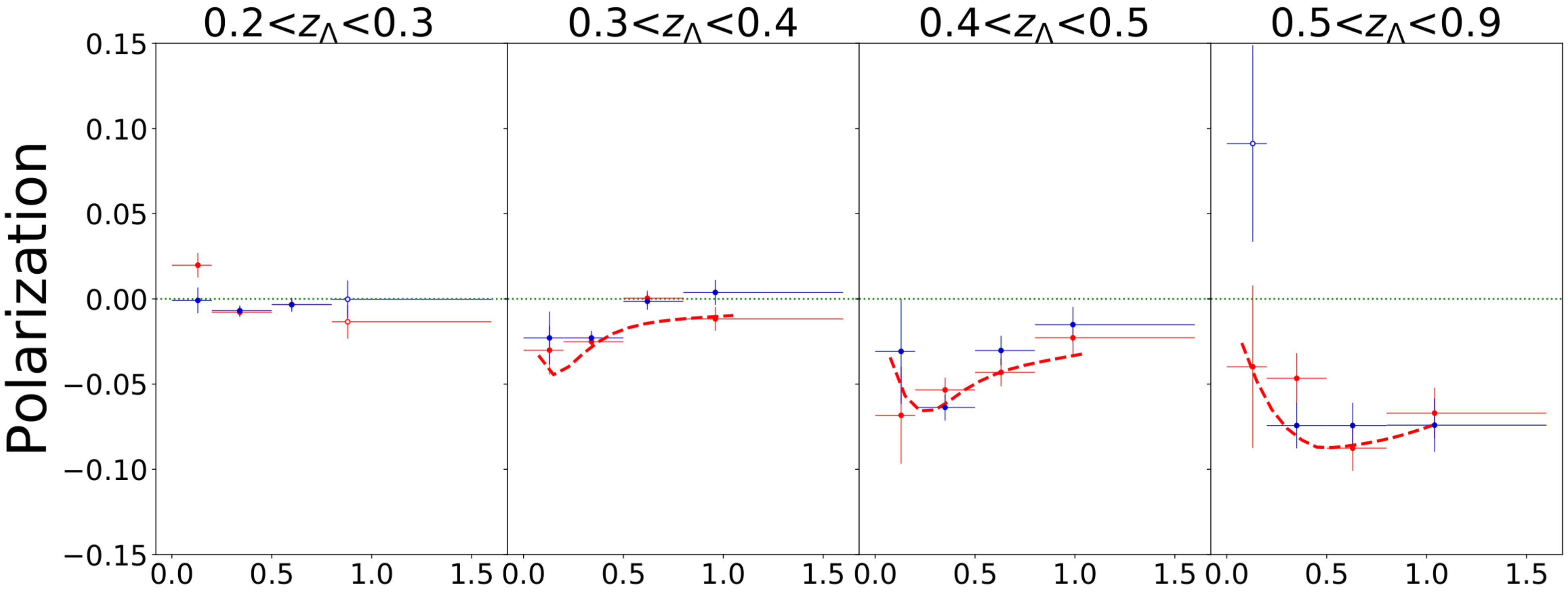
Bin excluded

$z_K = [0.5 \ - 0.9]$

Power Law Model

$\chi^2_{dof} = 1.58$

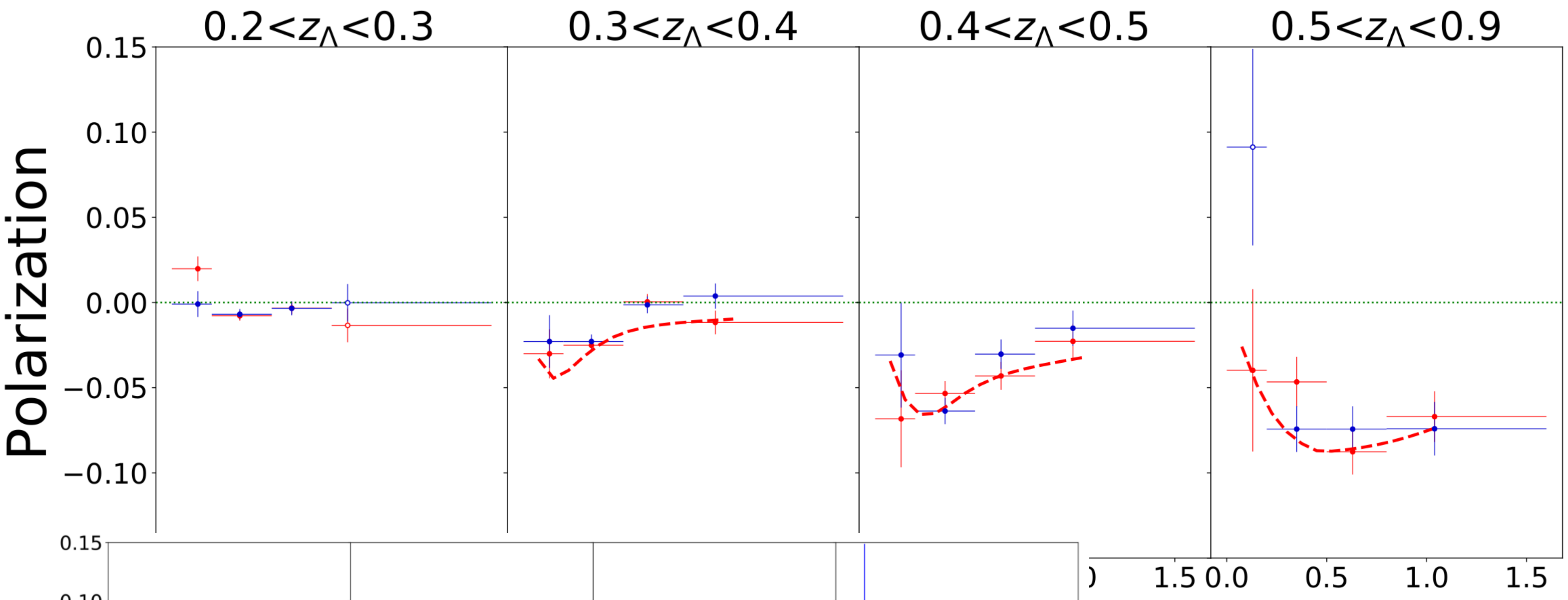
Lambda - thrust: fit comparison

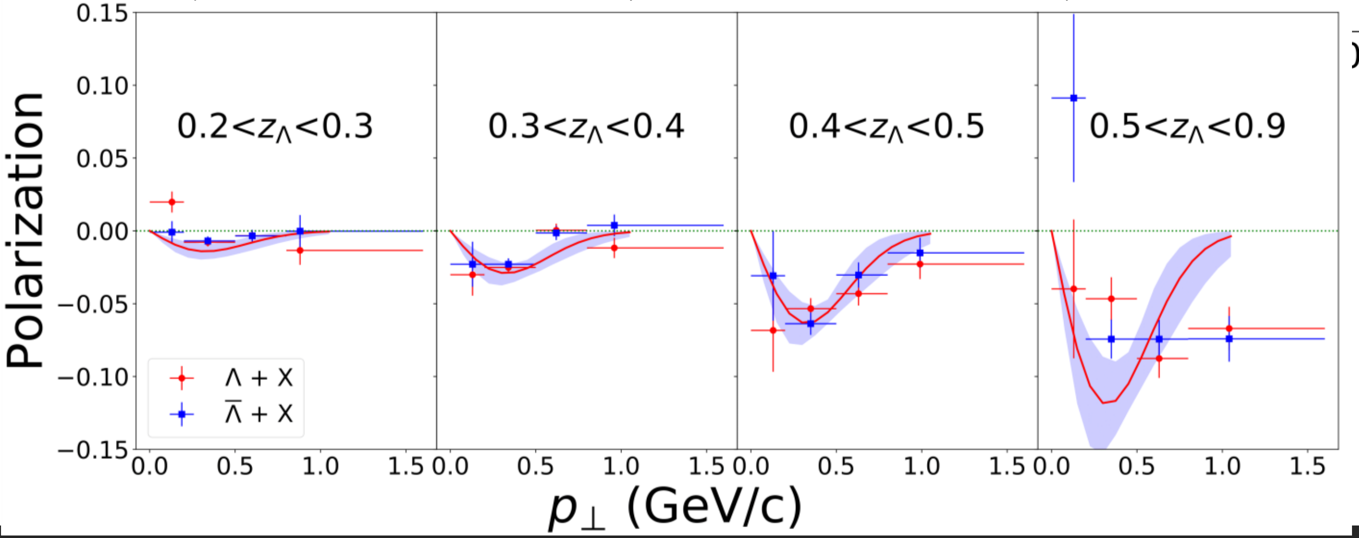


Bin excluded
$z_{\Lambda} = [0.2 \ - 0.3]$

$\chi^2_{dof} = 1.58$

Lambda - thrust: fit comparison



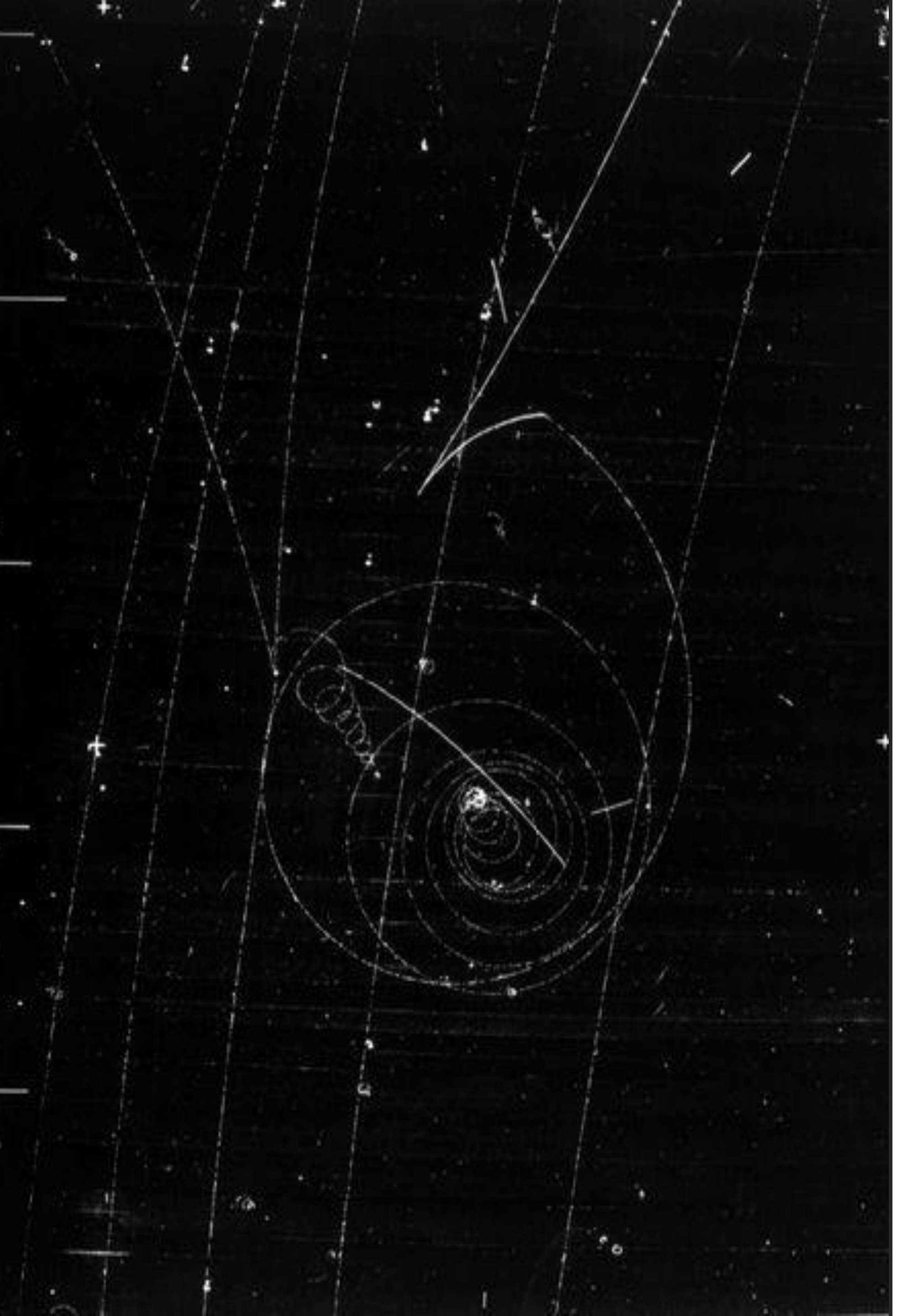


Bin excluded
$z_\Lambda = [0.2 - 0.3]$

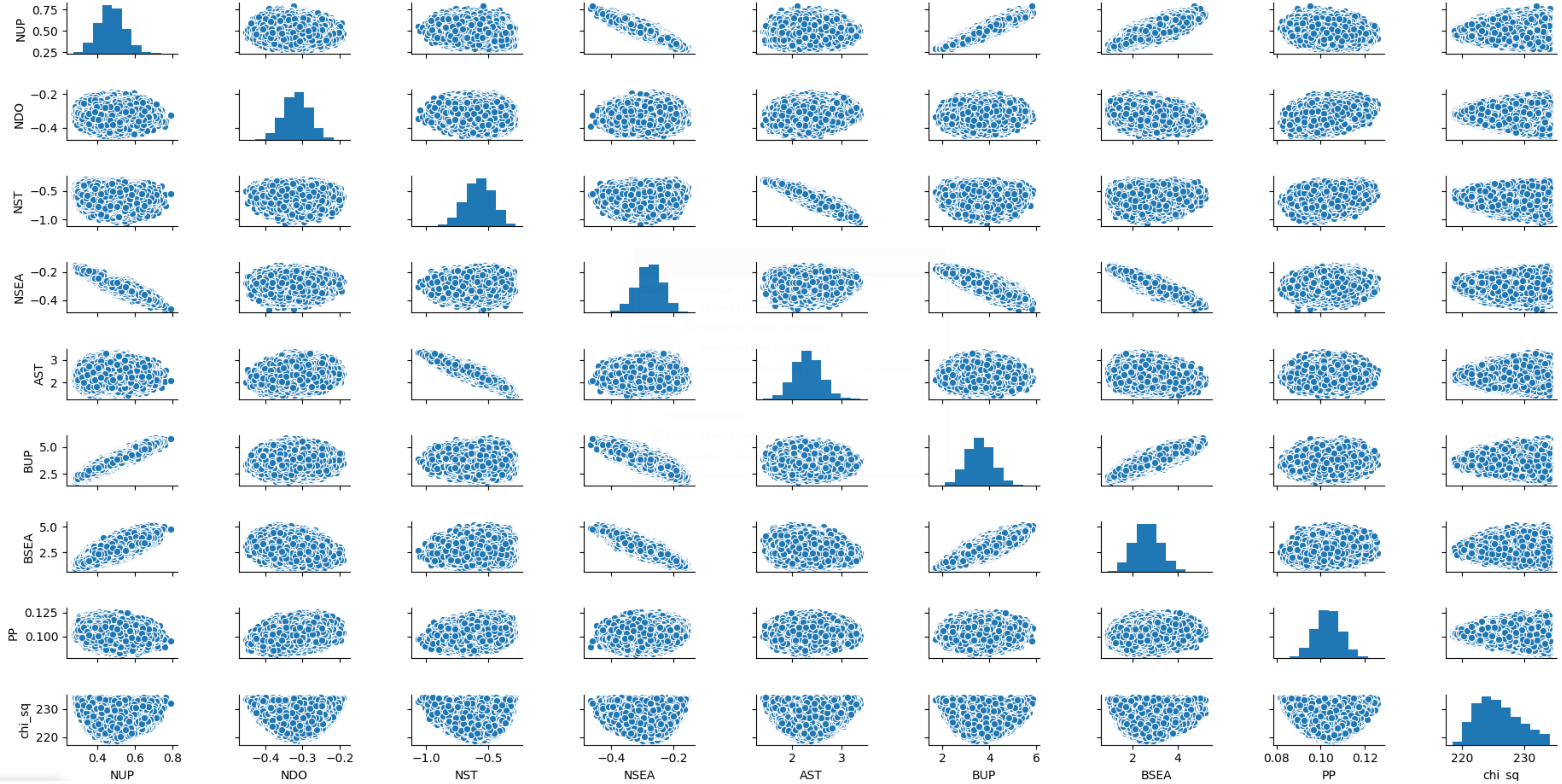
$\chi^2_{dof} = 1.58$

Conclusions

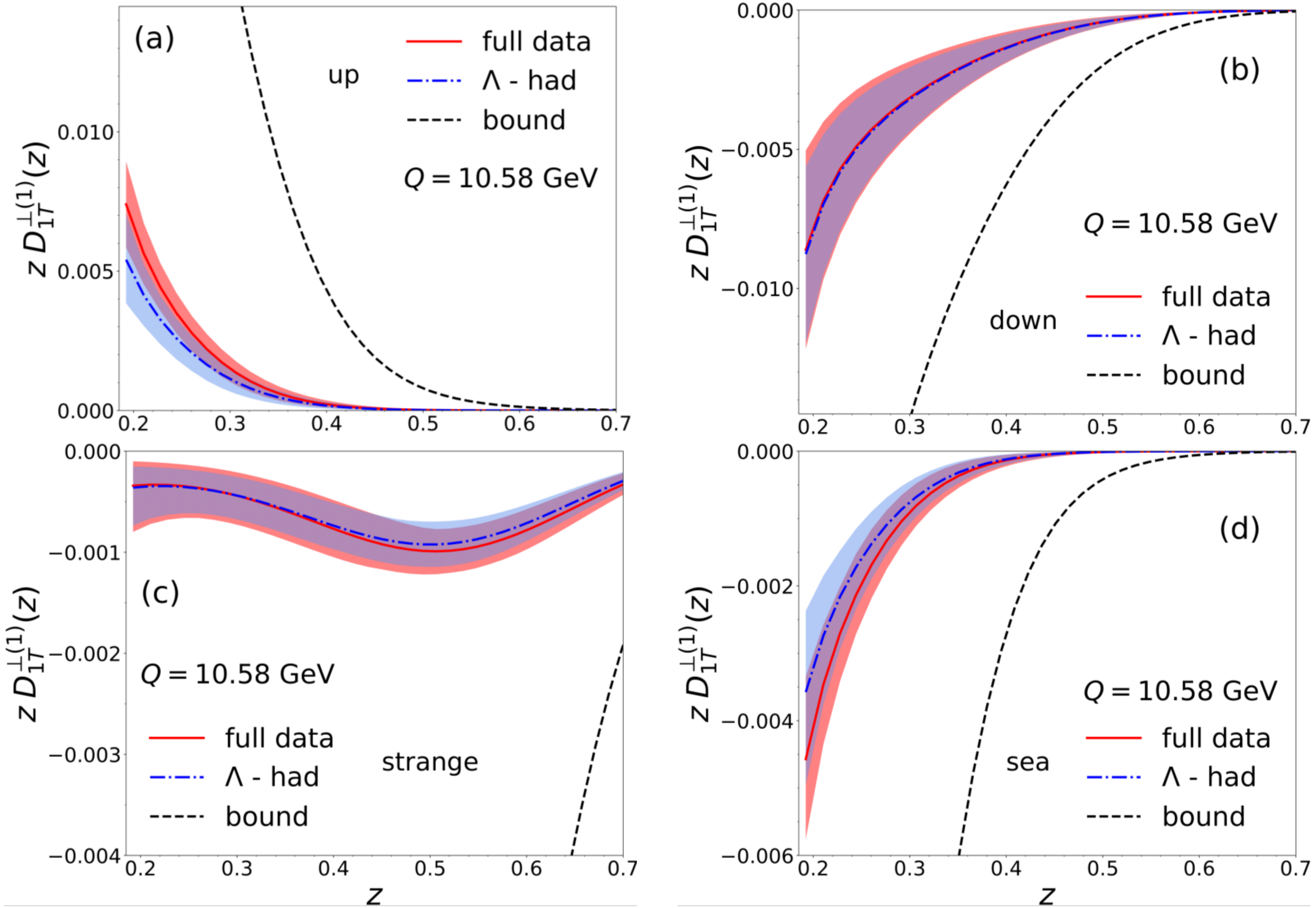
- First extraction of the Λ polarizing fragmentation function from Belle e^+e^- data;
- Clear separation in flavours: three different valence pFF needed;
- Preliminary results using TMD evolution;
- Fit of 2-h data with different models and g_K;
- Fit of 2-h and 1-h data with Power Law model;
- Comparison with previous fit.



Backup Slides


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
chi_sq

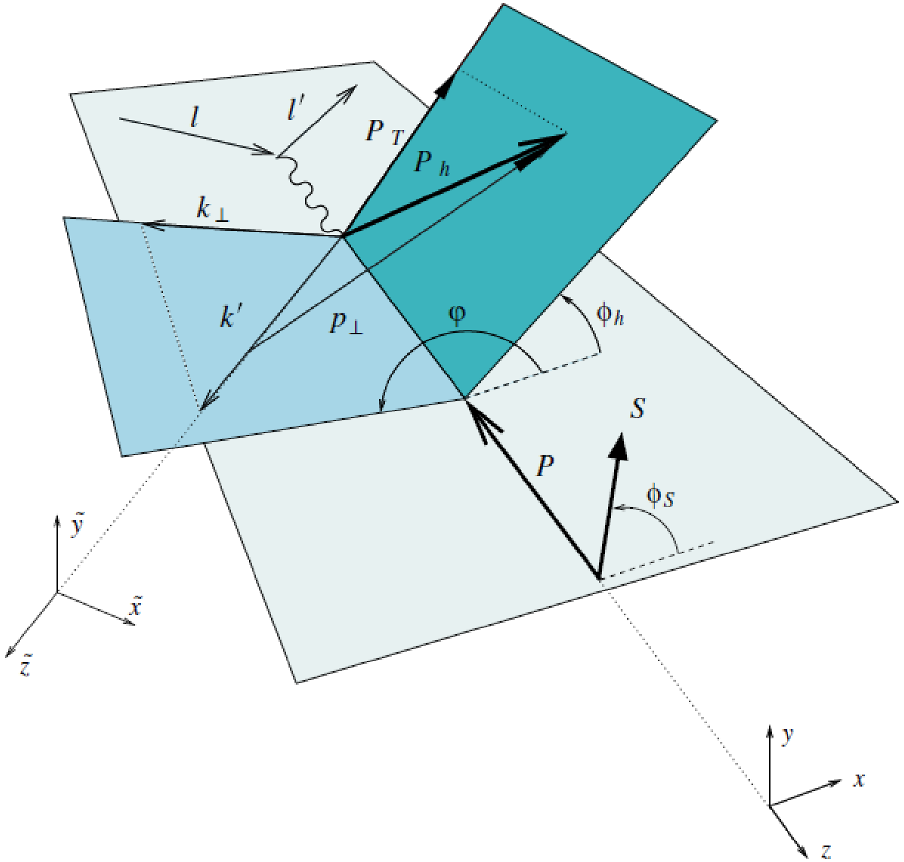
First moments



$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$

SiDIS – Polarized Lambda Production

$e^- P \rightarrow e^- \Lambda$



$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p} \frac{< p_\perp^2 >_p^2}{< p_\perp^2 >} \frac{1}{\sqrt{< p_\perp^2 >_p + \xi_p^2 < k_\perp^2 >}} \times \frac{\sum_q f_{q/P}(x_B) \Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

Proton PDF

CTEQ6L1

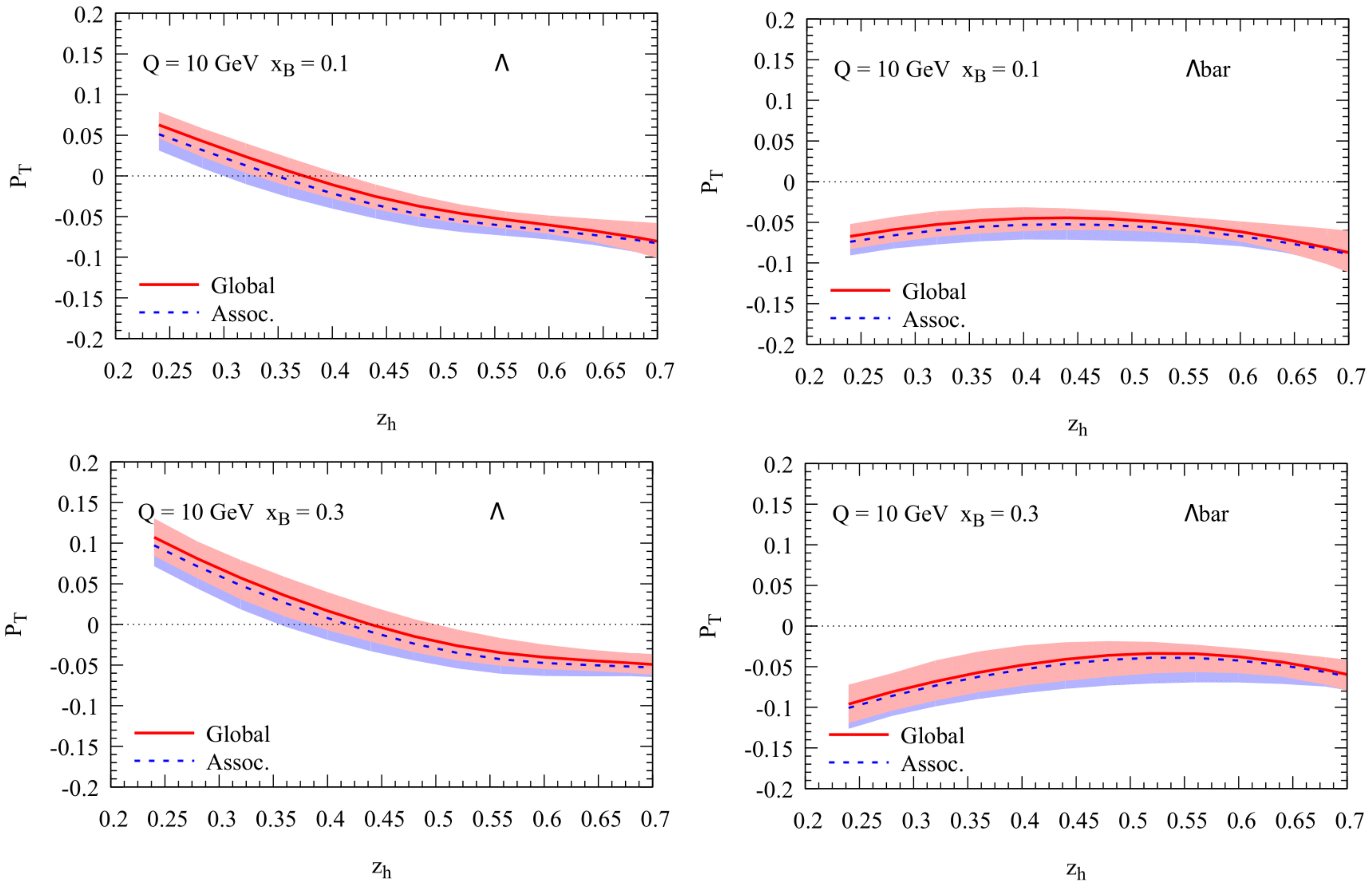
Lambda Polarizing FF

x_B Bjorken-x

z_h energy fraction

$$\xi_p = z_h \left(1 - \frac{m_h^2}{z_h^2 Q^2} \frac{x_B}{1 - x_B}\right)$$

SiDIS – Polarized Lambda Production



Prediction for the Λ polarization:

- x_B = 0.1
- x_B = 0.3

SiDIS – Polarized Lambda Production

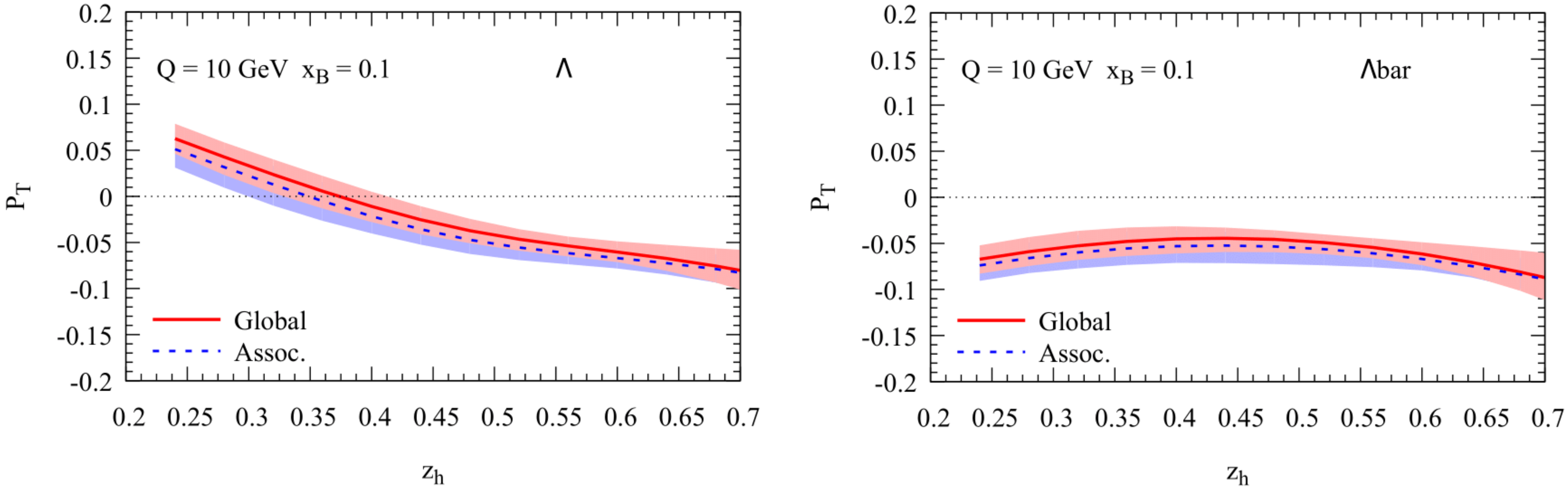
$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p}\frac{< p_\perp^2 >_p^2}{< p_\perp^2 >}\frac{1}{\sqrt{< p_\perp^2 >_p + \xi_p^2 < k_\perp^2 >}} \times \frac{\sum_q f_{q/P}(x_B)\Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

Kinematical configuration compatible with EIC

Prediction for the Λ polarization:

- x_B = 0.1

$$\xi_p = z_h\left(1 - \frac{m_h^2}{z_h^2 Q^2}\frac{x_B}{1-x_B}\right)$$



$g_K(b_T)$: 1 hadron

$$\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right)\right\}$$

$g_2 = 0{,}84$
$Q_0^2 = 2{,}4\ GeV^2$

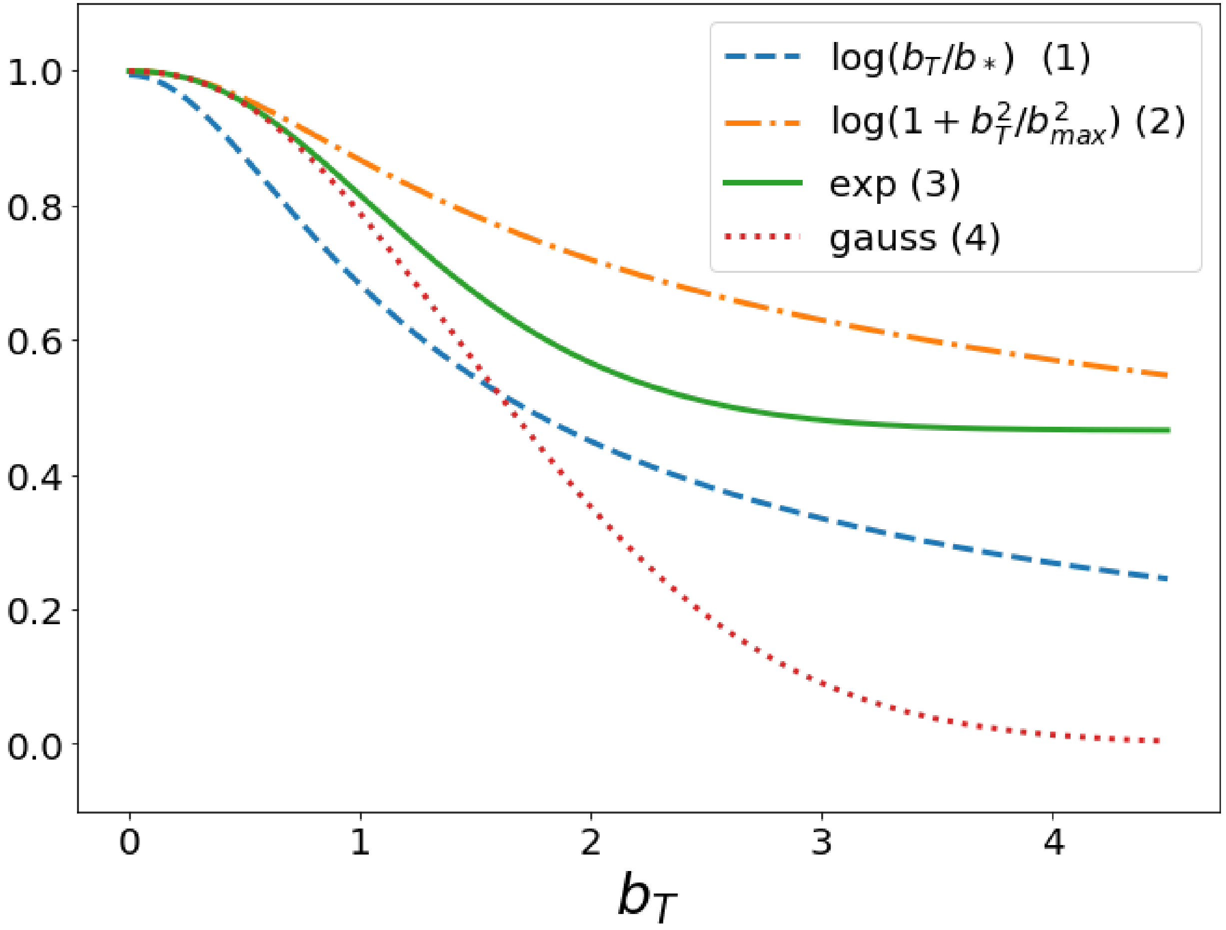
$$\frac{g_2}{2}\ln\left(\frac{b_T}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)$$

$$\frac{\alpha_s(C_1/b_*)C_F}{\pi}\ln(1+b_T^2/b_{max}^2)\ln\left(\frac{Qz_1}{M_{h1}}\right)$$

$$g_0(b_{max})\left(1-\exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{max})b_{max}^2}\right]\right)\ln\left(\frac{Qz_1}{M_{h1}}\right)$$

$g_0 = 0{,}55$
$b_{max} = 0{,}8$

$$\frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*})\ln\left(\frac{Qz_1}{M_{h1}}\right)$$



Statistical Uncertainty Band

Multivariate Normal Distribution

MINUIT:
- Best fit parameters
- Covariance matrix
- Minimum Chi-square χ^2

$$\mu: \mathcal{N}_q^p \, \alpha_q \, \beta_q \, \langle p_\perp^2 \rangle_p \quad \Sigma \quad \longrightarrow \quad \mu \ , \ \Sigma$$

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Generate a random set of parameter

$x: (\mathcal{N}_q^p \ \ \alpha_q \ \ \beta_q \ \ \langle p_\perp^2 \rangle_p \ )$ $\longrightarrow$

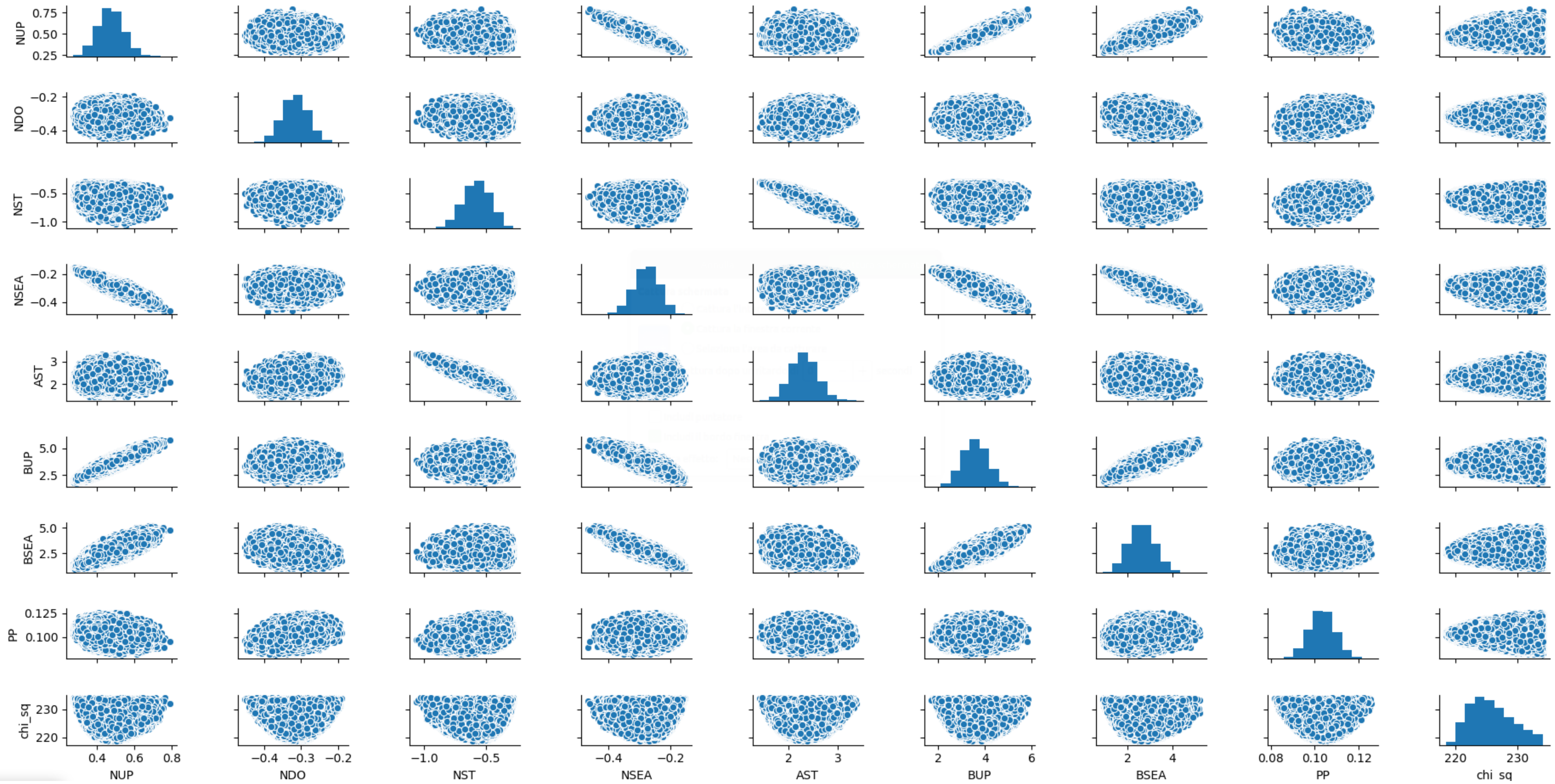
Calculate their own Chi-square

χ'^2 $\longrightarrow$

Keep set if

$$\chi^2 \leq \chi'^2 \leq \chi^2 + \Delta\chi^2$$

- Minimum Chi-square: χ^2 = 232,8
- Confidence interval $2\sigma \rightarrow$ 95,5% : $\Delta\chi^2$ = 15,79 for 8 parameters (χ^2 -distribution)


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
chi_sq

$e^+e^- \to h^\uparrow{}_1 h_2 X$

h_1, h_2 unpolarized

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$

$$= \frac{6e^4 e_q^2}{64\pi\hat{s}}\Big\{D_{h_1/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$

$$+\frac{1}{4}\sin^2\theta\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

Unpolarized FFs

Collins FFs

h_1 polarized: X , h_2 unpolarized

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$

$$= \frac{3e^4 e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_X/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

Collins FF

FF h_1 transv. Pol.

h_1 polarized: Y , h_2 unpolarized

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$

Polarizing FF

Unpolarized FF

$$= \frac{6e^4 e_q^2}{64\pi\hat{s}}\Big\{\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$

$$+\frac{1}{2}\sin^2\theta\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

Collins FF

FF h_1 transv. Pol.

h_1 polarized: Z , h_2 unpolarized

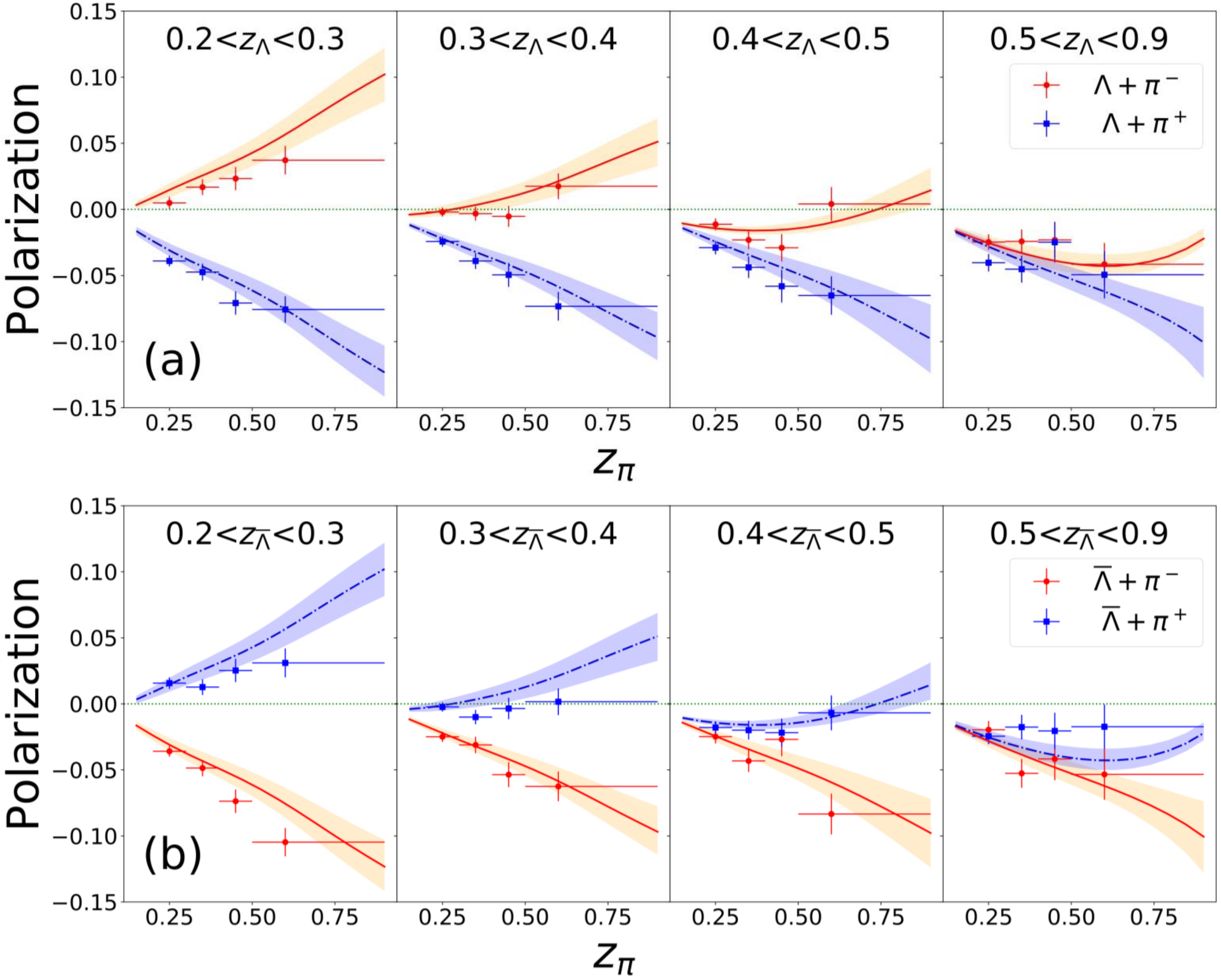
$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$

$$= \frac{3e^4 e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_Z/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

FF h_1 long. Pol.

Collins FF

Lambda-pion



Data fitted:

- $z_\pi < 0.50$

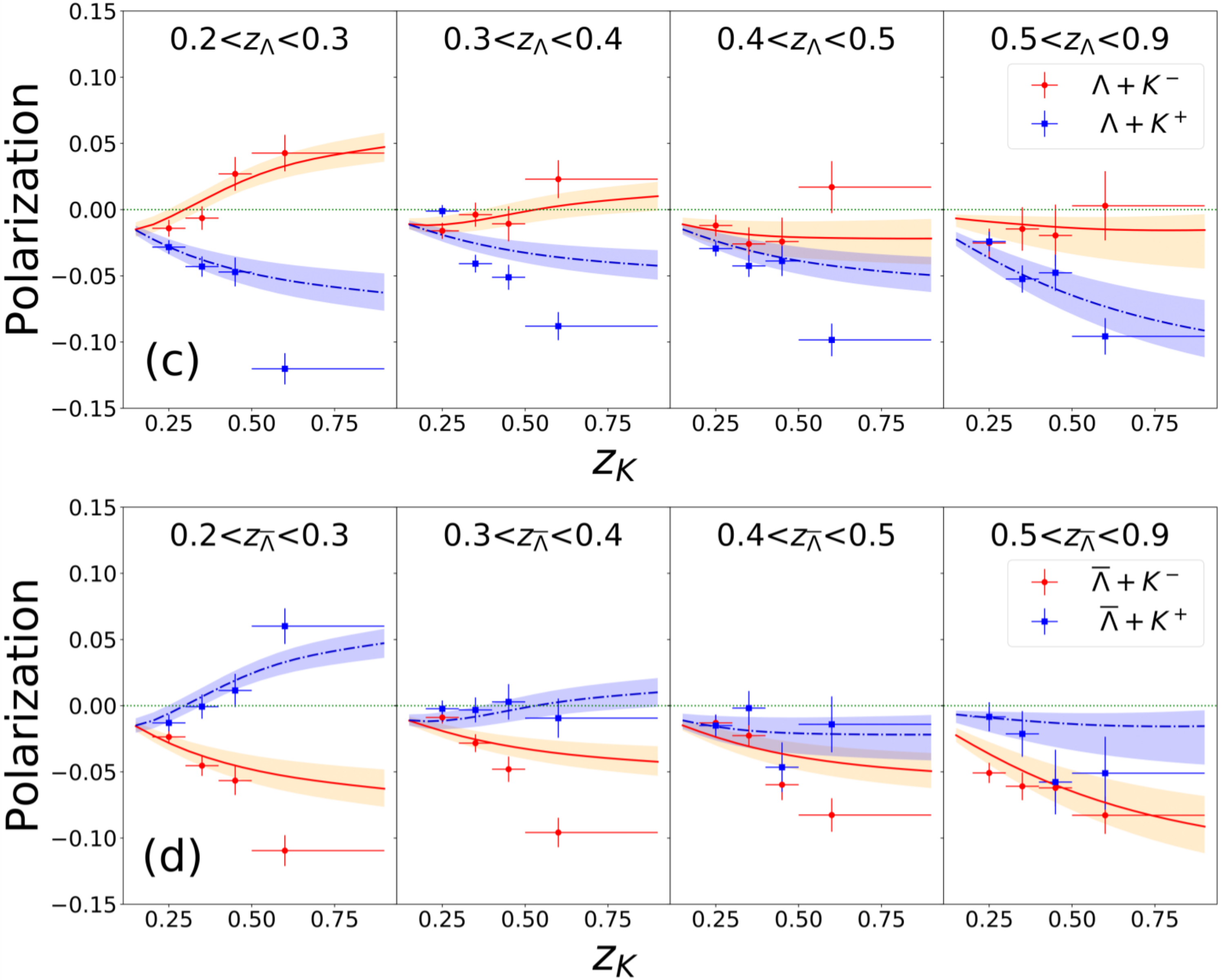
$\chi^2_{dof} = 1.94$

Lambda pion

- Charge-conjugation symmetry $P_n(\Lambda\pi^+) = P_n(\bar{\Lambda}\pi^-)$
- Data give information on the pFFs for u and d
- $P_n(\Lambda\pi^+)$ negative, dominated by down pFF
- $P_n(\Lambda\pi^-)$ positive, dominated by up pFF
- $P_n(\Lambda\pi^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,
- $P_n(\Lambda\pi^+)$ up and down cancel each other, sea pFF leads to large, and negative, values of the transverse polarization
- $P_n(\Lambda\pi^-)$ sea pFF partial reduction of the up pFF

Lambda-kaon



Data fitted:

- $z_K < 0.50$

$\chi^2_{dof} = 1.94$

Lambda-kaon

- Charge-conjugation symmetry $P_n(\Lambda K^+) = P_n(\bar{\Lambda} K^-)$
- Similar pattern pion
- Data give information on the pFFs for u and s
- $P_n(\Lambda K^+)$ negative, dominated by strange pFF
- $P_n(\Lambda K^-)$ positive, dominated by up pFF
- $P_n(\Lambda K^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,

$e^+e^- \to h_1(\text{jet})X$

For Spin-½ hadron production, two possibile cross sections :

Unpolarised hadron

$$\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta\, dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta) \hat{D}_{h/q}(z_1, p_{1\perp h_1})$$

Transversely polarised hadron

$$P_Y^{h_1} \frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta\, dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta) \Delta D^{h_1}_{S_Y/q}(z_1, p_{1\perp h_1})$$

ratio →

Hadron polarisation

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2\, D_{h_1/q}(z_1, p_{\perp 1})}$$

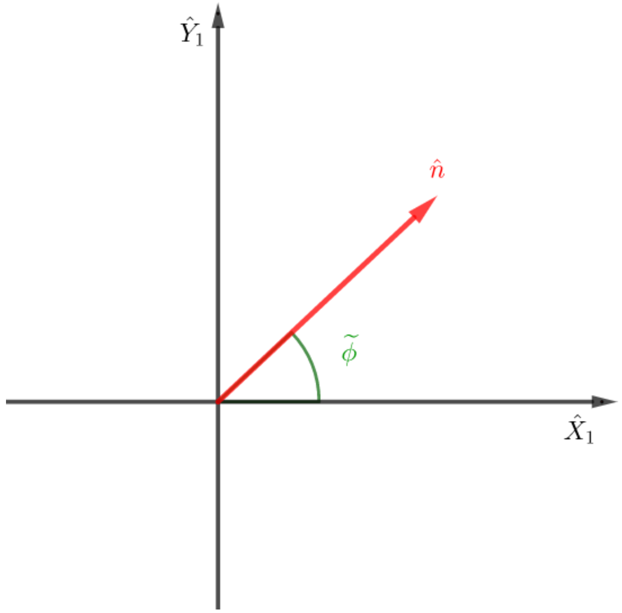
$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

$$\mathcal{P}^{h_1} = P_x^{h_1} \hat{X}_1 + P_y^{h_1} \hat{Y}_1 + P_z^{h_1} \hat{Z}_1$$

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$



$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos \widetilde{\phi} + P_y^{h_1} \sin \widetilde{\phi}$$

$$\cos \widetilde{\phi} \simeq \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \sin(\phi_1 - \varphi_2)$$

$$\sin \widetilde{\phi} \simeq \frac{P_{1T}}{p_{\perp 1}} - \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \cos(\phi_1 - \varphi_2)$$