Phenomenology of hadron production in e+e- collisions

UNIVERSITÀ
DEGLI STUDI
DI TORINO

J Osvaldo Gonzalez-Hernandez
University of Turin

Sar WorS 2021

Outlook

- Theoretical framework for e+e- -> h X
- Global fits
- Phenomenological analysis of recent BELLE data
  - zh-dependence and choice of collinear FFs
  - Model for PT-dependence
  - Hypotheses for gK (work in progress)

Theoretical framework for e+e- -> h X

M. Boglione, A. Simonelli *JHEP* 02 (2021) 076

$$\frac{d\sigma^{\text{NLO, NLL}}}{dz_h\, dT\, dP_T^2} =$$

$$= -\sigma_B\, \pi\, N_C\, \frac{\alpha_S(Q)}{4\pi}\, C_F\, \frac{3 + 8\log(1-T)}{1-T}\, \exp\left\{ -\frac{\alpha_S(Q)}{4\pi}\, 3C_F\, \left(\log(1-T)\right)^2 \right\} \times$$

$$\times \sum_f e_f^2 \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\frac{\vec{P}_T}{z_h}\cdot\vec{b}_T}\, \widetilde{D}_{1,\,H/f}^{\text{NLL}}(z_h,\, b_T,\, Q,\, (1-T)\,Q^2)\, \left[1 + \mathcal{O}\left(\frac{M_H^2}{Q^2}\right)\right]$$

$$\widetilde{D}_{1,\,H/f}(z,\, b_T;\, \mu,\, \zeta) = \underbrace{\frac{1}{z^2} \sum_k \int_z^1 \frac{d\rho}{\rho}\, d_{H/k}(z/\rho,\, \mu_b)\, \left[\rho^2\, \mathcal{C}_{k/f}\,(\rho,\, \alpha_S(\mu_b))\right]}_{\text{TMD at reference scale}} \times$$

$$\times \underbrace{\exp\left\{ \frac{1}{4}\, \widetilde{K}(b_T^\star;\, \mu_b)\, \log\frac{\zeta}{\mu_b^2} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\, \left[\gamma_D(\alpha_S(\mu'),\, 1) - \frac{1}{4}\, \gamma_K(\alpha_S(\mu'))\, \log\frac{\zeta}{\mu'^2}\right]\right\}}_{\text{Perturbative Sudakov Factor}}$$

$$\times \underbrace{(M_D)_{j,\,H}\,(z,\, b_T)\, \exp\left\{ -\frac{1}{4}\, g_K(b_T)\, \log\frac{z_h^2\, \zeta}{M_H^2} \right\}}_{\text{Non-Perturbative content}}.$$

Theoretical framework for e+e- -> h X

$$D = D^{\star}\sqrt{M_S}$$

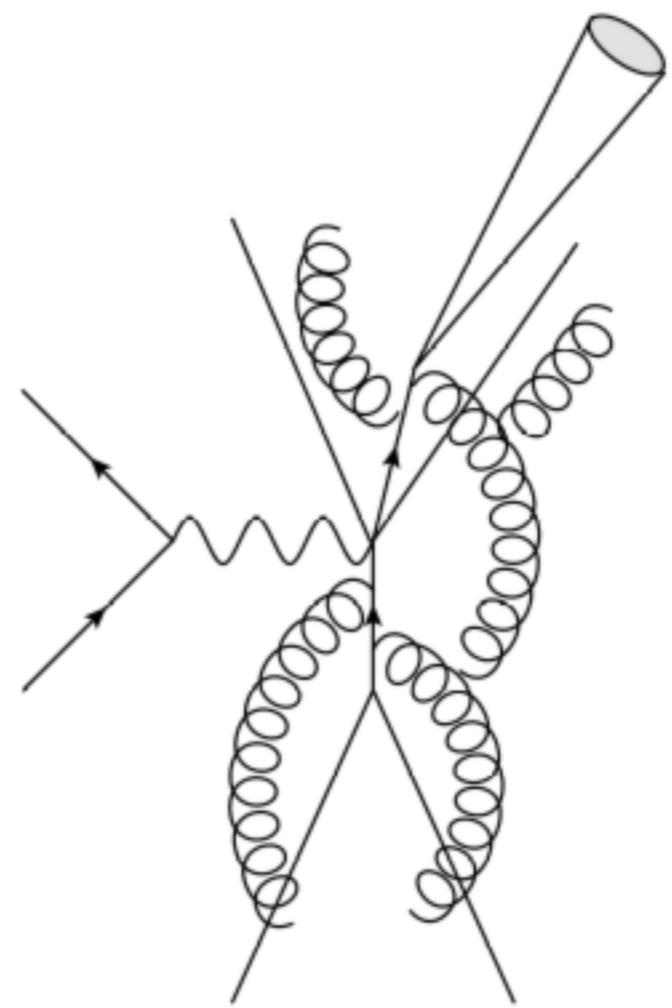
Usual definition TMD FF

For this process TMD FF

Soft non-perturbative Function

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^{\star}(P_T)$$

Theoretical framework for e+e- -> h X

$$D = D^{\star} \sqrt{M_S}$$

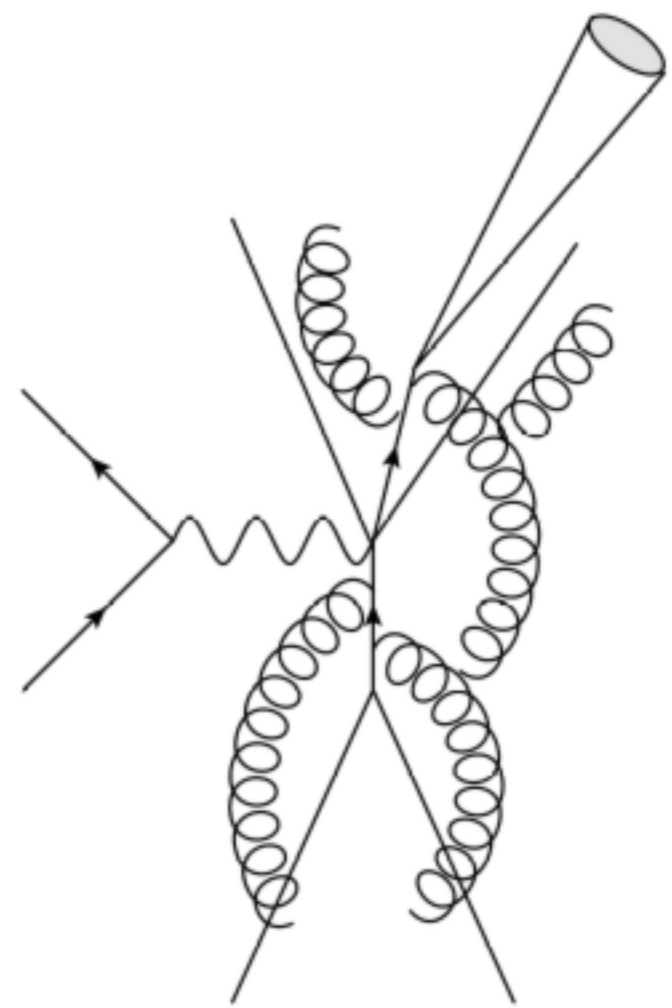
Same constraints to collinear FF

$$g_K(b_T) = \widetilde{K}(b_T^{\star};\, \mu) - \widetilde{K}(b_T;\, \mu)$$

Same function for non-perturbative evolution

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^{\star}(P_T)$$

Theoretical framework for e+e- -> h X

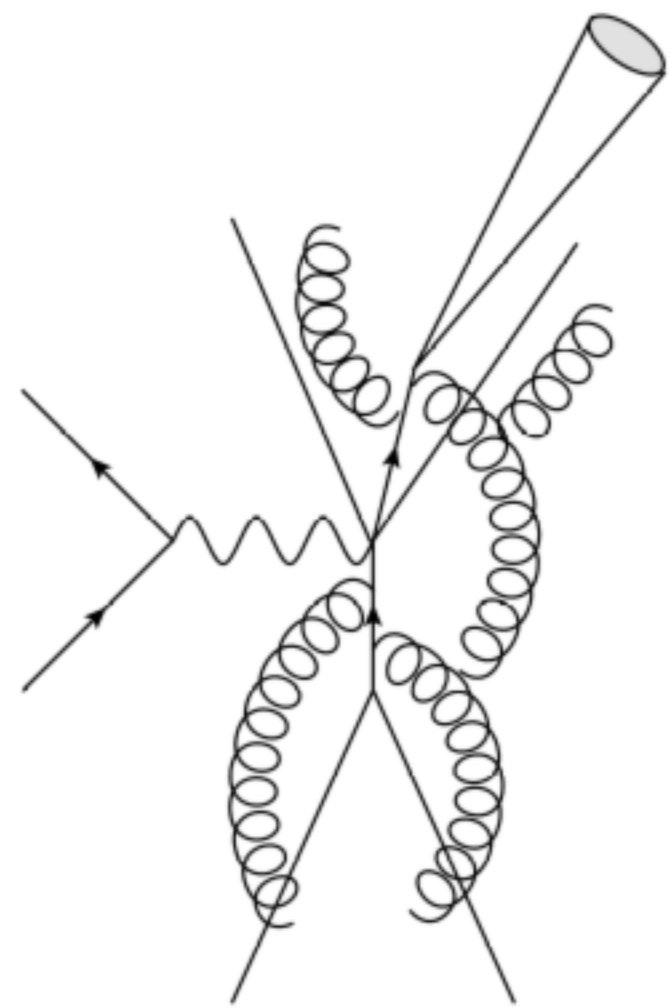
$$D = D^{\star}\,\sqrt{M_S}$$

What is the effect of the collinear FFs (and PDFs in general) ?

Large-bT behaviour of gK ?

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$

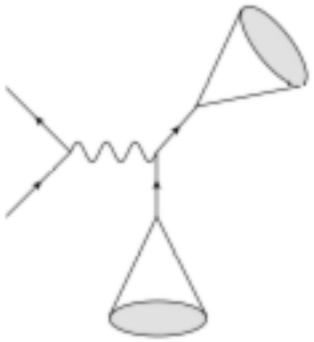


$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^{\star}(P_T)$$

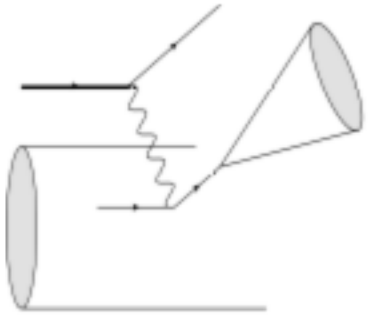
Global Fits

Possible roadmap

1. Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_S



3. SIDIS
   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_S, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, $F^* \neq F$).

Taken from M. Boglione DIS 2021

(Global) Fits

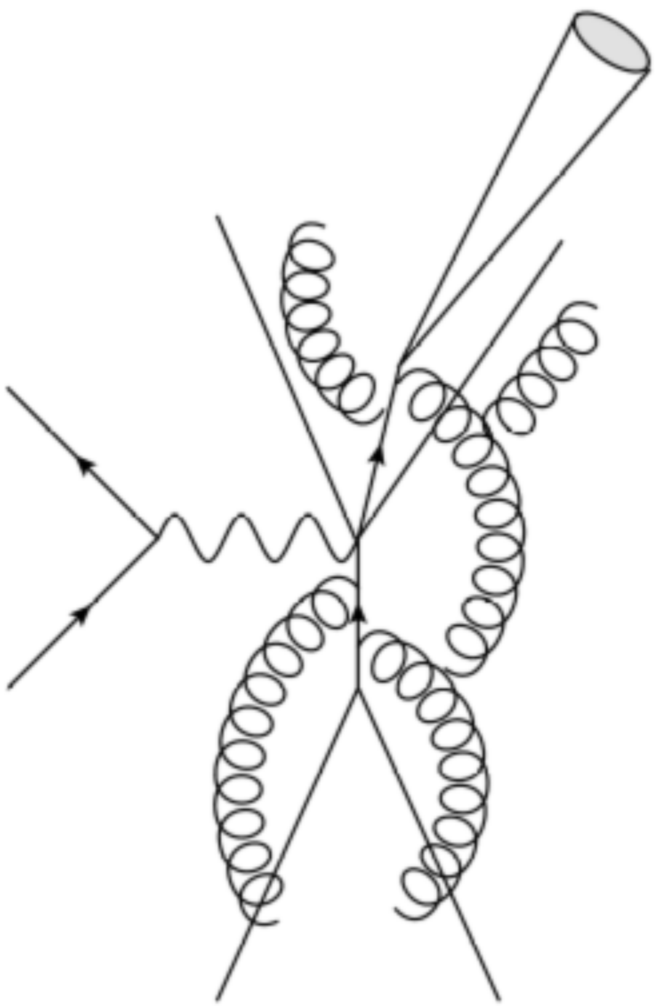
Some important aspects to consider:

- Which collinear functions are more appropriate?
- Which regions in bT are being mapped by extractions.
- Constraints of bT-behaviour for TMDs.
- Physical pictures/theoretical arguments /models (not parametrizations)
- Non perturbative evolution (gK) should be consistent with SIDIS, DY, e+e- two-hadron production.

Phenomenological analysis of recent BELLE data

Data overview

$e^+e^- \rightarrow hX$ (Charged pions)



Binned in PT, zh and T (thrust)

0.06<PT<2.5 GeV

0.125<zh<0.975

0.6<T<0.975

zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

$$\widetilde{D}_{1,\,H/f}(z,\,b_T;\,\mu,\,\zeta) = \underbrace{\frac{1}{z^2}\sum_k \int_z^1 \frac{d\rho}{\rho}\,\boxed{d_{H/k}(z/\rho,\,\mu_b)}\,\left[\rho^2\,\mathcal{C}_{k/f}\,(\rho,\,\alpha_S(\mu_b))\right]}_{\text{TMD at reference scale}} \times$$

$$\times \underbrace{\exp\left\{\frac{1}{4}\,\widetilde{K}(b_T^\star;\,\mu_b)\,\log\frac{\zeta}{\mu_b^2} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_D(\alpha_S(\mu'),\,1) - \frac{1}{4}\,\gamma_K(\alpha_S(\mu'))\,\log\frac{\zeta}{\mu'^2}\right]\right\}}_{\text{Perturbative Sudakov Factor}}$$

$$\times \underbrace{(M_D)_{j,\,H}\,(z,\,b_T)\,\exp\left\{-\frac{1}{4}\,g_K(b_T)\,\log\frac{z_h^2\,\zeta}{M_H^2}\right\}}_{\text{Non-Perturbative content}}.$$

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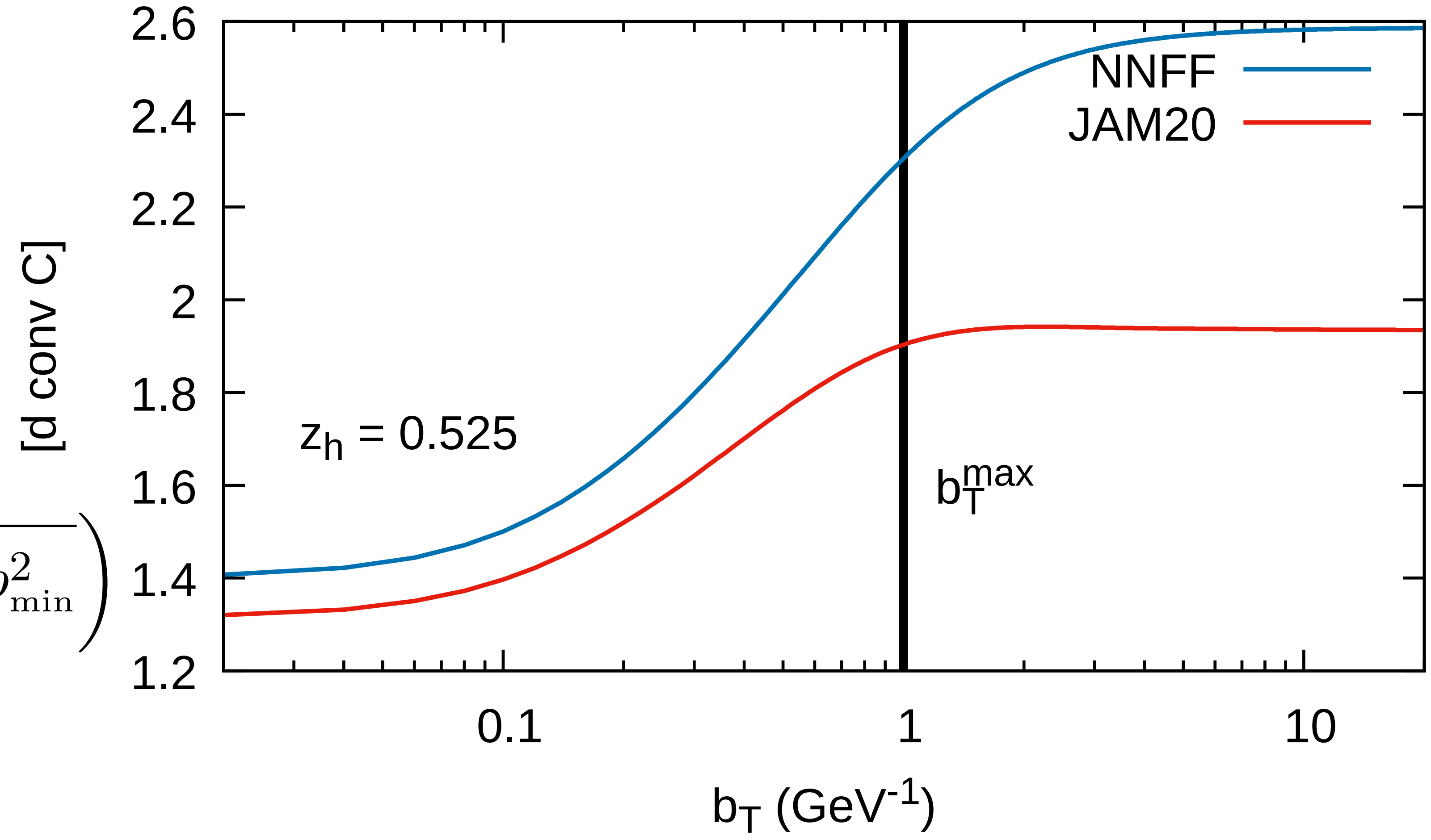
$$\frac{1}{z^2}\sum_k \int_z^1 \frac{d\rho}{\rho}\, d_{H/k}(z/\rho,\, \mu_b)\; \left[\rho^2\, \mathcal{C}_{k/f}\,(\rho,\, \alpha_S(\mu_b))\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T^\star(b_T)}$$

$$\vec{b}_T^\star\,(b_T) = \frac{\vec{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

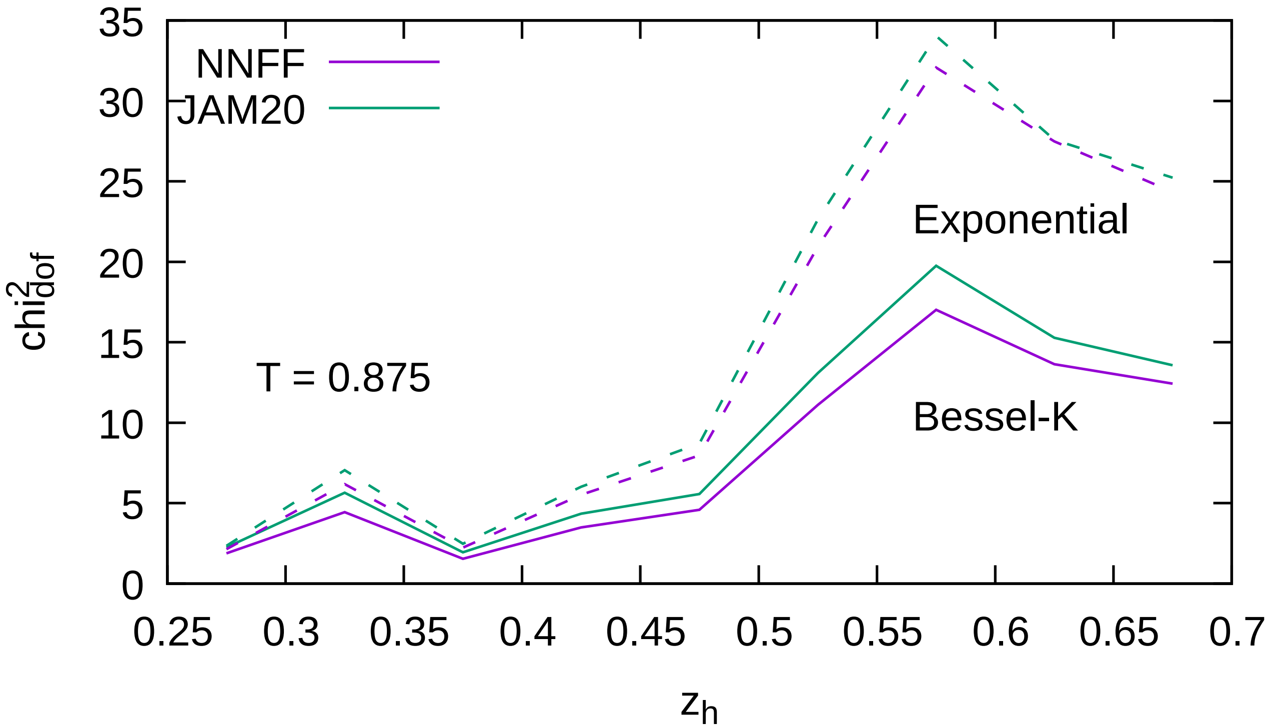
$$\vec{b}_T^\star\,(b_c(b_T)) = \vec{b}_T^\star\left(\sqrt{b_T^2 + b_{\min}^2}\right)$$



zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

Nomenclature	$M_{\rm D}$-model	parameters
z_h-independent models		
1)Exponential-q	$e^{-(M_0 b_{\rm T})^q}$	$M_0,\ q$
2)Bessel-K	$\dfrac{2^{2-p}(b_{\rm T}M_0)^{p-1}}{\Gamma(p-1)}K_{p-1}(b_{\rm T}M_0)$	$M_0,\ p$



Proxy models: performed fits at fixed T=0.875. One INDEPENDENT fit for each zh-bin in the range 0.25<zh<0.7

NNFF perform better. (non-conclusive due to large values of chi^2/dof)

zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

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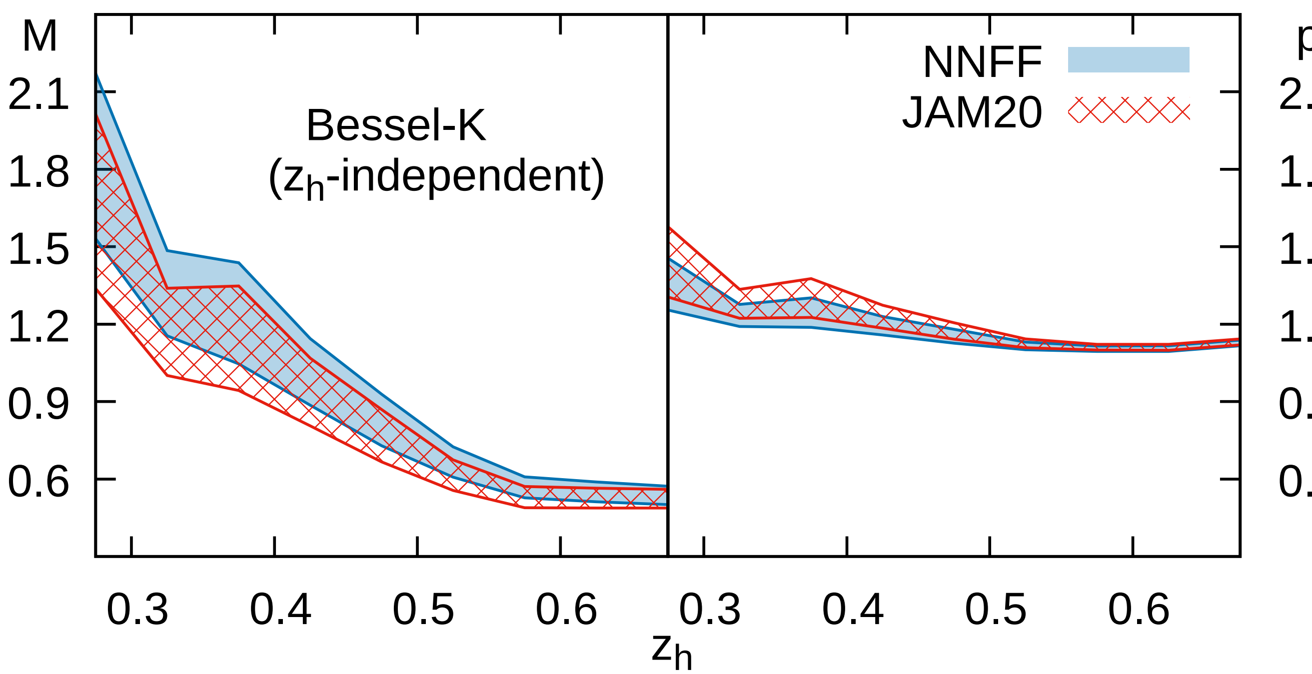

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Stronger zh-dependence in *dimensionfull* parameter

zh-dependence and choice of collinear FFs

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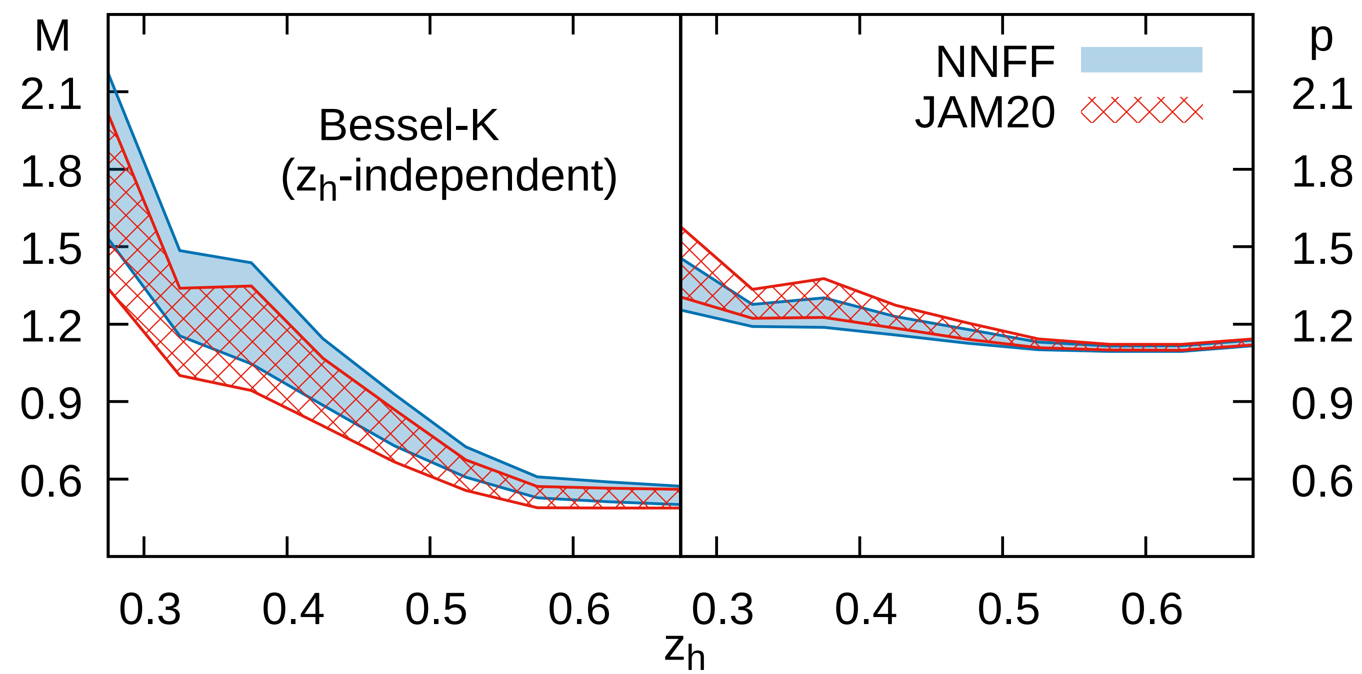

Stronger zh-dependence in *dimensionfull* parameter

Next Step, try fitting zh-bins simultaneously (fixed T=0.875)

zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

Nomenclature	$M_{\rm D}$-model	parameters
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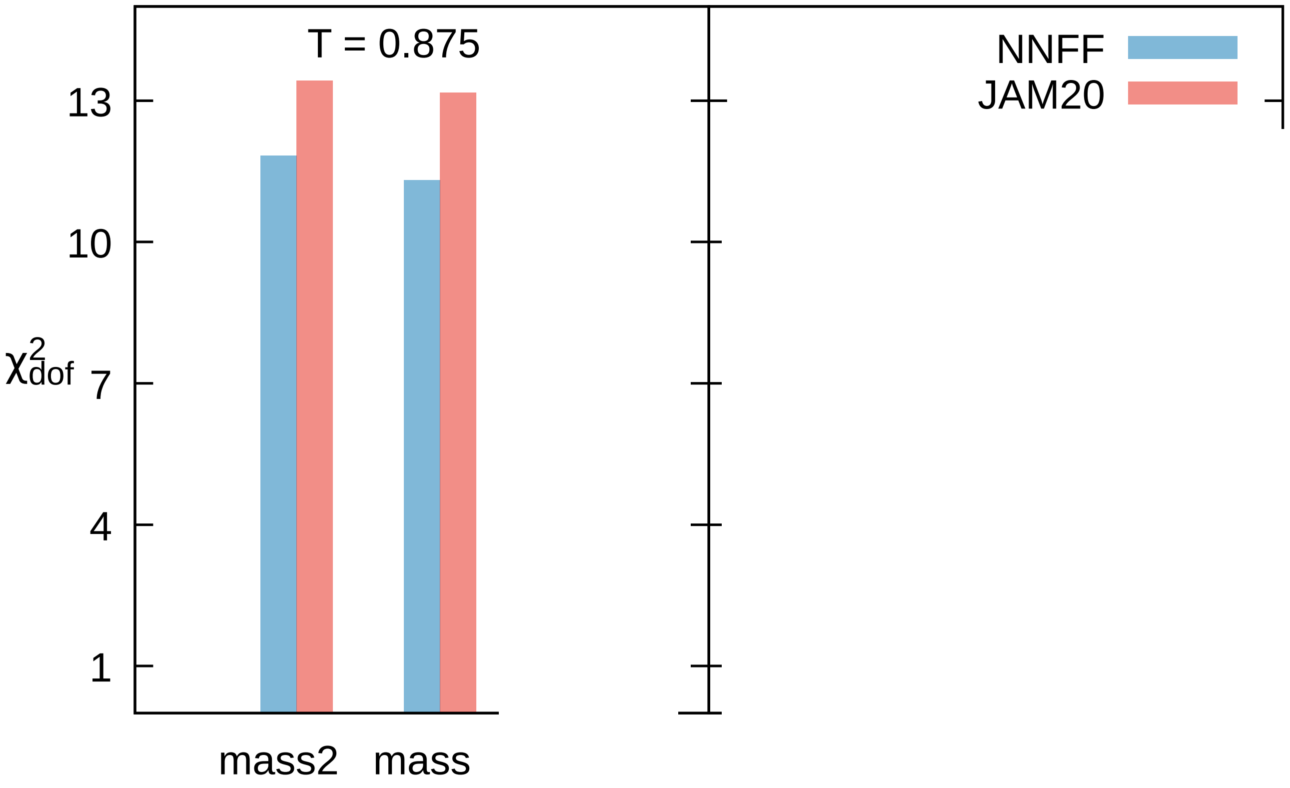
1)Exponential-q	$e^{-(M_0 b_{\rm T})^q}$	M_0, q
2)Bessel-K	$\frac{2^{2-p}(b_{\rm T}M_0)^{p-1}}{\Gamma(p-1)}K_{p-1}(b_{\rm T}M_0)$	M_0, p
z_h-dependent models		
3)Bessel-K-M_1^z	$M_0 \to M_1\,(1-\eta_1 \log(z_h))$	M_1, η_1, p
4)Bessel-K-M_2^z	$M_0 \to M_2\left(1+\frac{\eta_2}{z_h^2}\right)$	M_2, η_2, p



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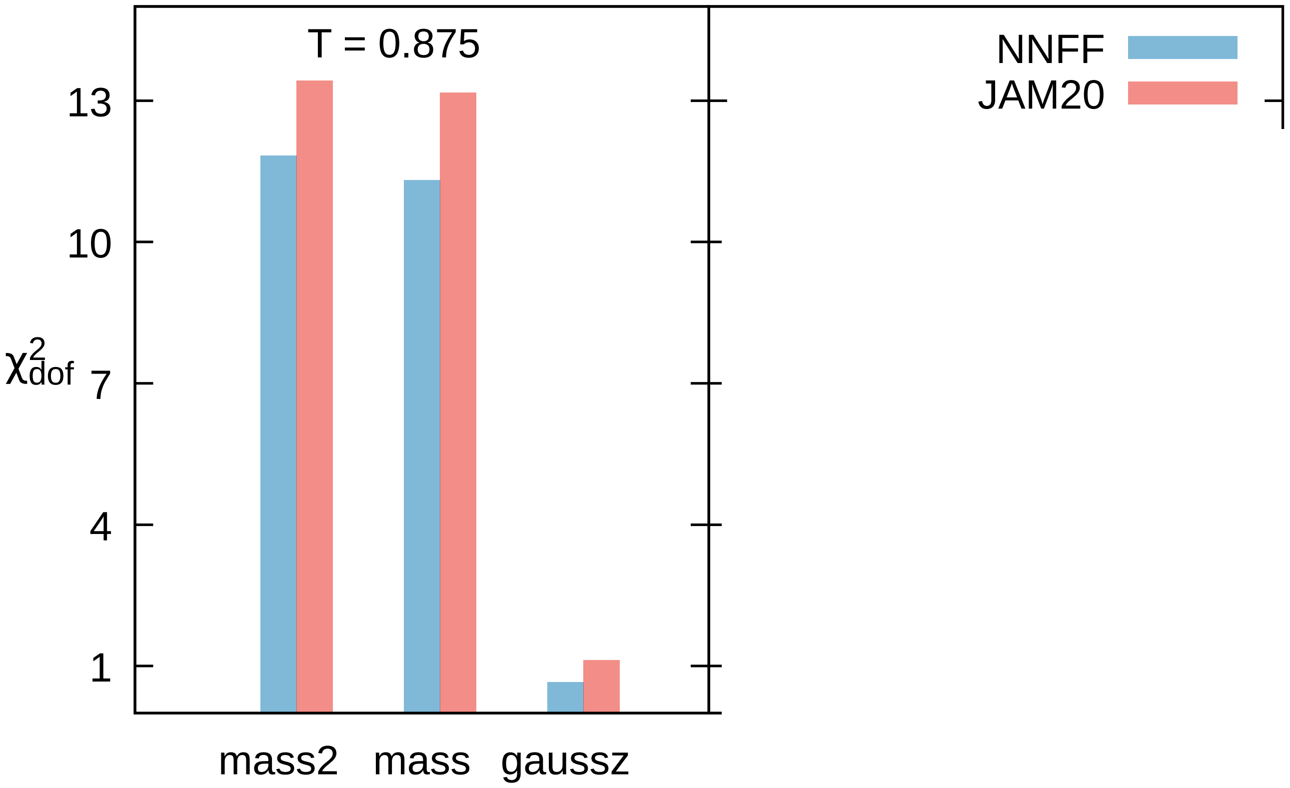
5)Bessel-K-M_g^z	$e^{(M_g b_{\mathrm{T}})^2 \log(z_h)} \times$ Bessel-K	$M_g,\ M_0,\ p$



Next Step, try fitting
zh-bins simultaneously
(fixed T=0.875)

+1 dimensionfull parameter

zh-dependence and choice of collinear FFs

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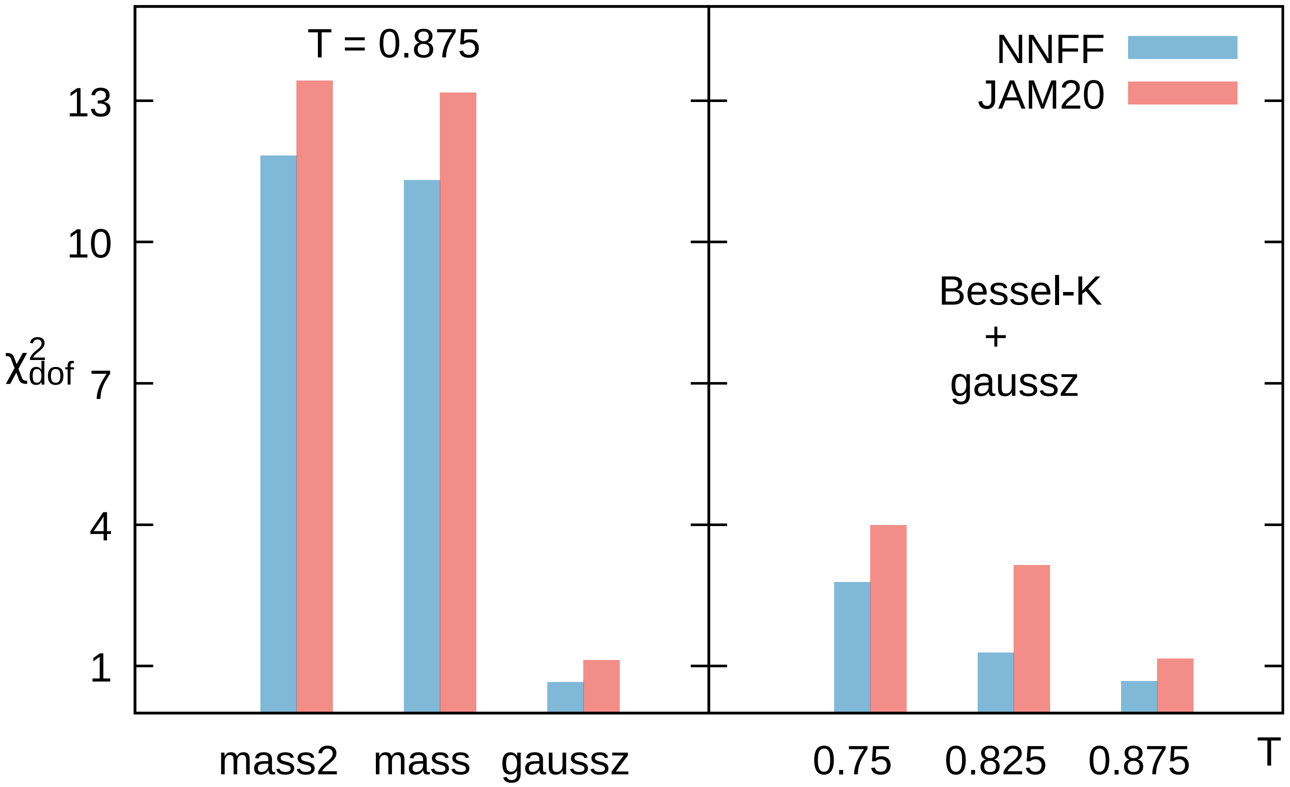

Next Step, try fitting zh-bins simultaneously (fixed T=0.750, 0.825, 0.875)

Systematically NNFFs outperform

zh-dependence and choice of collinear FFs

- Where did the improvement come from?

Nomenclature	M_{D}-model	parameters
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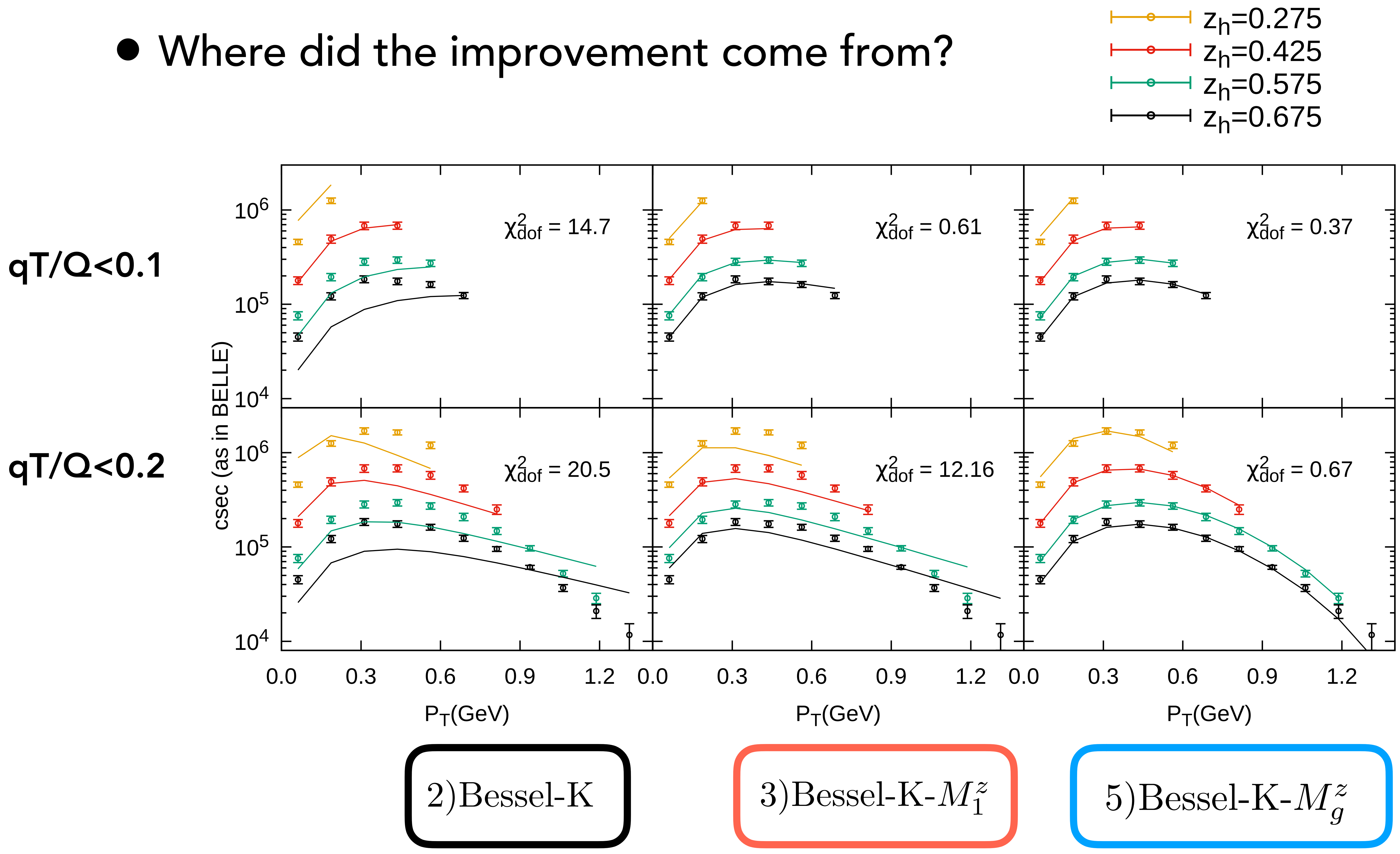
zh-dependence and choice of collinear FFs

- Where did the improvement come from?



Model for PT-dependence

$$M_{\rm D} = \frac{2^{2-p}(b_{\rm T} M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_{\rm T} M_0) \;\times\; F(b_{\rm T}, z_h)$$

$$M_z = M_0\big(1 - \eta \log(z_h)\big)$$

Nomenclature	F-model	parameters
1)Bessel-K-F_1	$F = \left(\frac{1+\log\left(1+b_{\rm T}M_z\right)}{1+(b_{\rm T}M_z)}\right)^q$	$M_0,\ \eta,\ p\,,q$
2)Bessel-K-F_2	$F = \frac{1}{1+(b_{\rm T}M_z)^q}$	$M_0,\ \eta,\ p\,,q$
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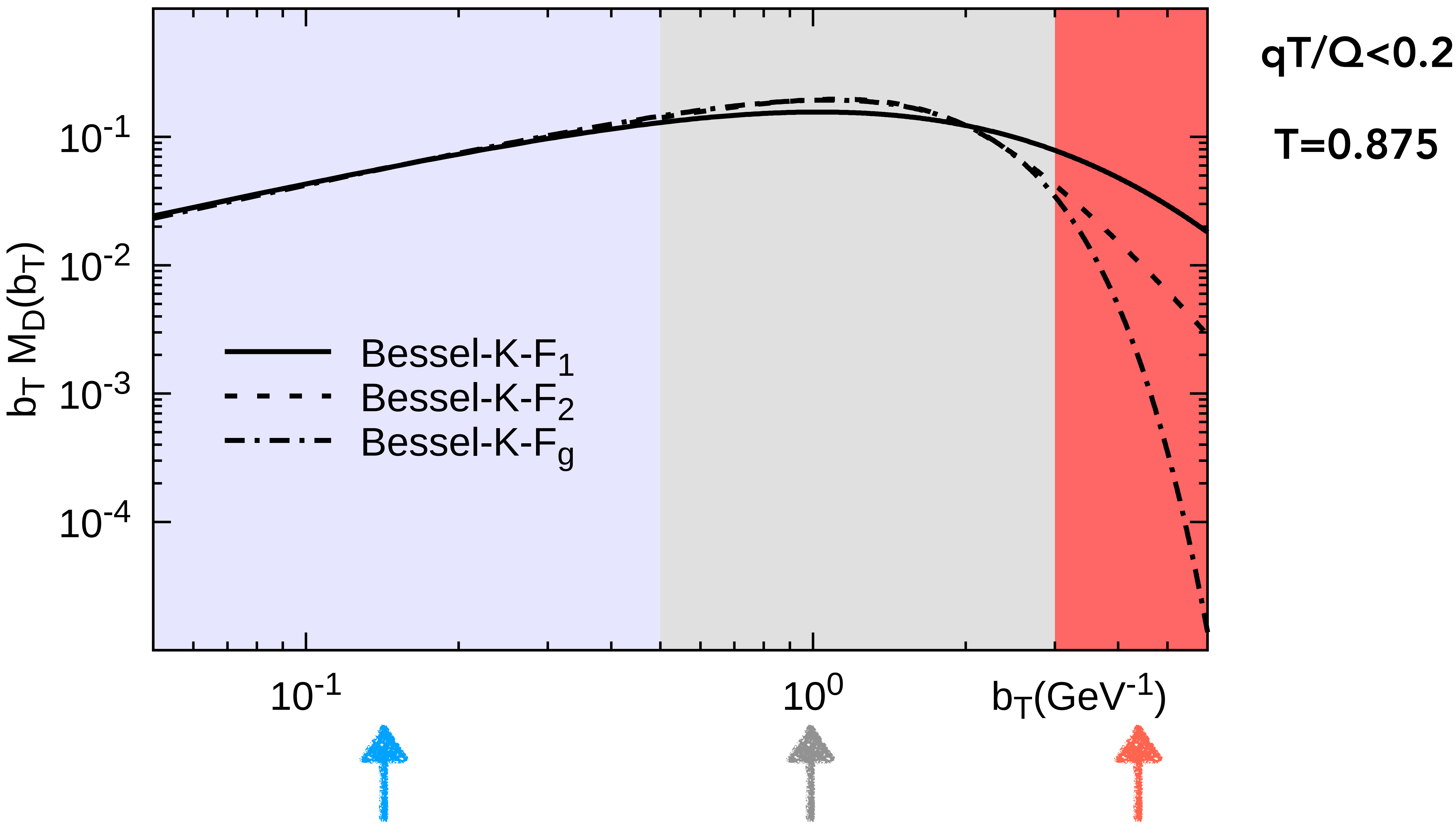
$\chi^2_{\rm d.o.f.}$ (fixed-T fits)			
$T =$	0.750	0.825	0.875
Bessel-K-F_1	3.06	1.24	0.65
Bessel-K-F_2	3.02	1.26	0.97
Bessel-K-F_g	2.82	1.29	0.68

Fit all zh-bins in the range 0.25<zh<0.7

qT/Q<0.2

Hard to discriminate bT-models

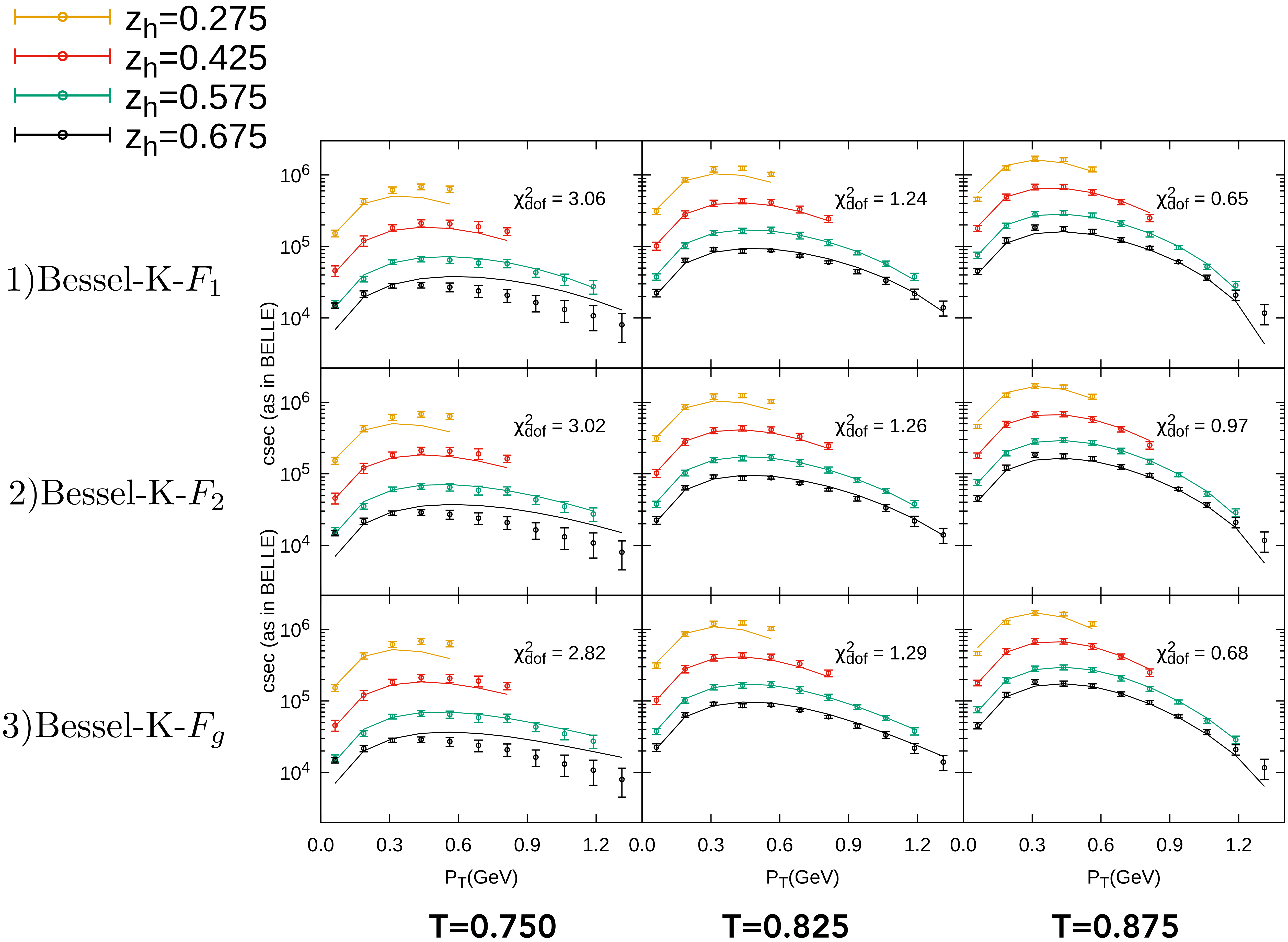
Model for PT-dependence



MD —>1 for bT—>0

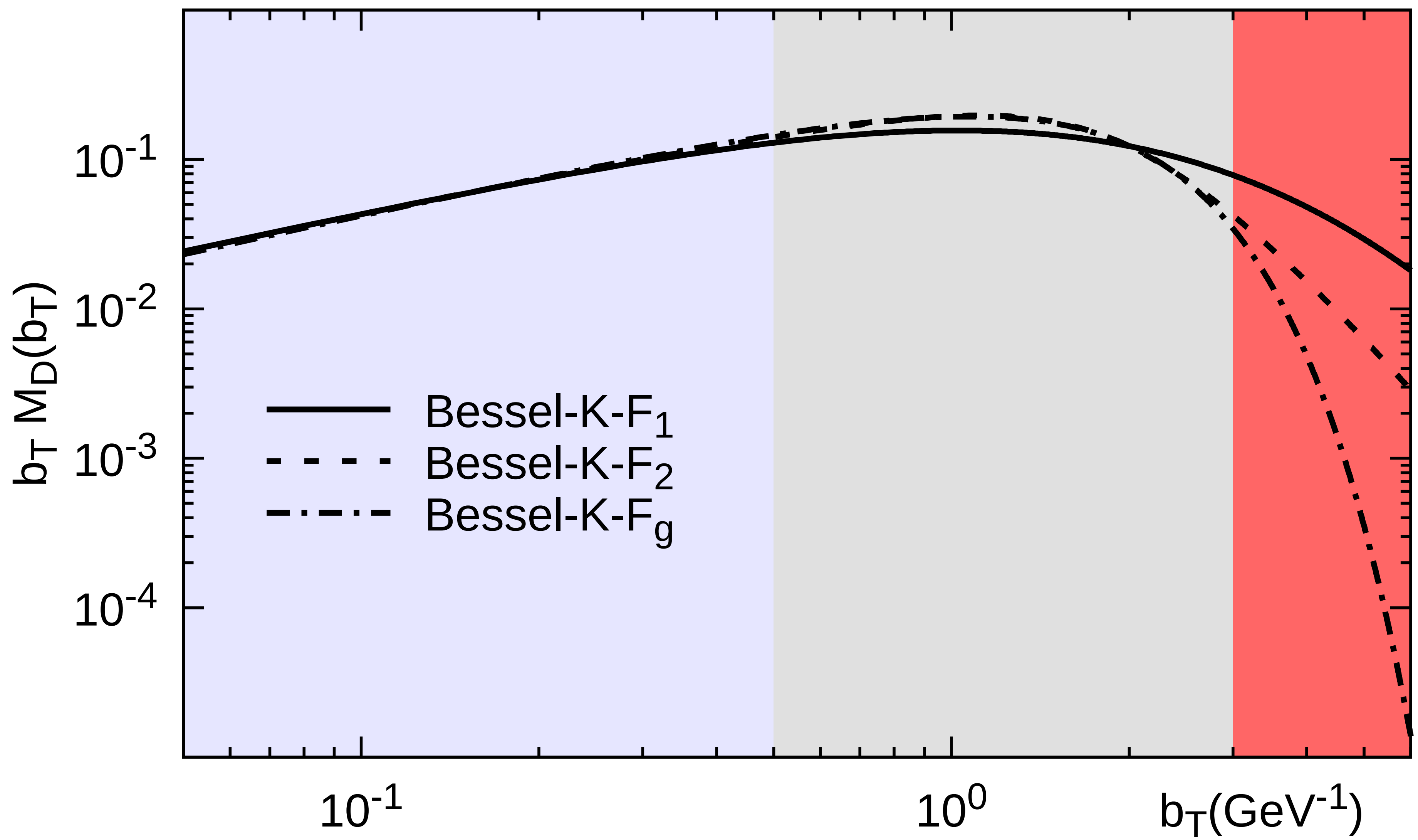
Largest proved distance (somewhere here)

Different asymptotic behaviour



z_h=0.275
z_h=0.425
z_h=0.575
z_h=0.675
1)Bessel-K-F_1
2)Bessel-K-F_2
3)Bessel-K-F_g
csec (as in BELLE)
P_T(GeV)
χ^2_{dof} = 3.06
χ^2_{dof} = 1.24
χ^2_{dof} = 0.65
χ^2_{dof} = 3.02
χ^2_{dof} = 1.26
χ^2_{dof} = 0.97
χ^2_{dof} = 2.82
χ^2_{dof} = 1.29
χ^2_{dof} = 0.68
T=0.750
T=0.825
T=0.875

Model for PT-dependence



qT/Q<0.2

T=0.875

All three models match the data, different asymptotic behaviour
Which choice?

Model for PT-dependence

P. Schweitzer, M. Strikman, and C. Weiss, JHEP 1301, 163 (2013)

J. Collins and T. Rogers, Phys. Rev. D91 (2015) 074020, [1412.3820].

TMDs $\longrightarrow \quad \frac{1}{{b_{\mathrm{T}}}^{\alpha}} e^{-m b_{\mathrm{T}}}$ as $\quad b_{\mathrm{T}} \to \infty.$

We consider $\quad \log(M_{\mathrm{D}}) \sim -C\, b_{\mathrm{T}} + o\,(b_{\mathrm{T}})$

Non-perturbative function

Mass parameter

Model for PT-dependence

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Focus on these

We consider $\log(M_{\mathrm{D}}) \sim -C\,b_{\mathrm{T}} + o\,(b_{\mathrm{T}})$

Non-perturbative function

Mass parameter

Hypotheses for gK (work in progress)

We consider $$\log(M_{\mathrm{D}}) \sim -C\, b_{\mathrm{T}} + o\,(b_{\mathrm{T}})$$

$$\mathrm{TMDFF} \sim M_{\mathrm{D}}\ \exp\left(-\frac{g_{\mathrm{K}}(b_{\mathrm{T}})}{4}\log\left(\frac{\zeta}{\zeta_0}\right)\right)$$

$$\log\left(\mathrm{TMDFF}\right) \sim -C b_{\mathrm{T}} - \frac{g_{\mathrm{K}}(b_{\mathrm{T}})}{4}\log\left(\frac{\zeta}{\zeta_0}\right) + o\,(b_{\mathrm{T}})$$

Hypotheses for gK (work in progress)

We consider $$\log(M_{\rm D}) \sim -C\, b_{\rm T} + o\,(b_{\rm T})$$

$$\text{TMDFF} \sim M_{\rm D}\ \exp\left(-\frac{g_{\rm K}(b_{\rm T})}{4}\log\left(\frac{\zeta}{\zeta_0}\right)\right)$$

$$\log\left(\text{TMDFF}\right) \sim -C b_{\rm T} - \frac{g_{\rm K}(b_{\rm T})}{4}\log\left(\frac{\zeta}{\zeta_0}\right) + o\,(b_{\rm T})$$

If $g_{\rm K}(b_{\rm T}) = O\,(b_{\rm T})$, the argument of exponential may flip sign depending on ζ_0 and ζ (relevant for global fits)

Hypotheses for gK (work in progress)

We consider $\log(M_{\rm D}) \sim -C\, b_{\rm T} + o\,(b_{\rm T})$

$$\text{TMDFF} \sim M_{\rm D}\ \exp\left(-\frac{g_{\rm K}(b_{\rm T})}{4}\log\left(\frac{\zeta}{\zeta_0}\right)\right)$$

$$\log\left(\text{TMDFF}\right) \sim -Cb_{\rm T} - \underbrace{\frac{g_{\rm K}(b_{\rm T})}{4}\log\left(\frac{\zeta}{\zeta_0}\right)} + o\,(b_{\rm T})$$

If $g_{\rm K}(b_{\rm T}) = o\,(b_{\rm T})$ limit of TMDFF always zero for $b_{\rm T} \to \infty$

This avoids a fast decaying behaviour for $g_{\rm K}(b_{\rm T})$ as $b_{\rm T} \to \infty$

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We consider $\log(M_{\mathrm{D}}) \sim -C\, b_{\mathrm{T}} + o\,(b_{\mathrm{T}})$

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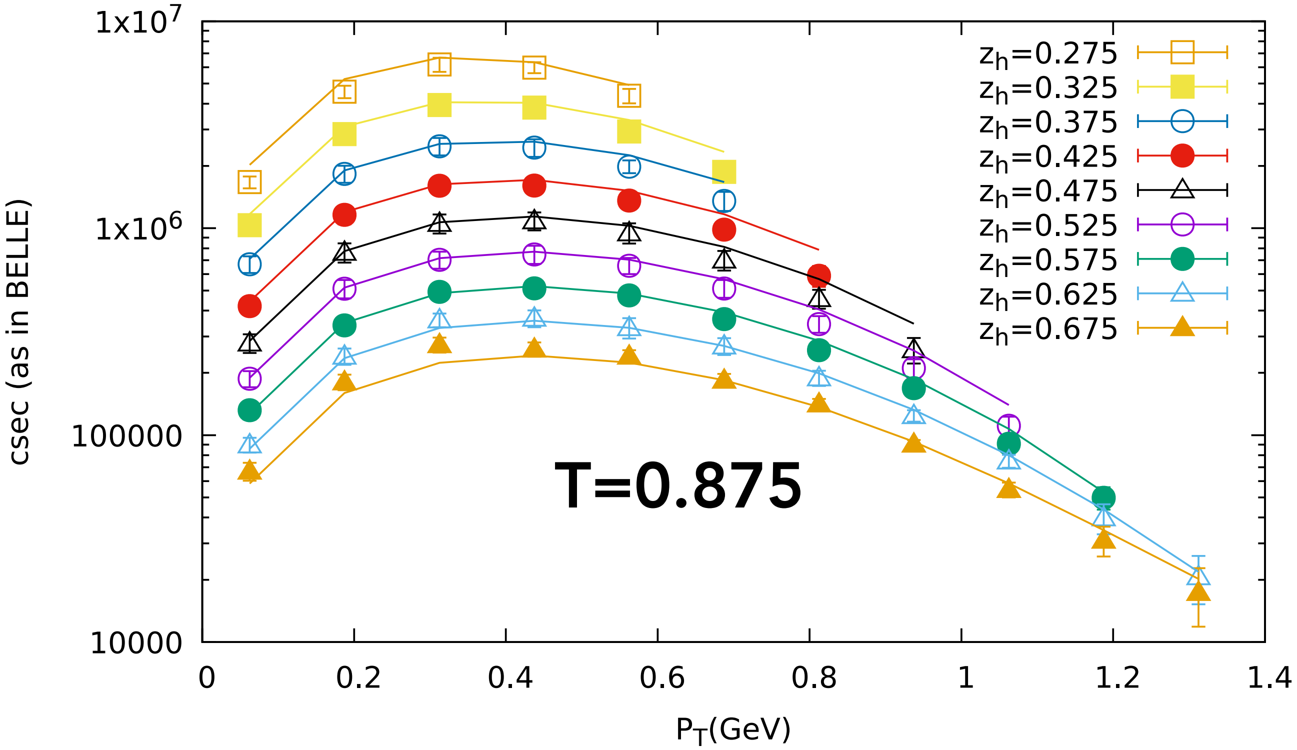
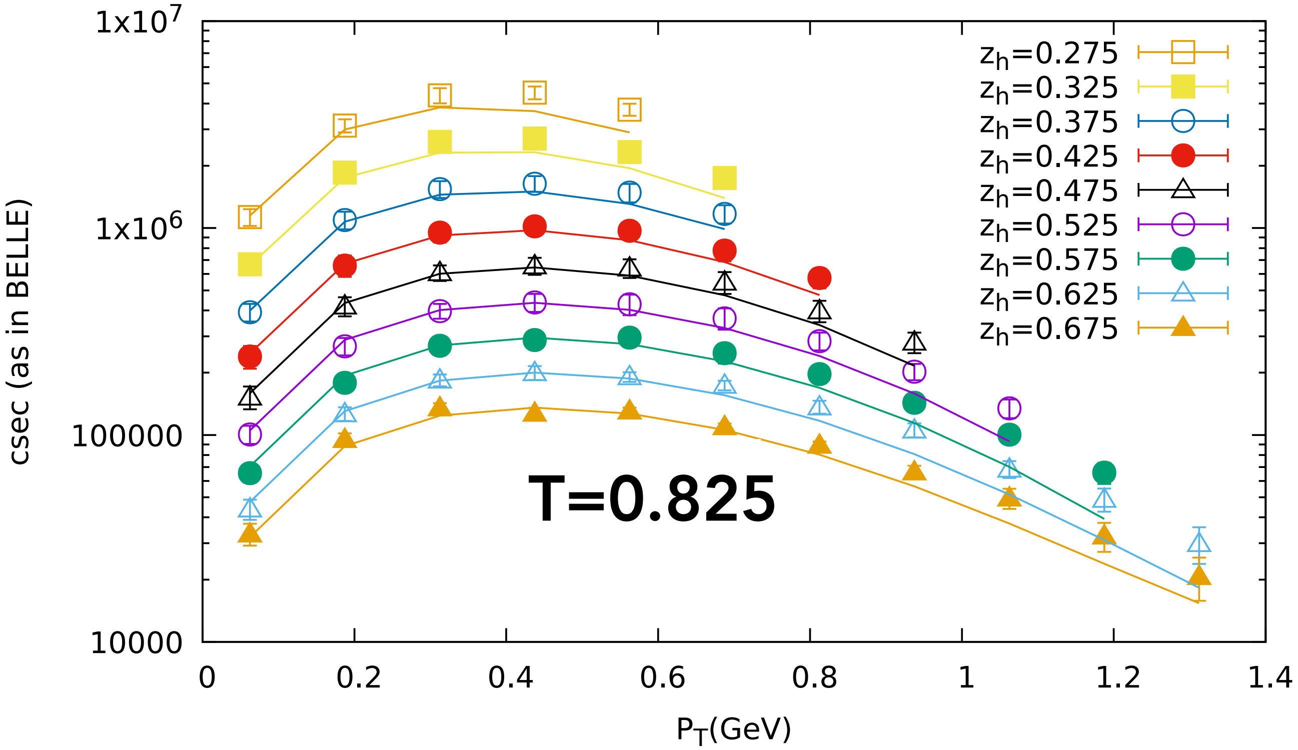
$$\log\left(\mathrm{TMDFF}\right) \sim -Cb_{\mathrm{T}} - \underbrace{\frac{g_{\mathrm{K}}(b_{\mathrm{T}})}{4}\log\left(\frac{\zeta}{\zeta_0}\right)} + o\,(b_{\mathrm{T}})$$

If $g_{\mathrm{K}}(b_{\mathrm{T}}) = o\,(b_{\mathrm{T}})$ limit of TMDFF always zero for $b_{\mathrm{T}} \to \infty$

This avoids a fast decaying behaviour for $g_{\mathrm{K}}(b_{\mathrm{T}})$ as $b_{\mathrm{T}} \to \infty$

For instance $g_{\mathrm{K}} = \log\left(1 + (M_k b_{\mathrm{T}})^{p_k}\right)$





qT/Q<0.2

chi2/dof = 2.1

Final remarks

- NNFF seem to work well to extract TMDFF in Belle data
- Gaussian behaviour in bT describes the data but NOT because of its asymptotic behaviour
- Cannot constrain phenomenologically the asymptotic behaviour of the TMDFF
- Instead we use the hypothesis that its logarithm should be dominantly linear.
- We attempted to describe multidimensional data without modifying such leading behaviour of the TMDFF by parameterising gk with a logarithmic asymptotic behaviour.
- Results are promising but must improve limited range of T

Thanks