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In collaboration with M. Boglione, O. Gonzalez

Jets and TMD Fragmentation Functions in e^+e^- processes

SAR WorS 2021 Sardinian Workshop on Spin

TMD factorization: three benchmark processes

Until the end of 2018, TMD factorization was known to be proved for:

1. $e^+e^- \rightarrow h_1 h_2 X$ with the two hadrons back-to-back,
2. Semi Inclusive DIS (SIDIS) for small values of momentum transfer
3. Drell-Yan scattering with the lepton pair generated back-to-back

$$q_T \ll Q$$

Same factorized cross-section structure:

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} F(b_T) S(b_T) D(b_T)$$

Diagram illustrating the factorized cross-section structure:

- First Hadron (points to $F(b_T)$)
- No direct experimental probe (points to $S(b_T)$)
- Second Hadron (points to $D(b_T)$)

Here the TMDs F and D are defined by the
Factorization Definition

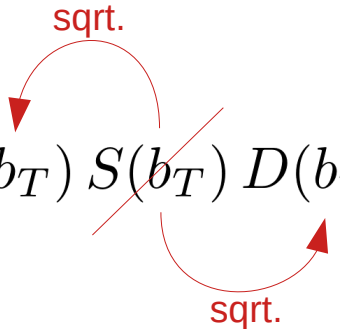
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$$\longleftrightarrow \quad q_T \ll Q$$

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$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} F^{\text{sqrt}}(b_T) D^{\text{sqrt}}(b_T)$$

First Hadron

Second Hadron

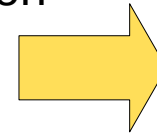
Square Root
definition of TMDs



Past ~10 years of
phenomenology

Beyond the benchmark processes: motivations

- The three benchmark processes offer very rich information about many spin effects,
- In principle, they allow to access all the (quark) TMDs.



Enormous efforts into developing more and more competitive **models, codes...**

... So **why** do we have to extend TMD factorization outside the benchmark processes?

- ◆ A practical reason: the cross section structure presents a convolution of **two** TMDs, not easy to be disentagled

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} F^{\text{sqrt}}(b_T) D^{\text{sqrt}}(b_T)$$



Are there TMD processes that present a **single** TMD parton density?► $e^+e^- \rightarrow h X$

- ◆ We have to keep up with experiments! EIC...

- ◆ Many theoretical reasons, for instance:

- Gluon TMDs investigation
- Some aspect of confinement /hadronization may be unaccessible due to the benchmark cross section structure

Beyond the benchmark processes: complications

1) The benchmark processes involve **two reference hadrons**

Universality-breaking effects in e^+e^- hadronic production processes, M. Boggione, A. Simonelli, *Eur.Phys.J.C* **81** (2021) **1**, 96

└─► Soft radiation is represented by a 2-h soft factor \mathbb{S}_{2-h}

Soft radiation contribution depends on the number N_h of the reference hadrons
 N_h -Universality

$N_h \neq 2$ ───► Different Soft Factor ───► **Sqrt Def.** of TMDs cannot be extended straightforwardly

Better use **Factorization Def.** of TMDs for extension beyond the benchmarks

2) Any dependence on rapidity cut-offs disappears in the cross section structure of benchmarks:

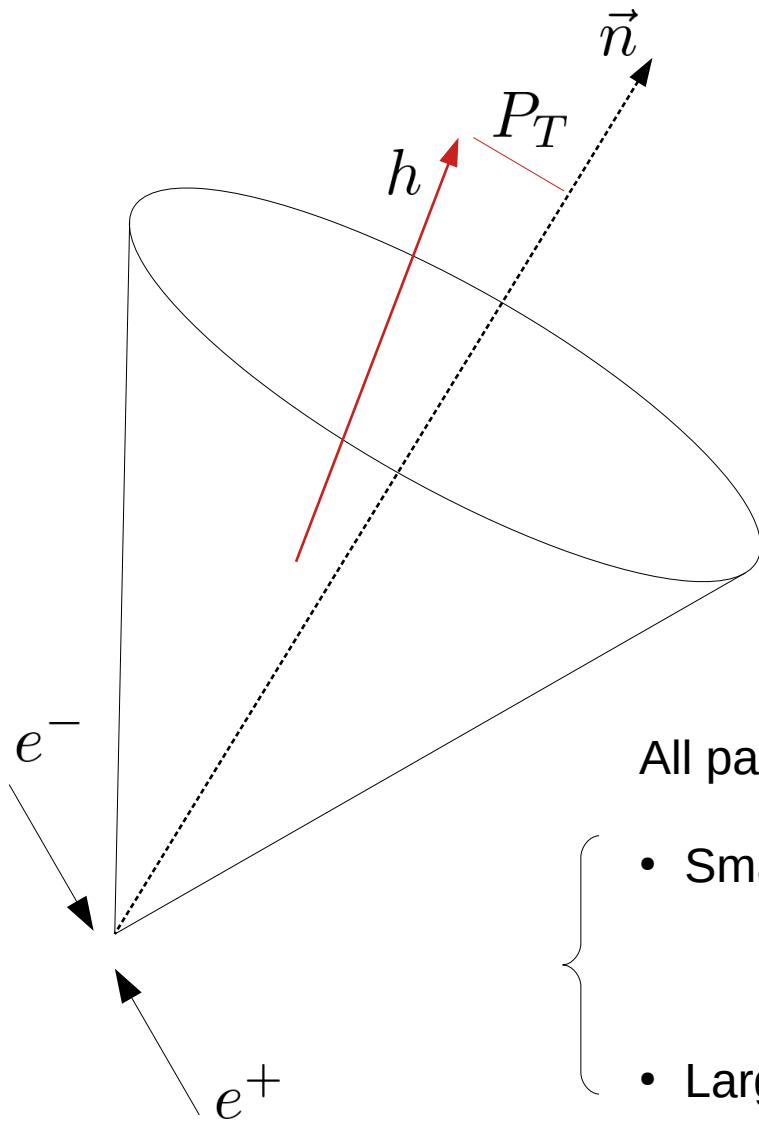
$$\frac{\partial}{\partial y_{1,2}} F(b_T, y_2) \mathbb{S}_{2-h}(b_T, y_1 - y_2) D(b_T, y_1) = 0$$

This may be not verified for different structures of factorized cross sections

The process $e^+ e^- \rightarrow h X$ (thrust)

The cross section is differential in:

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\text{c.m.}), i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}), i}|}, \quad P_T \text{ w.r.t } \vec{n}$$



$0.5 \leq T \leq 1$

Spherical distribution \leftarrow \rightarrow **2-jet limit**

2-jet final state is the most probable topology configuration

All particles inside the jet in which h is detected must have:

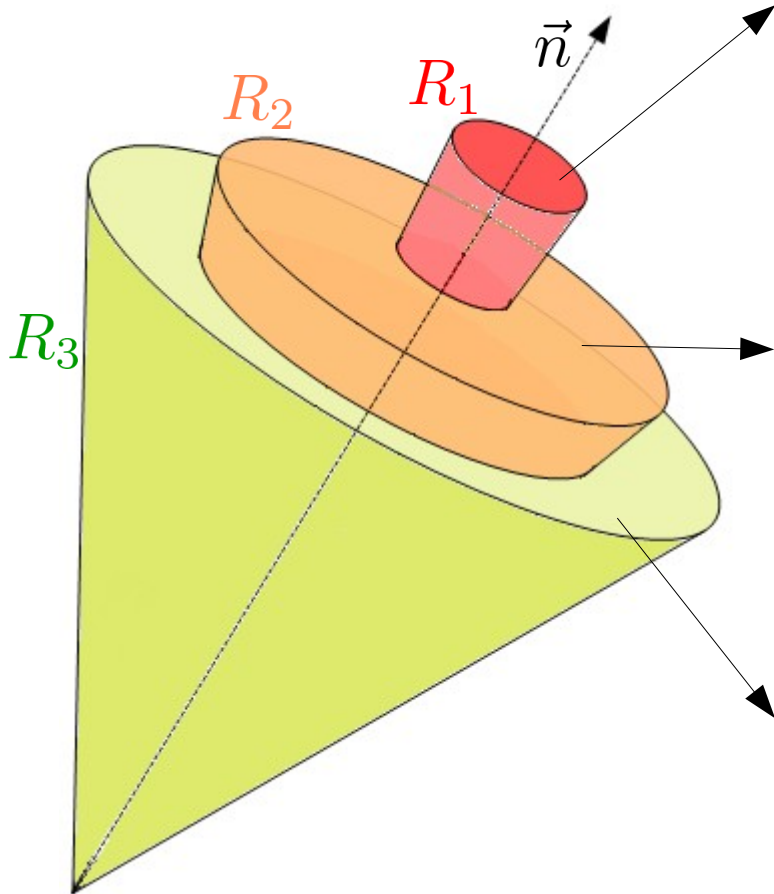
- Small transverse momentum $P_T \ll P^+ = z_h \frac{Q}{\sqrt{2}}$

- Large rapidity $y_P = \frac{1}{2} \log \frac{2(P^+)^2}{P_T^2 + M_h^2} \gg 0$

Kinematic regions of $e^+ e^- \rightarrow h X$ (thrust)

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems

Three Regions:



The hadron is detected very close to the **axis** of the jet:

- Extremely small P_T
- Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

TMD FF + non-pert. SOFT contribution

The hadron is detected in the **central region** of the jet:

- Most common scenario
- Majority of experimental data fall into this case

TMD FF

The hadron is detected near the **boundary** of the jet:

- Moderately small P_T
- The hadron P_T causes the spread of the jet affecting the topology of the final state (i.e. the value of thrust)

Generalized FJF

Kinematic Regions in $e^+ e^- \rightarrow h X$ process in a 2-jet topology, M. Boglione, A. Simonelli, arxiv:2109.xxyy (in preparation)

Identify the regions: kinematic requirements

We are primarily interested into the **central region R_2** where the hadron is detected. It corresponds to the region where the following requirements hold true:

- **(H1)** not too close to the jet boundary, hence its transverse momentum P_T should be small compared to the maximum transverse spreading of the jet.
- **(H2)** not too close to the thrust axis, hence its transverse momentum P_T should not be considerably small. The hadron belongs to the jet mostly because it has a very large rapidity rather than because of the small size of its transverse deviation from the thrust axis.

(H1)	(H2)	reg.
True	False	R_1
True	True	R_2
False	True	R_3

How can these requirements be used into the proof of factorization theorems?

TMD relevance

Consider 2-jet final state topology



Two hemispheres defined by the thrust axis

- S_A (forward radiation)
- S_B (backward radiation)

Not all the contributions singled out by the factorization procedure are relevant for the study of TMD effects.

TMD-relevant contributions

$$\int d^{D-2} \vec{k}_T e^{-i \vec{k}_T \cdot \vec{b}_T}$$

E.g. A-coll radiation

TMD-irrelevant contributions

$$\int d^{D-2} \vec{k}_T$$

E.g. all radiation flowing into S_B

This is particularly relevant for soft and soft-collinear S_A radiation

	soft	soft-coll	coll
R_1	Relevant	Relevant	Relevant
R_2	Irrelevant	Relevant	Relevant
R_3	Irrelevant	Irrelevant	Relevant

“Bottom-up” Factorization Proof

TMD relevance is an “easy” way to implement the kinematic requirements and **(H1)** and **(H2)** into perturbative computations.



“BOTTOM-UP” approach to factorization
Proceeding order by order in pQCD and
then deducing the all-orders cross section

ADVANTAGES:

- Easy way to approach the problem, one should simply apply the factorization procedure (power counting, approximators...)
- the steps of the factorization proof are much clearer, as the various contributions can be readily disentangled and made more explicit transparent

COMPLICATIONS:

- Explicit pQCD computations may be potentially very difficult. In this case:
 - Non-trivial implementation of 2-jet limit in b_T -space
 - Non-standard mathematical tools required to compute some of the integrals, e.g.

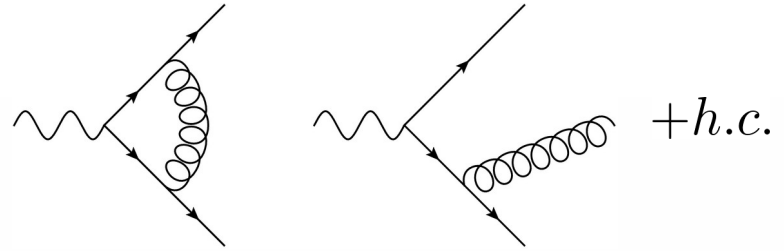
$$\left(\frac{1}{\tau}\right)_+ \left(\frac{1}{1-z}\right)_+ {}_0F_1\left(1-\varepsilon; -\tau\frac{1-zb^2}{z4}\right) \quad I(a, r) = \int_0^1 dx \frac{x^{\varepsilon/2}}{x-r} J_{-\varepsilon}\left(\frac{a}{\sqrt{x}}\right)$$

- Blindness to effects arising at higher orders

Region 2 at 1-loop

$$\frac{1}{\sigma_B} \frac{d\sigma_{R_2}^{[1]}}{dz db_T d\tau} =$$

Virtual gluon




$$= N_C \left[\delta(1-z) \left(\delta(\tau) V(\varepsilon) + \underbrace{J^{[1]}(\varepsilon; \tau) + S_-^{[1]}(\varepsilon; \tau)}_{\text{Backward gluon}} + \underbrace{S_+^{[1]}(\varepsilon; \tau, y_1)}_{\text{Forward gluon}} \right) + \delta(\tau) z \tilde{D}_{q/q}^{[1],(0)}(\varepsilon; z, b_T, y_1) \right]$$

Backward gluon:

- Jet Thrust Function $J^{[1]}(\varepsilon, \tau)$ (from B-collinear approximator)
- Soft Thrust Function in the S_B -hem. $S_-^{[1]}(\varepsilon; \tau)$ (from combination of soft and softcollinear approximators)

Forward gluon:

- Generalized Soft Thrust Function $S_+^{[1]}(\varepsilon; \tau, y_1)$ (from soft approximator) 
- q-from.q (bare) TMD FF $\tilde{D}_{q/q}^{[1],(0)}(\varepsilon; z, b_T, y_1)$
in **factorization definition** (from combination of soft and softcollinear approximators)

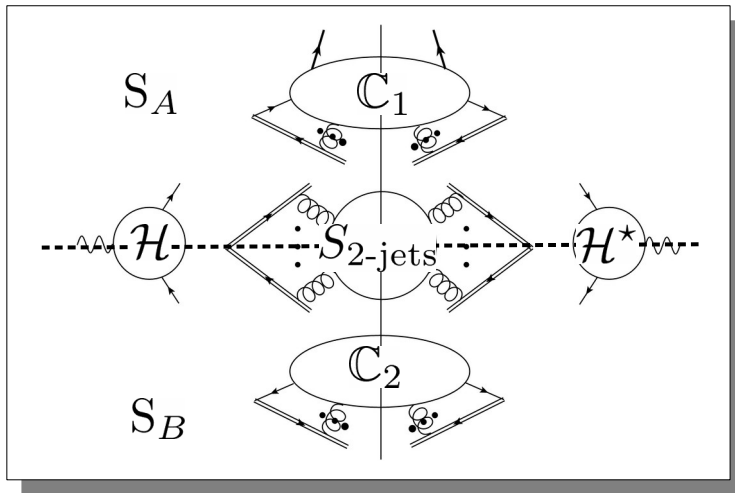
- ✓ Finite (no poles in ε)
- ✓ RG-invariant ($\mu = Q$)
- ✗ Not CS-invariant → The final result depends explicitly on y_1

Region 2: Factorized Cross Section

$$\frac{d\sigma_{R_2}}{dz_h dP_T^2 dT} = \sigma_B \pi N_C \int d\tau_{S_+} d\tau_{S_-} d\tau_B \delta(\tau - \tau_{S_+} - \tau_{S_-} - \tau_B) \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i \frac{\vec{P}_T}{z_h} \cdot \vec{b}_T} \sum_f e_f^2 \times$$

$$\times V \underbrace{J(\tau_B) S_-(\tau_{S_-}) S_+(\tau_{S_+}, y_1)}_{\text{Backward radiation}} \underbrace{\tilde{D}_{h/f}(z_h, b_T, y_1)}_{\text{Forward radiation}}$$

Virtual radiation



The same result can be obtained adopting a “top-down” approach to factorization

$$W_h^{\mu\nu}(z_h, \vec{P}_T, T) = \sum_{j_1} \int \frac{d^D k_1}{(2\pi)^D} \delta \left(\vec{P}_T \left[1 + \mathcal{O} \left(\frac{P_T^2}{Q^2} \right) \right] + \frac{P^+}{k_1^+} \vec{k}_{1,T} \right) \sum_{j_2} \int \frac{d^D k_2}{(2\pi)^D}$$

$$\times \text{Tr}_D \left\{ P_1 C_1(k_1, P)_{j_1} \bar{P}_1 \mathcal{H}_{j_1 j_2}^\mu(\hat{k}_1, \hat{k}_2) P_2 C_2(k_2)_{j_2} \bar{P}_2 (\mathcal{H}^\dagger)_{j_1 j_2}^\nu(\hat{k}_1, \hat{k}_2) \right\}$$

$$\times \int \frac{d^D k_S}{(2\pi)^D} S_{2\text{jets}; j_1 j_2}(k_S) \delta(q - k_1 - k_2 - k_S) \delta(T - T_{\text{def.}}(k_1, k_2, k_S))$$

(H2)
(H1)

Region 2: Collinear-TMD Factorization



$$\frac{d\sigma_{R_2}}{dz_h dP_T^2 dT} = \sigma_B \pi N_C \int d\tau_{S_+} d\tau_{S_-} d\tau_B \delta(\tau - \tau_{S_+} - \tau_{S_-} - \tau_B) \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i \frac{\vec{P}_T}{z_h} \cdot \vec{b}_T} \sum_f e_f^2 \times$$

Re-organization
into a partonic
cross-section

$$\times V J(\tau_B) S_-(\tau_{S_-}) \mathcal{S}_+(\tau_{S_+}, y_1) \tilde{D}_{h/f}(z_h, b_T, y_1) =$$

Fully perturbative!

$$= \pi \sum_f \int_{z_h}^1 \frac{d\hat{z}}{\hat{z}} \frac{d\hat{\sigma}_f(y_1)}{d(z_h/\hat{z}) dT} \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i \frac{\vec{P}_T}{\hat{z}} \cdot \vec{b}_T} \tilde{D}_{f,h}(\hat{z}, b_T, y_1)$$

$$\sim \sum_f \hat{\sigma}_f(T, y_1) \otimes \tilde{D}_{f,h}(b_T, y_1)$$

A **new kind** of factorization theorem (never encountered before in literature)

- Same structure of collinear factorized cross sections
- The convolution involves TMDs, in **Factorization Definition**
- Non-trivial role of rapidity cut-off

} *Hybrid nature*
between collinear and
TMD factorization

Region 2: Collinear-TMD Factorization

Universality-breaking effects in e^+e^- hadronic production processes, M. Boglione, A. Simonelli, Eur.Phys.J.C 81 (2021) 1, 96

- The convolution involves TMDs, in **Factorization Definition**
How to match with usual sqrt. definition of TMDs?

$$\tilde{D}_{h/f}^{\text{sqrt}}(z_h, b_T, \dots) = \tilde{D}_{h/f}(z_h, b_T, \dots) \sqrt{M_S(b_T)} \leftarrow$$

Which means:

$$M_D^{\text{sqrt}}(z_h, b_T, \dots) = M_D(z_h, b_T, \dots) \sqrt{M_S(b_T)}$$

SOFT MODEL

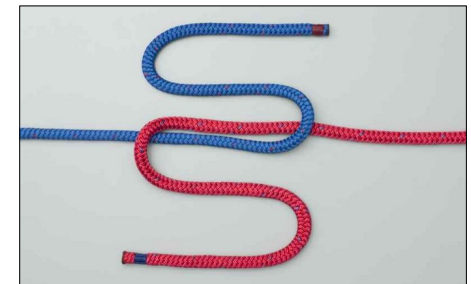
Non-Perturbative function describing the long-distance behavior of 2-h soft factor. It depends ONLY on b_T .



This highlights a strategy for extracting TMDs:

1. Extract M_D from $e^+e^- \rightarrow h X$ in Region 2 \longrightarrow See talk by O. Gonzalez
2. Compare with M_D^{sqrt} (benchmark SIDIS, $e^+e^- \rightarrow h_1 h_2 X$) to obtain the soft model as the ratio $M_S = D^{\text{sqrt}}/D$.

In this scheme the Soft Factor acquires a central and active role in phenomenological analyses.



3. The only remaining unknowns are the models of TMD PDFs, (benchmark SIDIS, Drell-Yan)

Region 2: Collinear-TMD Factorization

- Non-trivial role of rapidity cut-off $\frac{\partial}{\partial y_1} \sum_f \hat{\sigma}_f(y_1) \otimes \tilde{D}_{f,h}(b_T, y_1) \neq 0$

The rapidity cut-off is *intimately* related to the value of thrust:

$$y_P = \frac{1}{2} \log \frac{P^+}{P^-} = \log \frac{z_h Q}{\sqrt{P_T^2 + M_h^2}} \geq -\frac{1}{2} \log(1 - T) + \mathcal{O}\left(\frac{M_h^2}{P_T^2}\right),$$

- Since y_1 is related to a measured quantity, it cannot be just a mere computational tool and **it must acquire a real physical meaning**.
- However, in Collins factorization formalism the exact relation between rapidity cut-off and thrust cannot be made explicit: a **new** way of regularizing rapidity divergences is required.
- Without an explicit relation between y_1 and thrust, the resummation on T is compromised. A shortcut to circumvent this issue is:
 - Neglect all the T-divergent terms
 - Fix the rapidity cut-off according to the kinematic argument above: $y_1 = -\frac{1}{2} \log(1 - T)$

Factorization of $e^+e^- \rightarrow hX$ cross section,
differential in z_h, P_T and thrust, in the 2-jet limit,
M. Boglione, A. Simonelli, **JHEP 02 (2021) 076**

In Region 2 the TMD Ffs depend on thrust through their rapidity cut-off

Region 1: TMD Factorization

$$\frac{d\sigma_{R_1}}{dz_h dP_T^2 dT} = \sigma_B \pi N_C \int d\tau_S d\tau_B \delta(\tau - \tau_S - \tau_B) \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i \frac{\vec{P}_T \cdot \vec{b}_T}{z_h}} \sum_f e_f^2 \times \\ \times V J(\tau_B) S_-(\tau_S) \tilde{\mathbb{S}}_{2-h,+}(b_T, y_1) \tilde{D}_{h/f}(z_h, b_T, y_1)$$

A TMD factorization theorem:

- “half” of the structure present in SIDIS, $e^+e^- \rightarrow h_1 h_2 X$
- Non-trivial non-perturbative contribution of soft radiation
- CS-invariance
- Possible Non-Global Logs effects should be considered

The combination of non-perturbative functions in the cross section behaves as if the TMD FF was defined by the **square root def.**

$$\tilde{\mathbb{S}}_{2-h,+}(b_T, y_1) \tilde{D}_{h/f}(z_h, b_T, y_1) \propto \sqrt{M_S(b_T)} M_D(b_T, \dots) = M_D^{\text{sqrt}}(b_T, \dots)$$

Region 1: TMD Factorization

$$\frac{d\sigma_{R_1}}{dz_h dP_T^2 dT} = \sigma_B \pi N_C \int d\tau_S d\tau_B \delta(\tau - \tau_S - \tau_B) \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\frac{\vec{P}_T \cdot \vec{b}_T}{z_h}} \sum_f e_f^2 \times \\ \times V J(\tau_B) S_-(\tau_S) \tilde{\mathbb{S}}_{2-h,+}(b_T, y_1) \tilde{D}_{h/f}(z_h, b_T, y_1)$$

In the literature (SCET):

- The same cross section is presented in:
Joint thrust and TMD resummation in electron-positron and electron-proton collisions, Y. Makris, F. Ringer, W.J. Waalewijn **JHEP 02 (2021) 070**
- A similar structure is also presented in:
QCD resummation on single hadron transverse momentum distribution with the thrust axis, Z. Kang, D. Shao, F. Zhao, **JHEP 12 (2020) 127**.
However their cross section is integrated over thrust. It can be recovered as:

$$\int_0^1 d\tau d\sigma_{R_1} + \text{Contribution of Fragmenting gluons} = \text{Kang's cross section}$$

However, in the literature there is no mention to the different definition of the TMD we must adopt

Region 3: Generalized Collinear Factorization

$$\frac{d\sigma_{R_3}}{dz_h dP_T^2 dT} = \sigma_B \pi N_C \int d\tau_S d\tau_A d\tau_B \delta(\tau - \tau_S - \tau_A - \tau_B) \sum_f e_f^2 \times \\ \times V J(\tau_B) S(\tau_S) \Gamma_{h/f} \left(z_h, \frac{P_T}{z_h}, \tau_A \right)$$

Generalized Fragmenting Jet Function

- A lot in common with collinear FFs (e.g. DGLAP) but carries TMD information
- Its non-perturbative part depends EXPLICITLY on the thrust (invariant mass of the jet)

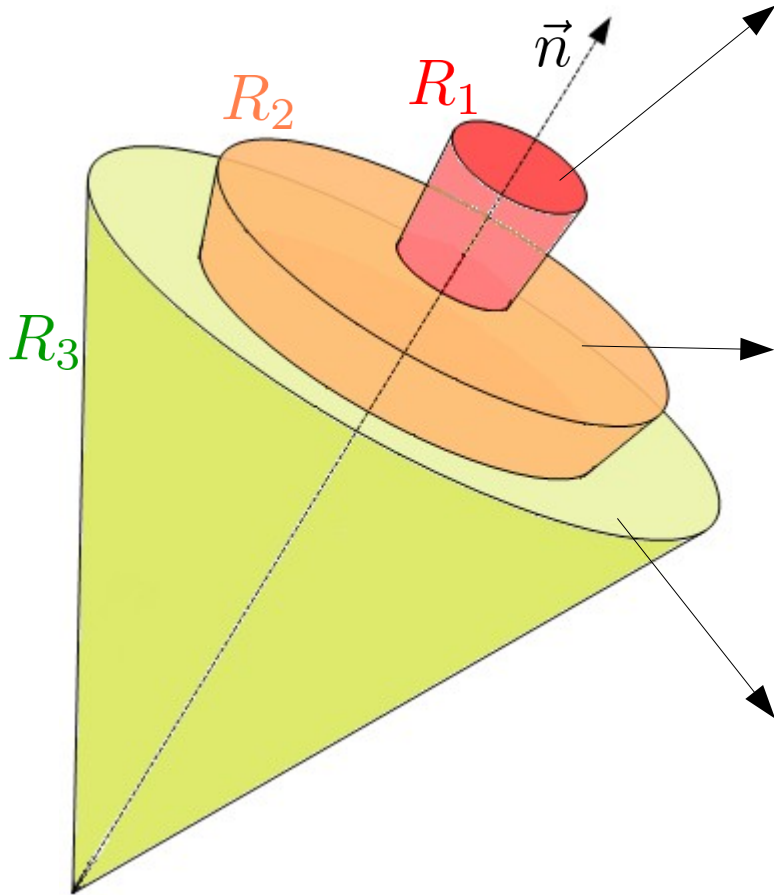
A generalized collinear factorization theorem:

- None of the functions in the cross section depend on a rapidity cut-off
- “Clean” way to access GFJFs

In the literature (SCET):

- The same cross section is presented in:
Joint thrust and TMD resummation in electron-positron and electron-proton collisions, Y. Makris, F. Ringer, W.J. Waalewijn **JHEP 02 (2021) 070**

Three distinct factorization theorems



Kinematic Regions in $e^+e^- \rightarrow h X$ process in a 2-jet topology, M. Boglione, A. Simonelli, arxiv:2109.xxyy (in preparation)

TMD Factorization (similar to benchmarks).

TMD FFs are contaminated by soft radiation as in the **square root definition**. Access to:

$$M_S, M_D, g_K$$

collinear-TMD Factorization

TMD FFs are pure collinear objects, defined through the **factorization definition**. Access to:

$$M_D, g_K$$

NEW!

Generalized collinear Factorization

The hadronization is described by GFJFs, which depend explicitly on thrust.

How to distinguish between the regions

- The transverse momentum of the detected hadron increases as we move from Region 1 to Region 3.



- Three variables to take into account z_h, T, P_T
- The boundaries of the three regions are not sharply defined, making the description of data difficult, especially in the overlapping regions.

→ **Problem of matching**
(c.f. e.g. SIDIS benchmark cases)

- TMDs (and GFJFs) are well defined only where $P_T \ll P^+$

We need an ALGORITHM

Kinematic Regions in $e^+e^- \rightarrow h X$ process in a 2-jet topology, M. Boglione, A. Simonelli, [arxiv:2109.xxyy](#) (in preparation)

Power-counting algorithm

The easiest algorithm is entirely based on power counting:

R_1	R_2	R_3
$\frac{P_T}{z_h} < \tau Q$	$\tau Q < \frac{P_T}{z_h} < \sqrt{\tau} Q$	$\frac{P_T}{z_h} \sim \sqrt{\tau} Q$

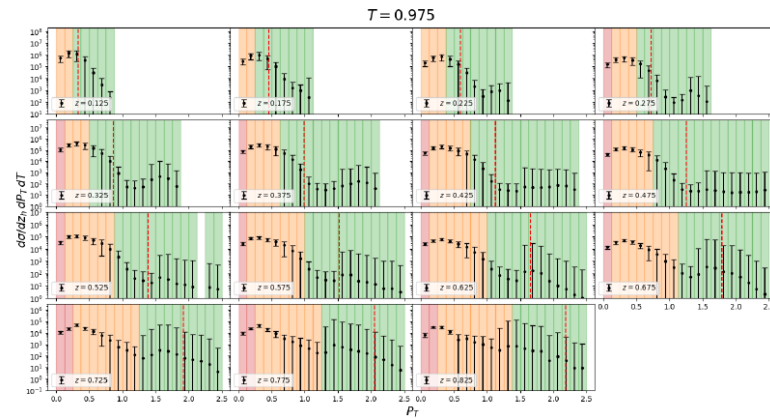
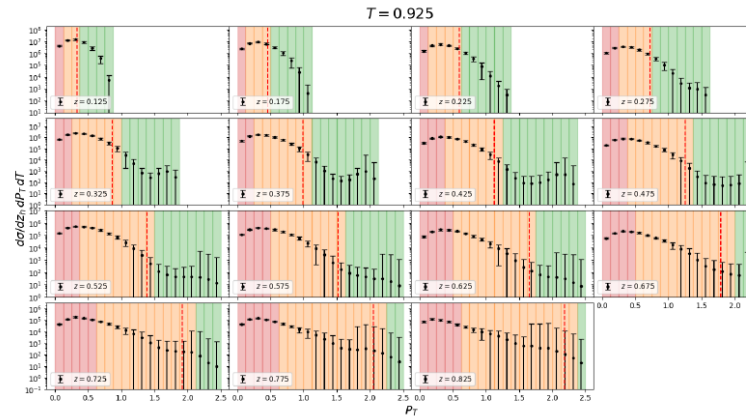
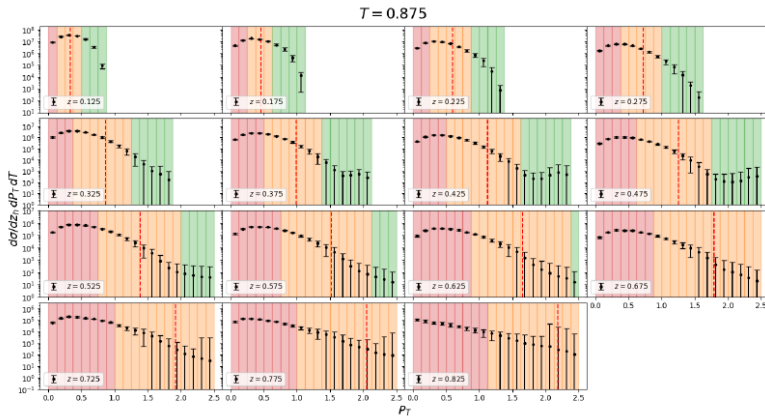
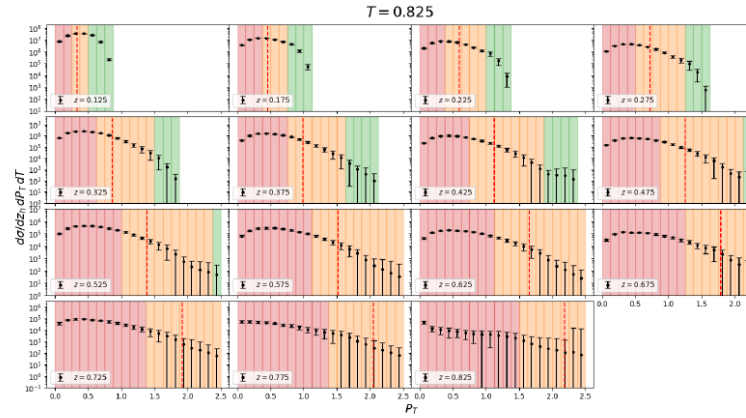
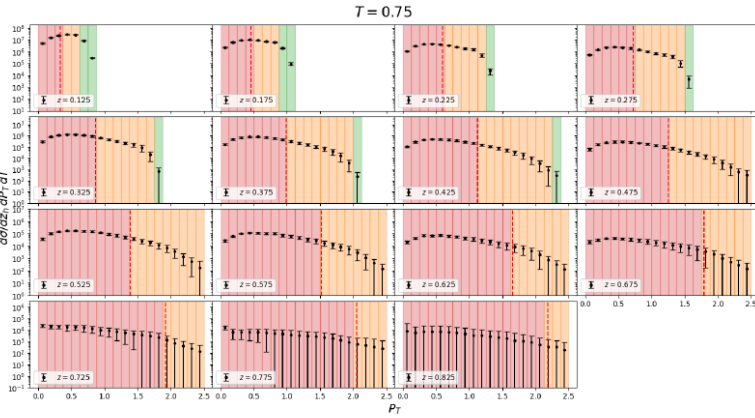
Proposed in Y. Makris, F. Ringer, W.J. Waalewijn
JHEP 02 (2021) 070

- ◆ Based on very general assumptions
- ◆ Very simple implementation
- ◆ The rapidity of the hadron is not taken into account
- ◆ Region 1 is overestimated
- ◆ Phenomenology cannot be performed without a well defined matching procedure (many bins are **multicolor**)

Difficult distinction
between R_1 and R_2

Power-counting algorithm

$$\frac{d\sigma}{dz_h dT P_T}$$



Ratios algorithm

- ◆ Based on the comparison of ratios that describe the kinematics of each region. Such ratios are inspired by 1-loop explicit computation.

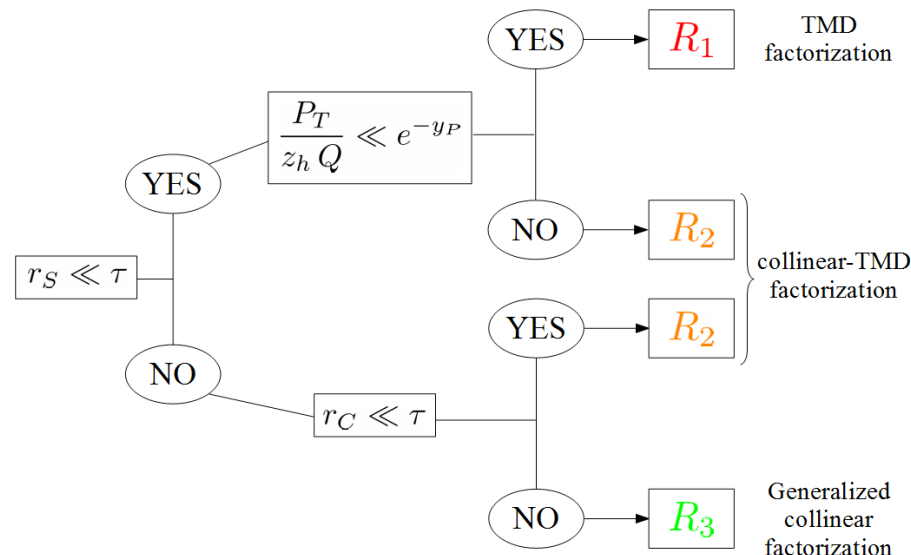
SOFT RATIO

$$r_S = \frac{P_T}{z_h Q} e^{-y_P}$$

COLLINEAR RATIO

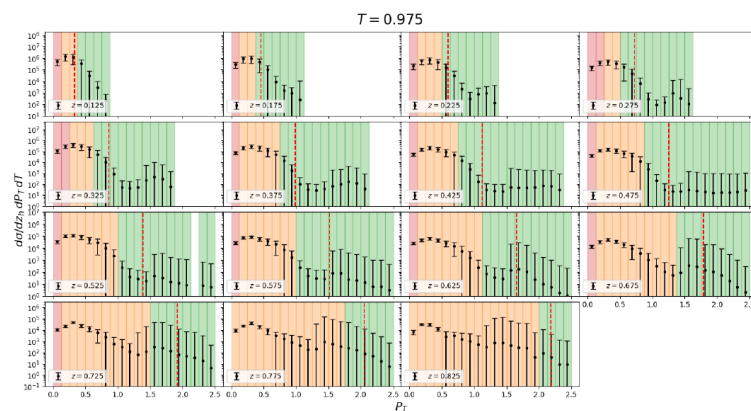
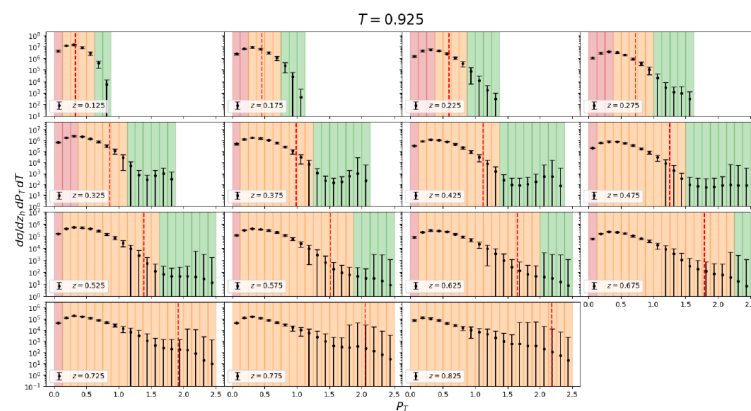
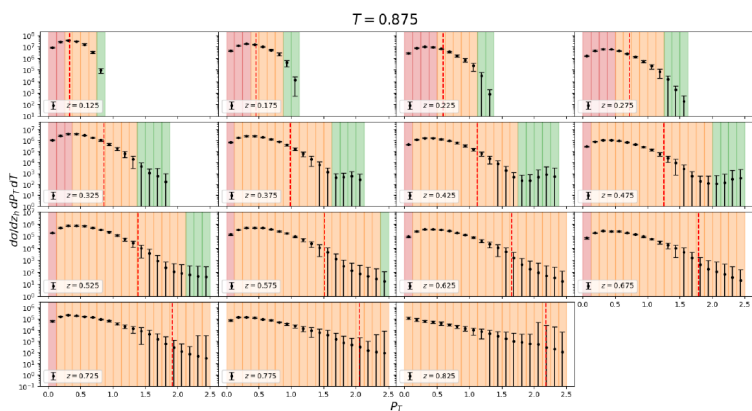
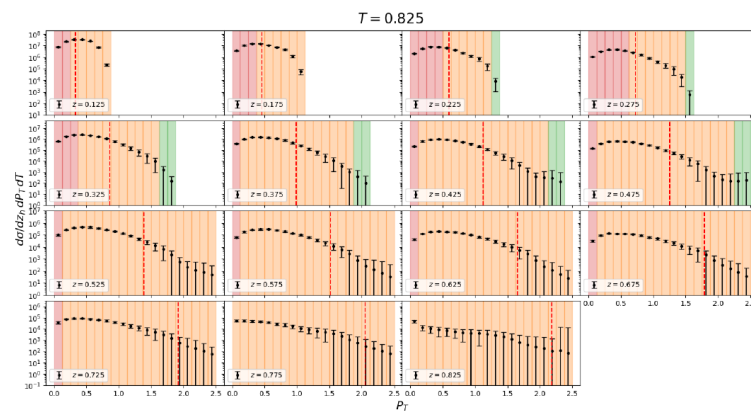
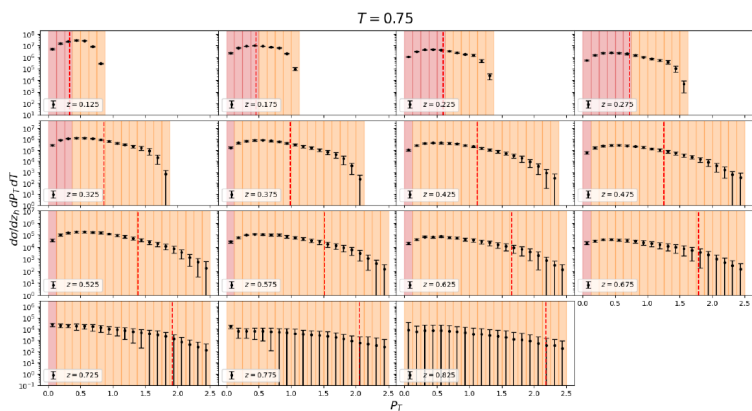
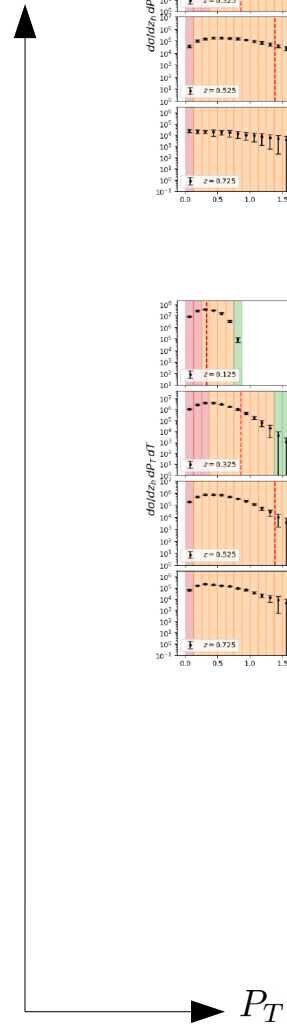
$$r_C = z_h (1 - z_h) e^{-2y_P}$$

- ◆ The rapidity of the hadron is taken explicitly into account
- ◆ Region 2 is the widest region as expected
- ◆ The problem of the matching is not urgent, as there are many **monochromatic** bins



Ratios algorithm

$$\frac{d\sigma}{dz_h dT P_T}$$



Conclusions

- ◆ The process $e^+e^- \rightarrow hX$ encodes a lot of incredibly interesting information regarding TMD physics.
- ◆ Three distinct factorization theorems, each associated to a different kinematic region.

R_1

TMD
factorization

R_2

collinear-TMD
factorization

R_3

Generalized collinear
factorization

- ◆ Collinear-TMD factorized cross sections (R_2) are a new kind of observables, where rapidity cut-offs show their hidden physical meaning. This may shed light on on totally new aspects of the hadronization mechanism and on the confinement of partons.

THANK YOU FOR YOUR ATTENTION!