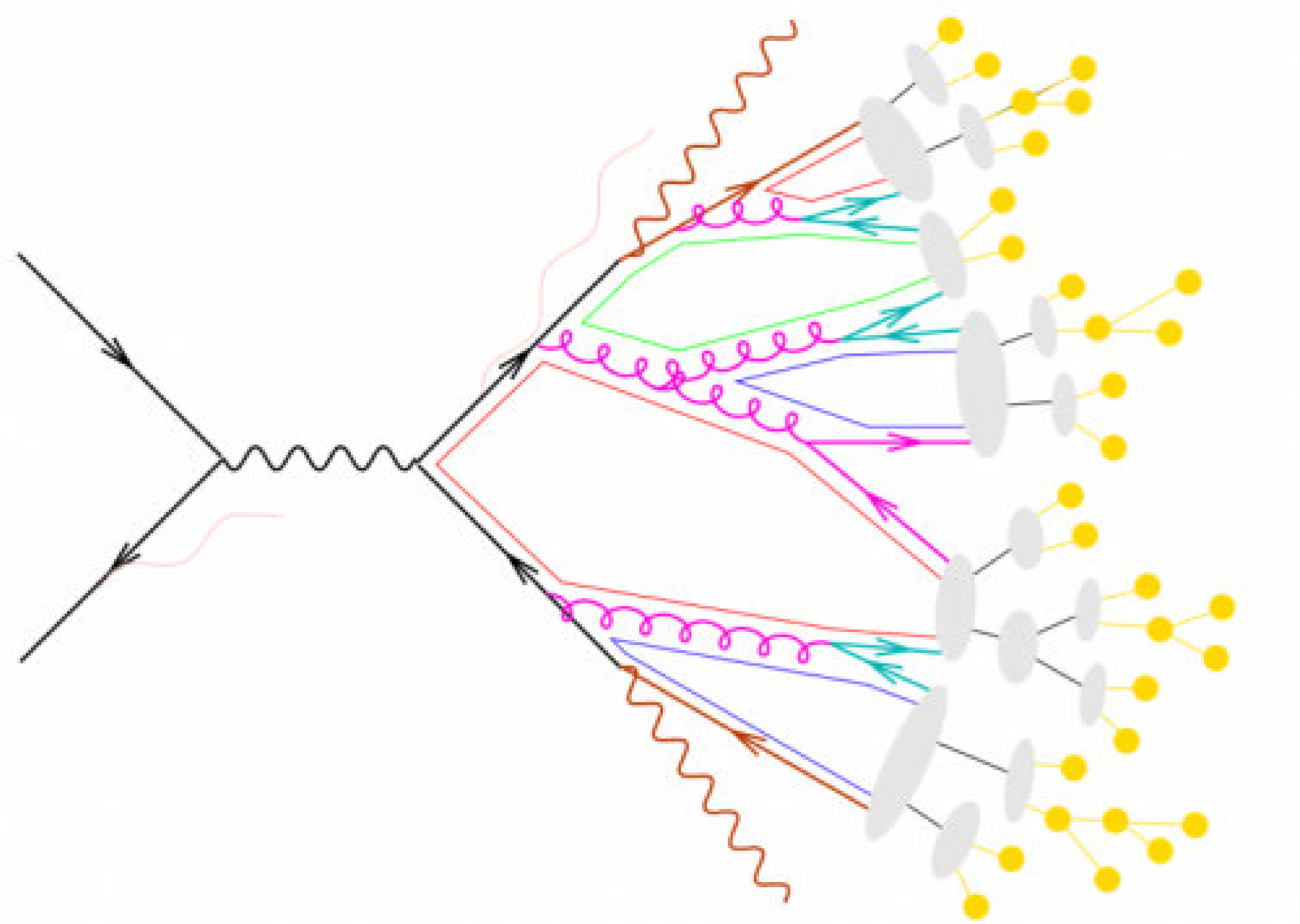
Sardinian Workshop on Spin Studies - Cagliari, September 6-8, 2021


[courtesy A. Signori]

TMD fragmentation functions from electron-positron annihilation: experimental results

This work is part of a project that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement STRONG-2020 - No 824093

Gunar.Schnell @ DESY.de

single-hadron*) (TMD) fragmentation functions

*) complemented by rich world of di-hadron FFs

quark pol.

hadron pol.

	U	L	T
U	D_1		$H_1^{\perp}$
L		G_1	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	$G_{1T}^{\perp}$	H_1 $H_{1T}^{\perp}$

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☛ relevant for unpolarized final state

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☛ relevant for unpolarized final state

Collins FF: $H_1^{\perp,q\to h}$

ordinary FF: $D_1^{q\to h}$

FF … fragmentation function

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polarized final-state hadrons

polarizing FF

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☛ relevant for unpolarized final state

polarized final-state hadrons

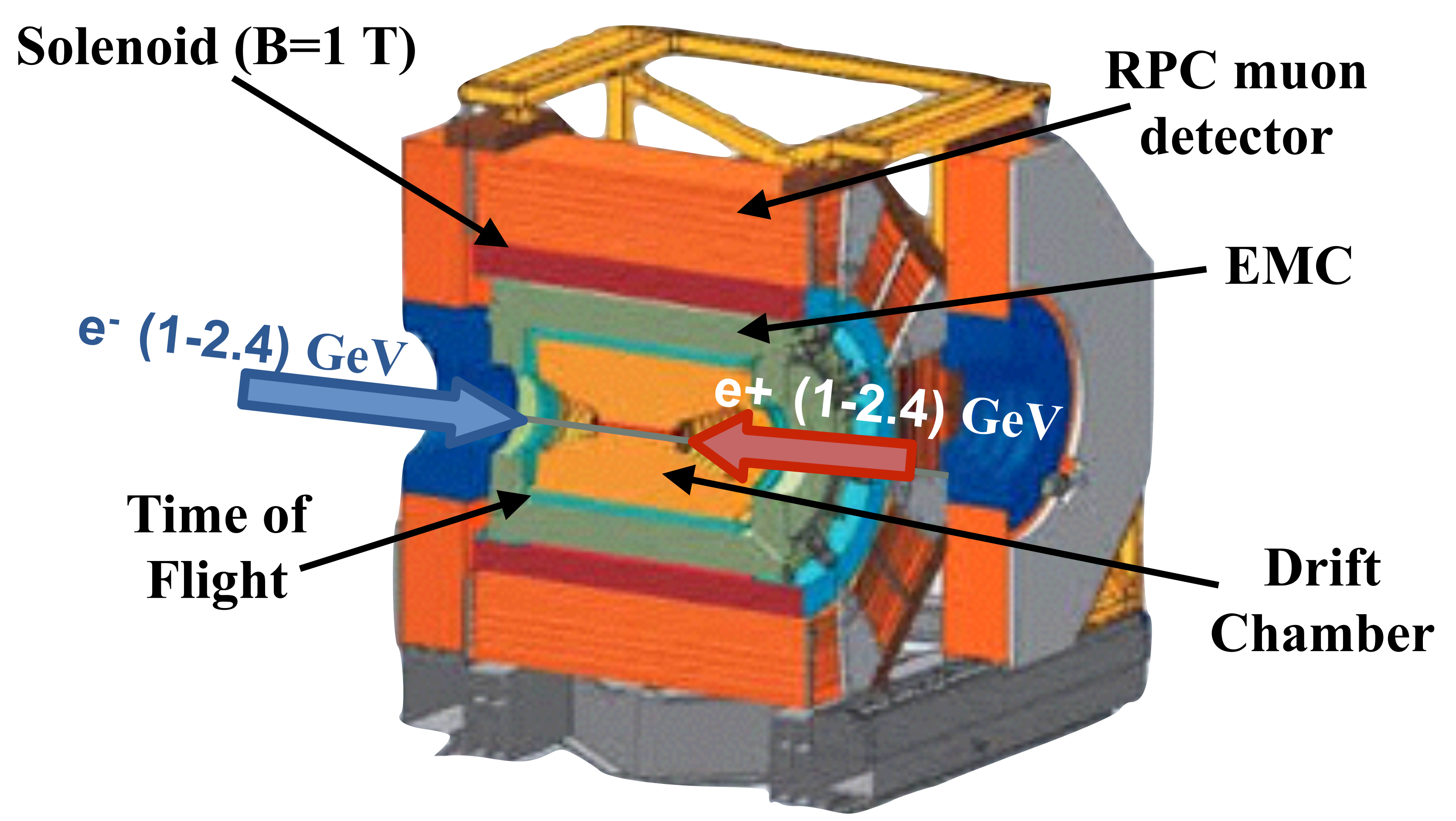
polarizing FF

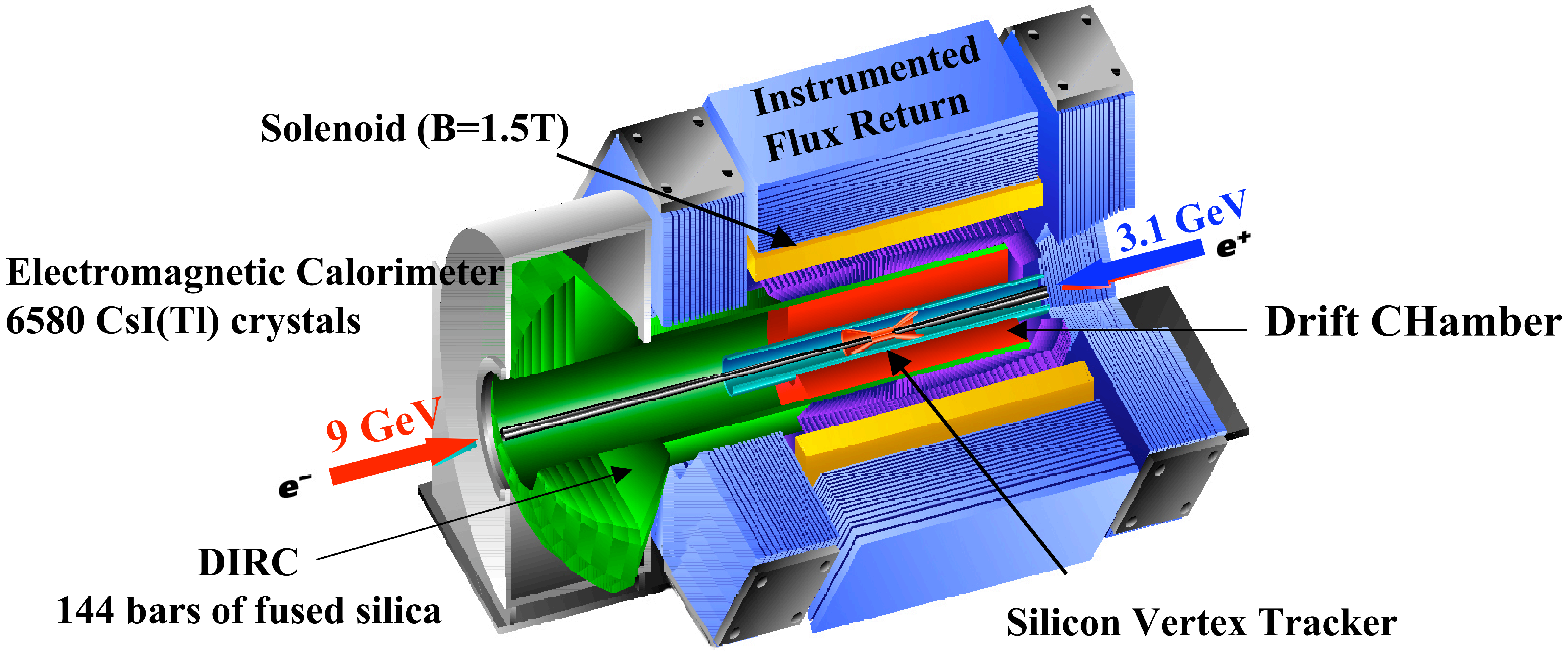
➡ FFs act as quark flavor-tagger and polarimeter

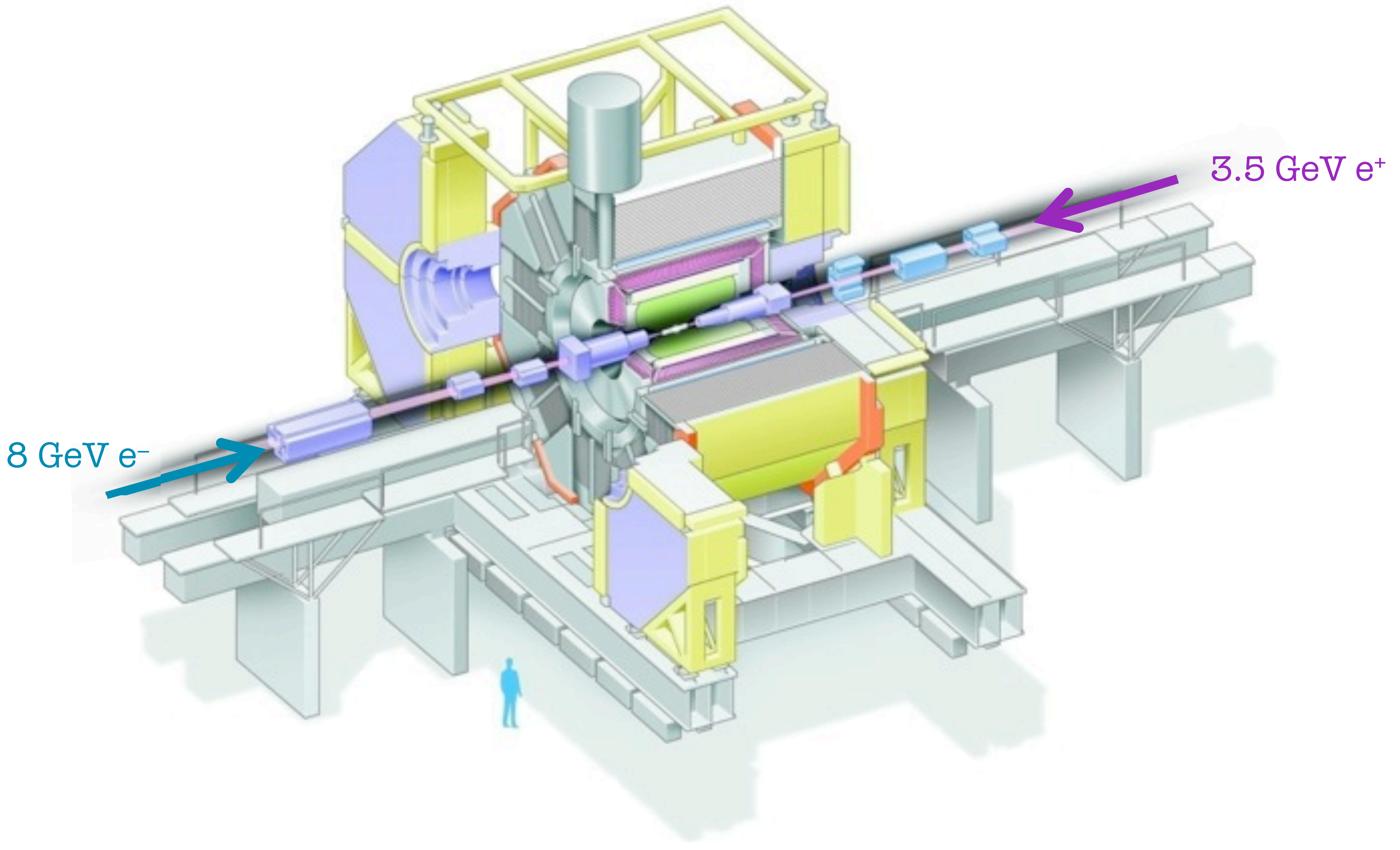
FF … fragmentation function

e^+e^- annihilation at BESIII, BaBar & Belle

- BESIII: symmetric collider (E_e=1…2.4 GeV)
- BaBar/Belle: asymmetric beam-energy e^+e^- collider near/at $\Upsilon(4S)$ resonance





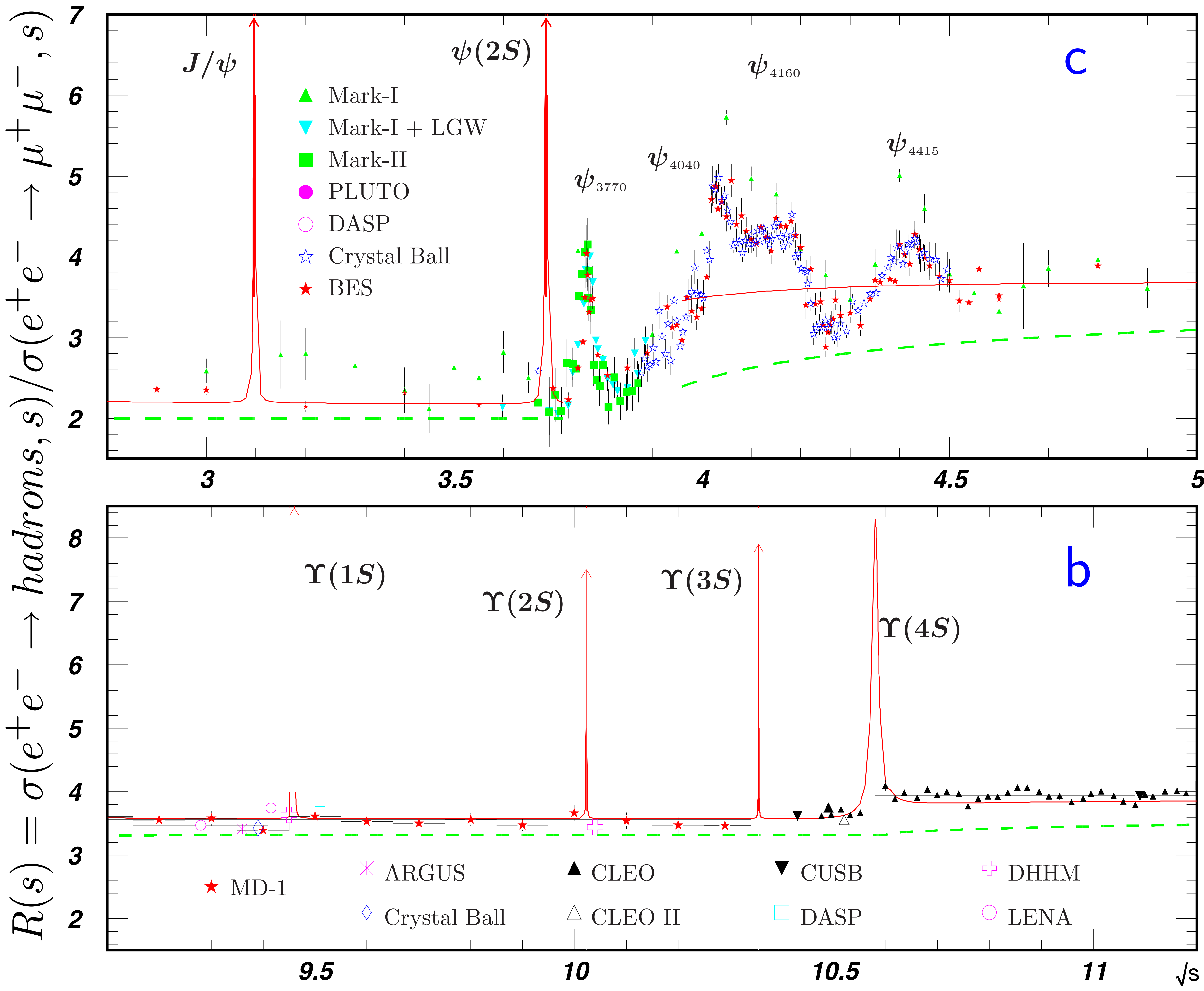


e^+e^- annihilation at BESIII, BaBar & Belle

- BESIII: symmetric collider (E_e=1…2.4 GeV)
- BaBar/Belle: asymmetric beam-energy e^+e^- collider near/at Υ(4S) resonance
- different scales (➠ QCD evolution) and sensitivities to quark flavor
- integrated lumi used for FF analyses:

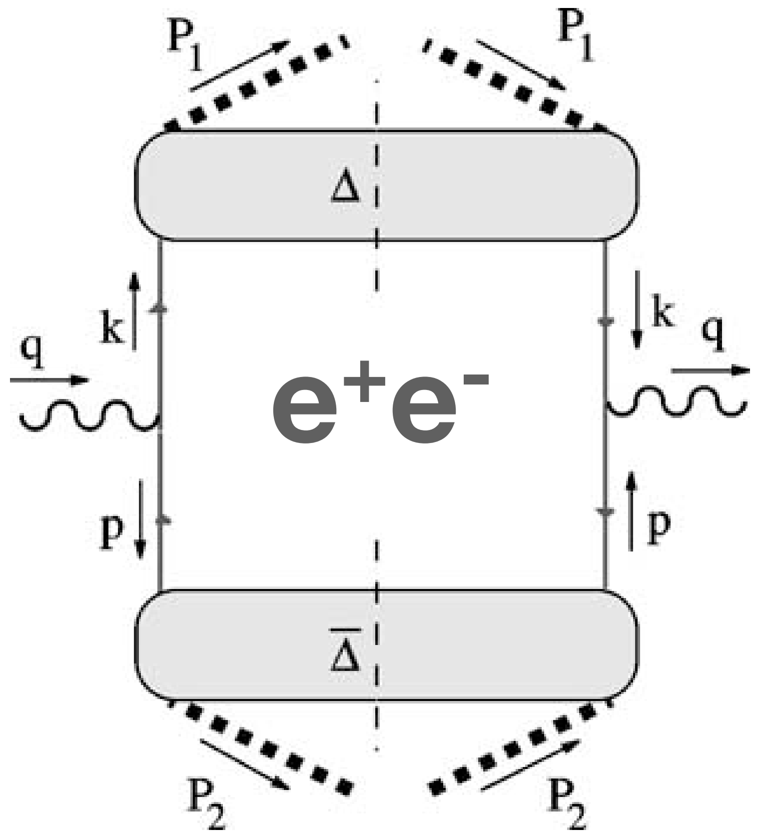
	Υ(4S) on resonance	Υ(4S) off resonance	other
BaBar	424.2 fb^{-1}	43.9 fb^{-1}	
Belle	(140+571) fb^{-1}	(15.6+73.8) fb^{-1}	~180 fb^{-1} @Υ(nS)
BESIII			~62 pb^{-1} @3.65 GeV *)

*) used for the Collins analysis



fragmentation in e^+e^- annihilation

- single-inclusive hadron production, **e^+e^- → hX**
  - D_1 fragmentation function
  - ($D_{1T}^{\perp}$ spontaneous transv. polarization)



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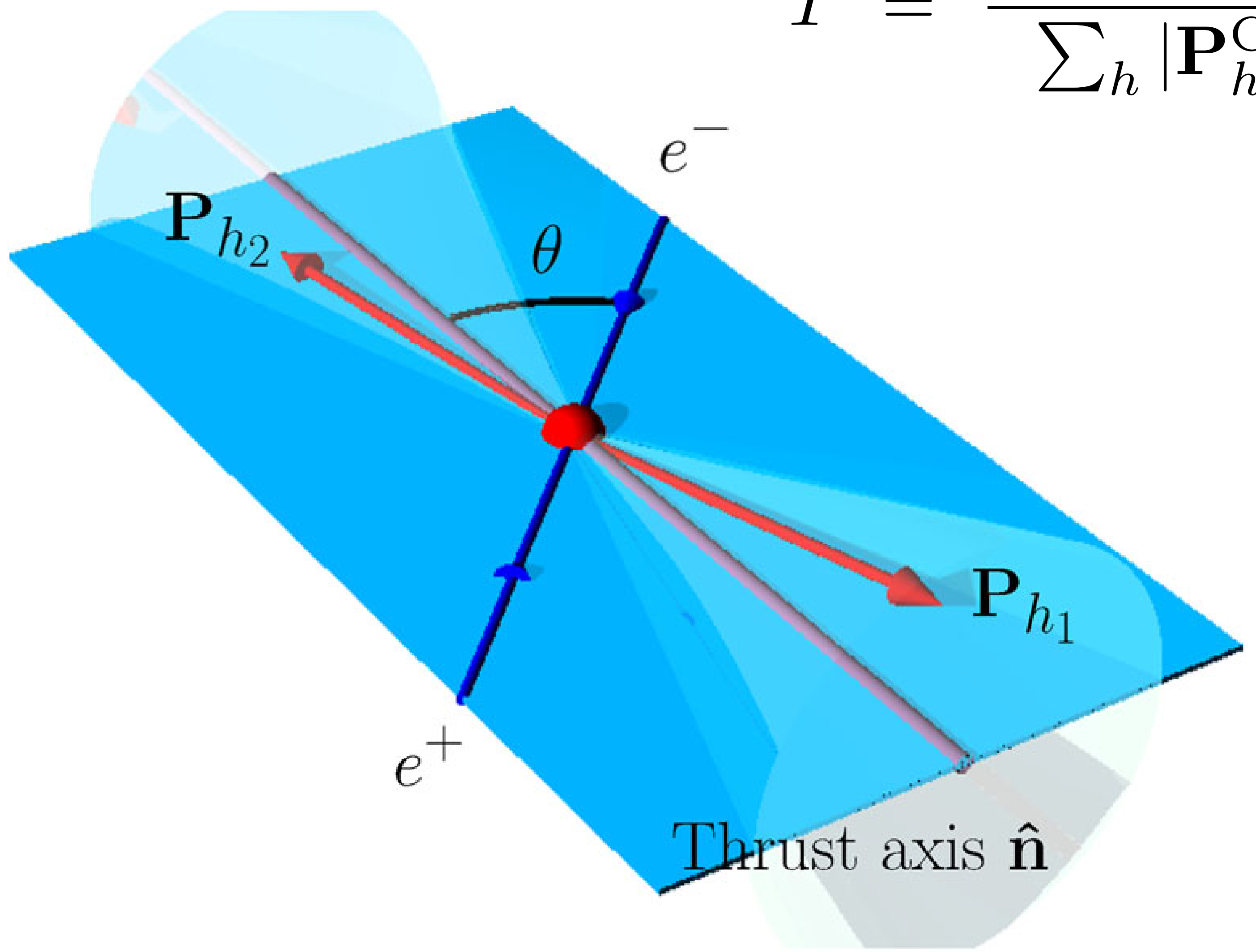
  - product of fragmentation functions
  - flavor, transverse-momentum, and/or polarization tagging

Thrust (axis):

$$T \overset{\max}{=} \frac{\sum_h |\mathbf{P}_h^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\mathrm{CMS}}|}$$



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  - flavor, transverse-momentum, and/or polarization tagging
- inclusive same-hemisphere hadron pairs, **e^+e^- → h_1h_2X**
  - di-hadron fragmentation

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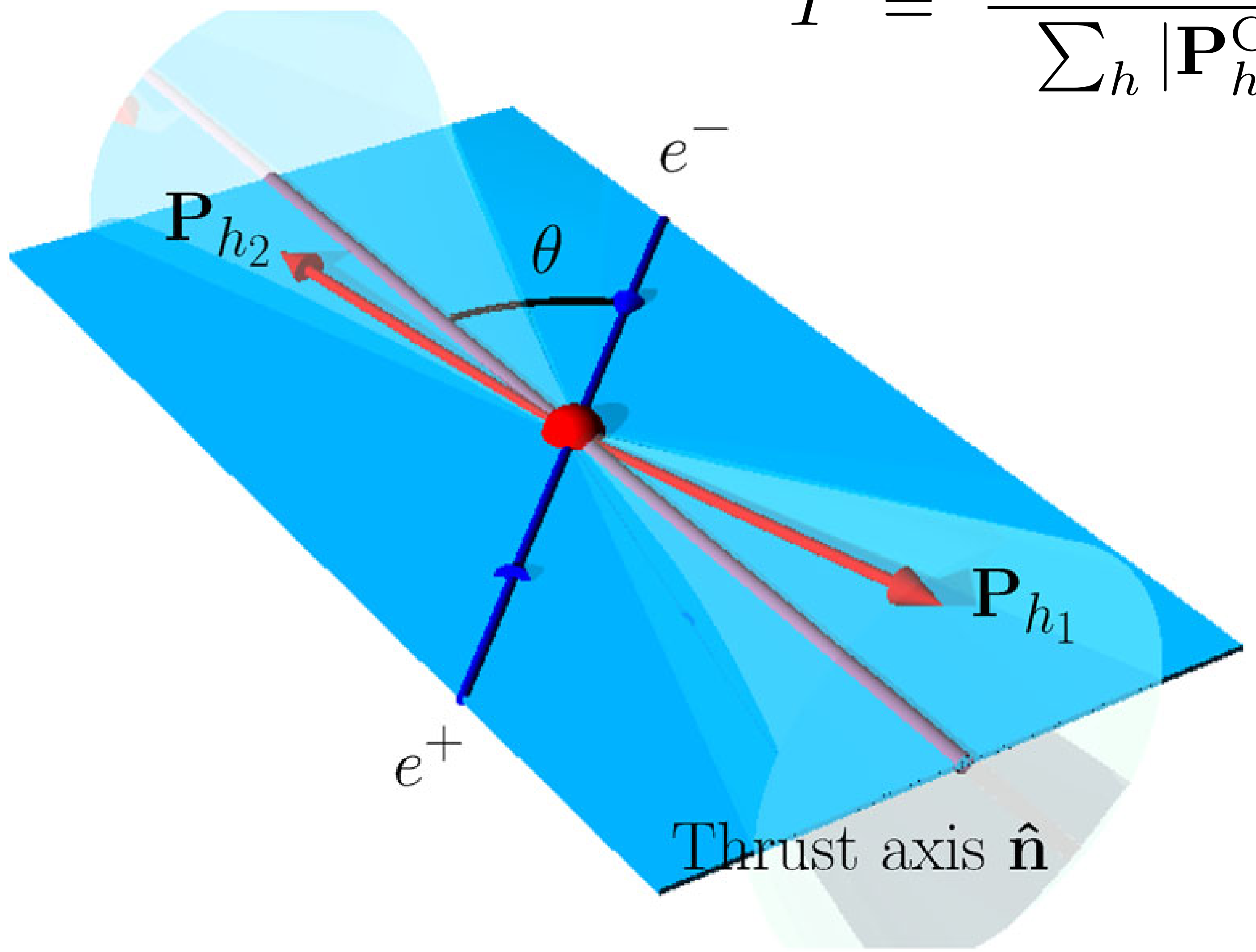
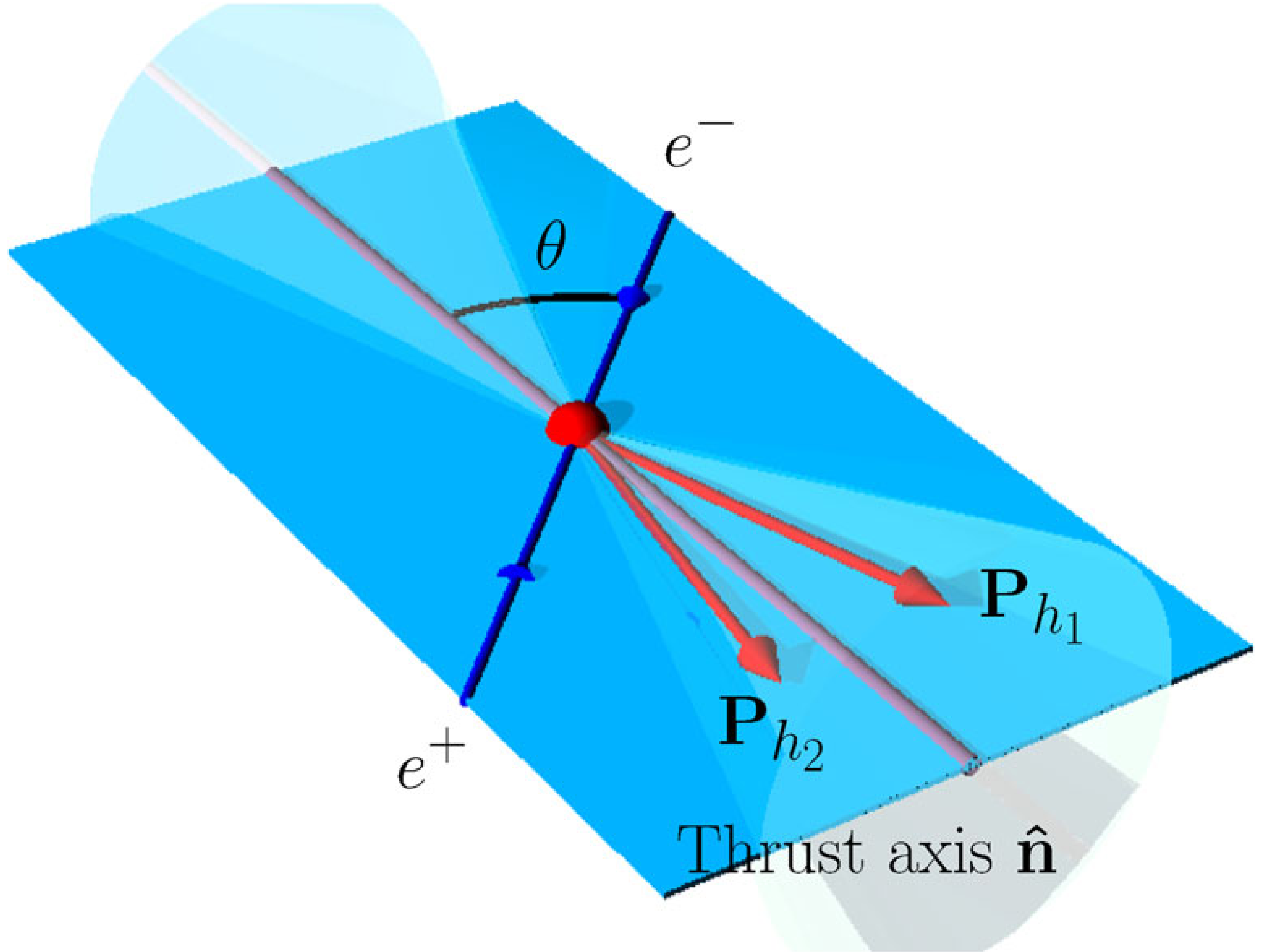
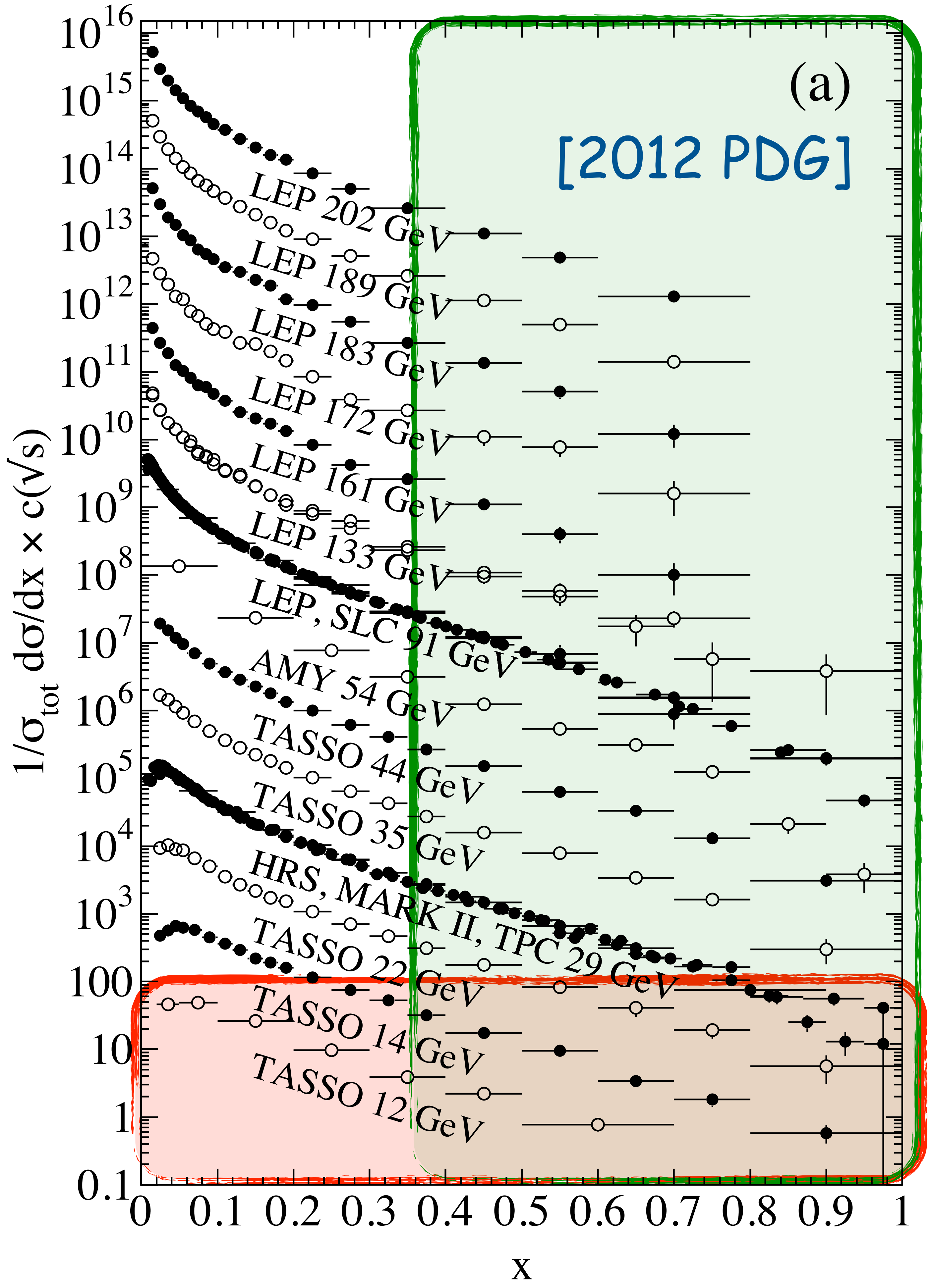




the collinear case

single-hadron production

- before 2013: lack of precision data at (moderately) high z and low √s
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS



single-hadron production

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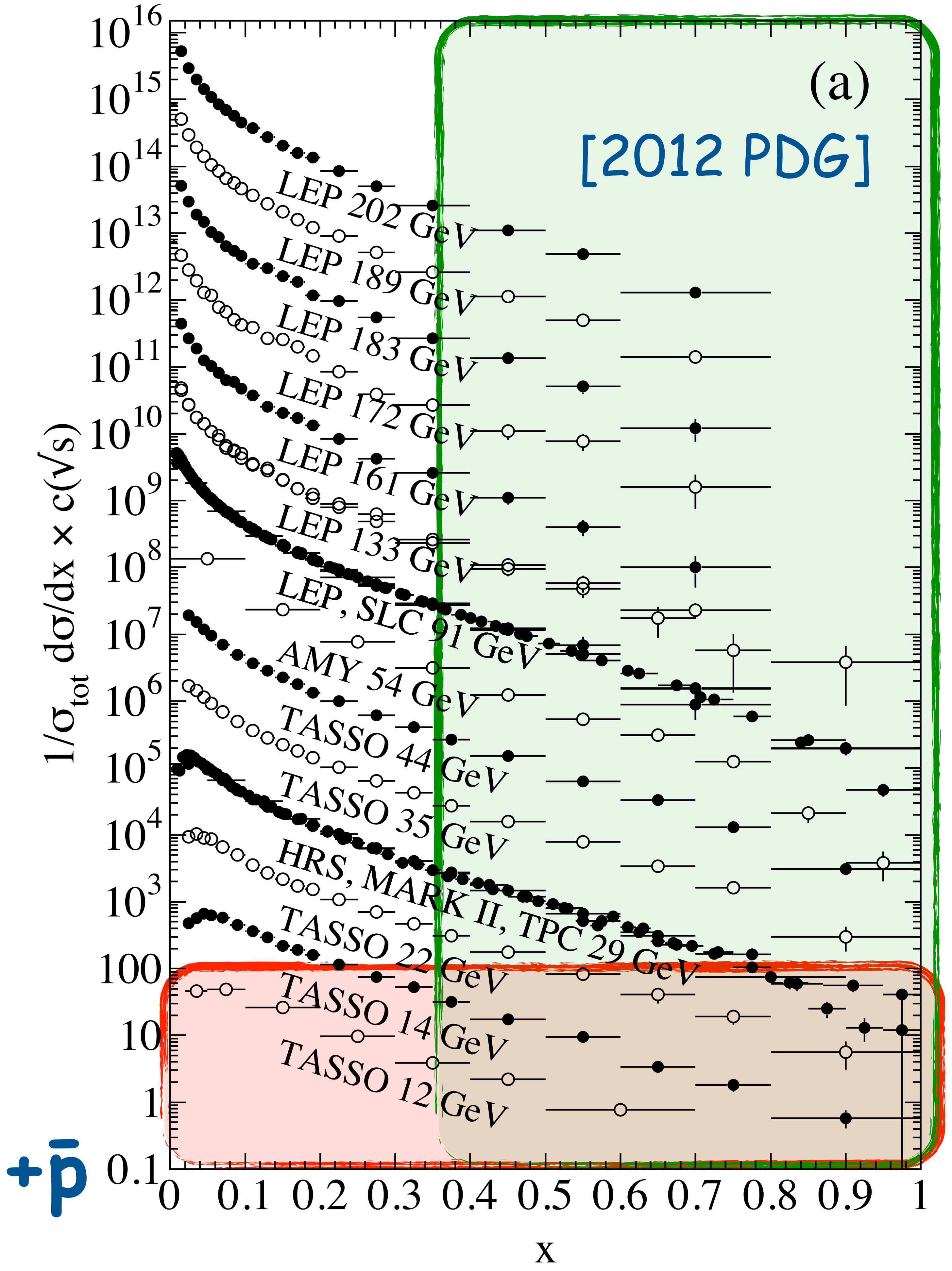
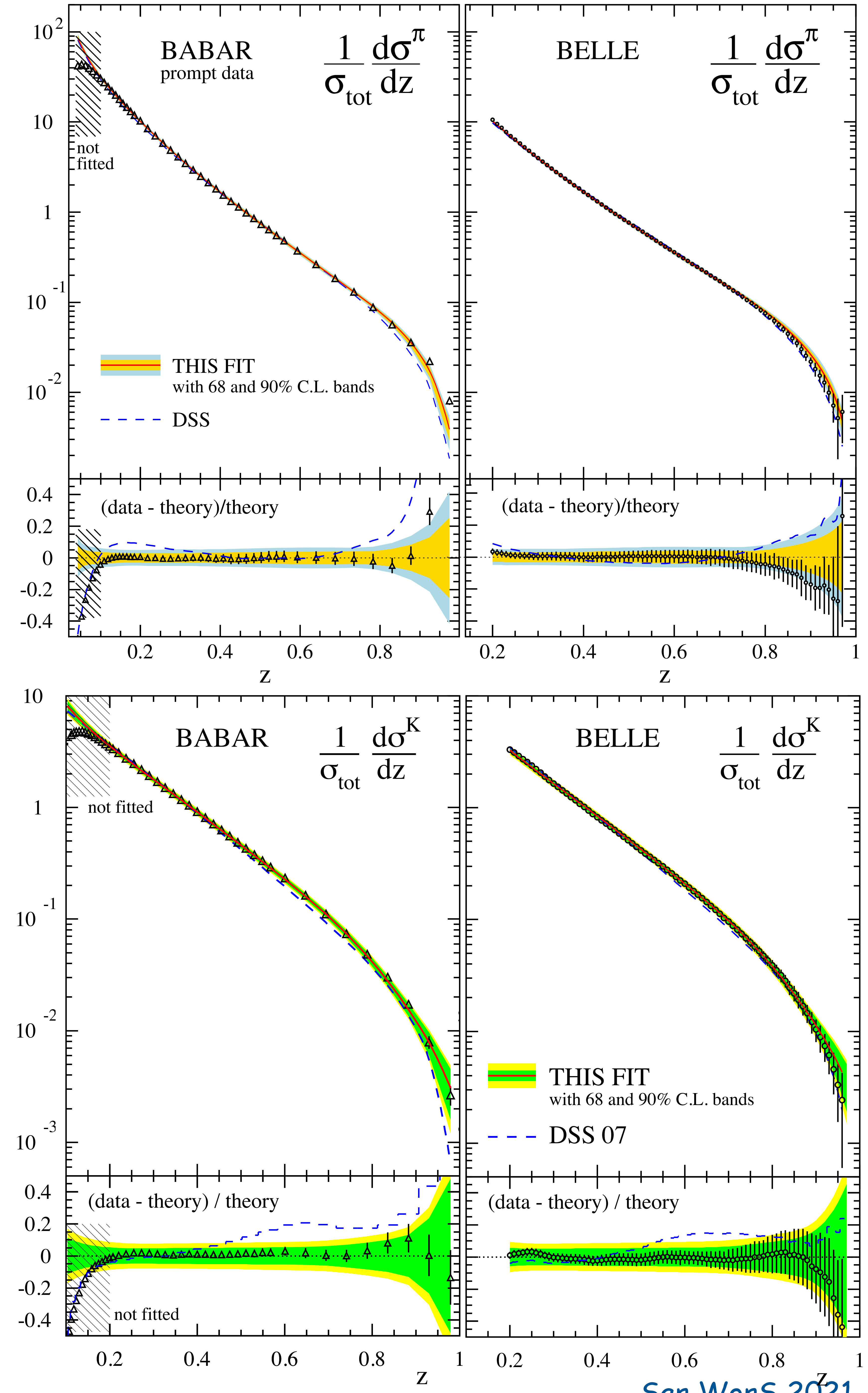
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS
- by now also results from BaBar and Belle:
  - BaBar Collaboration, Phys. Rev. D88 (2013) 032011: **$\pi^\pm$, $K^\pm$, $p+\bar{p}$**
  - Belle Collaboration, Phys. Rev. Lett. 111 (2013) 062002: **$\pi^\pm$, $K^\pm$**
  - Belle Collaboration, Phys. Rev. D92 (2015) 092007: **$\pi^\pm$, $K^\pm$, $p+\bar{p}$**
  - **NEW:** Belle Collaboration, Phys. Rev. D101 (2020) 092004: **$\pi^\pm$, $K^\pm$, $p+\bar{p}$**



single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in 2015 DEHSS fits [e.g., PRD91 (2015) 014035]
  - slight tension at low-z for BaBar and high-z for Belle



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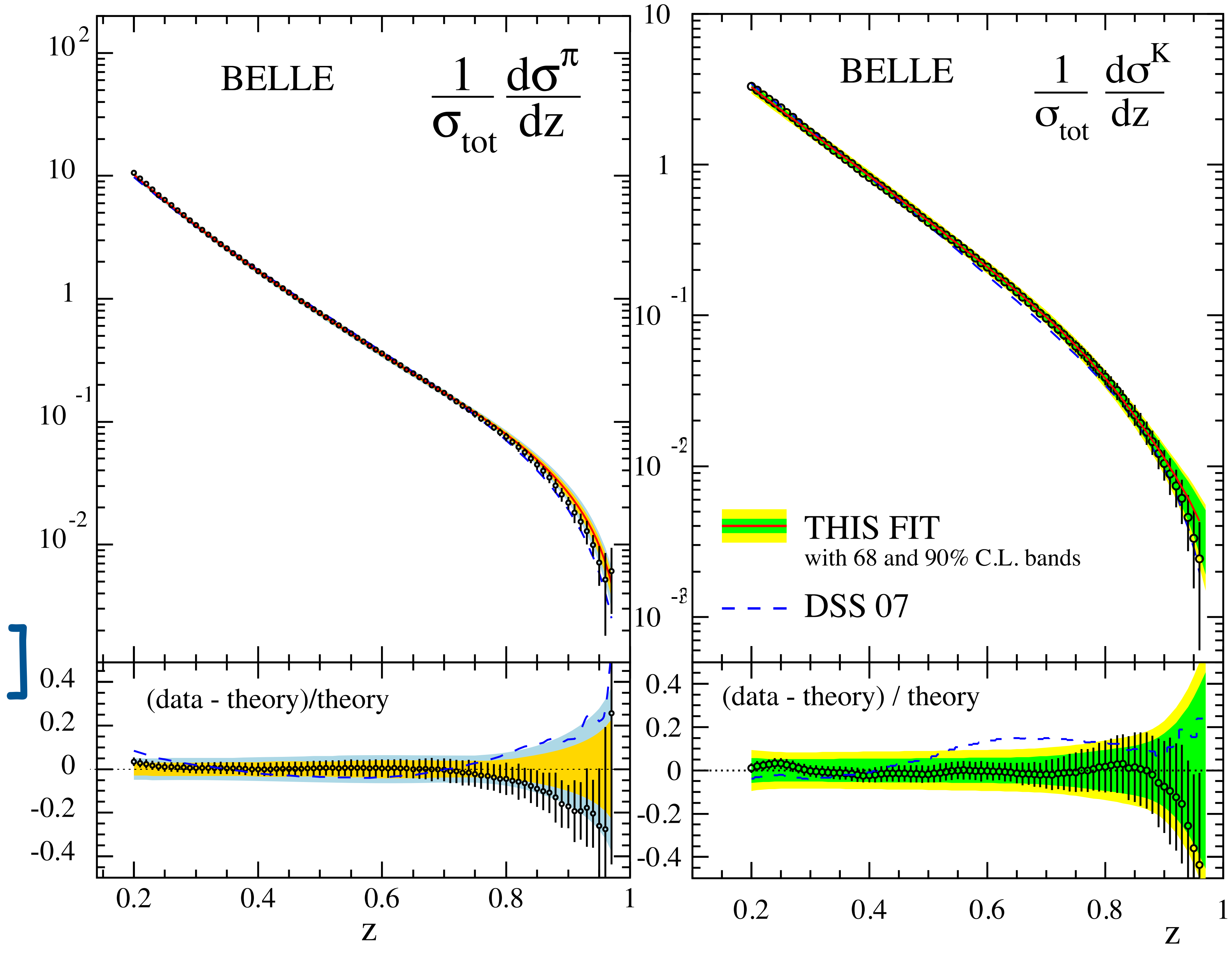

[EPJC 77 (2017) 516, NNFF1.0]

In the case of the BELLE experiment we multiply all data points by a factor $1/c$, with $c = 0.65$ for charged pions and kaons [69] and with c a function of z for protons/antiprotons [53]. This correction is required in order to treat the BELLE data consistently with all the other SIA measurements included in NNFF1.0. The reason is that a kinematic cut on radiative photon events was applied to the BELLE data sample in the original analysis instead of unfolding the radiative QED effects. Specifically, the energy scales

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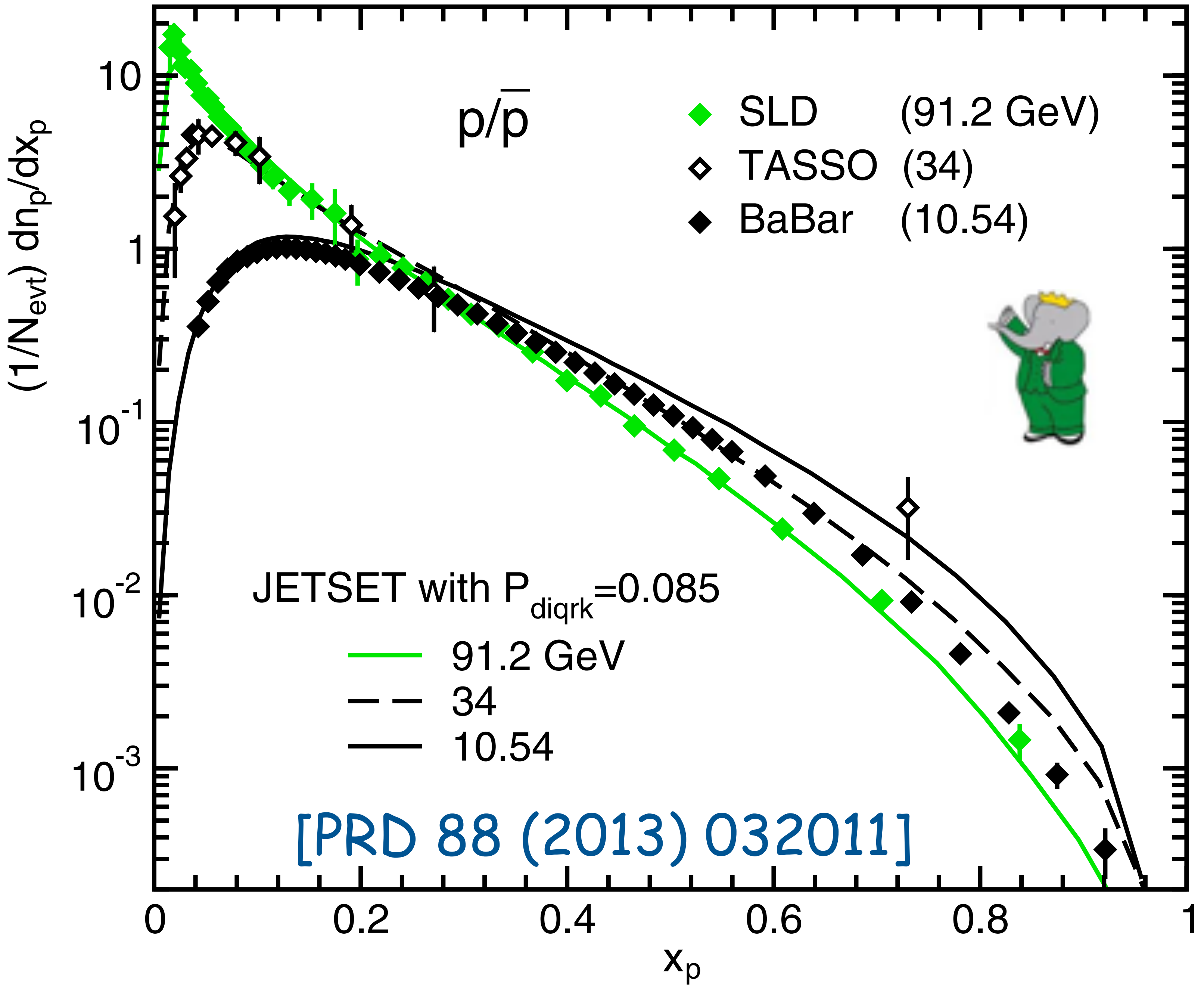
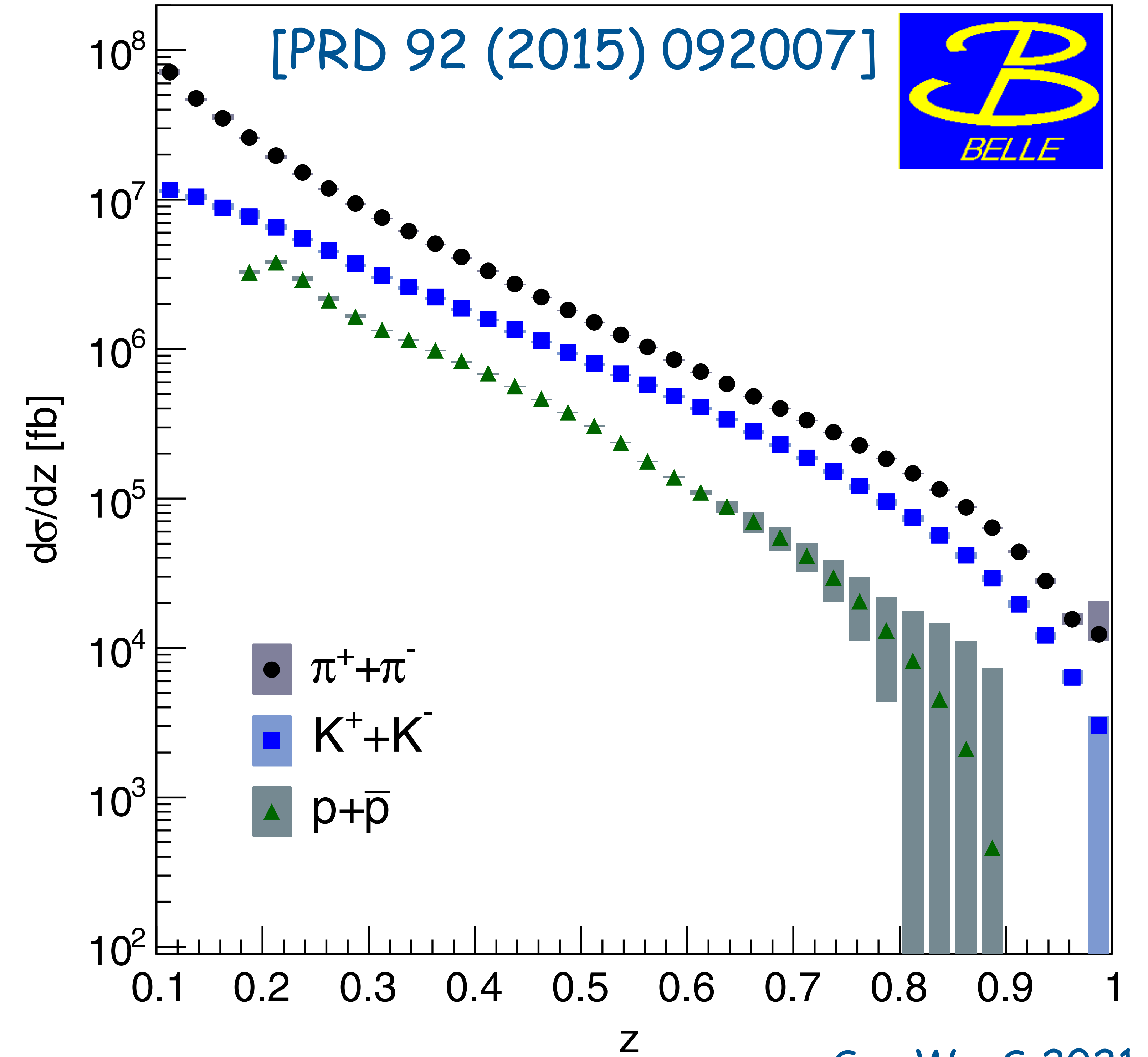
  - not (yet) included in DEHSS, but in NNFF 1.0 [EPJC 77 (2017) 516]
  - similar z dependence as pions
  - about ~⅕ of pion cross sections



[PRD 88 (2013) 032011]



[PRD 92 (2015) 092007]

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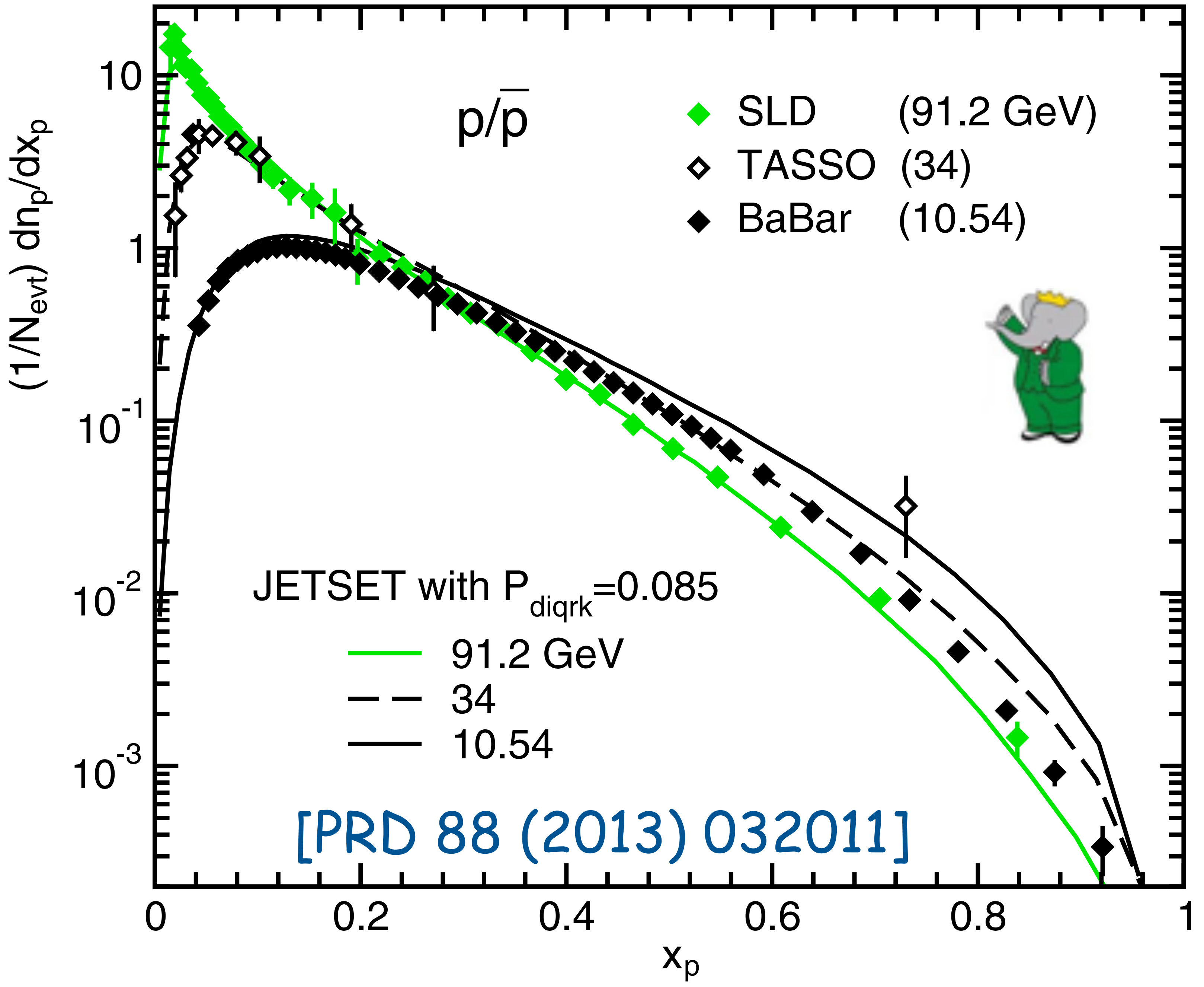
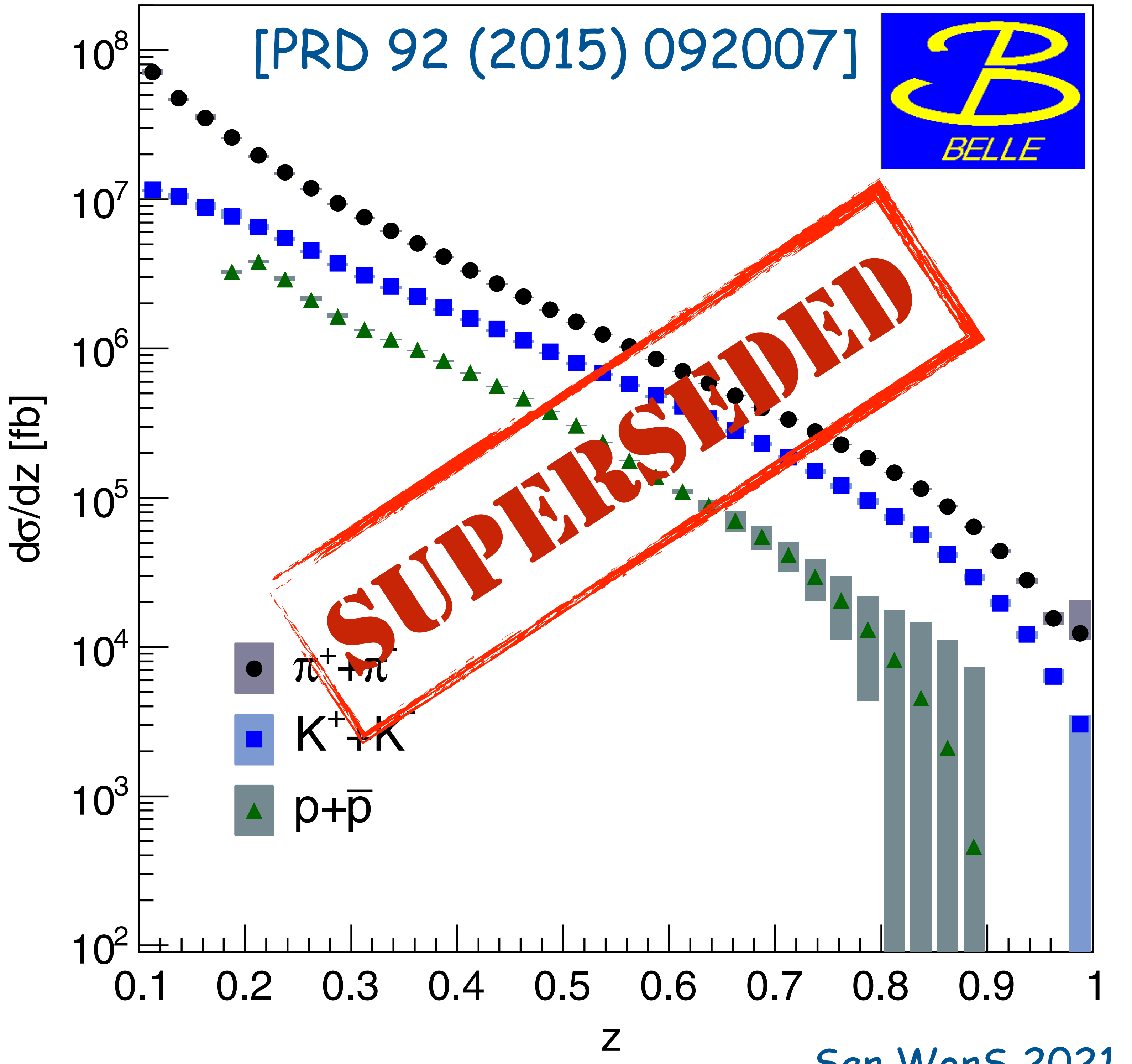
- Belle **re-analysis** presented in **PRD 101 (2020) 092004**





ISR

- what to do with hadrons that have (somewhere) an ISR photon
  - nothing! — leave it to phenomenology to deal with QED corrections
  - reject all events that have an isolated photon?
    - detectors almost never fully hermetic, many ISR photons travel down the beam pipe
    - still fully inclusive reaction?
  - use some Monte Carlo to estimate event fraction with an ISR photon that carries away more than x% of total available energy (e.g., 0.5% as in earlier Belle analyses)
    - what is a reasonable choice for x?
    - ISR treatment model dependent, indeed depends on annihilation cross section (imagine sitting on 2-pion threshold, no phase space to radiate ISR photon and produce hadrons at then lower s)
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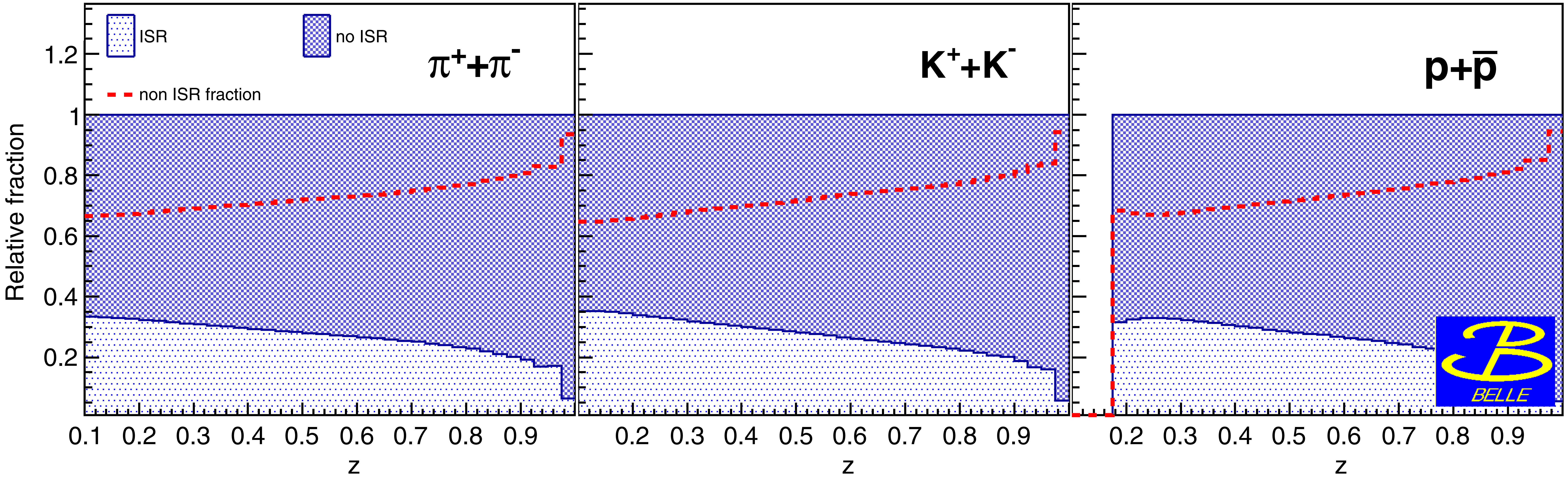
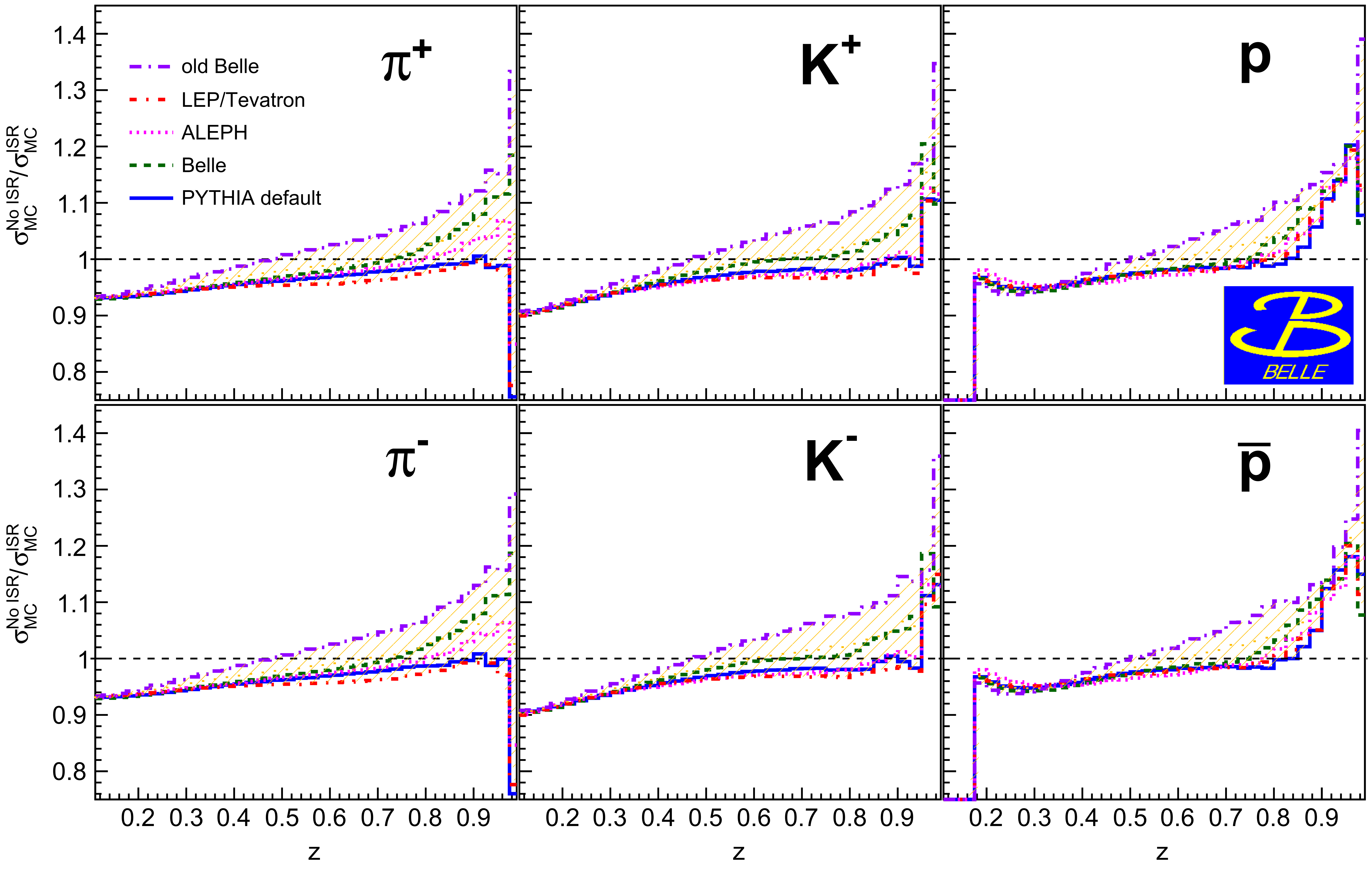
ISR corrections - PRD 92 (2015) 092007



- relative fractions of hadrons as a function of z originating from ISR or non-ISR events (≡ energy loss less than 0.5%)
  - large non-ISR fraction at large z, as otherwise not kinematically reachable (remember $z = E_h / 0.5\sqrt{s_{nominal}}$)
  - keep only fraction of the events -> strictly speaking not single-inclusive annihilation

ISR corrections - PRD 101 (2020) 092004



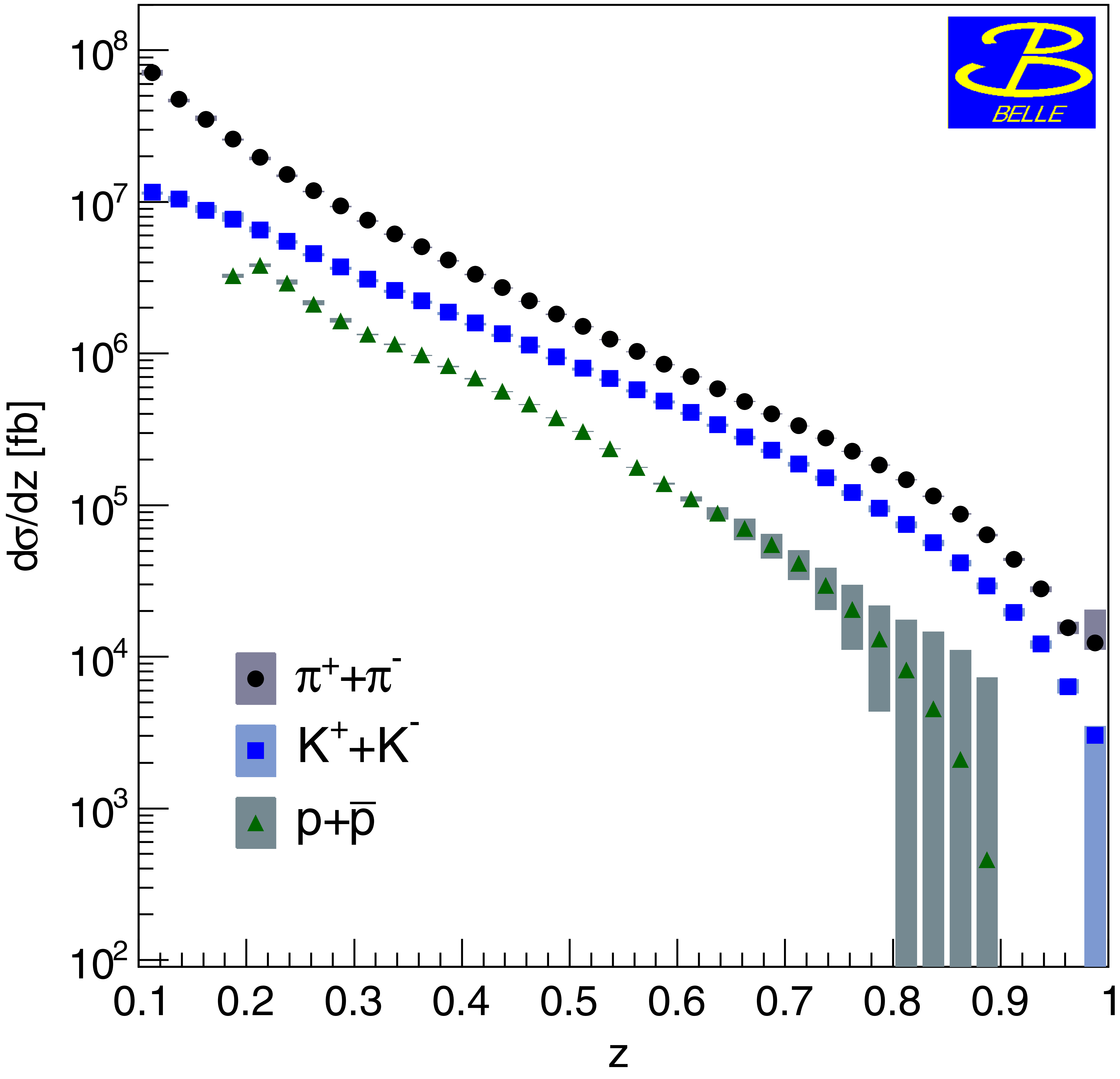
[PRD 101 (2020) 092004]

- non-ISR / ISR fractions based on PYTHIA switch MSTP(11)
- PYTHIA model dependence; absorbed in systematics by variation of tunes

comparison old&new Belle single-hadron cross sections

- previous analysis

[PRD 92 (2015) 092007]



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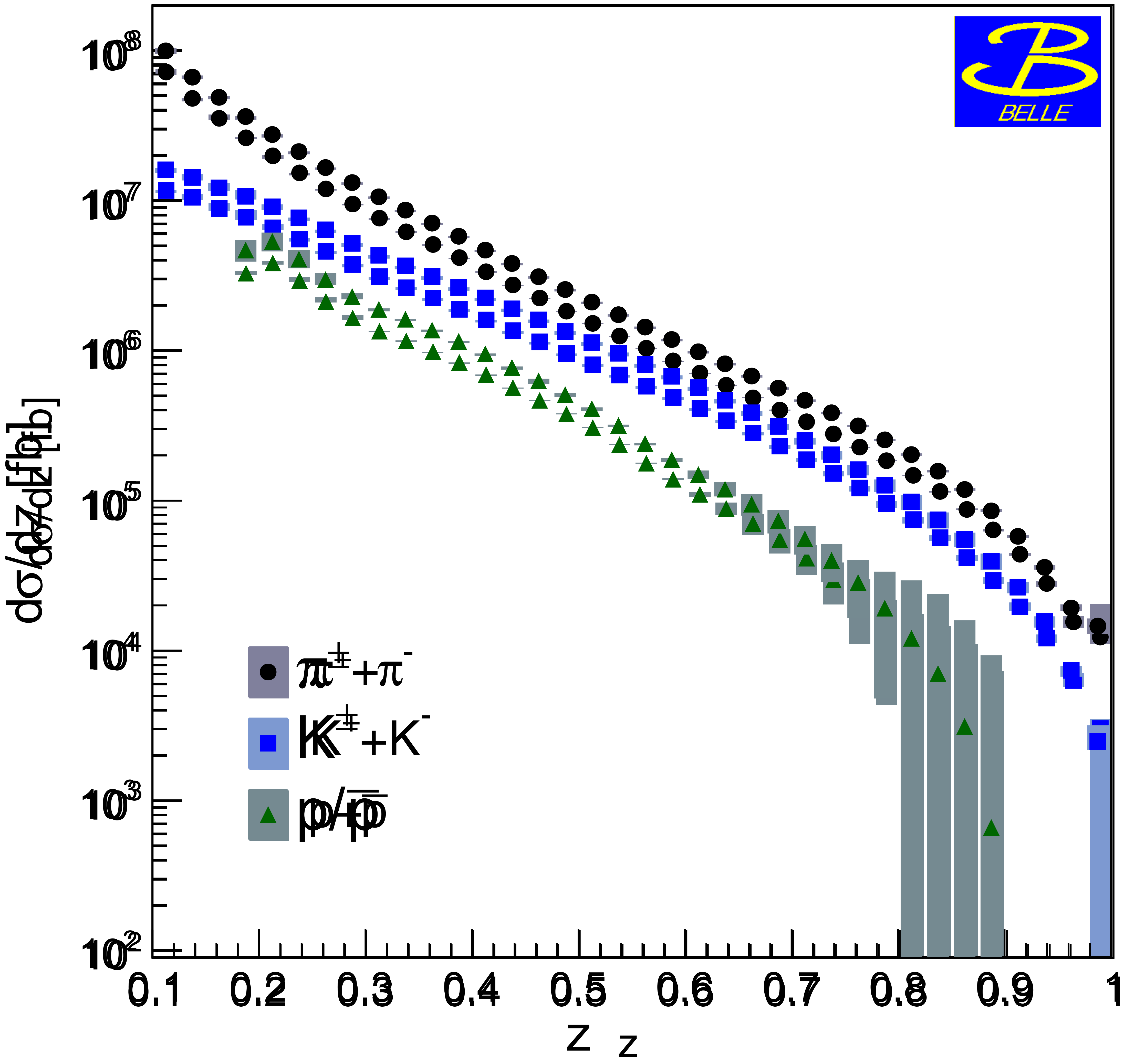
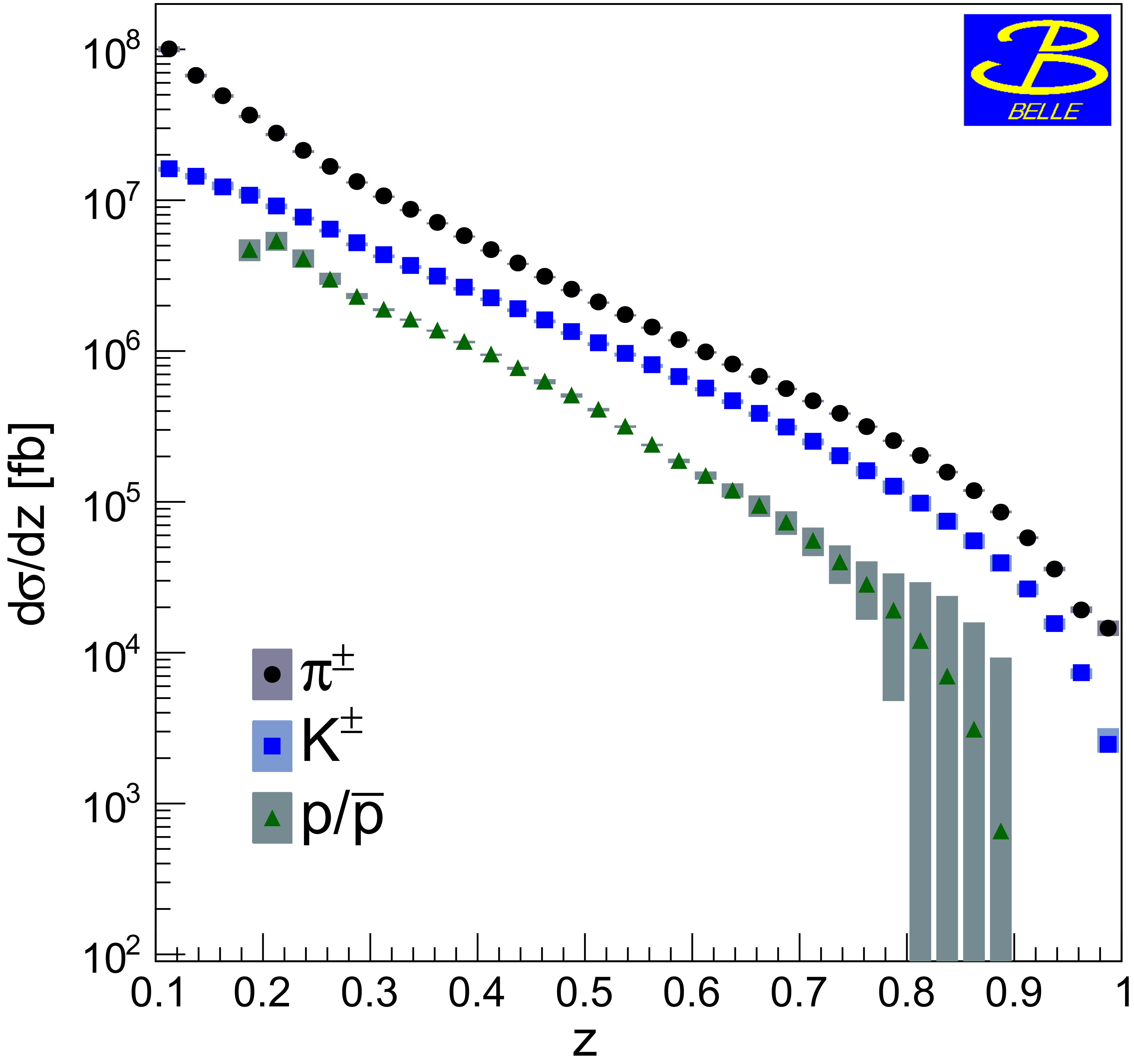
[PRD 92 (2015) 092007] [PRD 101 (2020) 092004]



- updated analysis

comparison old&new Belle single-hadron cross sections

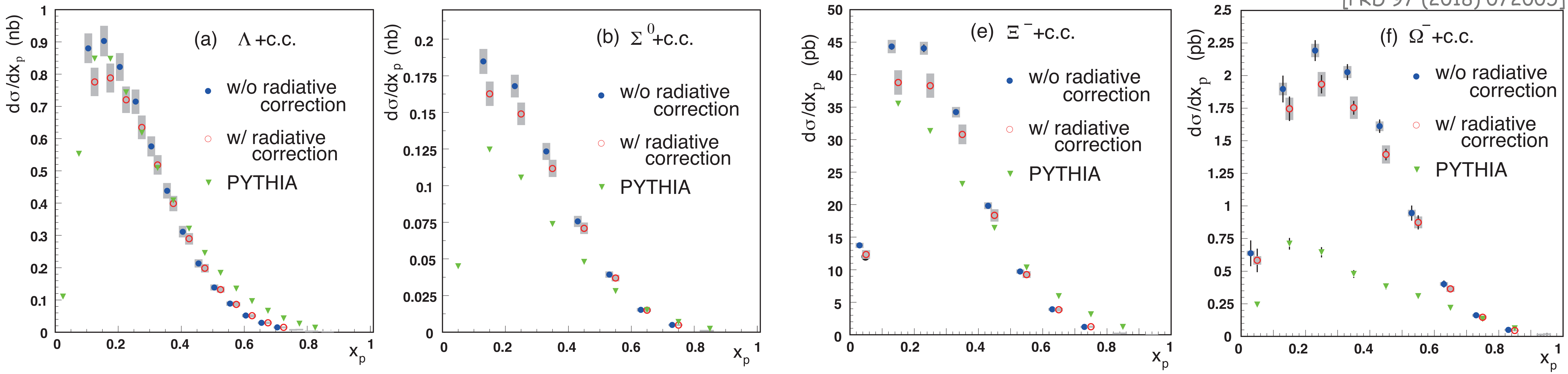
[PRD 101 (2020) 092004]

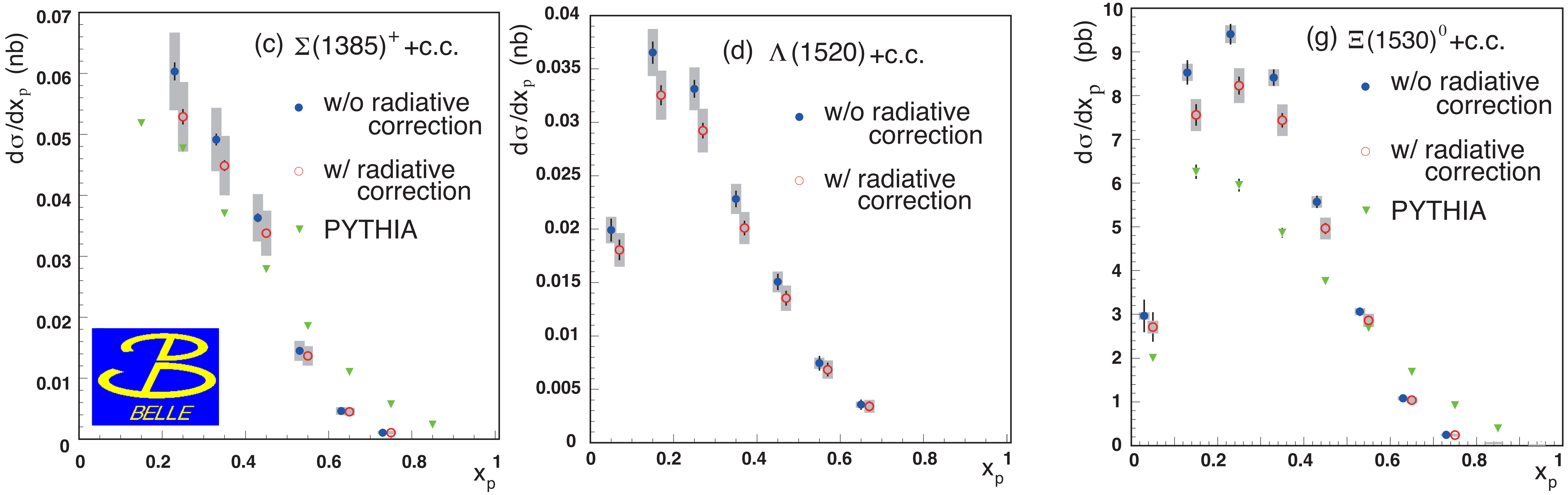


- updated analysis

single-hadron production: hyperons

[PRD 97 (2018) 072005]



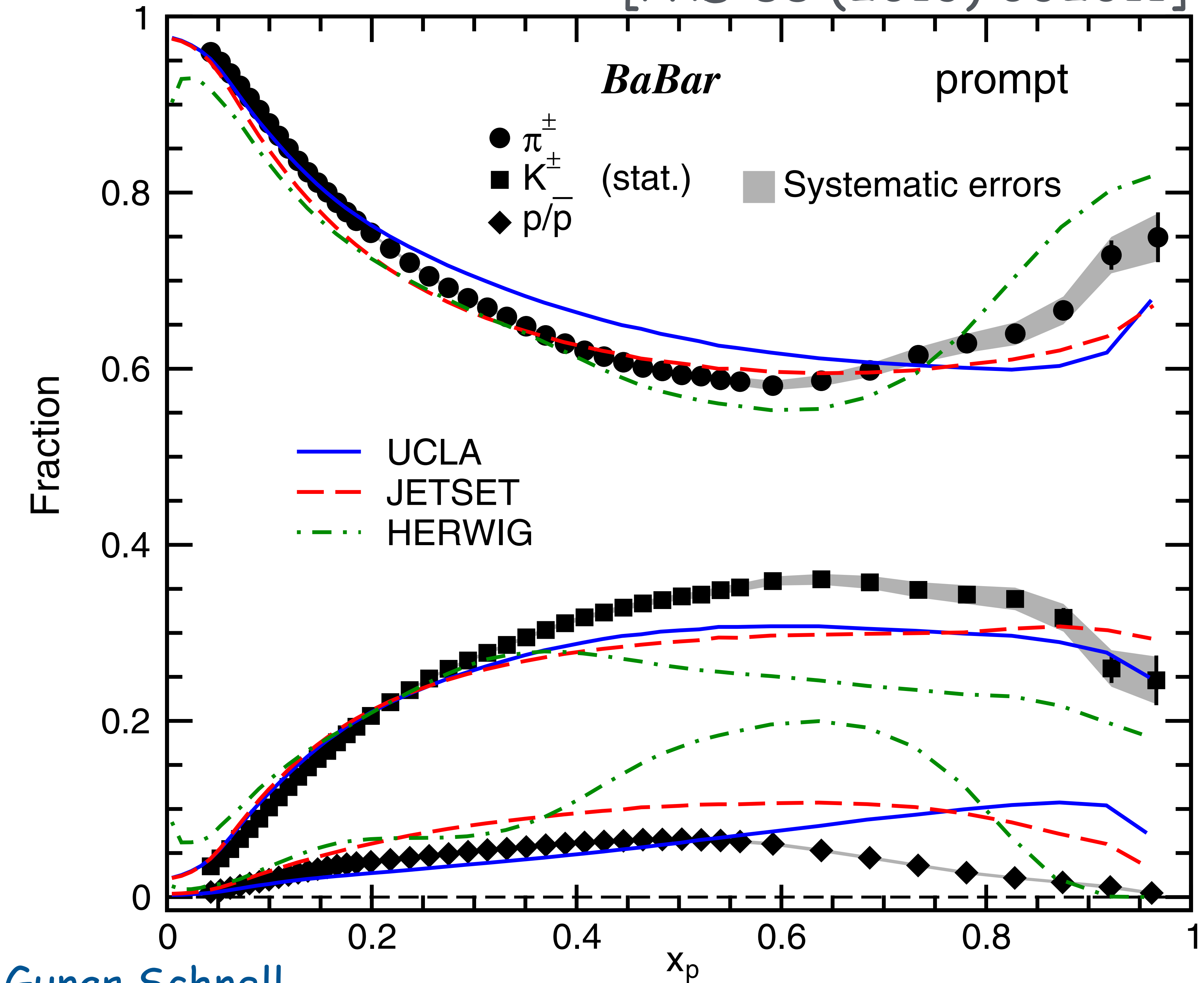


- Λ production reasonably well described by PYTHIA
- less satisfactory for heavier hyperons (a quite common problem)
- basically fails to describe Ω^- production

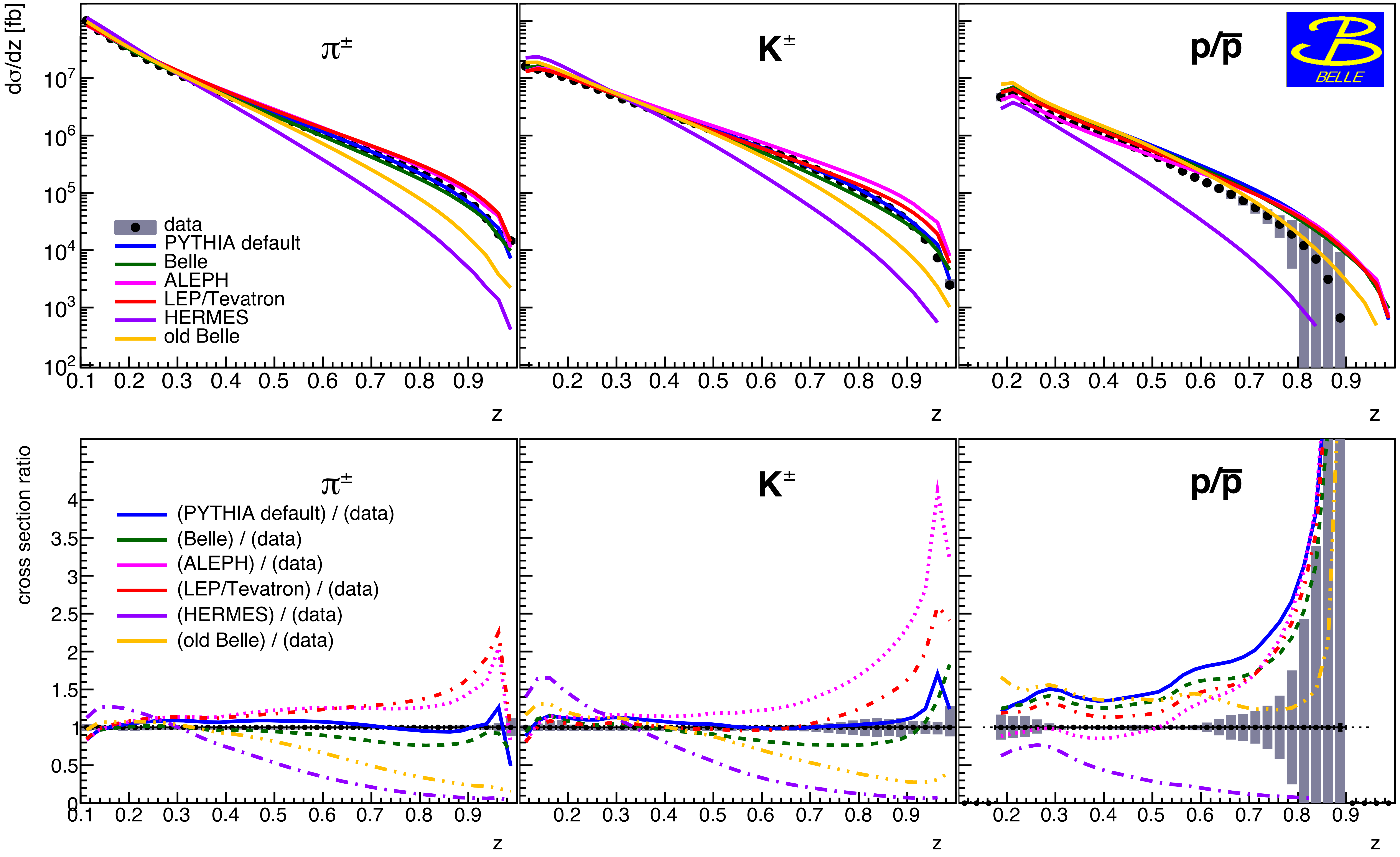
single-hadron production: data-MC comparison

- pion and(?) kaon data reasonably well described by Jetset
- protons difficult to reproduce, especially at large z
  - MC overshoots data

[PRD 88 (2013) 032011]



[PRD 101 (2020) 092004]

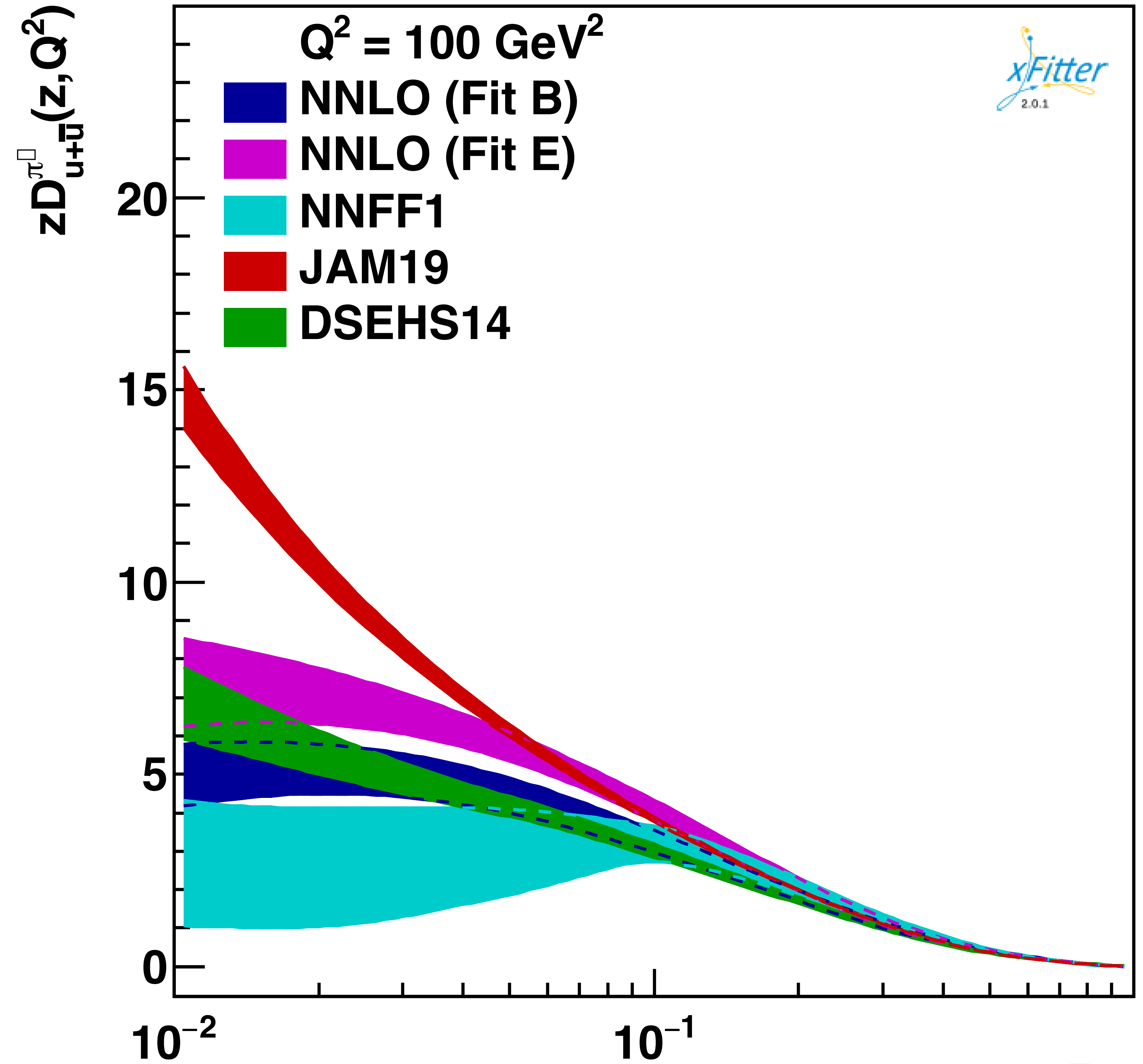


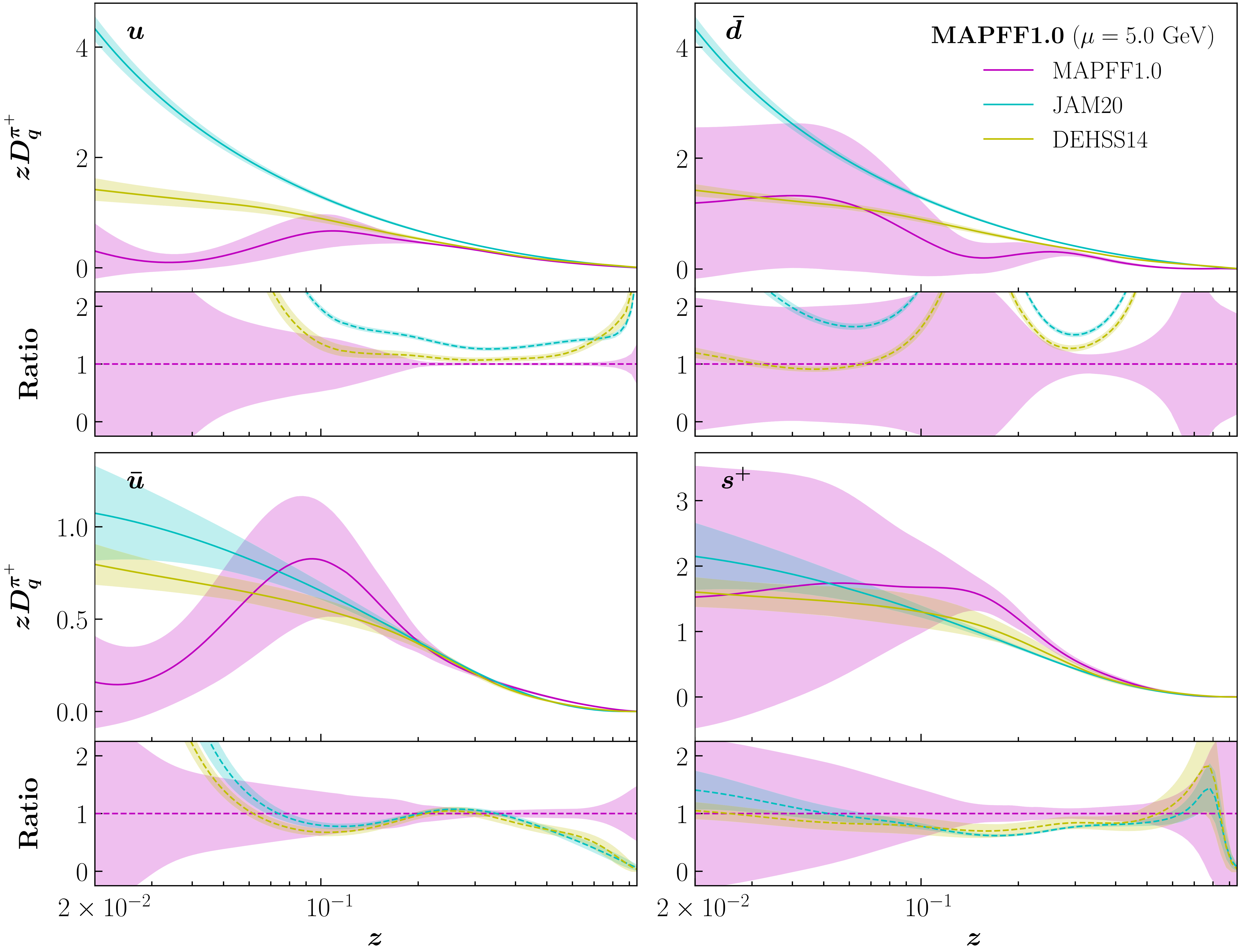
pion fragmentation functions: fit comparisons

[PRD 104 (2021) 034007]

- still large differences in FF extractions
  - also in "SIDIS" region, where needed as flavor tagger

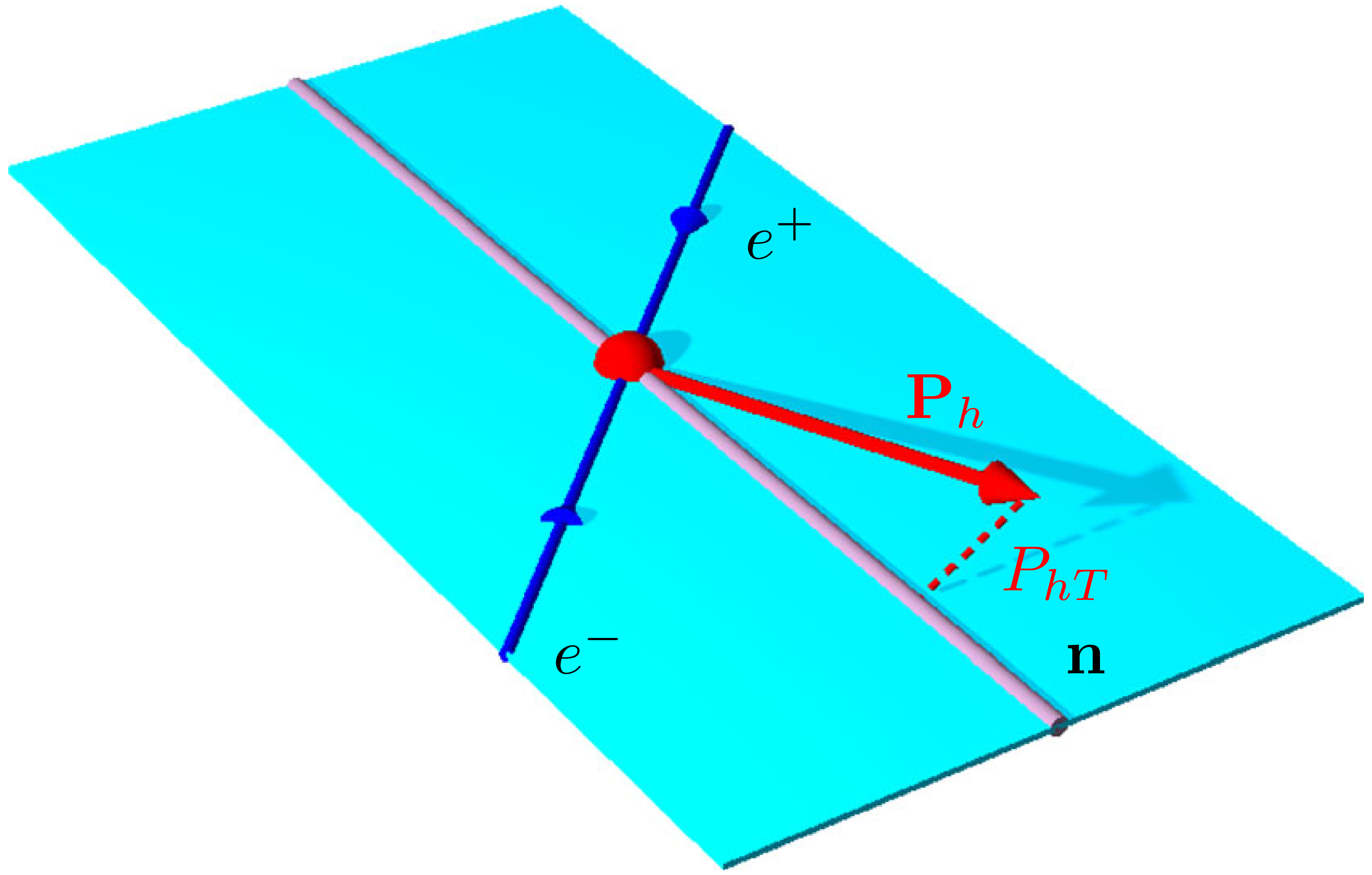
[arXiv:2105.11306v1]





inclusive hadrons - transverse momentum

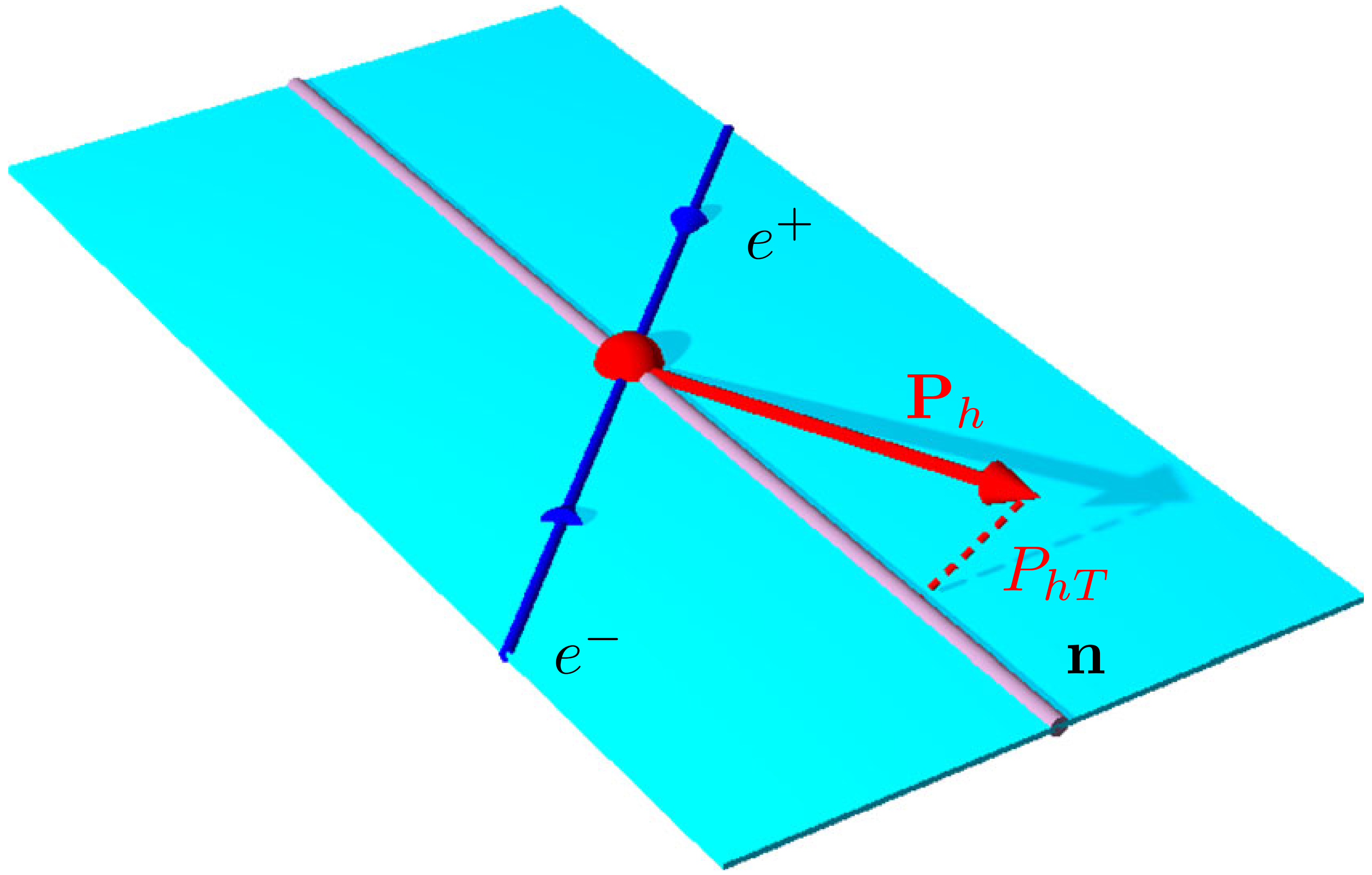
- quasi-inclusive hadron production gives access to transverse momentum in fragmentation
- transverse momentum measured with respect to thrust axis n
  - involves sum over all final-state particles in event
  - event selection and hadron distributions dependent on thrust value T required
    - low thrust -> more spherical
    - high thrust -> highly collimated



$$T \overset{\max}{=} \frac{\sum_h |\mathbf{P}_h^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\mathrm{CMS}}|}$$

inclusive hadrons - transverse momentum

- quasi-inclusive hadron production gives access to transverse momentum in fragmentation
- transverse momentum measured with respect to thrust axis n
- analysis performed differential in z & P_{hT}, in various slices in thrust T (➠ 18x20x6 bins)
- correction steps similar as for P_{hT}-integrated cross sections
- Gaussian fits to transverse-momentum distribution provided for all hadrons in (z,T)-bins



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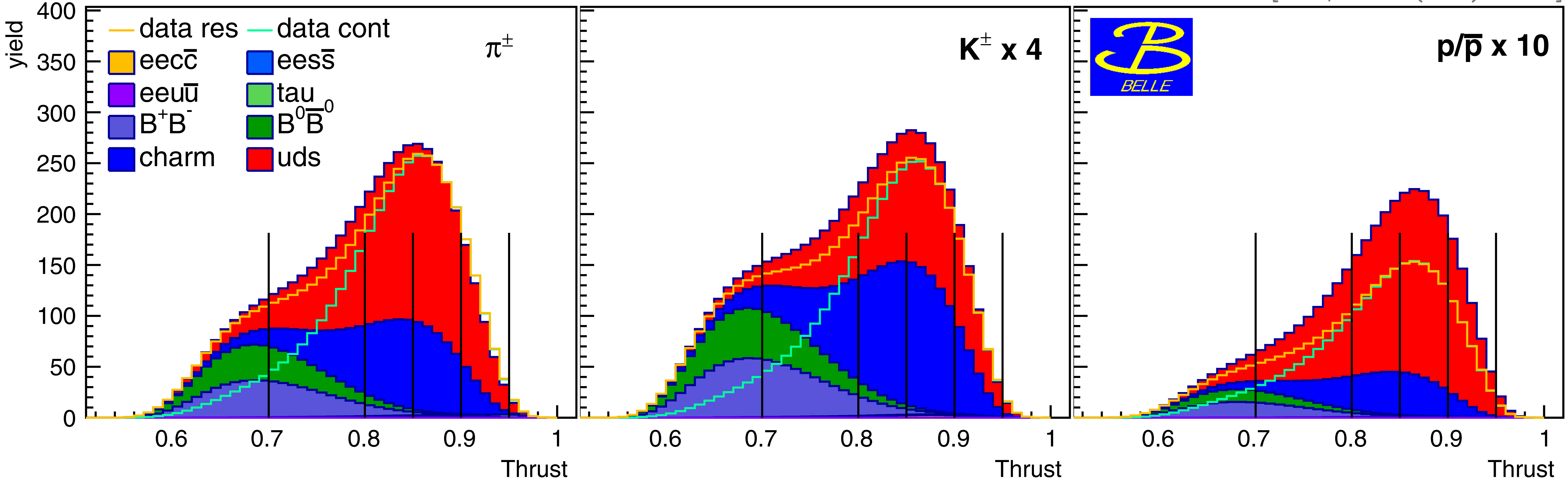
thrust distribution: process contributions

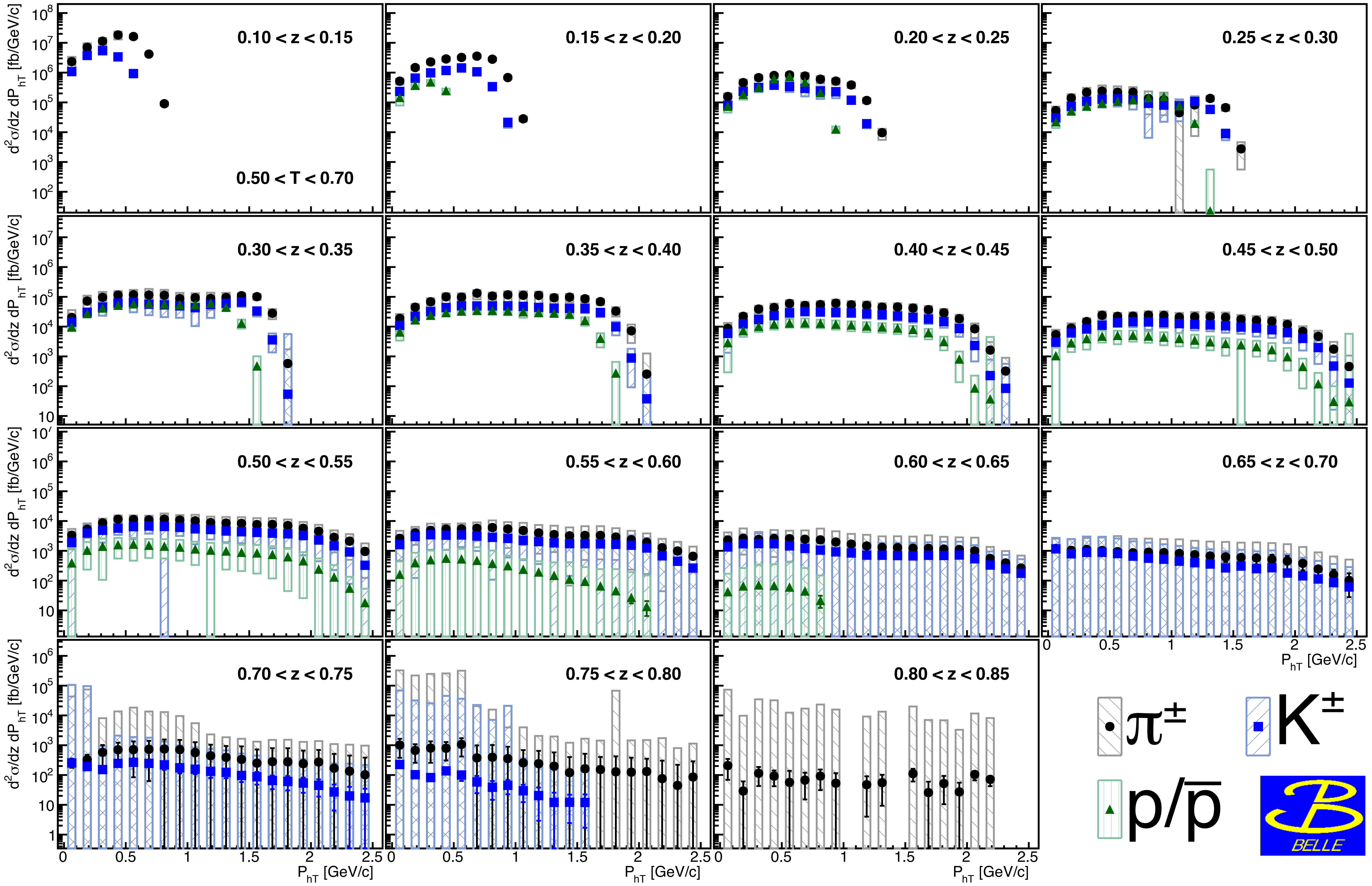
[Belle, PRD 99 (2019) 112006]



- large contribution from BB at lower thrust
- large thrust dominated by uds and charm fragmentation (at very large T significant τ contribution for pions, not visible here)
- will concentrate mainly on 0.85<T<0.9 bin, though others available as well

transverse-momentum distributions

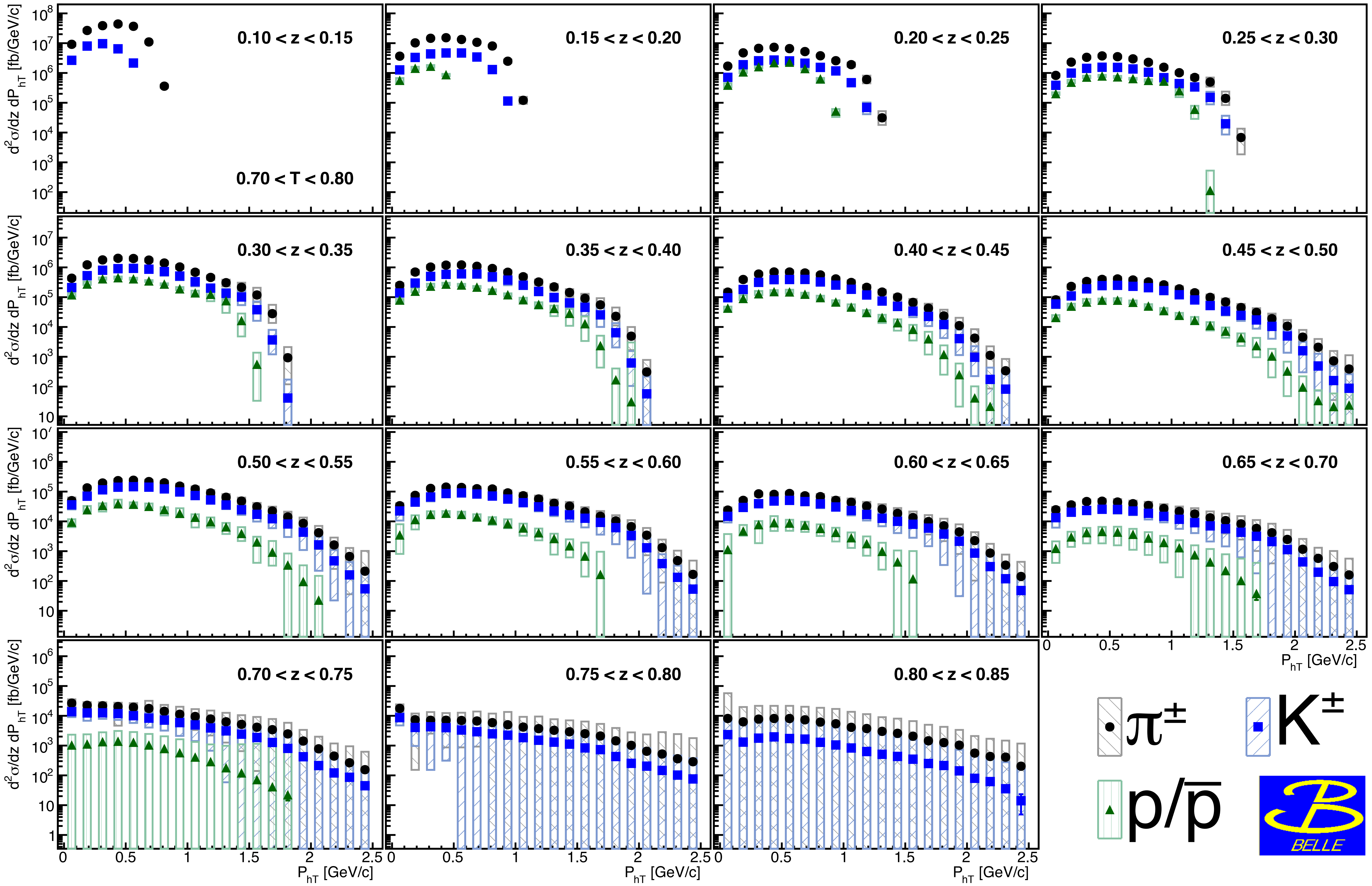
- **lowest T bin** -> rather spherical events
  - transverse momenta almost uniformly distributed in medium-z bins
  - faster drop for heavier hadrons



[PRD 99 (2019) 112006]

transverse-momentum distributions

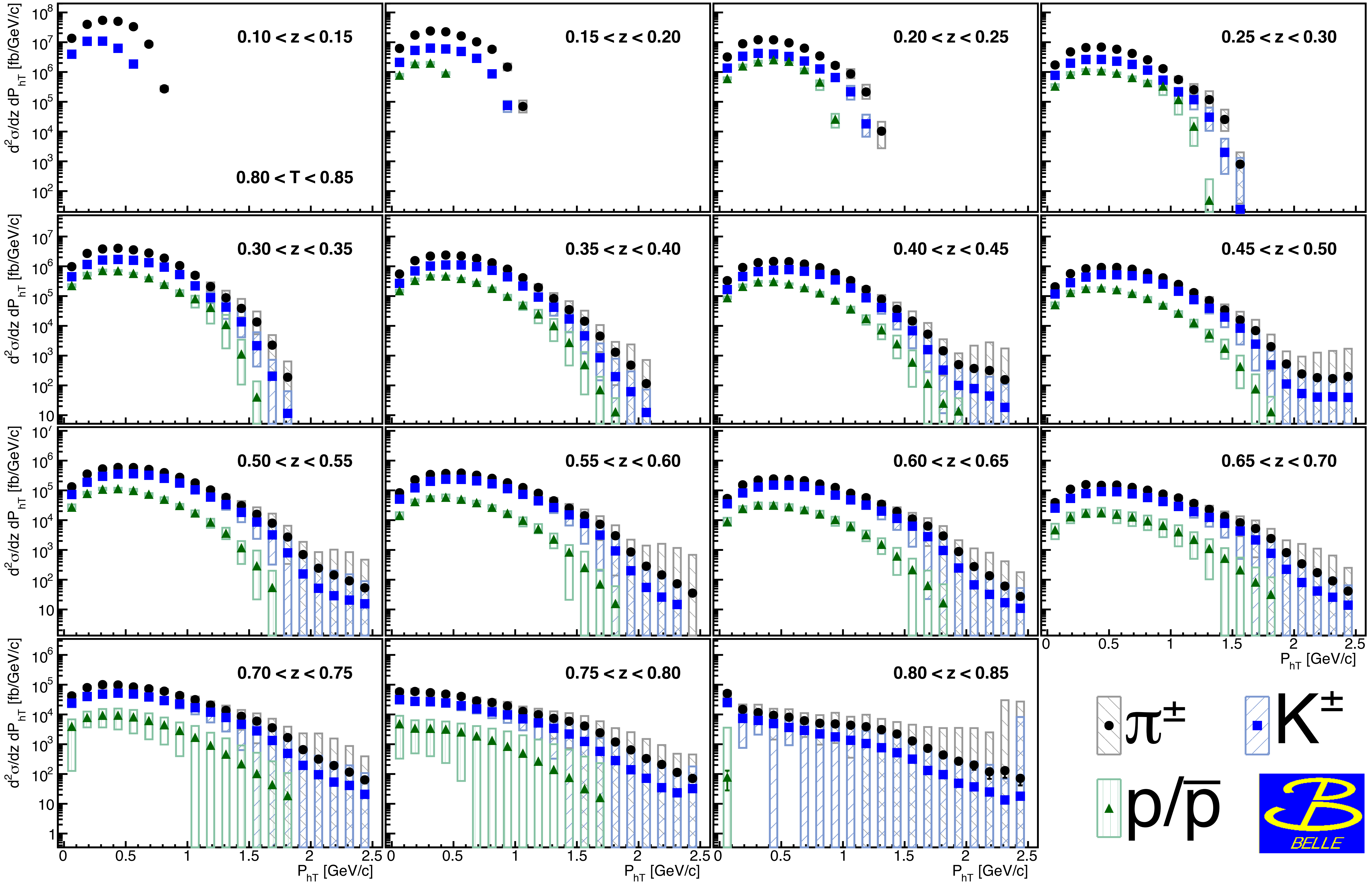
- **0.7<T<0.8** -> particles already more collimated
  - transverse momenta more Gaussian distributed
- large-z region with large uncertainties



[PRD 99 (2019) 112006]

transverse-momentum distributions

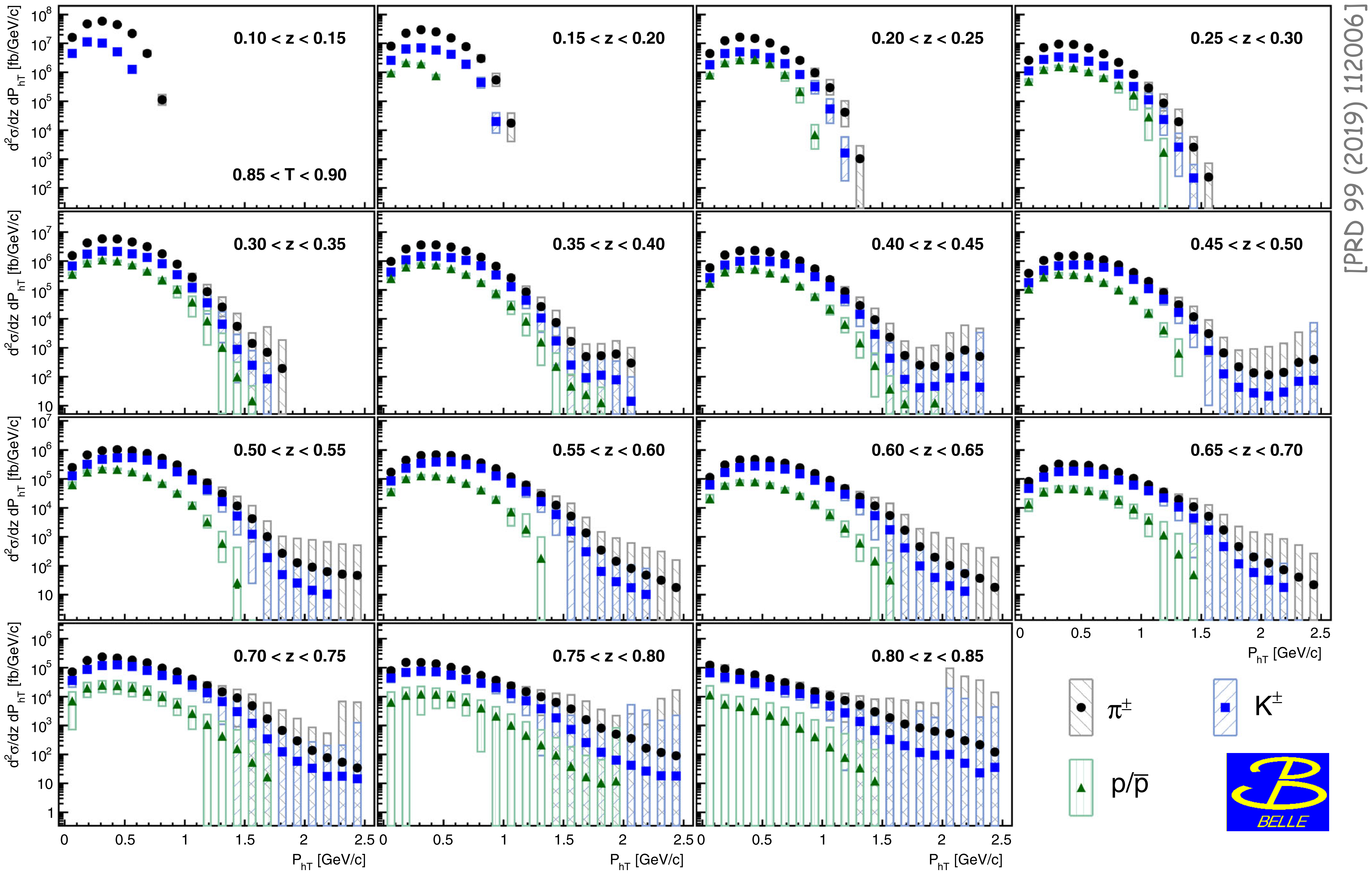
- **0.8<T<0.85**
  - transverse momenta mostly Gaussian distributed
  - possible deviations for large-P_{hT} tails [but also larger uncertainties]



[PRD 99 (2019) 112006]

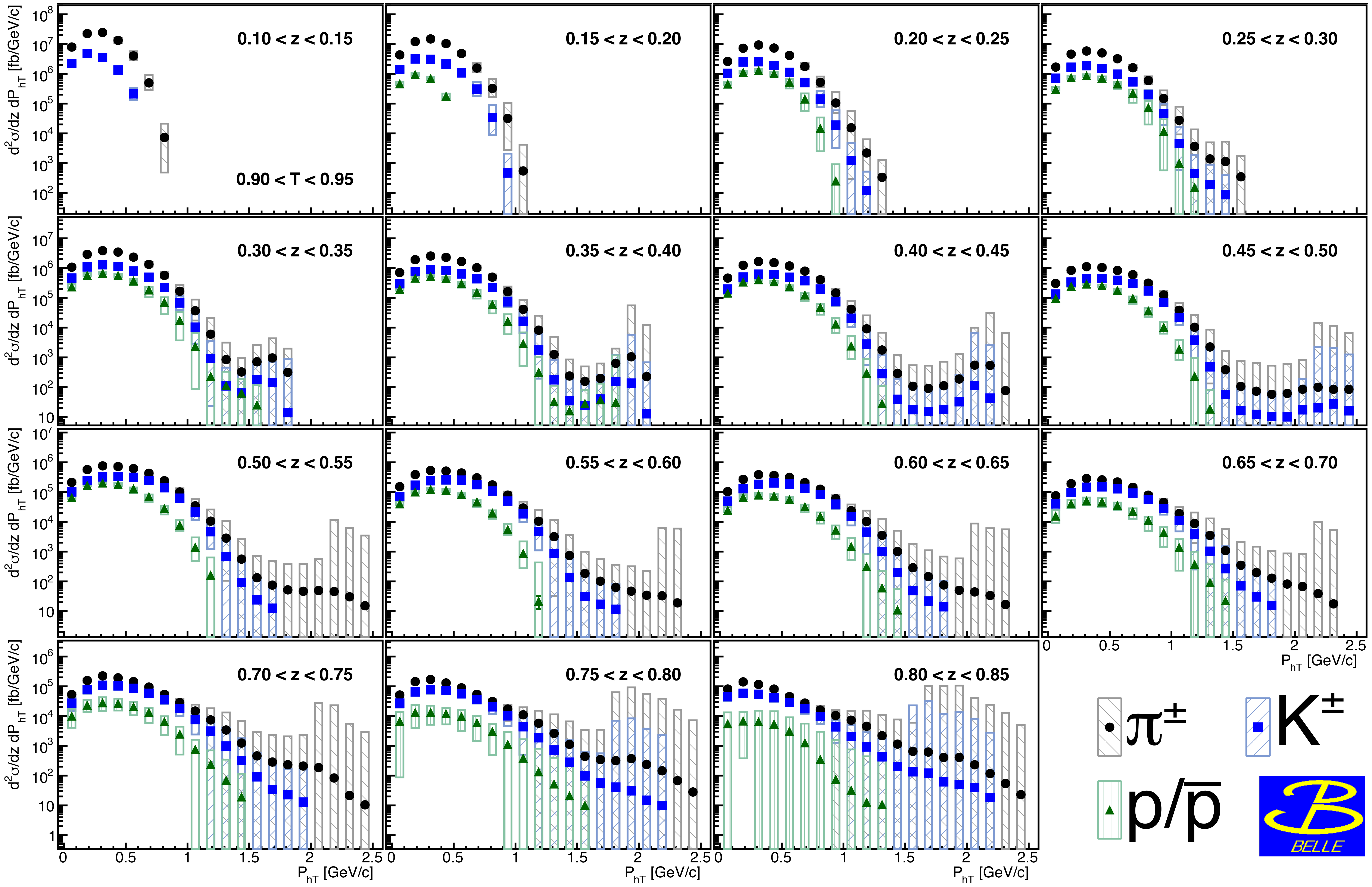
transverse-momentum distributions

- **0.85<T<0.9**
  - transverse momenta mostly Gaussian distributed; widths narrowing
  - possible deviations for large-P_{hT} tails [but also larger uncertainties]



transverse-momentum distributions

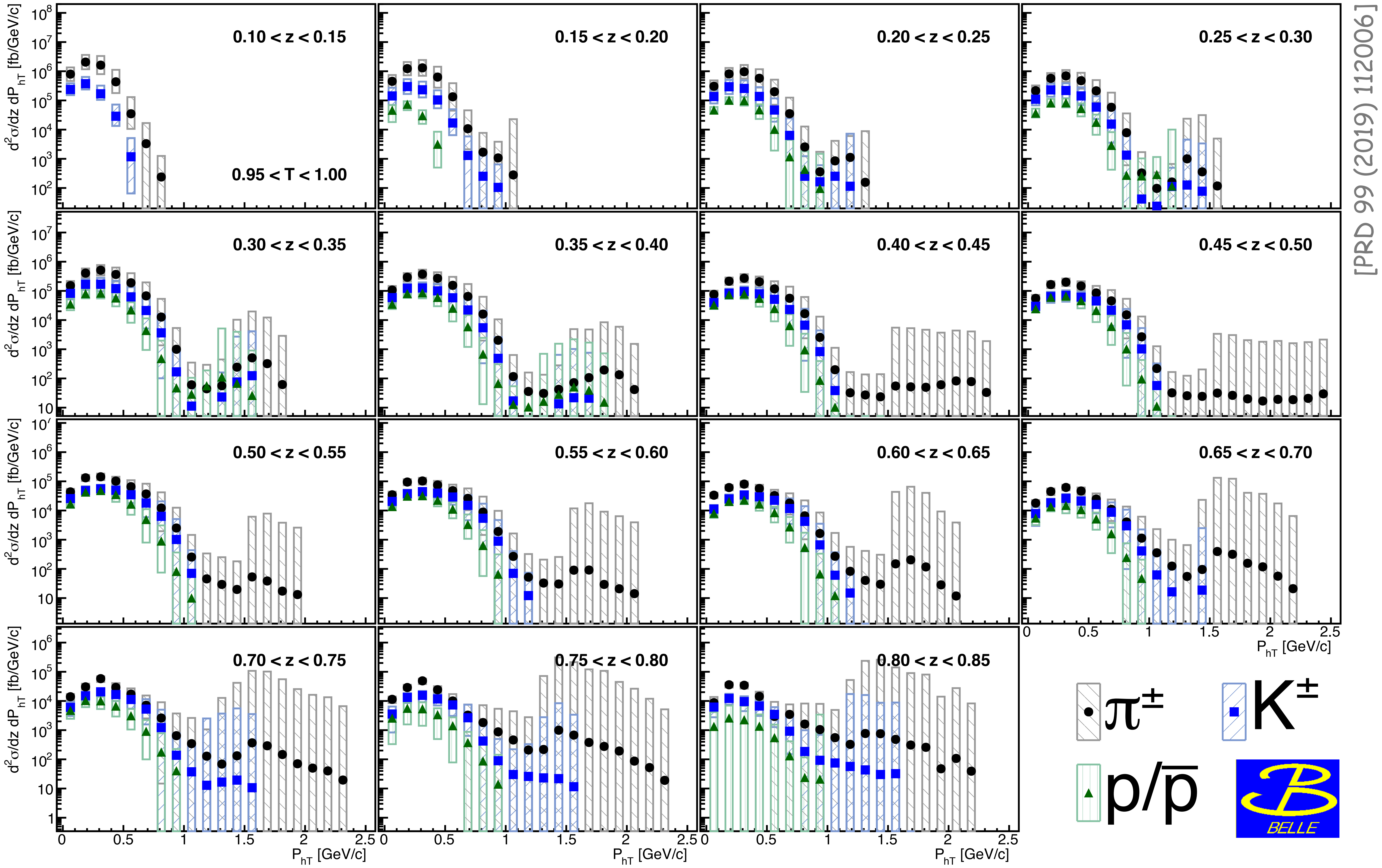
- **0.9<T<0.95**
  - transverse momenta mostly Gaussian distributed; widths even narrower
  - possible deviations for large-P_{hT} tails [but also larger uncertainties]



[PRD 99 (2019) 112006]

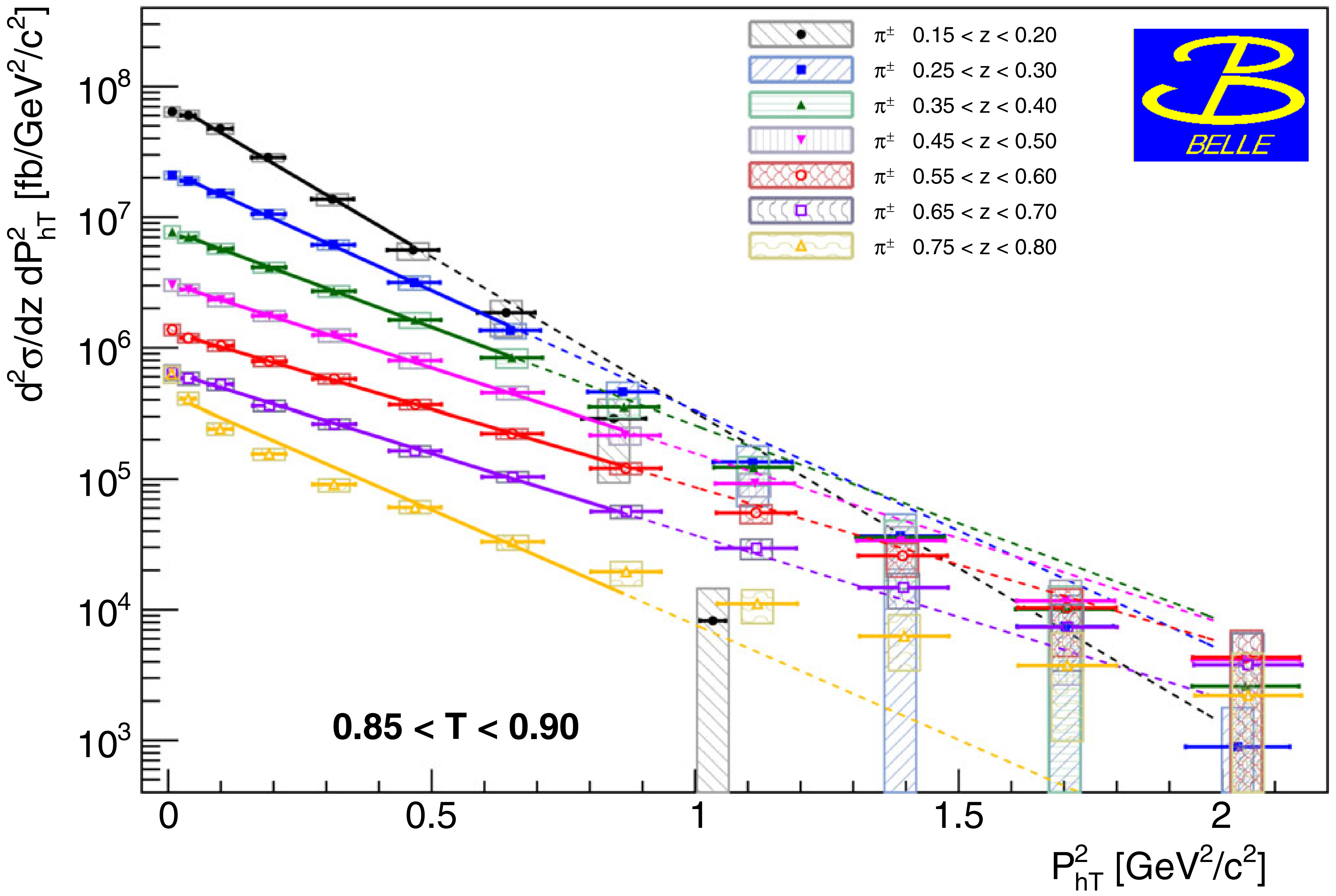
transverse-momentum distributions

- **0.95<T<1.0**
  - transverse momenta mostly Gaussian distributed
  - widths very narrow as particles now very collimated

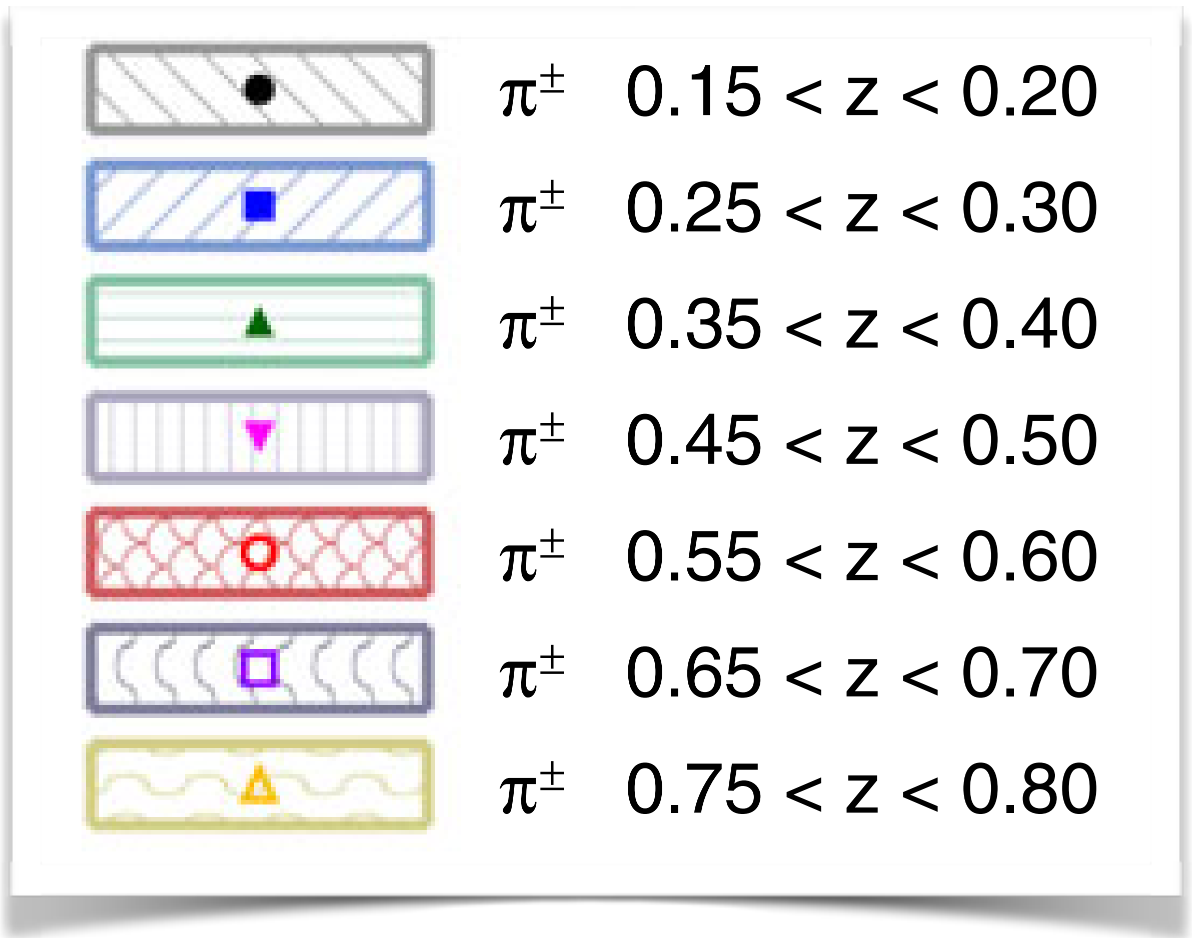


transverse-momentum: Gaussian widths

- **0.85<T<0.90**
  - fit Gauss to low-P_{hT} data
  - mostly well described with possible exception at high z
  - deviation from Gauss at large P_{hT}
  - clear increase of width with z for low values of z



[PRD 99 (2019) 112006]



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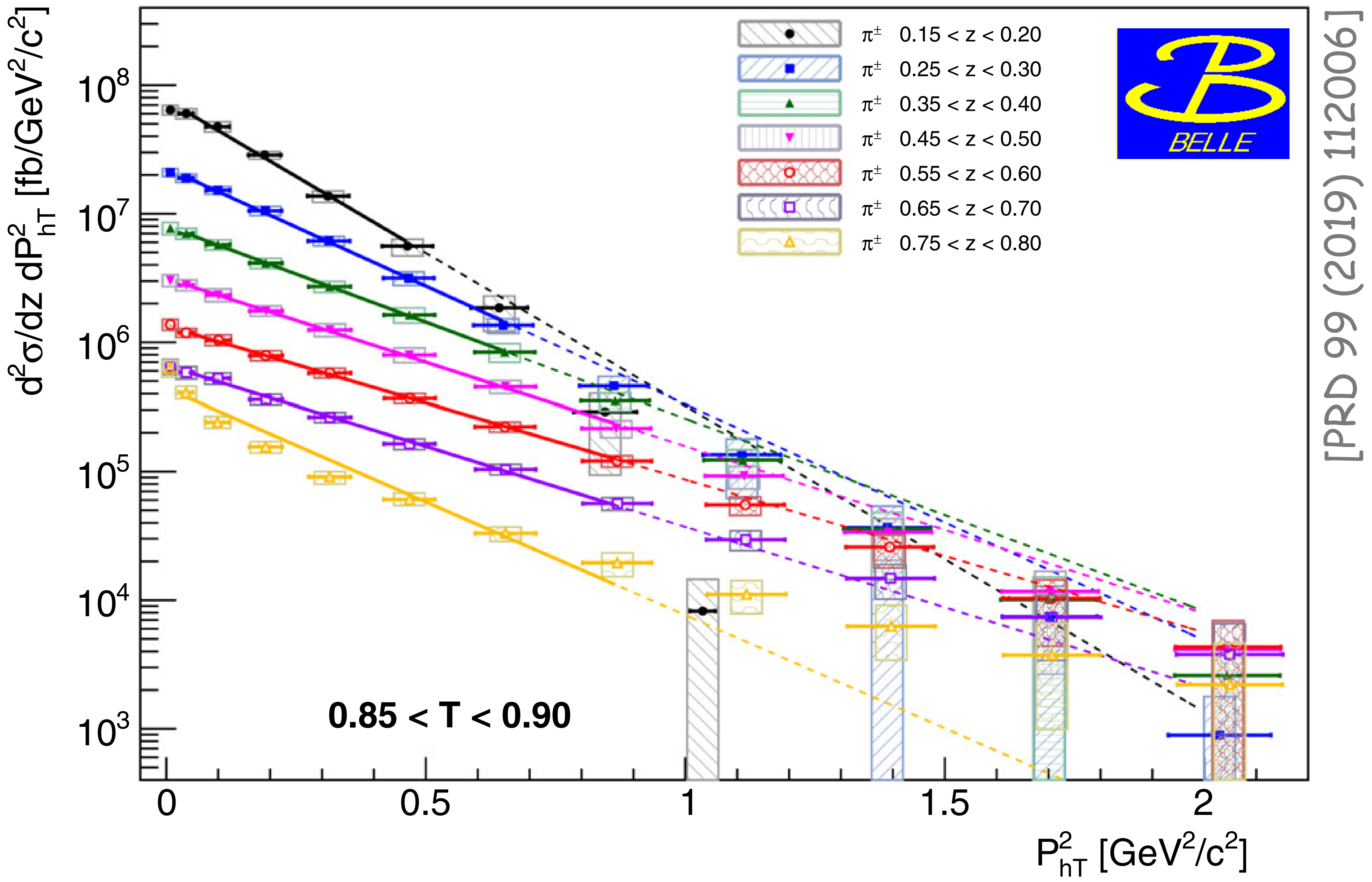
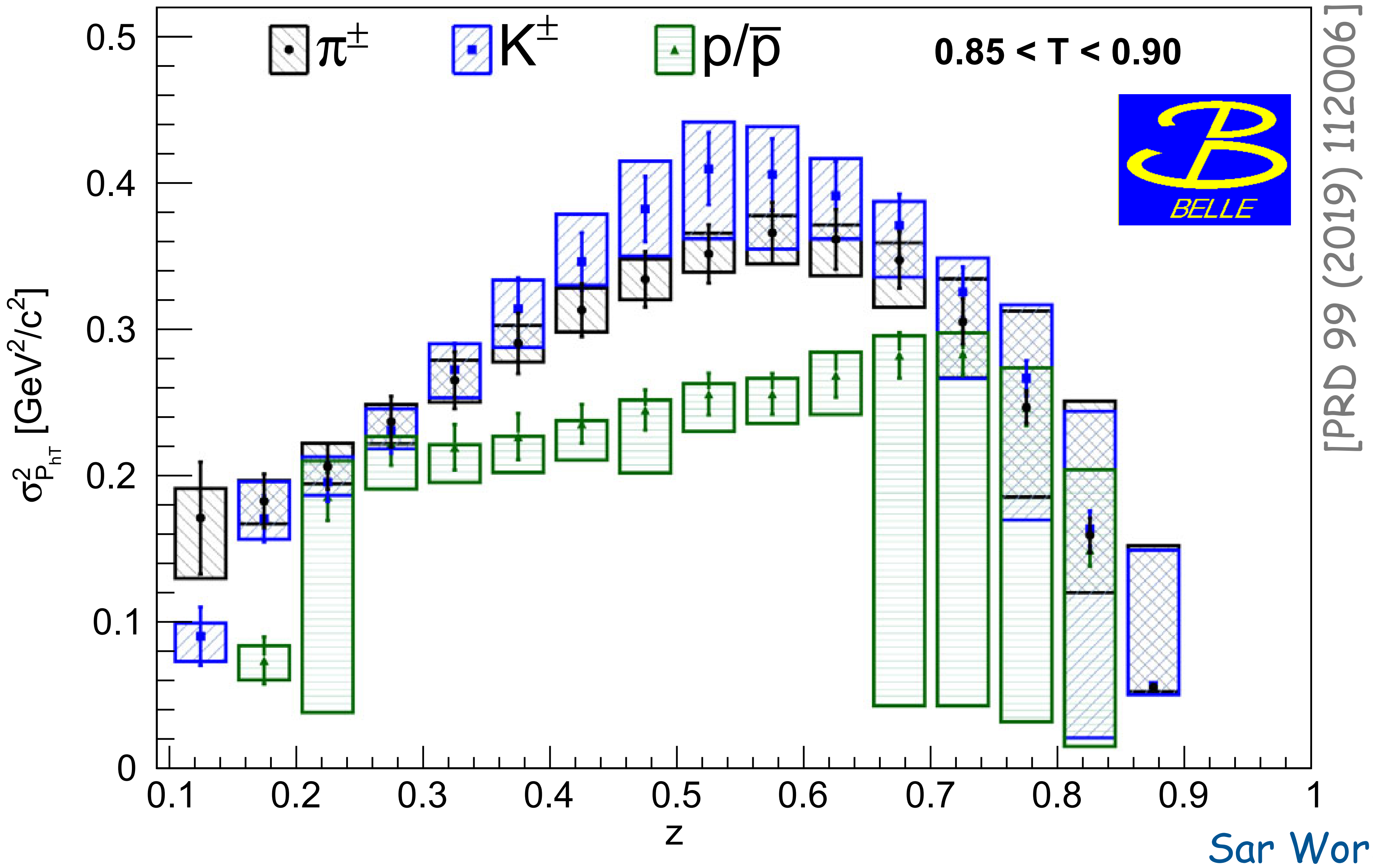
- Gaussian widths as function of z
  - general increase with z with turnover at larger values of z for mesons
  - protons with smaller width and a more linear rise with z



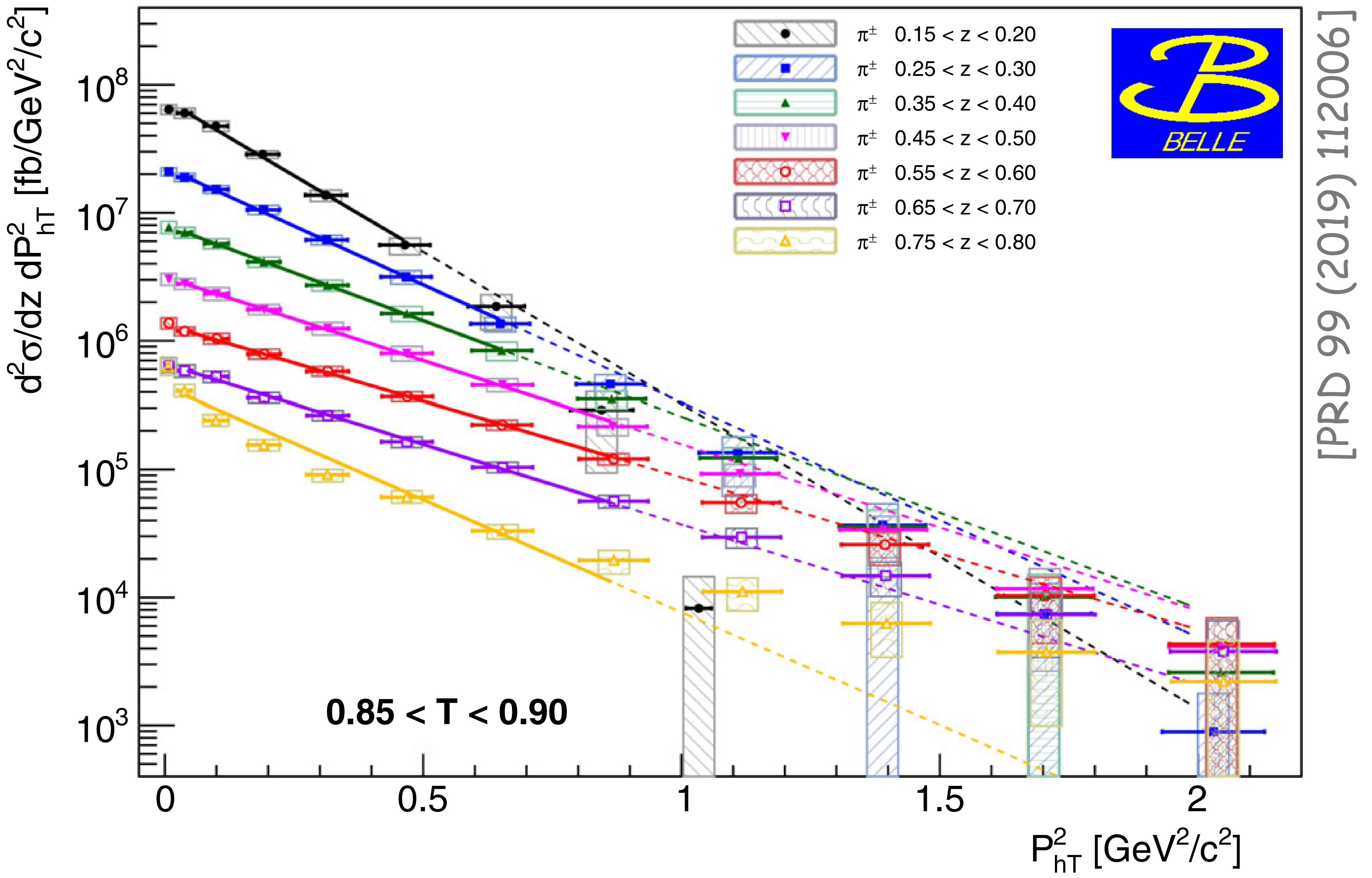
[PRD 99 (2019) 112006]



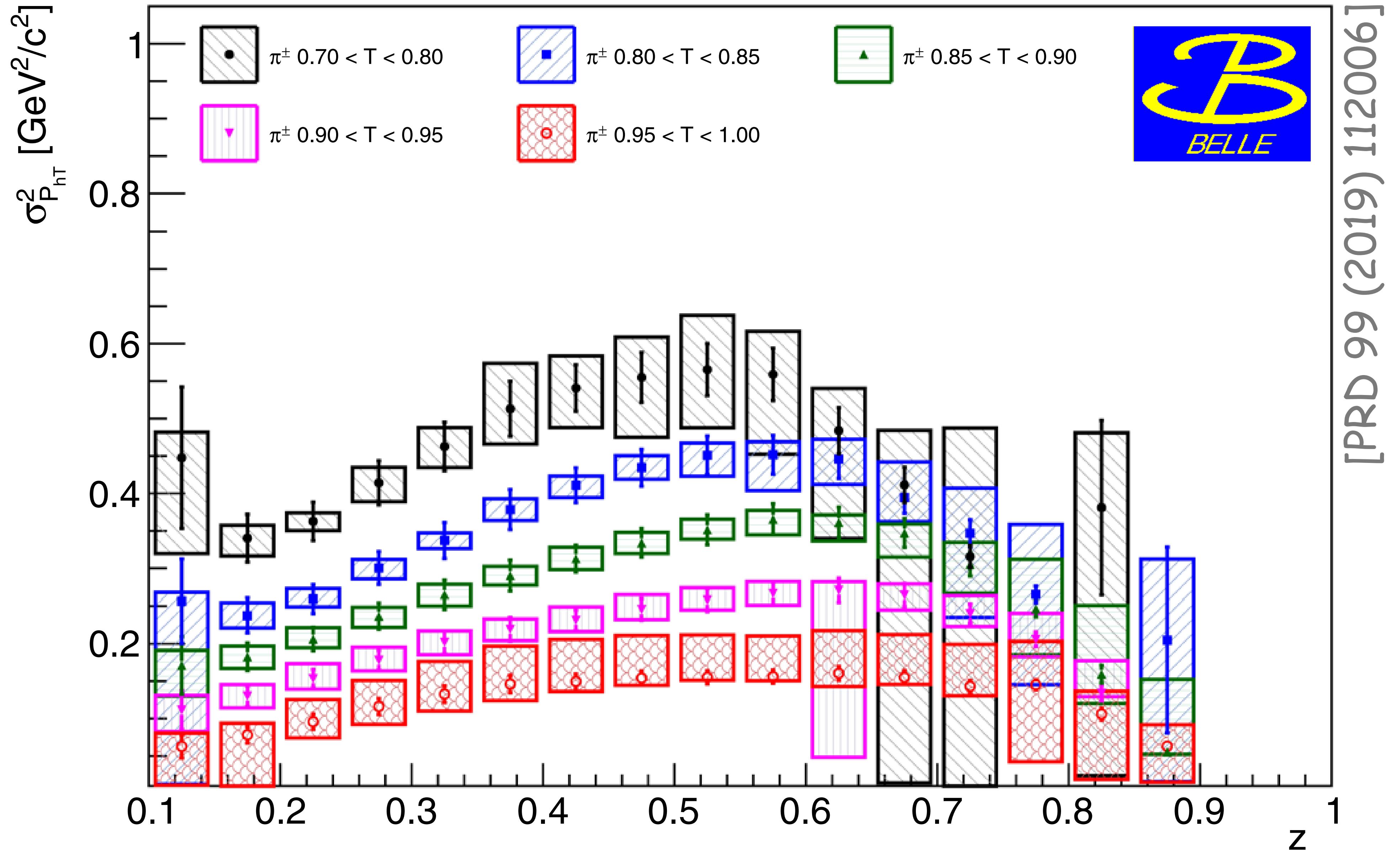
[PRD 99 (2019) 112006]

transverse-momentum: Gaussian widths

- **$0.85<T<0.90$**
  - fit Gauss to low-P_{hT} data
  - mostly well described with possible exception at high z
  - deviation from Gauss at large P_{hT}
  - clear increase of width with z for low values of z
- Gaussian widths depend on z and T
  - general increase with z with turnover at larger values of z
  - clear decrease of widths with increase of T
    - particles more and more collimated



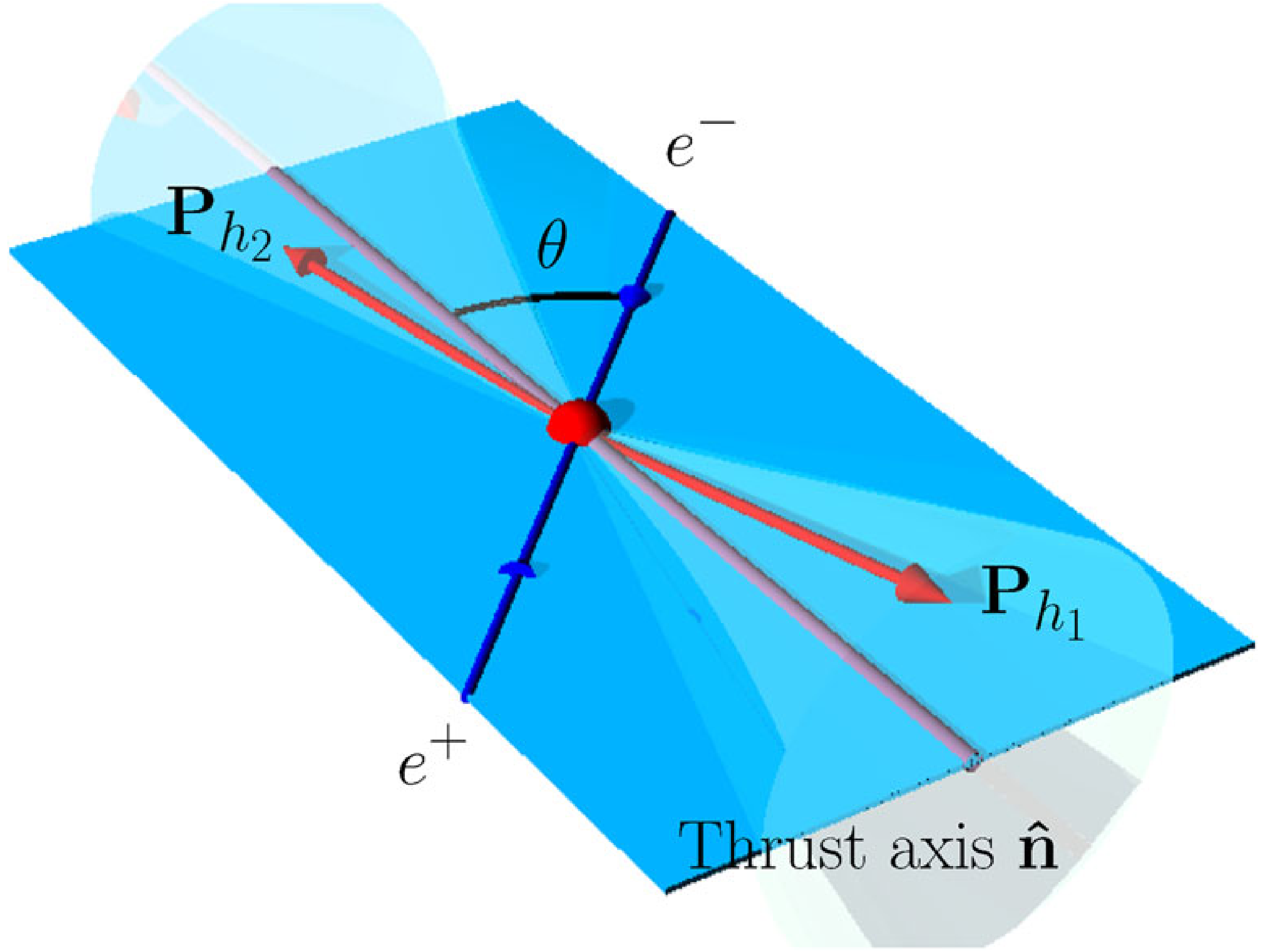
[PRD 99 (2019) 112006]

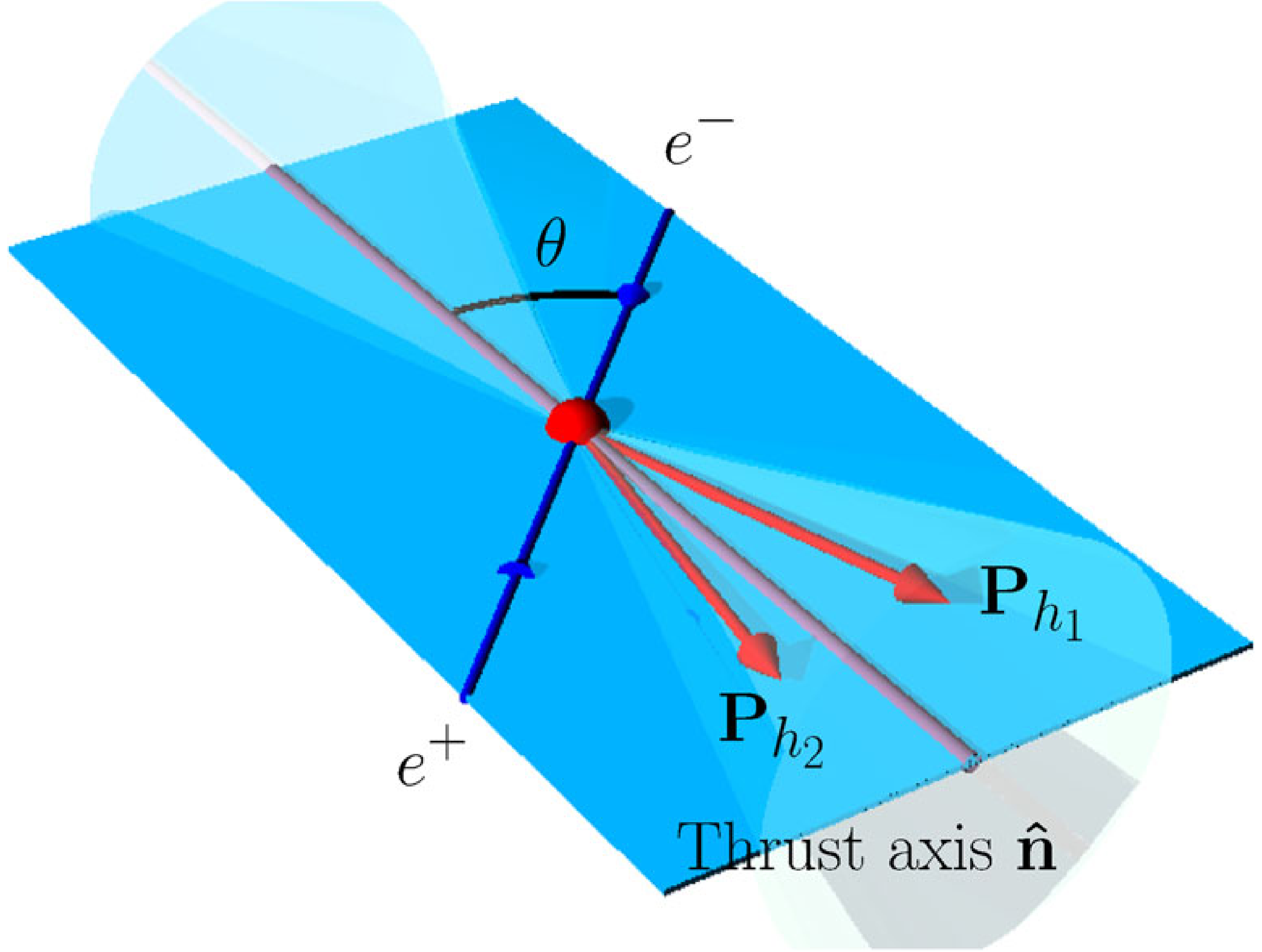


[PRD 99 (2019) 112006]

hadron-pair production

- single-hadron production has low discriminating power for parton flavor
- can use 2nd hadron in opposite hemisphere to "tag" flavor, transverse momentum, as well as polarization
  - mainly sensitive to product of single-hadron FFs
- if hadrons in same hemisphere: **dihadron fragmentation**
  - a la de Florian & Vanni [Phys. Lett. B 578 (2004) 139]
  - a la Collins, Heppelmann & Ladinsky [NPB 420 (1994) 565]; Boer, Jacobs & Radici [PRD 67 (2003) 094003]
- raises question of defining hemispheres
  - common choices: separation by plane normal to i) thrust axis or to ii) one of the two hadrons (back-to-back case)
  - alternatively, via relevant kinematic variables





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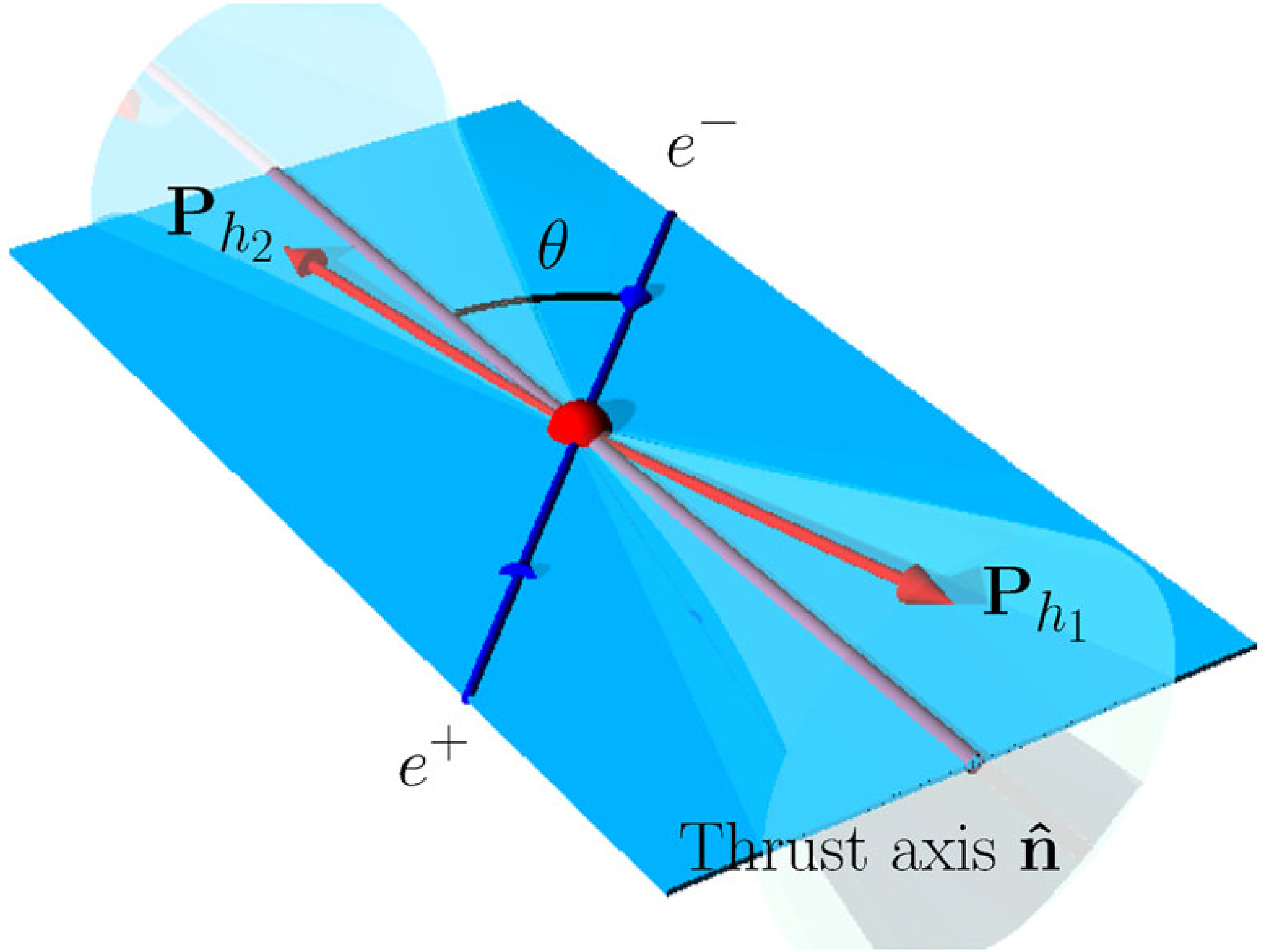
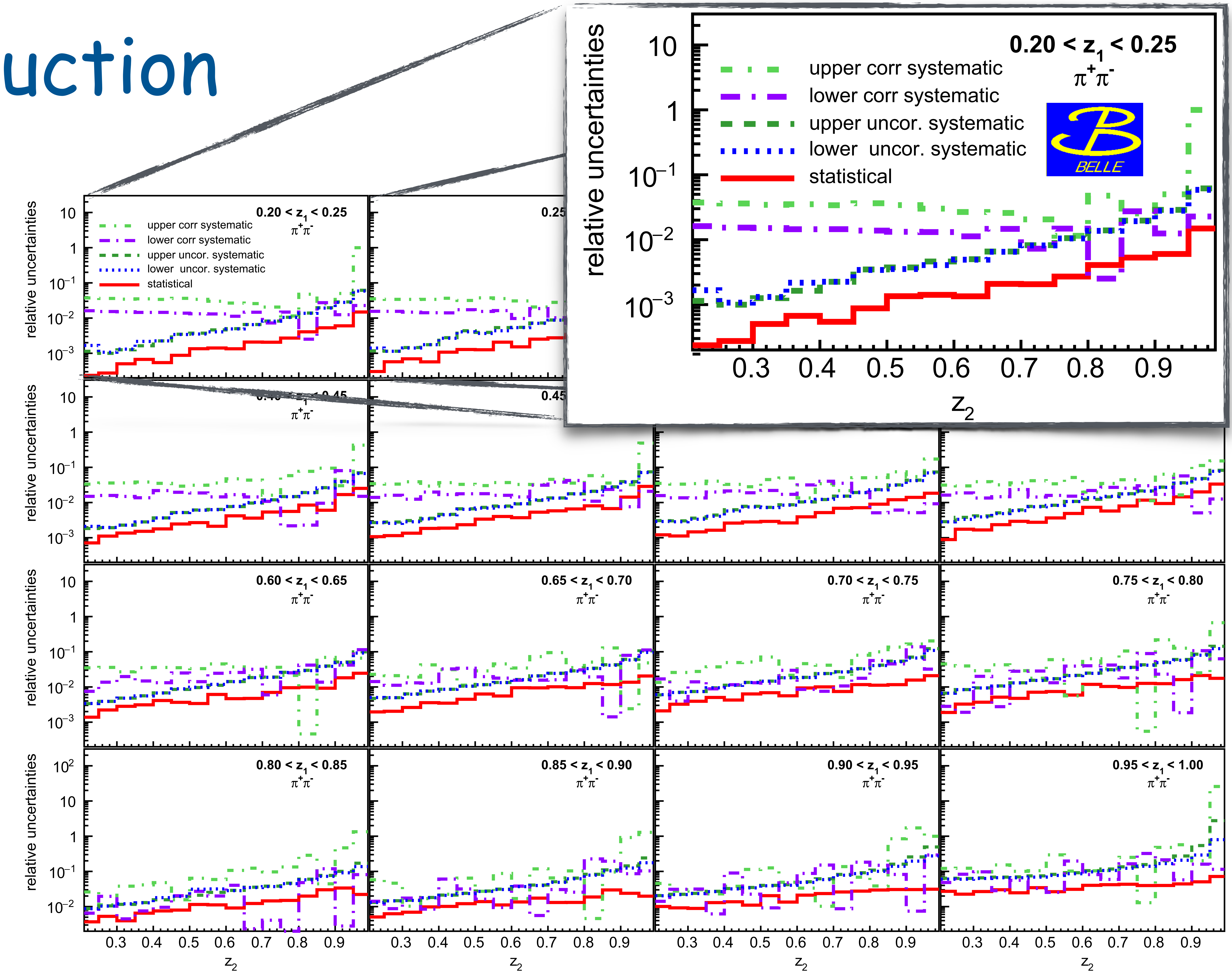


- various definitions for scaling variable
  - traditional z ("std"): $$z_i = \frac{2P_i \cdot q}{q^2} \qquad (i = 1, 2)$$
  - Altarelli et al. ("AEMP"): [Nucl. Phys. B160 (1979) 301] $$z_1 = \frac{2P_1 \cdot q}{q^2} \qquad z_2 = \frac{P_1 \cdot P_2}{P_1 \cdot q}$$
  - Mulders & van Hulse ("MVH"): [PRD 100 (2019) 034011] $$z_1 = \left(P_1 \cdot P_2 - \frac{M_{h1}^2 M_{h2}^2}{P_1 \cdot P_2}\right) \frac{1}{P_2 \cdot q - M_{h2}^2 \frac{P_1 \cdot q}{P_1 \cdot P_2}}$$

light-meson pair production

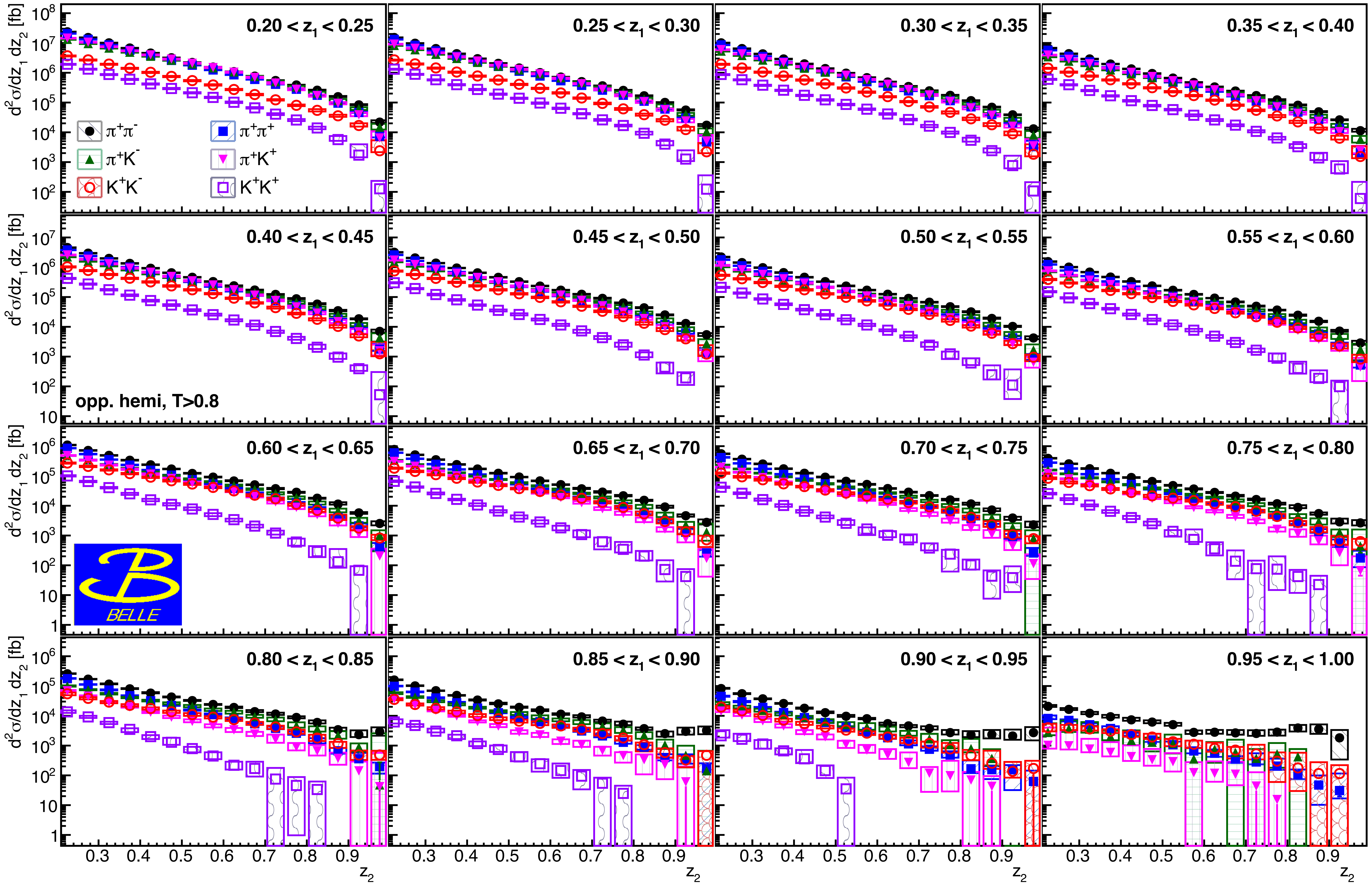
- systematics dominated over entire kinematic range
- strongly asymmetric systematics
    - no straightforward use in fits
- systematics dominated over entire kinematic range



light-meson pair production

- systematics dominated over entire kinematic range
- strongly asymmetric systematics
  - no straightforward use in fits
- clear flavor dependence
  - suppression of like-sign pairs
  - suppression of kaons
  - more pronounced at large z (stronger flavor sensitivity)

[PRD 101 (2020) 092004]



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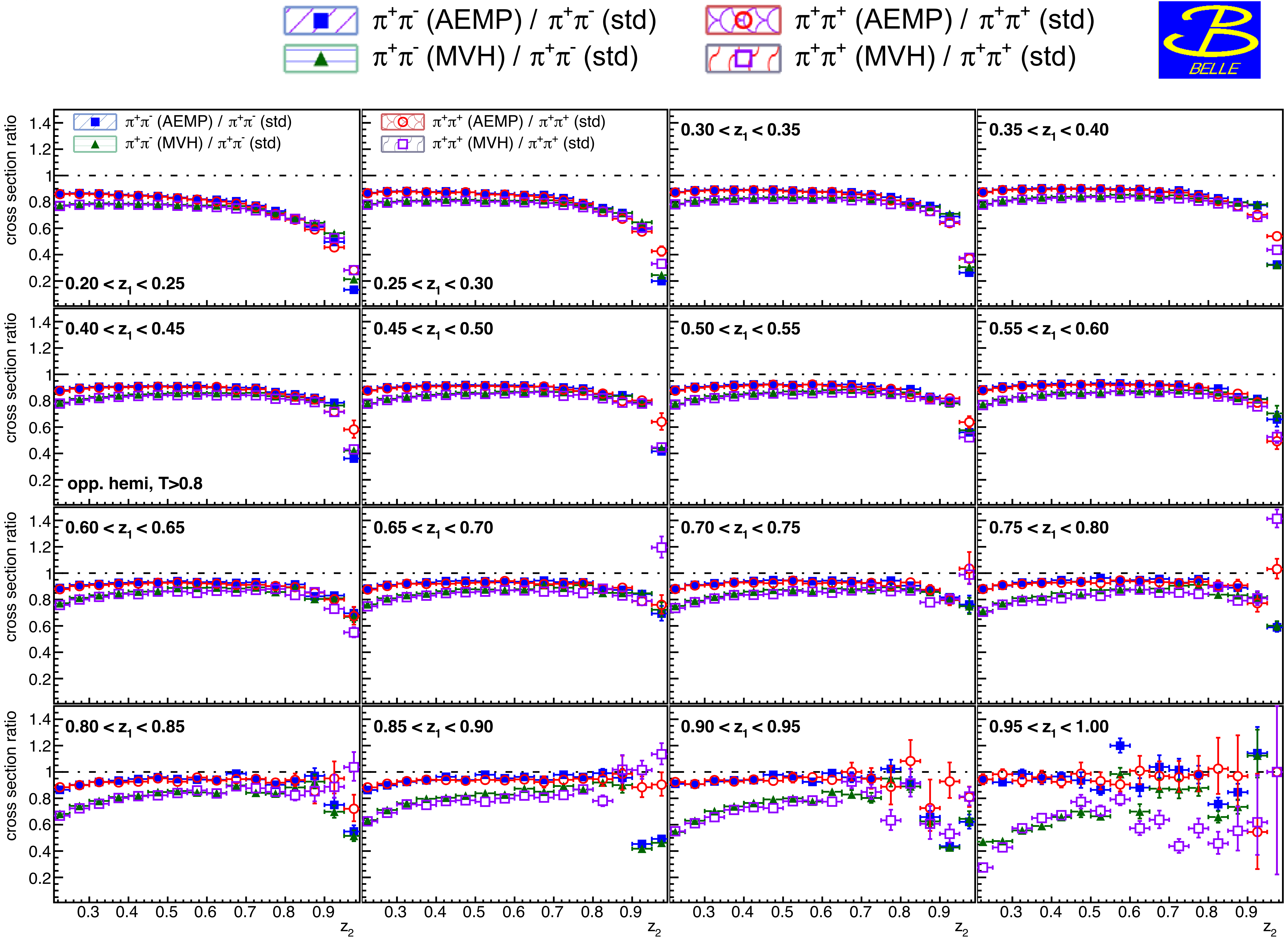
- similar behavior for different z definitions when imposing T>0.8



[PRD 101 (2020) 092004]

light-meson pair production

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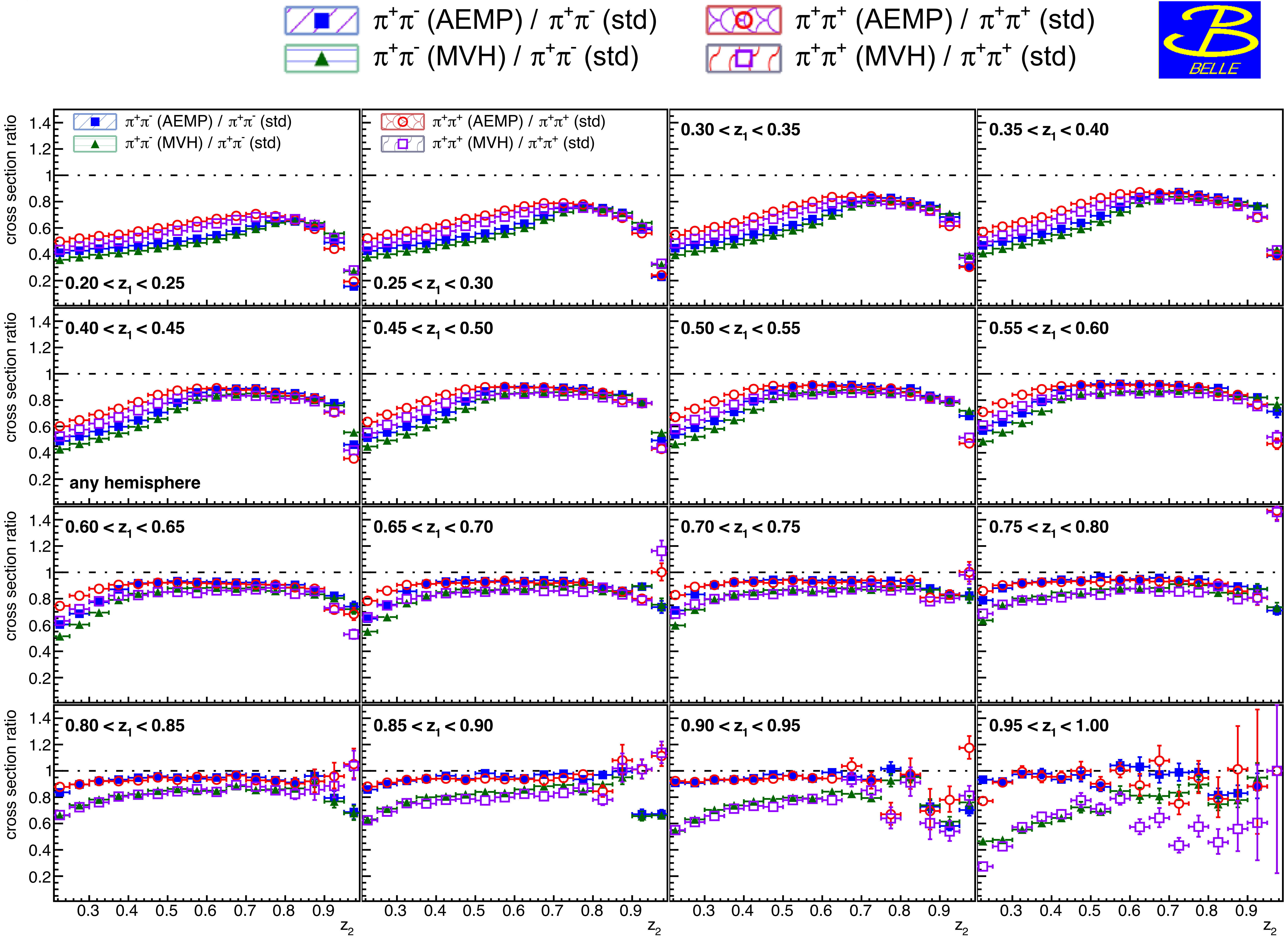
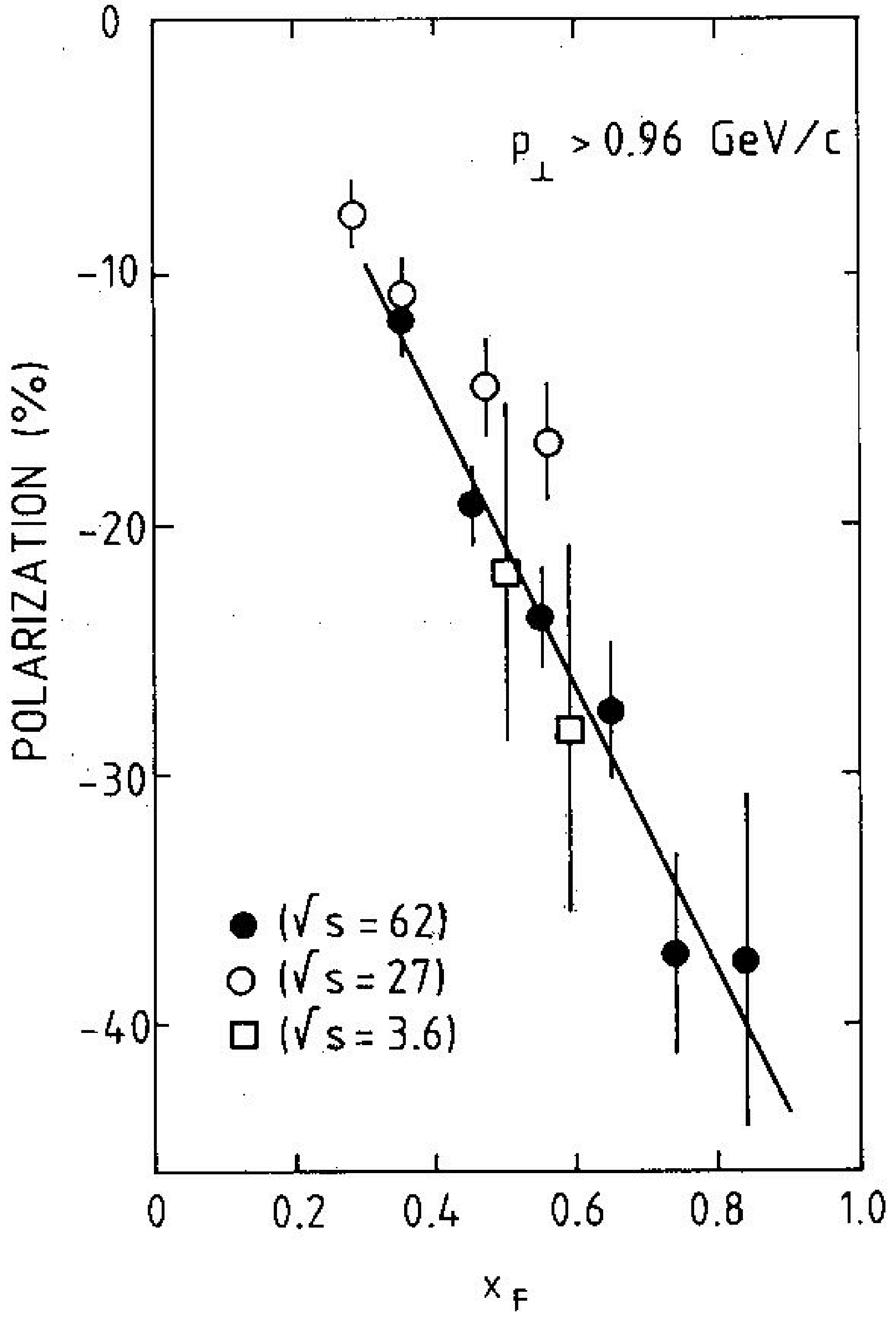
- strongly asymmetric systematics
  - no straightforward use in fits
- clear flavor dependence
  - suppression of like-sign pairs
  - suppression of kaons
  - more pronounced at large z (stronger flavor sensitivity)
- similar behavior for different z definitions when imposing T>0.8
- larger suppression (low z) for fully inclusive pairs (“any hemisphere”)



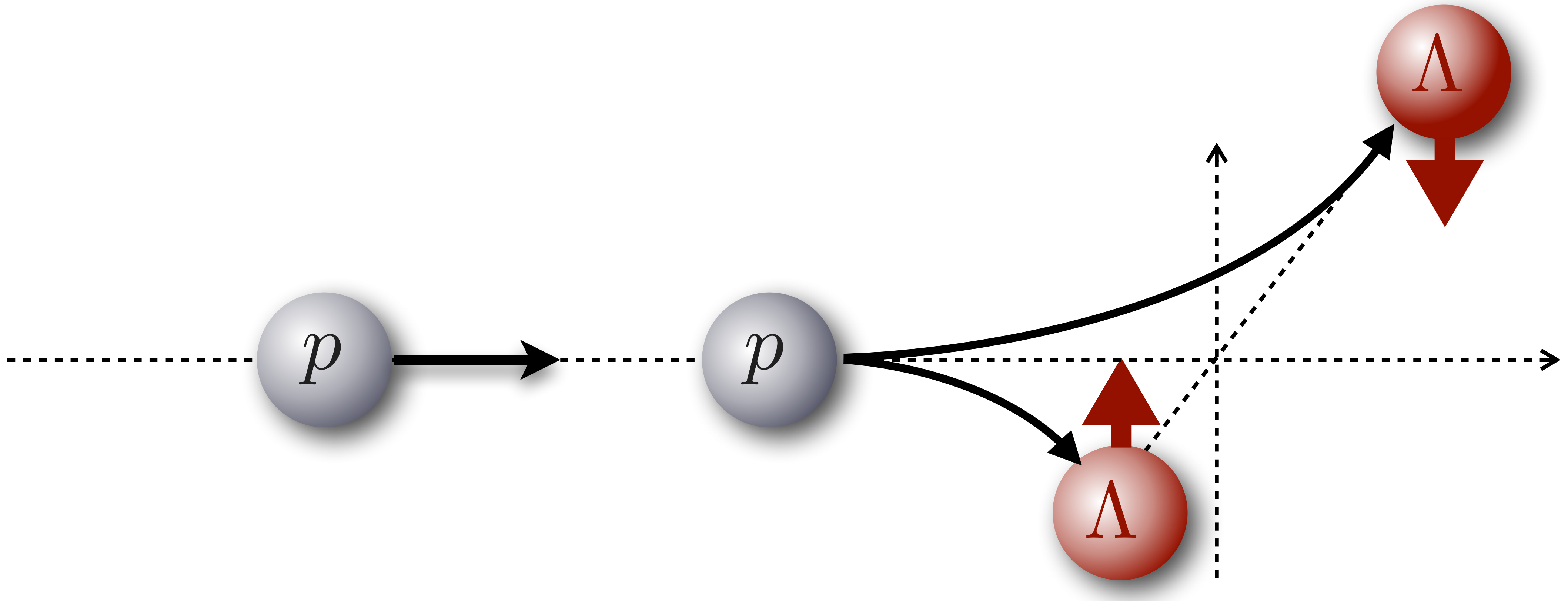
[PRD 101 (2020) 092004]

polarization

despite unpolarized initial state

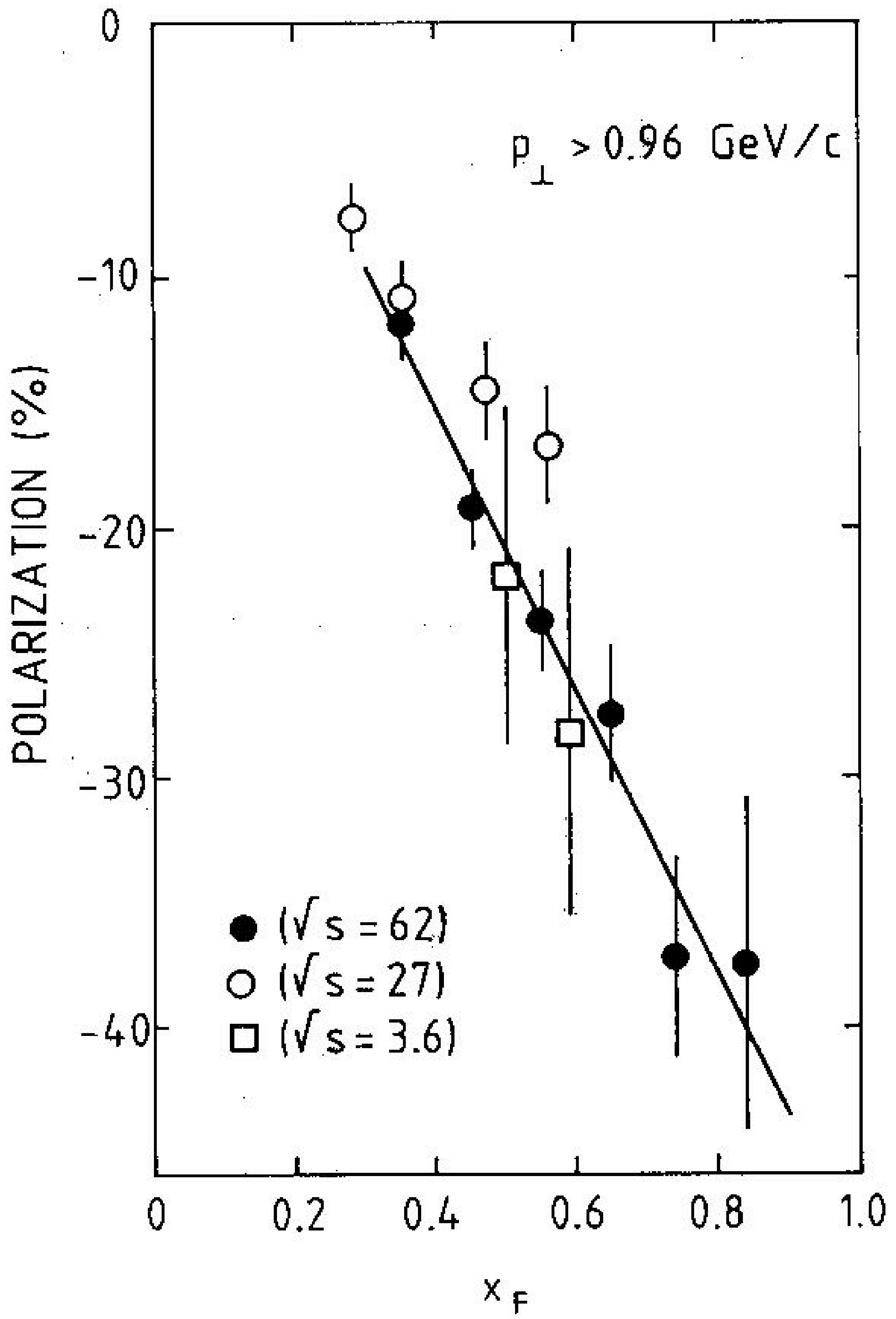


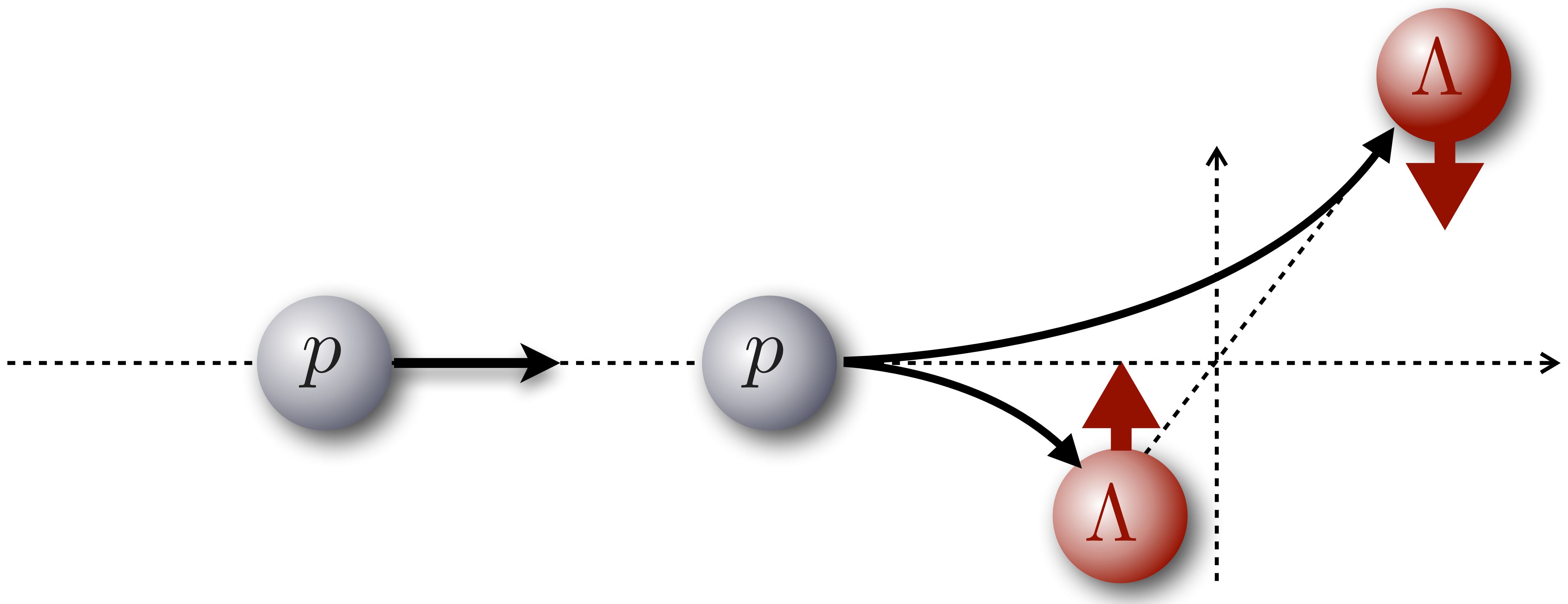
polarizing fragmentation



- large hyperon polarization in unpolarized hadron collision observed

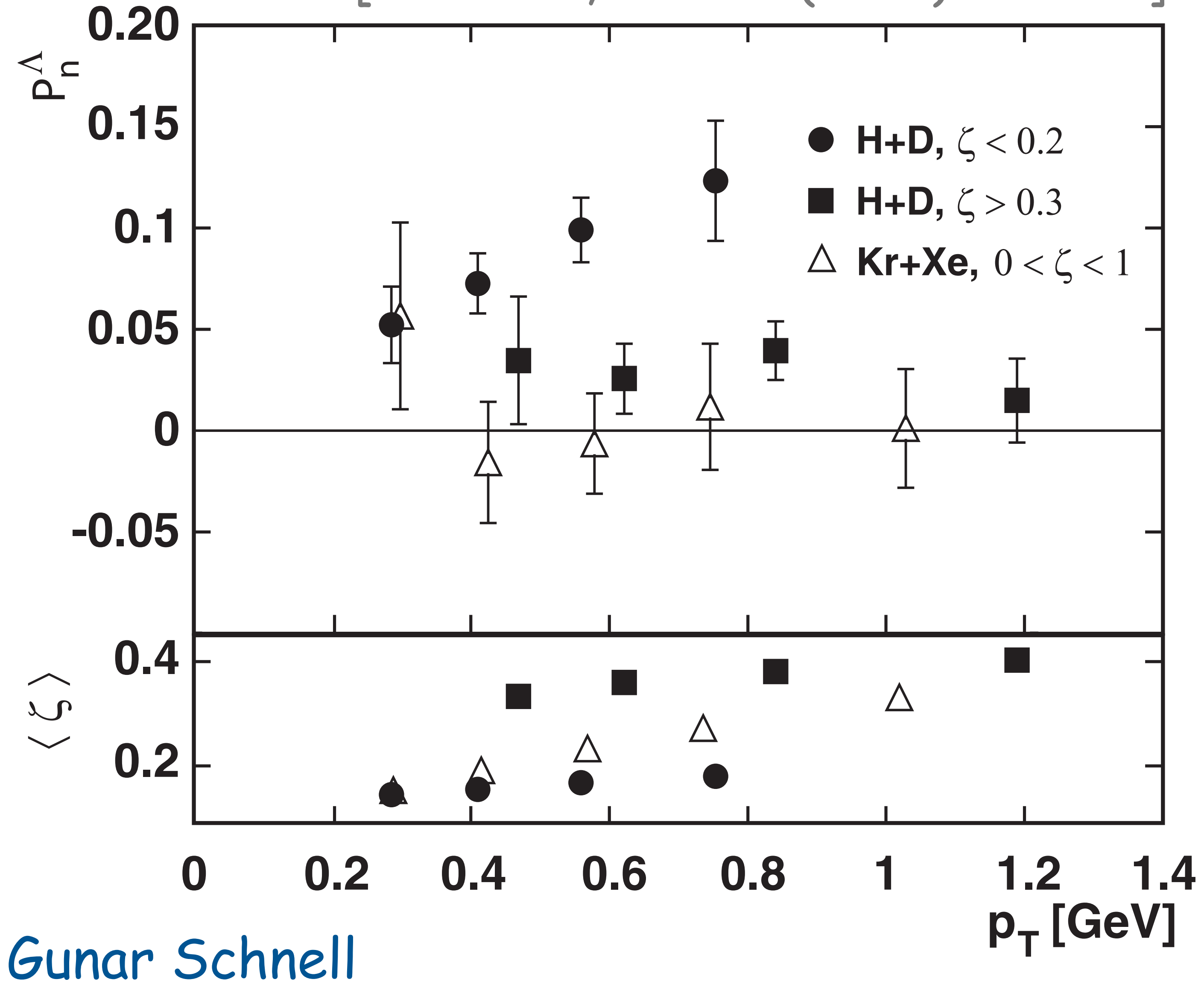
polarizing fragmentation



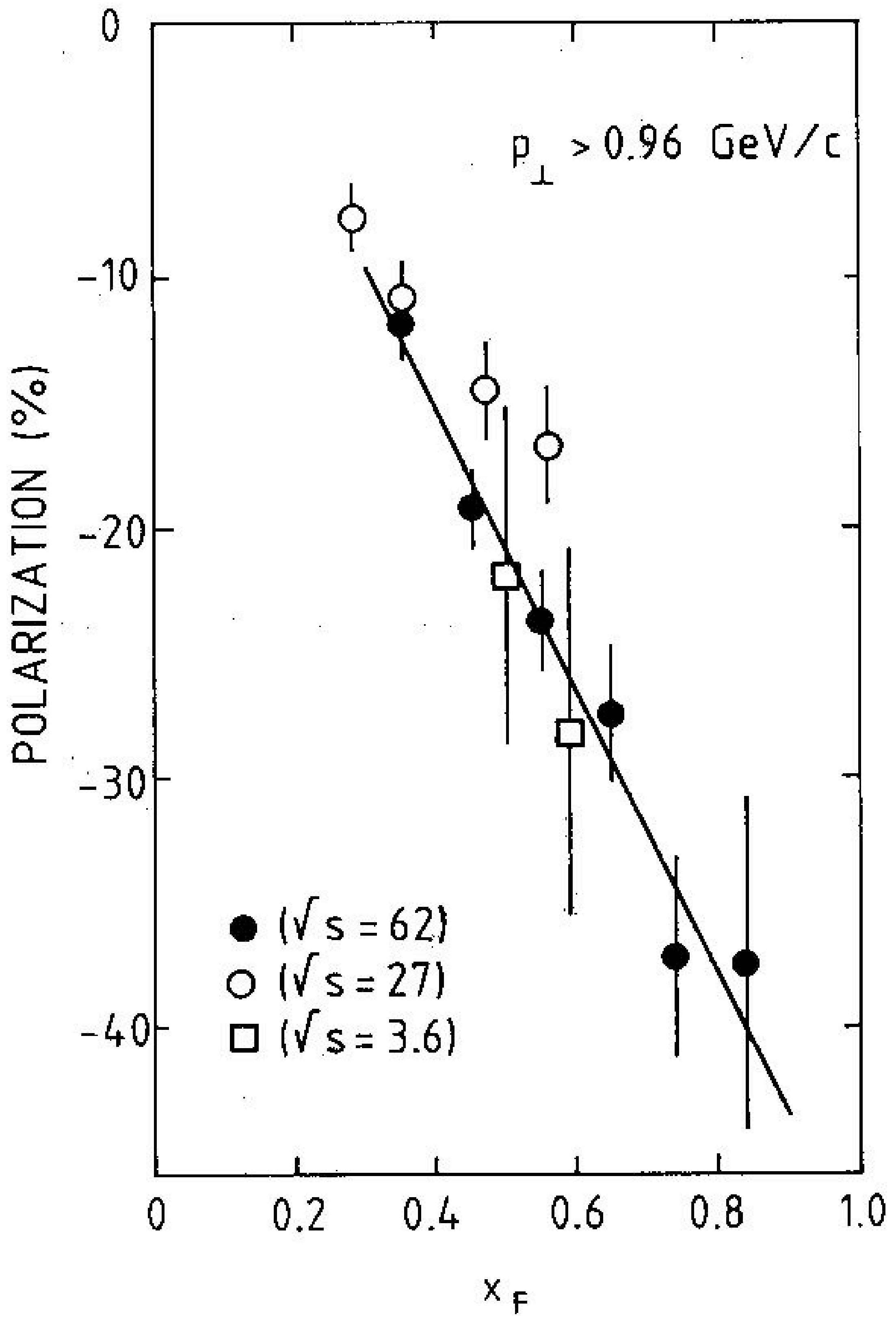


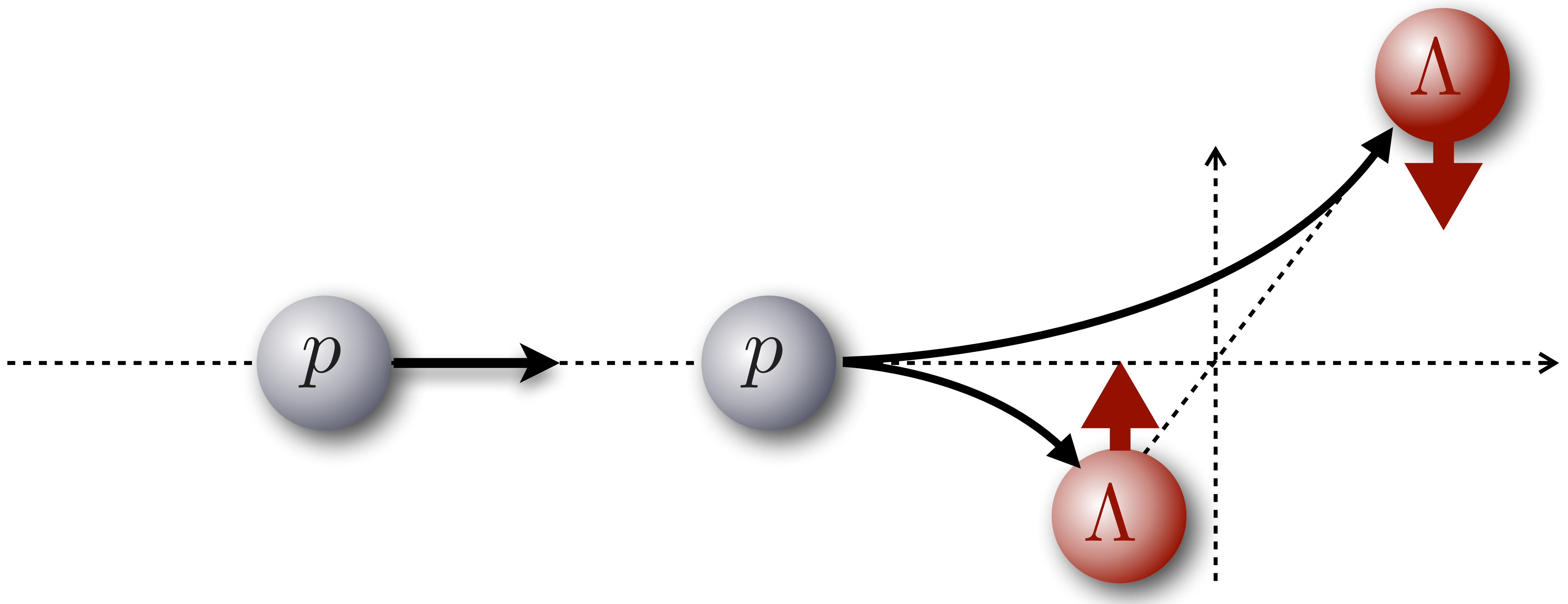
- large hyperon polarization in unpolarized hadron collision observed
- … as well as in inclusive lepto-production

[HERMES, PRD 90 (2014) 072007]



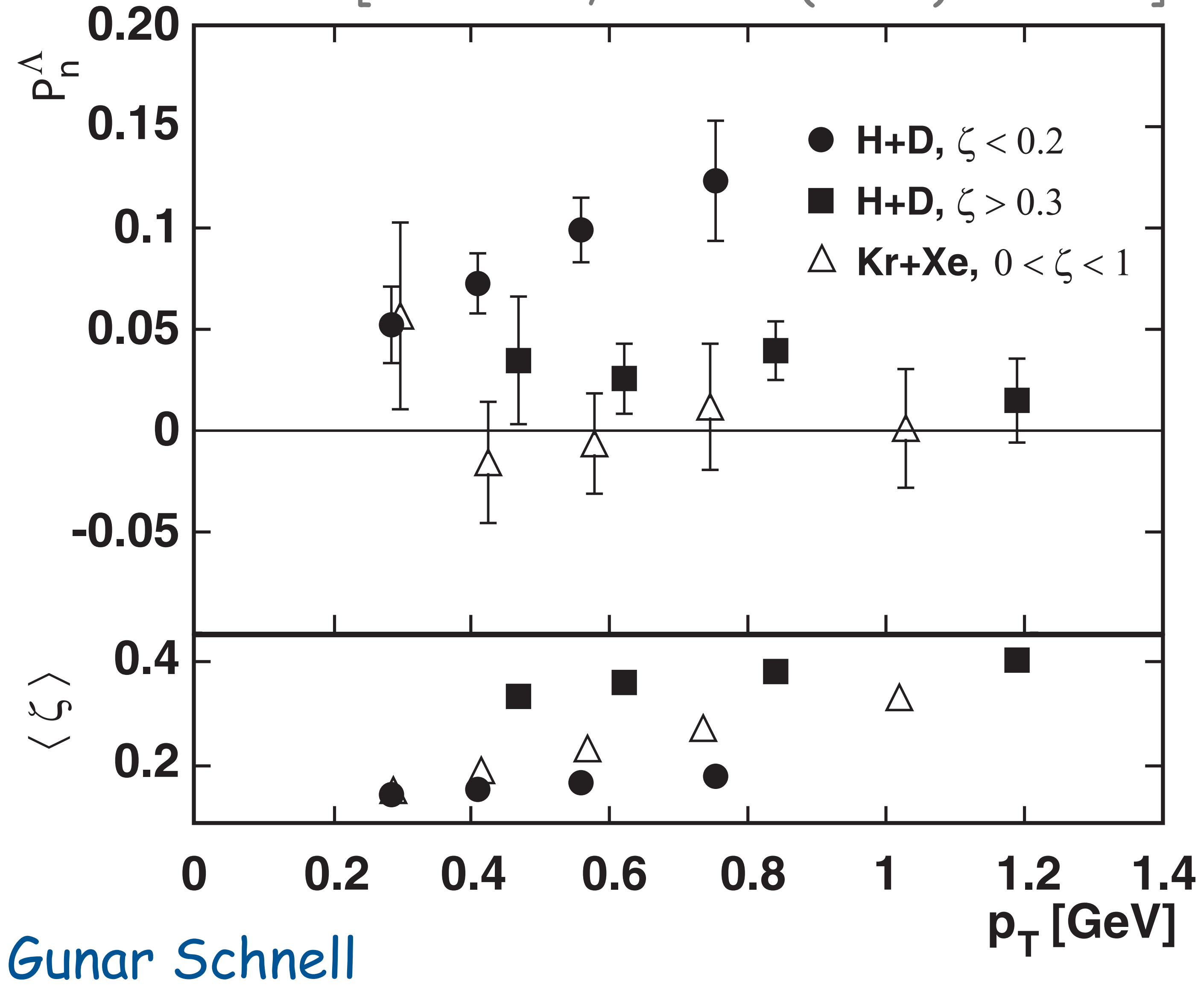
polarizing fragmentation

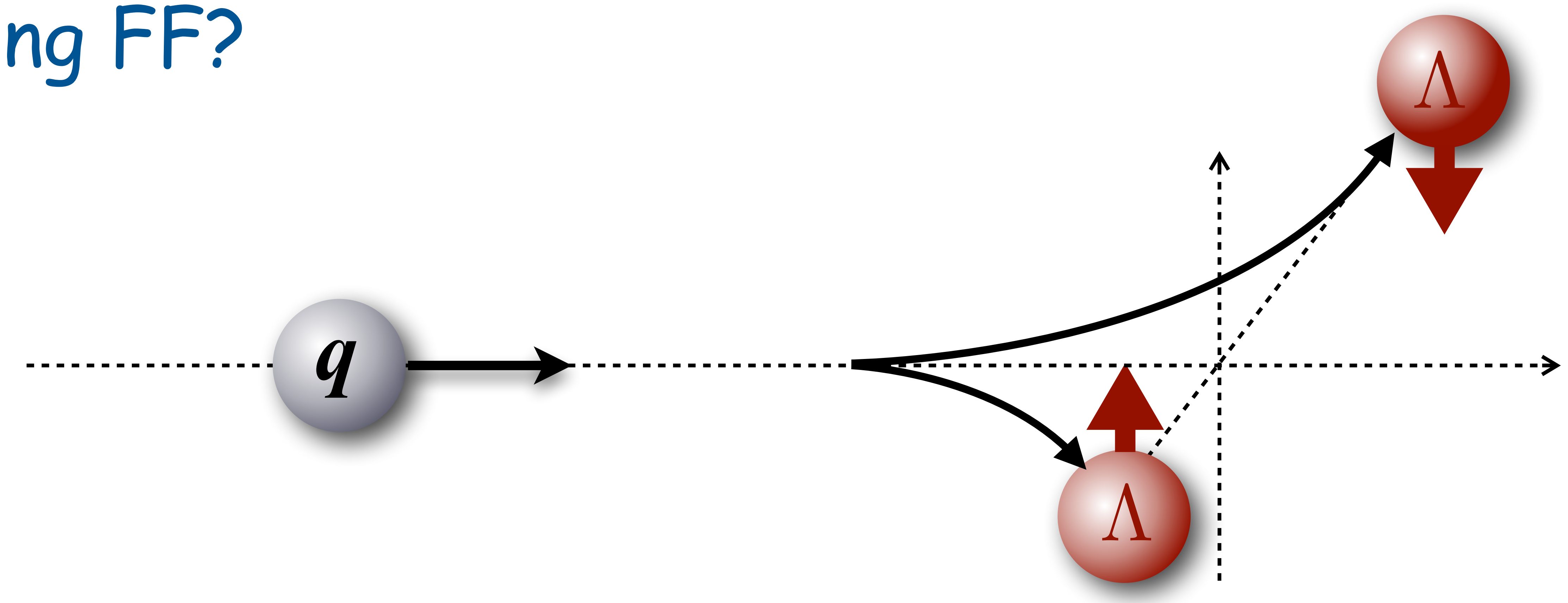




- large hyperon polarization in unpolarized hadron collision observed
- … as well as in inclusive lepto-production
- caused by polarizing FF?

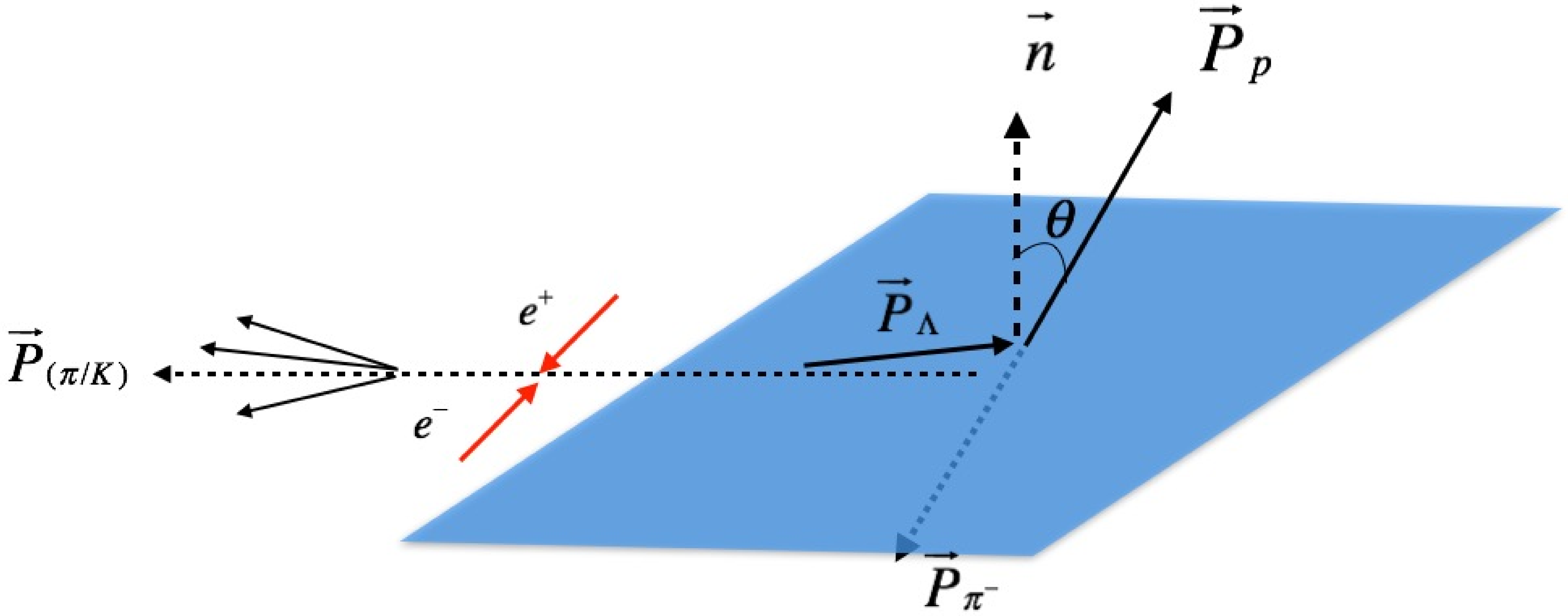
[HERMES, PRD 90 (2014) 072007]





polarizing fragmentation function

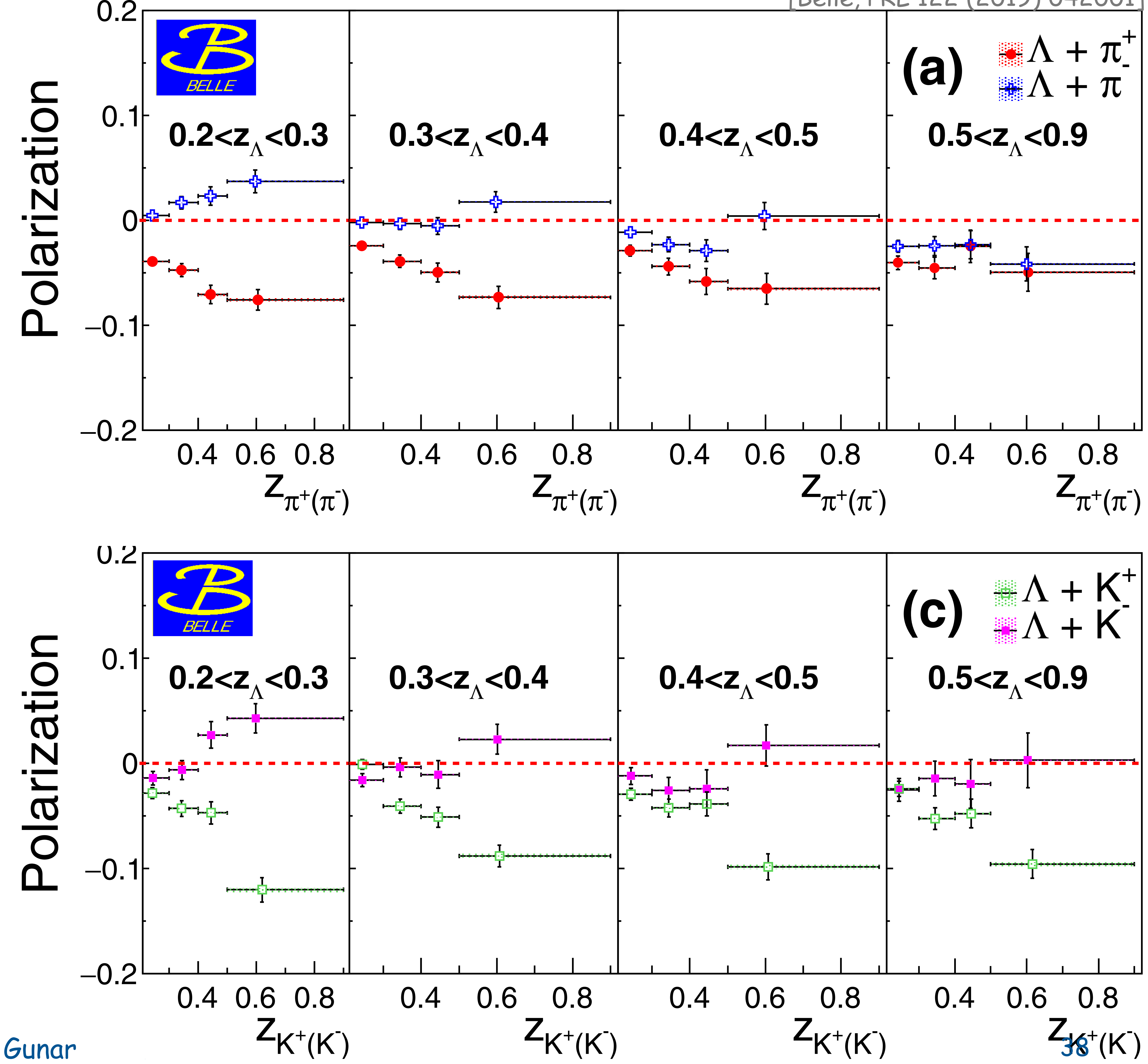
- polarization measured normal to production plane, i.e. $\propto$ ("P_q" × P_Λ)



- reference axis to define transverse momentum:
  - "hadron frame" - use momentum direction of "back-to-back" hadron
  - "thrust frame" - use thrust axis
- exploit self-analyzing weak decay of Λ to determine polarization

polarizing fragmentation function

[Belle, PRL 122 (2019) 042001]



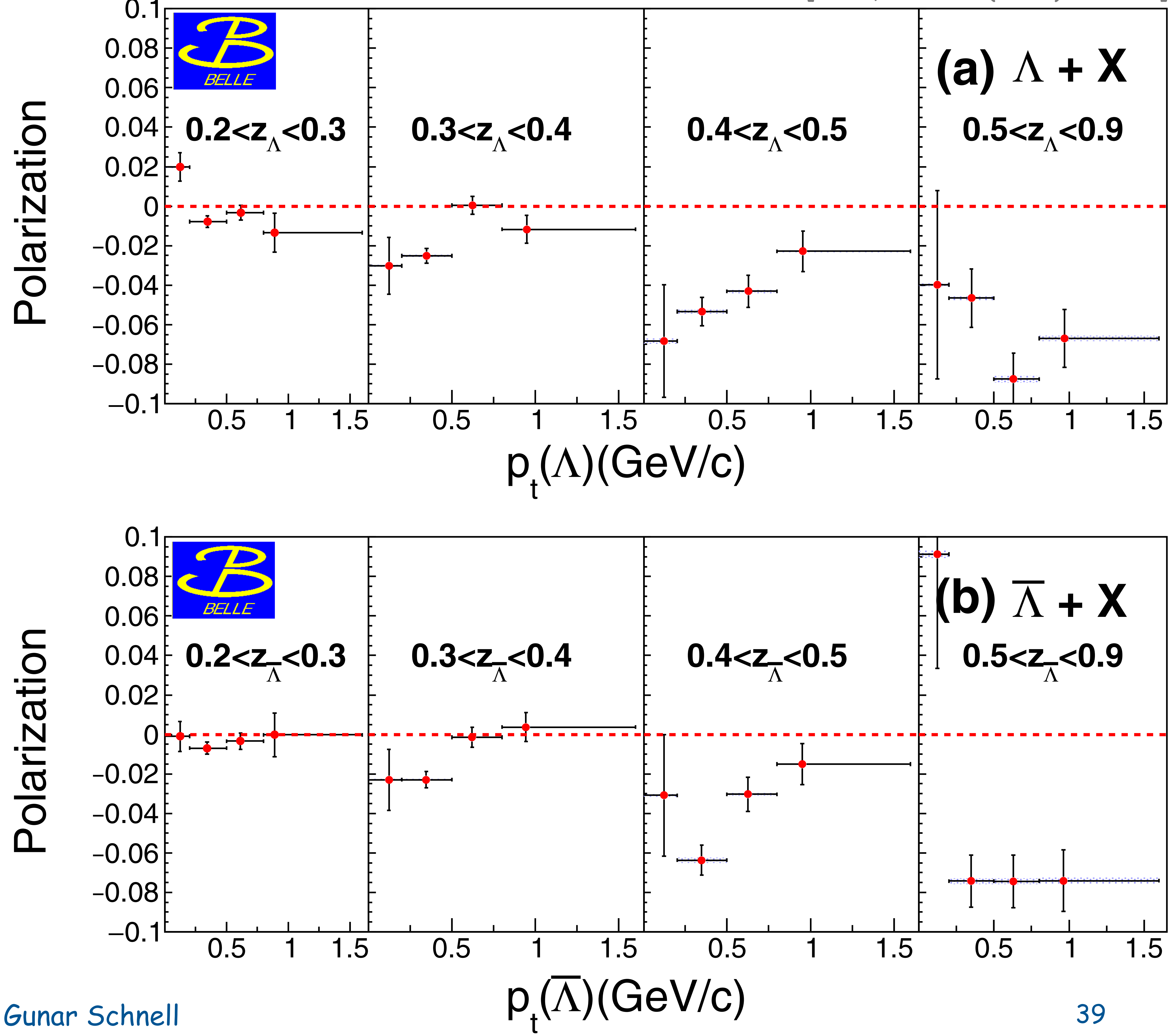
- flavor tagging through hadrons in opposite hemisphere:
  - large-z_h hadrons tag quark flavor more efficiently

    ➡ enlarges differences between oppositely charged hadrons
  - MC-based quark-flavor decomposition in backup

$$z_h = \frac{E_h}{\sqrt{s}/2}$$

polarizing fragmentation function

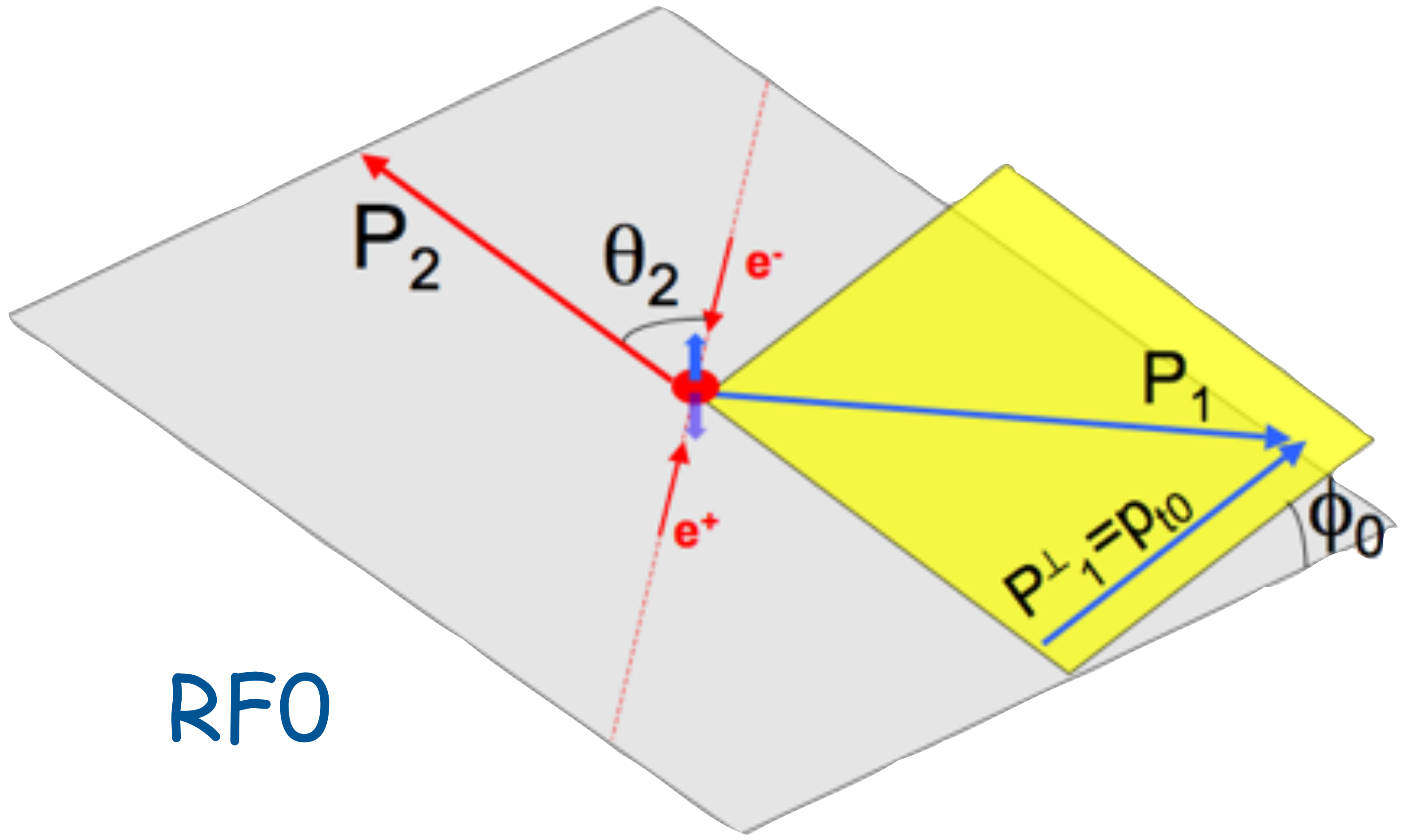
[Belle, PRL 122 (2019) 042001]



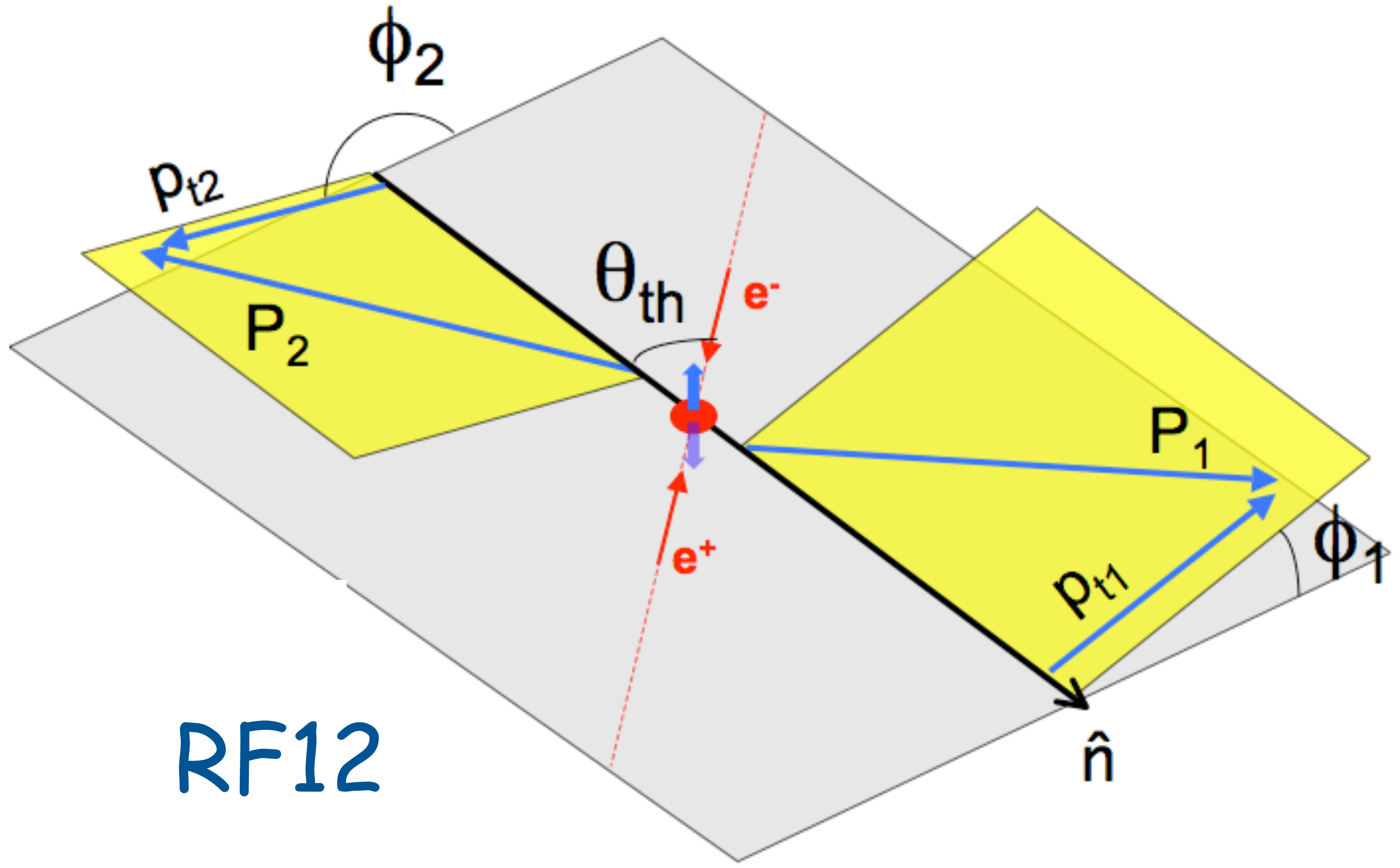
- polarization measured as function of z and p_T
- strong dependence on both kinematics
  - somewhat unexpected behavior for p_T -> 0

hadron pairs: angular correlations

- angular correlations between nearly back-to-back hadrons used to tag transverse quark polarization -> Collins fragmentation functions
  - RF0: one hadron as reference axis -> $\cos(2\phi_0)$ modulation
  - RF12: thrust (or similar) axis -> $\cos(\phi_1+\phi_2)$ modulation



RF0

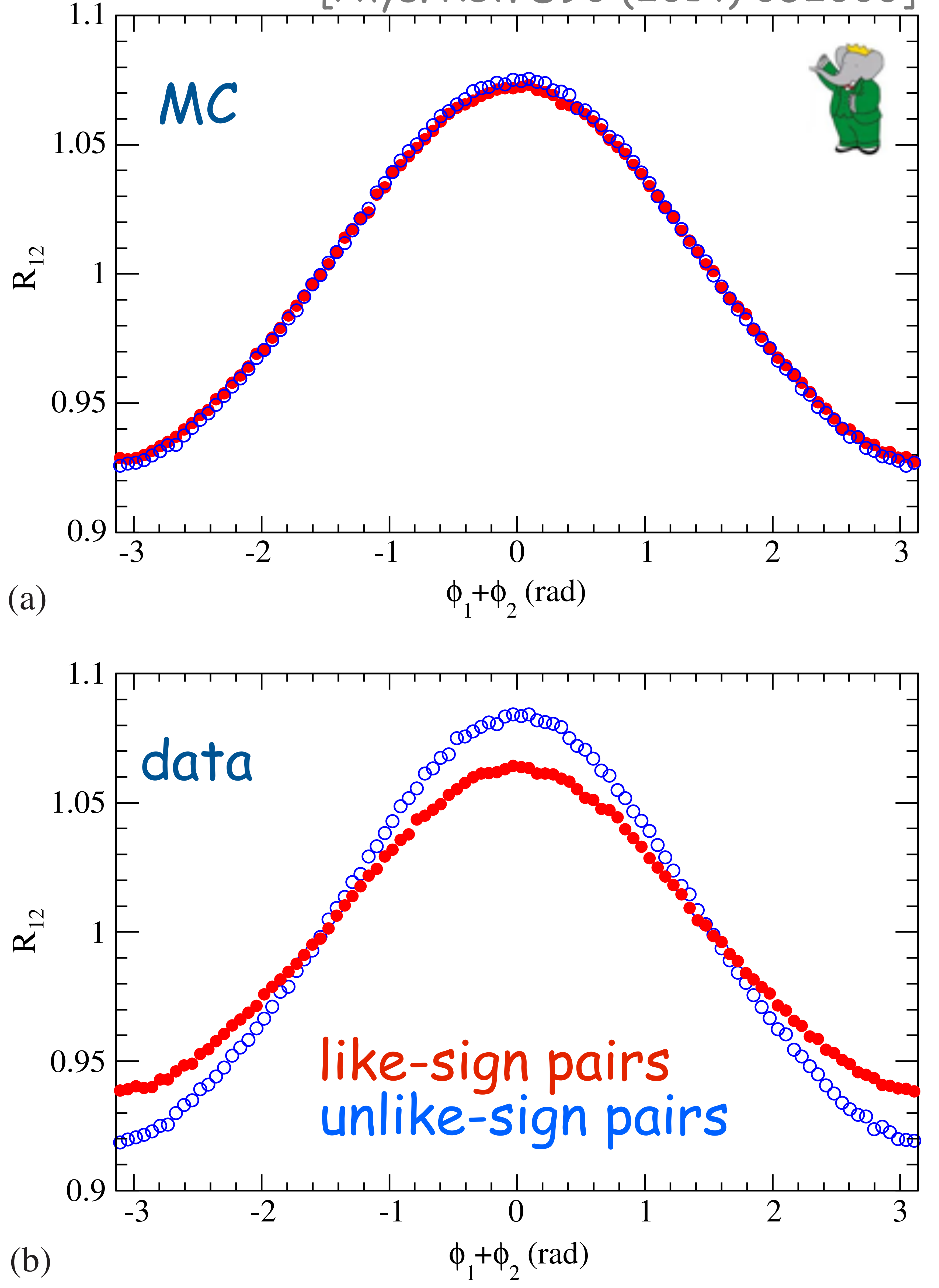


RF12

- RF0 and RF12: different convolutions over transverse momenta
- debatable: MC used to “correct” thrust axis to $q\bar{q}$ axis

hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)



[Phys. Rev. D90 (2014) 052003]

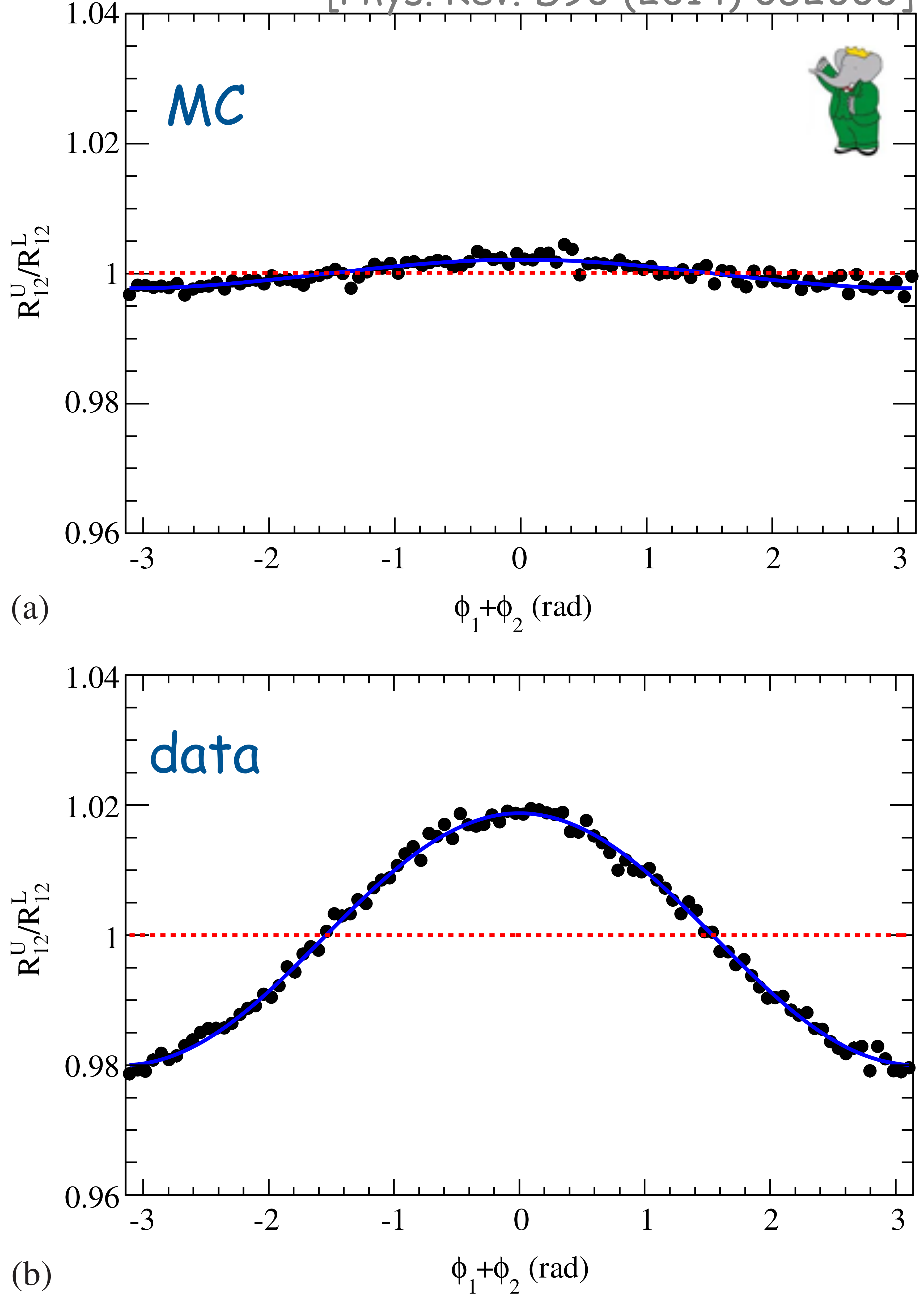
hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)
- construct double ratio of normalized-yield distributions R_{12}, e.g. unlike-/like-sign:

$$\frac{R_{12}^U}{R_{12}^L} \simeq \frac{1+\langle\frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}}\rangle G^U\cos(\phi_1+\phi_2)}{1+\langle\frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}}\rangle G^L\cos(\phi_1+\phi_2)}$$

$$\simeq 1+\left\langle\frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}}\right\rangle\{G^U-G^L\}\cos(\phi_1+\phi_2)$$

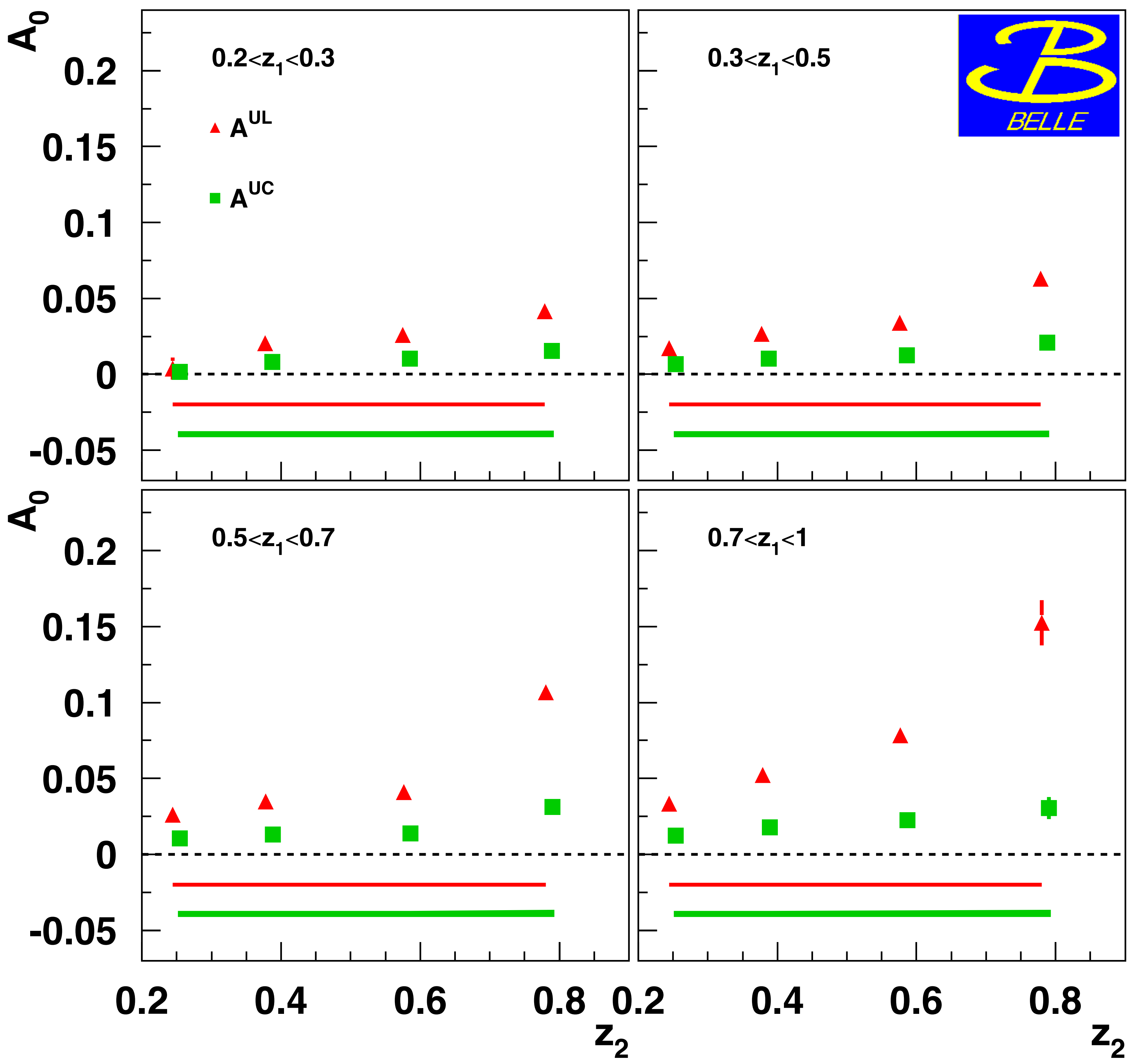
- suppresses flavor-independent sources of modulations
- $G^{U/L}$: specific combinations of FFs
- remaining MC asymmetries ➠ systematics



[Phys. Rev. D90 (2014) 052003]

Collins asymmetries (RF0)

- first measurement of Collins asymmetries by Belle [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
  - significant asymmetries rising with z
  - used for first transversity and Collins FF extractions



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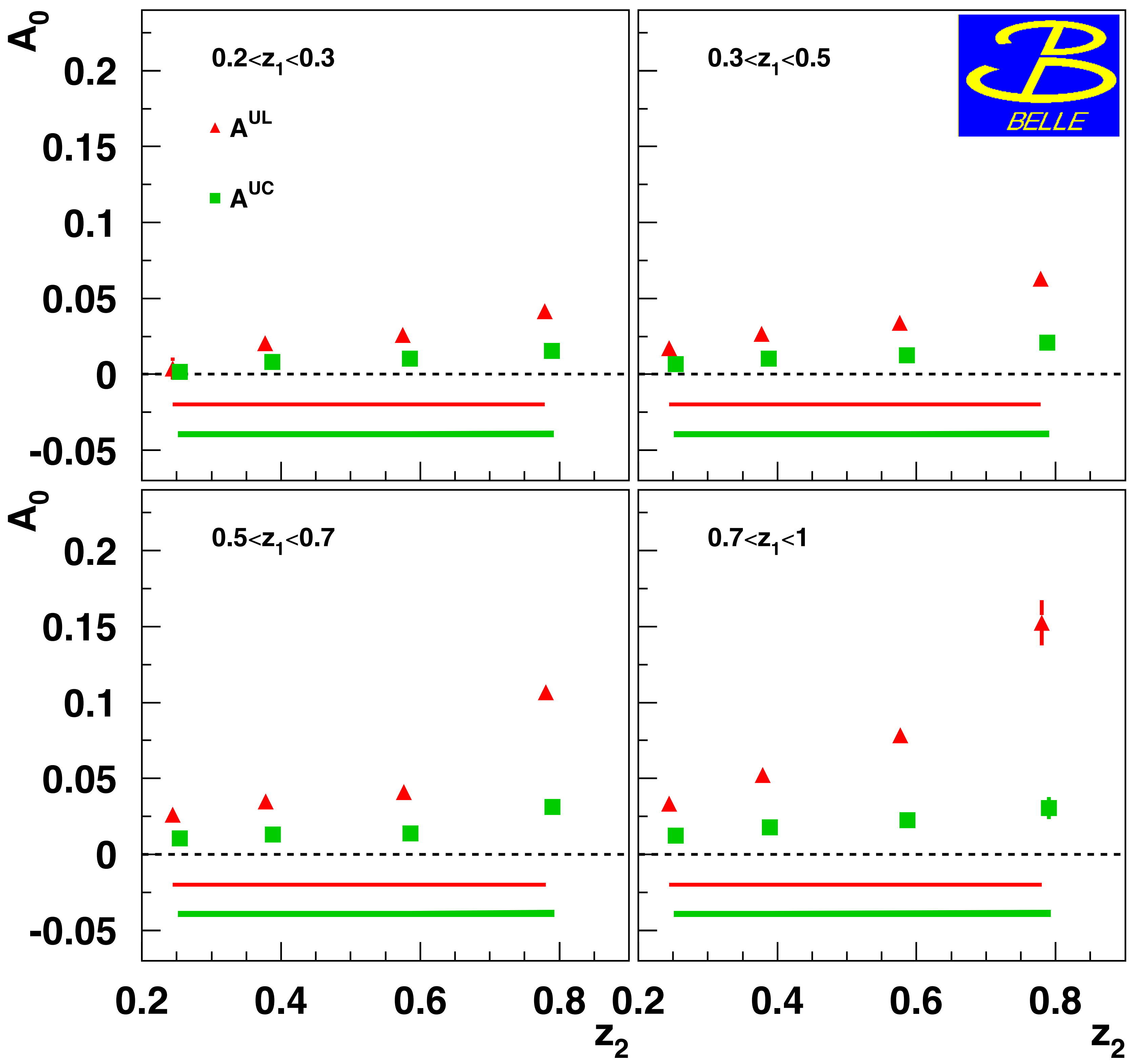

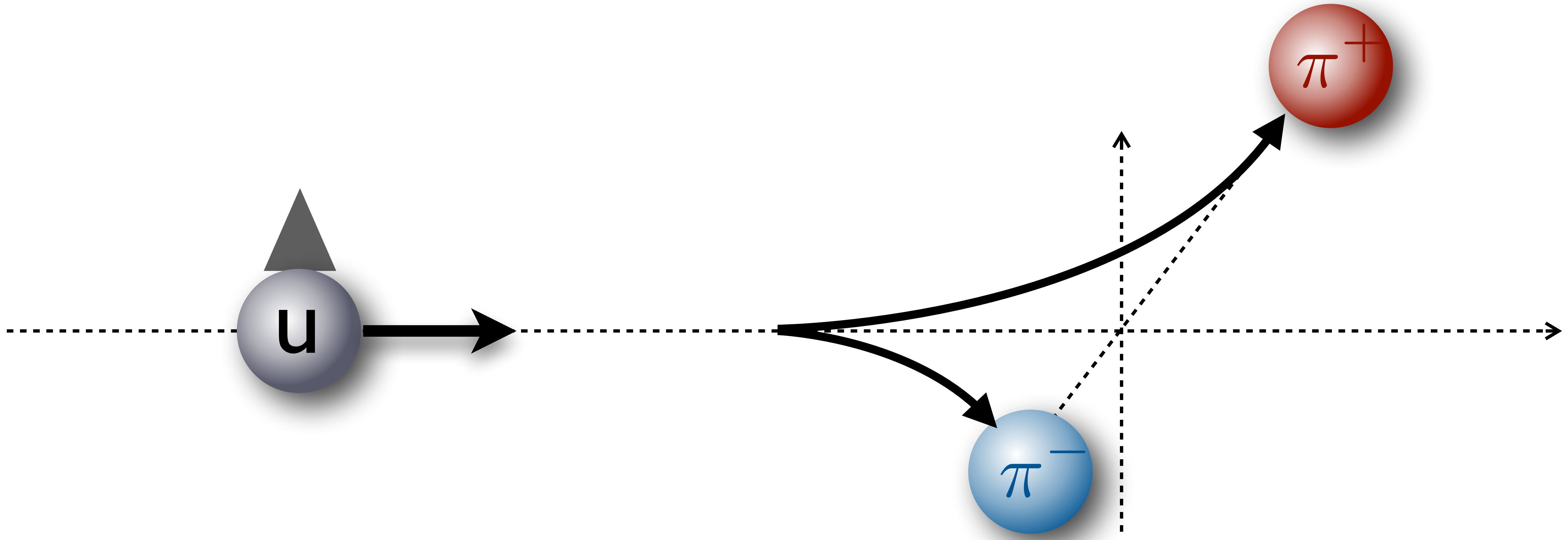

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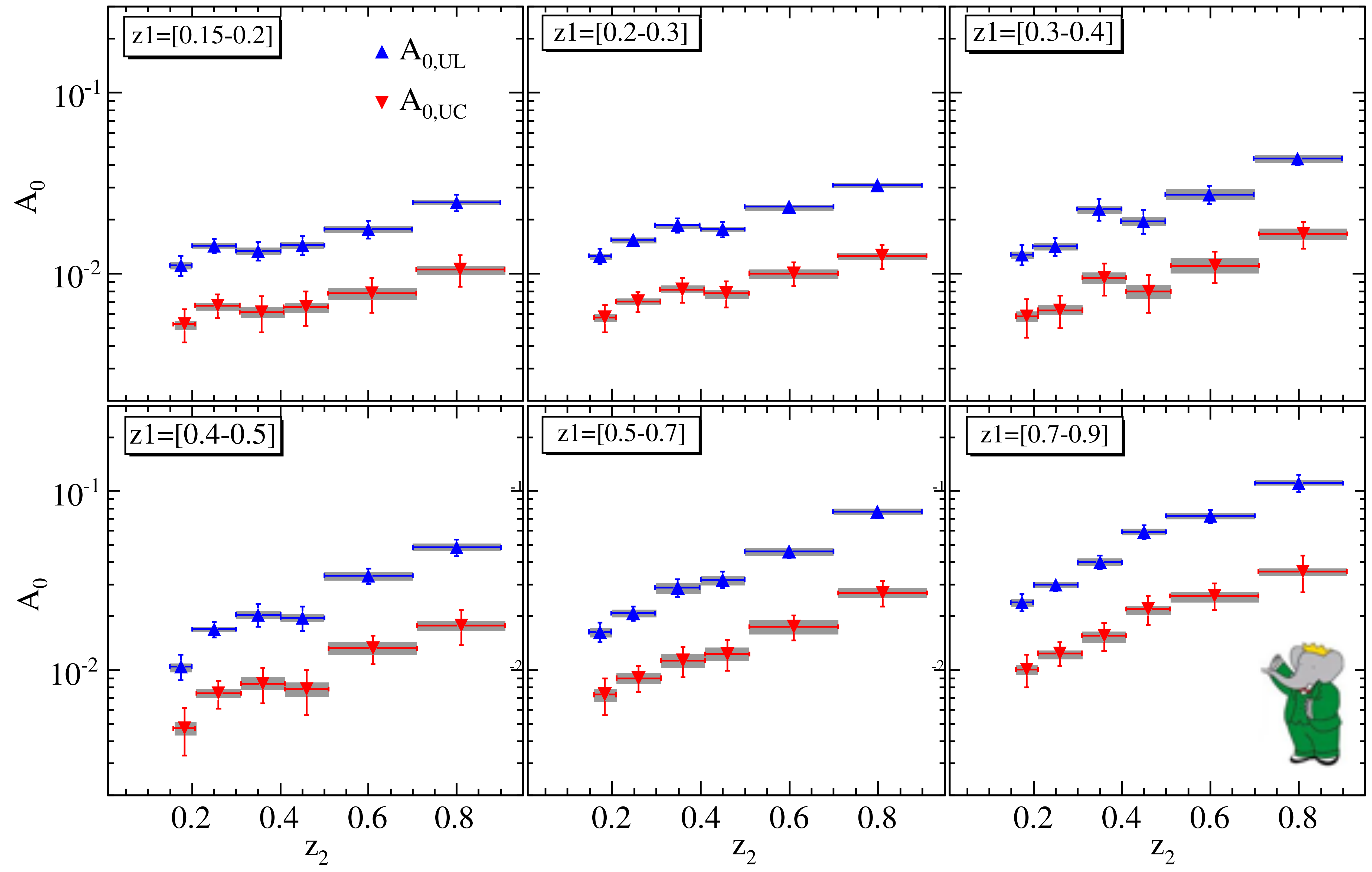

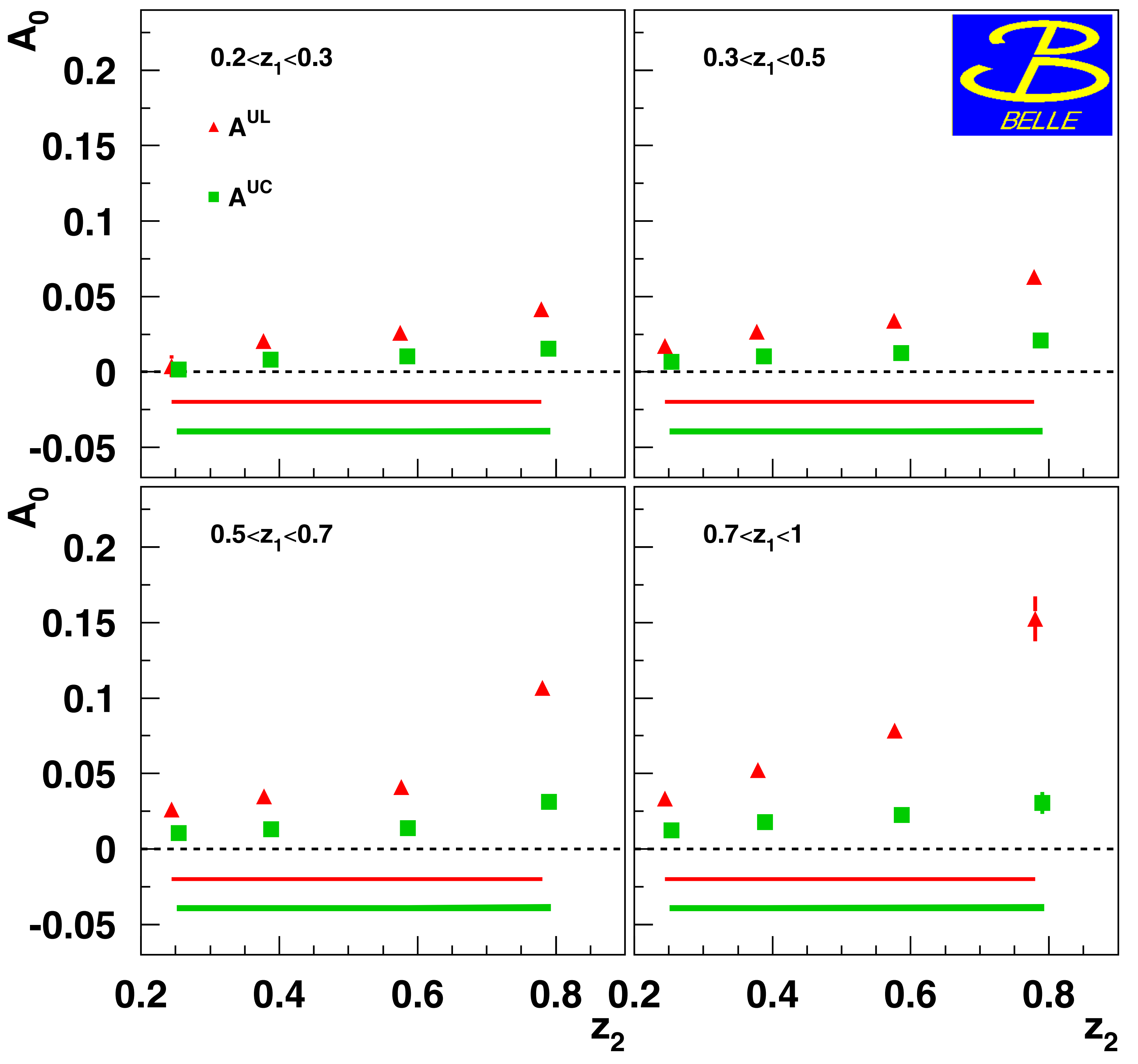

- BaBar results [PRD 90 (2014) 052003] consistent with Belle

Collins asymmetries (RF0)

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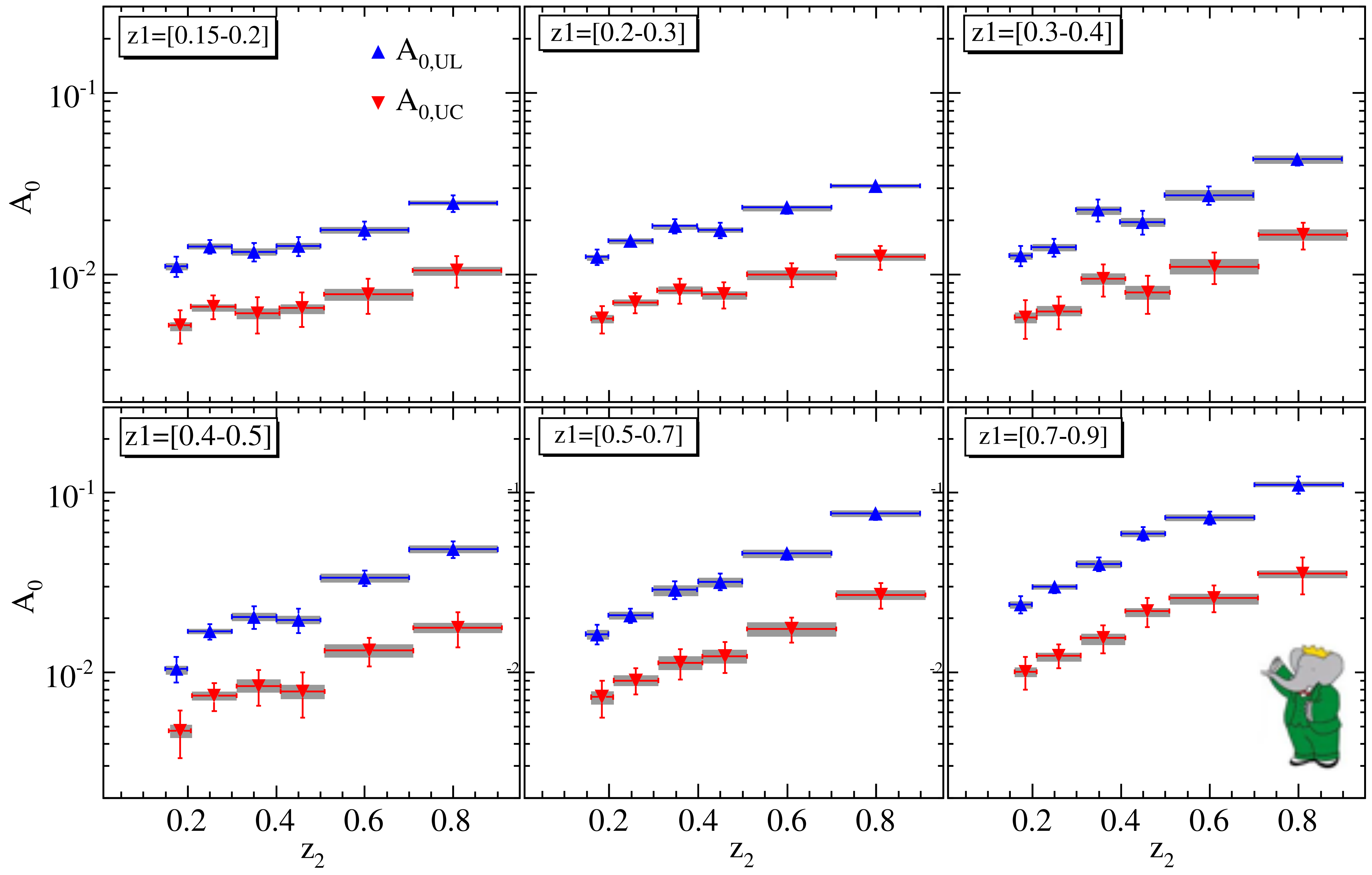
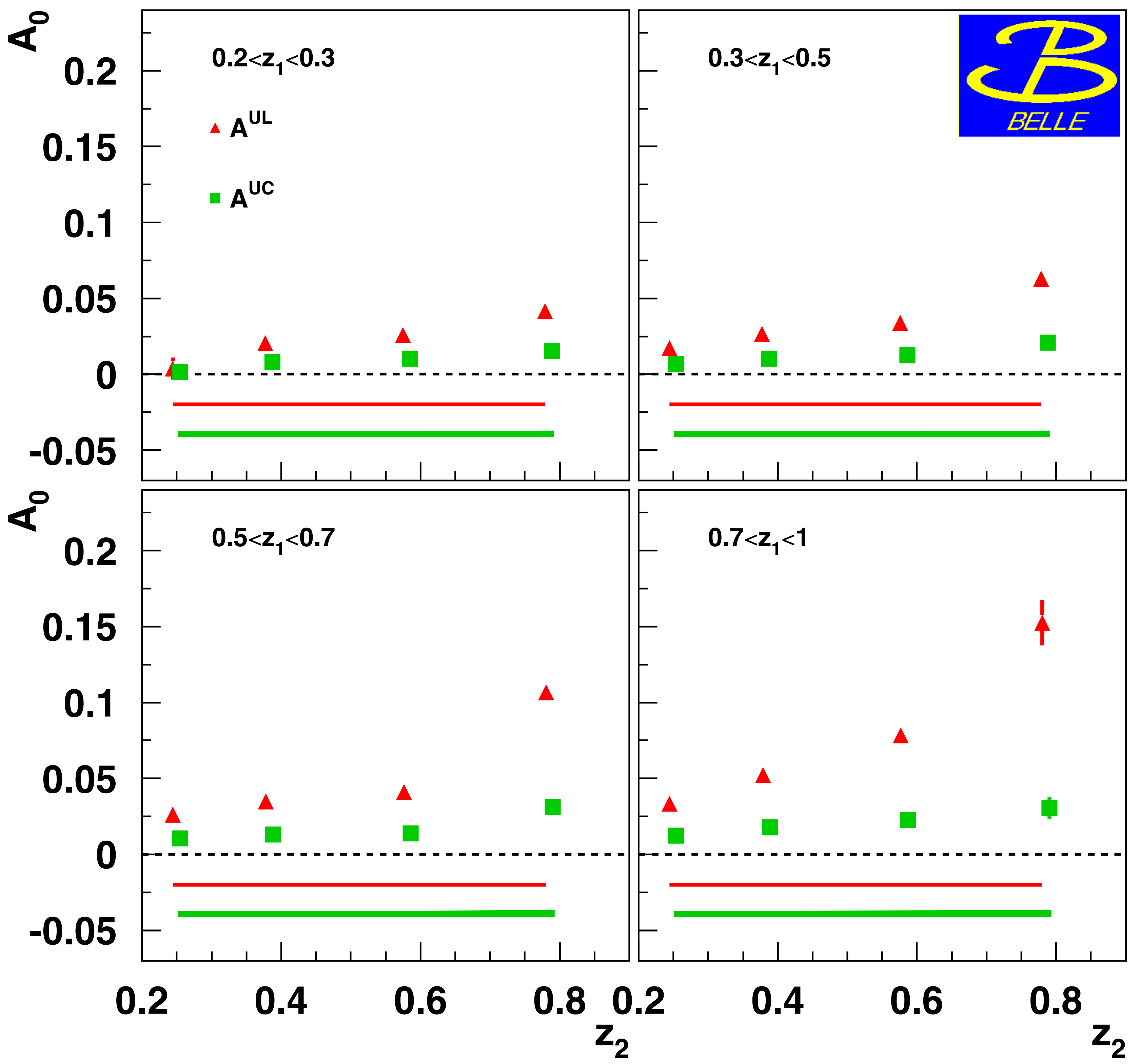
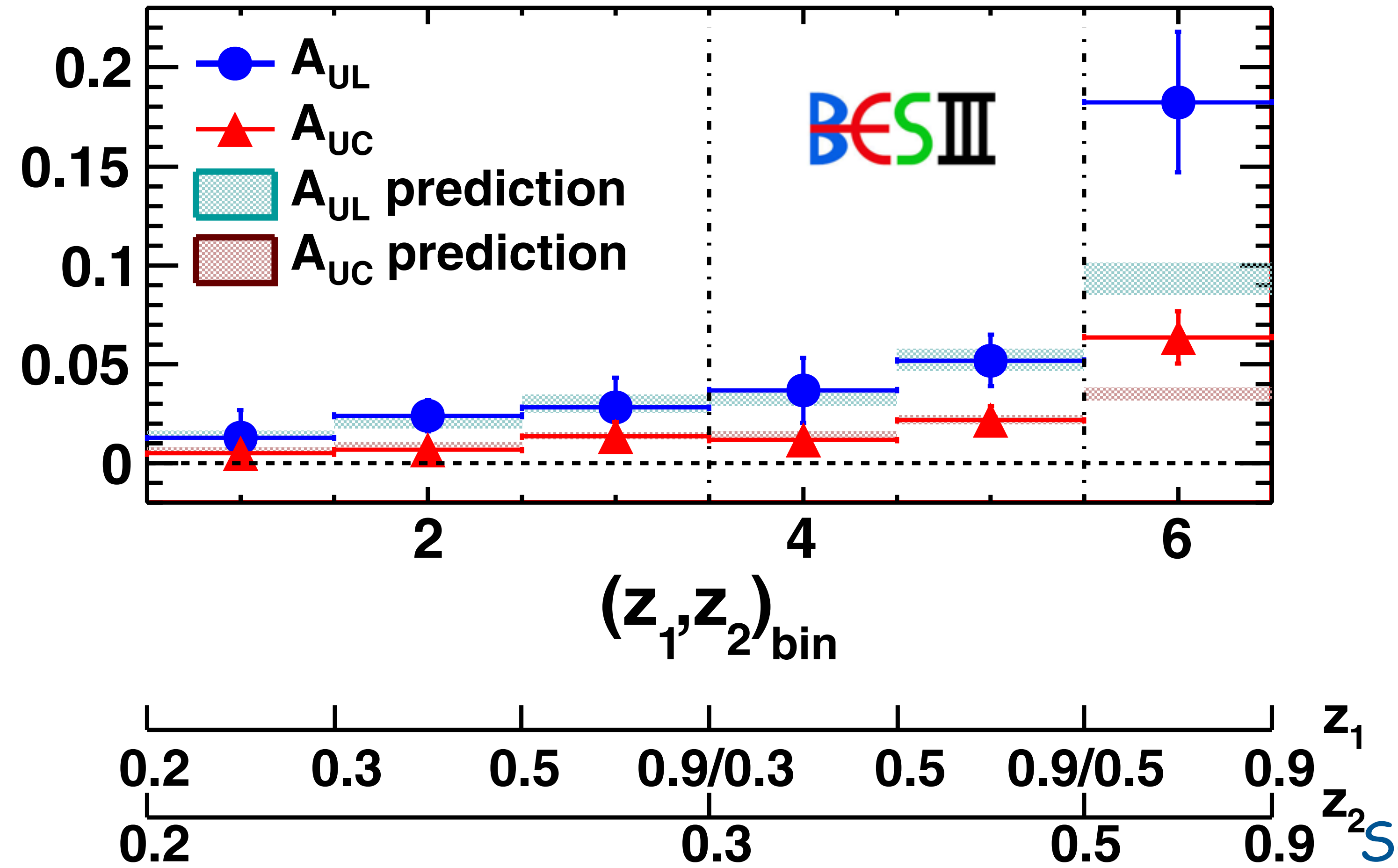


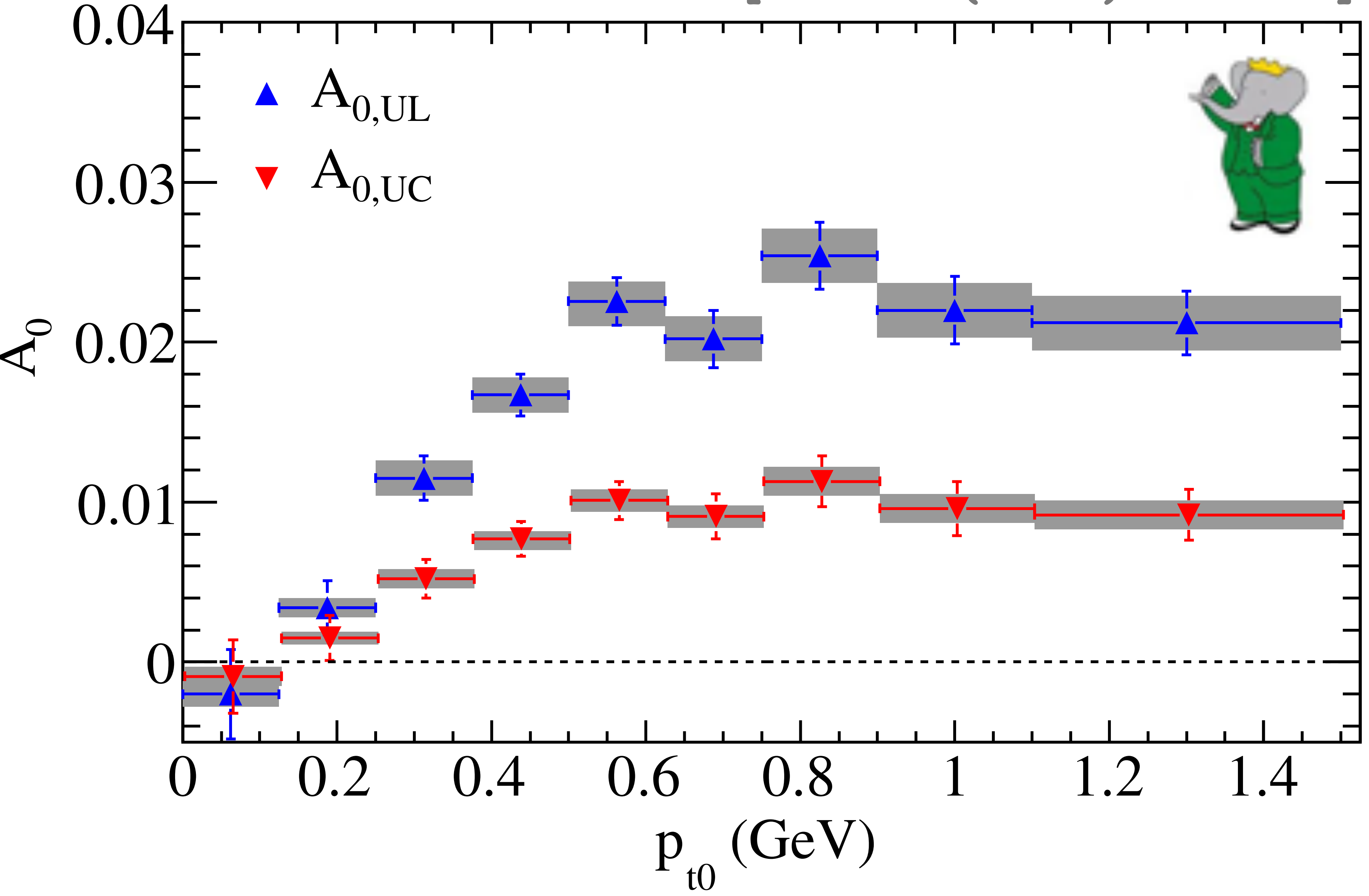


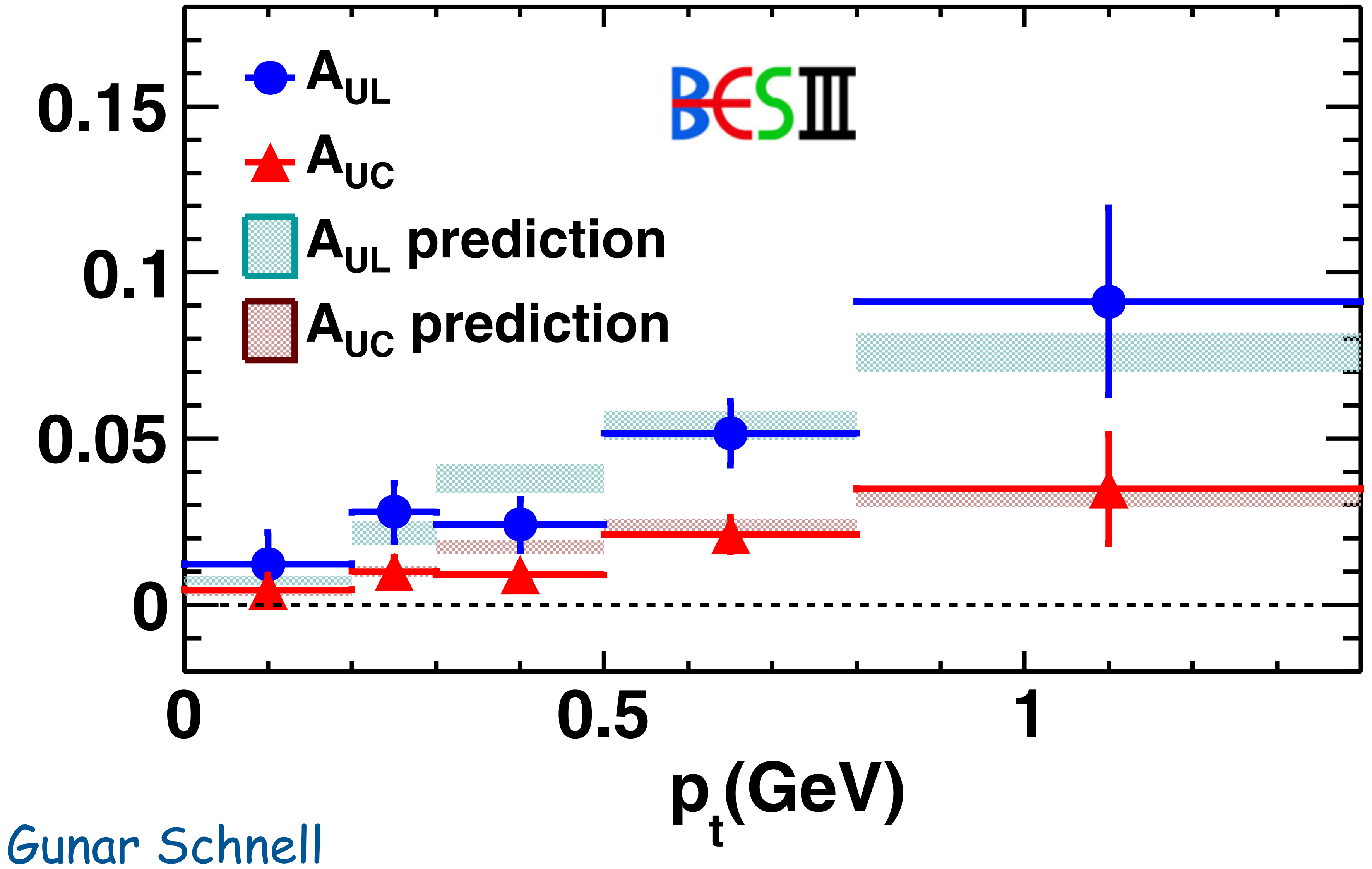


- BaBar results [PRD 90 (2014) 052003] consistent with Belle
- BESIII [PRL 116 (2016) 042001] (at smaller s) consistent with TMD evolution [Kang et al., PRD 93 (2016) 014009]

Collins asymmetries - going further

[PRD 90 (2014) 052003]



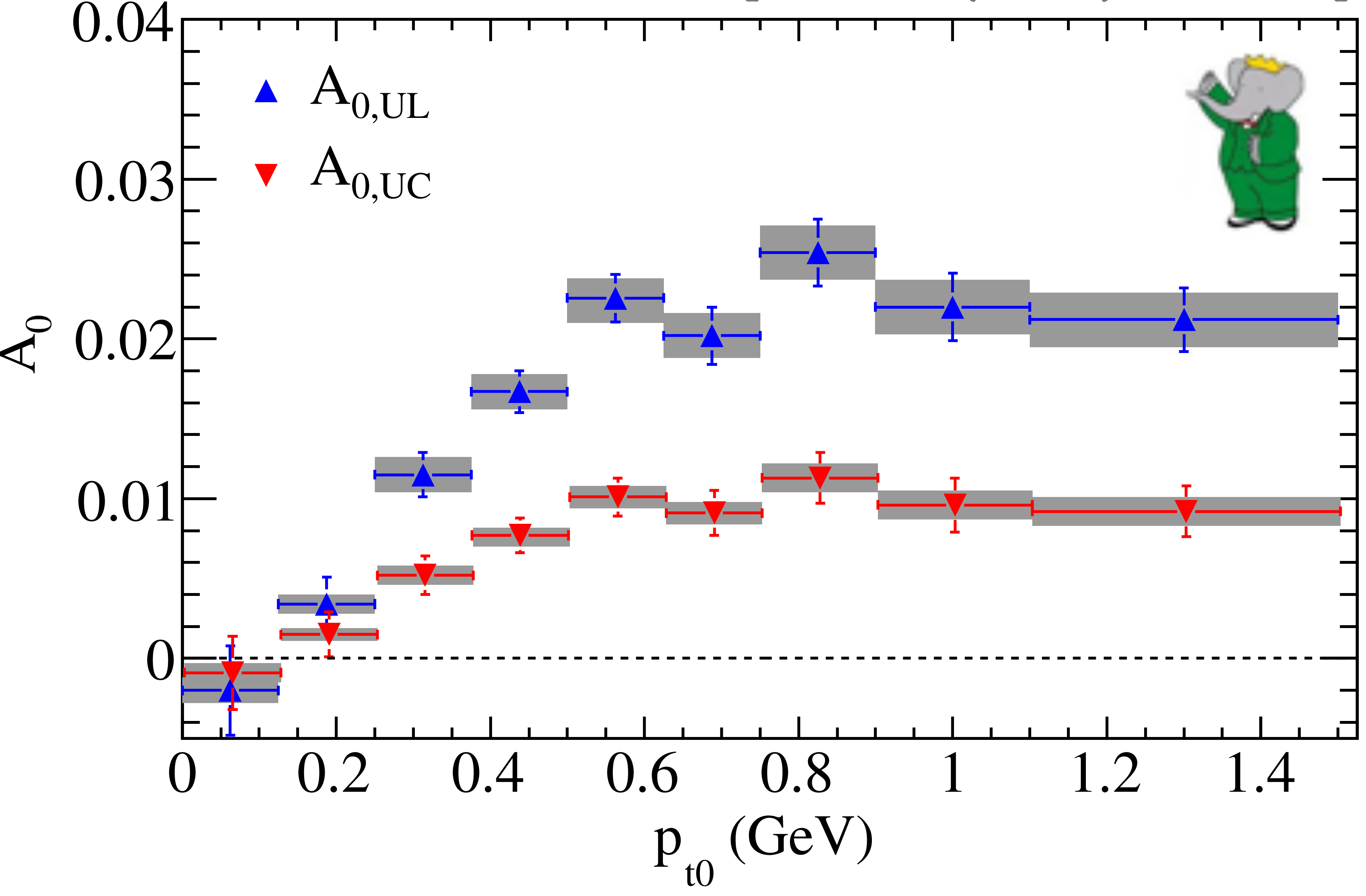


[PRL 116 (2016) 042001]

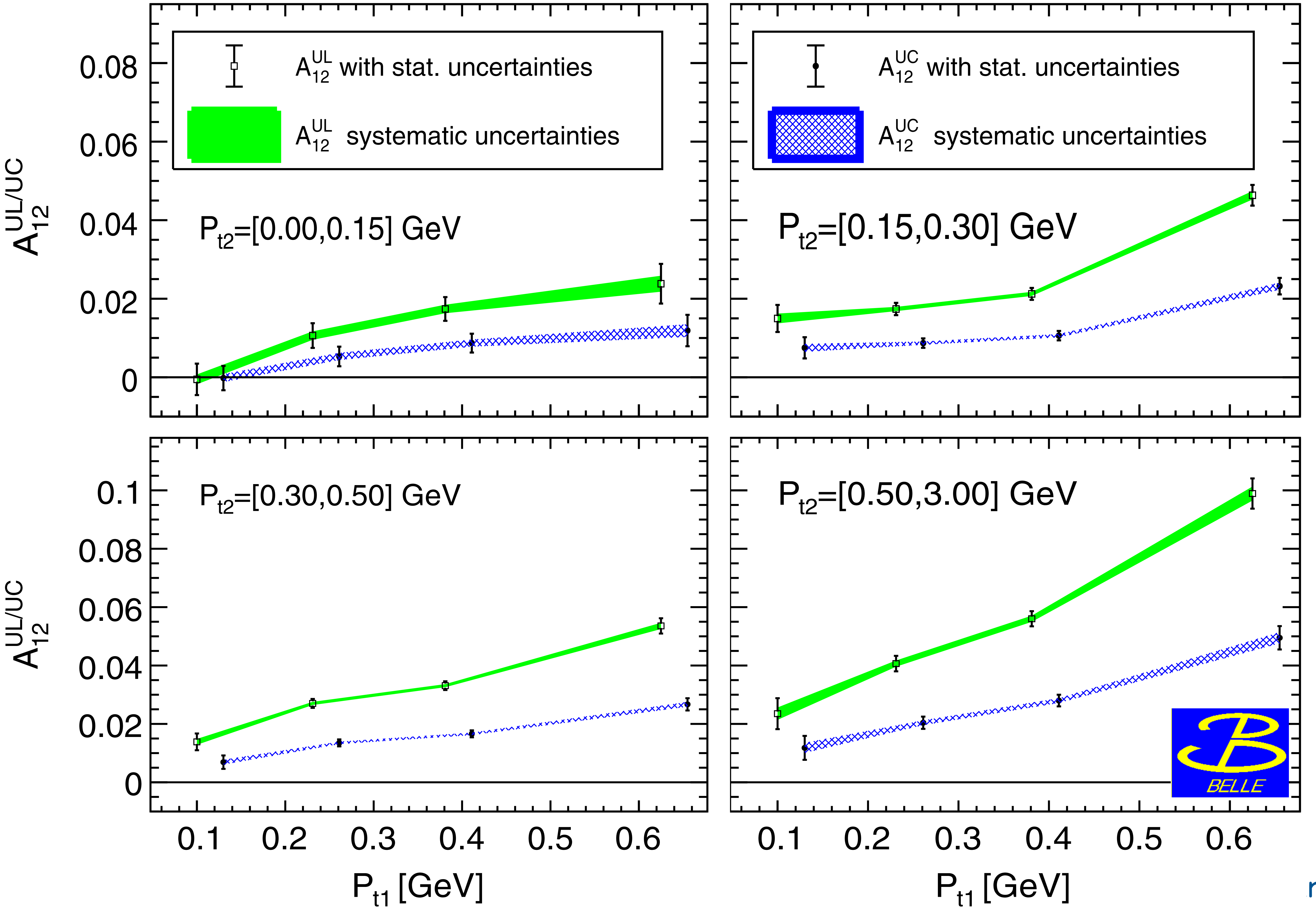
- p_T dependence for charged pions from BaBar and BES
  - typical rise with p_T; turnover around 0.8 GeV

Collins asymmetries - going further

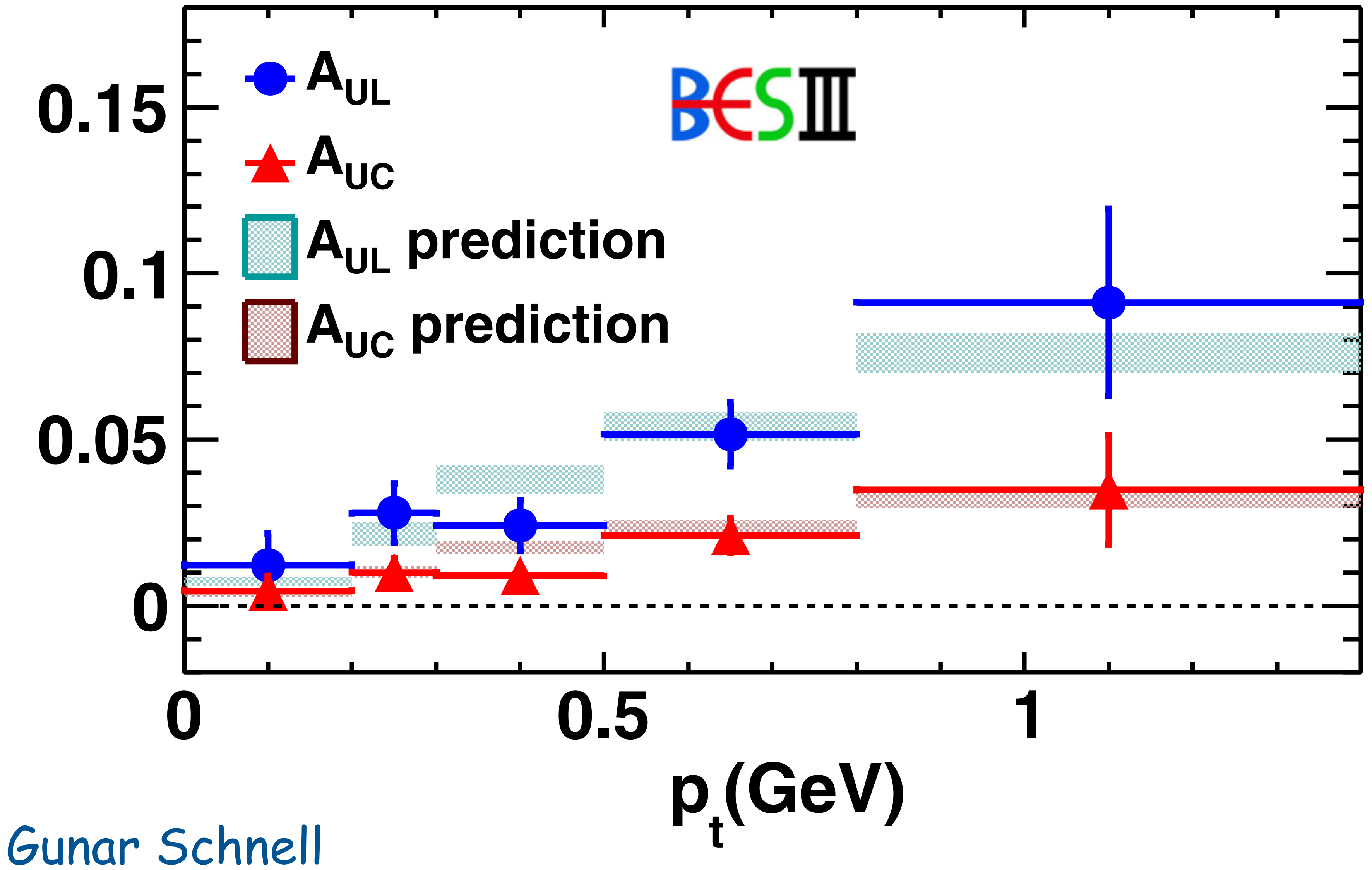
[PRD 90 (2014) 052003]



- p_T dependence for charged pions from BaBar and BES
  - typical rise with p_T; turnover around 0.8 GeV
- … now also from Belle in R12 frame:



[PRD 100 (2019) 92008]



[PRL 116 (2016) 042001]

Collins asymmetries - going further

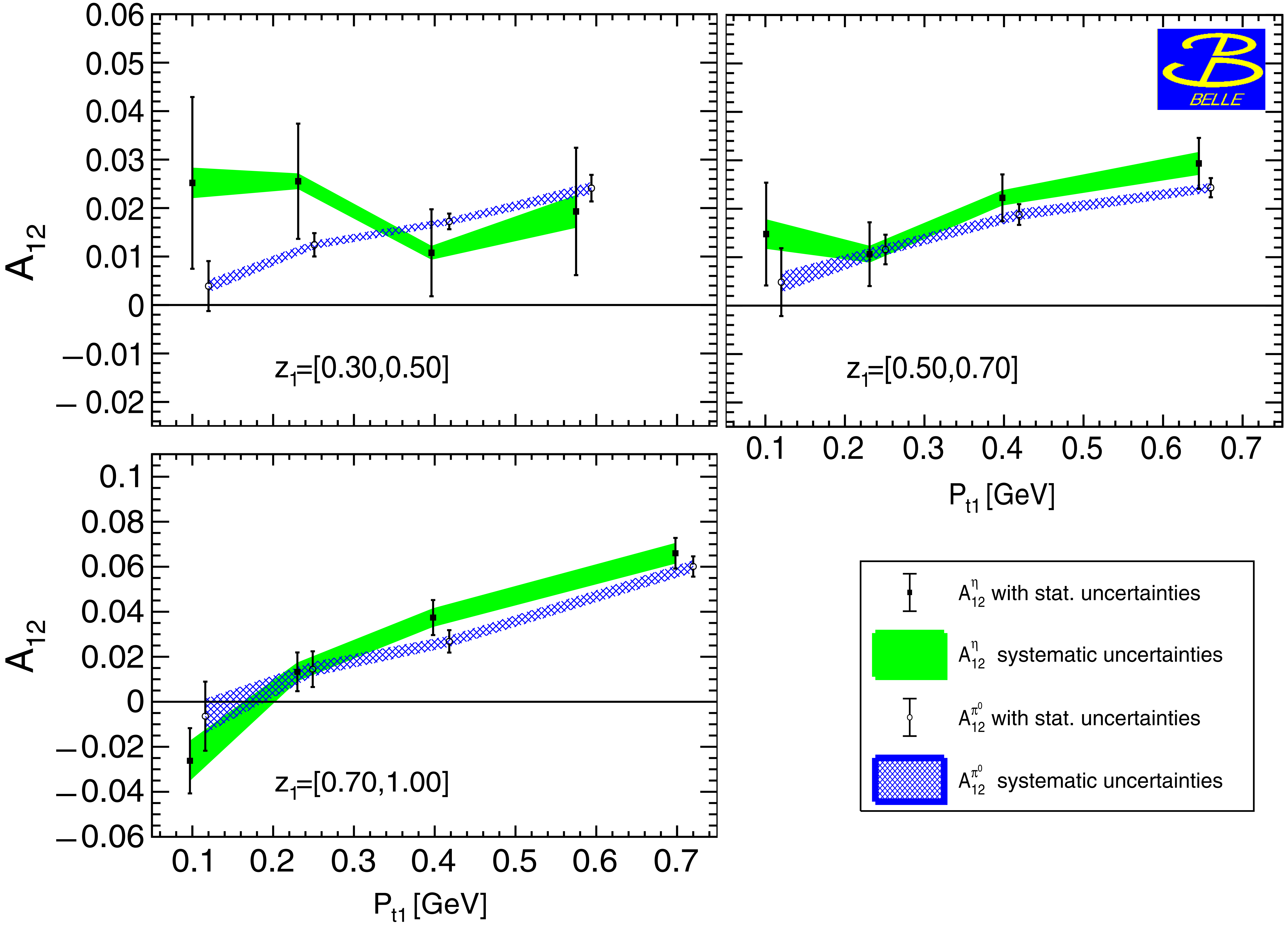
- … as well as for neutral pion and eta

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} = \frac{\pi^0\pi^+ + \pi^0\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

$$R_{12}^{\eta} = \frac{R_{12}^{\eta\pm}}{R_{12}^{L}} = \frac{\eta\pi^+ + \eta\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

- no significant differences observed

[PRD 100 (2019) 92008]



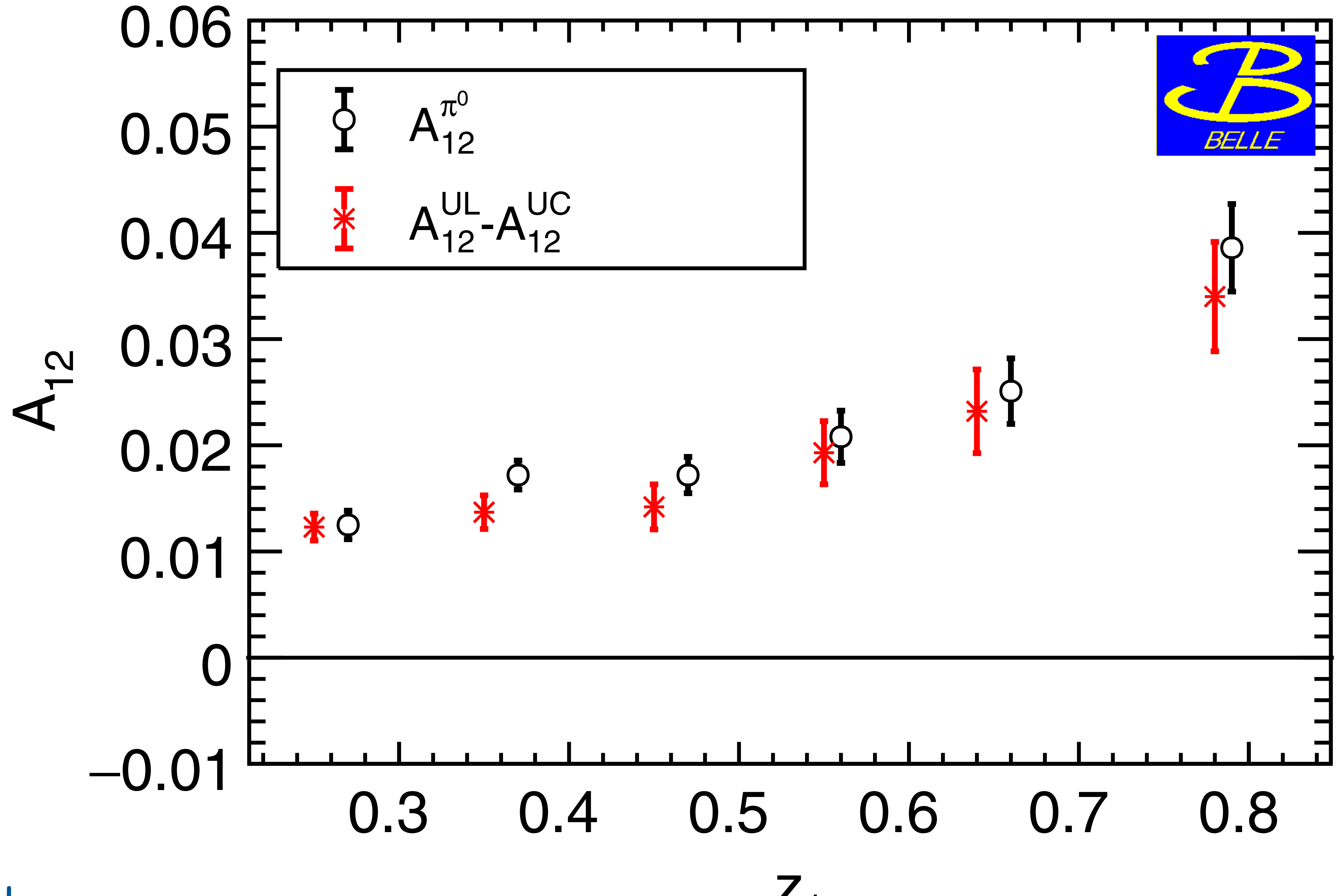
Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)}$$

$$\left.\begin{aligned} &\times \Bigg\{ \frac{5(H_1^{\perp,fav} + H_1^{\perp,dis}) \otimes (H_1^{\perp,fav} + H_1^{\perp,dis}) + 4H_{1,s\to\pi}^{\perp,dis} \otimes H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis})} \\ &- \frac{5(H_1^{\perp,fav} \otimes H_1^{\perp,dis} + H_1^{\perp,dis} \otimes H_1^{\perp,fav}) + 2H_{1,s\to\pi}^{\perp,dis} H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis}} \Bigg\}. \end{aligned}\right\} \overset{\text{isospin}}{=} A_{12}^{UL} - A_{12}^{UC}$$

Collins asymmetries - going further

$$\left.\begin{aligned} R_{12}^{\pi^0} &= \frac{R_{12}^{0\pm}}{R_{12}^{L}} \approx 1+\cos(\phi_{12})\frac{\sin^2(\theta)}{1+\cos^2(\theta)} \\ &\times\left\{\frac{5(H_1^{\perp,fav}+H_1^{\perp,dis})\otimes(H_1^{\perp,fav}+H_1^{\perp,dis})+4H_{1,s\to\pi}^{\perp,dis}\otimes H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav}+D_1^{dis})\otimes(D_1^{fav}+D_1^{dis})+4D_{1,s\to\pi}^{dis}\otimes D_{1,s\to\pi}^{dis})}\right. \\ &\left.-\frac{5(H_1^{\perp,fav}\otimes H_1^{\perp,dis}+H_1^{\perp,dis}\otimes H_1^{\perp,fav})+2H_{1,s\to\pi}^{\perp,dis}H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav}\otimes D_1^{dis}+D_1^{dis}\otimes D_1^{fav})+2D_{1,s\to\pi}^{dis}\otimes D_{1,s\to\pi}^{dis}}\right\}. \end{aligned}\right\} \overset{\text{isospin}}{=} A_{12}^{UL}-A_{12}^{UC}$$

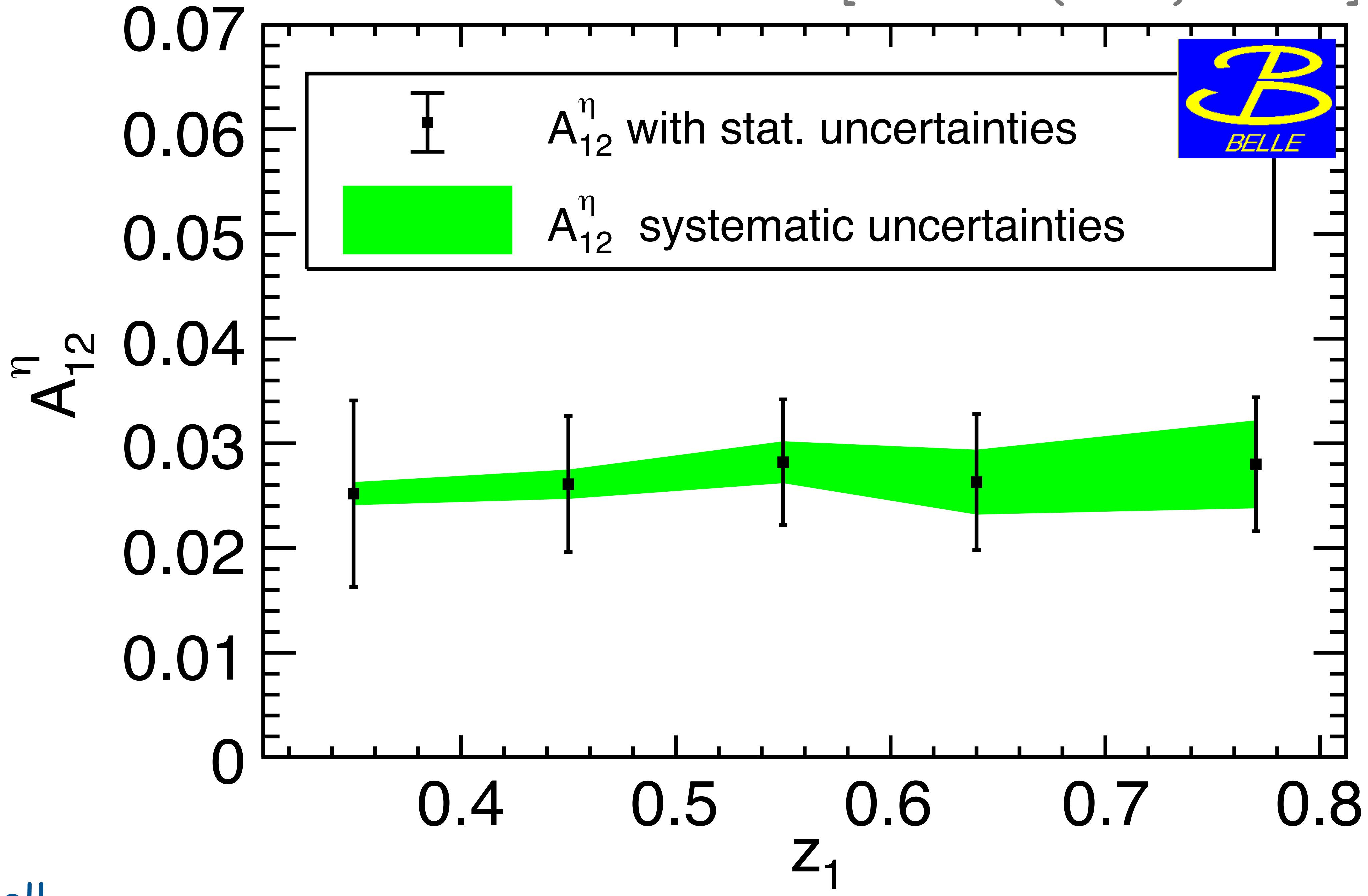


- consistency between neutral and charged pions
  - typical rise with z also seen for neutral pions

Collins asymmetries - going further

$$\left.\begin{aligned} R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \\ \times \left\{ \frac{5(H_1^{\perp,fav} + H_1^{\perp,dis}) \otimes (H_1^{\perp,fav} + H_1^{\perp,dis}) + 4H_{1,s\to\pi}^{\perp,dis} \otimes H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis})} \right. \\ \left. - \frac{5(H_1^{\perp,fav} \otimes H_1^{\perp,dis} + H_1^{\perp,dis} \otimes H_1^{\perp,fav}) + 2H_{1,s\to\pi}^{\perp,dis} H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis}} \right\}. \end{aligned}\right\} \overset{\text{isospin}}{=} A_{12}^{UL} - A_{12}^{UC}$$

[PRD 100 (2019) 92008]

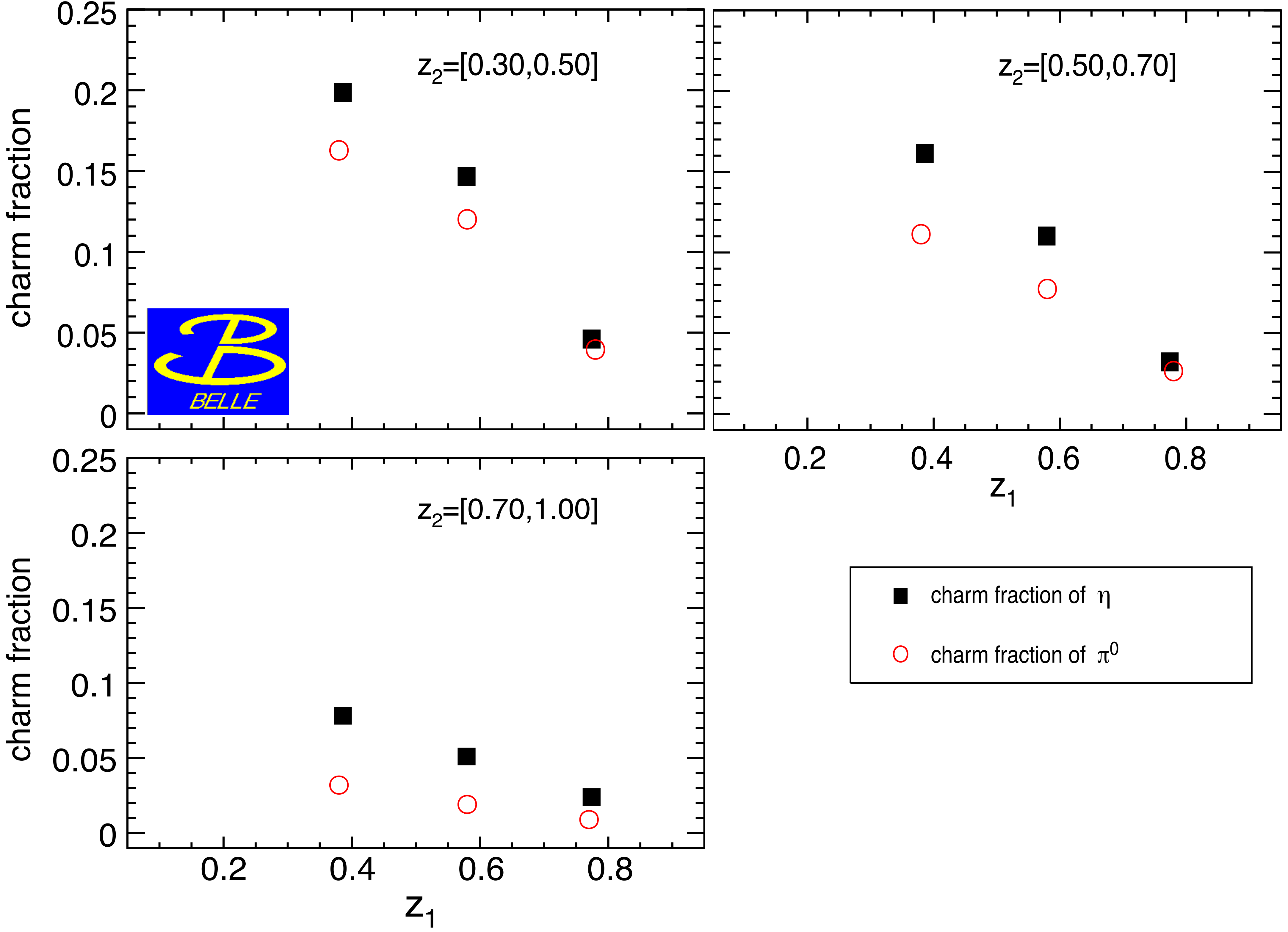


- consistency between neutral and charged pions
  - typical rise with z also seen for neutral pions
  - … while basically flat for eta

Collins asymmetries - going further

- qualitative changes in 2019 Belle analysis w.r.t. previous Belle analyses:
  - no correction to $q\bar{q}$ axis;
    ➟ rather to thrust axis, which is observable
  - upper limit on opening angle imposed
  - no correction for charm contribution;
    ➟ provide charm fraction

[PRD 100 (2019) 92008]



the future

- several analyses still in the pipeline, e.g.,
  - k_T-dependent D1 FFs (back-to-back hadrons) (Belle, possibly BESIII & BaBar)
  - Collins asymmetries:
    - pion update w/ increased statistics (BESIII)
    - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
    - Collins asymmetries w/o double ratios (BaBar)
  - single-hadron production
    - lower-s data (BESIII)

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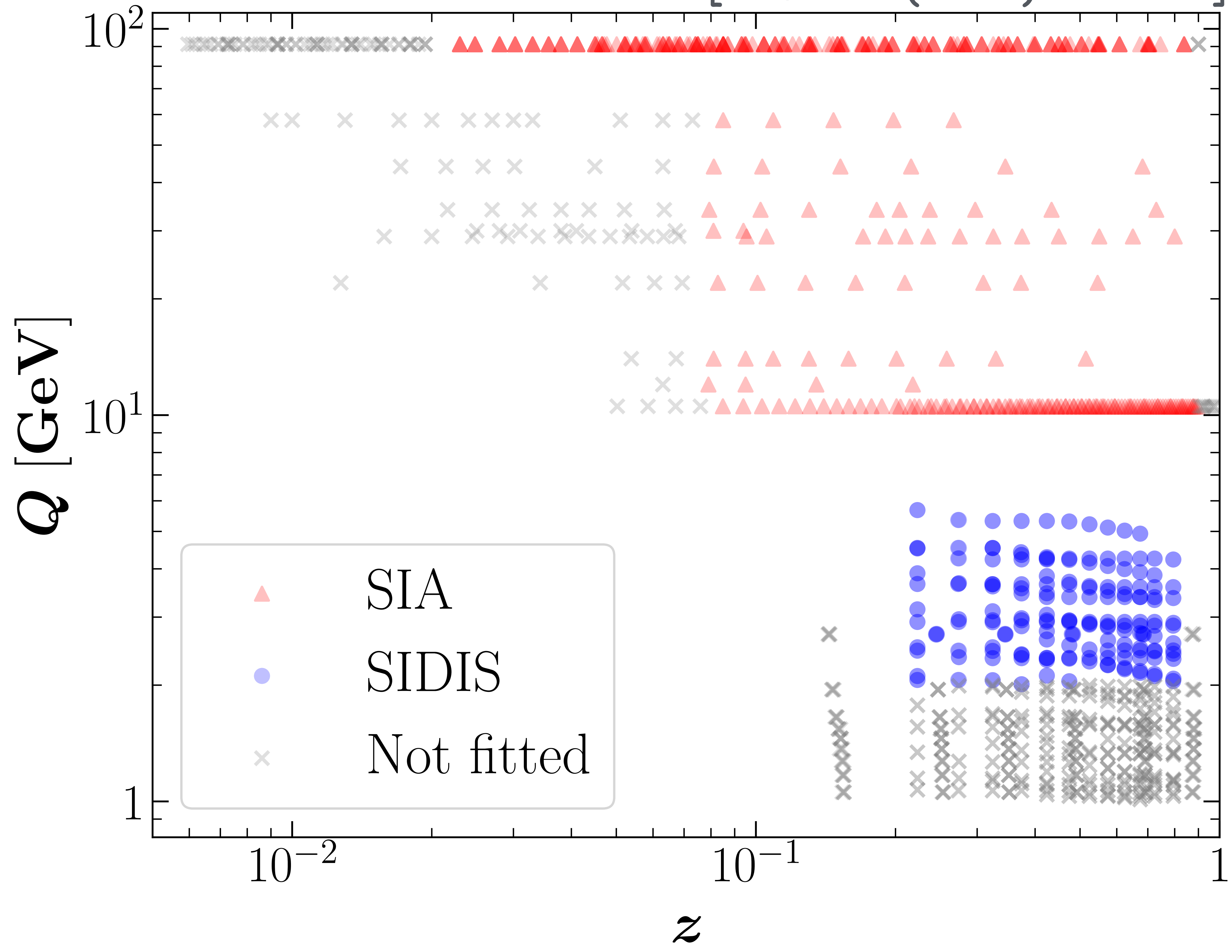
[PRD 104 (2021) 034007]



the future

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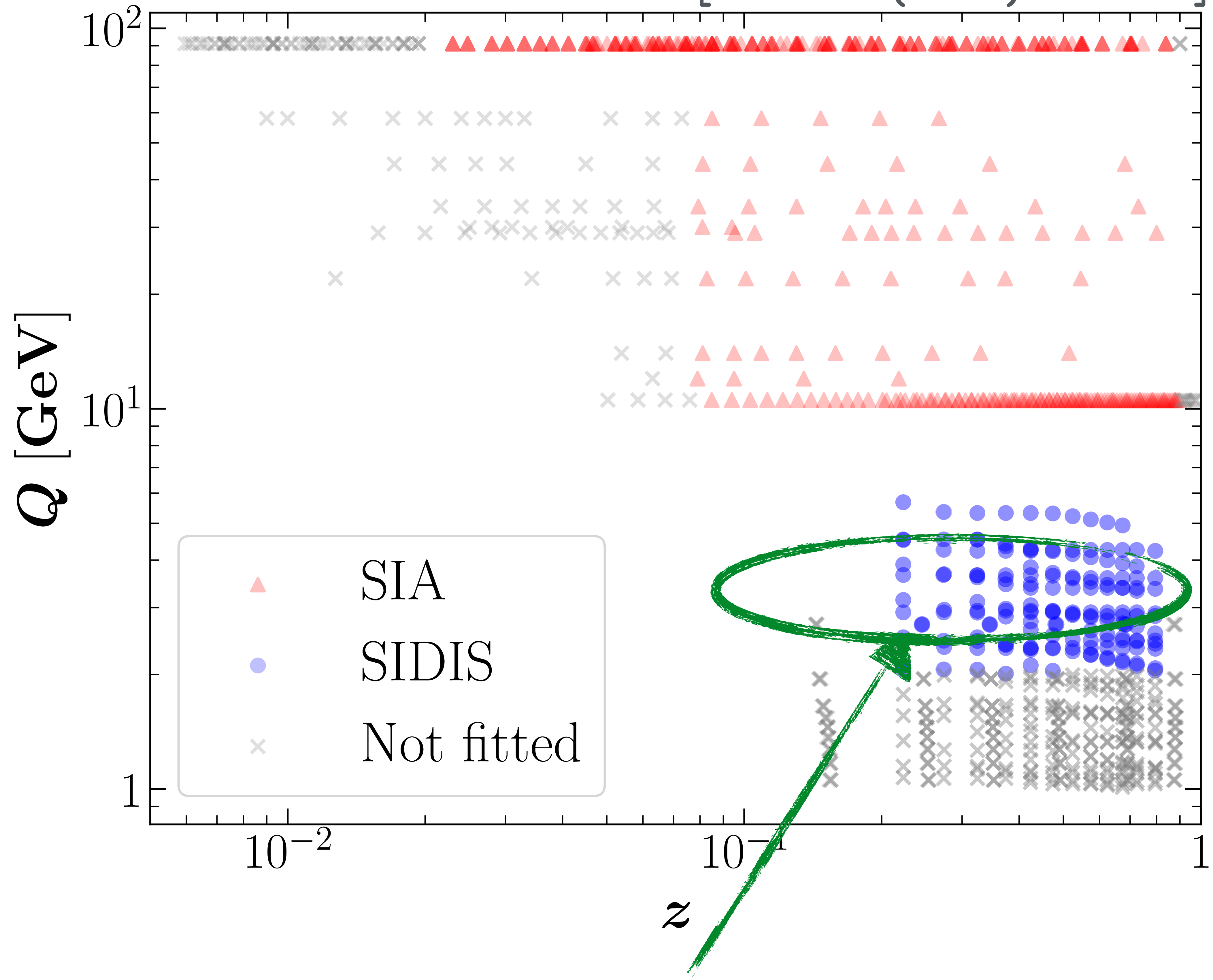
[PRD 104 (2021) 034007]



BESIII region

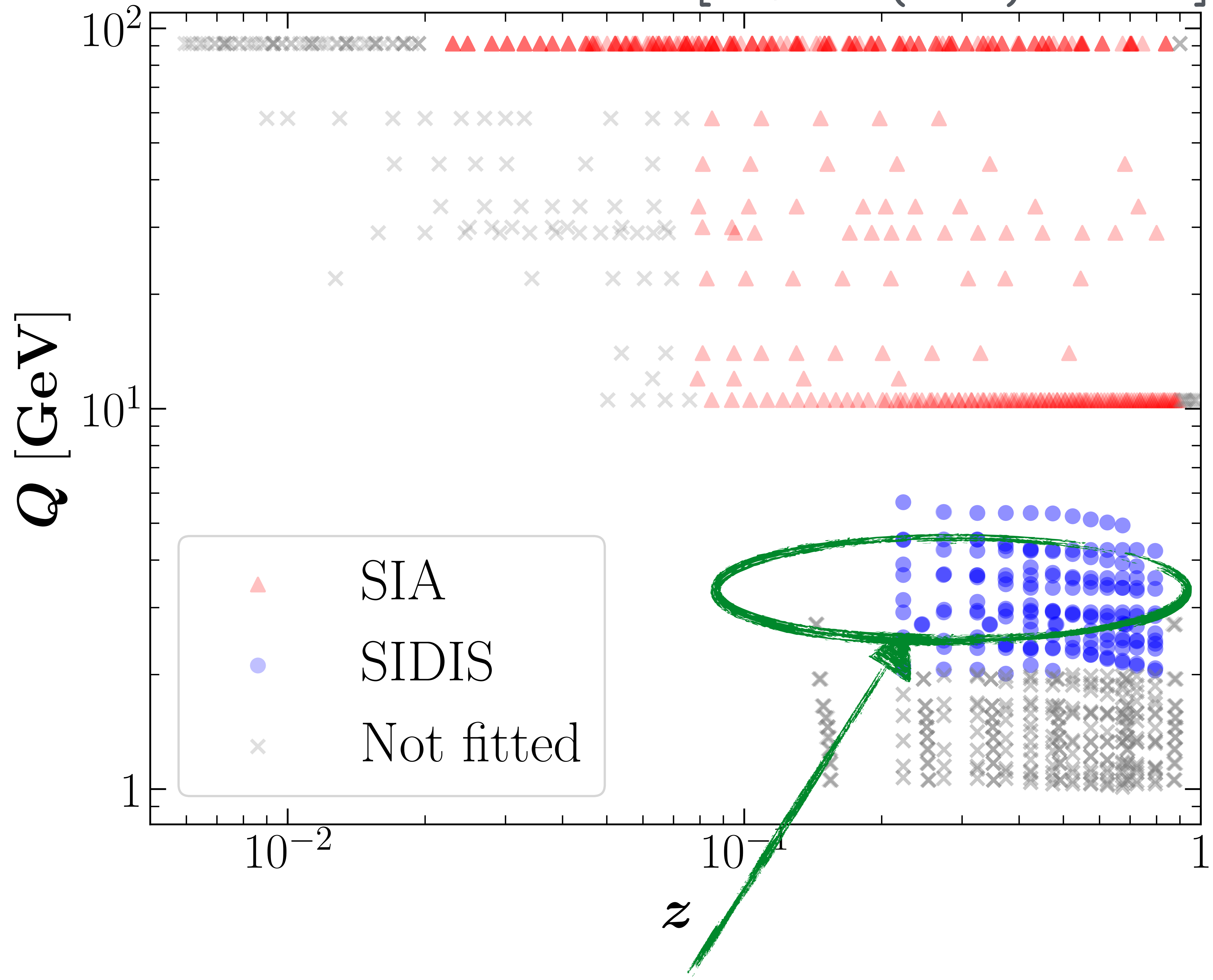
~62pb^{-1} @3.52 GeV used for Collins asym's

aim at **250pb^{-1}** data set

the future

- several analyses still in the pipeline, e.g.,
  - k_T-dependent D1 FFs (back-to-back hadrons) (Belle, possibly BESIII & BaBar)
  - Collins asymmetries:
    - pion update w/ increased statistics (BESIII)
    - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
    - Collins asymmetries w/o double ratios (BaBar)
  - single-hadron production
    - lower-s data (BESIII)
- new data from Belle II

[PRD 104 (2021) 034007]



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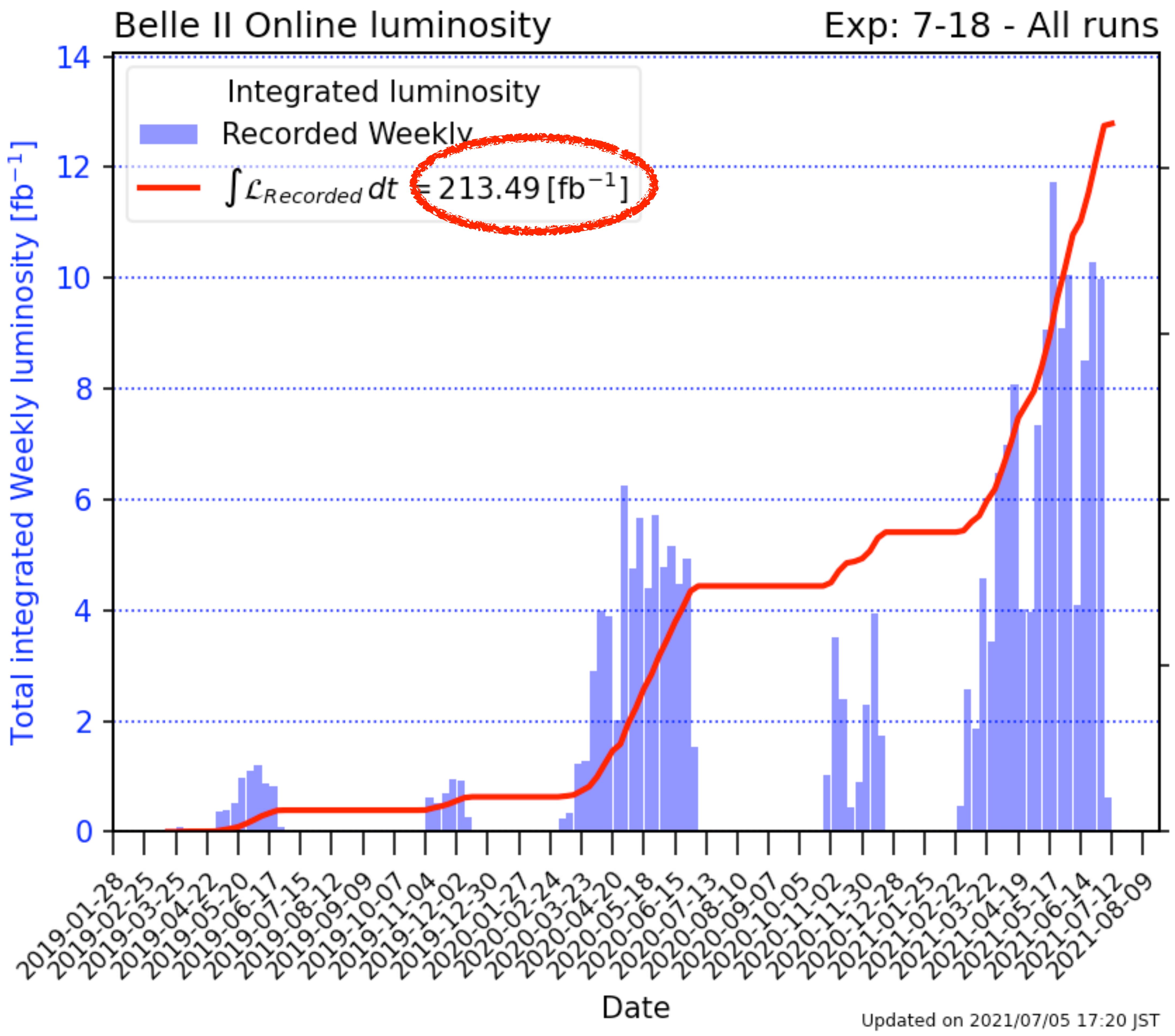
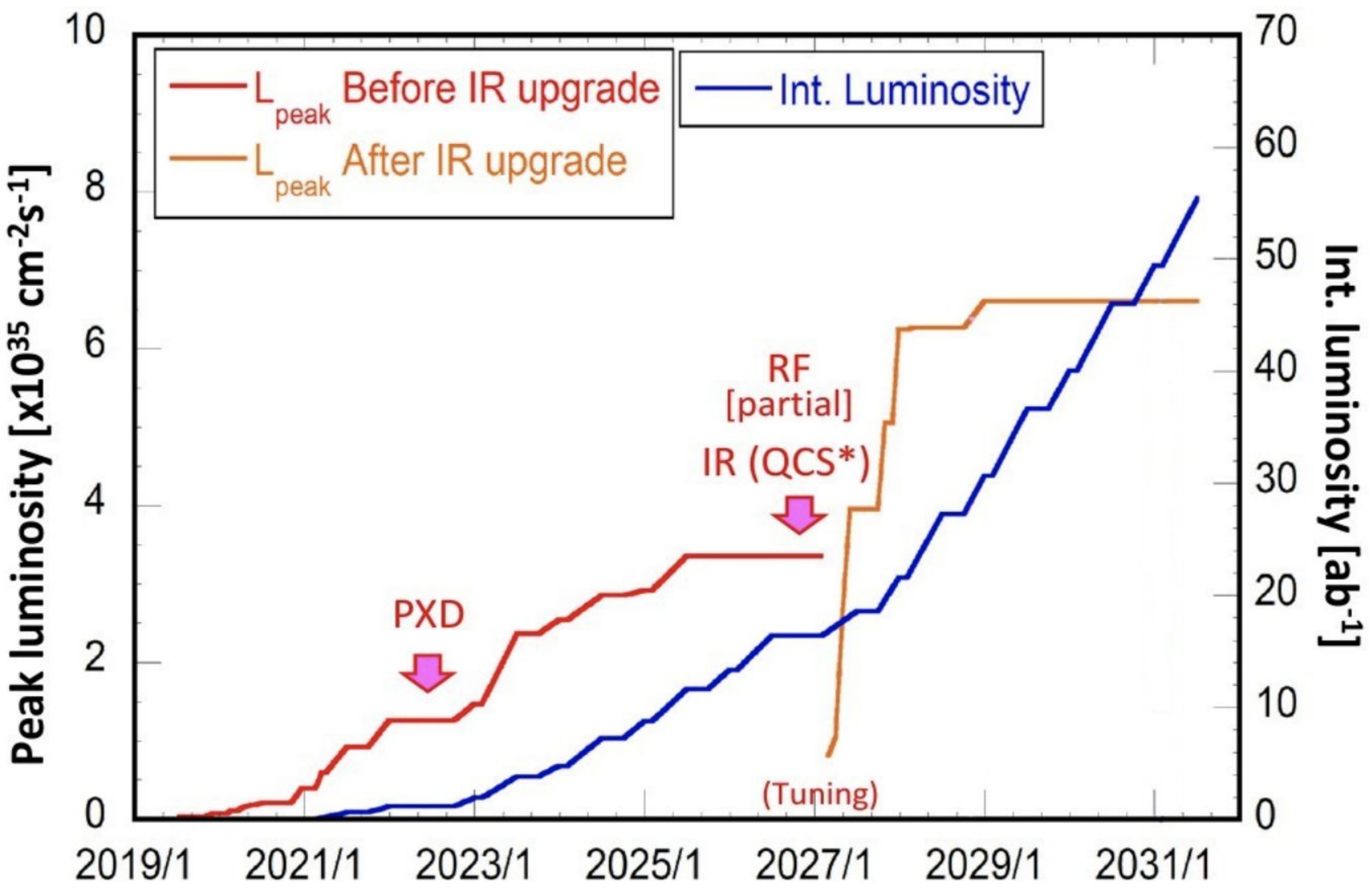


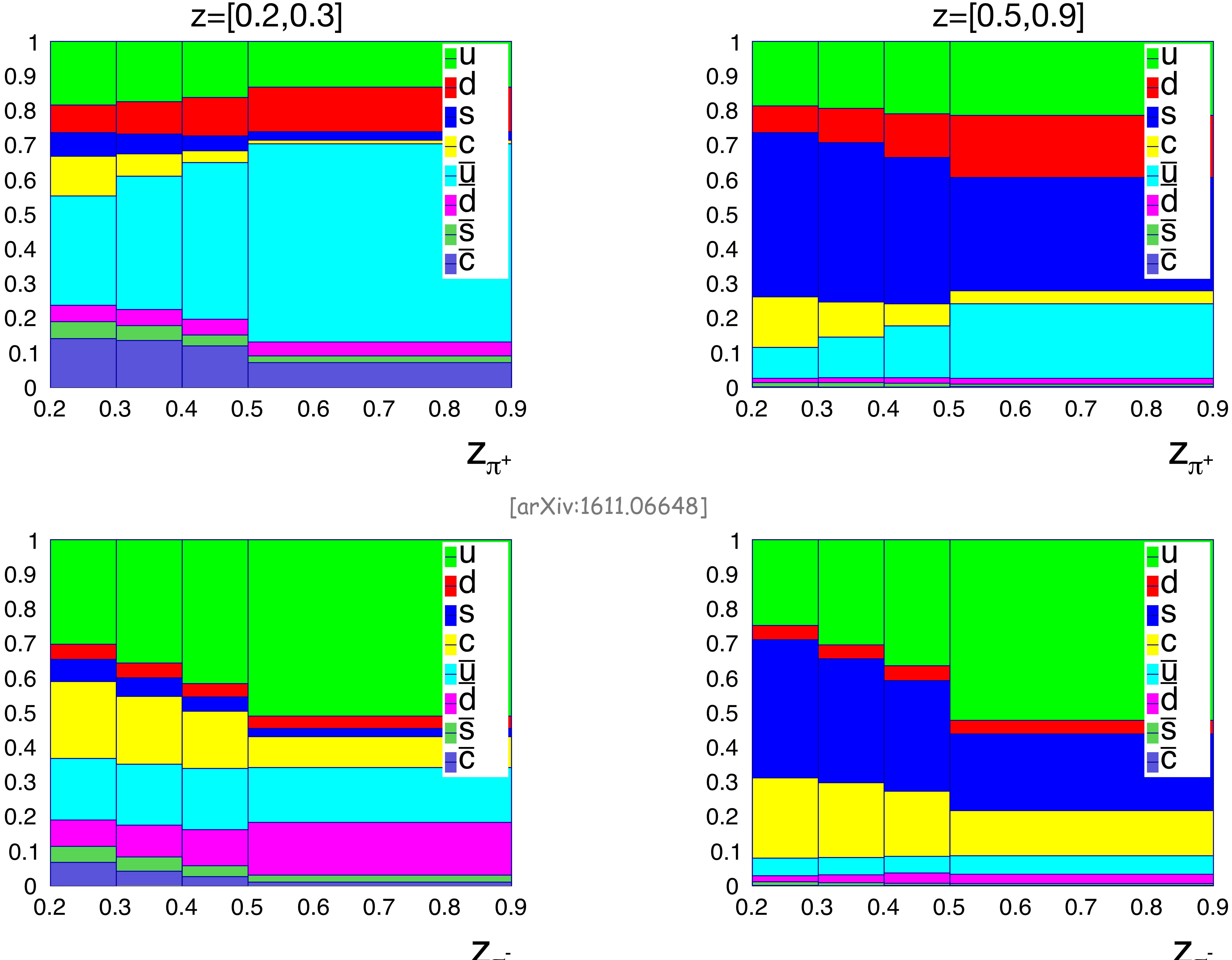


➡ similar data sample as at 1st generation B-factories by 2022

backup

quark-flavor contributions to Lambda prod.

- flavor tagging through opposite-hemisphere hadrons



[arXiv:1611.06648]