Di-Jet production and Wigner distributions

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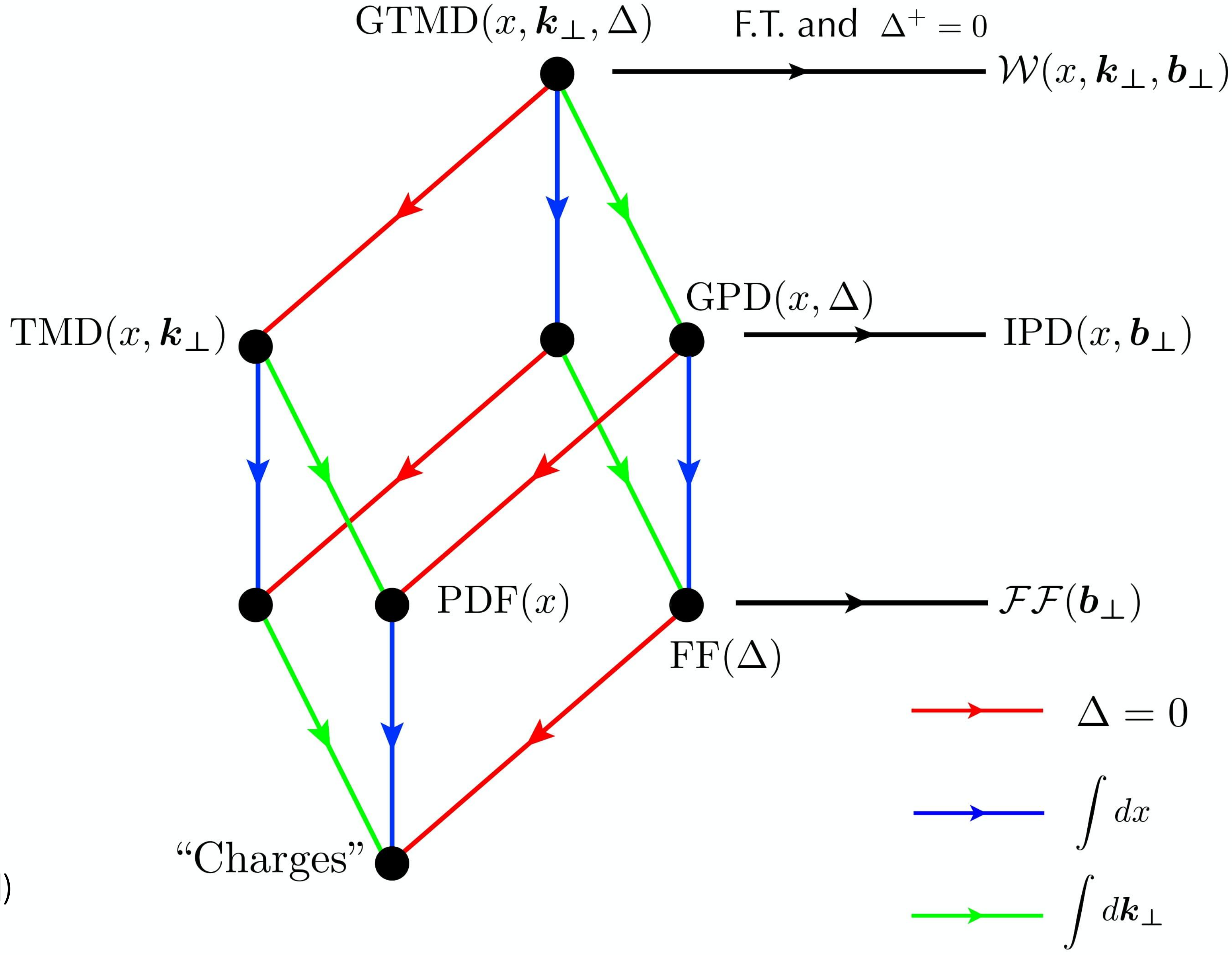
Outline

Wigner distributions and angular correlations

Overview of processes to access the Wigner distributions

Exclusive dijet production: models and small-x regime

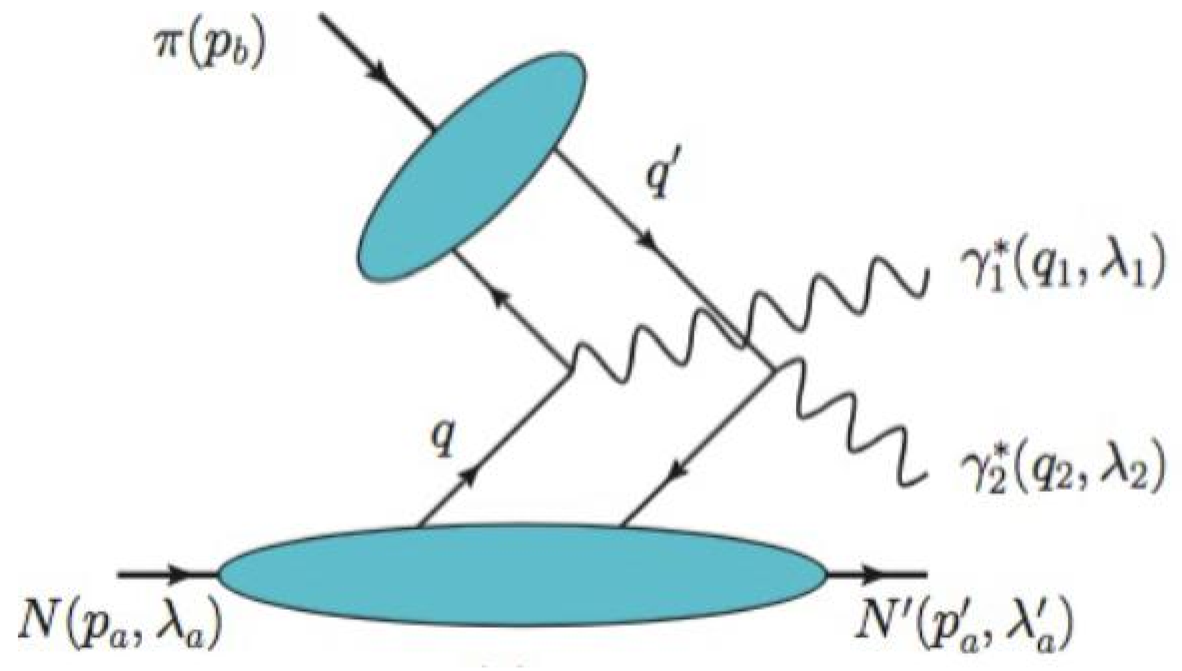
First steps beyond the small-x approximation



Lorcé, Pasquini, PRD 84 (2011)

Lorcé et al, JHEP 05 (2011)

Exclusive pion-nucleon double Drell-Yan (quark)

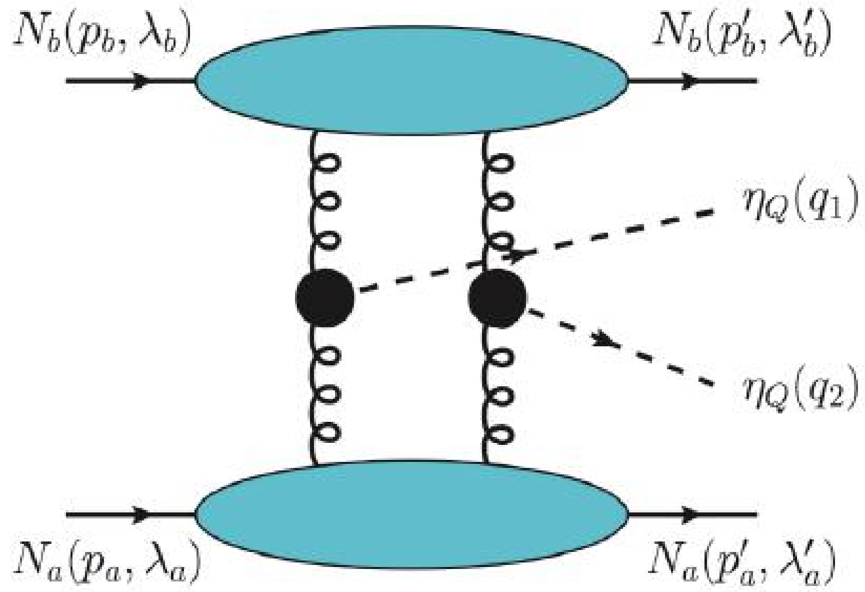


Bhattacharya et al, PLB 771 (2017)

At present, the only known process that is sensitive to quark GTMDs

At leading order, sensibility to ERBL region only

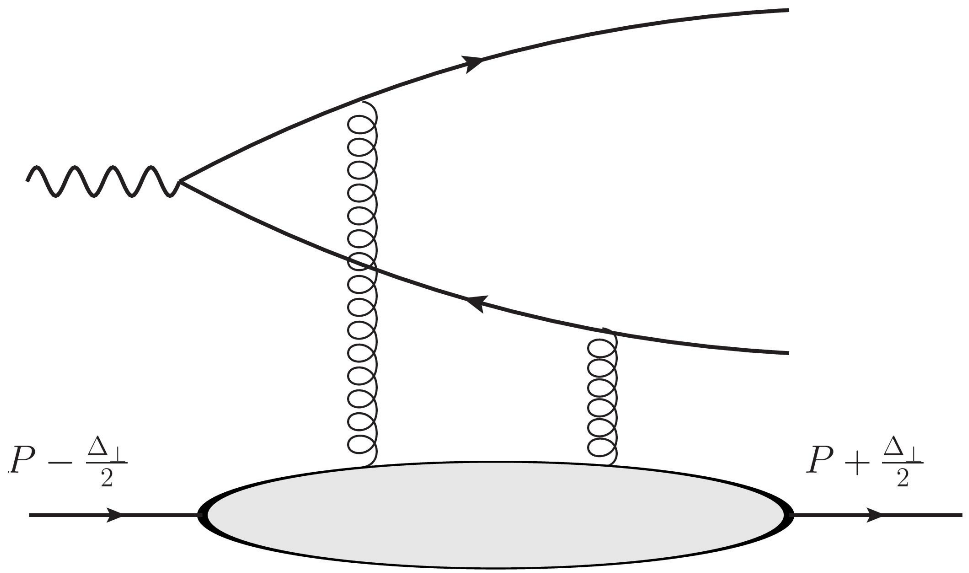
Exclusive double quarkonia production in hadronic collisions (gluon)



Boussarie et al, PRD 98 (2015) 074015

Bhattacharya et al, arXiv:1802.10550

Exclusive dijet production in ep/eA and ultra peripheral Ap collisions (gluon)



Hatta et al, PRL 116 (2016)

Hatta et al. PRD 95 (2017)

Hagiwara et al, PRD 96 (2016)

Why the Wigner distributions?

Angular correlations

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

Lorcé, Pasquini, PRD 93 (2016)

Why the Wigner distributions?

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

In red vanishing if
no orbital angular momenum

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, \ h_{1T}^\perp$

$$L_g(x) = 2 \int d\boldsymbol{k}_\perp d\boldsymbol{b}_\perp \ \epsilon_{ij} b_\perp^i k_\perp^j \ W^u(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

1) How are they defined?
2) How can we measure them?

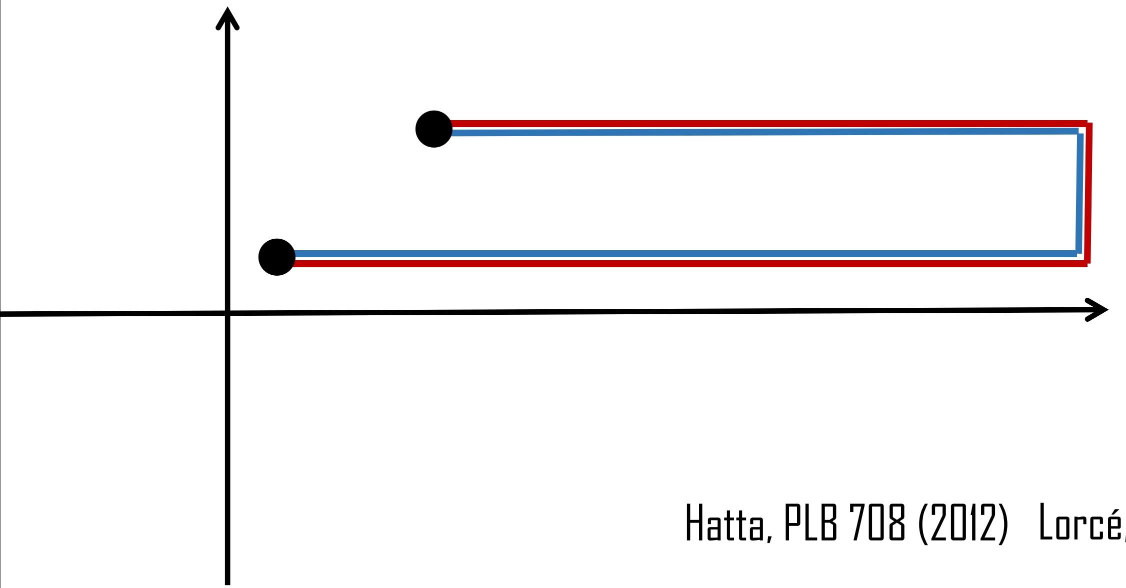
Lorcé, Pasquini, PRD 84 (2011)   Lorcé et al, PRD 85 (2012)   Hatta, et al. PRD 95 (2017)

The Wigner distributions depends on the gauge link paths!

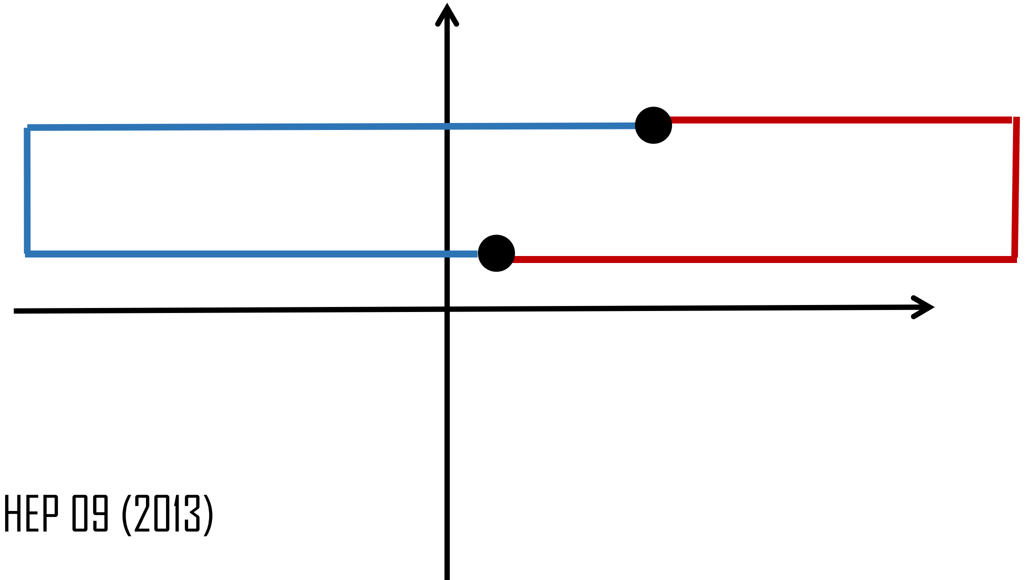
$$W^{u}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{2}{xP^+}\int\frac{dz^-d\boldsymbol{z}_\perp}{(2\pi)^3}\int\frac{d\boldsymbol{\Delta}_\perp}{(2\pi)^2}e^{-ixP^+z^-+i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp}$$

$$\times\left\langle P+\frac{\boldsymbol{\Delta}_\perp}{2},S\right|\mathrm{Tr}\left(F^{+i}\left(z+\boldsymbol{b}_\perp\right)\mathcal{U}(z+\boldsymbol{b}_\perp,\boldsymbol{b}_\perp)F^{+i}\left(\boldsymbol{b}_\perp\right)\mathcal{U}(\boldsymbol{b}_\perp,z+\boldsymbol{b}_\perp)\right)\left|P-\frac{\boldsymbol{\Delta}_\perp}{2},S\right\rangle_{z^+=0}$$

Weizsäcker-Williams



Dipole



Hatta, PLB 708 (2012)   Lorcé, Pasquini, JHEP 09 (2013)

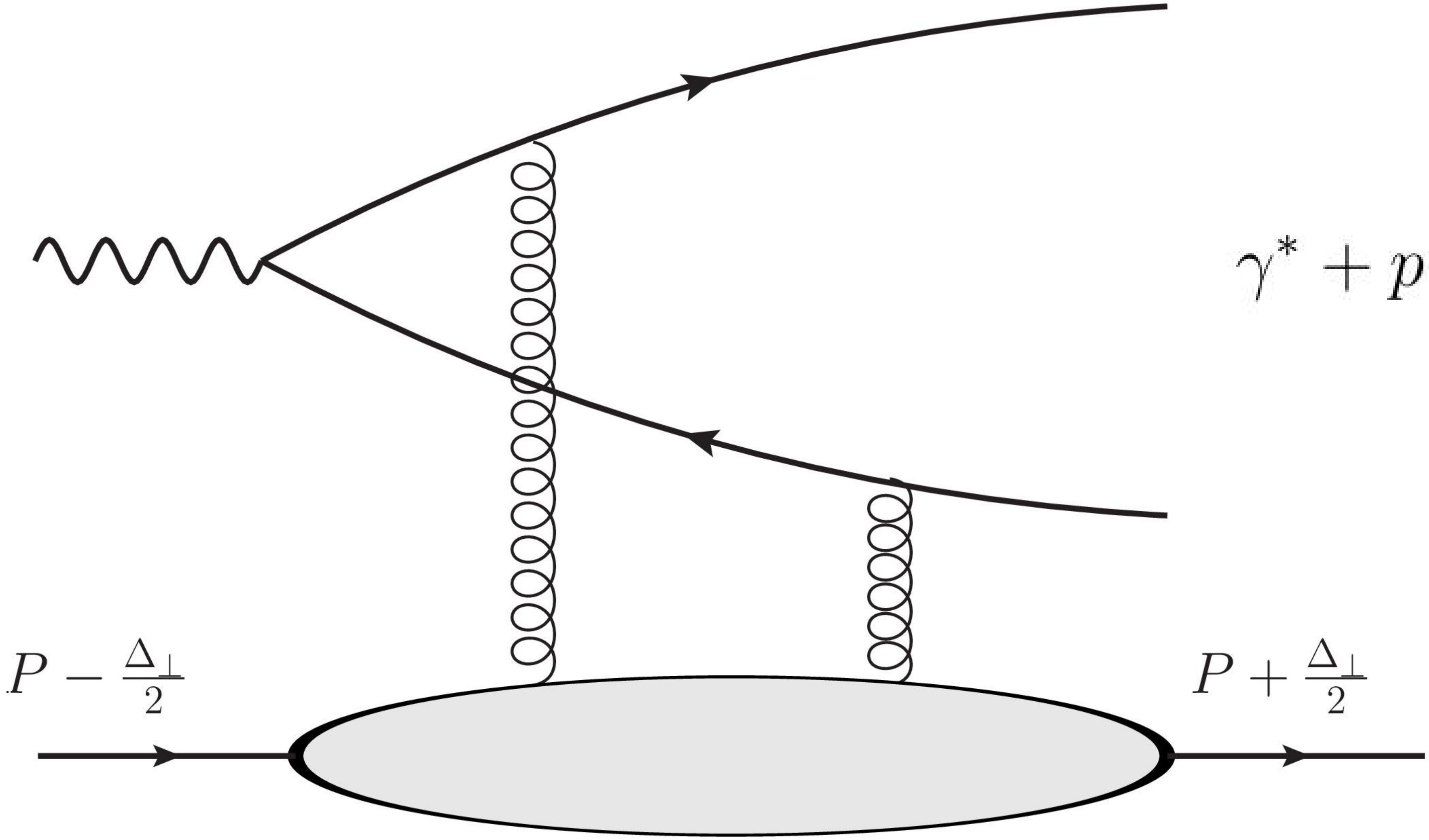
Does this constitue a problem for the definition of L_g ?

No!

The proof is general and uses
just the PT transformation properties of the Wigner distributions

This entails that one can use one path
to establish the physical interpretation of L_g
and another path to extract it.

Exclusive dijet production



$$\gamma^* + p \longrightarrow 2J + p$$

Mantysaari et al, PRD 99 (2019)

Boer. Setyadi 2106.15148

Hatta et al. PRD 95 (2017)

Hagiwara et al, PRD 96 (2016)

Why the dijet?

Because it can be studied both:

at the future EIC with hard photons generated by the electron beam

and

at LHC, with very soft photons generated by radiation of heavy nuclei.

In the latter case, the invariant mass of the jets provide the necessary hard scale to apply the partonic formalism.

Moreover, ultraperipheral collisions have the advantage of verly large photon flux.

Hatta et al. PRD 95 (2017)

Hagiwara et al, PRD 96 (2016)

To study the dijet we can use a ‘small-x’ approach .

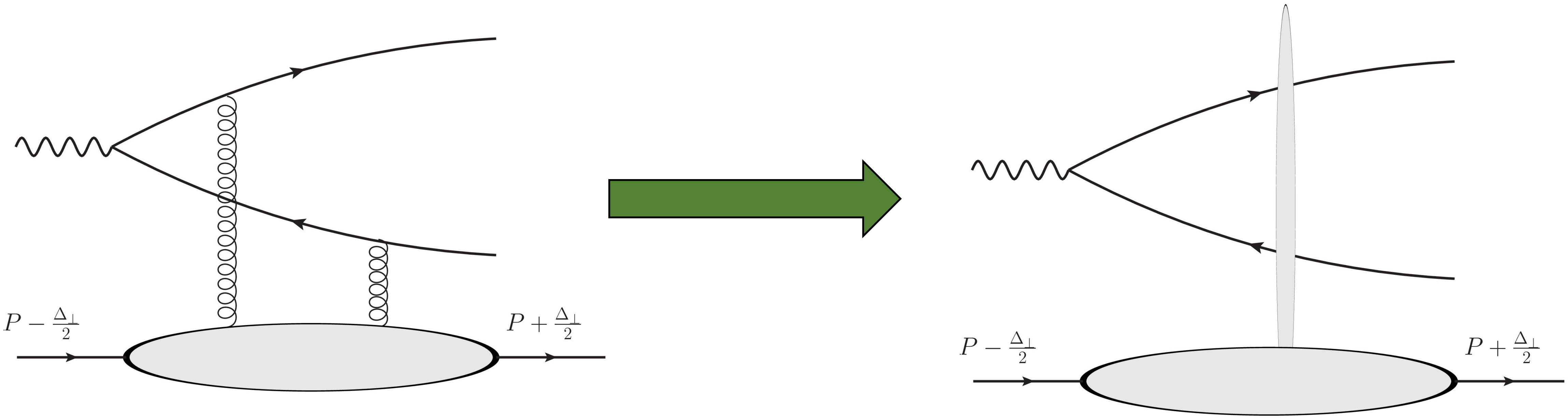
One expands the exponential in the definition of the Wigner functions in power of x, keeping only the leading terms.

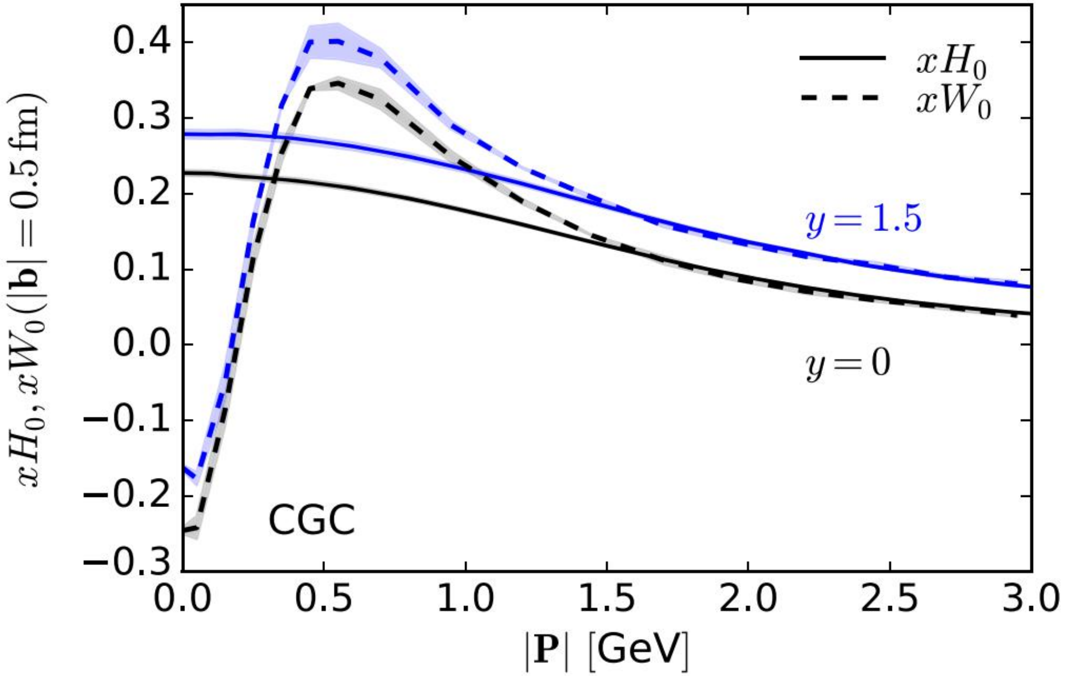
This leads to the Color Glass Condensate limit

Studying the cross-section give some information on the size of the angular correlations

Mantysaari et al, PRD 99 (2019)

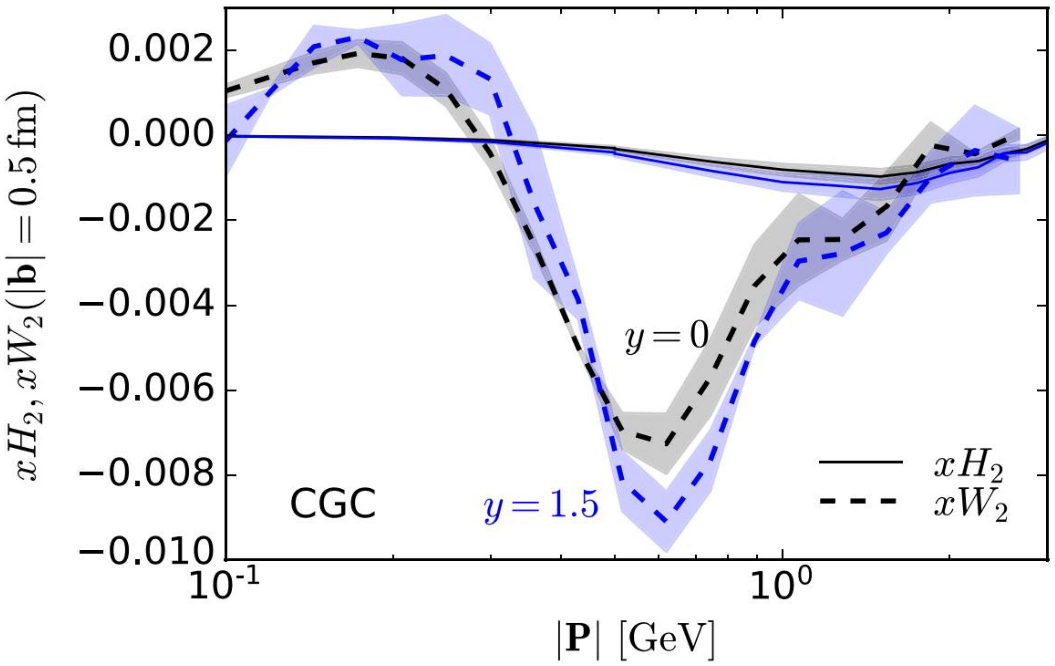
The CGC approach is equivalent to replacing the physical gluons with just gauge links, i.e. the q-qbar pair scatters off the background gluon medium inside the proton.

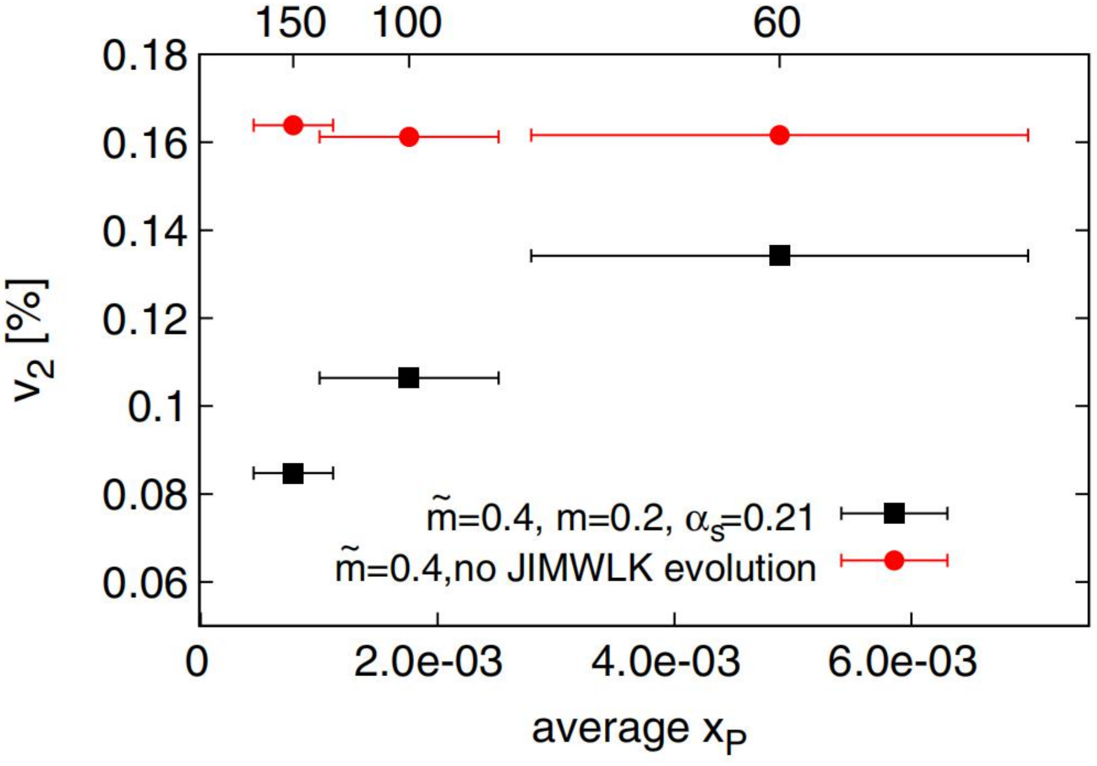




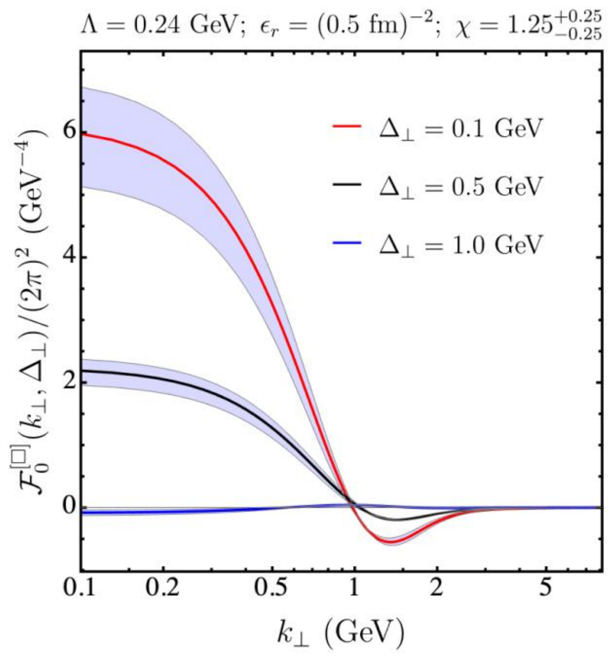
Mantysaari et al, PRD 99 (2019)

$$xW = xW_0 + 2xW_2 \cos\left(2\theta_{\boldsymbol{P}_\perp,\boldsymbol{b}_\perp}\right)$$



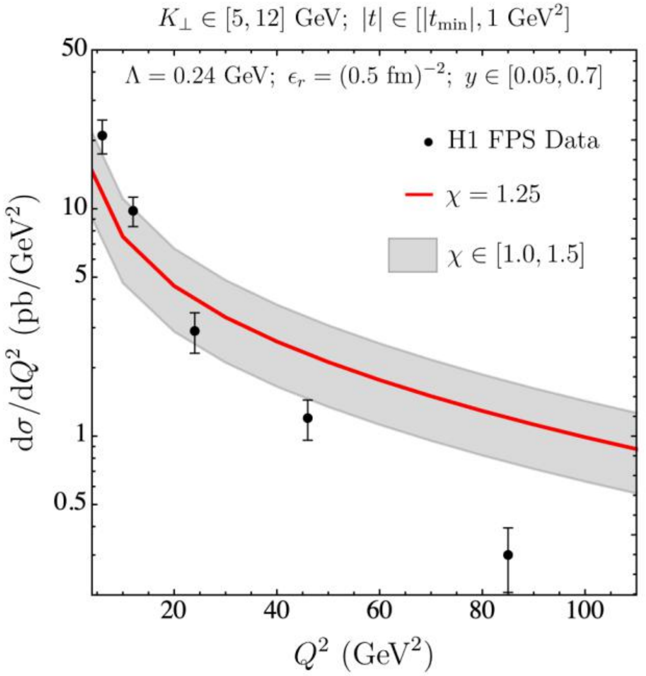


A recent fit of the HERA data has as a starting point
a small-x model for the Wigner functions



Boer. Setyadi 2106.15148

No elliptic modulation considered
(estimated to be 10-30%)
since we have too large data uncertanties



By going beyond the small-x limit, including the first order corrections,
leads to the interesting (yet approximated) relation:

$$L_g(x) \simeq -\Delta G(x)$$

Hatta et al. PRD 95 (2017)

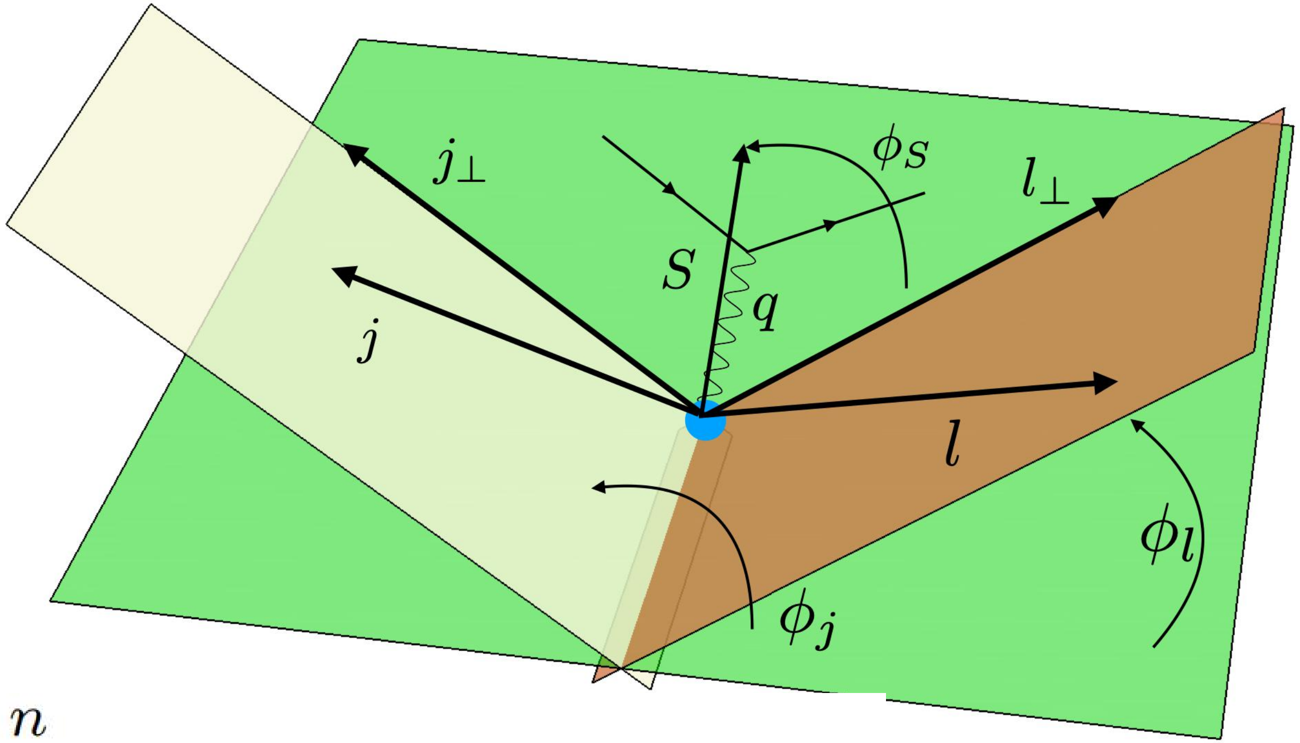
We would like to move away from the small-x approximation for the study of the dijet processes, since it is one of the very few processes that is, to date, a candidate to extract information on the Wigner distributions.

To study the dijet processes for all values of x, we first must obtain the complete cross section. Then, one can move to an analysis of the process, identifying the leading contributions and the associated angular structures.

The full angular dependence is potentially interesting for the small-x studies as well, since there can be effects that appears beyond the leading small-x approximation

Fully differential cross-section

$$\frac{d\sigma}{dj_\perp^2 d\phi_j dz_j dl_\perp^2 d\phi_l dz_l dx dy d\psi}$$
$$= W_{\mu\nu} L^{\mu\nu} \frac{\gamma y}{4MQ} \frac{Q^2}{4x} \frac{Q^2 z_j}{2\gamma^2 j^3} \frac{Q^2 z_l}{2\gamma^2 l^3} \frac{\alpha^2 16\pi^2}{Q^4} \frac{1}{(2\pi)^8}$$



$$(j \cdot l)^n = F_0^{(n)} + \sum_{k=1}^{n} F_k^{(n)} \cos\left(k(\phi_j - \phi_l)\right)$$

$$T_n(\cos(\theta)) = \cos(n\theta)$$

Expansion in Čebyšëv polynomials

Jet angular excitations

Metz, Pasquini, Rodini, in preparation

$$\text{parity: } W^{\mu\nu}(P,q,j,l,S) = \Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta}W^{\alpha\beta}\left(\tilde{P},\tilde{q},\tilde{j},\tilde{l},-\tilde{S}\right),$$

$$\text{hermiticity: } W^{\mu\nu}(P,q,j,l,S) = \left[W^{\nu\mu}(P,q,j,l,S)\right]^{*},$$

$$\text{gauge invariance: } q_{\mu}W^{\mu\nu}(P,q,j,l,S) = q_{\nu}W^{\mu\nu}(P,q,j,l,S) = 0.$$

ε : ratio between longitudinal and transverse photon flux

$$\begin{aligned}
\mathcal{I}m\left(W^{\mu\nu}\right)\mathcal{I}m\left(L_{\mu\nu}\right) = \lambda_e \frac{1}{1-\epsilon}\sum_{k=0}^{\infty}\cos\left(k\phi_j - k\phi_l\right)\Big\{ & \sqrt{\epsilon(1-\epsilon)}\sin\left(\phi_j\right)H_1^{(k)} + \sqrt{\epsilon(1-\epsilon)}\sin\left(\phi_l\right)H_2^{(k)} \\
& + \sqrt{1-\epsilon^2}\sin\left(\phi_j-\phi_l\right)H_3^{(k)} + \sqrt{\epsilon(1-\epsilon)}S_{\perp}\cos\left(\phi_S\right)H_4^{(k)} + \sqrt{1-\epsilon^2}S_{\perp}\cos\left(\phi_S-\phi_j\right)H_5^{(k)} \\
& + \sqrt{1-\epsilon^2}S_{\perp}\cos\left(\phi_S-\phi_l\right)H_6^{(k)} + \sqrt{\epsilon(1-\epsilon)}S_{\perp}\cos\left(\phi_S-2\phi_j\right)H_7^{(k)} \\
& + \sqrt{\epsilon(1-\epsilon)}S_{\perp}\cos\left(\phi_S-2\phi_l\right)H_8^{(k)} + \sqrt{\epsilon(1-\epsilon)}S_{\perp}\sin\left(\phi_j\right)\sin\left(\phi_S-\phi_l\right)H_9^{(k)} \\
& + \sqrt{\epsilon(1-\epsilon)}S_{\perp}\sin\left(\phi_l\right)\sin\left(\phi_S-\phi_j\right)H_{10}^{(k)} + \sqrt{1-\epsilon^2}S_{\perp}\cos\left(\phi_S-2\phi_j+\phi_l\right)H_{11}^{(k)} \\
& + \sqrt{1-\epsilon^2}S_{\perp}\cos\left(\phi_S-2\phi_l+\phi_j\right)H_{12}^{(k)} + \sqrt{\epsilon(1-\epsilon)}S_L\cos\left(\phi_j\right)H_{13}^{(k)} \\
& + \sqrt{\epsilon(1-\epsilon)}S_L\cos\left(\phi_l\right)H_{14}^{(k)} + \sqrt{\epsilon(1-\epsilon)}S_L\cos\left(\phi_j-2\phi_l\right)H_{15}^{(k)} \\
& + \sqrt{\epsilon(1-\epsilon)}S_L\cos\left(\phi_l-2\phi_j\right)H_{16}^{(k)} + \sqrt{1-\epsilon^2}S_L H_{17}^{(k)}\Big\}
\end{aligned}$$

Conclusions

Wigner distributions encode the most general information about the partonic structure of the proton

They offer the preferential tool to study the orbital angular momentum distribution inside the proton

Very few processes are known to be able to access the Wigner distributions

Increasingly more work is being put into the study of exclusive dijet production as it grants access to the gluon Wigner distributions

We are at a stage in which the full differential cross-section is necessary to go beyond the small-x approximation