

**Sar WorS 2021**

**Cagliari, September 7th, 2021**

# **Gluon TMDs and hadronic collisions**

**Francesco Giovanni Celiberto**

**ECT\*/FBK Trento & INFN-TIFPA**

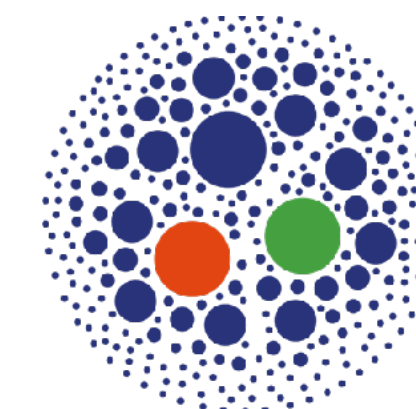
In collaboration with **A. Bacchetta, C. Bissolotti, M. Radici** and **A. Signori**

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



Trento Institute for  
Fundamental Physics  
and Applications



**HAS QCD**

HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: gauge links and modified universality

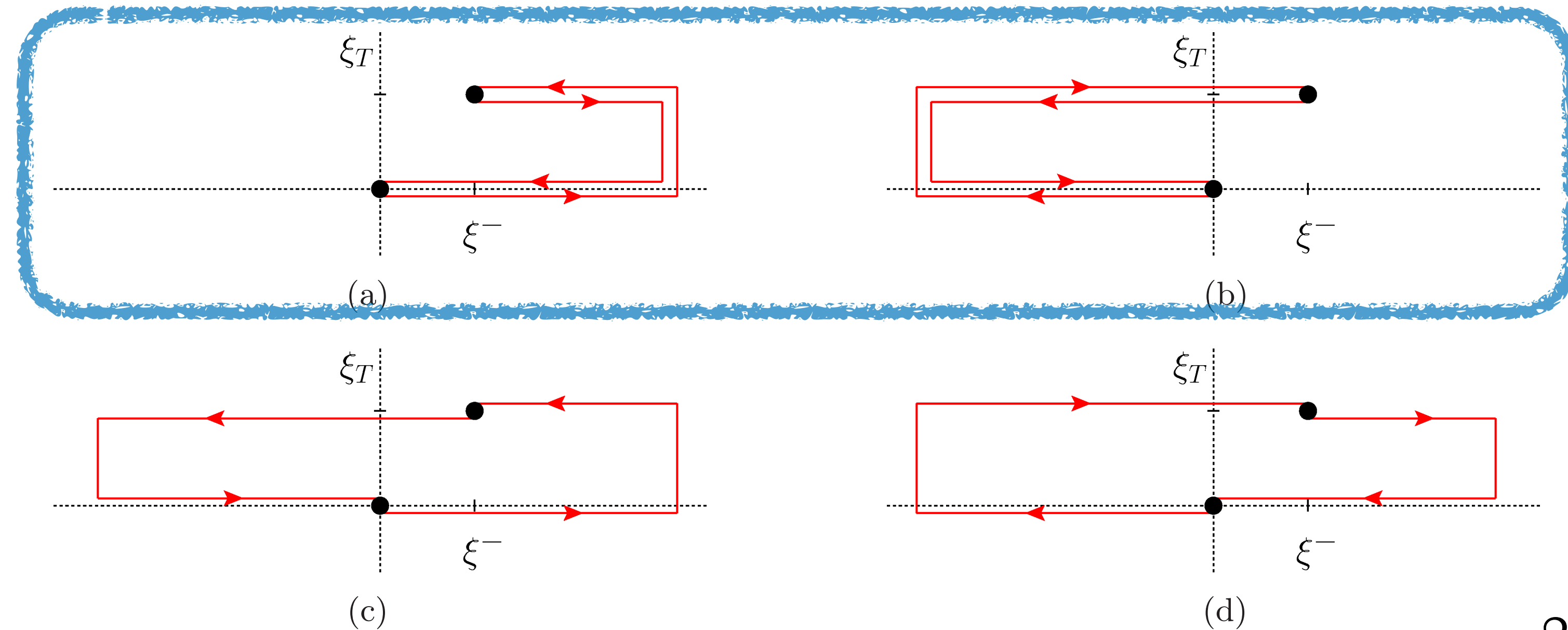
- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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## Weizsäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

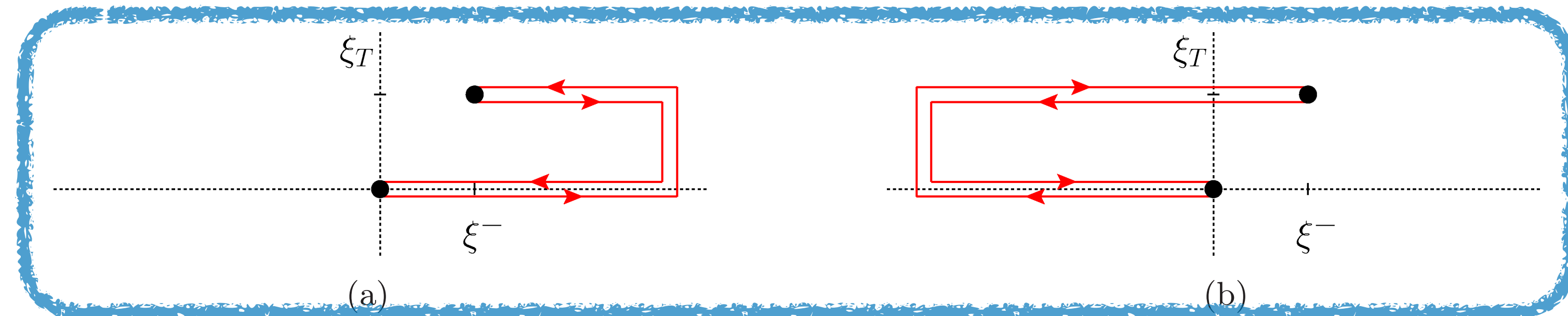


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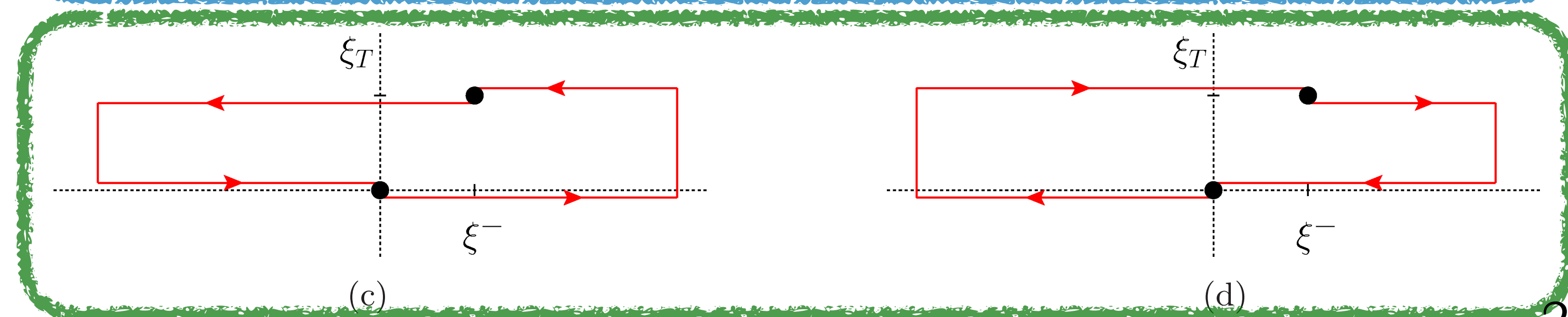
## Weizsäcker-Williams (WW)

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## Dipole (DP)

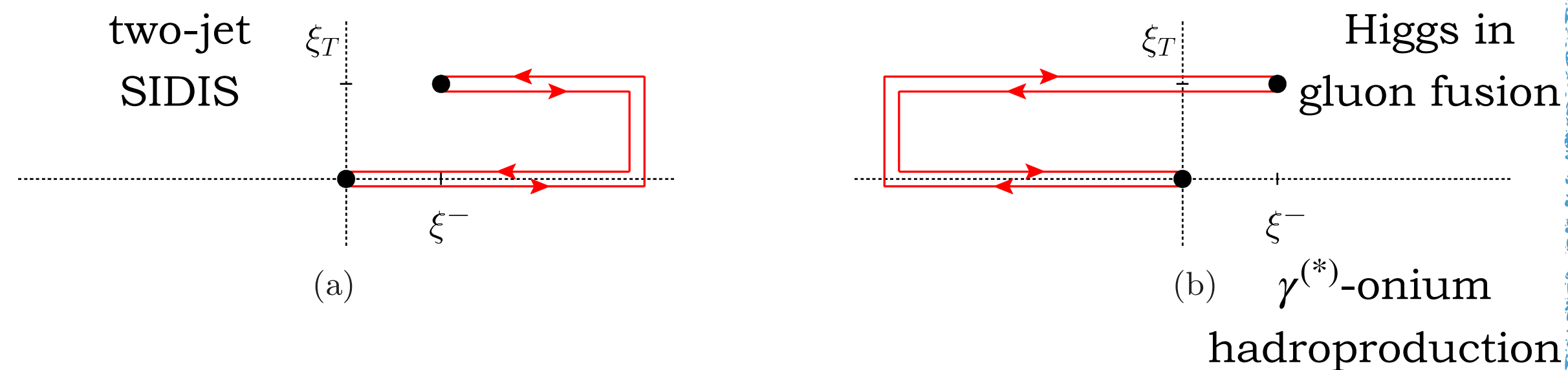
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



\* Color flow annihilated within final/initial state

\*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure

\* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$

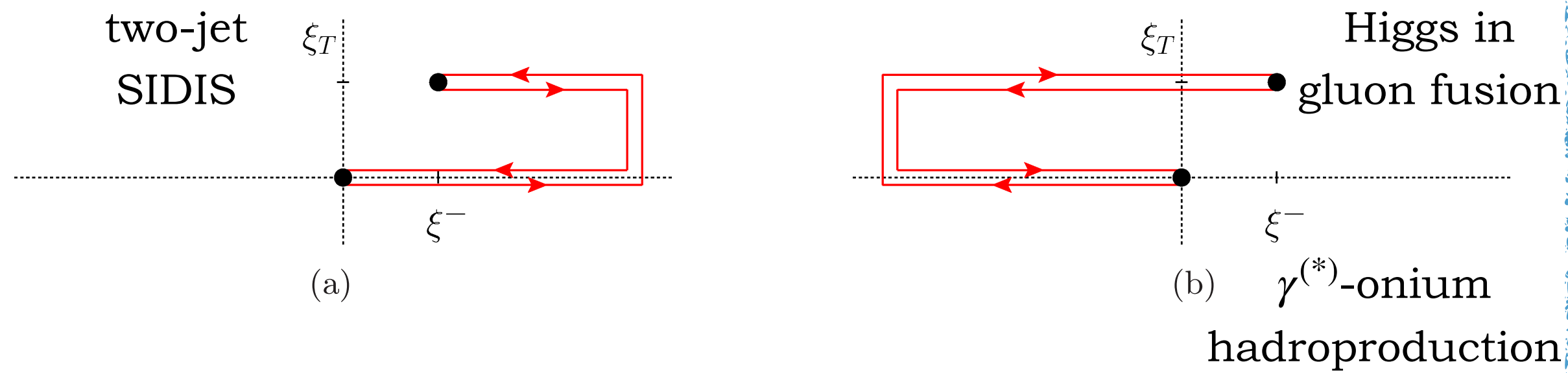
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

\* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

# Accessing WW and DP gluon TMDs

## Weizsäcker-Williams (WW)

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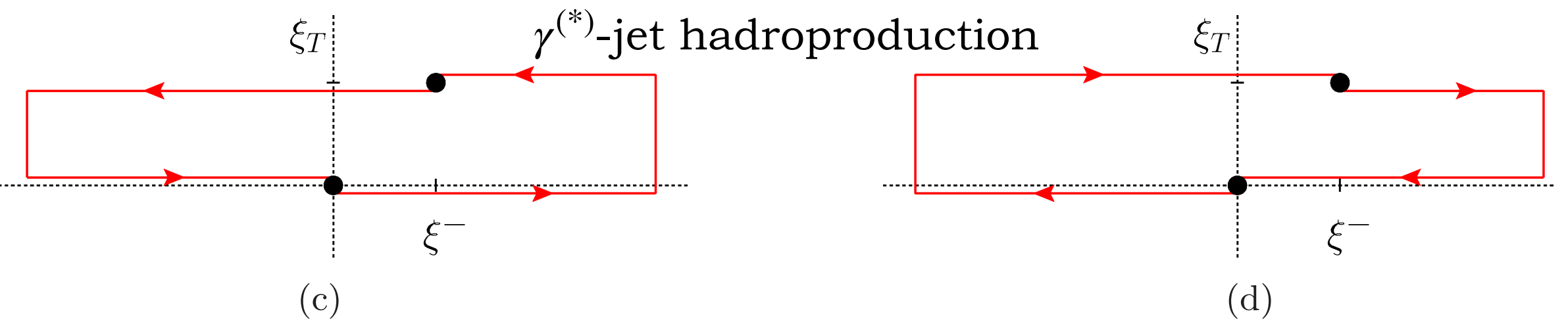
$$f_1^{[+,+]} = f_1^{[-,-]},$$

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- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

## Dipole (DP)

(c) [ + , - ] or (d) [ - , + ]



- \* Color flow involving both initial and final states

- \*  $d$ -type gluon TMDs  $\rightarrow d^{abc}$  color structure

- \* Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]},$$

$$f_{1T}^\perp[+,-] = -f_{1T}^\perp[-,+]$$

- \* Phenomenology: single hadron or  $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

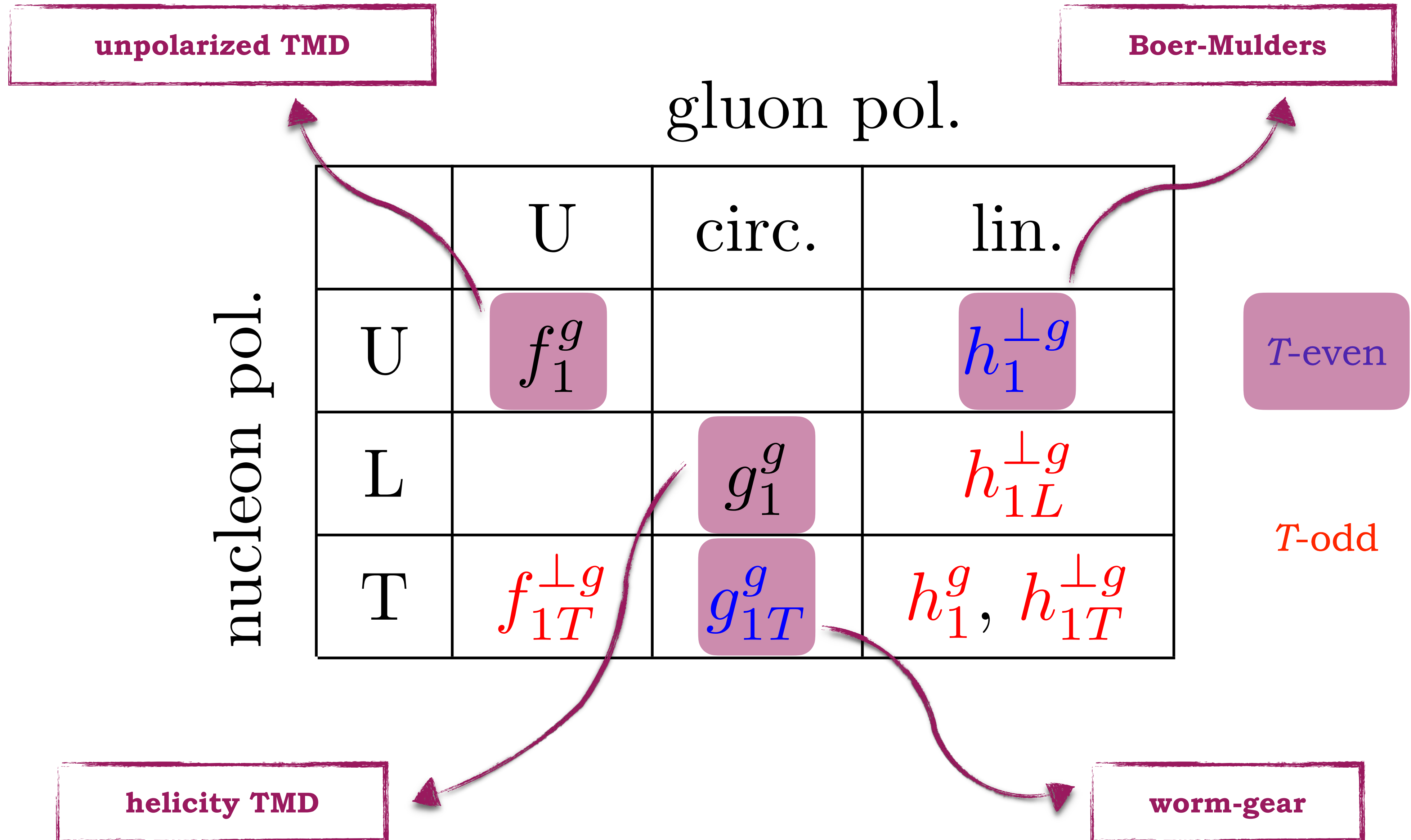
Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# ***T*-even and *T*-odd gluon TMD PDFs at twist-2**

gluon pol.

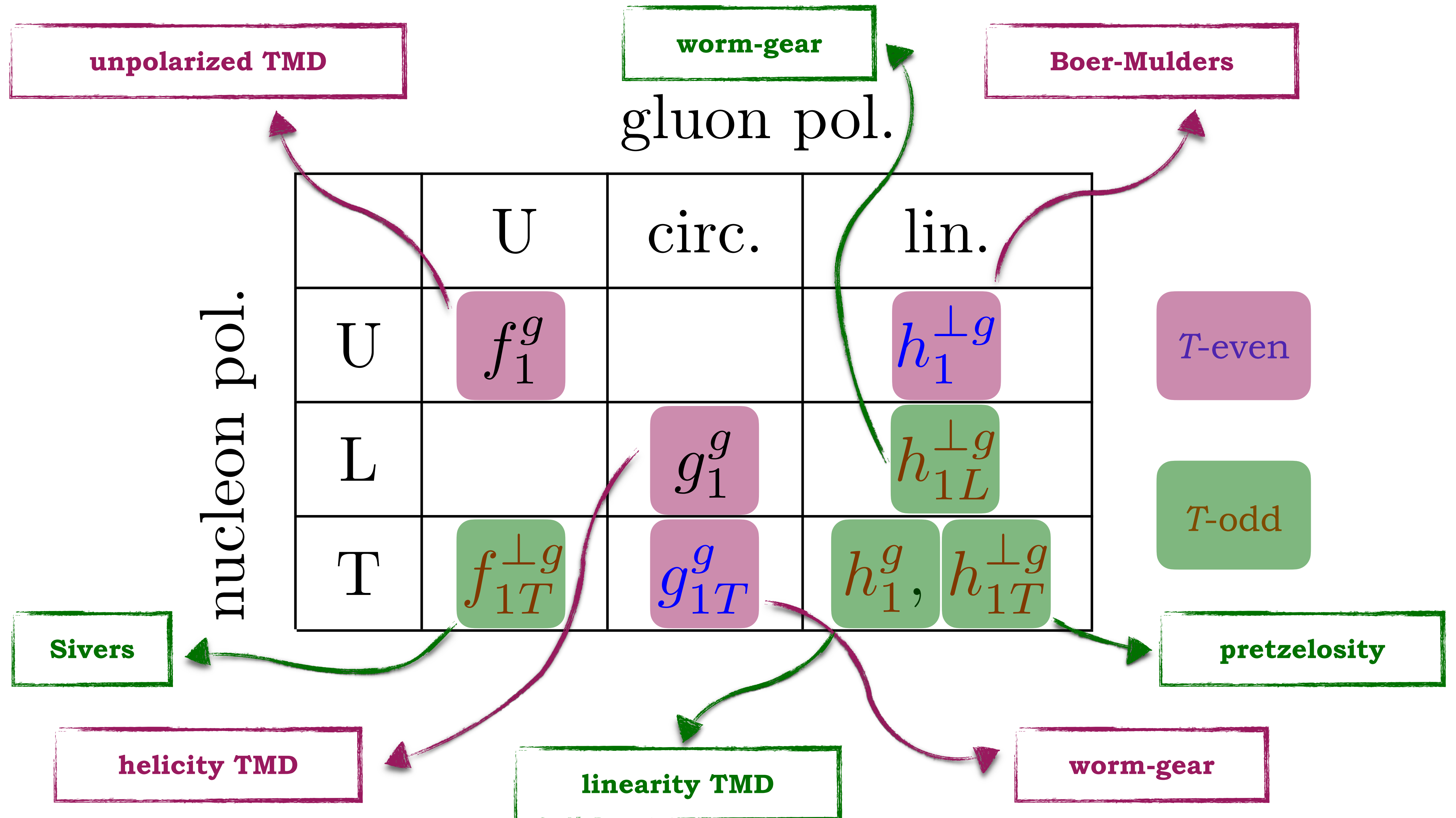
nucleon pol.		U	circ.	lin.	
	U	$f_1^g$		$h_1^{\perp g}$	<i>T</i> -even
	L		$g_1^g$	$h_{1L}^{\perp g}$	<i>T</i> -odd
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$	

# $T$ -even and $T$ -odd gluon TMD PDFs at twist-2



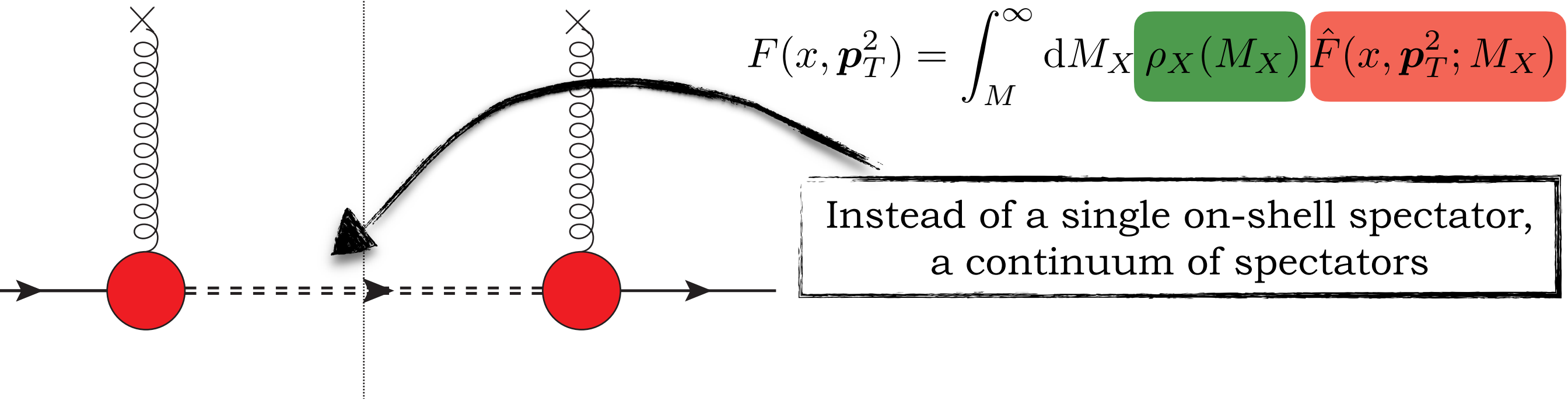


# $T$ -even and $T$ -odd gluon TMD PDFs at twist-2



# Our model at a glance

## Spectator-system spectral-mass function

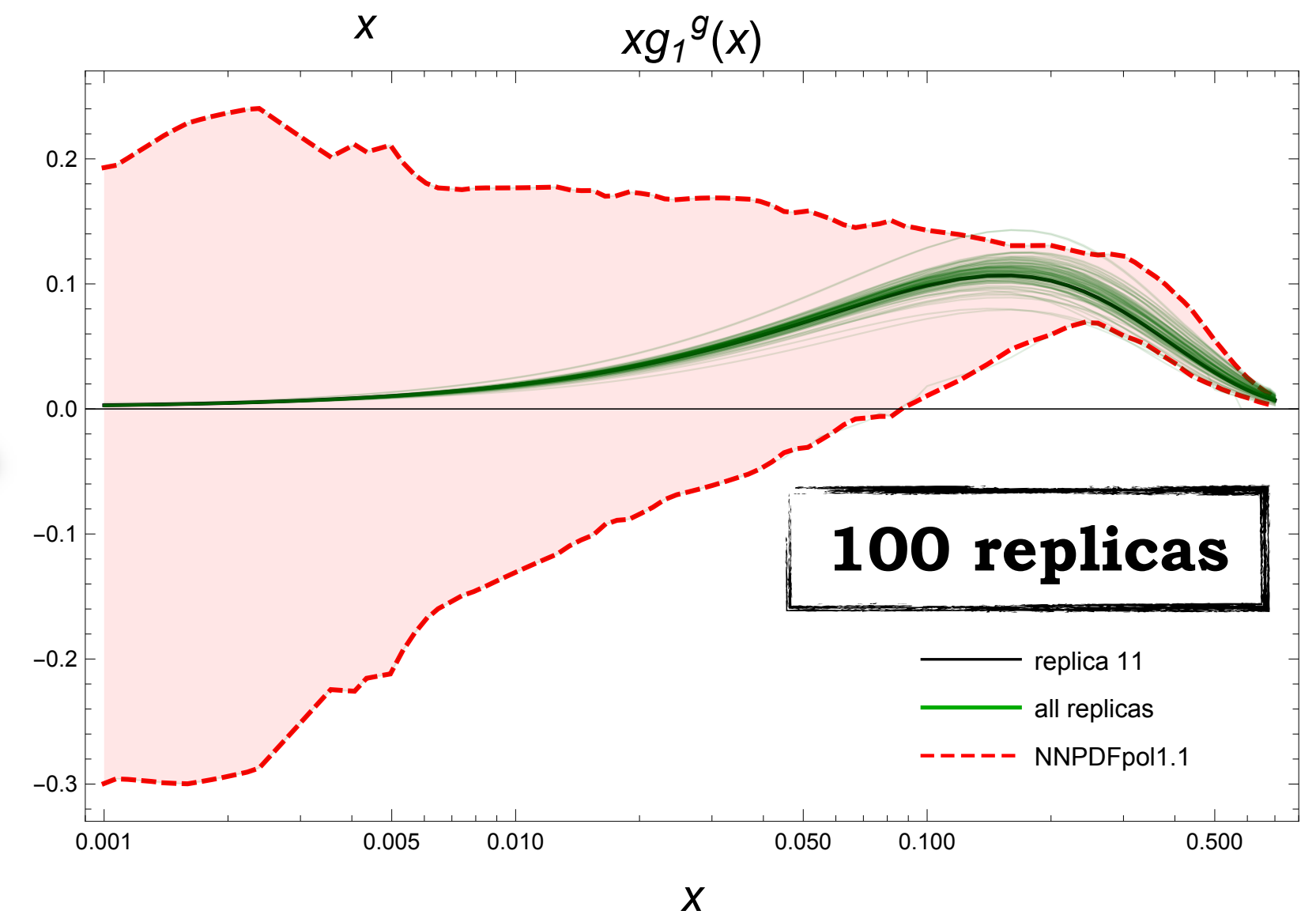
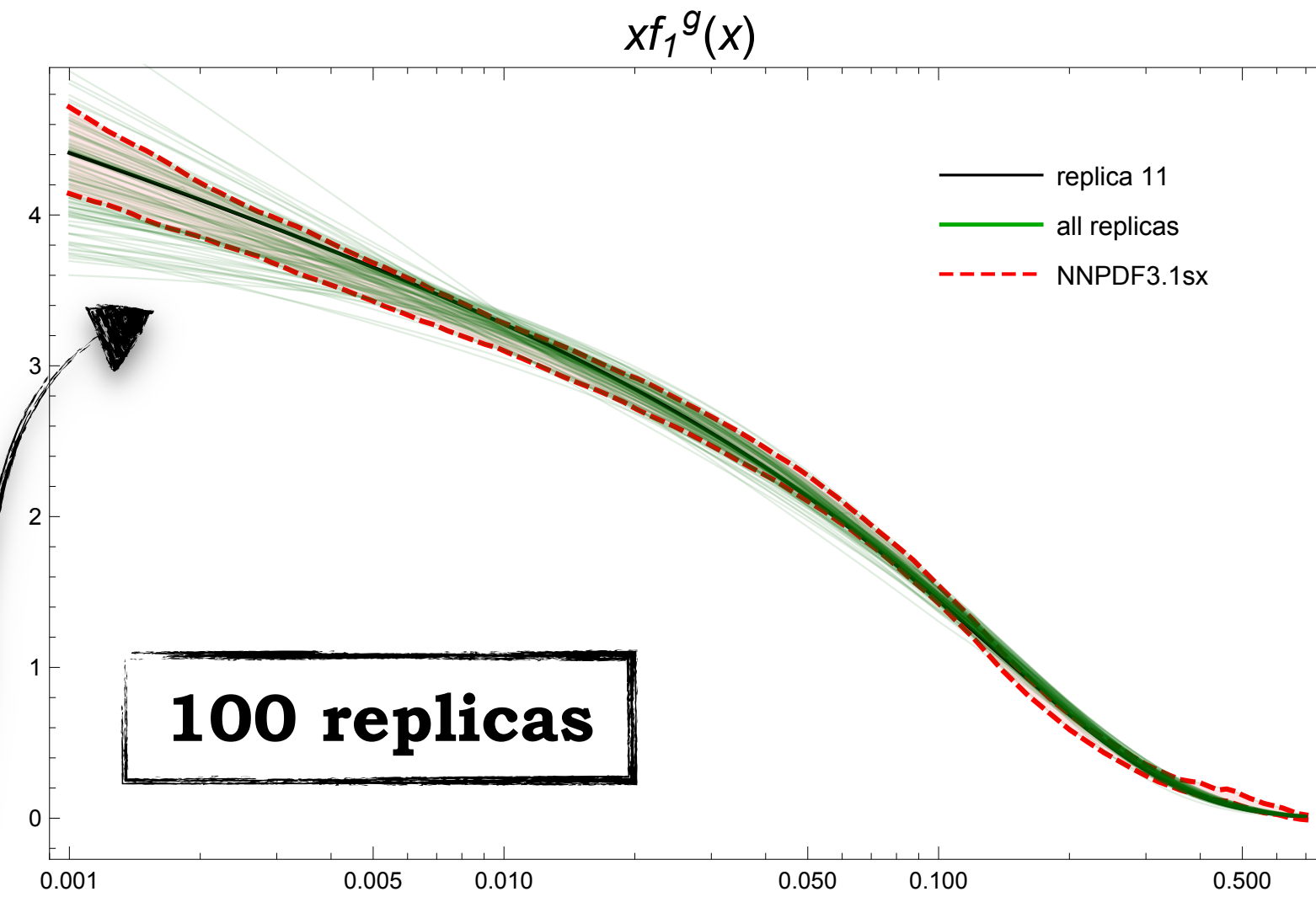


Spectral function **learns** small- and moderate- $x$  info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)

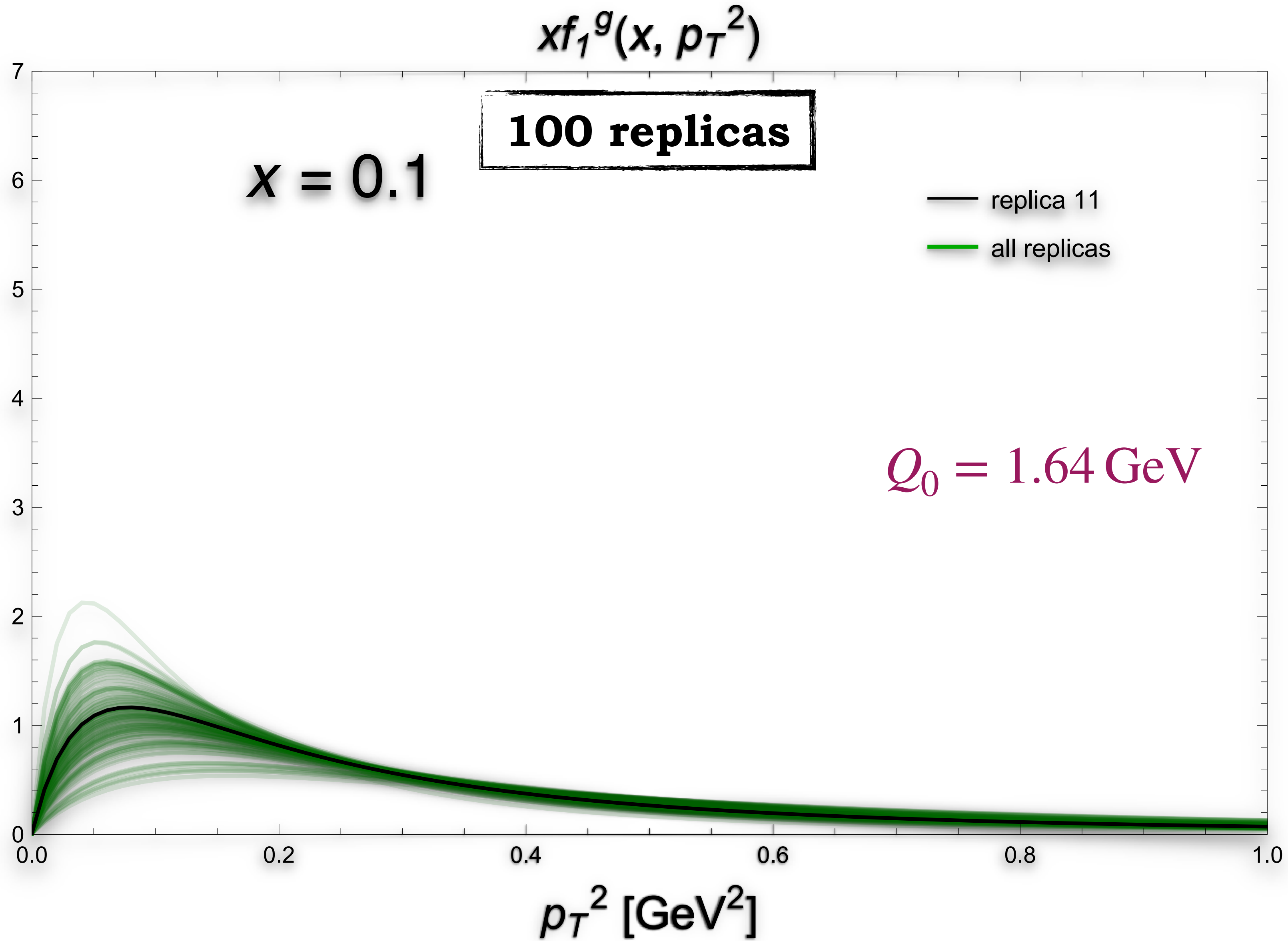
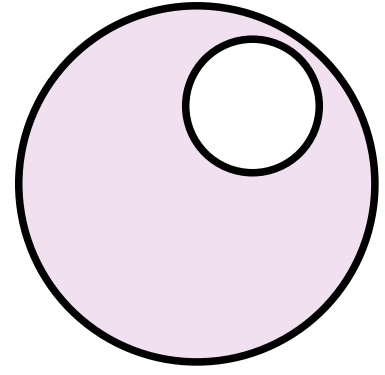
- ✓ **Simultaneous fit** of  $f_1$  and  $g_1$  PDFs
- ✓ Inclusion of small- $x$  resummation effects (**BFKL**)
- ✓ Calculation of all twist-2  $T$ -even gluon TMDs

## Link with collinear factorization

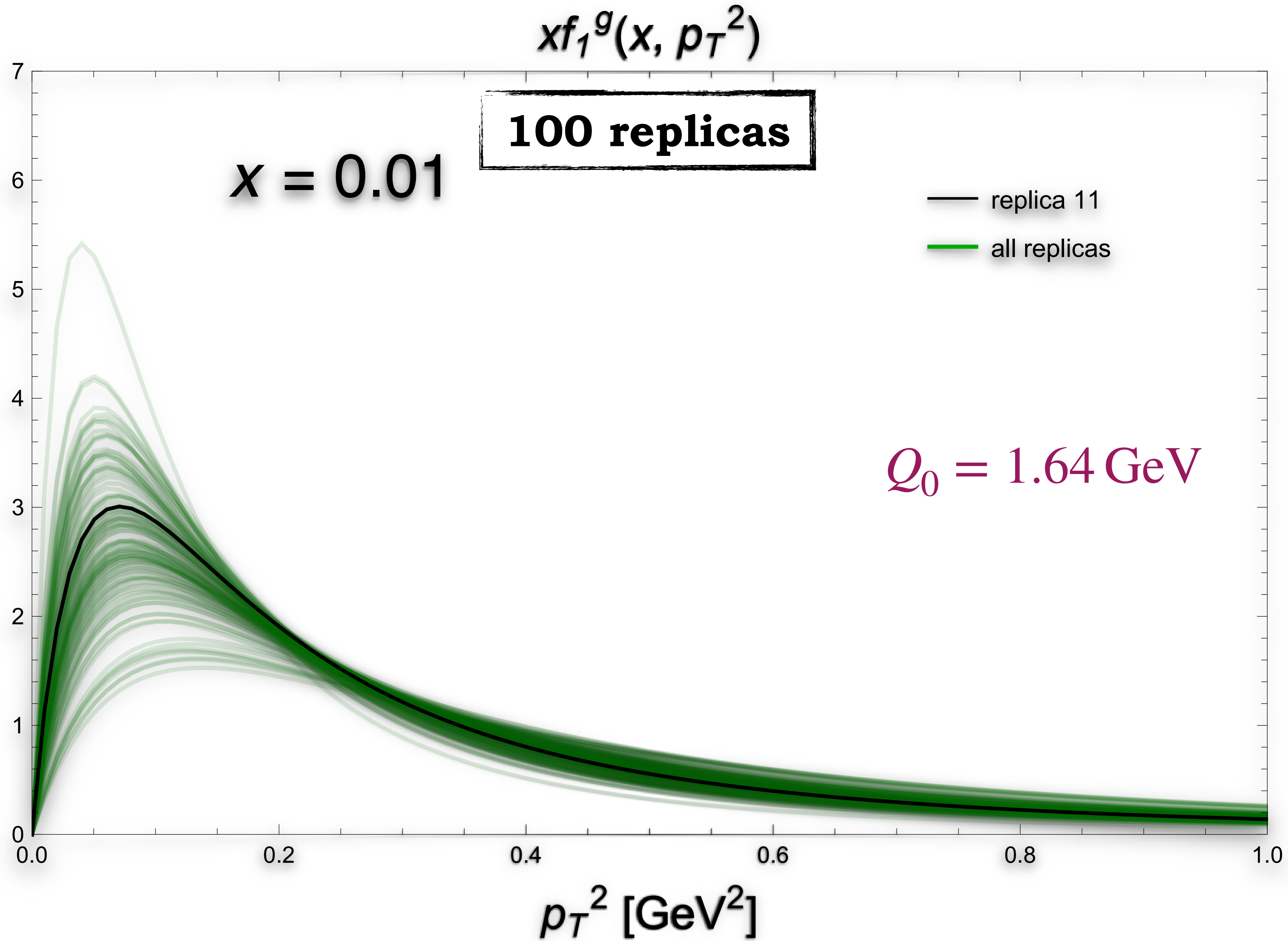
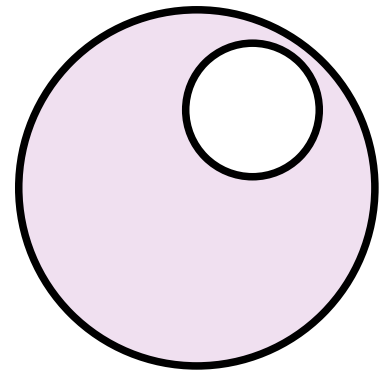
$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution



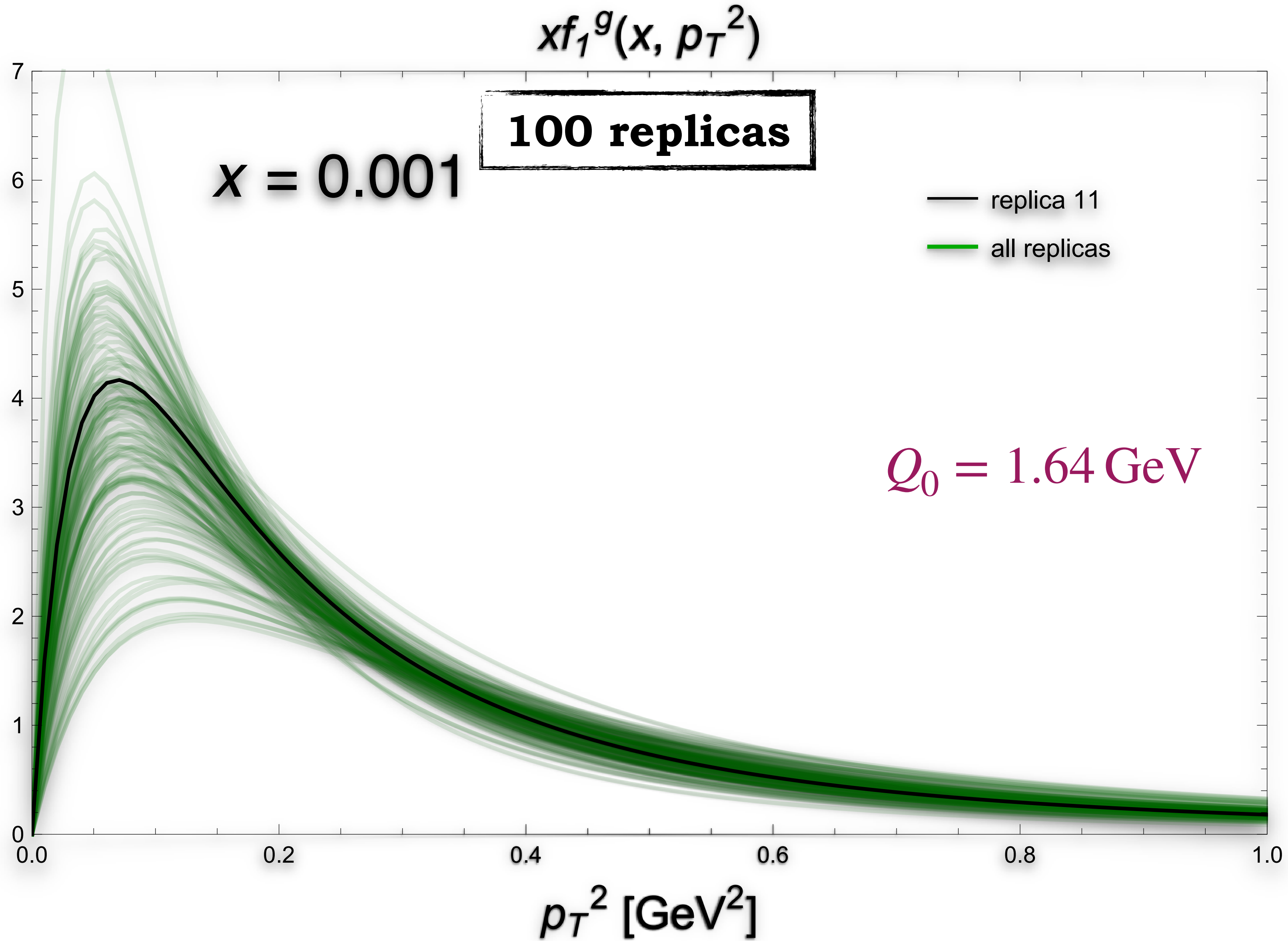
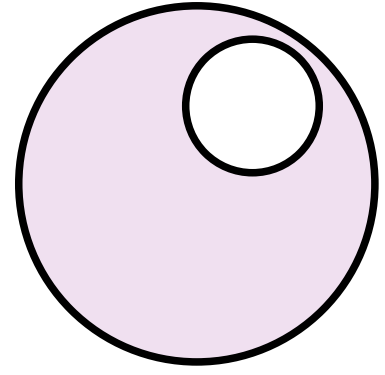
# Unpolarized gluon TMD



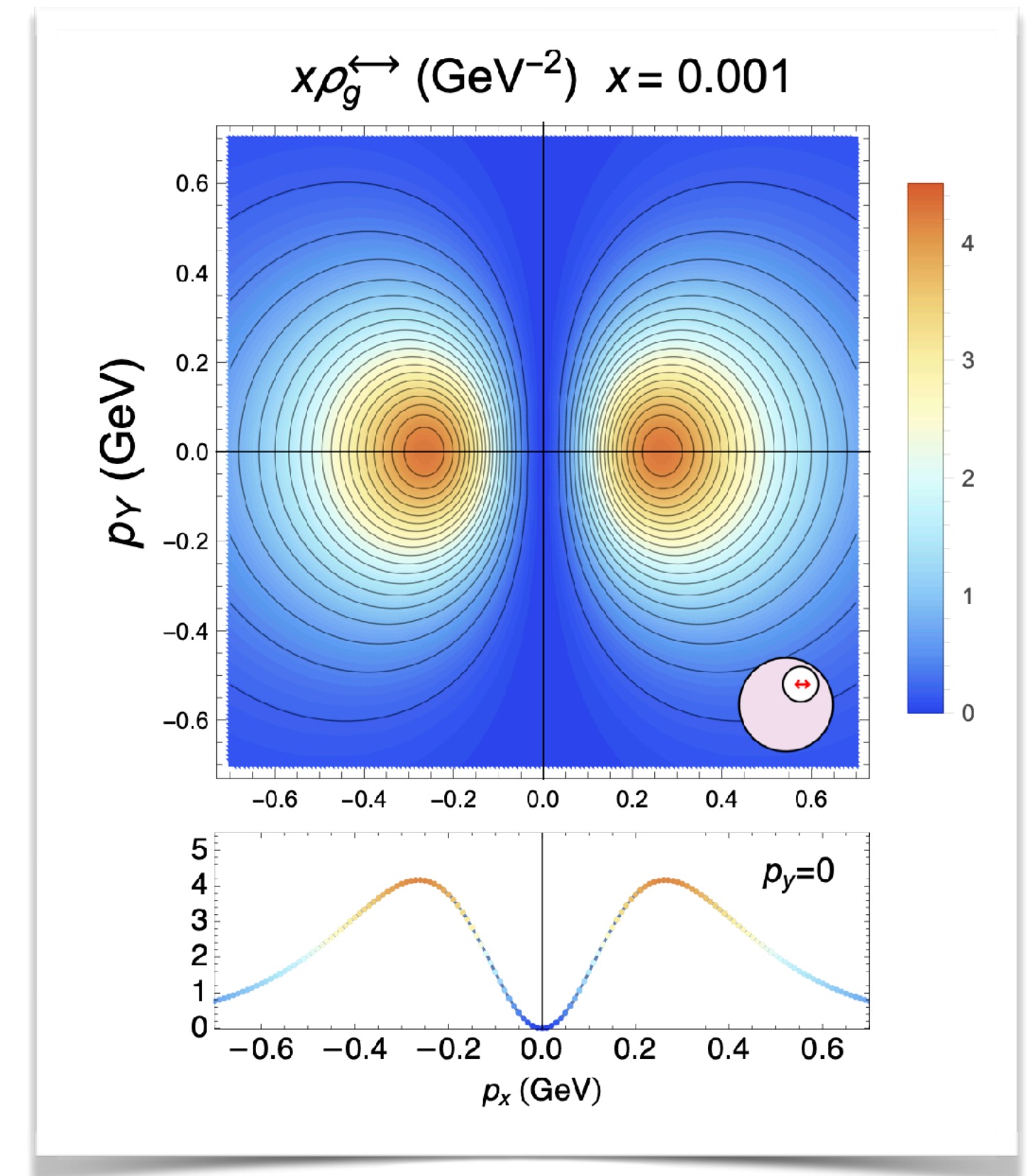
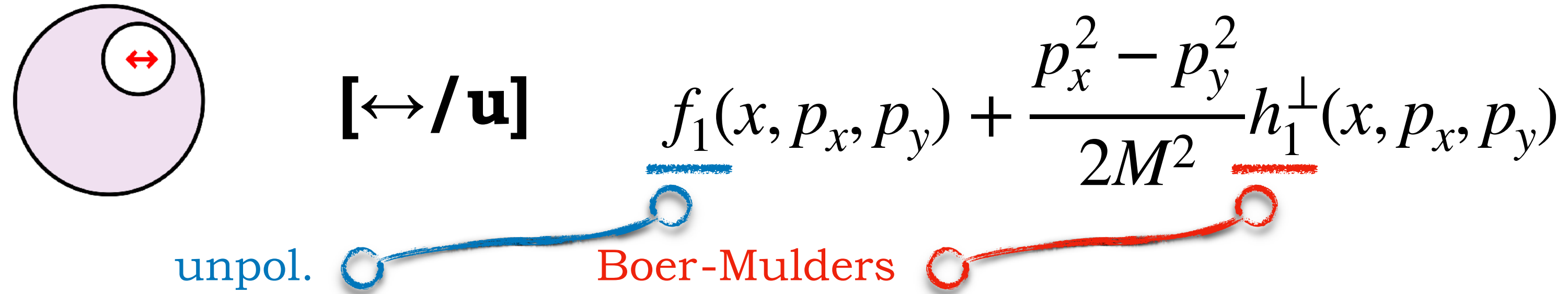
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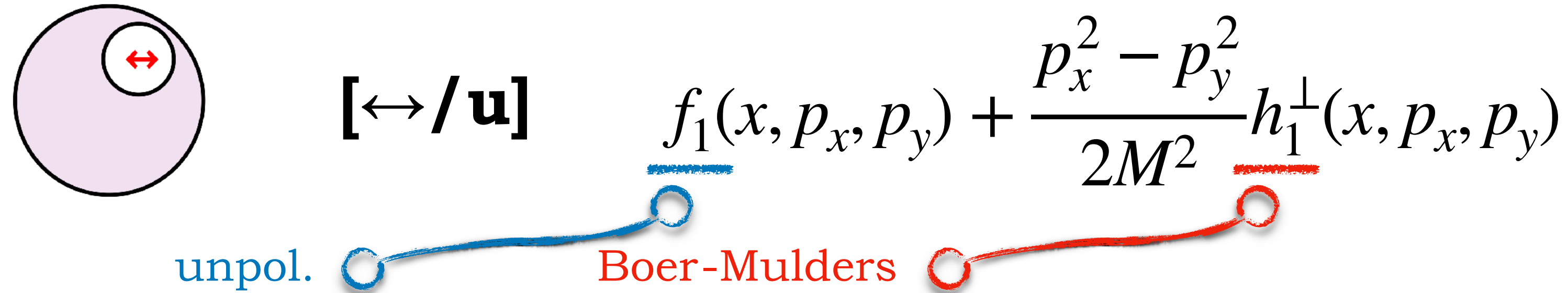


# Boer-Mulders effect in unpolarized $pp$ collisions



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

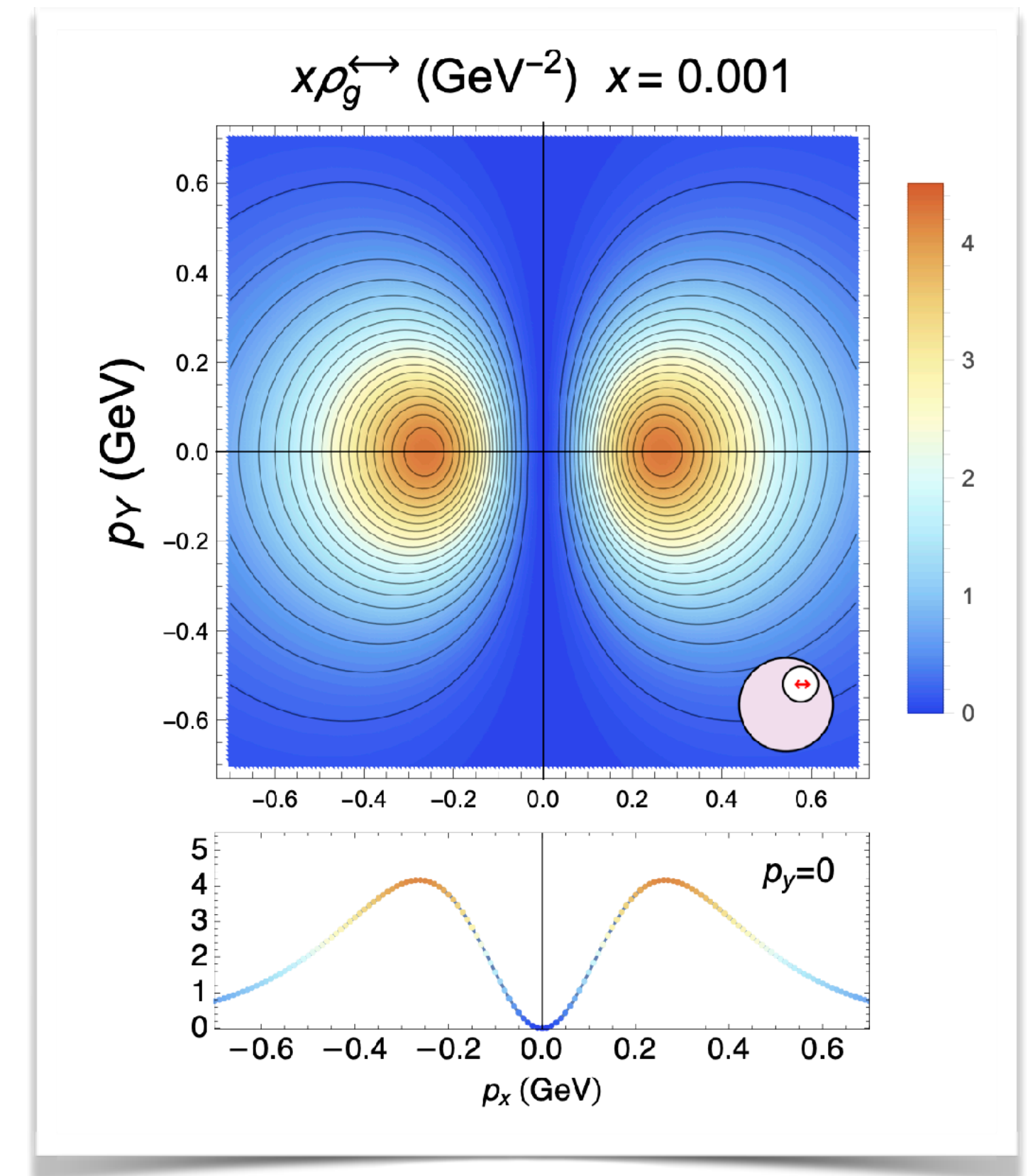
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## (Pseudo)scalar Higgs $p_T$ -distribution

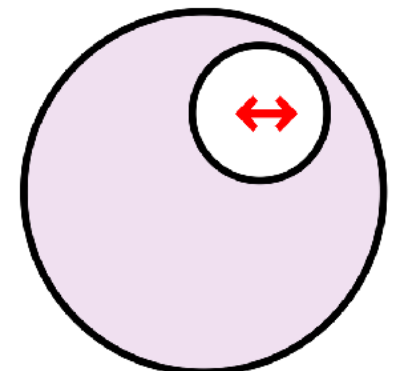
$$\frac{E d\sigma^{H(A)}}{d^3\vec{q}} \Big|_{q_T \ll m_H} = \frac{\pi\sqrt{2}G_F}{128m_H^2 S} \left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_{H(A)}(\tau)|^2 \times \left( C [f_1^g f_1^g] \pm C [w_H h_1^{\perp g} h_1^{\perp g}] \right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

# Boer-Mulders effect in unpolarized $pp$ collisions



$$[\leftrightarrow / \mathbf{u}] \quad \underbrace{f_1(x, p_x, p_y)}_{\text{unpol.}} + \frac{p_x^2 - p_y^2}{2M^2} \underbrace{h_1^\perp(x, p_x, p_y)}_{\text{Boer-Mulders}}$$

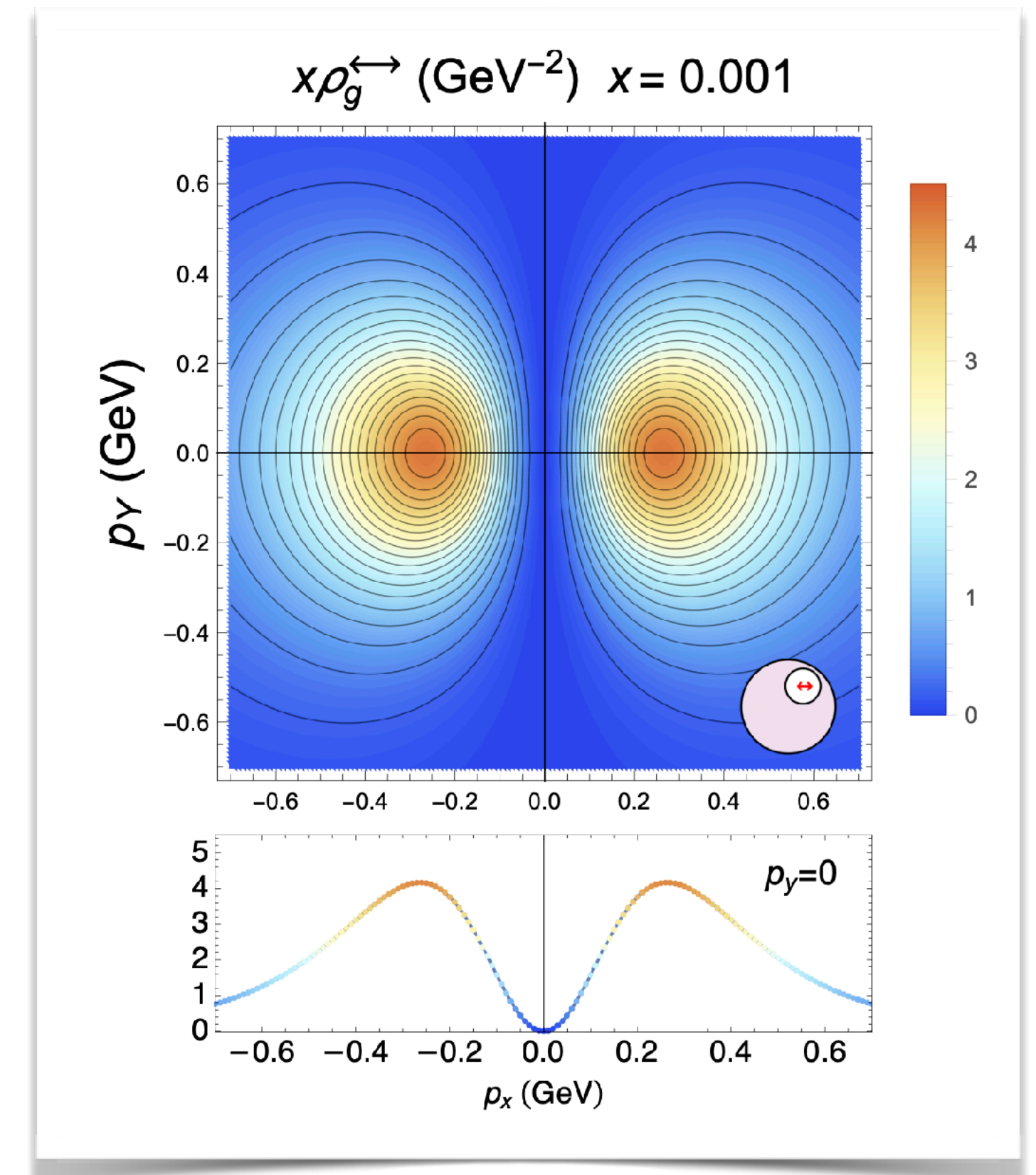
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## Model prediction at low- $x$

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$





⌋ [A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

see yesterday's talk by **Daniel**



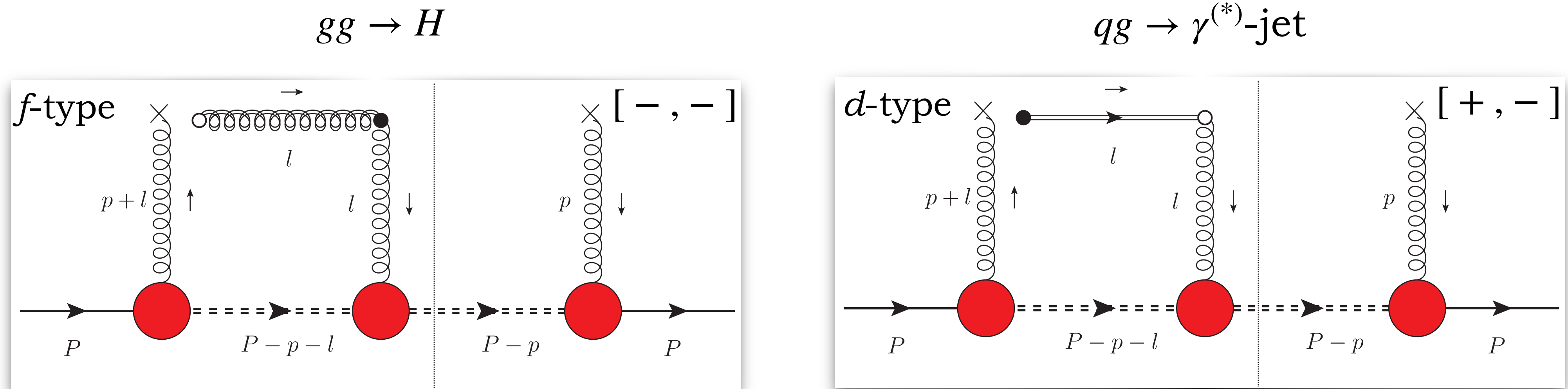
**...towards twist-2**  
***T*-odd gluon TMDs**

# ***T*-odd gluon TMDs in a spectator model**

-  No residual gluon-spectator interaction at tree level
-  *Interference* with one-gluon exchange (*eikonal*)

# $T$ -odd gluon TMDs in a spectator model

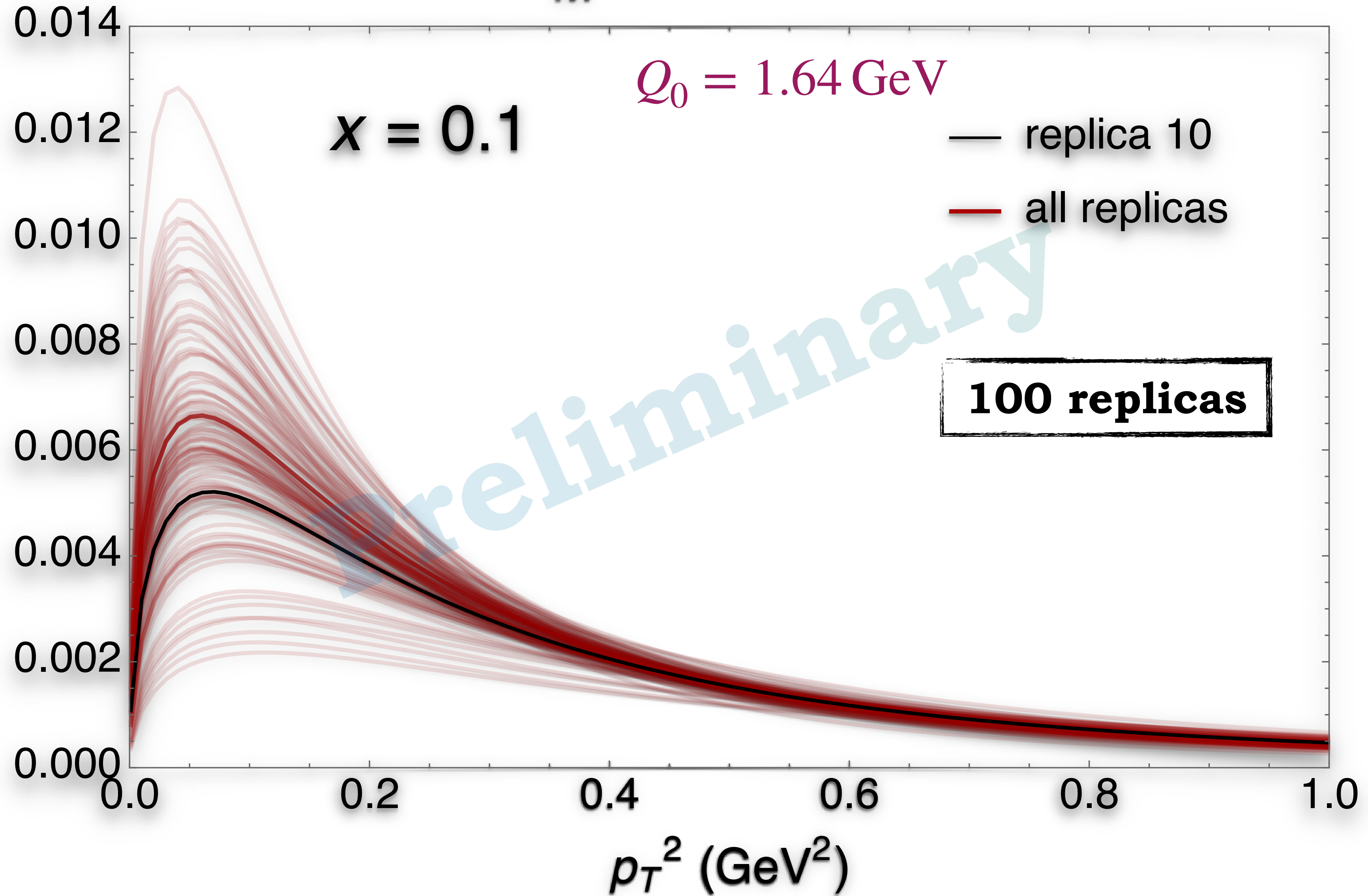
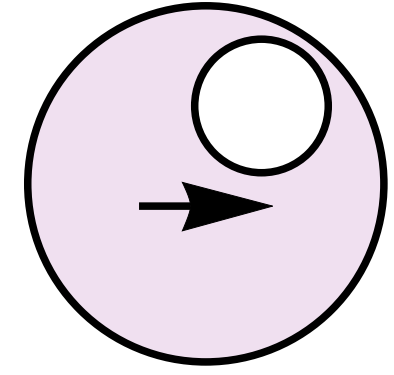
- No residual gluon-spectator interaction at tree level
- *Interference* with one-gluon exchange (*eikonal*)



- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

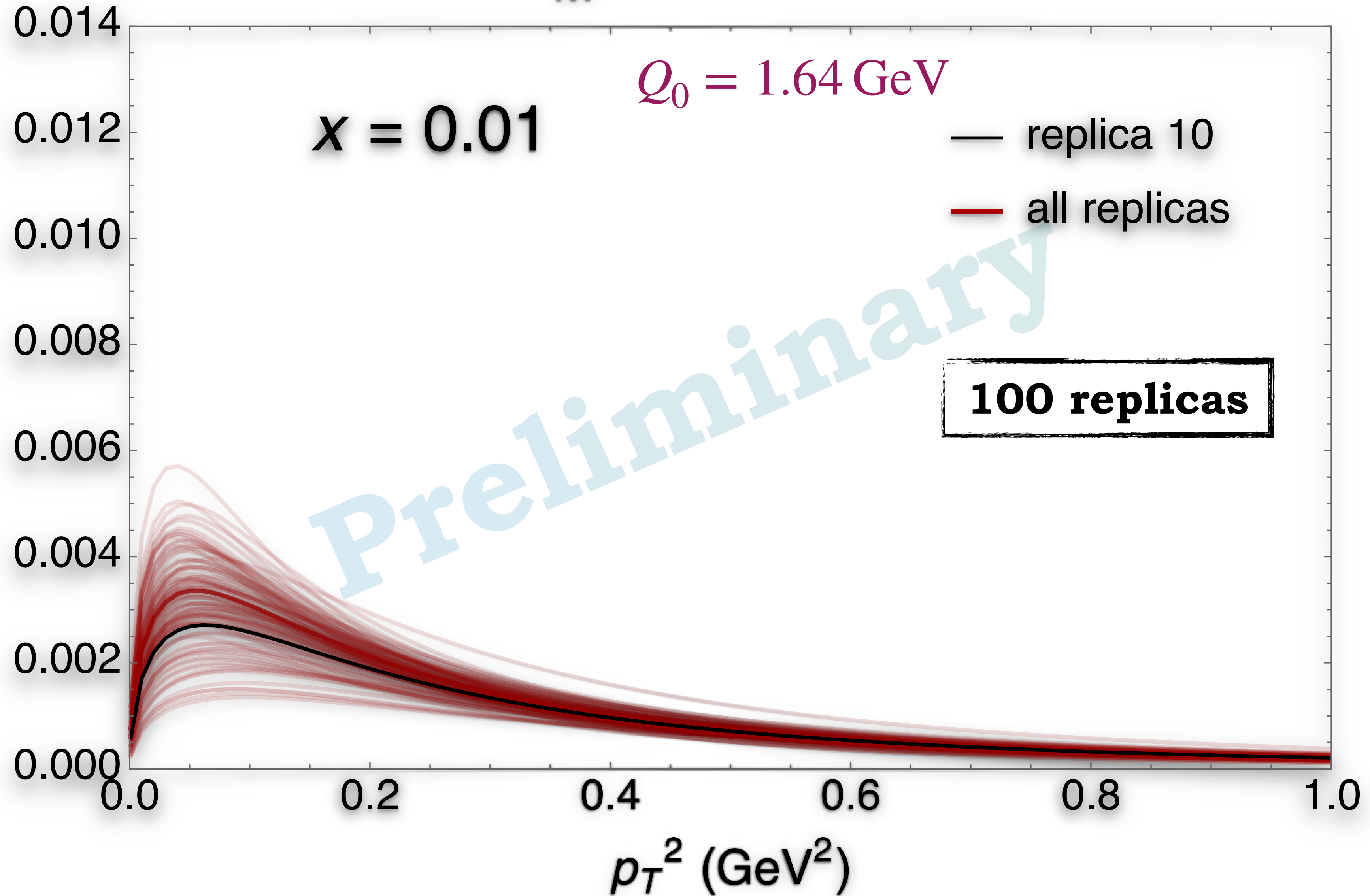
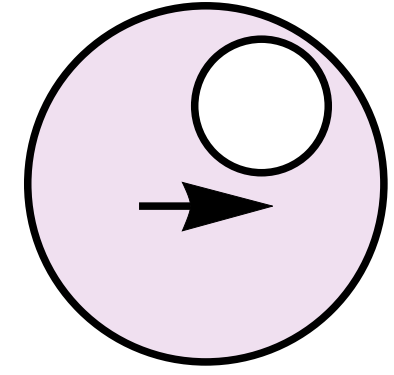
# *f*-type Sivers gluon TMD

$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



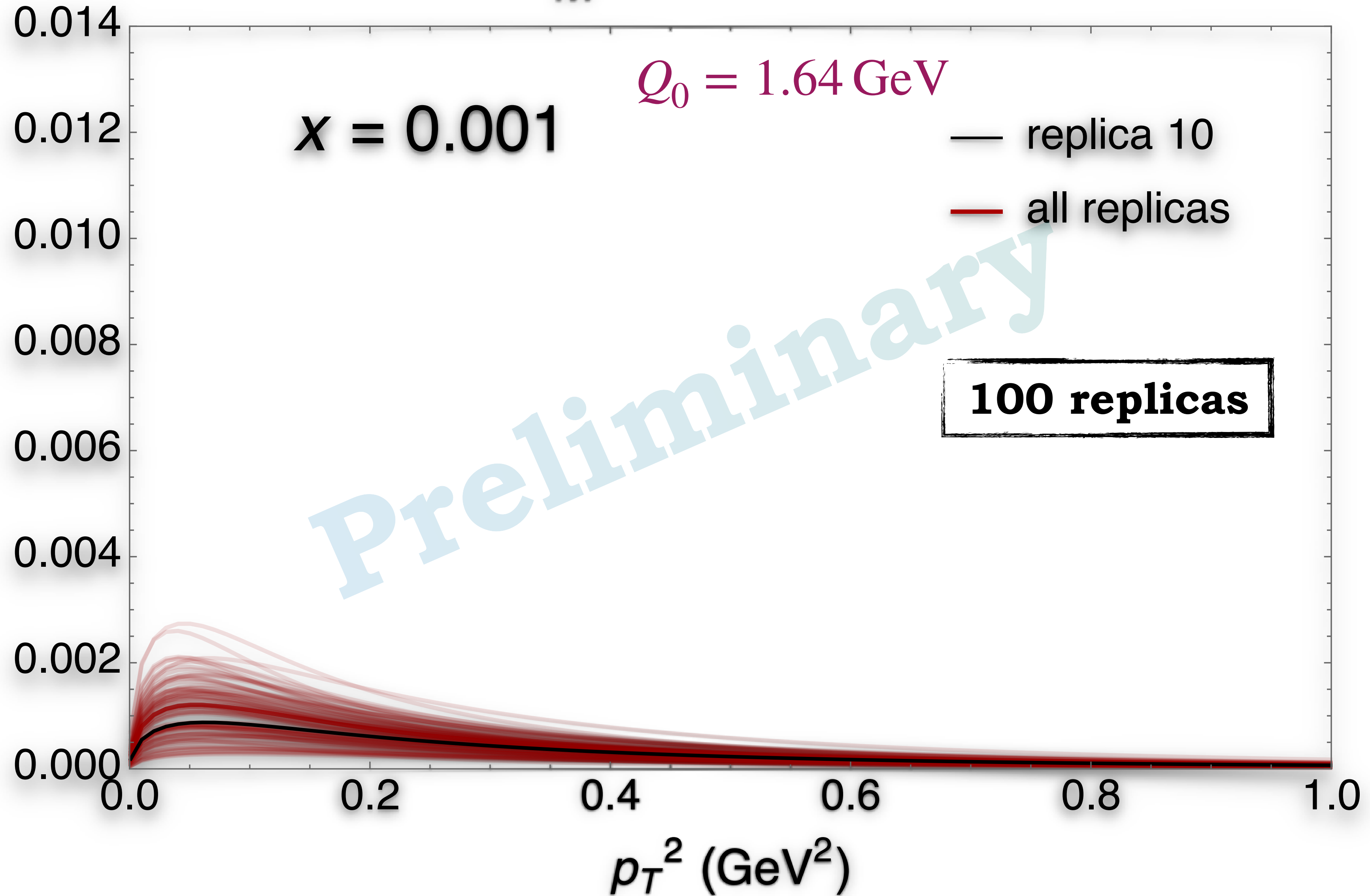
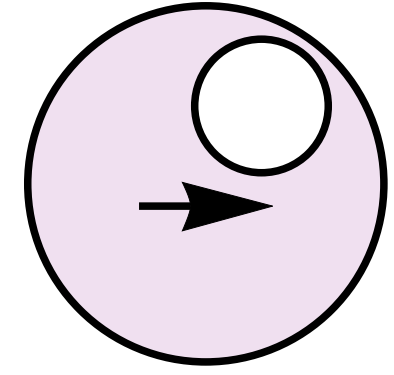
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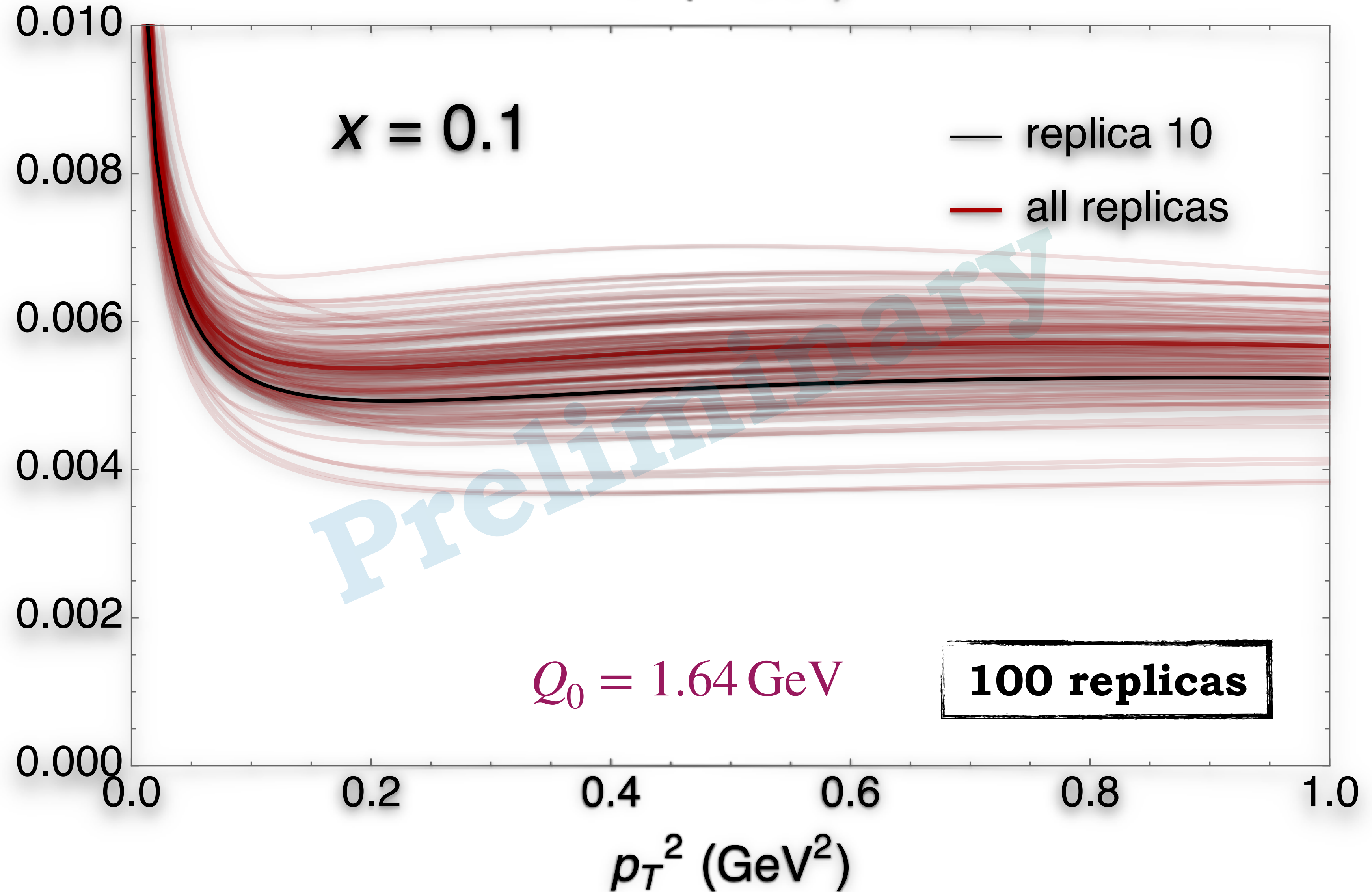
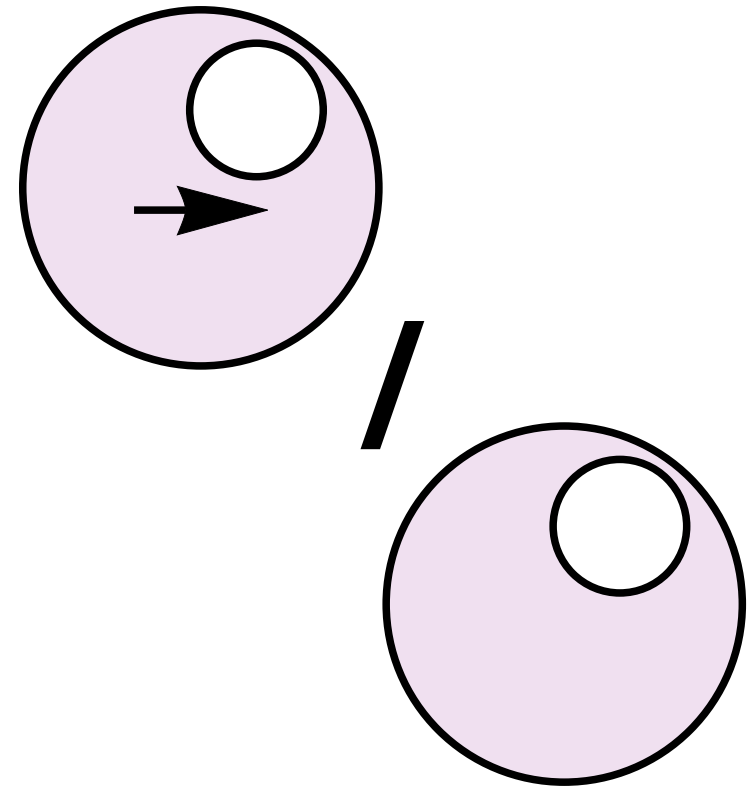
# *f*-type Siverson gluon TMD

$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



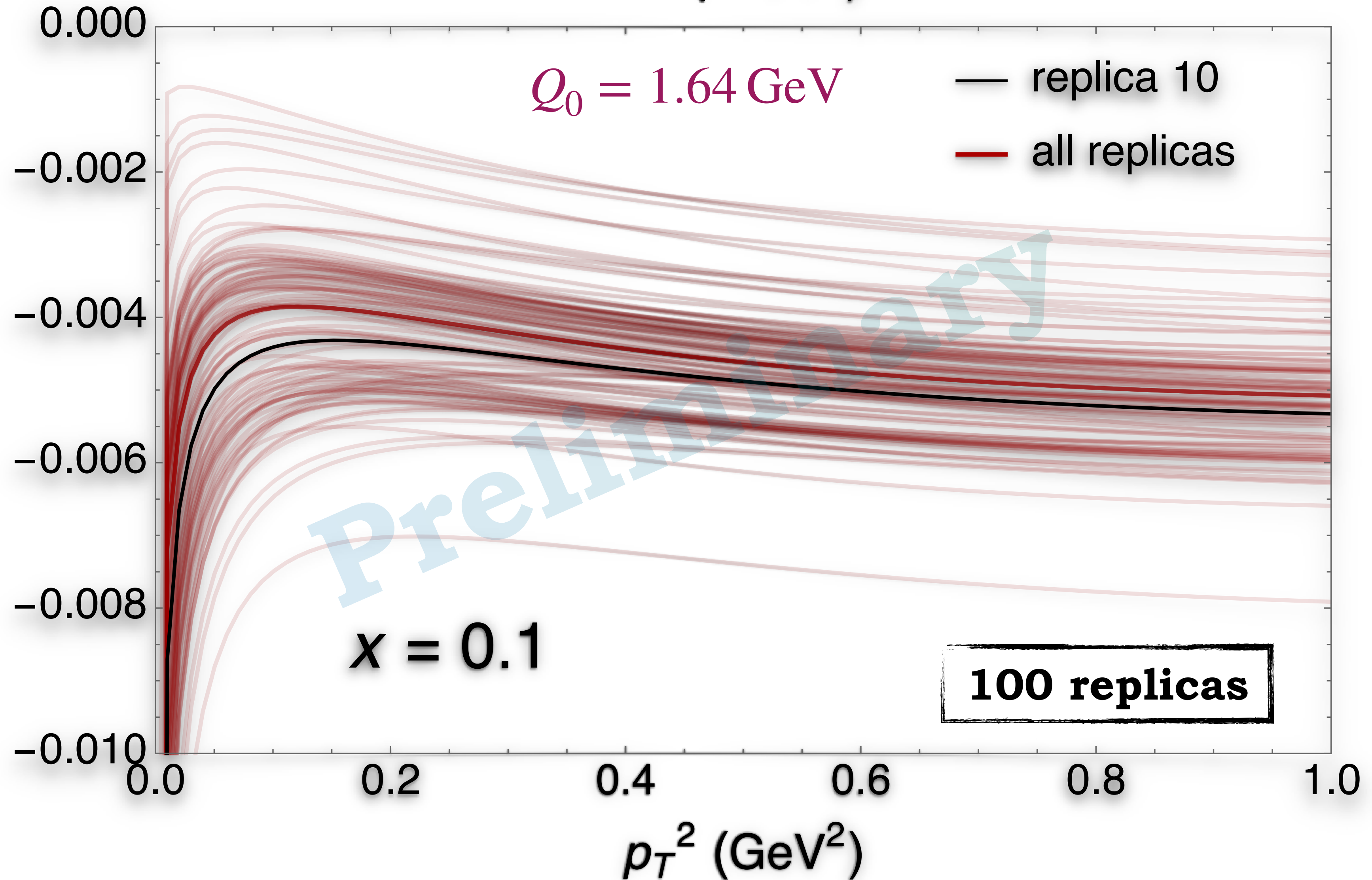
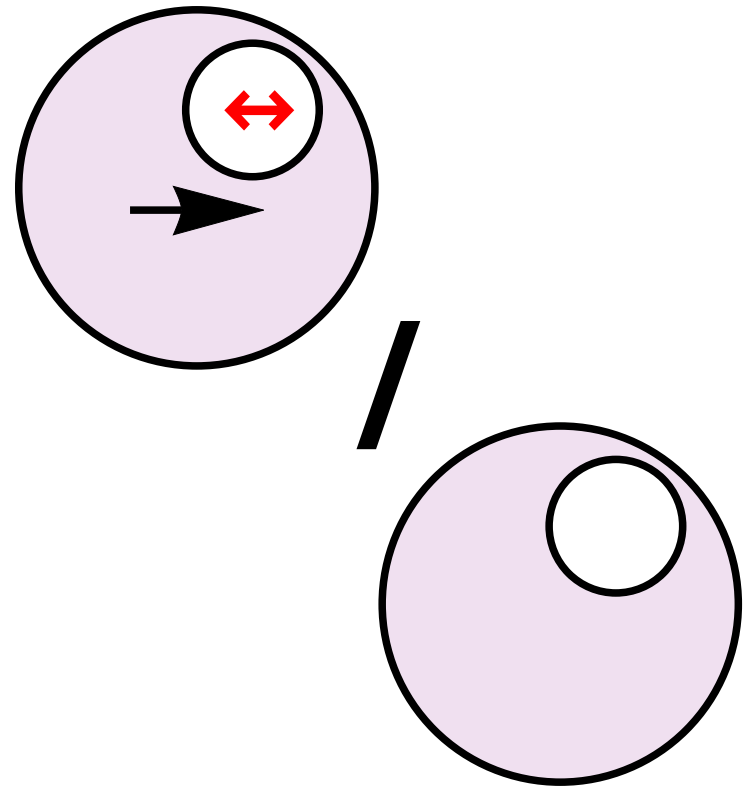
# *f*-type Sivers / unpol.

$$\frac{\frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



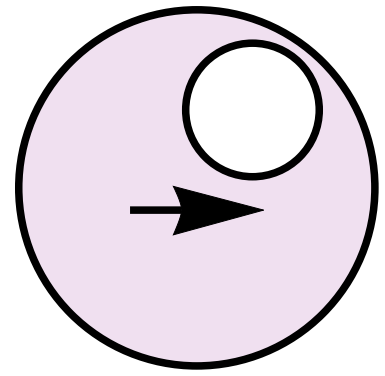
# $f$ -type linearity / unpol.

$$\frac{\frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$

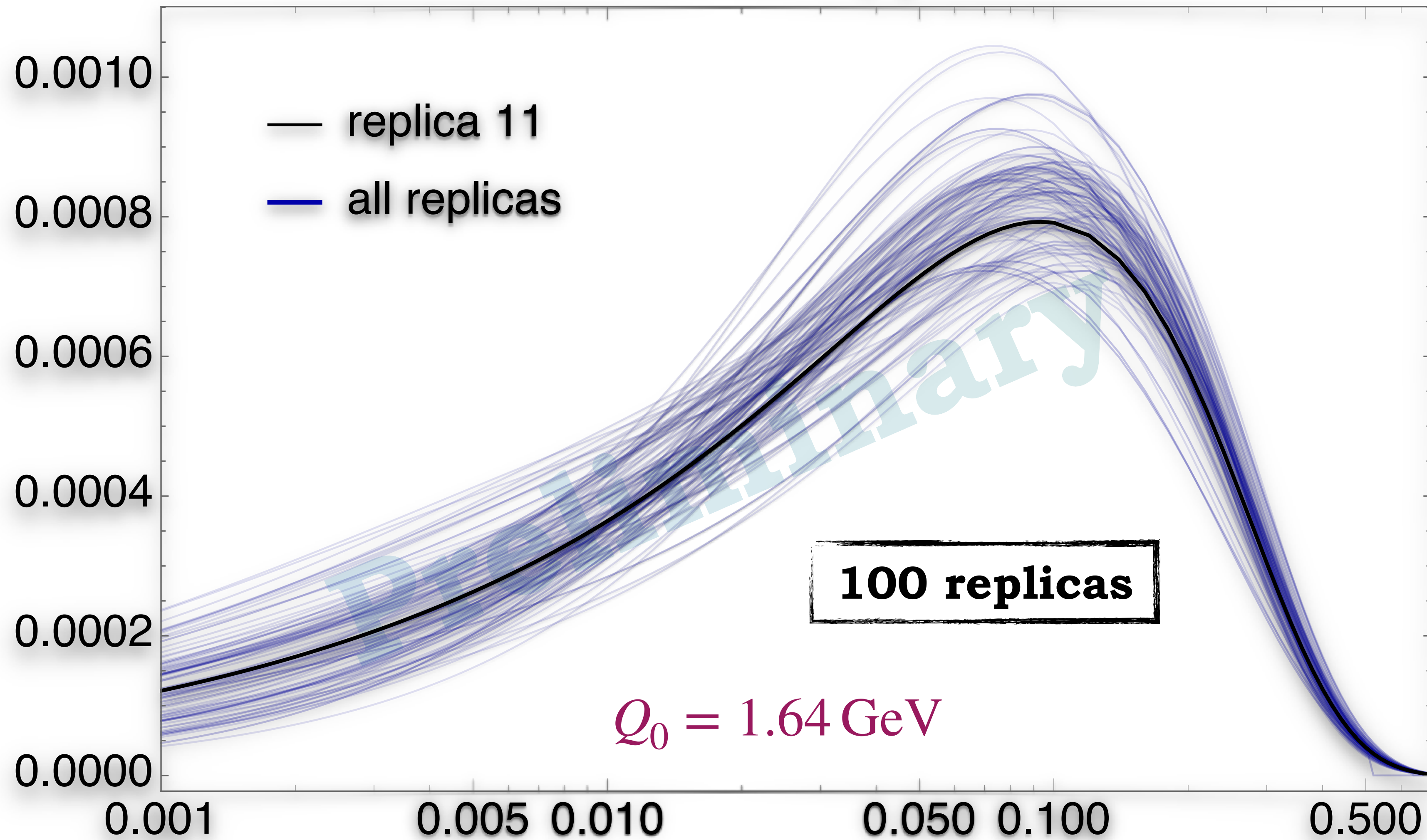




# $f$ -type Qiu-Sterman twist-3 gluon PDF



$$xf_{1T}^{\perp(f)}(x)$$



$$f_{1T}^{\perp(f)}(x) = \int d^2p_T \frac{p_T^2}{2M^2} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

$x$

# Checkpoints and further steps

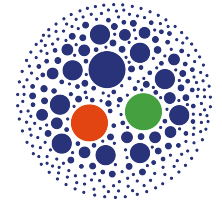
- ☑ Systematic calculation of all twist-2  $T$ -even gluon TMDs
- ☑ Spectral mass to catch small- and large- $x$  effects
- ☑ **Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**

# Checkpoints and further steps

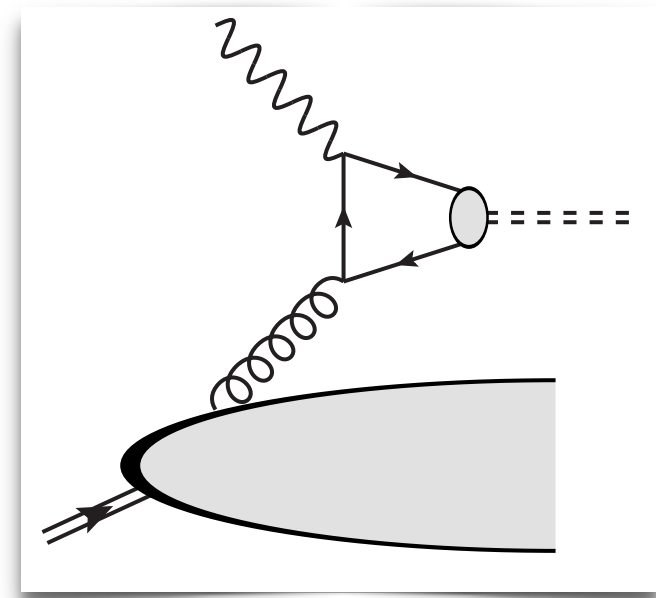
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- Spectral mass to catch small- and large- $x$  effects
- Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**
- Twist-2  $T$ -odd gluon TMDs (**Sivers**, etc.) in progress!
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Evolution: extension to quark TMDs in the same framework
- Explorative studies on gauge-link sensitivity and factorization
- Studies on GPD and small- $x$  UGD sectors

**...gluon TMDs  
and quarkonia**

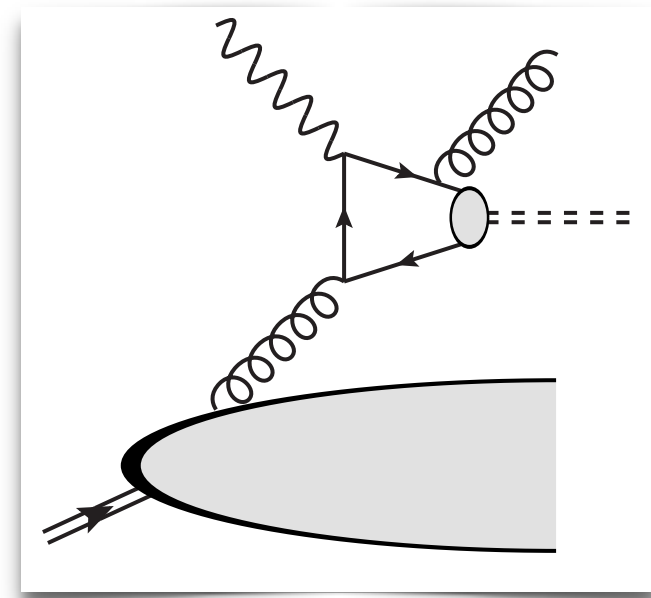
# Hadronic structure and quarkonia



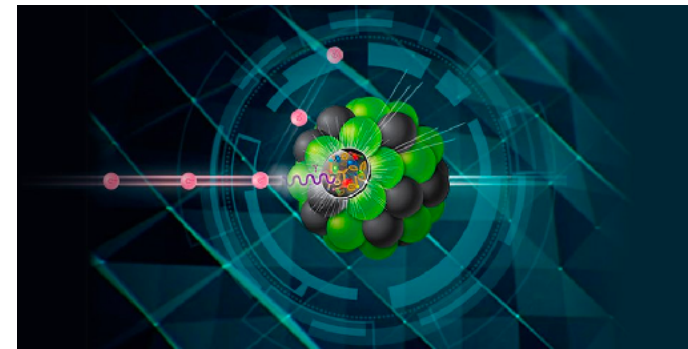
## SIDIS



CO



CO + CS

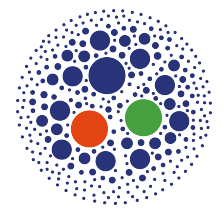


EIC Yellow Report Document  
[\(EICUG website\)](#)

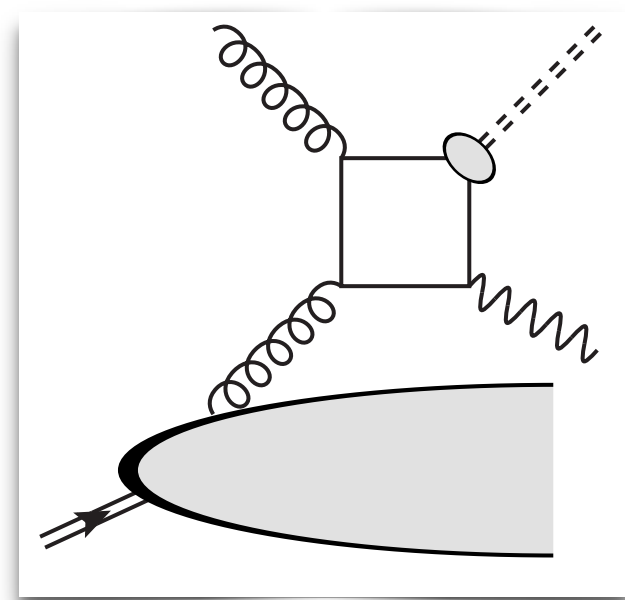
[\[EICUG \[arXiv:2103.05419\]\]](#)

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

[\[NICA Collaboration \[arXiv:2011.15005\]\]](#)



## Hadroproduction



CO + CS

Prospects for quarkonium studies at the high-luminosity LHC  
[\(QAT 2021 Workshop\)](#)

[\[Quarkonia As Tools Collaboration \[arXiv:2012.14161\]\]](#)

# $\eta_{b,c}$ production in unpolarized $pp$ collisions

## TMD phenomenology: from JLab to the LHC

Andrea Signori

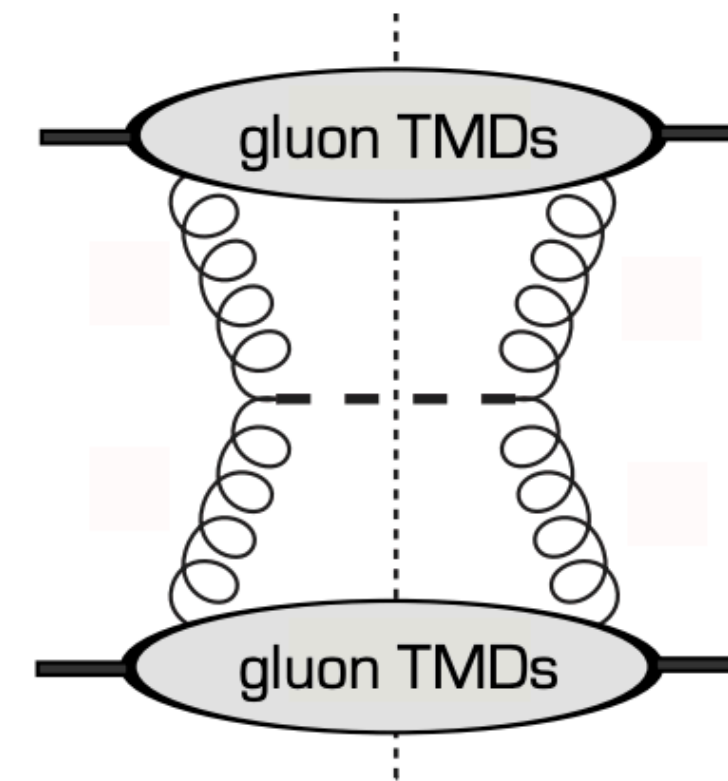
Spatial and momentum  
tomography  
of hadrons and nuclei

INT 17-3  
Sept 25 2017

### NRQCD

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$

## gluon TMD PDFs



pseudoscalar quarkonium production:

$$p p \rightarrow \eta_b X \quad M = 9.39 \text{ GeV}$$

$$p p \rightarrow \eta_c X \quad M = 2.98 \text{ GeV}$$

(see also talk by C. Pisano week 4)

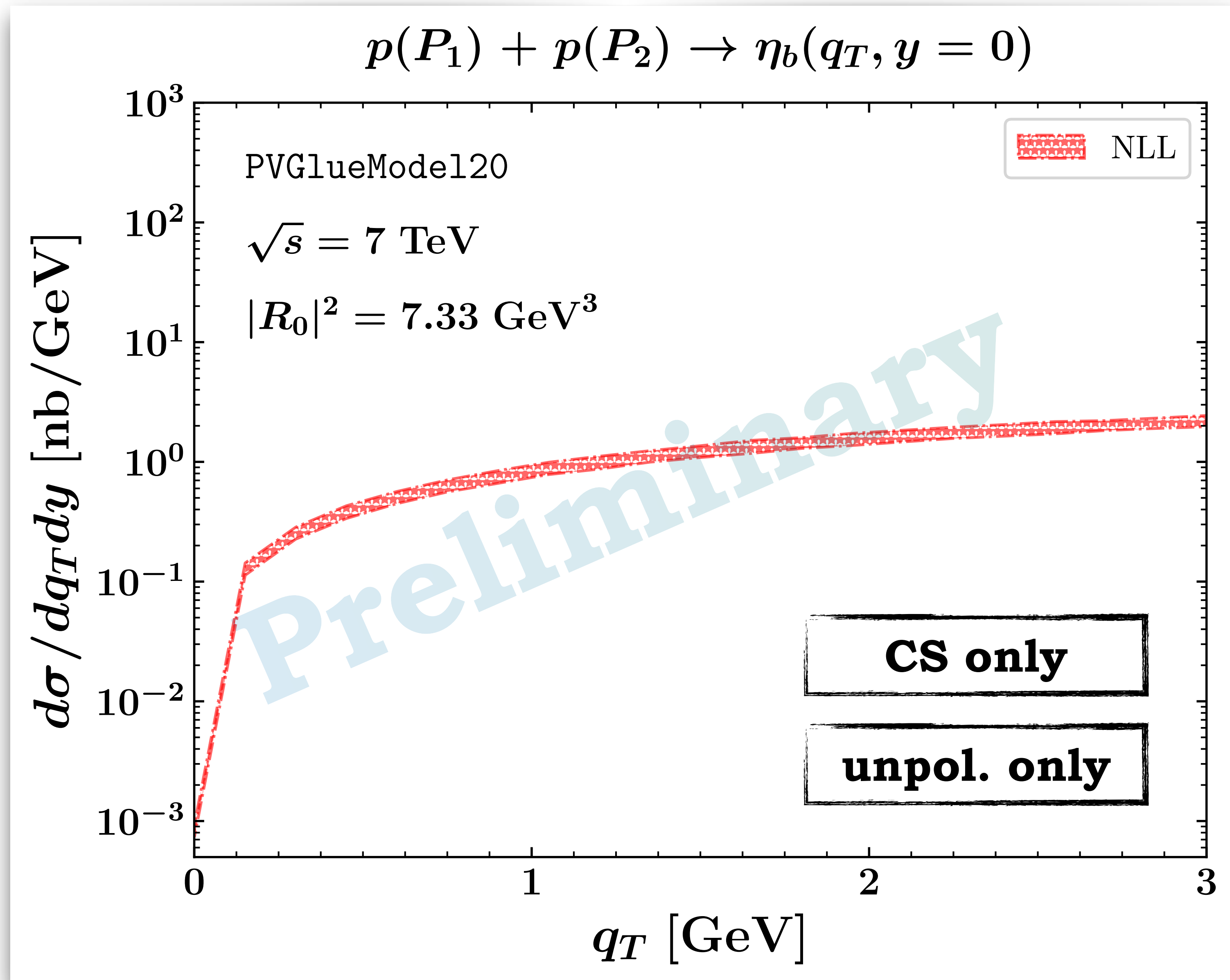
$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

$$\sim \mathcal{C} \left[ \begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[ \begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

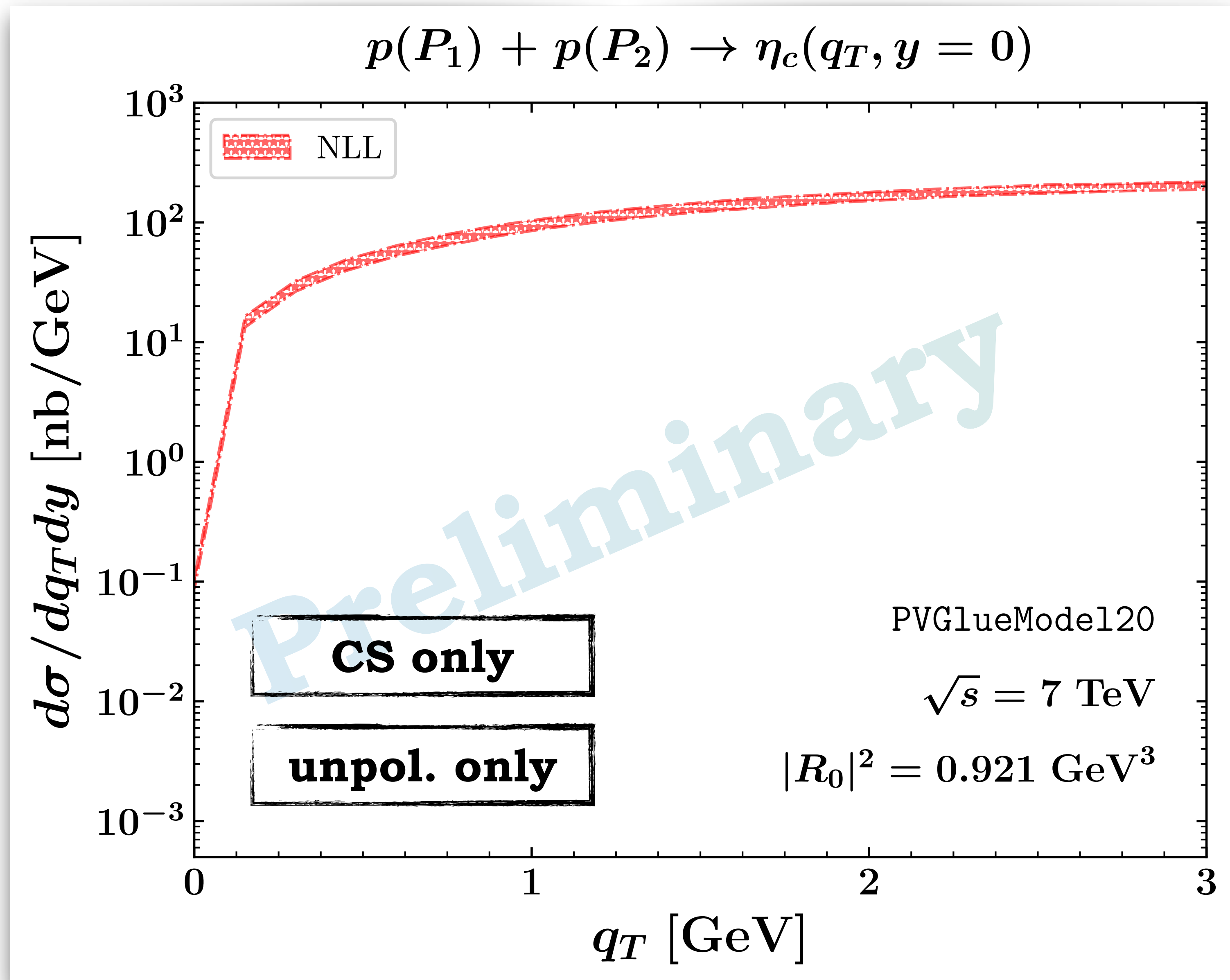
unpolarized gluons                      lin. polarized gluons

unpolarized cross section  
at low transverse momentum  
for (pseudo)scalar state

# $\eta_b$ production @ 7TeV LHC



# $\eta_c$ production @ 7TeV LHC

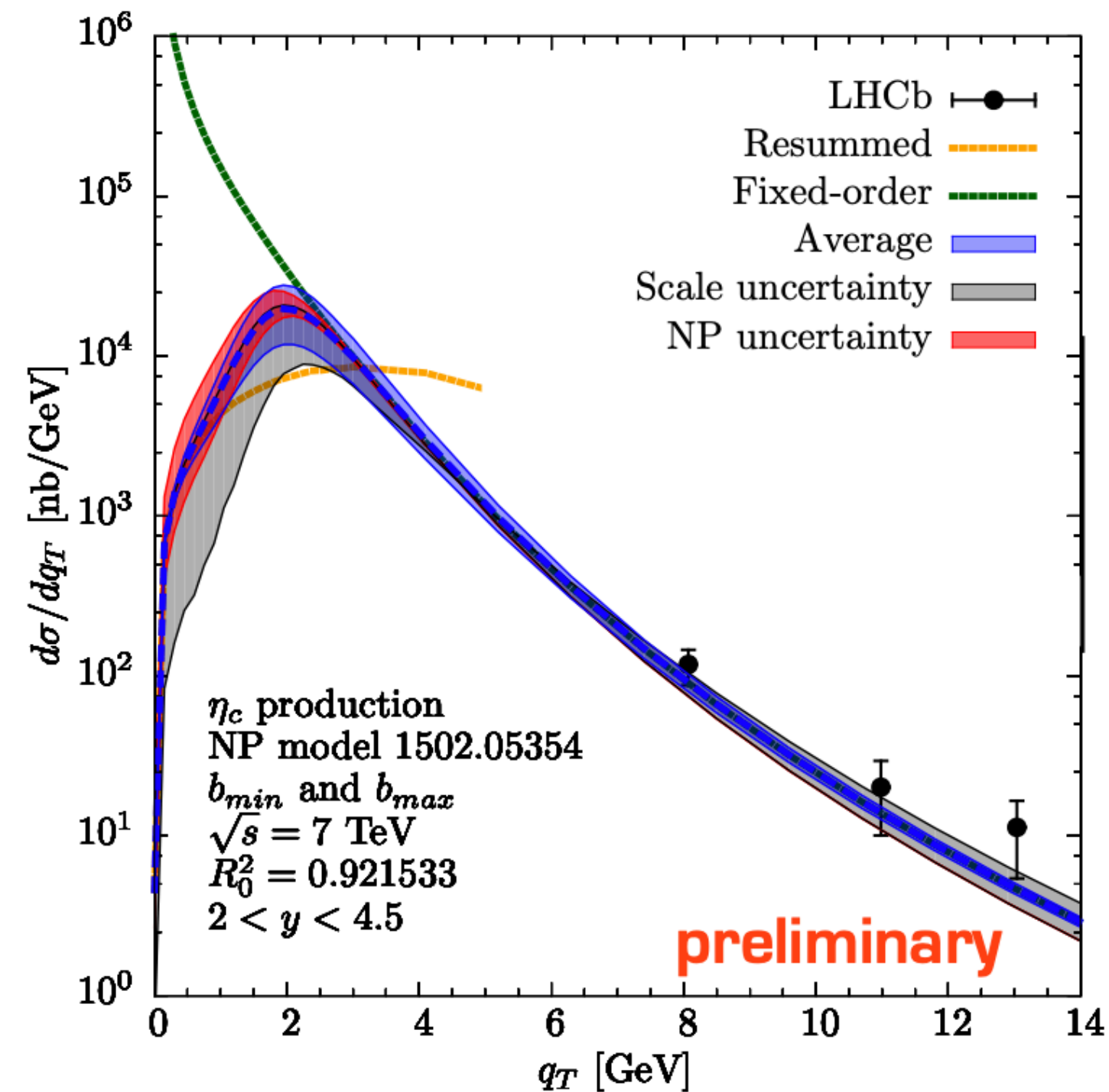




# $\eta_c$ production @ 7TeV LHC

## $\eta_c$ production at LHC

full transverse momentum spectrum:  
low  $q_T$  matched with high  $q_T$  region



**blue band:** uncertainty from matching

**grey band:** scale uncertainty

**red band:** nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[ \frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5 \text{ GeV}^2$ , var. 50%, envelope

both for unpolarized and  
linearly polarized distributions

**the formalism is in good shape!**  
we need the data at low  $q_T$

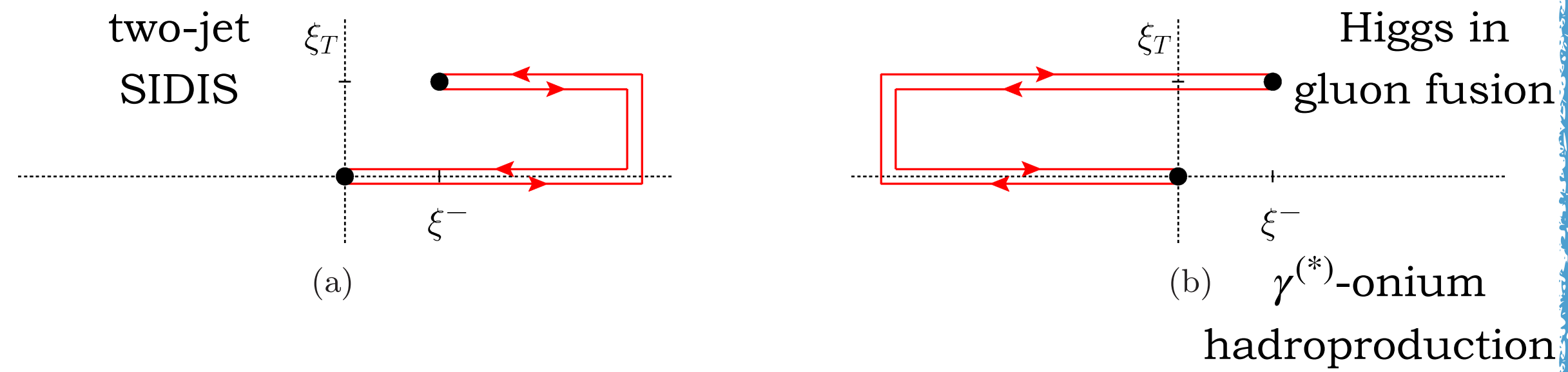
Jefferson Lab

**Backup  
slides**

# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

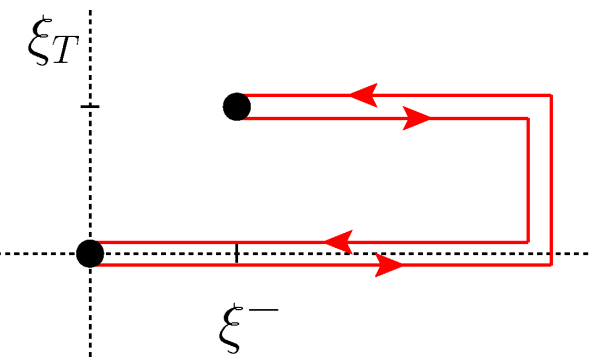


# Accessing WW and DP gluon TMDs

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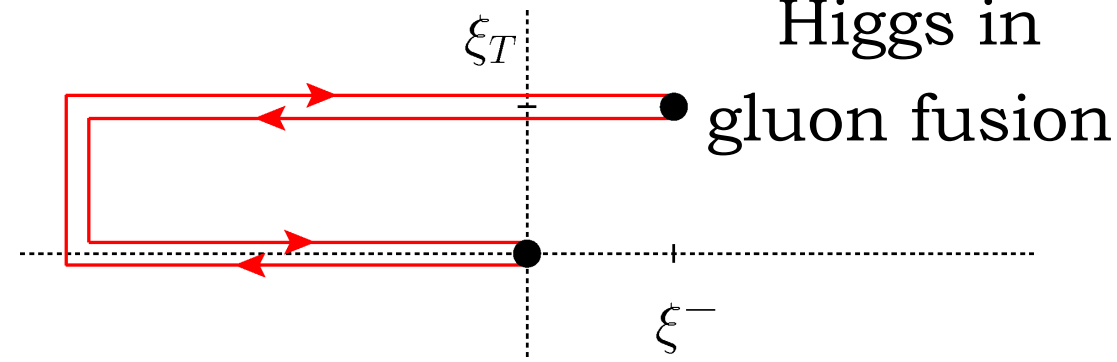
(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

Higgs in  
gluon fusion

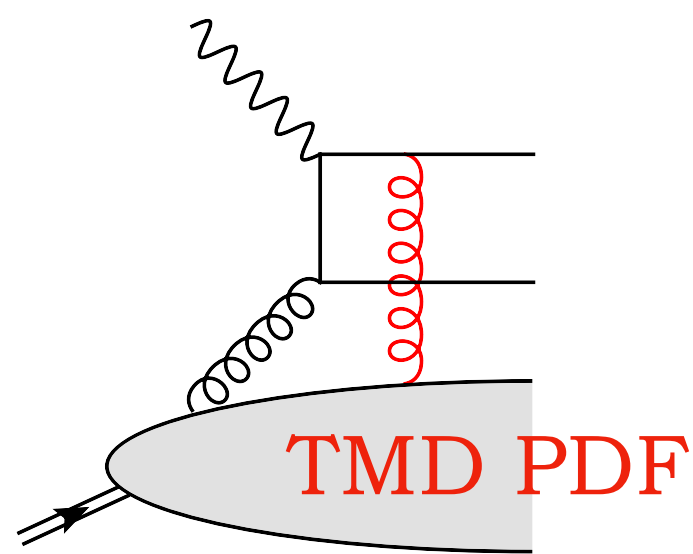


(b)

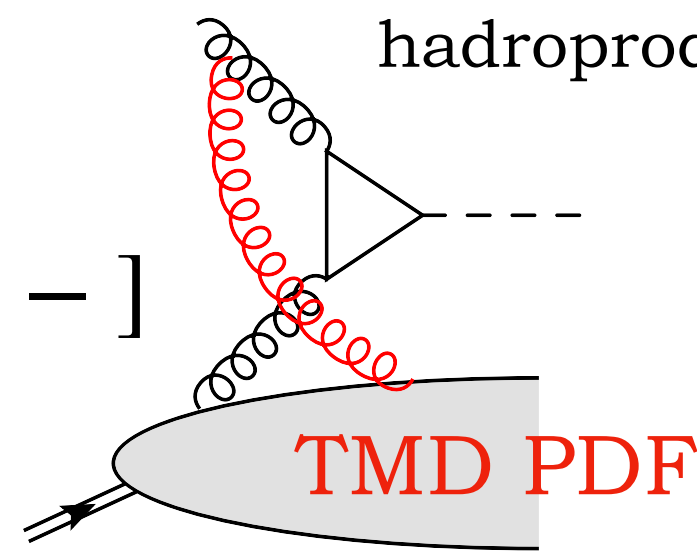
$\gamma^{(*)}$ -onium

hadroproduction

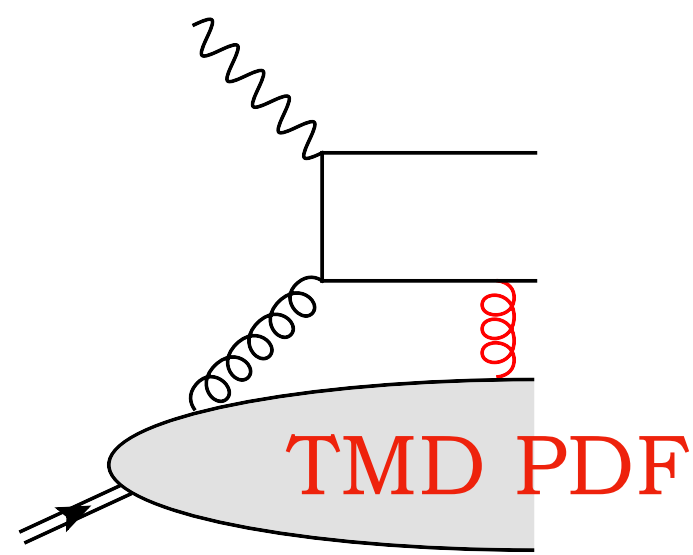
[+]



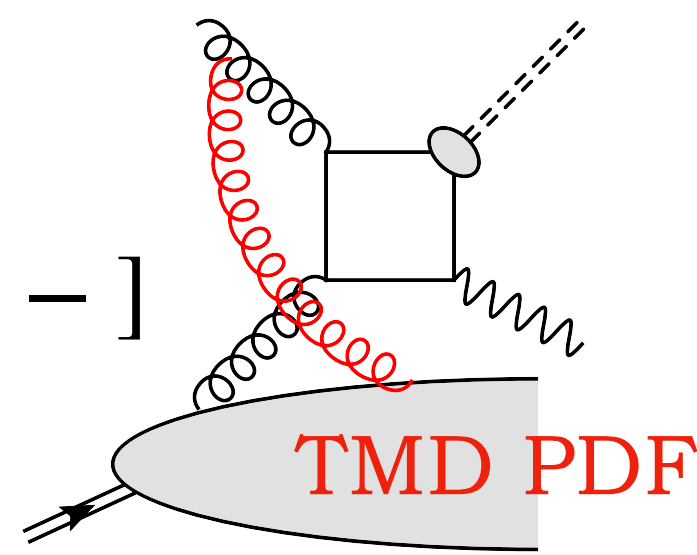
[-, -]



[+]



[-, -]

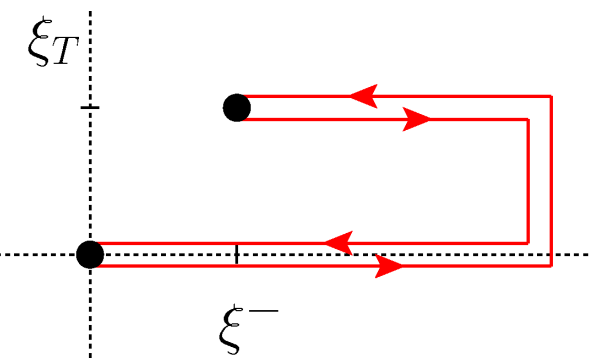


# Accessing WW and DP gluon TMDs

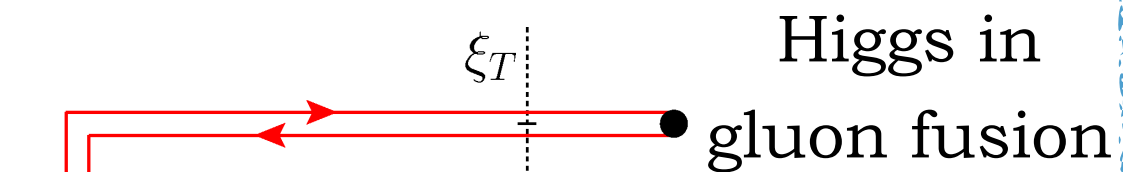
## Weizsäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

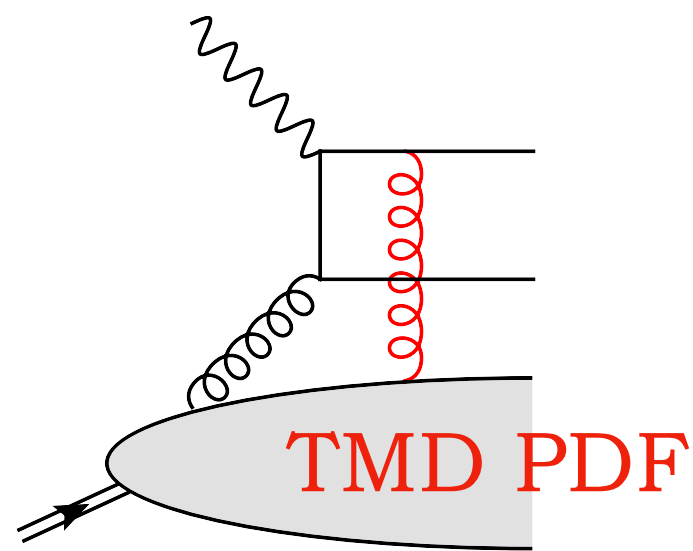


(b)

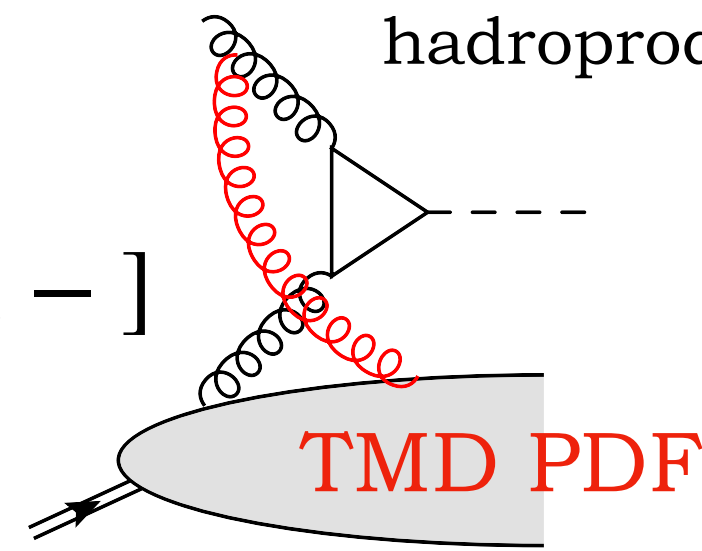
Higgs in  
gluon fusion

$\gamma^{(*)}$ -onium  
hadroproduction

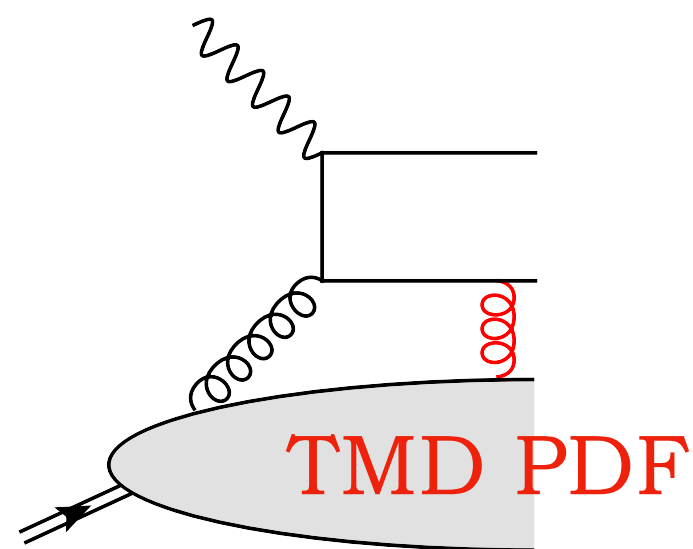
[+]



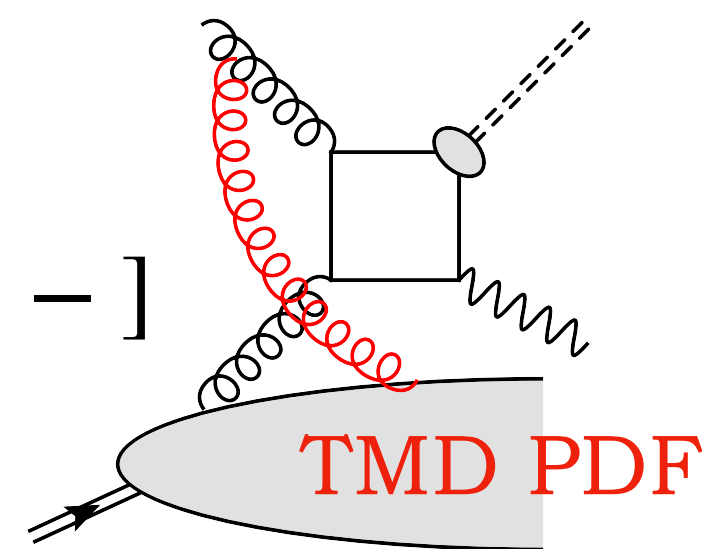
[-, -]



+] ]

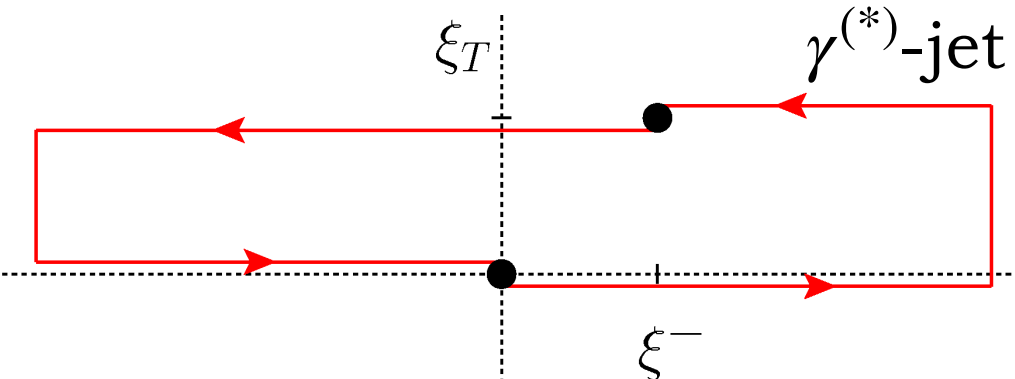


[-, -]



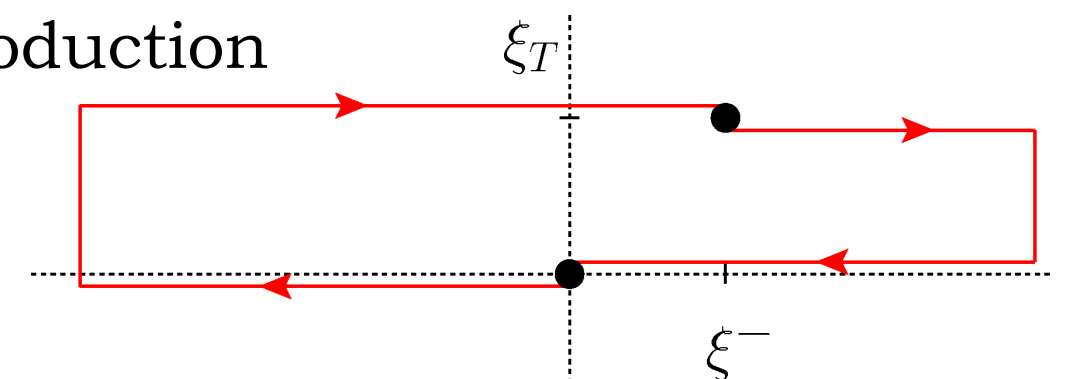
## Dipole (DP)

(c)  $[+, -]$  or (d)  $[-, +]$



(c)

$\gamma^{(*)}$ -jet hadroproduction



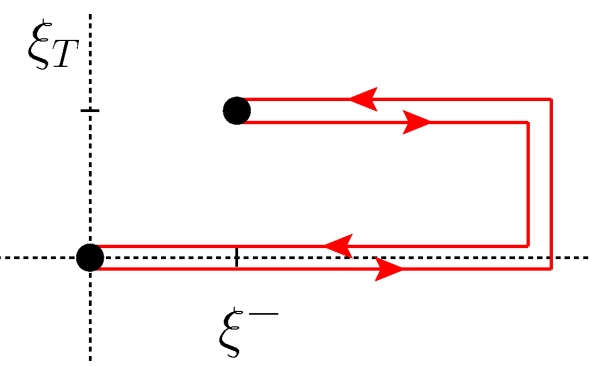
(d)

# Accessing WW and DP gluon TMDs

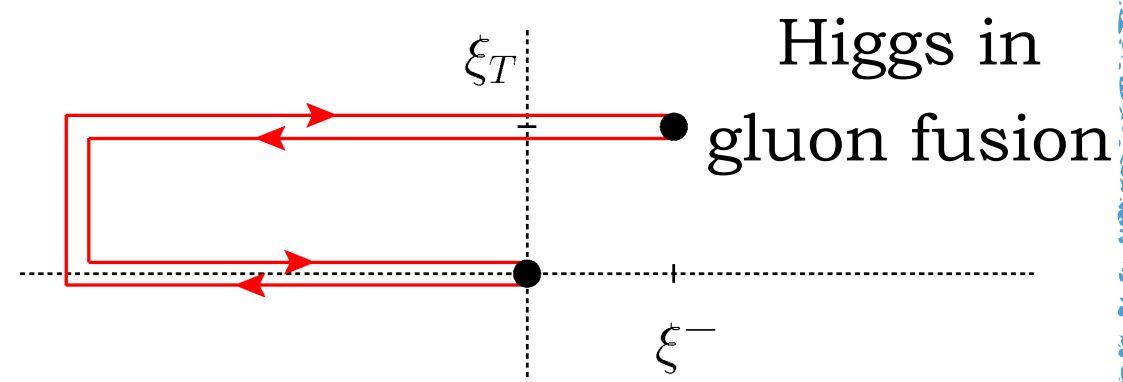
## Weizsäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

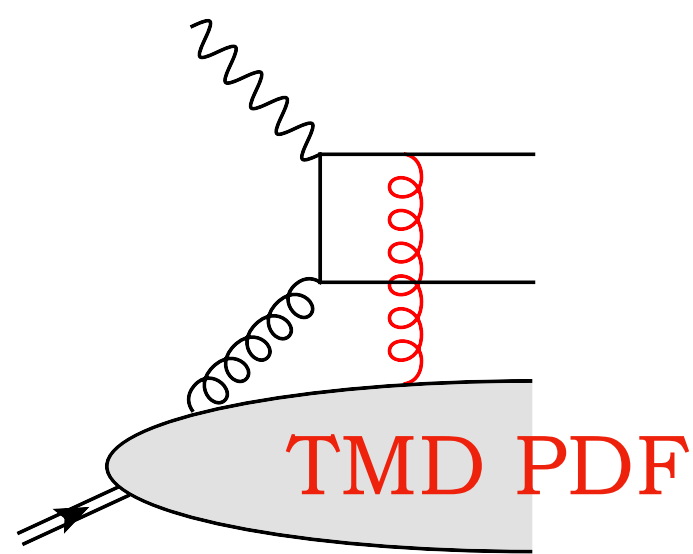


(b)

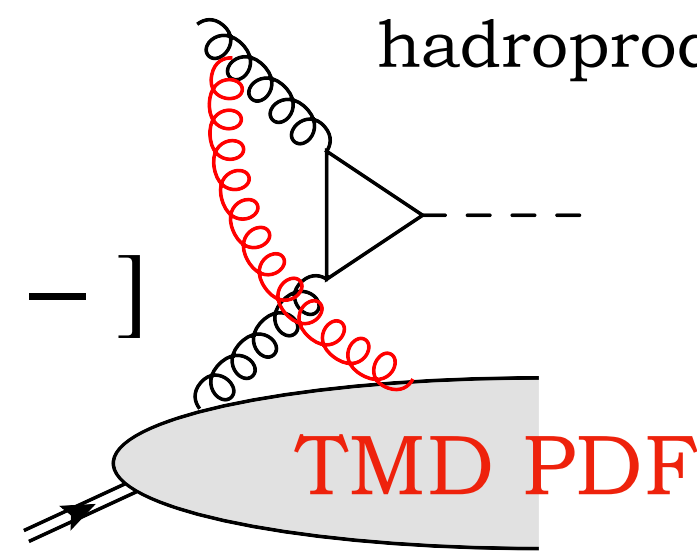
Higgs in  
gluon fusion

$\gamma^{(*)}$ -onium  
hadroproduction

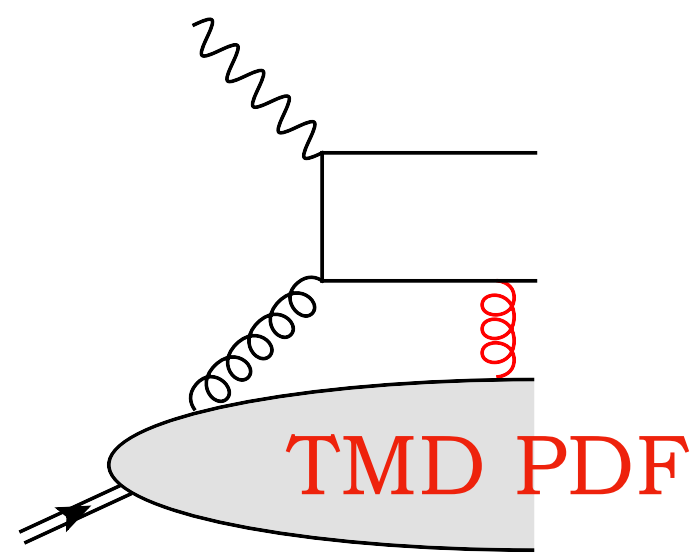
[+]



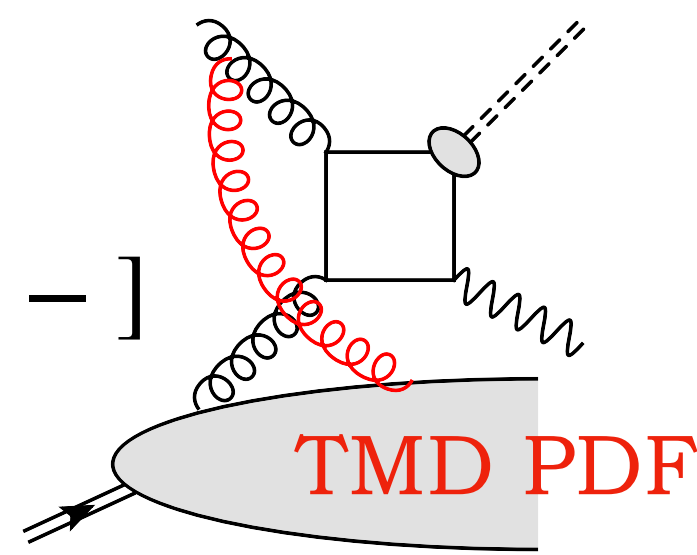
[-, -]



[+]

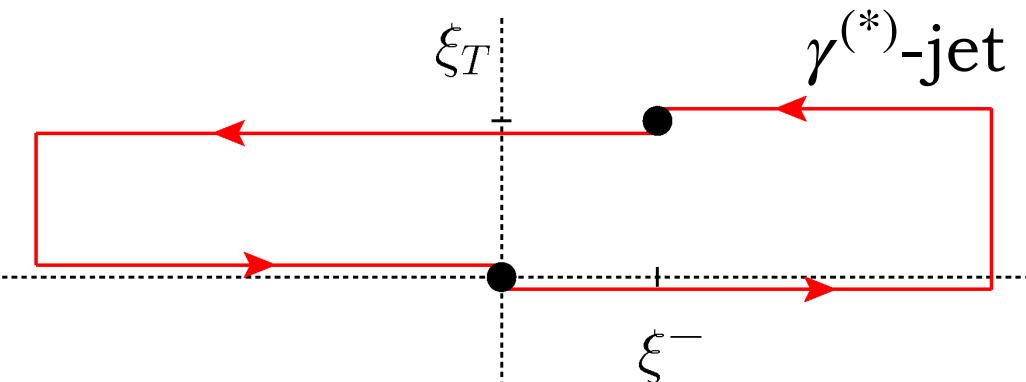


[-, -]

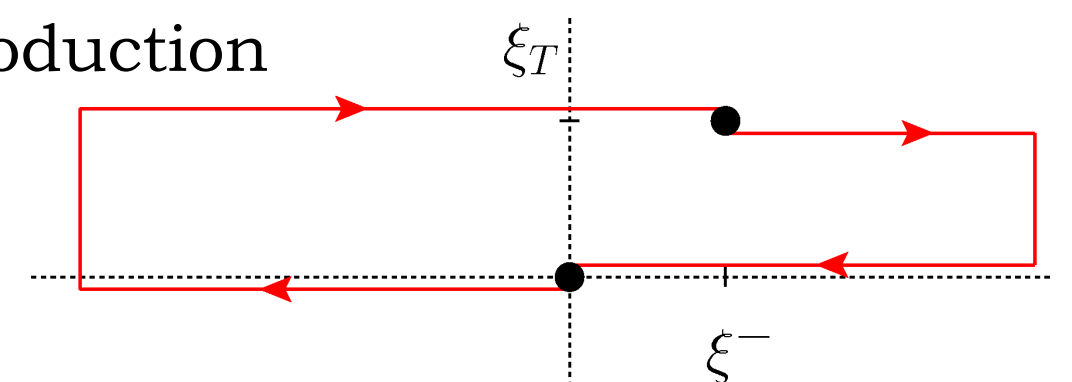


## Dipole (DP)

(c)  $[+, -]$  or (d)  $[-, +]$



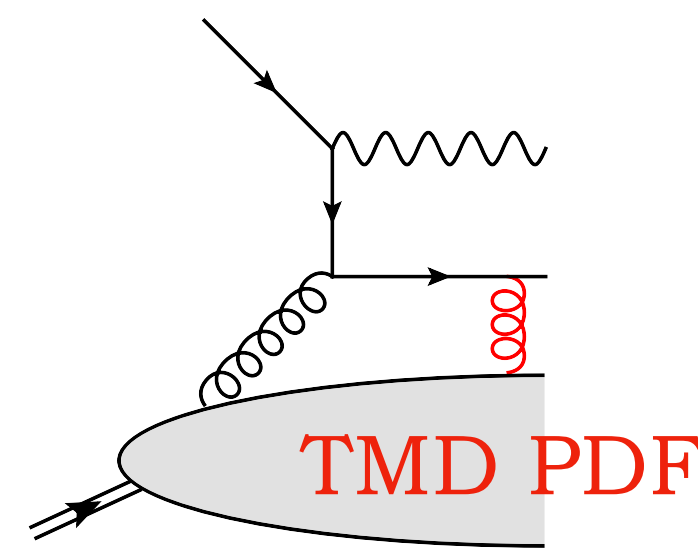
(c)



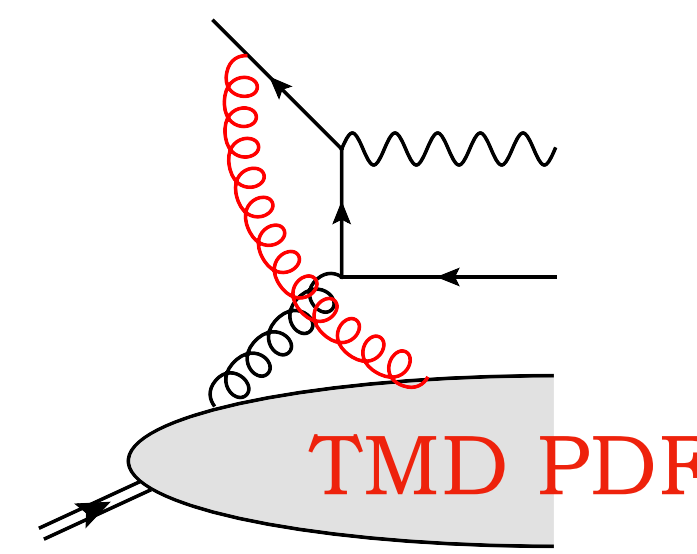
(d)

$\gamma^{(*)}$ -jet hadroproduction

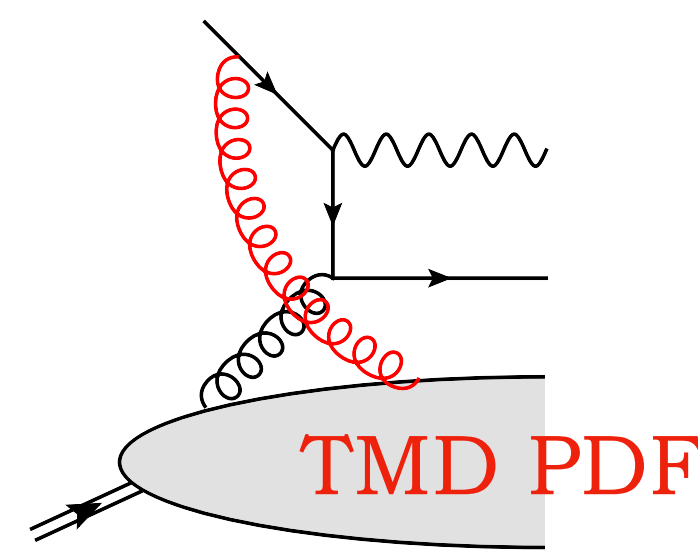
[+]



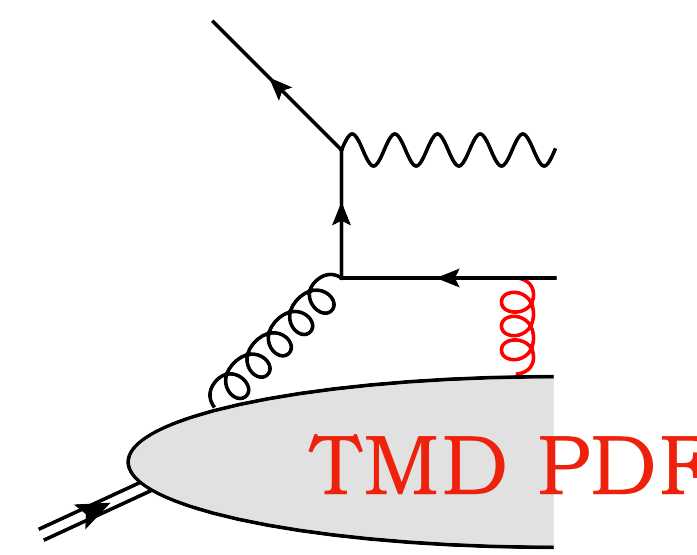
[-]



[-]



[+]



# Dihadron hadroproduction and factorization breaking

\* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

\* Assumed factorization in SCET and CGC

\* Significance of low- $x$  studies

\* Size of factorization-breaking effects small?

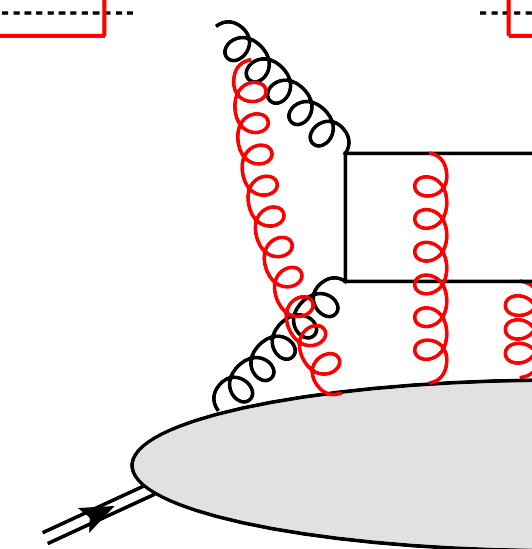
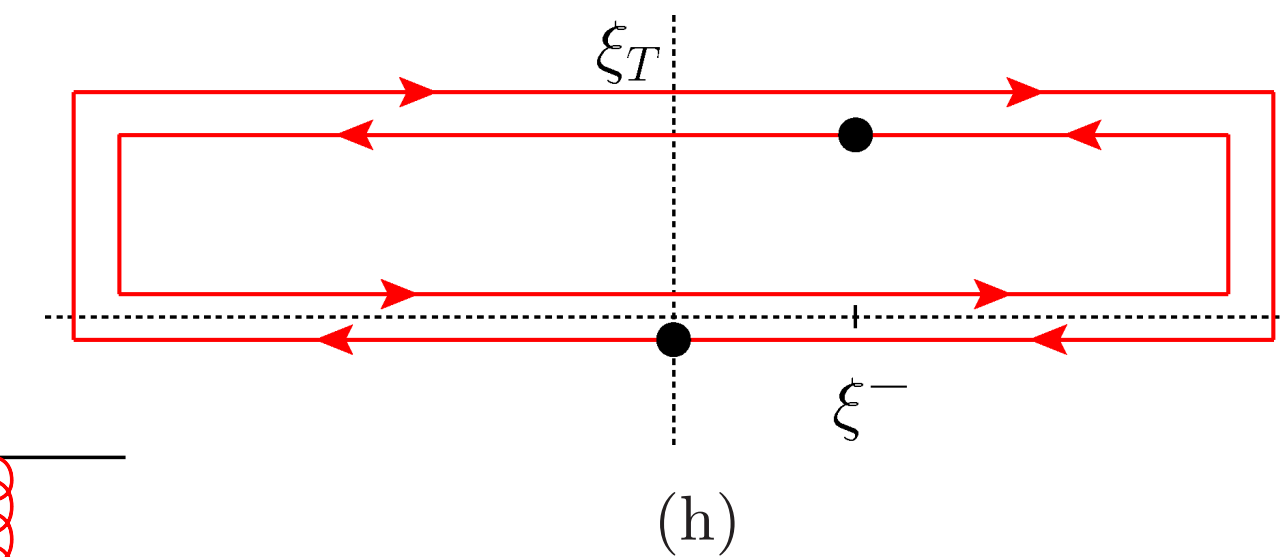
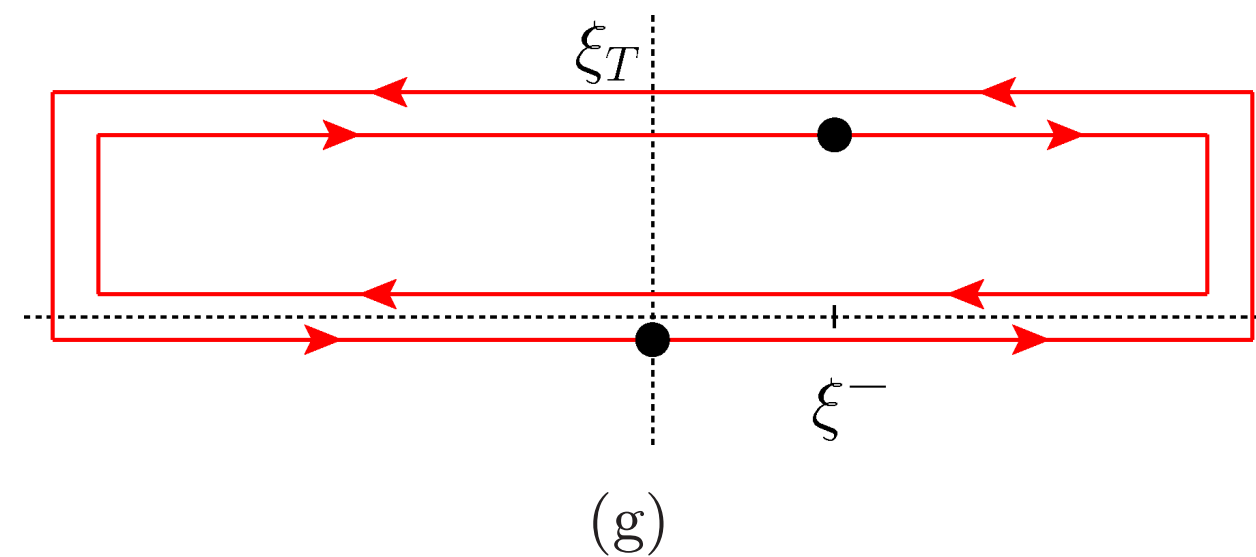
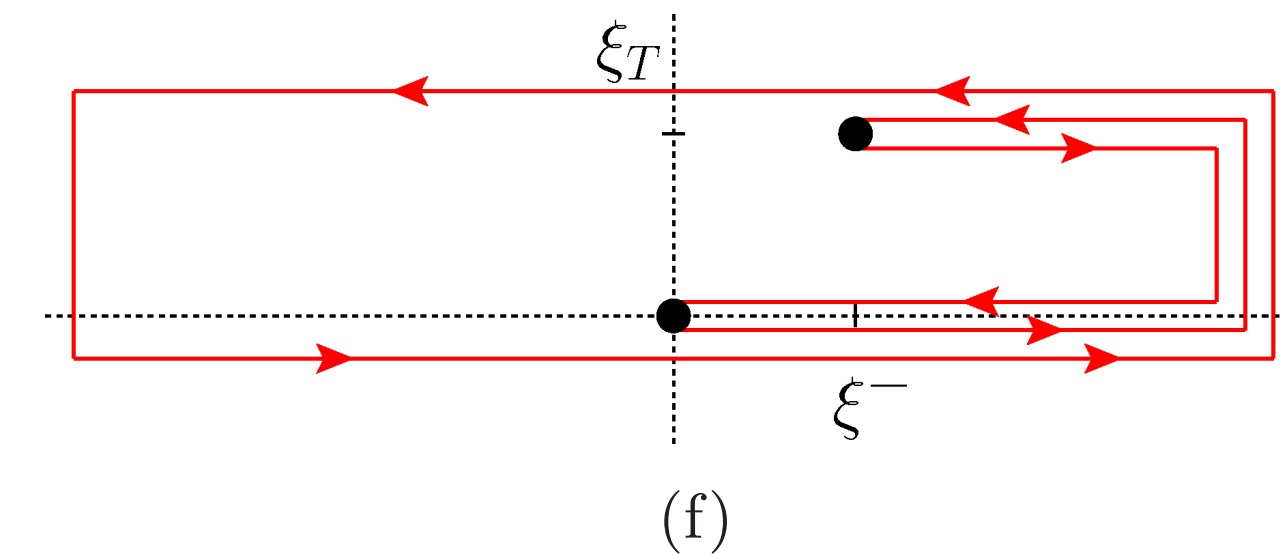
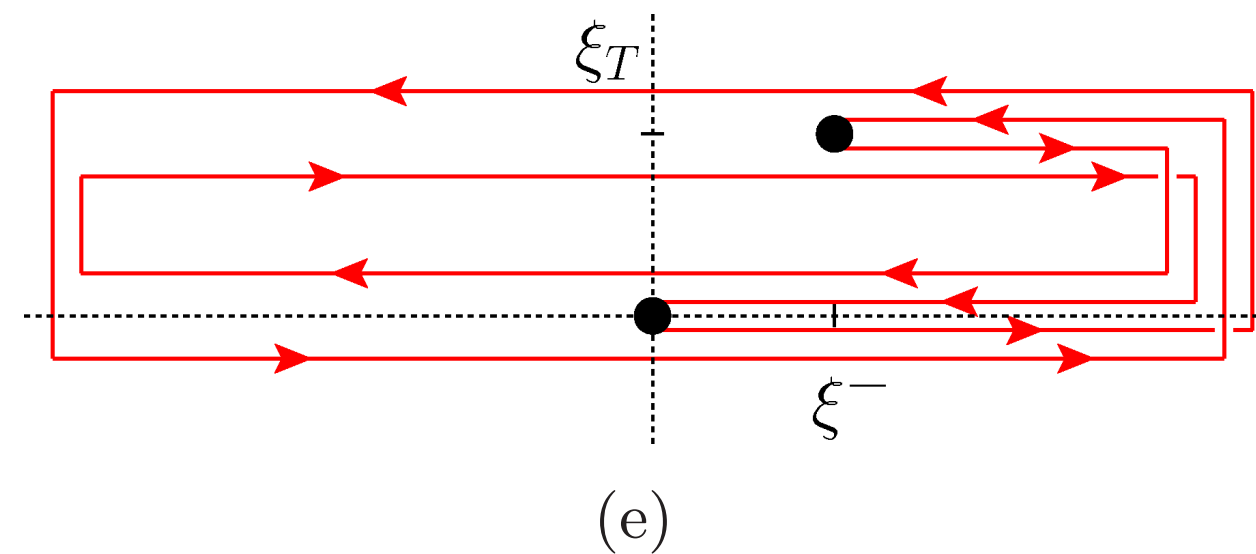
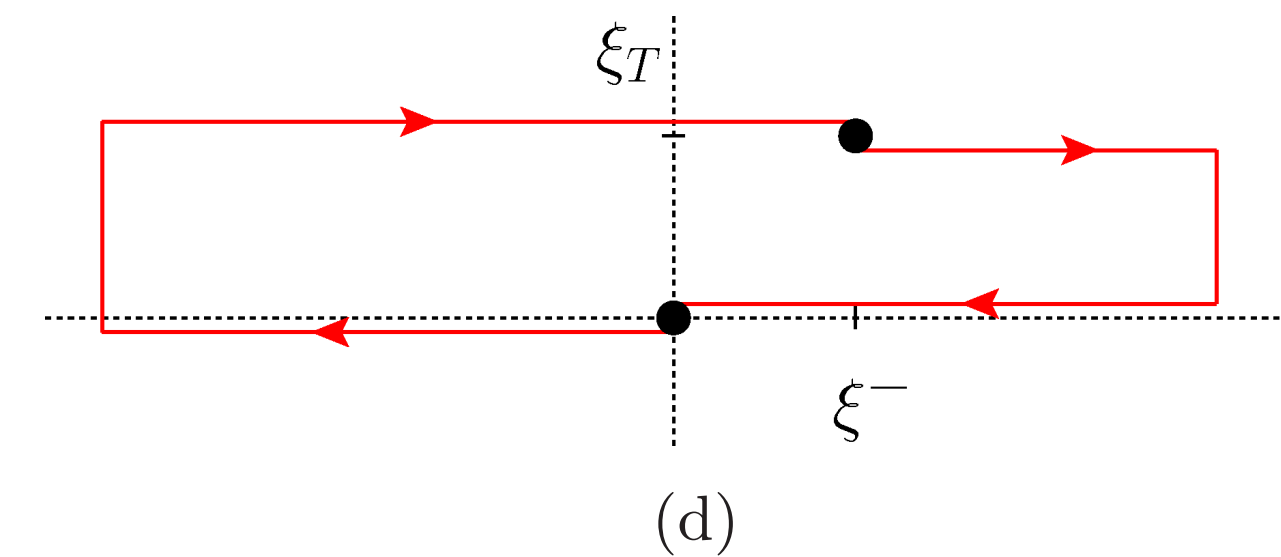
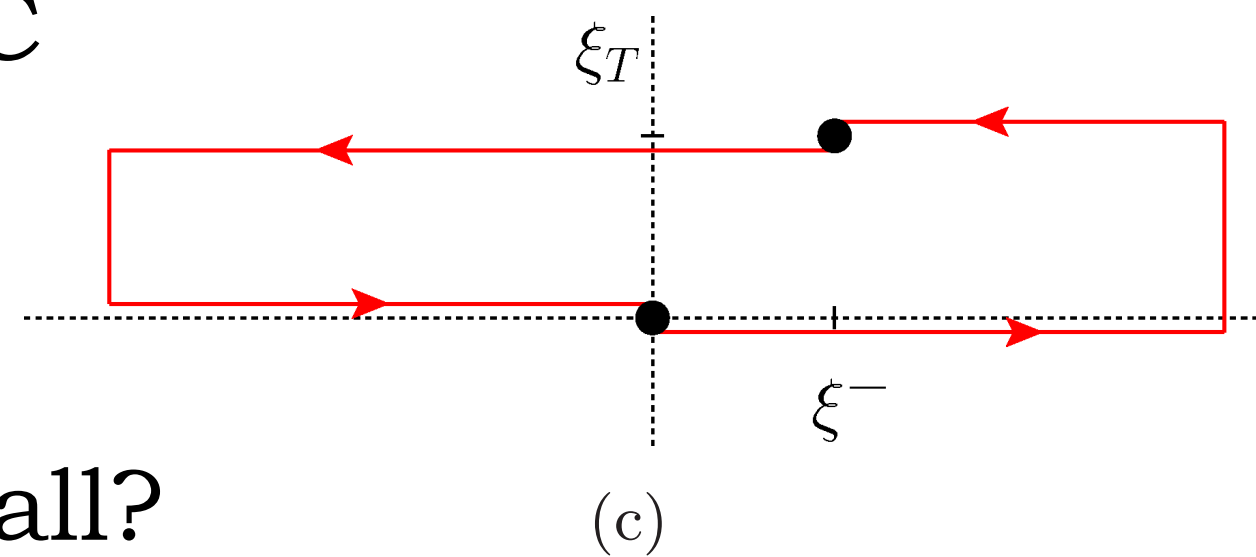
\* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

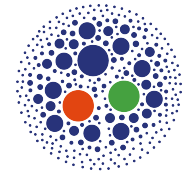
\* Appearance of new gauge **loop links**:

(e)  $[+\square, +\square]$ , (f)  $[+, +\square]$ ,

(g)  $[\square, \square]$ , and (h)  $[\square, \square]$

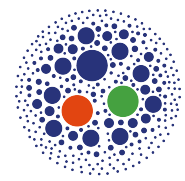
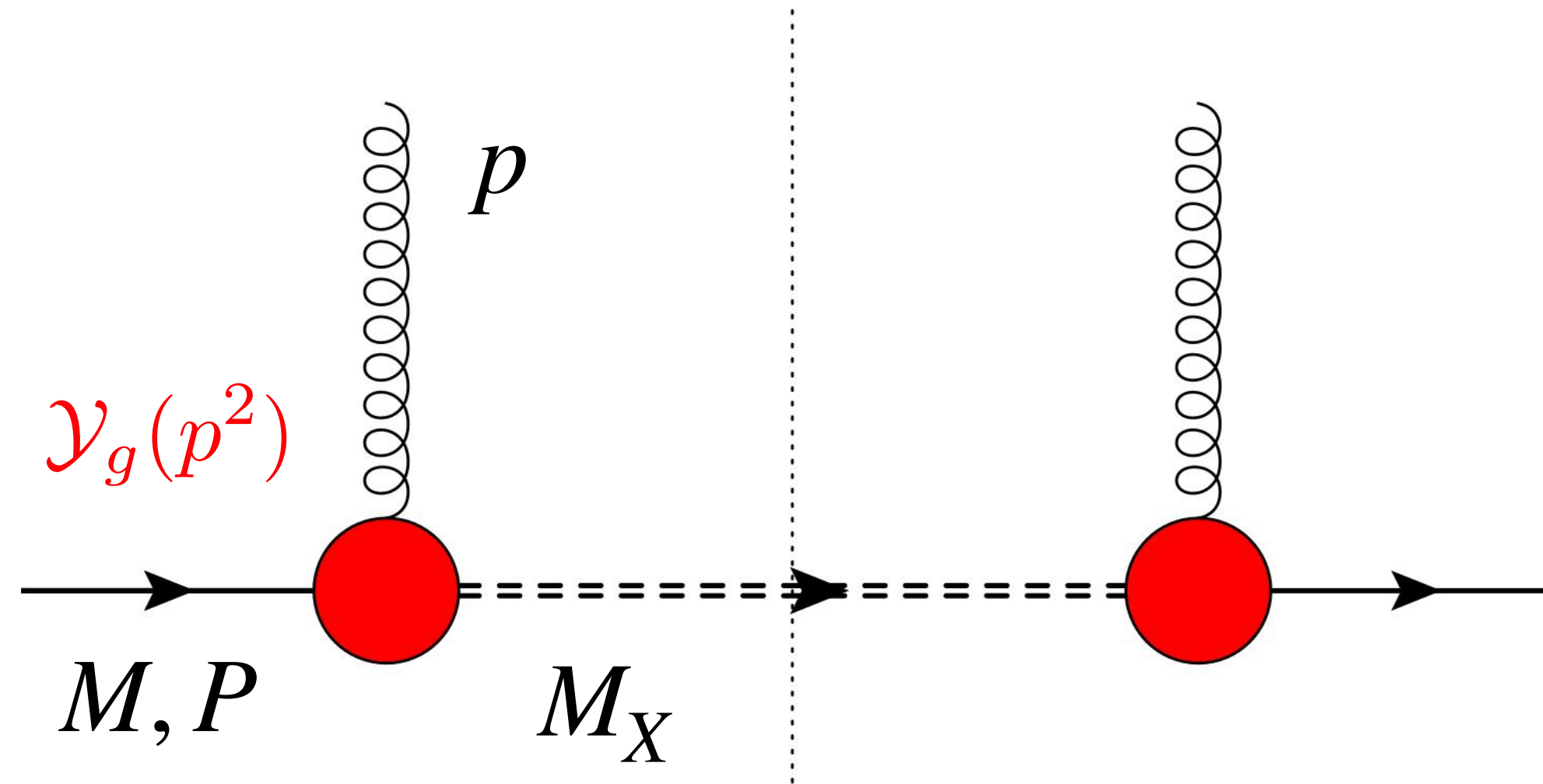


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark** spectator  
 on-shell and  
 with mass  $M_X$



## Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[ (\not{P} + M) \frac{1 + \gamma^5 \not{\not{p}}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

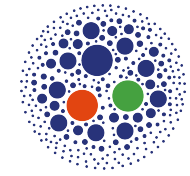
$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



mimics proton form factors  
 (conserved EM current  
 of a free nucleon)

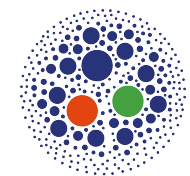


# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

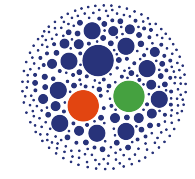


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



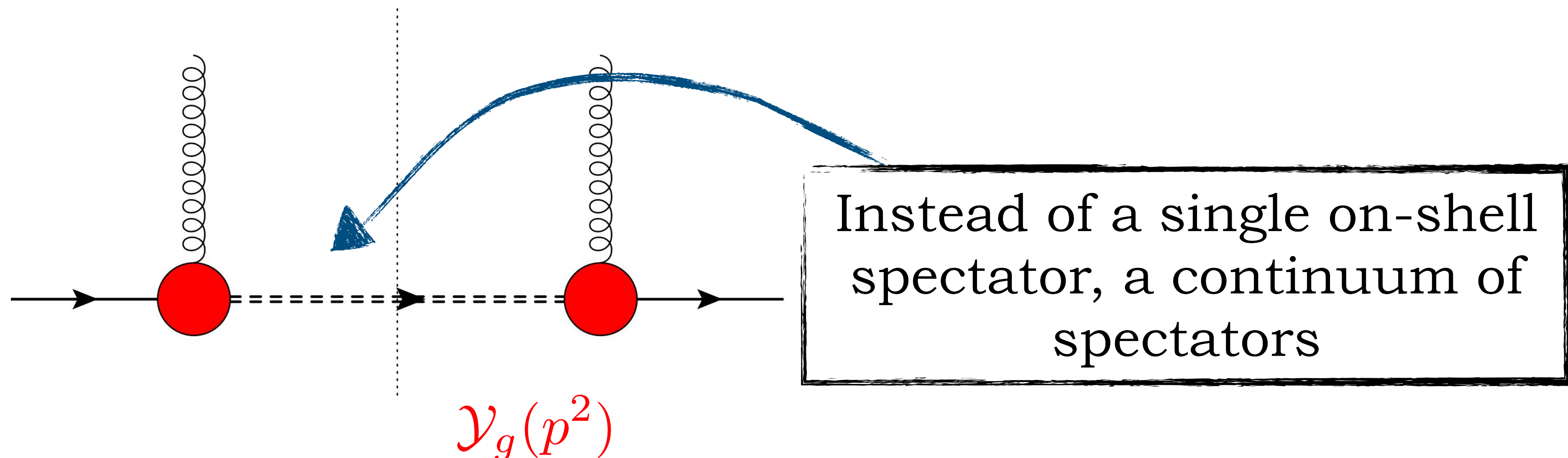
## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



# Assumptions of the model



## Spectator-system spectral-mass function

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spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left( M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

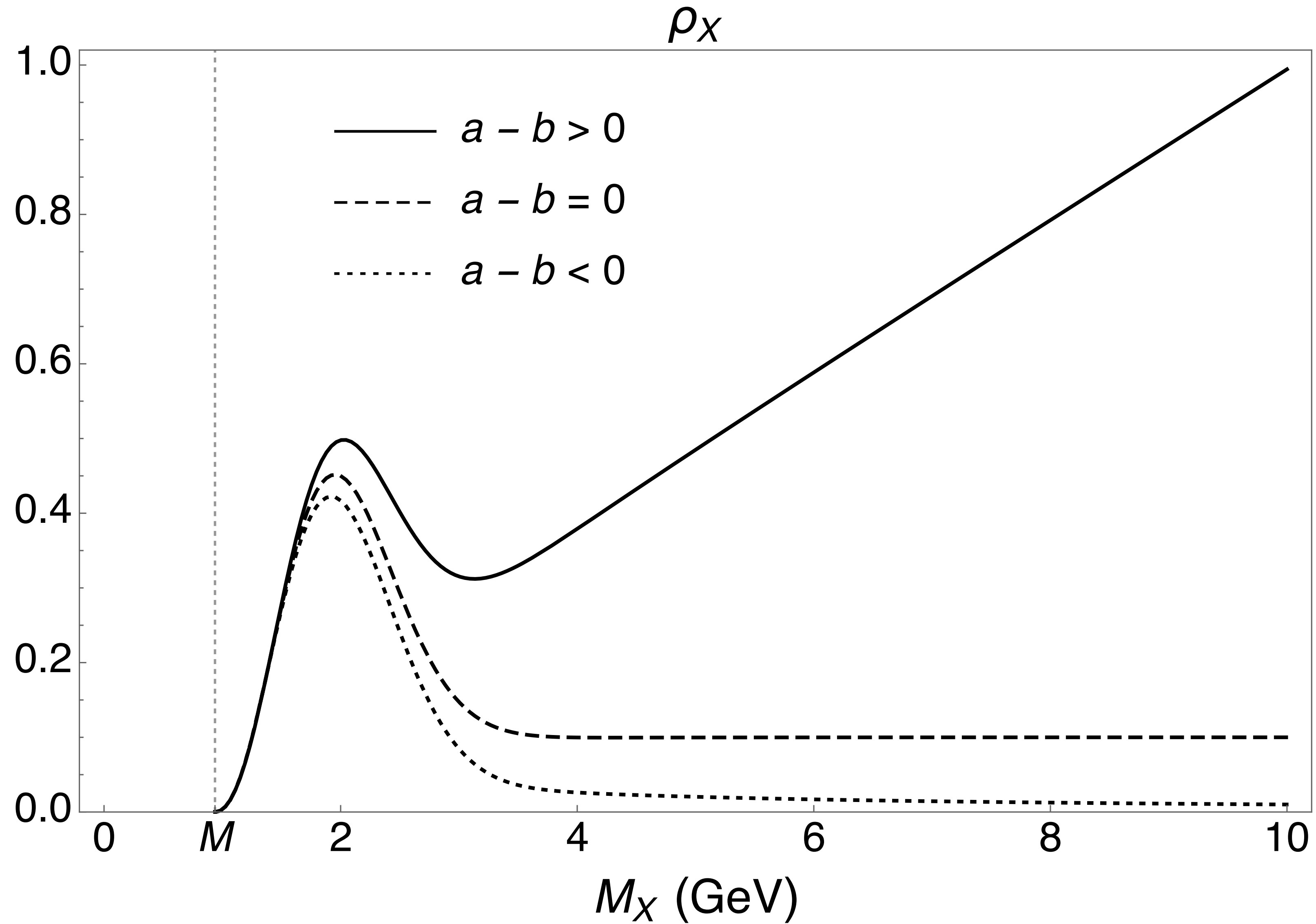
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

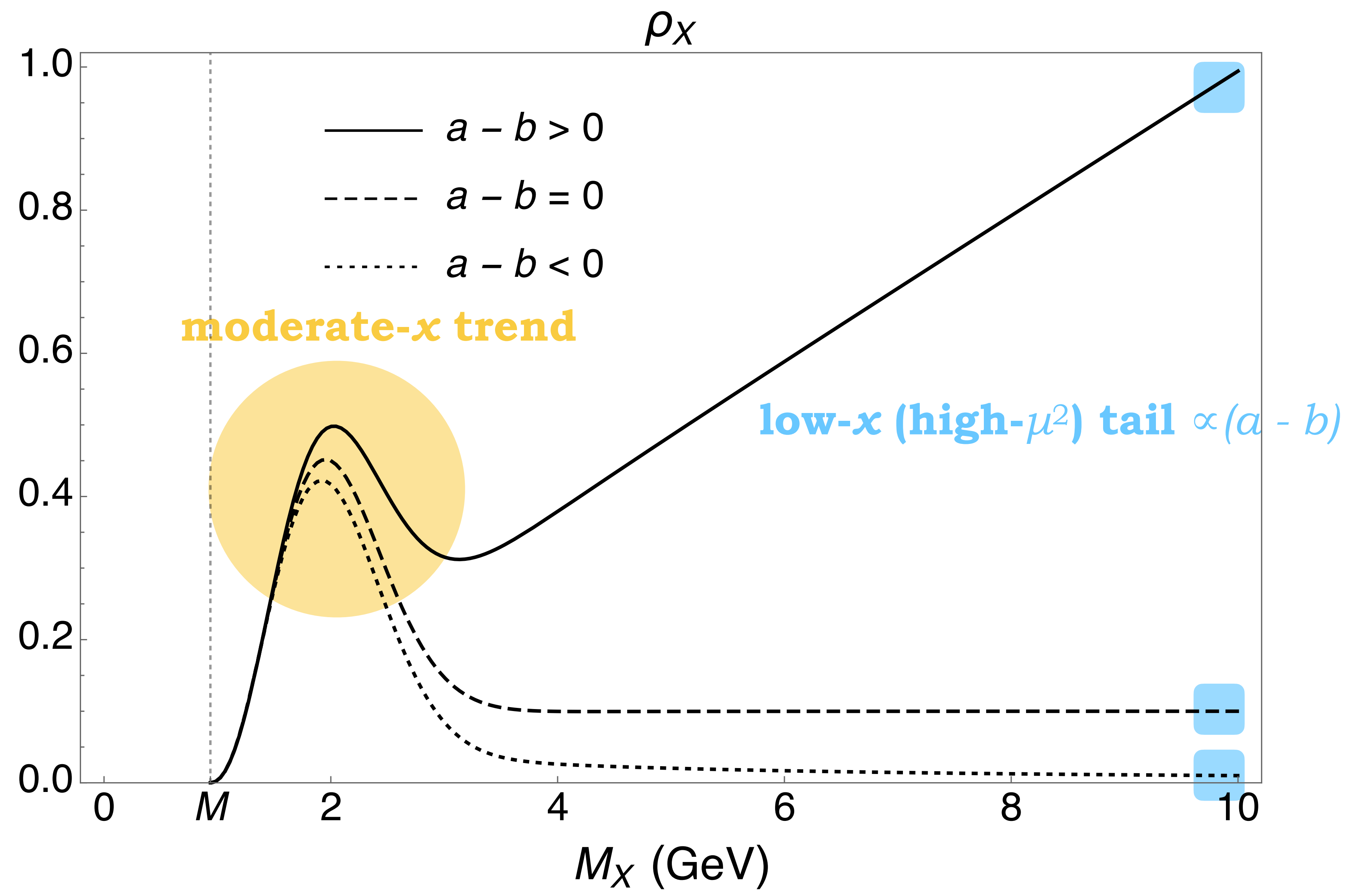
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

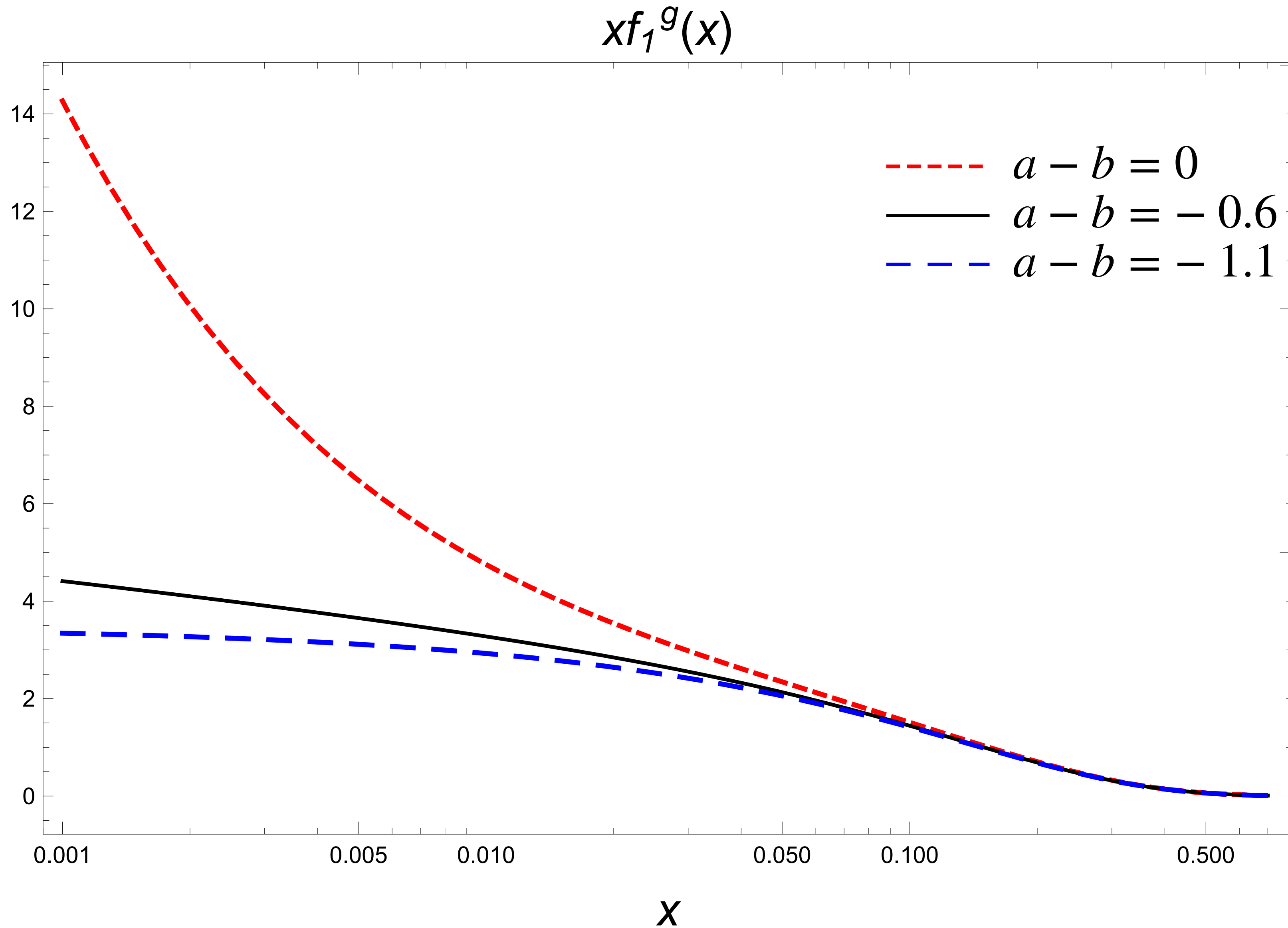
# Spectral function vs $(a - b)$



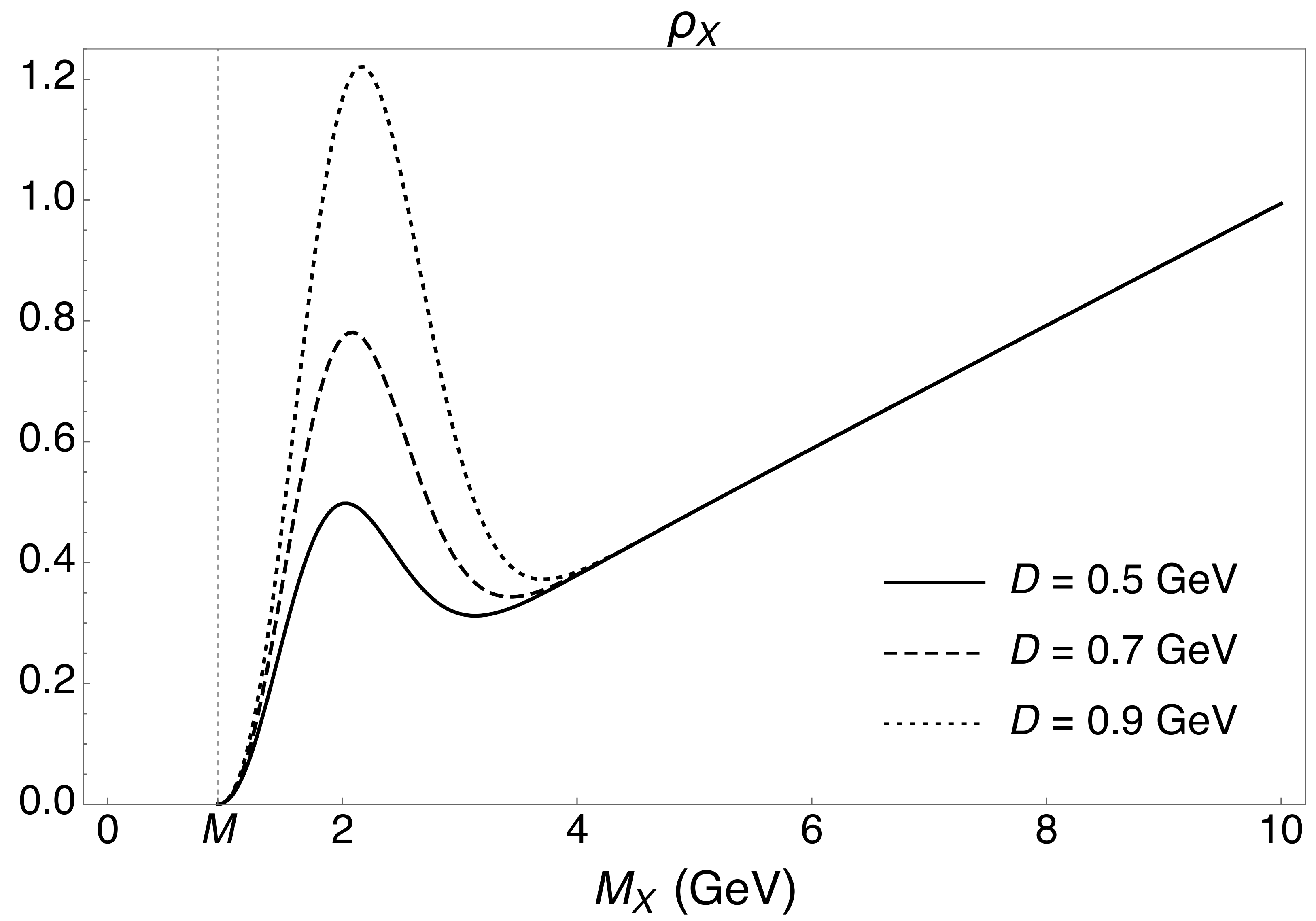
# Spectral function vs $(a - b)$



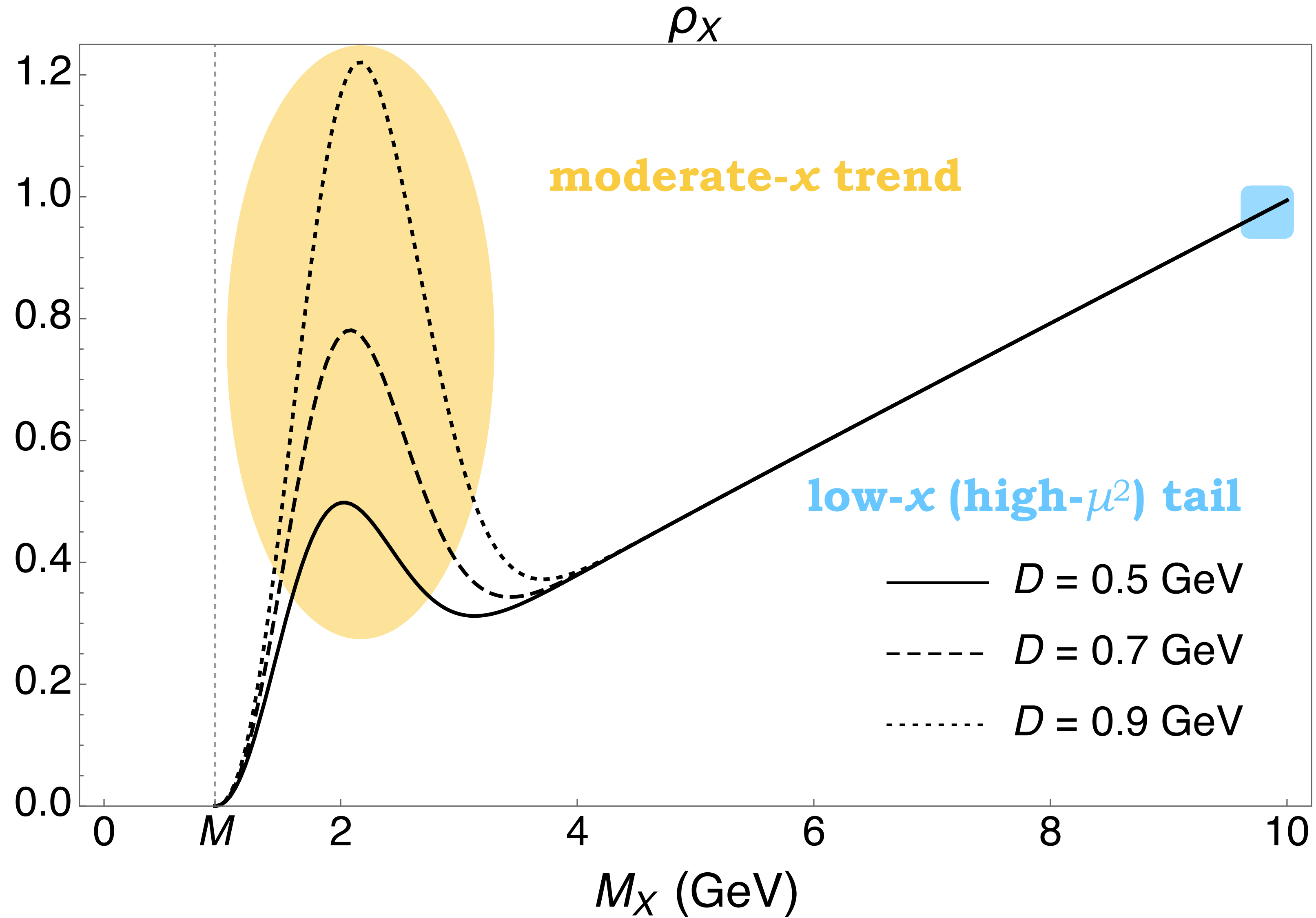
# $xf_1$ collinear PDF vs $(a - b)$



# Spectral function vs $D$



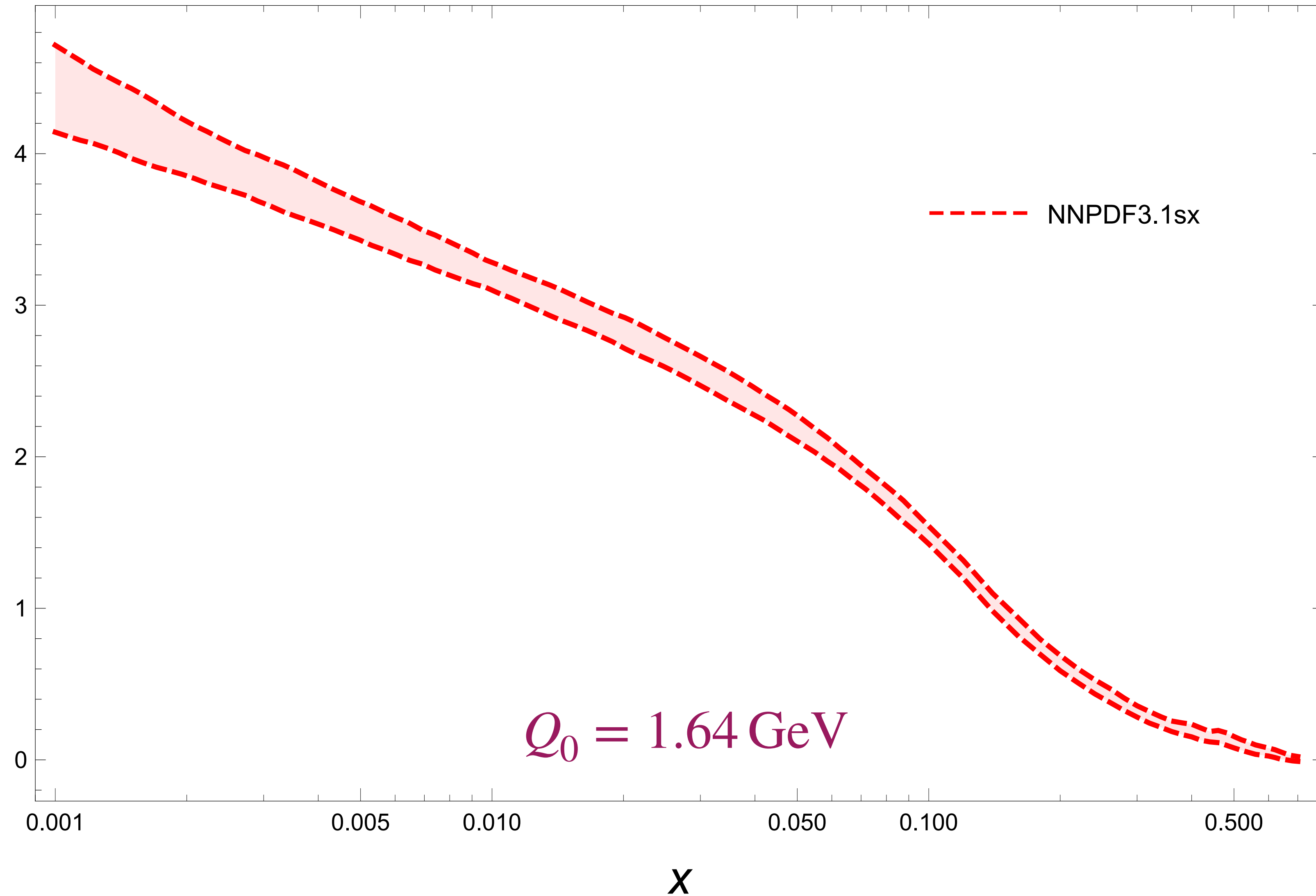
# Spectral function vs $D$





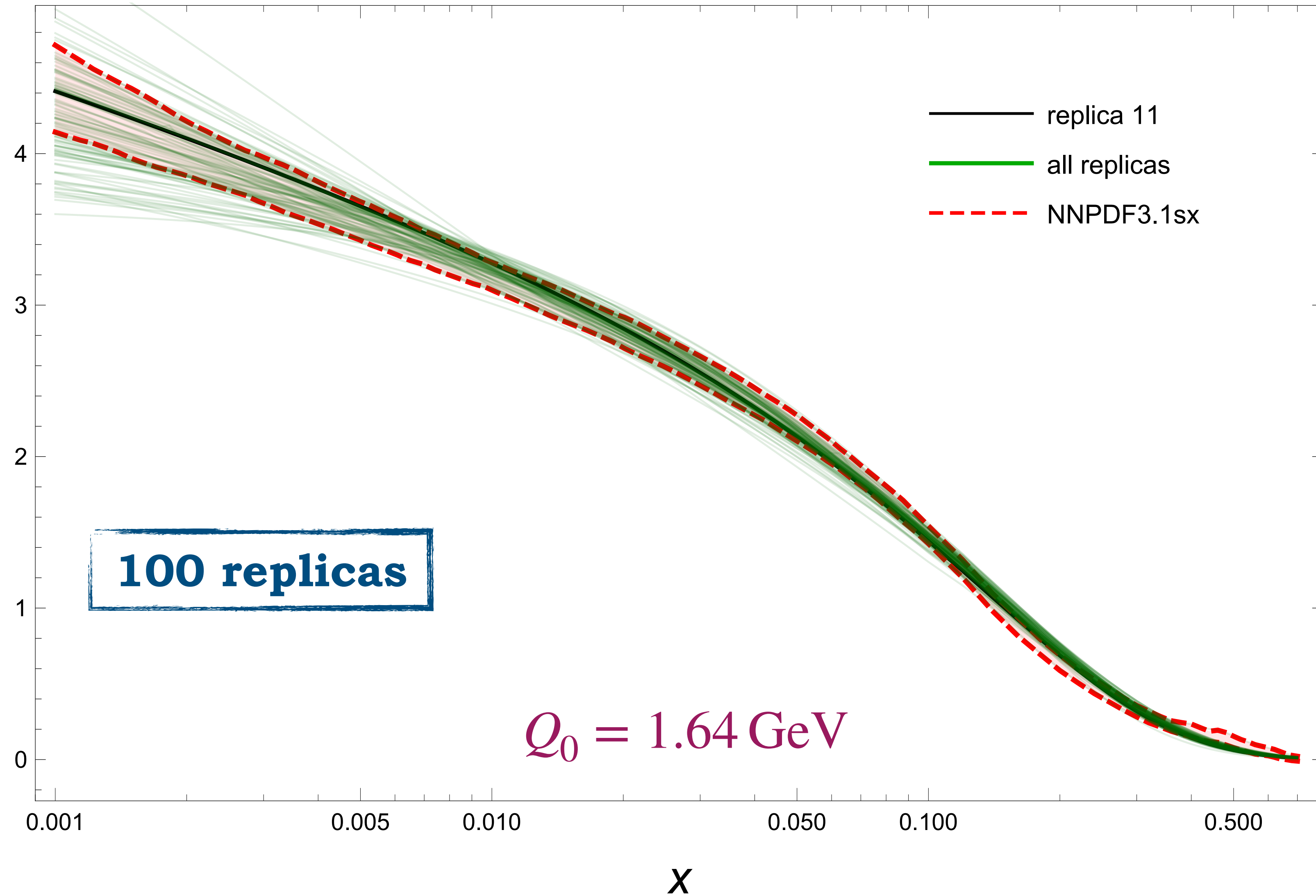
# Unpolarized gluon PDF

$$xf_1^g(x)$$



# Unpolarized gluon PDF

$$xf_1^g(x)$$

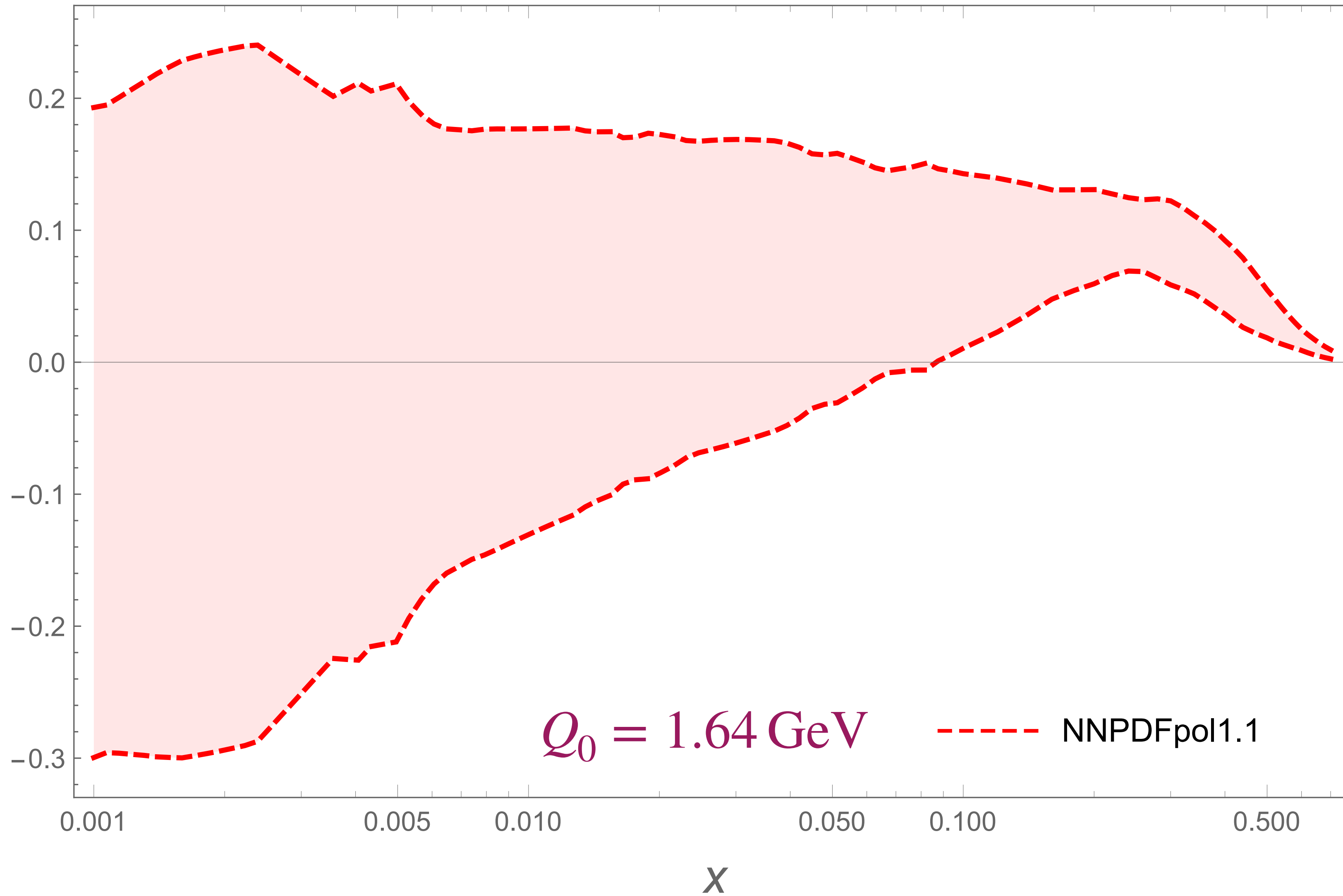


100 replicas

$Q_0 = 1.64 \text{ GeV}$

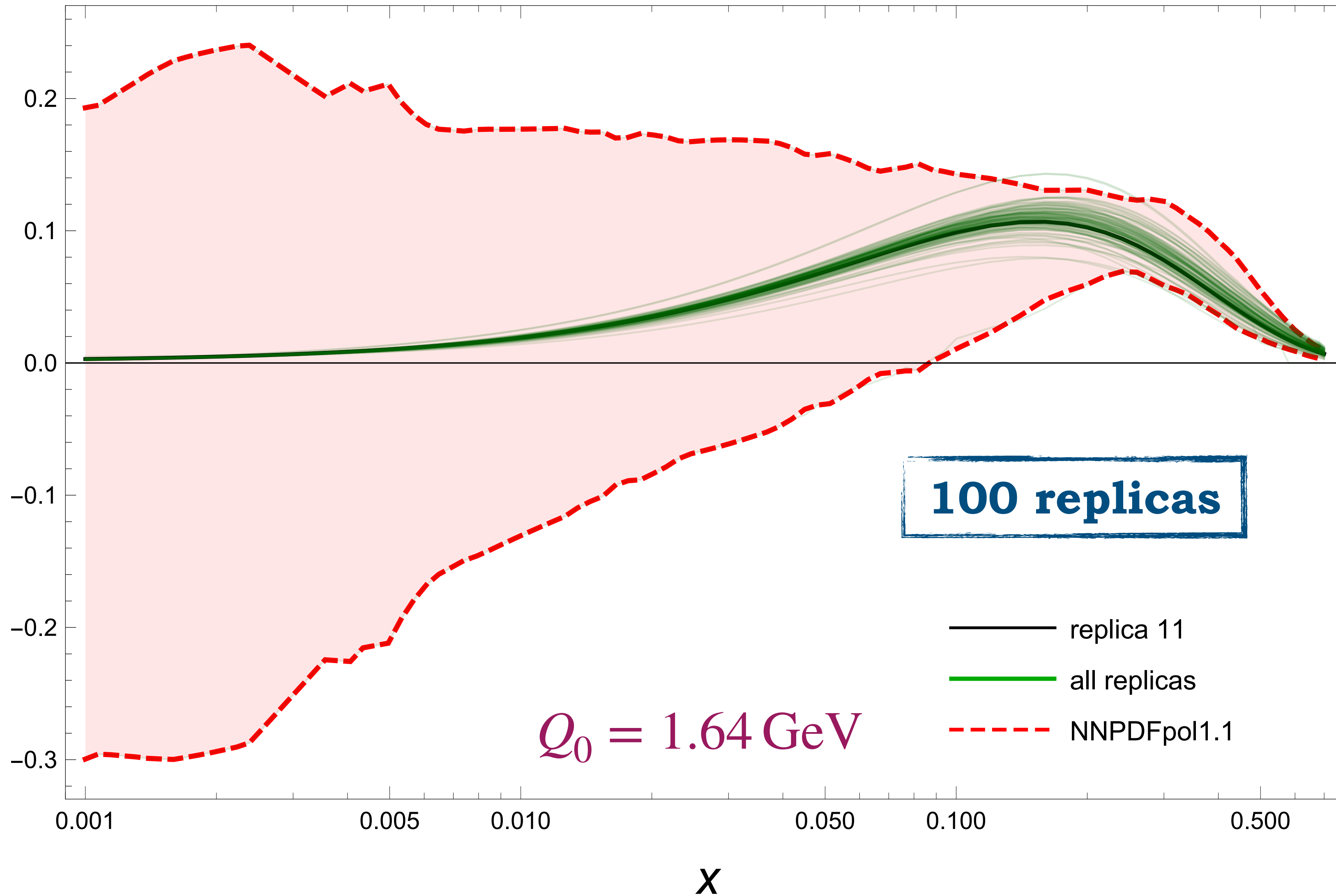
# Helicity gluon PDF

$$xg_1^g(x)$$



# Helicity gluon PDF

$$xg_1^g(x)$$



**100 replicas**

- replica 11
- all replicas
- - - NNPDFpol1.1

$Q_0 = 1.64 \text{ GeV}$

# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0) \qquad S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

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$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

Our model @  $Q_0 = 1.64$  GeV

Lattice @  $Q_0 = 2$  GeV

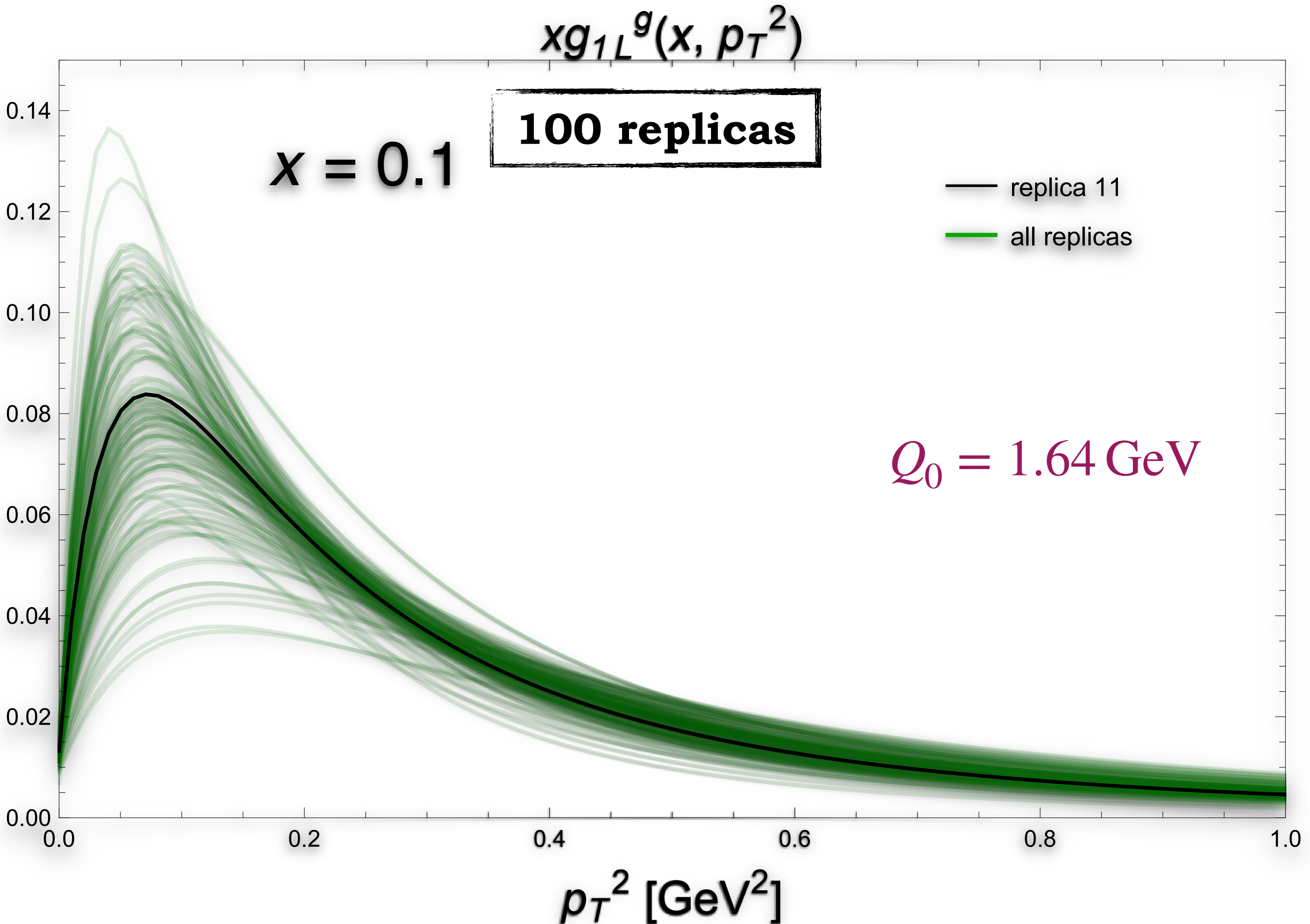
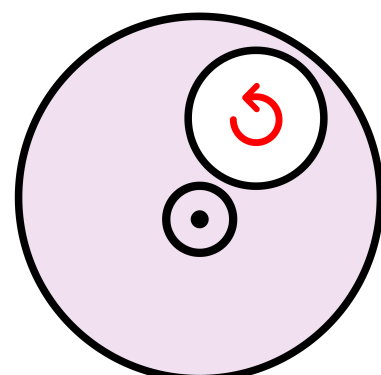
$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

$$\langle x \rangle_g = 0.427(92)$$

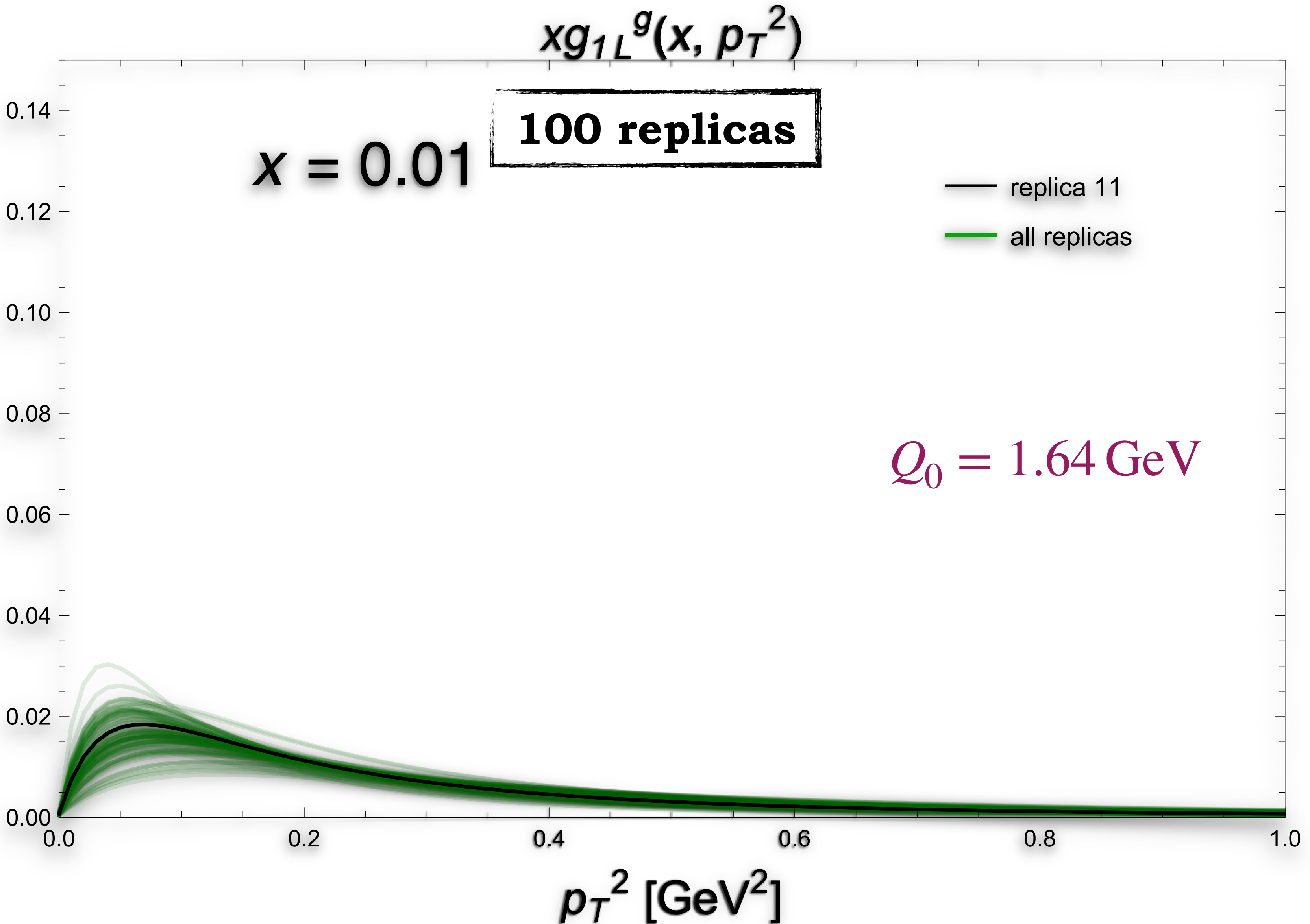
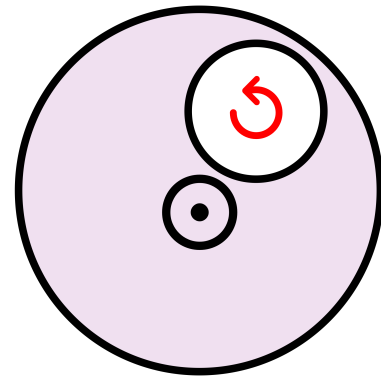
$$\langle J \rangle_g = 0.187(46)$$

# Helicity gluon TMD

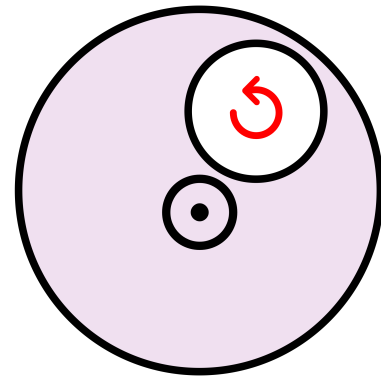




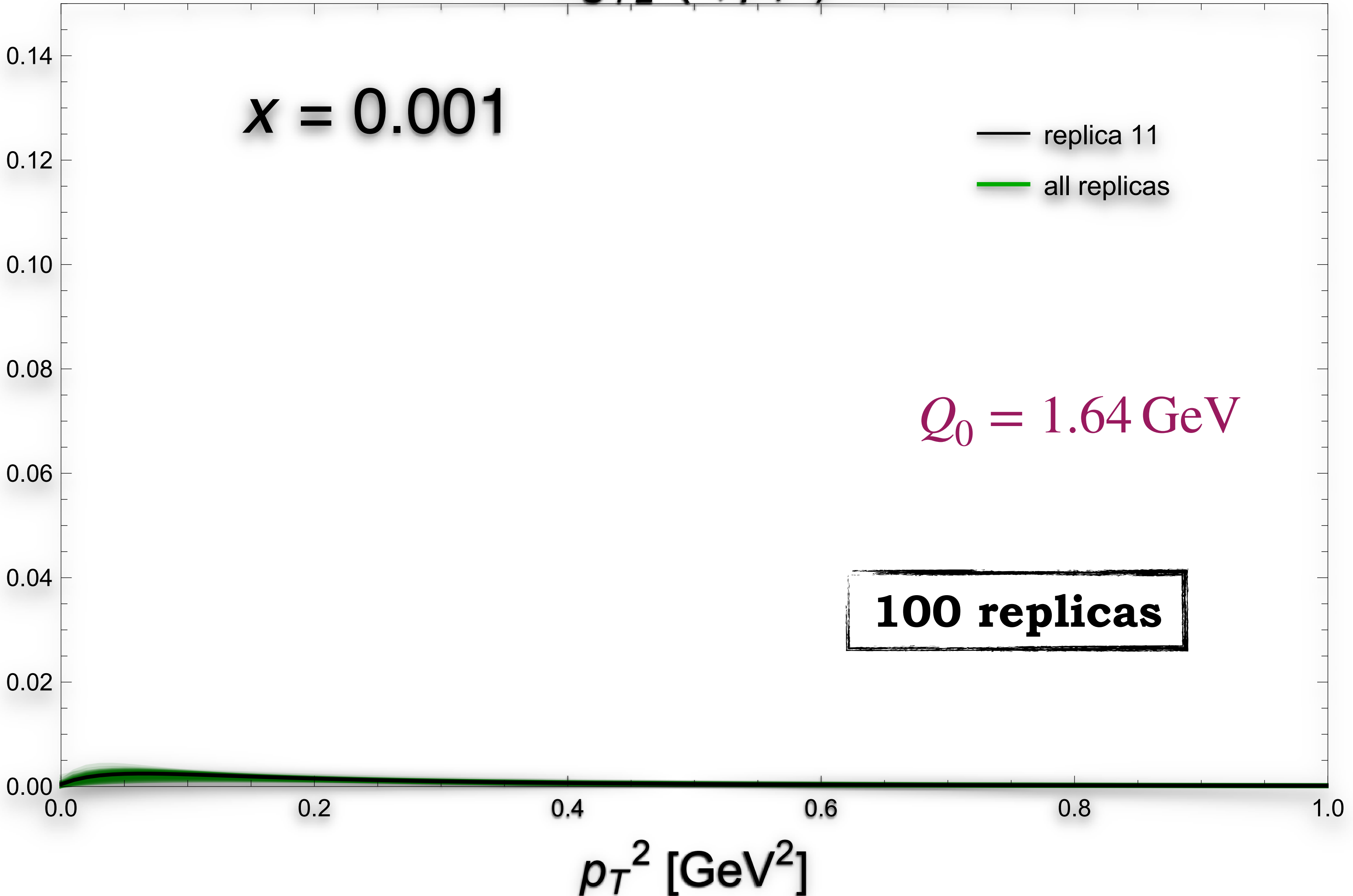
# Helicity gluon TMD



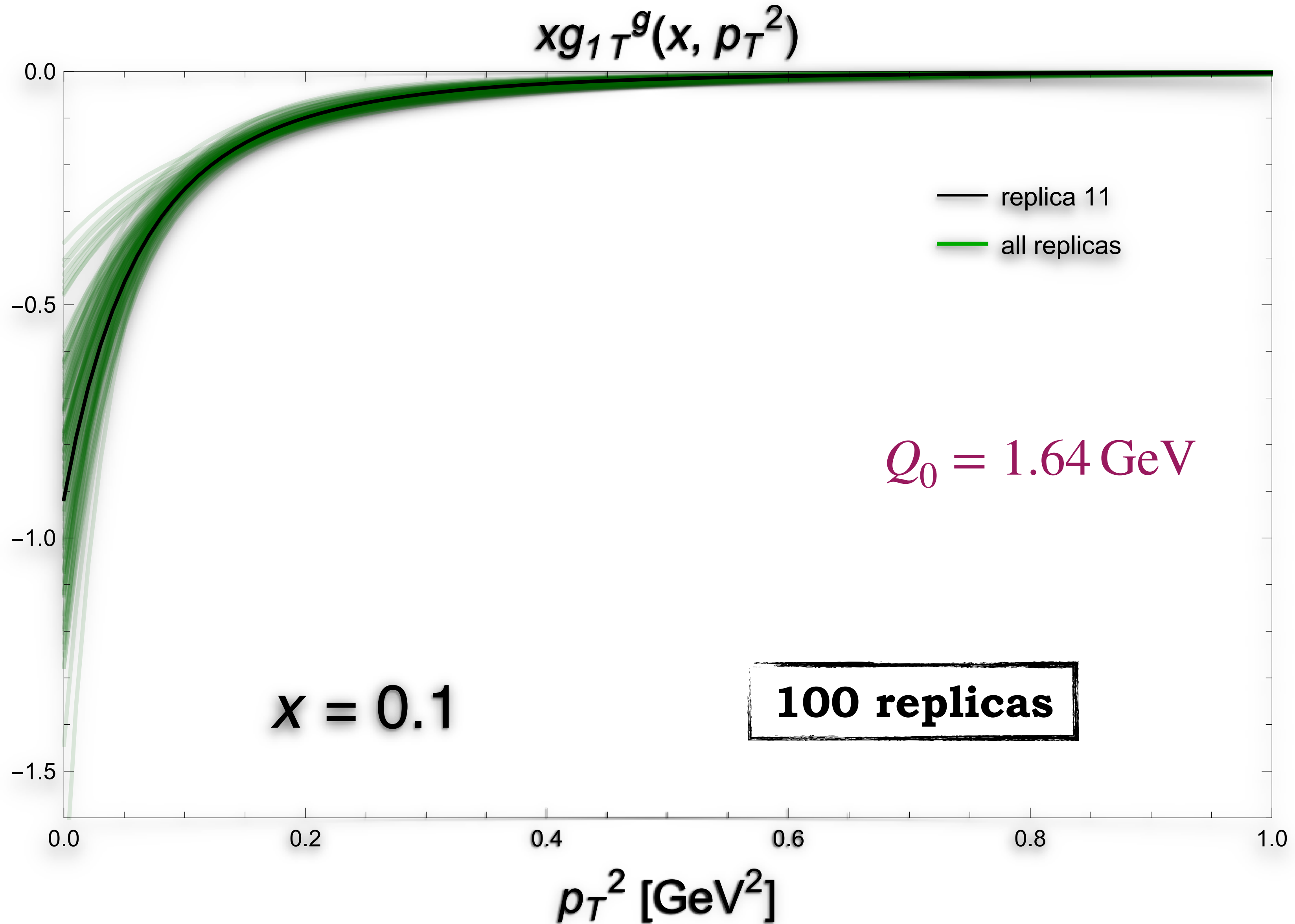
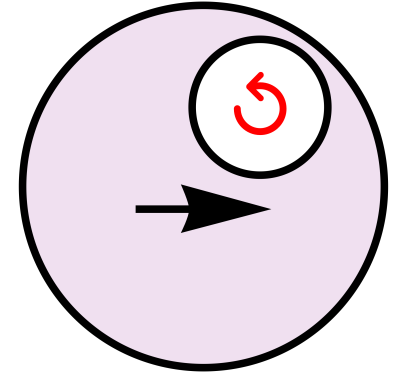
# Helicity gluon TMD



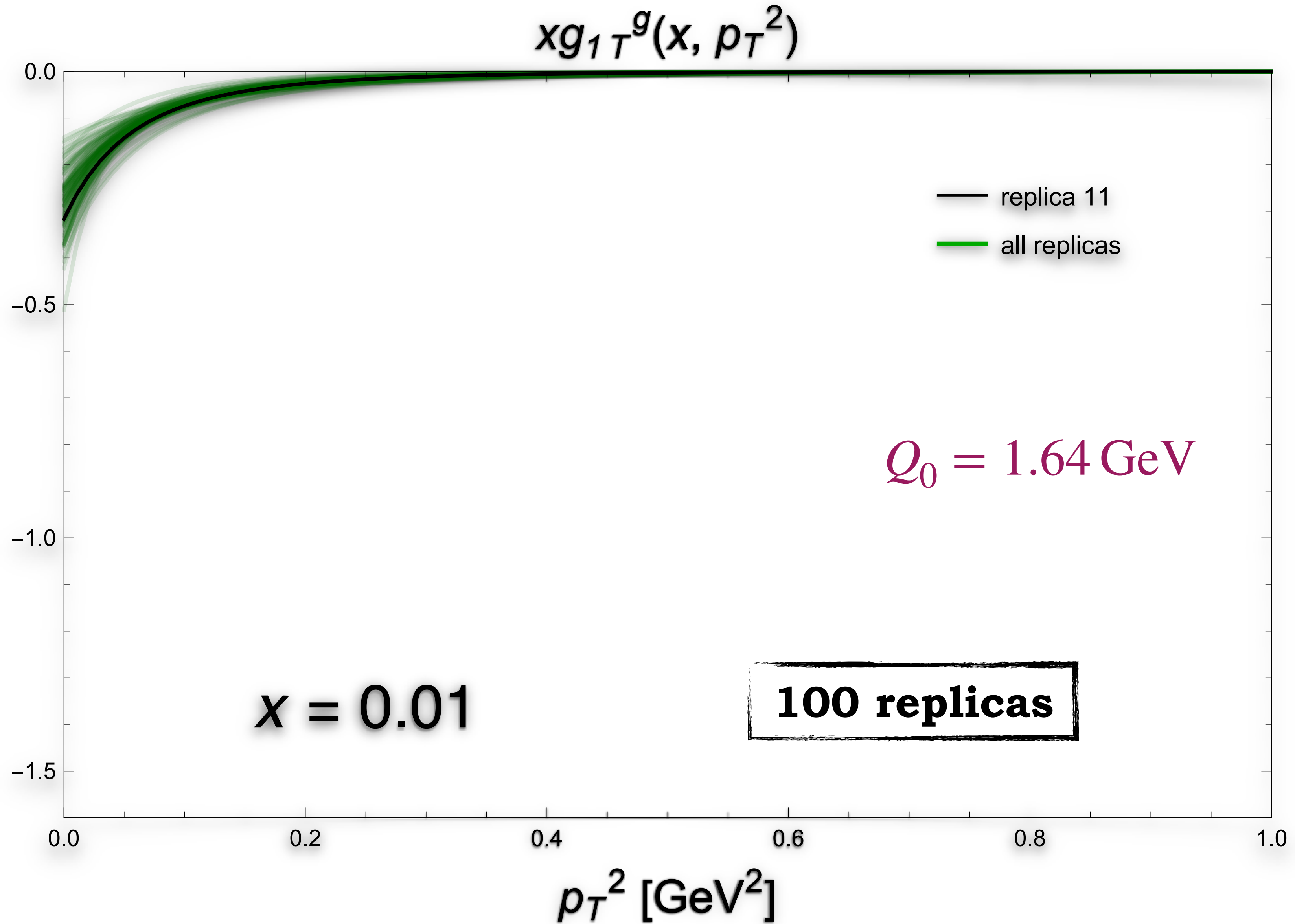
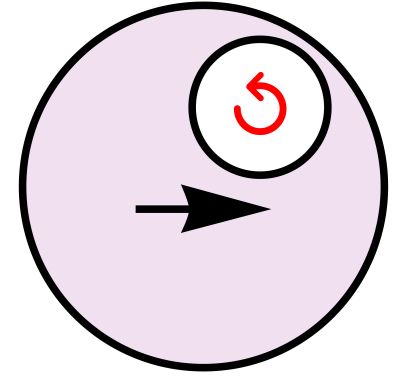
$$xg_{1L}^g(x, p_T^2)$$



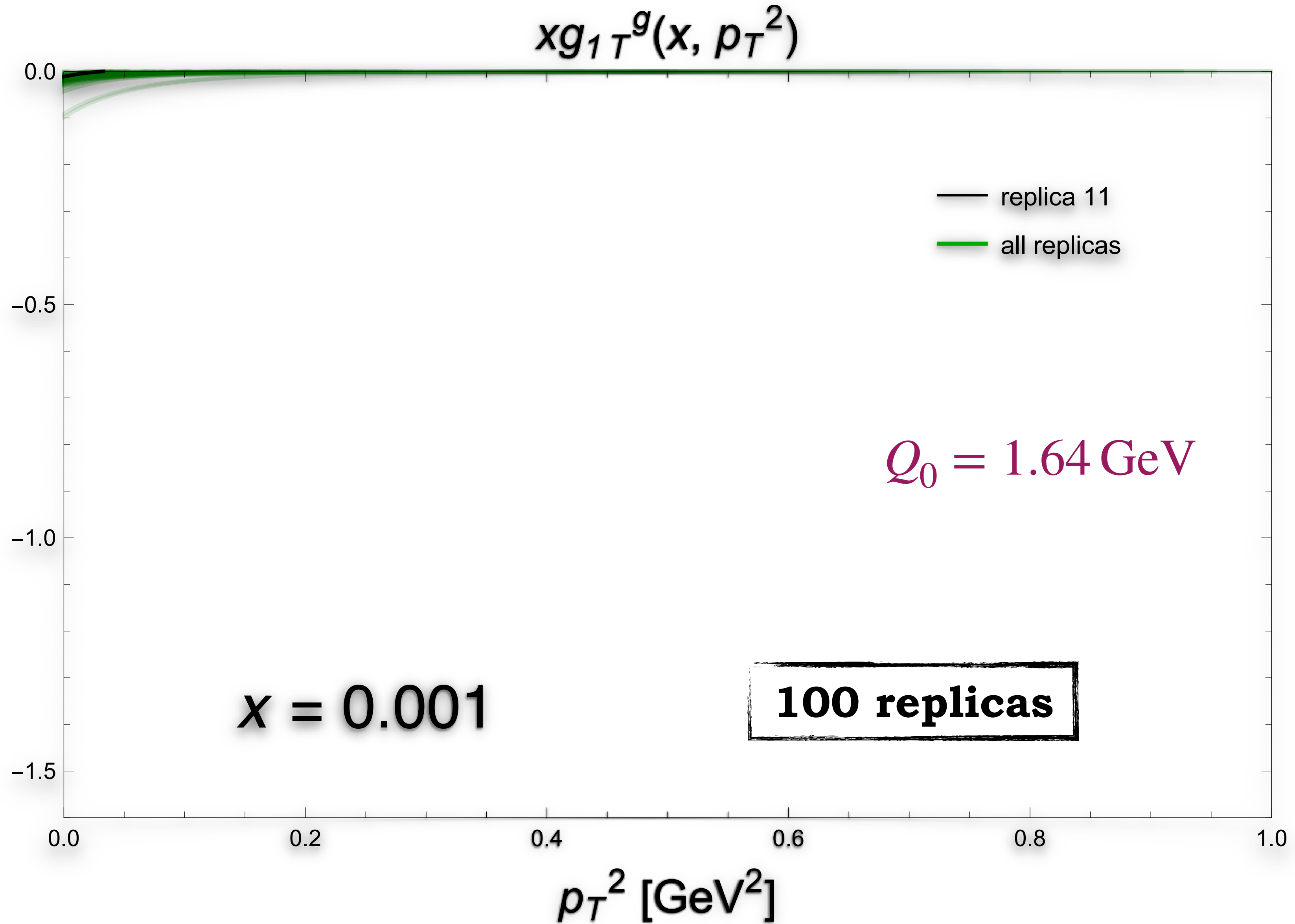
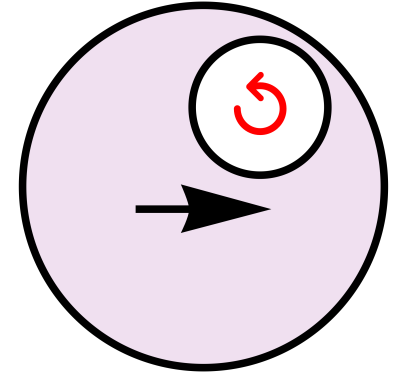
# Worm-gear gluon TMD



# Worm-gear gluon TMD

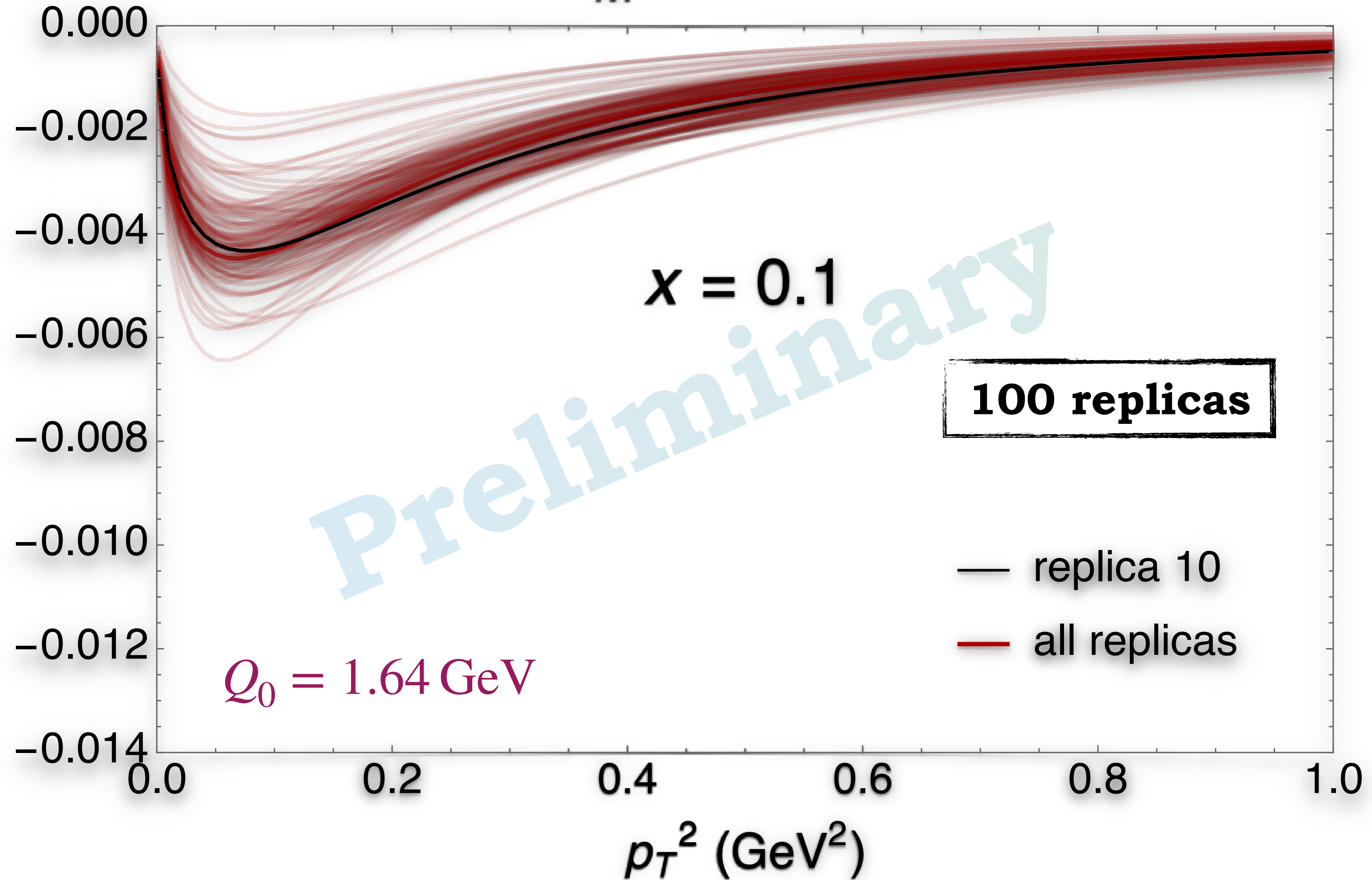
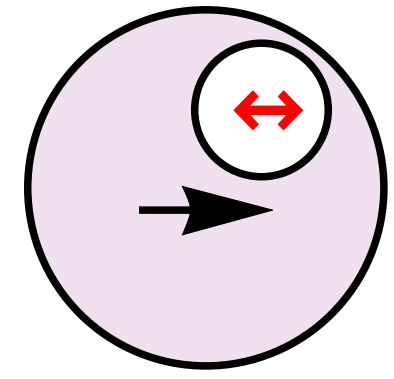


# Worm-gear gluon TMD



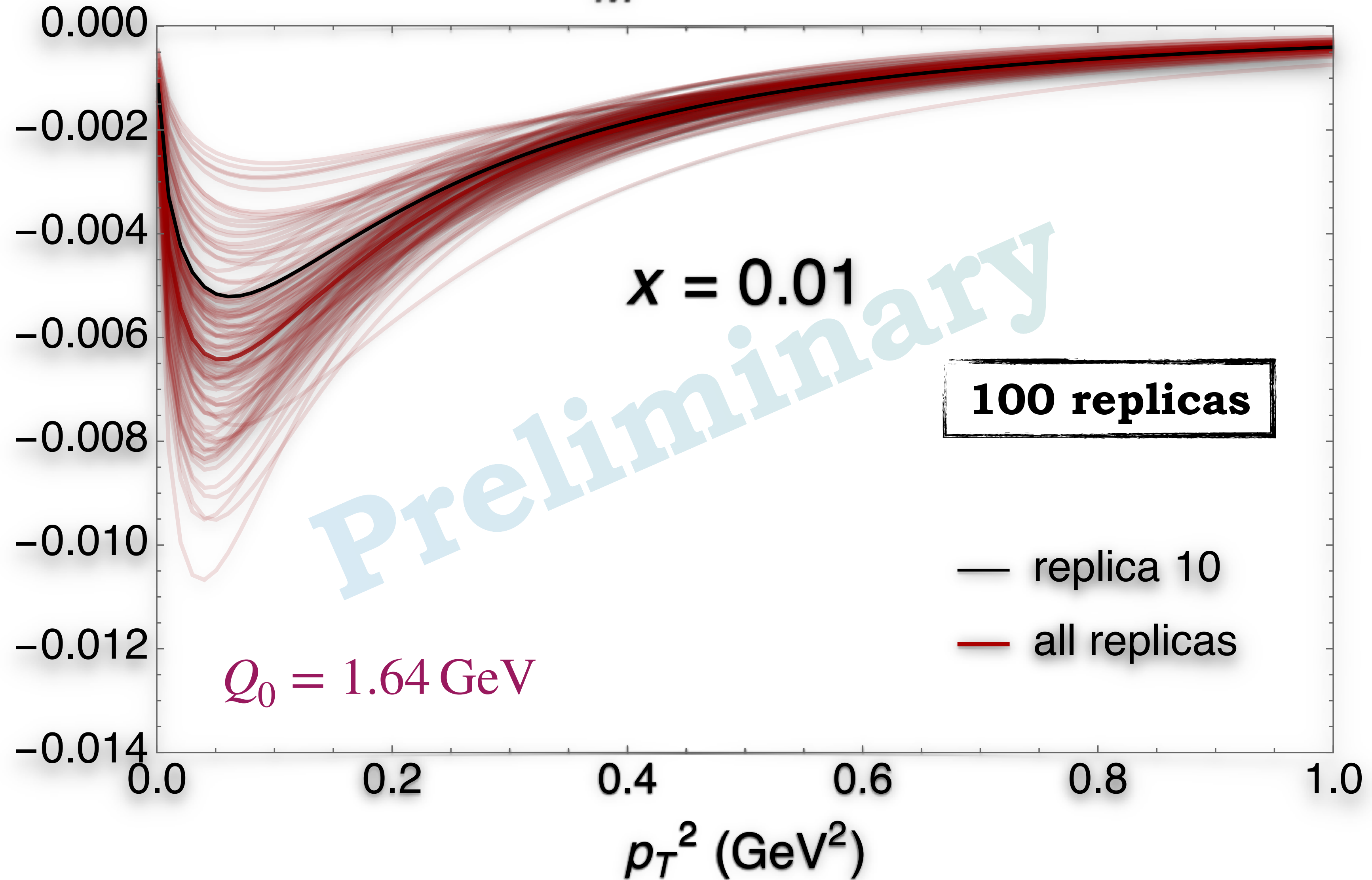
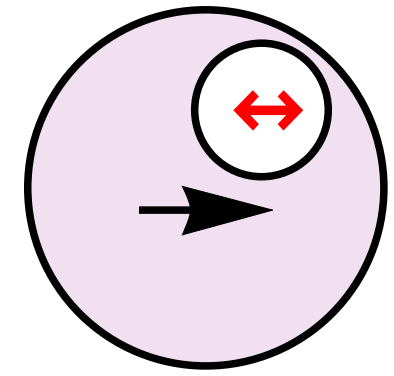
# *f*-type Linearity gluon TMD

$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



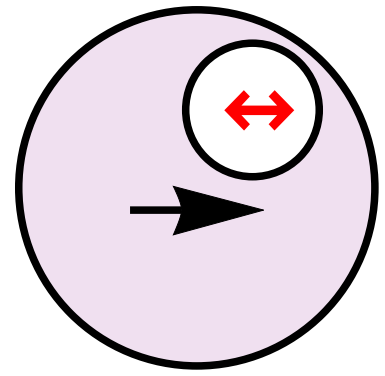
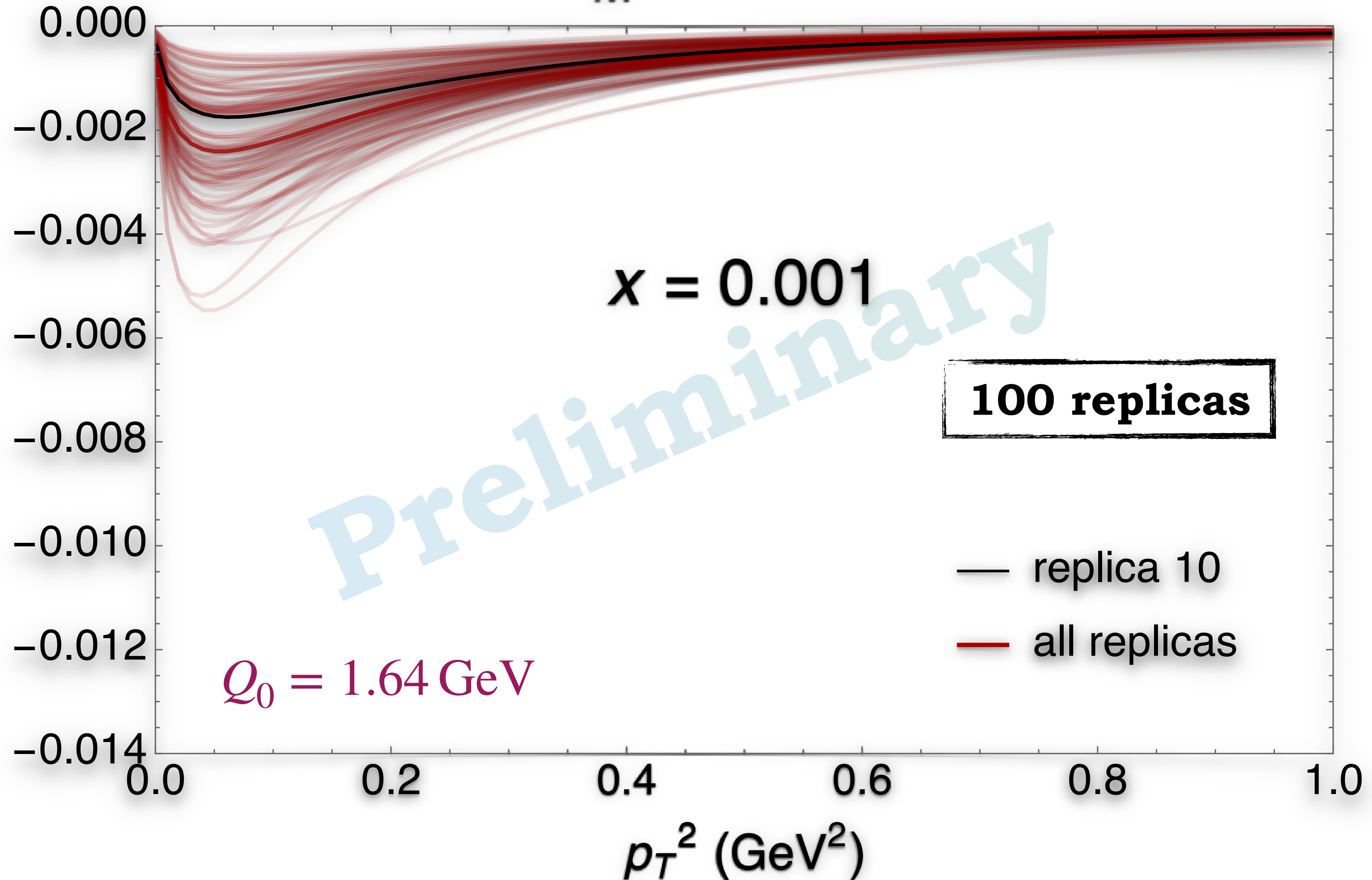
# *f*-type Linearity gluon TMD

$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



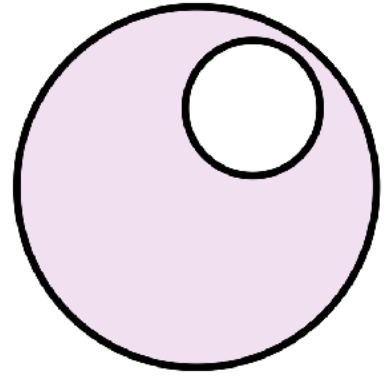
# *f*-type Linearity gluon TMD

$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$





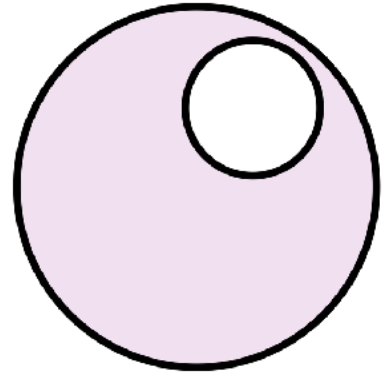
# $\rho$ -densities



**Unpolarized [u/u]**

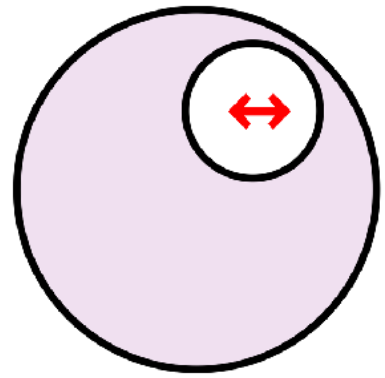
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized** [**u/u**]

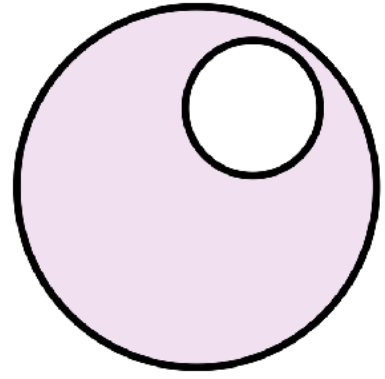
$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [**↔/u**]

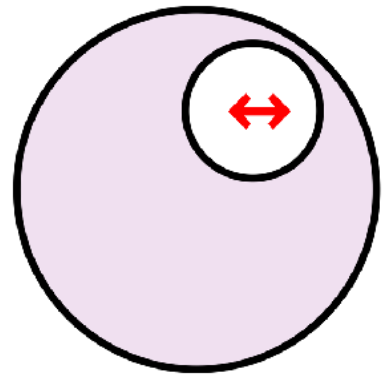
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



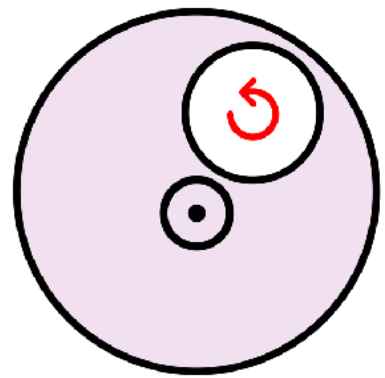
**Unpolarized** [ $\mathbf{u}/\mathbf{u}$ ]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/\mathbf{u}$ ]

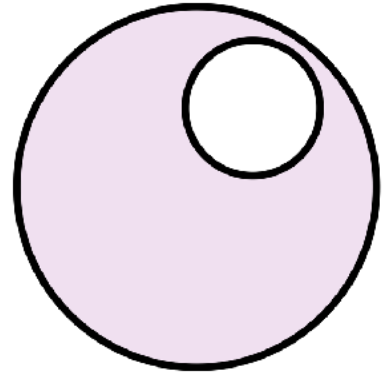
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

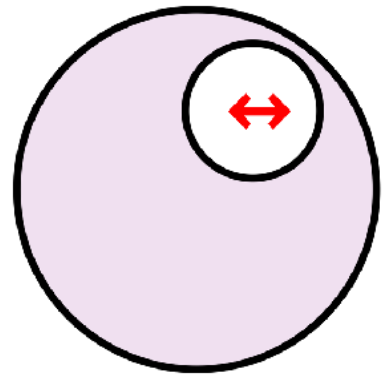
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



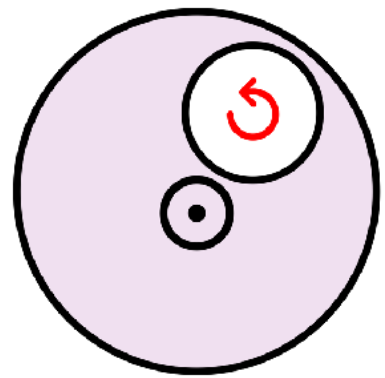
**Unpolarized** [ $\mathbf{u}/\mathbf{u}$ ]

$$f_1(x, p_x, p_y)$$



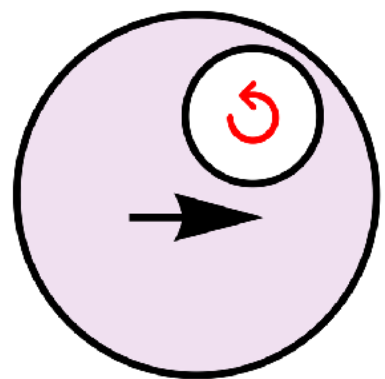
**Boer-Mulders** [ $\leftrightarrow/\mathbf{u}$ ]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$



**Worm-gear** [ $\cup/\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$