Spin alignment in quarkonium production in SIDIS

In collaboration with: U. D’Alesio, F. Murgia, C. Pisano, R. Sangem

Speaker: Luca Maxia
Università di Cagliari – INFN CA

eSarWorS 2021
Date: 07/09/2021

Istituto Nazionale di Fisica Nucleare

OUTLINE

- Quarkonium polarization within NRQCD
  - Formalism
  - TMD and collinear factorization
  - Matching between factorization schemes
- Considerations on EIC predictions

QUARKONIUM POLARIZATION

By measuring the polarization we can understand the angular momentum state in which the particle is produced

- Test of perturbative QCD
- Test of hadronization models (CSM vs NRQCD vs …?)

Color Singlet Model
(CSM)

Quarkonium produced perturbatively as *color-neutral* $Q\bar{Q}$-pair

→

Non-relativistic QCD approach
(NRQCD)

Quarkonium produced through *colored* $Q\bar{Q}$-pair that evolves non-pertubatively

→ LDME

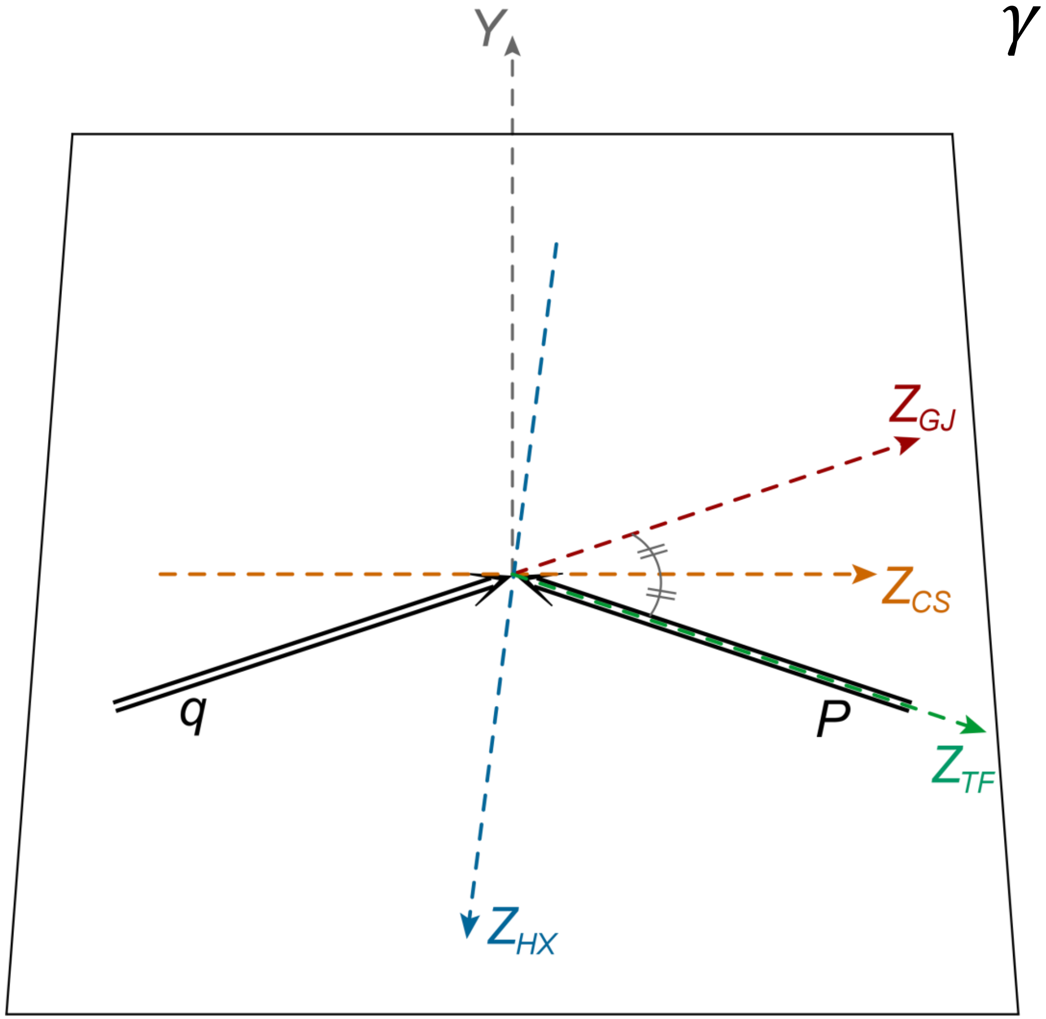
Baier Ruckl, Z.Phys.C 19 (1983)

Berger Jones, PRD 23 (1981)

Bodwin Braaten Lepage, PRD 55 (1997)

Cho Leibovich, PRD 53 (1996)

QUARKONIUM POLARIZATION IN *SIDIS*



$\gamma^*(q) + p(P) \rightarrow J/\psi(P_\psi) + X$ in J/ψ rest frame

Semi-Inclusive DIS (SIDIS)

Different choices for the reference frame

- GJ *Gottfried-Jackson* frame
- CS *Collins-Soper* frame
- HX *Helicity* frame
- TF *Target* frame

Frames are related by a rotation around *Y*-axis

HELICITY CONSERVATION IN LEPTON DECAY

Polarization of quarkonia is accessed by the angular distribution of its decay products

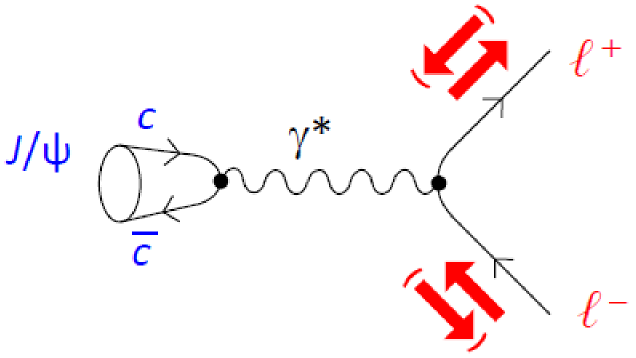
→ decay into fermion-antifermion is a clean case for $^3S_1$ states

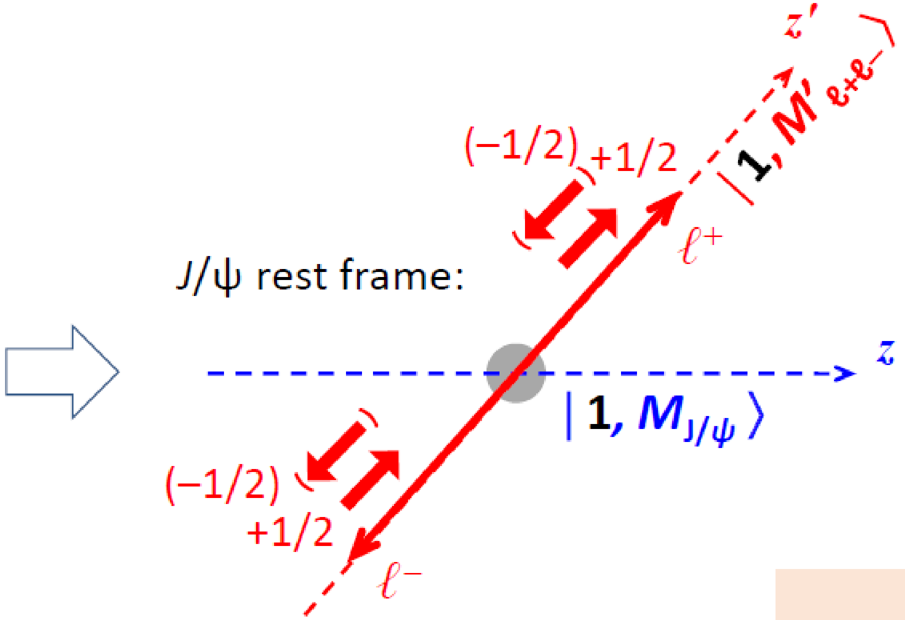
Electroweak and strong forces preserve *chirality*

→ *chirality = helicity = spin alignment*

(relativistic limit)

Example $J/\psi \rightarrow l^+l^-$





$M_{J/\psi} = +1, 0, -1$

↓ Wigner *D-matrix*

$M_{l^+l^-} = +1, -1$

P. Faccioli et al., EPJC 69 (2010)

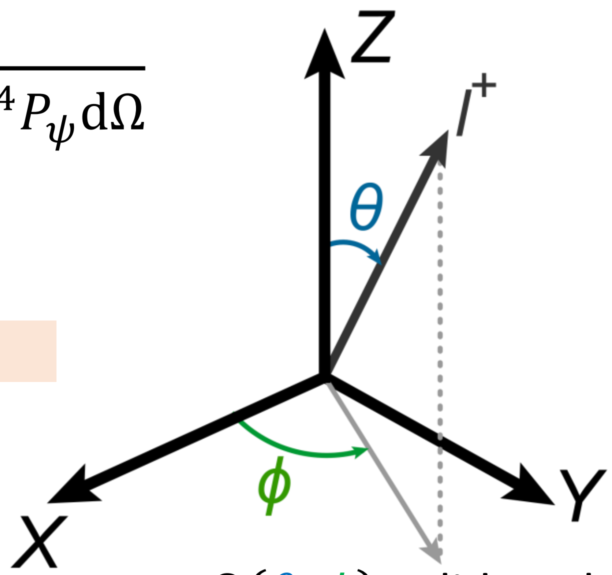
ANGULAR STRUCTURE OF THE CROSS SECTION

Parameterization of the SIDIS cross section

$$d\sigma \equiv \frac{d\sigma}{dx_B\,dy\,dz\,d^4P_\psi\,d\Omega}$$

$$d\sigma \propto \mathcal{W}_T(1+\cos^2\theta) + \mathcal{W}_L(1-\cos^2\theta) + \mathcal{W}_\Delta \sin 2\theta \cos\phi + \mathcal{W}_{\Delta\Delta} \sin^2\theta \cos 2\phi$$

Boer Vogelsang, PRD 74 (2006)



$\Omega(\theta,\phi)$ solid angle of l^+

obtained from model-independent arguments:

Hermiticity   Parity conservation   Gauge invariance

→ 8 *helicity structure functions* $\mathcal{W}_\Lambda^P$

with $\Lambda = T, L, \Delta, \Delta\Delta$ → J/ψ helicity

$P = \perp, \parallel$ → γ^* polarization

SIDIS variables

$Q^2 = -q^2,\ x_B = \frac{Q^2}{2P\cdot q},$

$y = \frac{P\cdot q}{P\cdot l},\ z = \frac{P\cdot P_\psi}{P\cdot q}$

Independent linear combination of $\mathcal{W}_\Lambda^P$

$$\mathcal{W}_\Lambda = [1+(1-y)^2]\mathcal{W}_\Lambda^\perp + (1-y)\mathcal{W}_\Lambda^\parallel$$

FACTORIZATION SCHEMES

TMD factorization for SIDIS is proven at leading twist

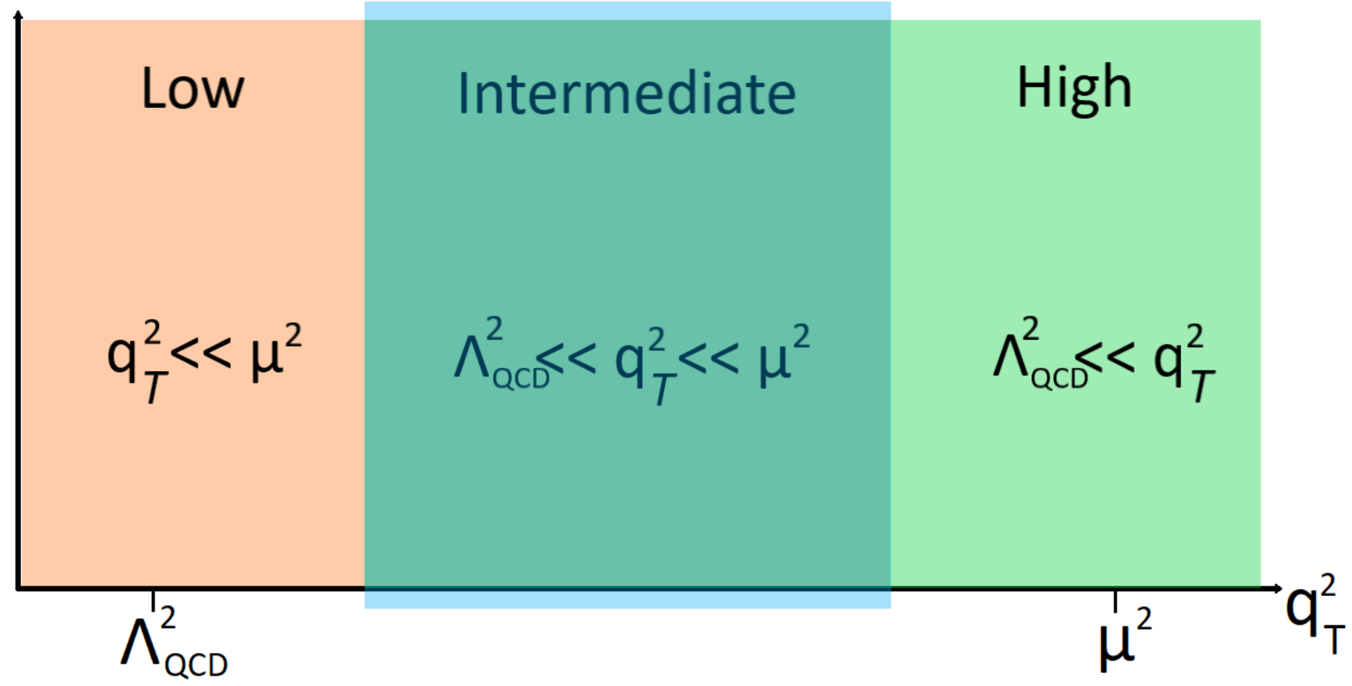
$$q_T^2 \ll \mu^2$$

Collins, Cambridge University Press (2011)

Collinear factorization is valid at high transverse momentum

$$q_T \gg \Lambda_{QCD}$$

q_T: photon TM w.r.t. P_ψ, P



It could exist a region where both factorization schemes are valid

$$\Lambda^2 \ll q_T^2 \ll \mu^2$$

Do they describe the same dynamics?

Bacchetta Boer Diehl Mulders, JHEP 08 (2008)

J/ψ POLARIZATION WITHIN *NRQCD*

In the NRQCD approach there is a double expansion: α_s and v

Collinear $\alpha\alpha_s^2$ subprocess $\gamma^* + a \to c\bar{c}[n](P_\psi) + a$ $a = q, \bar{q}, g$

TMD $\alpha\alpha_s$ subprocess $\gamma^* + g \to c\bar{c}[n](P_\psi)$

with Fock states $n = {}^{2S+1}L_J^{[c]}$

up to v^4 order ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$

${}^1S_0^{[8]}$ → unpolarized

${}^3P_J^{[8]}$ → $J = 0, 1, 2$

NRQCD waves with different L and S quantum numbers contribute separately to the polarization → interference between P-waves

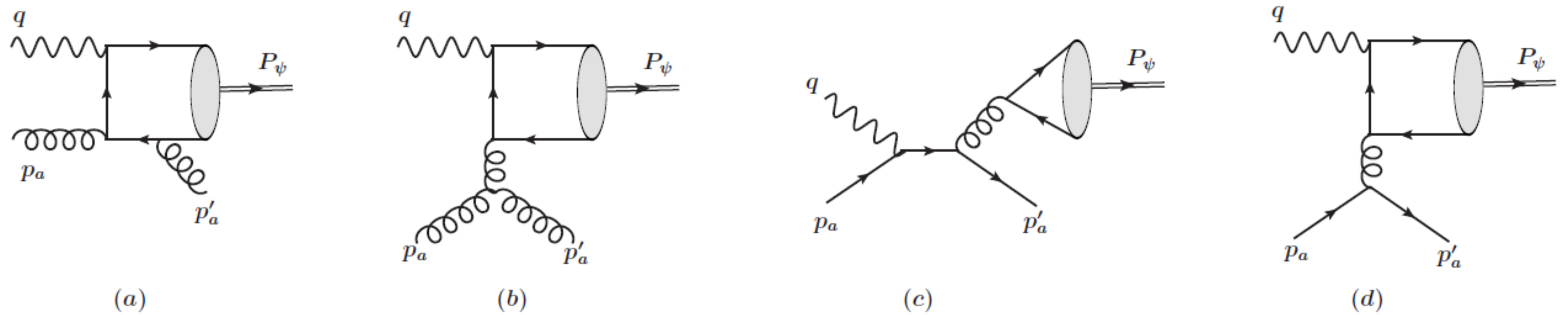
$$\mathcal{W}_{\Lambda\Lambda'} = \mathcal{W}_{\Lambda\Lambda'}\left[{}^3S_1^{[1]}\right] + \mathcal{W}_{\Lambda\Lambda'}\left[{}^1S_0^{[8]}\right] + \mathcal{W}_{\Lambda\Lambda'}\left[{}^3S_1^{[8]}\right] + \mathcal{W}_{\Lambda\Lambda'}\left[\{S=1, L=1\}^{[8]}\right]$$

→ $\Lambda, \Lambda' = -1, 0, +1$ J/ψ polarization

Beneke Krämer Vänttinen, PRD 57 (1998)

COLLINEAR FACTORIZATION

Partonic subprocesses at $\alpha\alpha_s^2$ $\gamma^*(q) + a(p_a) \to c\bar{c}[n](P_\psi) + a(p'_a)$ $a = q, \bar{q}, g$



$\hat{\sigma}_\Lambda^P$ pol. partonic cross section for $\gamma^* g \to J/\psi$

4 frame independent structure functions surviving in $\Lambda \ll q_T \ll Q$ up to $\mathcal{O}(\Lambda_{QCD}/q_T)$ and $\mathcal{O}(q_T/Q)$ → $\mathcal{W}_T^\perp,\ \mathcal{W}_L^\perp,\ \mathcal{W}_L^\parallel,\ \mathcal{W}_{\Delta\Delta}^\perp$

$$\mathcal{W}_{\Delta\Delta}^\perp = \hat{\sigma}_{\Delta\Delta}^\perp \frac{1}{\boldsymbol{q}_T^2}\left(\delta P_{gg} \otimes f_1^g + \delta P_{gi} \otimes f_1^g\right)(x,\mu^2)$$

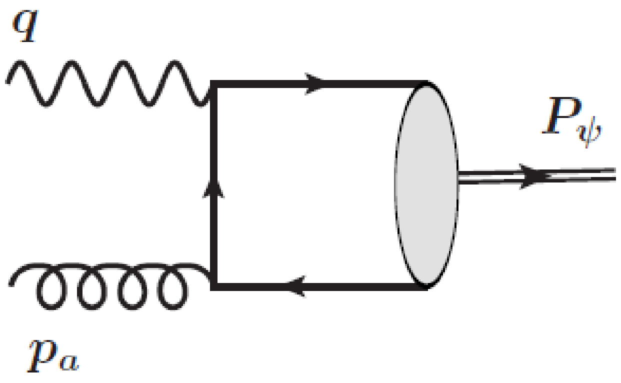
$$\mathcal{W}_\Lambda^P = \hat{\sigma}_\Lambda^P \frac{1}{\boldsymbol{q}_T^2}\left[L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) f_1^g(x,\mu^2) + \left(P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^g\right)(x,\mu^2)\right]$$

$$\longrightarrow\ L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) = 2C_A \ln\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) - \frac{11C_A - 4n_f T_R}{6}$$

TMD FACTORIZATION

Partonic subprocesses at $\alpha\alpha_s$

$$\gamma^*(q) + g(p_a) \rightarrow c\bar{c}[n](P_\psi)$$



Leading twist TMDs

gluon polar. / proton polar.	Unpolarized	Circular	Linear
Unpolarized	f_1		$h_1^\perp$
Longitudinal		g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Mulders Rodriguez, PRD 63 (2001)

Structure functions are combinations of pol. partonic cross section $\hat{\sigma}_\Lambda^P$ and *TMD PDF*

→ same as collinear at small q_T

<u>Neglecting smearing effects:</u>

$$\mathcal{W}_T^\perp = \hat{\sigma}_T^\perp f_1(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_L^\perp = \hat{\sigma}_L^\perp f_1(x, \boldsymbol{q}_T^2)$$

$$\mathcal{W}_L^\parallel = \hat{\sigma}_L^\parallel f_1(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_{\Delta\Delta}^\perp = \hat{\sigma}_{\Delta\Delta}^\perp h_1^\perp(x, \boldsymbol{q}_T^2)$$

MATCHING AND SMEARING EFFECTS

In the $\Lambda_{QCD} \ll q_T \ll Q$ region

$(\Lambda, P) = (L,\perp), (T,\perp), (L,\parallel)$

$$\mathcal{W}_{\Delta\Delta}^{\perp}\Big|_{coll.} - \mathcal{W}_{\Delta\Delta}^{\perp}\Big|_{TMD} = 0$$

$$\mathcal{W}_{\Lambda}^{P}\Big|_{coll.} - \mathcal{W}_{\Lambda}^{P}\Big|_{TMD} \neq 0$$

→ matching requires *shape functions*

Echevarria, JHEP 10 (2019)

Fleming Makris Mehen, JHEP 04 (2020)

Shape function $\Delta^{[n]}$ is a TMD generalization of NRQCD LDME

f_1^g → $\mathcal{C}[f_1^g \Delta^{[n]}]$

$$\Delta^{[n]}(\boldsymbol{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A < \mathcal{O}_8[n] > \ln\frac{\mu^2}{\boldsymbol{k}_T^2} \quad k_T^2 \gg m_p^2$$

$h_1^{\perp g}$ → $\mathcal{C}\left[w h_1^{\perp g} \Delta_h^{[n]}\right]$

$\Delta_h^{[n]}(\boldsymbol{k}_T^2, \mu^2)$ not observable at this α_s order

Boer D'Alesio Murgia Pisano Taels, JHEP 09 (2020)

D'Alesio LM Murgia Pisano Sangem, in preparation

EIC: COLLINEAR REGION PRELIMINARY RESULTS

Experimentally a different parameterization is usually adopted for $\mathrm{d}\sigma \equiv \dfrac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega}$

$$\mathrm{d}\sigma \propto 1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi$$

$$\lambda = \frac{\mathcal{W}_T - \mathcal{W}_L}{\mathcal{W}_T + \mathcal{W}_L} \qquad \mu = \frac{\mathcal{W}_\Delta}{\mathcal{W}_T + \mathcal{W}_L} \qquad \nu = \frac{2\mathcal{W}_{\Delta\Delta}}{\mathcal{W}_T + \mathcal{W}_L}$$

where $\lambda = +1$ → transverse, $\lambda = -1$ → longitudinal

→ easier to access

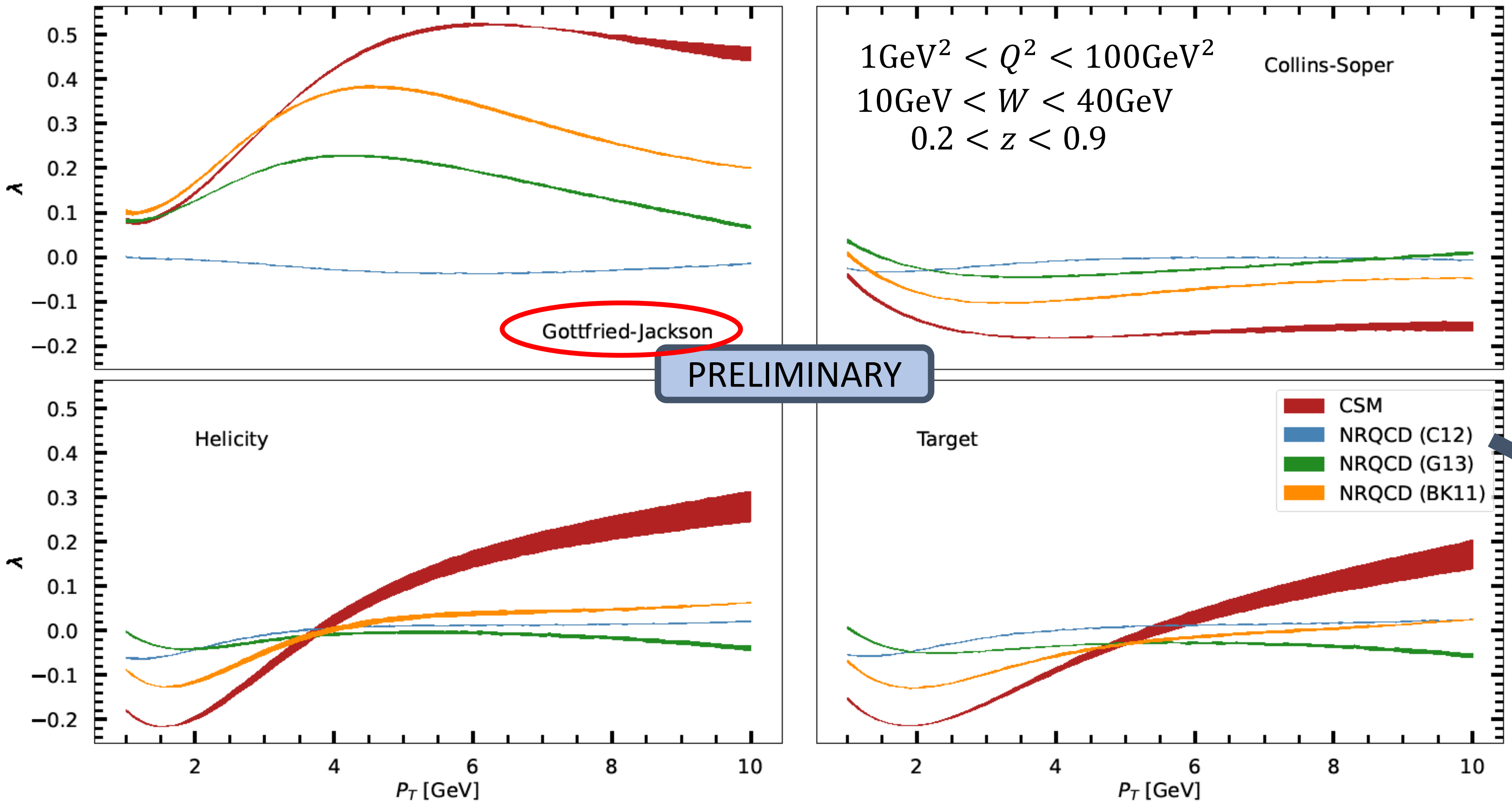
Next: focus on λ in CSM and NRQCD at scale $\mu_0/2 < \mu < 2\mu_0$ $\qquad \mu_0 = \sqrt{M_\psi^2 + Q^2}$

NRQCD with different LDME choices

C12	Chao Ma Shao Wang Zhang, PRL 108 (2012)	→ include polarization data
G13	Gong Wan Wang Zhang, PRL 110 (2013)	→ include polarization data
BK11	Butenschoen Kniehl, PRD 84 (2011)	→ include low P_T unpolarized data

PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$



CSM vs NRQCD

GJ best frame for λ

CSM

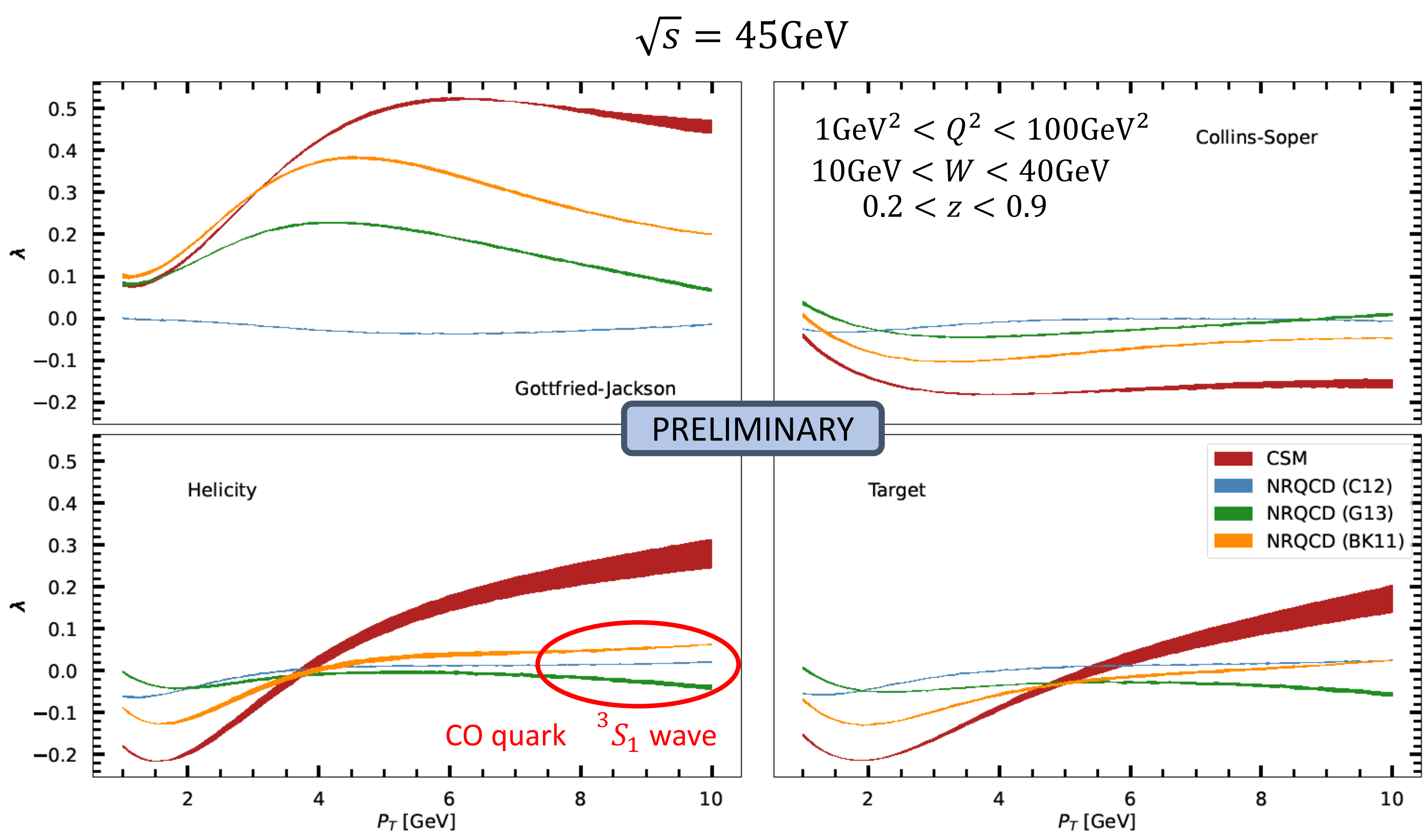
- dependence on scale μ

NRQCD

- dependence on LDME choices

C12
almost unpol. prod.

PREDICTIONS FOR *EIC*



Wave contributions

CS

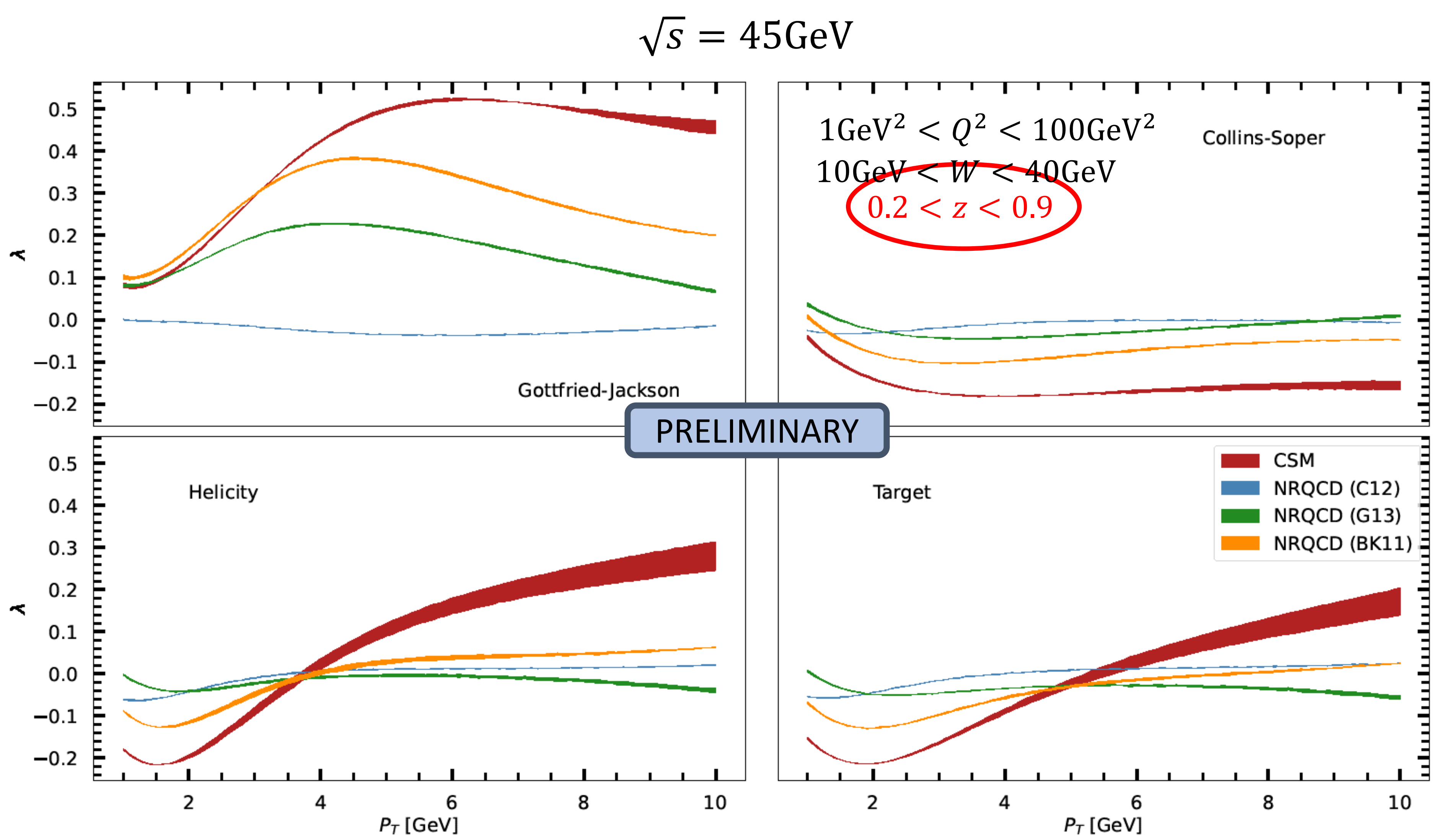
- CS wave is the main contribution up to mid P_T

CO

- *P* wave is the main CO contribution
  - → *gluon* at lower P_T
  - → *quark* at higher P_T

- *CO quark* $^3S_1$ wave is the main contribution at high P_T for a specific choice of frame and variable

PREDICTIONS FOR *EIC*



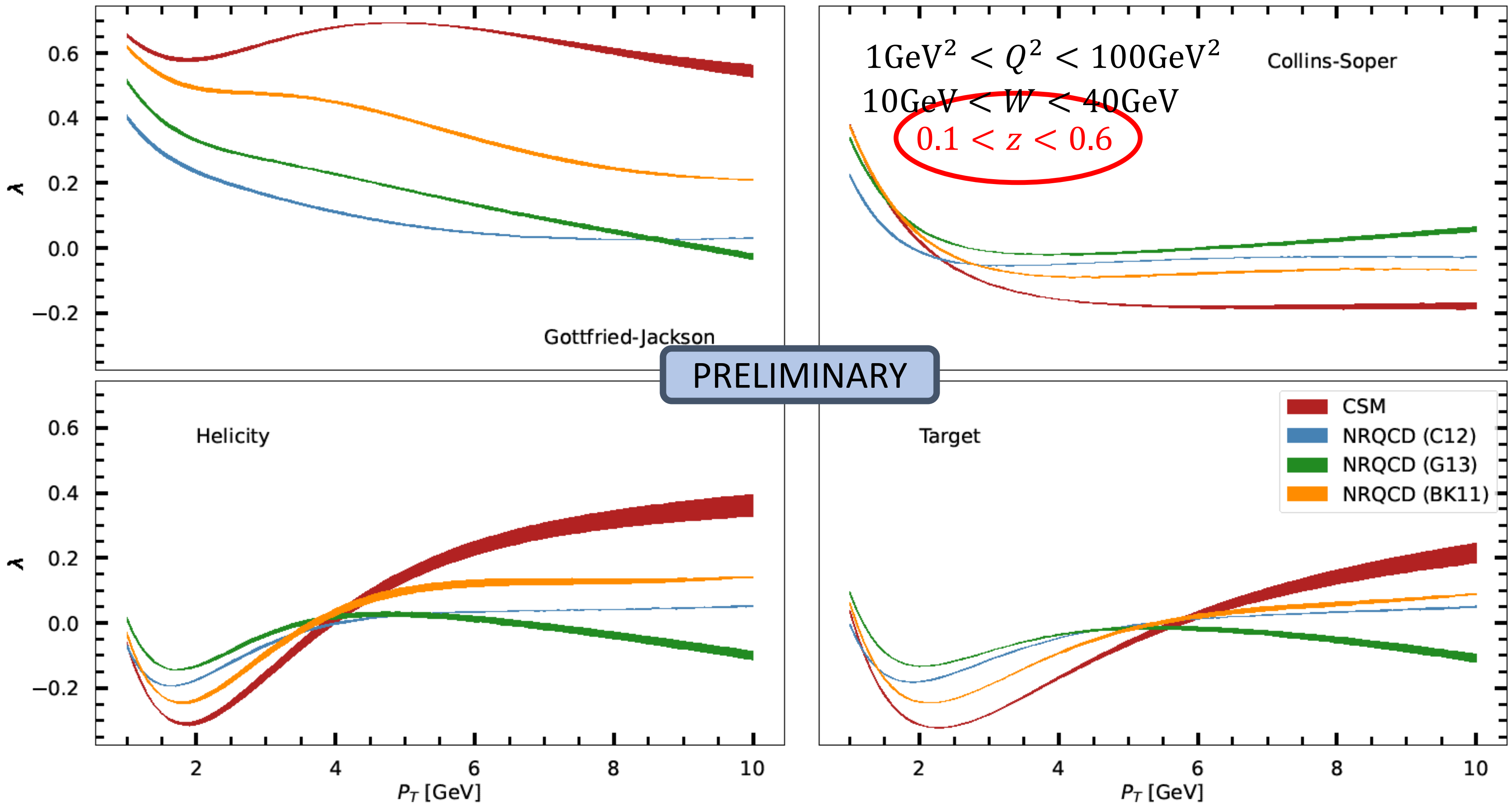
z cuts

- CO $^1S_0$ and ^3P_J waves diverge for $z \to 1$

Can we trust results in such case?

PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$



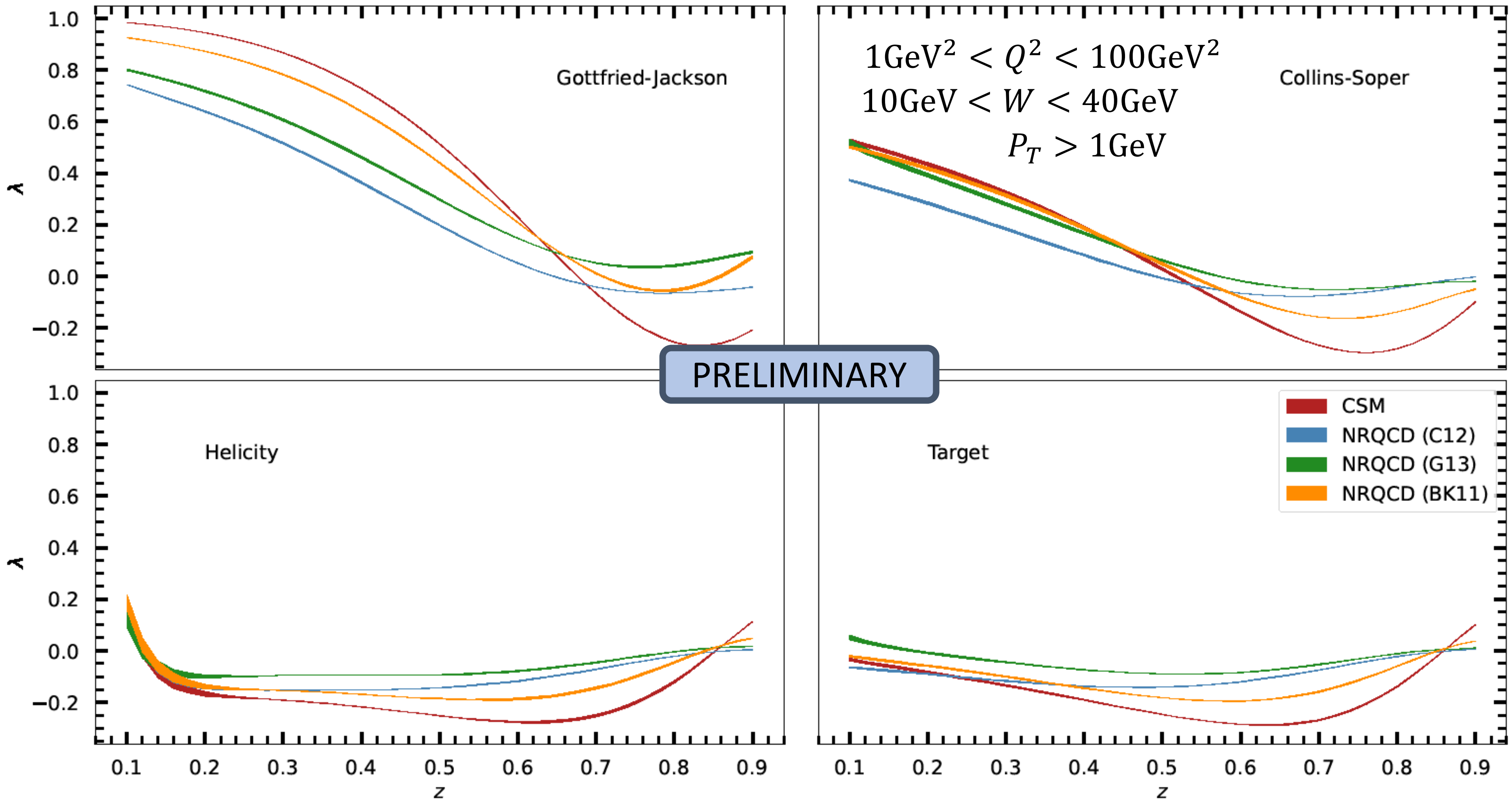
z cuts (new)

- Imposing $z < 0.6$ we get «NRQCD-safe» results
- Higher values at low P_T

→ main contribution *gluon CS wave*

PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$



λ vs z

- Less dependence on LDME choices for NRQCD results
- <u>No apparent divergence</u> for $z > 0.6$

CONCLUSIONS

- Study of J/ψ polarization states in different frames at EIC
- Information extraction regarding TMD PDFs
- In TMD region $\mathcal{W}_{\Delta\Delta}^{\perp}$ is related to the linearly polarized gluon distribution
- Proper shape functions are necessary to provide correct expressions in the intermediate q_T region
- Preliminary predictions for EIC in the collinear approach already highlight the importance of precise polarization data

Thanks for the attention

BACK-UP

HADRONIC TENSOR PARAMETERISATION

Properties of the hadronic tensor $W^{\mu\nu}$

Gauge-invariance

$$q^\mu W_{\mu\nu}(q, P, P_\psi) = q^\nu W_{\mu\nu}(q, P, P_\psi) = 0$$

projector to q orthogonal space

$$\widehat{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

Parity

$$W_{\mu\nu}(q, P, P_\psi) = W_{\mu\nu}(\overline{q}, \overline{P}, \overline{P}_\psi)$$

Hermeticity

$$W_{\mu\nu}(q, P, P_\psi) = W^*_{\nu\mu}(q, P, P_\psi)$$

General parameterisation of $W^{\mu\nu}$

$$W^{\mu\nu}(q, P, P_\psi) = -W_1\, \widehat{g}^{\mu\nu} + W_2\, \widehat{P}^\mu \widehat{P}^\nu - \frac{1}{2} W_3\, (\widehat{P}^\mu \widehat{P}^\nu_\psi + \widehat{P}^\mu_\psi \widehat{P}^\nu) + W_4\, \widehat{P}^\mu_\psi \widehat{P}^\nu_\psi$$

HELICITY STRUCTURE FUNCTIONS

$$\frac{1}{B_{\ell\ell}}\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega} = \frac{y}{(8\pi)^2}\frac{\alpha^2}{Q^4}L^{\mu\nu}W_{\mu\nu}$$ with $$L^{\mu\nu}W_{\mu\nu} = \frac{Q^2}{y^2}\left\{\left[1+(1-y)^2\right]\mathcal{W}^T + (1-y)\,\mathcal{W}^L\right\}$$

Introduction of helicity structure functions

$$\mathcal{W}^{\mathcal{P}} = \sum_{\lambda,\lambda'} W^{\mathcal{P}}_{\alpha\beta}\,\epsilon^{\alpha}_{\lambda}(P_\psi)\,\epsilon^{\beta\,*}_{\lambda'}(P_\psi)\,\delta_{\lambda\lambda'} = \sum_{\lambda,\lambda'}\mathcal{W}^{\mathcal{P}}_{\lambda\lambda'}\,\delta_{\lambda\lambda'}$$ where $$\mathcal{W}^{\mathcal{P}}_{\lambda\lambda'} \equiv \epsilon^{\alpha}_{\lambda}(P_\psi)\,\epsilon^{*\beta}_{\lambda'}(P_\psi)\,W^{\mathcal{P}}_{\alpha\beta}$$

From hadronic tensor conservation properties

hermeticity $\mathcal{W}^{\mathcal{P}}_{\lambda\lambda'} = \mathcal{W}^{\mathcal{P}\,*}_{\lambda'\lambda}$ parity $\mathcal{W}^{\mathcal{P}}_{\lambda\lambda'} = (-1)^{\lambda+\lambda'}\mathcal{W}^{\mathcal{P}}_{-\lambda-\lambda'}$

Generic form of the helicity structure tensor

$$W^{\mathcal{P}}_{\alpha\beta} = -(\mathcal{W}^{\mathcal{P}}_T + \mathcal{W}^{\mathcal{P}}_{\Delta\Delta})\,(g_{\alpha\beta} - T_\alpha T_\beta) + (\mathcal{W}^{\mathcal{P}}_L - \mathcal{W}^{\mathcal{P}}_T - \mathcal{W}^{\mathcal{P}}_{\Delta\Delta})\,Z_\alpha Z_\beta - \mathcal{W}^{\mathcal{P}}_{\Delta}\,(X_\alpha Z_\beta + Z_\alpha X_\beta) - 2\mathcal{W}^{\mathcal{P}}_{\Delta\Delta}\,X_\alpha X_\beta$$

Summing over the decaying lepton pol. vectors generates the cross section parameterisation

$$\mathcal{W}^{\mathcal{P}} = W^{\mathcal{P}}_{\alpha\beta}\sum_{\sigma=-1,1}\epsilon^{\alpha}_{\sigma}(P_\psi)\,\epsilon^{\beta\,*}_{\sigma}(P_\psi) = -W^{\mathcal{P}}_{\alpha\beta}\,g^{\alpha\beta}_{l\perp}$$

$$g^{\alpha\beta}_{l\perp} = g^{\alpha\beta} - \frac{1}{l\cdot P_\psi}\left(l^\alpha P^\beta_\psi + l^\beta P^\alpha_\psi\right) + \frac{M^2_\psi}{(l\cdot P_\psi)^2}\,l^\alpha l^\beta$$

PARTONIC CROSS SECTION

Polarization partonic cross section for $2 \to 1$ photon-gluon fusion $\gamma^* + g \to J/\psi$

$$\sigma_L^{\perp} = (4\pi)^2 \frac{\alpha\alpha_s e_c^2}{M_\psi} \left[\frac{1}{3} < \mathcal{O}_8(^1S_0) > + \frac{4}{M_\psi^2} < \mathcal{O}_8(^3P_0) > \right] \delta(1-z)$$

$$\sigma_T^{\perp} = (4\pi)^2 \frac{\alpha\alpha_s e_c^2}{M_\psi} \left[\frac{1}{3} < \mathcal{O}_8(^1S_0) > + \frac{4}{M_\psi^2} \frac{3M_\psi^4 + Q^4}{\left(M_\psi^2 + Q^2\right)^2} < \mathcal{O}_8(^3P_0) > \right] \delta(1-z)$$

$$\sigma_{\Delta\Delta}^{\perp} = -4(4\pi)^2 \frac{\alpha\alpha_s e_c^2}{M_\psi\left(M_\psi^2 + Q^2\right)} < \mathcal{O}_8(^3P_0) > \delta(1-z)$$

$$\sigma_{\mathrm{L}}^{\parallel} = 64(4\pi)^2 \frac{\alpha\alpha_s e_c^2\, Q^2}{M_\psi\left(M_\psi^2 + Q^2\right)^2} < \mathcal{O}_8(^3P_0) > \delta(1-z)$$

MISMATCH

Full form of the mismatch

$$\mathcal{W}_{\Delta\Delta}^{\perp}\Big|_{coll.} - \mathcal{W}_{\Delta\Delta}^{\perp}\Big|_{TMD} = 0$$

$$\mathcal{W}_{\Lambda}^{P}\Big|_{coll.} - \mathcal{W}_{\Lambda}^{P}\Big|_{TMD} = \sigma_{\Lambda}^{P}\frac{C_A}{\pi^2\left(M_\psi^2 + Q^2\right)\boldsymbol{q}_T^2}\ln\left(\frac{Q^2 + M_\psi^2}{\boldsymbol{q}_T^2}\right)$$

$$(\Lambda, P) = (L, \perp), (T, \perp), (L, \parallel)$$

where TMD result are obtain with the TMD-PDF expansion at $|\boldsymbol{q}_T| \gg \Lambda_{QCD}$

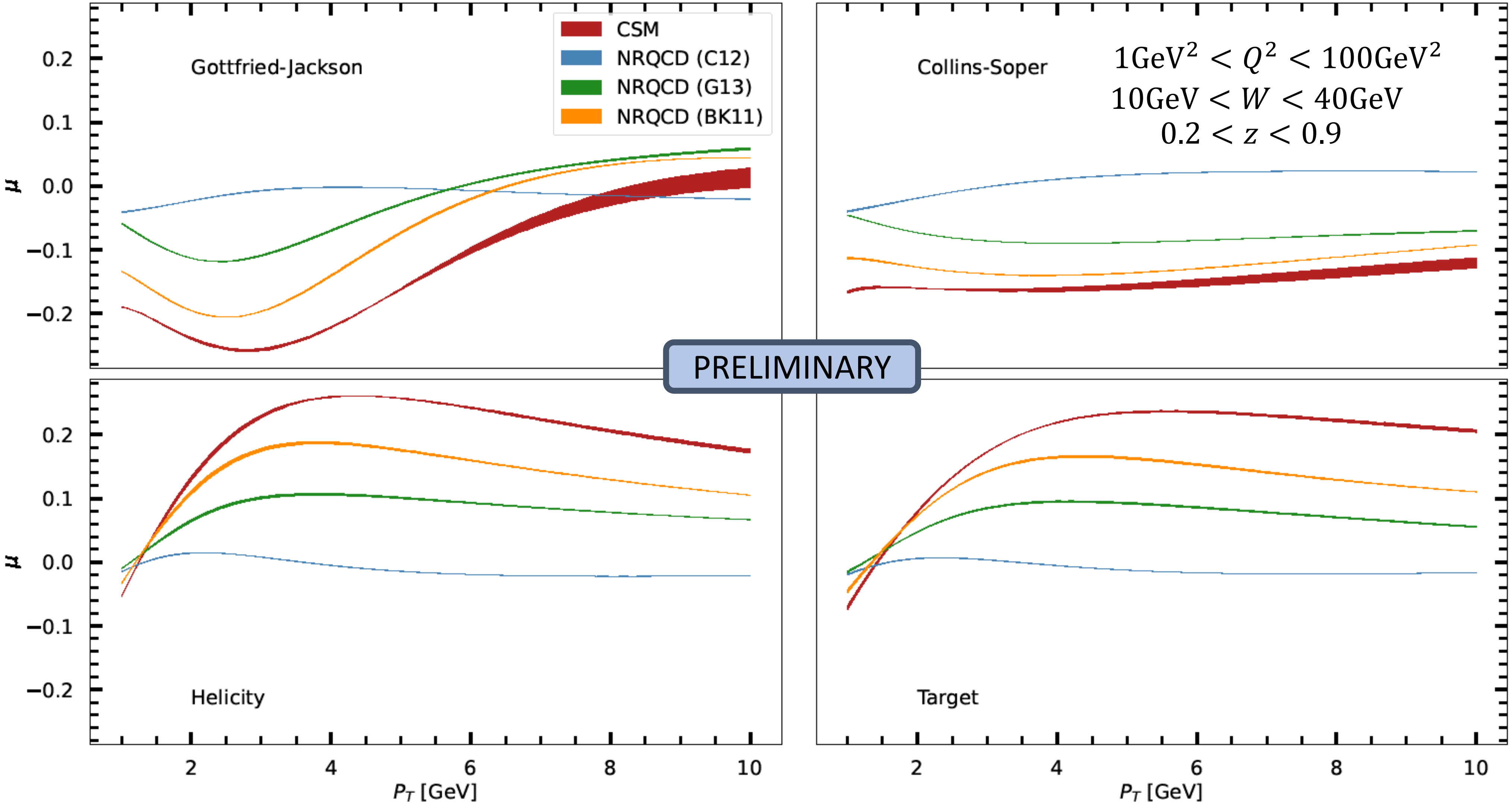
$$\frac{\boldsymbol{q}_T^2}{2m_P^2} h_1^{\perp g}(x, \boldsymbol{q}_T^2; \mu^2) = \frac{\alpha_s}{\pi^2}\frac{1}{\boldsymbol{q}_T^2}\left(\delta P_{gg} \otimes f_1^g + \delta P_{gi} \otimes f_1^g\right)(x, \mu^2)$$

$$f_1^g(x, \boldsymbol{q}_T^2; \mu^2) = \frac{\alpha_s}{2\pi^2\boldsymbol{q}_T^2}\left[\tilde{L}\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) f_1^g(x, \mu^2) + \left(P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^g\right)(x, \mu^2)\right]$$

$$\longrightarrow \quad \tilde{L}\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) = C_A \ln\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) - \frac{11C_A - 4n_f T_R}{6}$$

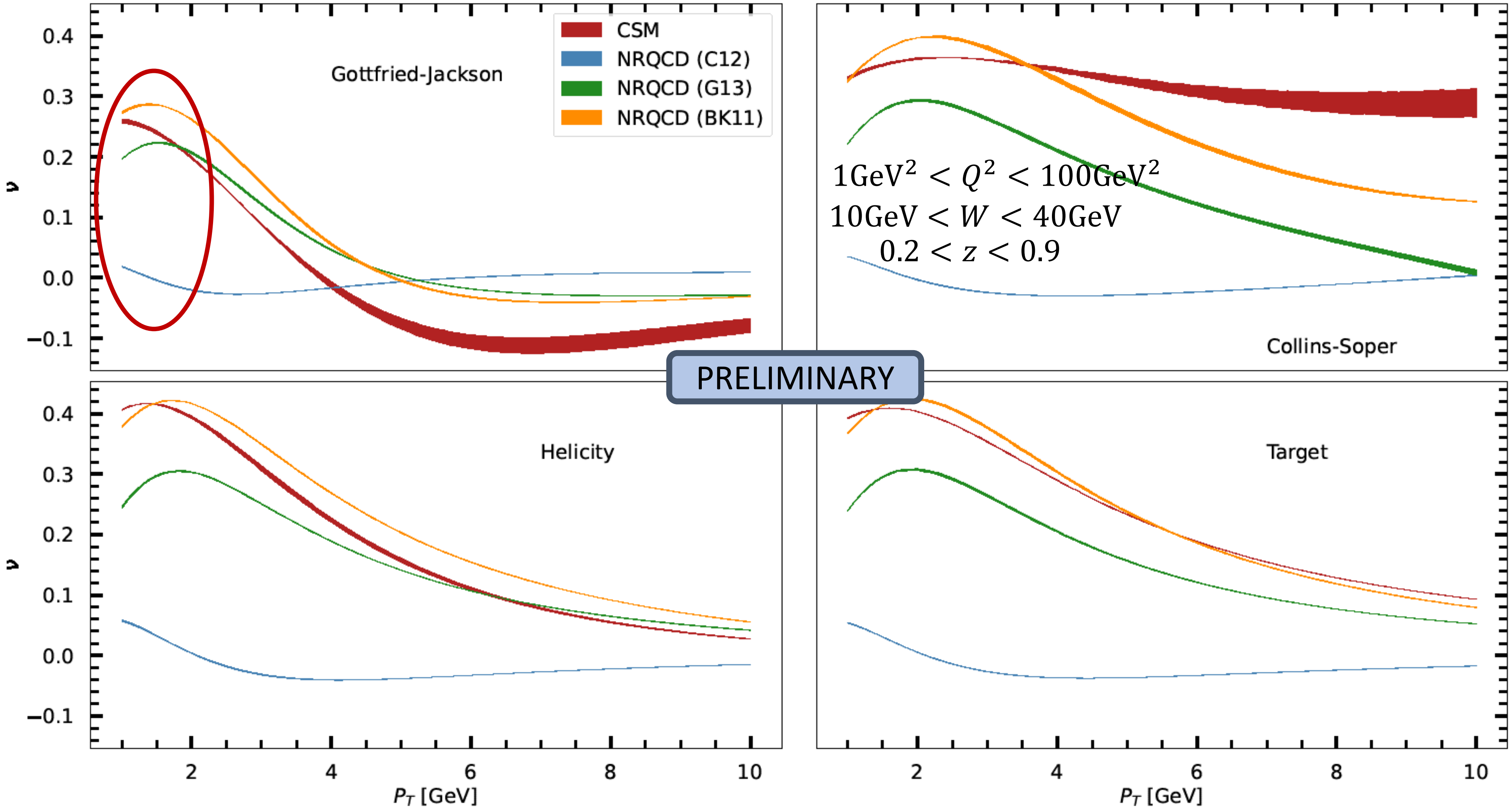
MORE PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$

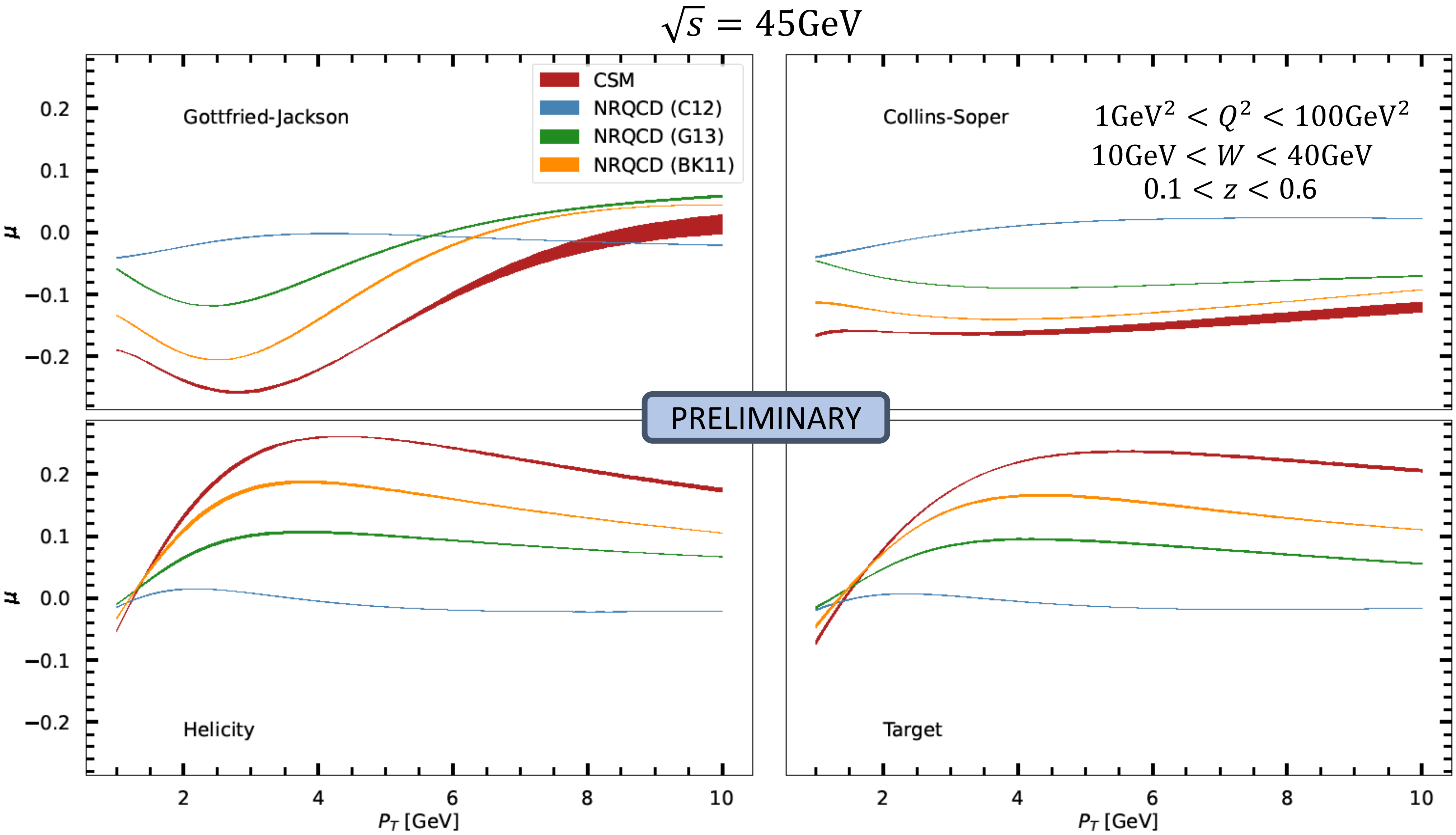


MORE PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$

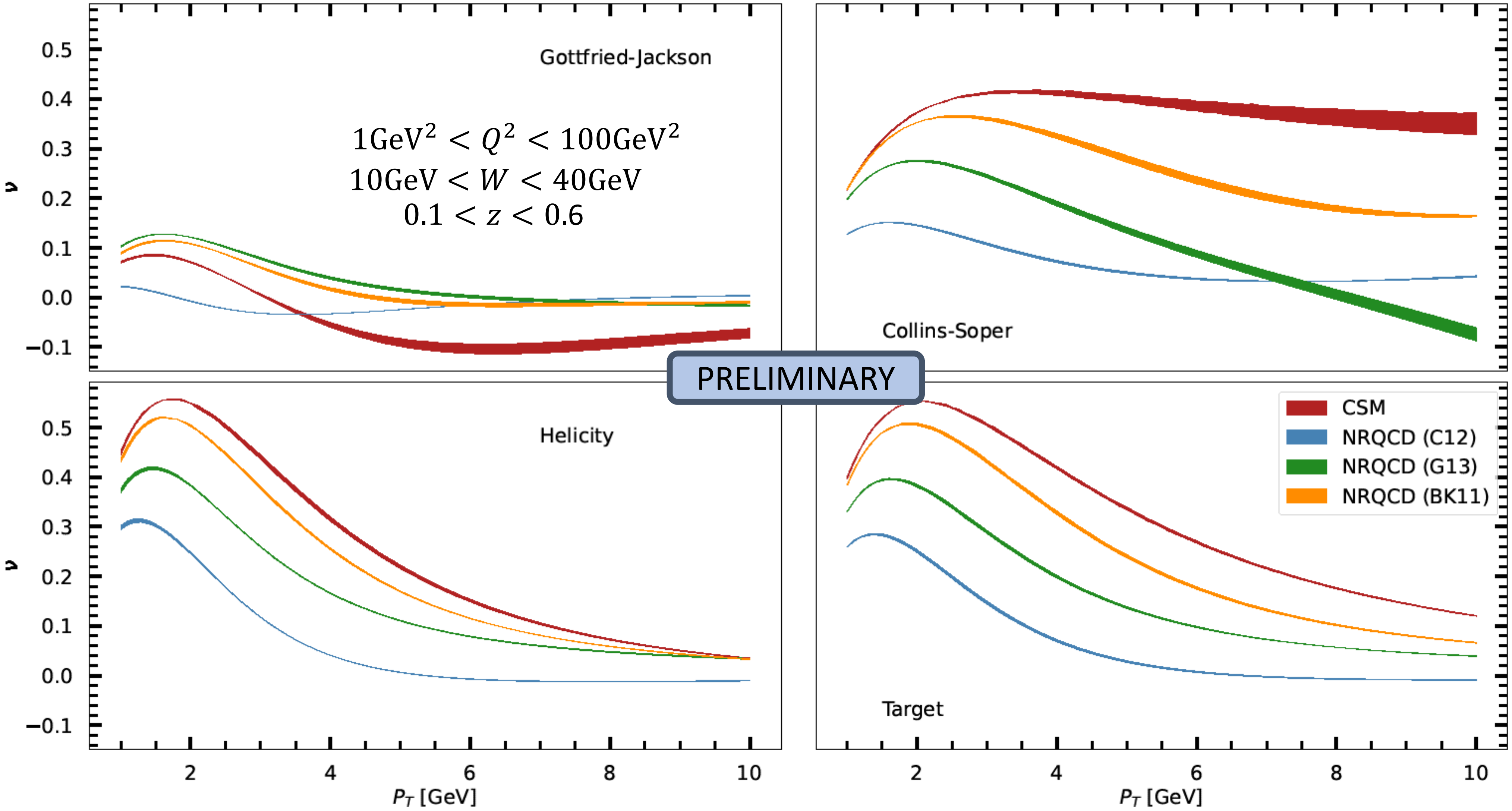


MORE PREDICTIONS FOR *EIC*



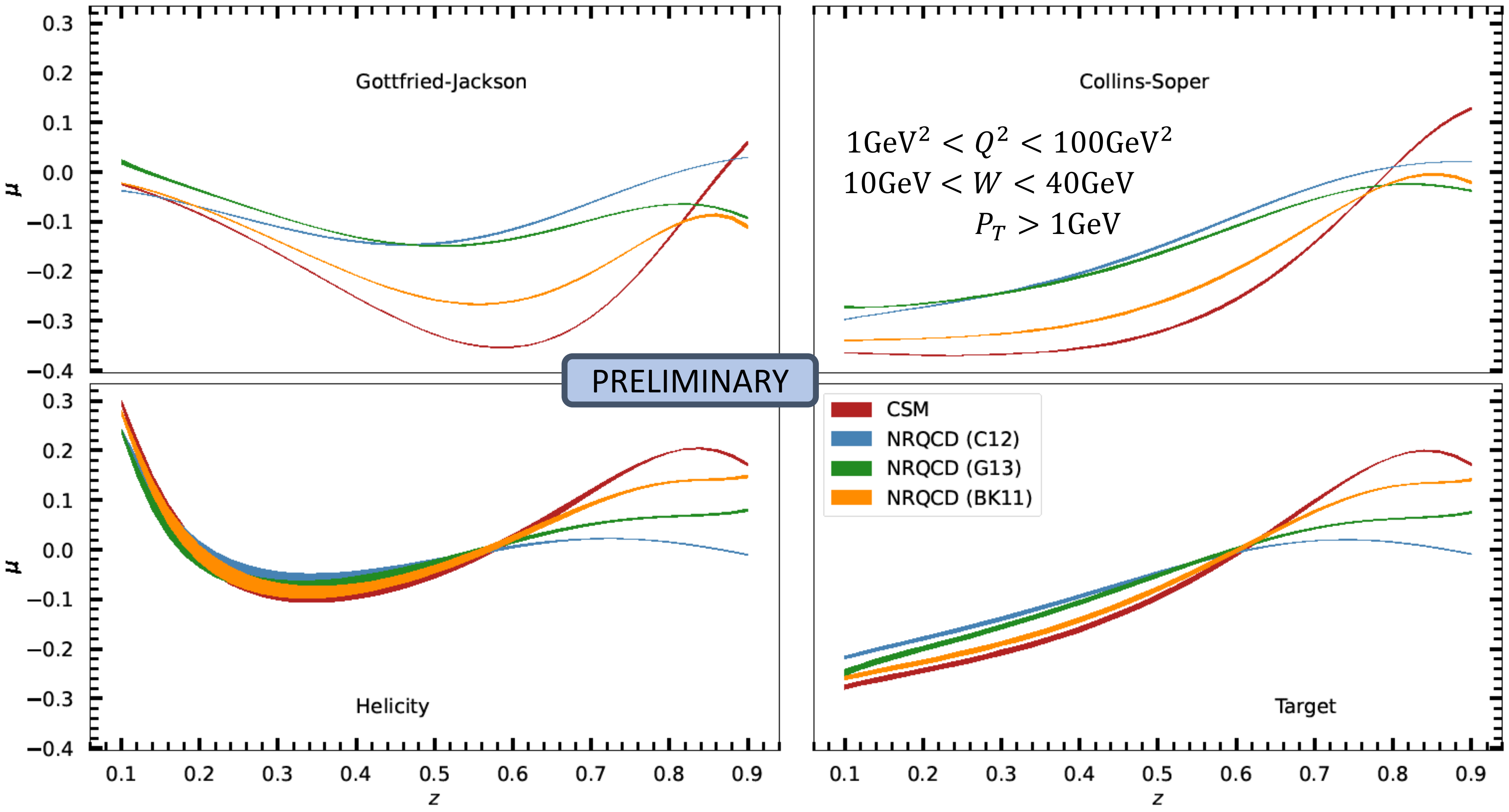
MORE PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$



MORE PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$



MORE PREDICTIONS FOR *EIC*

$\sqrt{s} = 45\text{GeV}$