String Theory

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Bruno and me

- I got to know Bruno following his course on Stat. Mech. in 1964.
- I got immediately fascinated by his way of doing physics, by his very informal personality and by his free spirit.
- Then, at the beginning of 1965, I asked him if I could do my thesis with him.
- He was very positive and he gave me to compute the double bremsstrahlung ($e^+e^- \rightarrow e^+e^- + 2\gamma$) because, at that time, he was thinking to use it as a monitor for ADONE.
- In spring 1965 I followed his course on QED, learned about the Bloch-Nordsieck approximation and I used this approximation to compute the amplitude for very soft photons.
- Then I started to compute it in QED and I obtained some partial results that I could use to get my LAUREA in Feb. 1966.
- In 1966 I got a one-year fellowship in Frascati and came in contact with M. Greco, who was trying to perform the same calculation.
- We joined the forces and after several months we managed to get the final result.

- In the meantime I got a permanent position in Frascati (CNEN).
- Since it was so difficult to compute just tree diagrams I did not want to continue to go into loop calculations as the others in the group.
- I started to work on FESR and S-matrix theory with the full support of Bruno.
- In 1968 I went for three months to Caltech discussing with Frautschi on S-matrix theory and followed the lectures by Feynman.
- In 1969 I got a NATO fellowship to go to work in USA.
- In Dec. 1969 I went to MIT for working with Sergio Fubini.
- I had leave of absence from LNF for one year, but, when I asked to continue it for a second year, I received a negative answer from the Director of LNF.
- In 1970 I resigned from LNF and stayed one more year at MIT.

From the Dual Resonance Model to String Theory

- The S-matrix is usually constructed from the Lagrangian.
- Due to the large value of $\frac{g^2_{\pi NN}}{4\pi} \sim 14$, in the sixties, a Lagrangian was considered useless for strong interactions.
- The idea was instead to write directly the S-matrix implementing the axioms of S-matrix theory through the bootstrap approach.
- The most successful result of S-matrix theory was the Veneziano Model (1968) for the amplitude of four scalars ($\alpha_0 + \alpha' m^2 = 0$):

$$B_4 \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} \underset{s>>|t|}{\sim} \Gamma(-\alpha(t))(-\alpha')s^{\alpha(t)} \; ; \; \alpha(t) = \alpha_0 + \alpha' s$$

  and its extension to the scattering of N scalar particles (1969)

$$B_N \sim \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{i=1}^{N} \left[(z_i - z_{i+1})^{\alpha_0 - 1} \right] \prod_{i<j} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

  that was called Dual Resonance Model.
- It satisfies the axioms of S-matrix theory with an infinite number of zero-width resonances lying on linearly rising Regge trajectories.

- Is it a consistent S-matrix? What is the underlying theory?
- Fubini, Gordon and Veneziano (1969) and also Nambu rewrote the previous amplitude introducing an infinite set of harmonic oscillators spaced by integers: $[a_{n\mu}, a^\dagger_{m\nu}] = \delta_{nm}\eta_{\mu\nu}$ with $n, m = 1, 2 \ldots$:

$$B_N = \int \frac{\prod_{i=1}^N dz_i}{dV_{abc}} \langle 0| \prod_{i=1}^N V(z_i, p_i)|0\rangle \;\; ; \;\; V(z,p) =: e^{ipQ(z)} :$$

$$Q^\mu(z) = \hat{q}^\mu - 2i\alpha' \hat{p}^\mu \log z + i\sqrt{2\alpha'} \sum_{n=1}^\infty \sqrt{n}\left(a_n^\mu z^{-n} - a_n^{\dagger\mu} z^n\right)$$

- Factorising the amplitude at the pole for $s_{ij} \sim m^2 = \frac{N-1}{\alpha'} (\alpha_0 = 1)$ $(N = 0, 1 \ldots)$ one gets the states $|\lambda\rangle$ with mass m^2 that contribute to its residue:

$$N|\lambda\rangle = \sum_{n=1}^\infty n a_n^\dagger \cdot a_n |\lambda\rangle \;\; ; \;\; \eta_{\mu\nu} = (-1, 1, 1, 1)$$

- Virasoro (1969) found that only for $\alpha_0 = 1$ there are extra conditions (Virasoro conditions) that possibly eliminate ghosts.

- For $\alpha_0 = 1$ together with Del Giudice (1970) we found that the on-shell physical states satisfy the conditions

$$L_n|Phys.\rangle = (L_0 - 1)|Phys.\rangle = 0 \ ; \ \ n > 0$$

generalising the Fermi condition in QED: $\partial^\mu A_\mu^{(+)}|Phys.\rangle = 0$.

- Fubini and Veneziano (1970) showed that the Virasoro generators L_n satisfy the conformal algebra in two space-time dimensions called Virasoro algebra:

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{D}{24} n(n^2 - 1)\delta_{n+m,0}$$

J. Weis (1970) found the central charge, using the expression of L_n in terms of the oscillators.

- Campagna, Fubini, Napolitano and Sciuto (1970) generalised the amplitude to include any physical state (not just the ground state).
- Each physical state is described by a vertex operator that is a conformal field with dimension $\Delta = 1$.
- For $\alpha_0 = 1$ the lowest state is a tachyon with mass $m^2 = -p^2 = -\frac{1}{\alpha'}$ and the next state is a massless photon with vertex operators

$$V(p, z) =: e^{ipQ(z)} : \; ; \quad V_i(k, z) = (\epsilon_i \frac{dQ(z)}{dz}) e^{ikQ(z)}$$

- The N-point amplitude involving N physical states can be written In terms of the vertex operators:

$$A_N = \int \frac{\prod_{i=1}^{N} dz_i}{dV_{abc}} \langle 0| \prod_{i=1}^{N} V_{\alpha_i}(z_i, p_i)|0\rangle$$

- Democracy among the physical states, as suggested by Chew in "The analytic S Matrix", a basis for nuclear democracy, 1966.

- The photon vertex operator was then used to construct the $(D-2)$-dimensional DDF operators:

$$A_{n,i} = \frac{i}{\sqrt{2\alpha'}} \oint_0 dz(\epsilon_i \frac{dQ(z)}{dz})e^{ikQ(z)} \; ; \; k^2 = \epsilon k = 0 \;\; ; \;\; [L_m, A_{n,i}] = 0$$

- They satisfy the algebra of the harmonic oscillators:

$$[A_{n,i}, A_{m,j}] = n\delta_{ij}\delta_{n+m;0} \;\; ; \;\; A_{-m,i} \equiv A^{\dagger}_{m,i} \;\; ; \;\; n > 0$$

and generate an infinite number of physical states with positive norm (no ghosts): but not all of them for arbitrary D (only for $D = 26$).

- Already in 1969 Nambu, Nielsen and Susskind suggested that the structure underlying the DRM was that of a string theory.
- In their formulation the Virasoro algebra was a classification algebra as in Conformal Field Theory, while in the DRM was a gauge algebra needed to eliminate ghost states.
- The Nambu-Goto action was written down in 1970

$$L_{part.} = -m \int \sqrt{-dx_\mu dx^\mu} \;\; ; \;\; L_{string} = -T \int \sqrt{-d\sigma_{\mu\nu} d\sigma^{\mu\nu}}$$

  m is the mass of the particle and $T = \frac{1}{2\pi\alpha'}$ is the string tension.
- But it took a while to understand how to use it.
- In terms of the world-sheet coordinates σ and τ one gets

$$L_{String} = -T \int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2(x')^2}$$

  where $x^\mu(\tau, \sigma)$ is the string coordinate, $\dot{x}^\mu = \frac{\partial x^\mu}{\partial \tau}$ and $(x')^\mu = \frac{\partial x^\mu}{\partial \sigma}$.
- Invariant under any choice of coordinates σ and τ.

- More convenient to use a simpler and classically equivalent action:

$$S(x,g) = -\frac{T}{2}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma \sqrt{-g} g^{ab}\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu}$$

  g^{ab} is the metric of the two-dim world-sheet(σ, τ).
- The Eq. of motion for g_{ab} implies the vanishing of the world-sheet energy-momentum tensor:

$$T_{ab} = \partial_a x \partial_b x - \frac{1}{2} g_{ab} g^{cd} \partial_c x \partial_d x = 0 \Longrightarrow \dot{x}^2 + (x')^2 = \dot{x}x' = 0$$

- Choosing the conformal gauge

$$g_{ab} = \rho(\sigma, \tau)\eta_{ab}$$

  we get a free action with the constraint of the vanishing of T_{ab}.
- The gauge is not completely fixed because we can still perform conformal transformation staying in this gauge.
- To fix the gauge completely we can go to the light-cone gauge:

$$x^+ = 2\alpha' p^+ \tau \;\; ; \;\; x^\pm = \frac{x^0 \pm x^{D-1}}{\sqrt{2}}$$

- Using the light-cone gauge condition and the vanishing of the two-dim energy momentum tensor, one can determine x^- in terms of the components $x^\perp$ (orthogonal to $x^\pm$).
- Only independent components are the $D-2$ transverse $x^\perp$.
- This analysis was performed by Goddard, Goldstone, Rebbi and Thorn (End of 1972).
- They checked that the D-dimensional Lorentz generators, written only in the terms of the $D-2$ transverse $x^\perp$, satisfy the Lorentz algebra only if

$$\alpha_0 = 1 \ \ ; \ \ D = 26$$

- For $D = 26$ the DDF operators generate a complete set of physical states implying that the bosonic string is ghost free.
- This shows that the theory underlying the DRM for $D = 26$ is the string theory described by the Nambu-Goto action.

- Even before the connection with string theory, the DRM, with the infinite set of zero width resonances, was considered a tree diagram of a unitary theory: no ghosts but $\Gamma_T \neq \sum_i \Gamma_i$.
- In order to implement unitarity one-loop and even multiloop amplitudes were constructed (Alessandrini, Amati, Lovelace).
- Lovelace (1970) showed that the non-planar loop had cuts violating unitarity unless $D = 26$.
- If $D = 26$ those cuts become poles that later turned out to be the states of a closed string also lying on linearly rising Regge trajectories:

$$\alpha_{open}(s) = 1 + \alpha' s \;\; ; \;\; \alpha_{closed}(s) = 2 + \frac{\alpha'}{2} s$$

- Unitarity requires that open strings always include closed strings
- Closed strings require open strings but only at non-perturbative level.
- Gauge theories always include Gravity and viceversa.

The Dual Pion Model

- A N-point amplitude for pions was proposed by Neveu and Schwarz (1971).
- The amplitude involving three pions vanishes as in pion physics (the pion has G-parity=-1)
- All previous analysis can be done for the NS model finding that it corresponds to a string having also spin degrees of freedom in it.
- It has no ghosts if

$$\alpha_0 = 1 \ ; \ D = 10$$

- Actually, if $\alpha_0 = \frac{1}{2}$, as for the ρ Regge trajectory (for $m_\pi = 0$), one gets the four-point Lovelace-Shapiro model (1969):

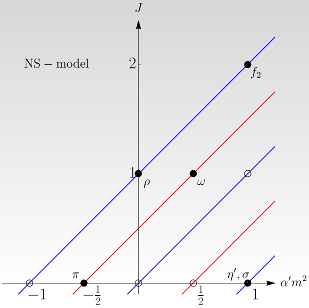
$$A(s,t) \sim \frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))} \ ; \ \alpha_\rho(s) = \frac{1}{2} + \alpha' s \ ; \ \alpha' = \frac{1}{2m_\rho^2}$$

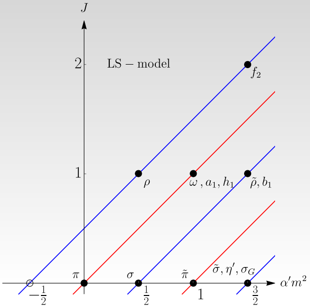
with Adler zeroes as expected for pions (A(s=0, t=0) =0).

- The N-point generalisation of the LS model is discussed in a recent paper with M. Bianchi and D. Consoli (2020):

$$A_N = \int \frac{\prod_{1=1}^{N} d\theta_i dz_i}{dV_{abc}} \prod_{i<j} (Z_i - Z_j)^{2\alpha' k_i k_j} \prod_{i=1}^{N} (Z_i - Z_{i+1})^{-\frac{1}{2} - \alpha' m_\pi^2}$$

  using a super-conformal formalism, where $Z_i = (z_i, \theta_i)$
- It has the correct Adler zeroes.
- It reduces to the non-linear σ model when $\alpha' \to 0$.
- But it has negative norm states: ghosts.
- It seems impossible to write a consistent N-pion amplitude with linearly rising Regge trajectories.





- The particles on the Regge trajectories with half-integer intercepts, as the pion, have G-parity (-1), while those on the Regge trajectories with integer intercepts have G-parity (+1).
- Amplitudes with an odd number of particles with G-parity (-1) vanish.

Unification of gauge theories and gravity

- In hadron physics only the pion is massless in the chiral limit.
- The consistent string theories that we have discussed do not allow for a massless pion, but contain massless gauge fields in the open string sector and a massless graviton in the closed string sector.
- In 1973 QCD was formulated and those interested in hadron physics left DRM (string theory) and joined QCD.
- In 1974 Scherk and Schwarz proposed to use string theory, not for hadrons, but as a theory consistently unifying gauge theories with gravity.
- Gauge theories and GR are not put by hand, but emerge together as an unavoidable part of the theory.
- Necessary for a consistent theory with massless spin 1 and 2.
- A single interaction, that among strings. Only parameter is α'.

- The string coupling constant g_s that enters in the loop expansion is not a free parameter but is given by the vacuum expectation value of a particular state of the closed string, called the dilaton, $g_s \sim e^{\phi}$.
- String theory is an extension of Field Theory: field theory amplitudes are recovered in the limit of $T \to \infty$ or $\alpha' \to 0$.
- The softness of string at high energy that was a problem for hadrons becomes now a virtue.

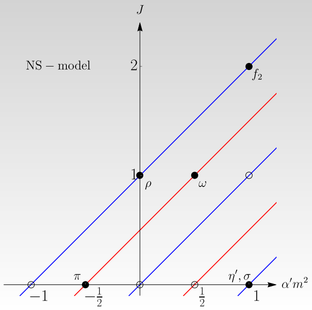
From the RNS model to superstring

- In addition to the bosonic oscillators the Ramond-Neveu-Schwarz model has also an infinite set of fermionic oscillators.
- It contains a sector with ten-dimensional fermions and a sector with ten-dimensional bosons.
- The spectrum of states in the bosonic sector is given by

$$\alpha' m_B^2 = N - \frac{1}{2} \quad ; \quad N|\lambda\rangle = \sum_{n=1}^{\infty} \left(n a_n^\dagger a_n + (n - \frac{1}{2}) \psi_{n-\frac{1}{2}}^\dagger \psi_{n-\frac{1}{2}} \right) |\lambda\rangle$$

where N can be integer and half-integer.

- The lowest state is still a tachyon ($|0\rangle$) and the next state is a massless gauge field $\psi_{\frac{1}{2};\mu}^\dagger |0\rangle$.



- One can define a world-sheet fermion number:

$$(-1)^F \ ; \ F = \sum_{n=1}^{\infty} \psi^{\dagger}_{n-\frac{1}{2}} \psi_{n-\frac{1}{2}} - 1$$

The states with an even (odd) number of fermionic oscillators are even (odd) under the action of $(-1)^F$.

- The spectrum of states in the fermionic sector is given by:

$$\alpha' m_F^2 = N \ ; \ \ N|\lambda\rangle = \sum_{n=1}^{\infty} \left(n a_n^\dagger a_n + n \psi_n^\dagger \psi_n \right) |\lambda\rangle$$

  where N is an integer.
- The lowest state is a massless ten-dimensional fermion.
- There is a fermionic zero mode that satisfies the same algebra as that of the Dirac Γ-matrices:

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

- This means that the ground vacuum state $|0, A\rangle$ has a ten-dimensional Dirac spinor index
- Also in this case we have a fermion number operator:

$$(-1)^F = \Gamma_{11}(-1)^{F_R} \ ; \ \ F_R = \sum_{n=1}^{\infty} \psi_n^\dagger \psi_n$$

- In 1976 Gliozzi-Scherk and Olive proposed to truncate the spectrum of the RNS model keeping only the states that are even under the action of the fermion number operator:

$$(-1)^F|\psi\rangle = |\psi\rangle$$

This is called GSO projection.

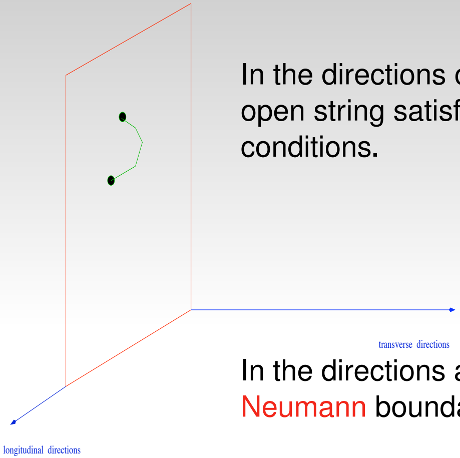
- The GSO projection eliminates the states in the NS sector that lie on half-integer Regge trajectories and in R sector imposes to the ground state to be a Weyl-Majorana fermions.
- This is the first string theory without a tachyon in the spectrum.
- The two lowest states are a gauge field in the bosonic and a massless Weyl-Majorana fermion in the fermionic sector.
- They have the same number of components 8 in $D = 10$.
- It turns out that, after the GSO projection, we get at each level the same number of bosons and fermions.
- One gets the spectrum of the open type I string theory that is supersymmetric in $D = 10$.
- There is also a supersymmetric closed string sector with gravitons, gravitinos and other massless states.

Superstring theories: type IIA, type IIB

- The two closed superstring have two bosonic sector (NS-NS and R-R) and two fermionic sectors (NS-R and R-NS).
- Type IIB
  NS-NS: $G_{\mu\nu}, \phi, B_{\mu\nu}$; R-R: $C_0, C_{2\mu\nu}, C_{4\mu\nu\rho\sigma}$
  R-NS+NS-R: 2 gravitinos+2 dilatinos
  Two gravitinos and dilatinos have the same chirality: chiral theory.
- Type IIA
  NS-NS: $G_{\mu\nu}, \phi, B_{\mu\nu}$; R-R: $C_{1\mu}, C_{3\mu\nu\rho}$
  R-NS+NS-R: 2 gravitinos+2 dilatinos
  Two gravitinos and dilatinos have opposite chirality: non-chiral theory.
- Those theories contain potentials with more than one index.
- Together with A. Cappelli, E. Castellani and F. Colomo we edited a book "The birth of string theory" where we discuss all previous developments between the middle of the sixties to the middle of the eighties.

D(irichlet)p-branes

- They are generalizations of the electromagnetic potential A_μ
$$\int A_\mu dx^\mu \Longrightarrow \int A_{\mu_1\mu_2\ldots\mu_{p+1}} d\sigma^{\mu_1\mu_2\cdots\mu_{p+1}}$$
As the electromagnetic field is coupled to point-like particles so they are coupled to p-dimensional objects.
- There exist classical solutions of the low-energy string effective action that are coupled to the metric, the dilaton and are charged with respect a RR field. For them we get
$$C_{01\ldots p} \sim \frac{1}{r^{D-3-p}} \Longleftrightarrow C_0 \sim \frac{1}{r} \;\; \text{if } D=4, p=0$$
They are additional non-perturbative states of string theory with tension and RR charge given by:
$$\tau_p = \frac{Mass}{p-volume} = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} \quad ; \quad \mu_p = \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p}$$
- g_s is the string coupling constant.
- Polchinski showed in 1994 that, in string theory, these objects are required by T-duality that changes the Neumann into Dirichlet boundary conditions for the open strings.
- Therefore they are called D(irichlet)p-branes.

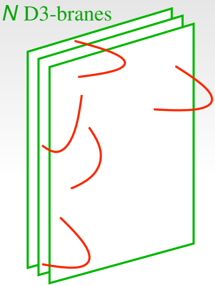


In the directions orthogonal to the brane the open string satisfies Dirichlet boundary conditions.

In the directions along the brane they satisfy Neumann boundary conditions.

- The open strings (gauge theory) live in the (p+1)-dim. worldvolume of a Dp-brane, while closed strings (gravity) live in the entire ten dimensional space.

- If we have a stack of N parallel D branes, then we have N^2 open strings having their endpoints on the D branes:



Open strings attached to a stack of parallel D branes transform according to the adjoint representation of $U(N)$

- Massless bosonic states $\Longrightarrow$ gauge fields of $U(N)$.
- Massless fermionic states $\Longrightarrow$ their supersymmetric partners, called gauginos.

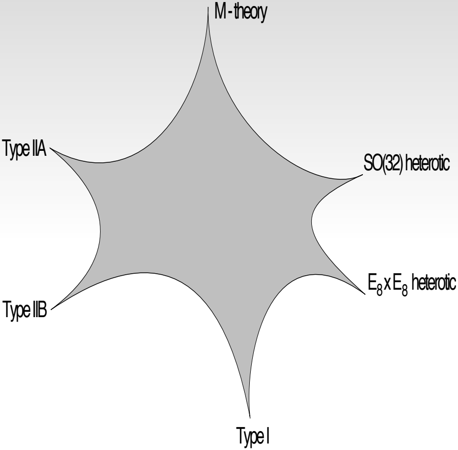
- The gauge theory leaving on N maximally supersymmetric D3-branes is the maximally supersymmetric $\mathcal{N} = 4$ super-Yang-Mills with $U(N)$ gauge group (one gluon, 6 scalars and 4 Majorana fermions).
- It is conformal invariant with $\beta = 0$.
- Maldacena conjectured that this theory is equivalent to 10-dim string theory on $AdS_5 \otimes S_5$.
- By now a lot of evidence for it and a lot of applications both for hadrons and condensed matter.
- How to extend it to non-conformal theories as QCD ?
- What is the string theory beyond the 't Hooft large N expansion of QCD ?

M-theory

- Up to now we have discussed the three superstring theories that were constructed before 1985.
- Only after the first string revolution two other superstring theories were constructed.
- They are called heterotic strings.
- They are closed string theories, but, unlike type II theories, they contain a gauge theory.
- There exist two heterotic string theories, one with gauge group $SO(32)$ and the other with gauge group $E_8 \times E_8$.
- With those gauge symmetries no gauge and gravitational anomalies.
- Those five superstring theories are all consistent string theories in ten-dimensional Minkowski space-time.
- They are perturbatively inequivalent and supersymmetric in $D = 10$.

- This has generated a puzzle for many years: If string theory is a unique theory why do we have 5 theories instead of just 1?
- At non-perturbative level they are related to each other through a web of weak-strong dualities.
- They are all part of a unique 11-dimensional theory, called M-theory, Witten, 1995.
- Not easy to investigate because an explicit formulation of M theory is lacking.
- At low energy it reduces to 11-dim supergravity.

- The five 10-dimensional string theories are actually part of a unique 11-dimensional theory called M theory.



Polchinski, hep-th/9607050

- $R_{11} \sim g_s \sqrt{\alpha'}$ not seen in string perturbation theory.

From string theory to real world

- We observe only 4 and not 10 or 11 non-compact directions.
- Therefore 6 of the 10 dimensions must be compactified and small: $R^{1,9} \to R^{1,3} \times M_6$ where M_6 is a compact manifold.
- In order to preserve at least $N = 1$ supersymmetry M_6 must be a Calabi-Yau manifold.
- Then low-energy physics will depend not only on α' and g_s , but also on the size and shape of the manifold M_6.
- Compactifying 6 of the 10 dimensions we generate a bunch of scalar fields (moduli) corresponding to the components g_{ij} of the metric ($g_{MN} = g_{\mu\nu}, g_{\mu i}, g_{ij}$) and of the other closed string fields in the extra dimensions.
- Their v.e.v., corresponding to the parameters of the compact manifold, not fixed in any order of perturbation theory because their potential is flat.
- This happens also for the dilaton: $\mathrm{e}^{<\phi>} \sim g_s$.

- We get a continuum of consistent and inequivalent string vacua for any value of the moduli ! No good for phenomenology !
- Problem of Moduli stabilization.
- Even if we stabilise the moduli we still have a discrete (and huge) quantity of string vacua: "Landscape Problem".
- How do we fix the vacuum we live in?
- This is still a big problem for String Theory!

THANK YOU !