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Spontaneous symmetry breaking in particle physics

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Abstract

This talk will be mainly historical. I will begin reviewing the appearance of spontaneous symmetry breaking (SSB) in particle physics at the end of the fifties and beginning of the sixties. I will recall Heisenberg non-linear spinor theory and the genesis of the first model (NJL) of fermion mass generation developed in collaboration with Yoichiro Nambu, based on the idea of spontaneous symmetry breaking. Both the non-linear spinor theory and the NJL model are invariant under a chiral transformation (γ_5 - invariance) which was introduced by Bruno Touschek in 1957 and named by Heisenberg the Touschek transformation. Then I will briefly describe the subsequent evolution where the NJL model has been used as an effective theory for low energy QCD and SSB was fundamental in the electroweak unification. Finally, time permitting, I will consider SSB in non-equilibrium. It may be of interest in connection with the matter-antimatter asymmetry in the universe.

CONTENTS

1. PREHISTORY

Spontaneous (dynamical) symmetry breaking

L. Euler, Memoires de l'Academie des Sciences de Berlin, **13***, 252 (1759)*

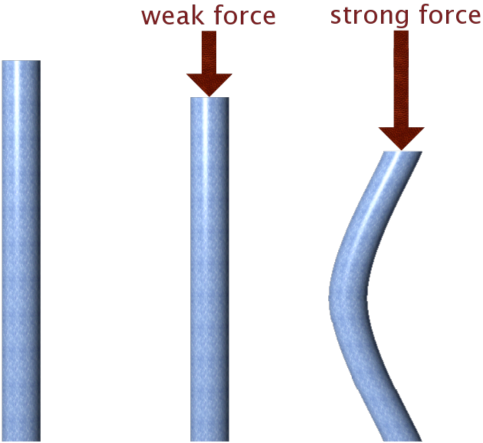

Figure: Elastic rod compressed by a force of increasing strength

Euler formula for the critical load

Euler made also a theory of the critical load

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$

where E is the modulus of elasticity, I the sectional area moment of inertia and L the length of the rod.

Heisenberg non-linear spinor theory

W. Heisenberg, Rev.Mod. Phys. ***29****, 269 (1957); W. Heisenberg, W. H. Dürr, H. Mitter, S. Schlieder, K. Yamazaki, Zeit. f. Naturf.* ***14****, 441 (1959)*

The basic equation of Heisenberg theory is

$$\gamma_\mu \partial_\mu \psi - l^2 \gamma_\mu \gamma_5 \psi (\bar{\psi} \gamma_\mu \gamma_5 \psi) = 0 \qquad (2)$$

which is invariant under the following transformation

$$\psi \to \exp\,[i\gamma_5 \alpha]\psi \qquad (3)$$

$$\bar{\psi} \to \bar{\psi} \exp\,[i\gamma_5 \alpha] \qquad (4)$$

referred to as the Touschek transformation.

IL NUOVO CIMENTO VOL. V, N. 3 1° Marzo 1957

Parity Conservation and the Mass of the Neutrino.

B. F. TOUSCHEK

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Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(ricevuto il 26 Gennaio 1957)

In the frame of the special theory of relativity there are two particles of zero mass: the photon and the neutrino. Whereas the photon mass is fixed to be zero by the principle of gauge invariance no similar principle is known for the neutrino. It is however, quite clear that the mass of the neutrino can be defined to be 0 by the requirement that the equations of motion—and with them the Lagrangian—of the free neutrino field are invariant under the transformation

$$\nu' = \exp[i\gamma_5\alpha]\nu \,. \tag{1}$$

Here ν is the waveoperator of the neutrino and α is a real number. Since γ_5 is a Hermitian matrix we have for $\bar{\nu} = \nu^+\gamma_4$:

field the invariance requirements (1) and (2) ensure that the neutrino mass be zero and that the sum of all Fermi particles minus the sum of all anti-Fermions remains constant. The latter is a property of all known strong interactions. A peculiar situation arises if one tries to construct a Lagrangian in which neutrinos can be singly produced or annihilated. For if one assumes that this Lagrangian is of the usual form

$$\mathfrak{L}' = f(\bar{\nu}\Gamma'\psi_1)(\bar{\psi}_2\Gamma\psi_3) + \text{herm. Conj.} \tag{3}$$

where $\psi_1\psi_2\psi_3$ are three not necessarily different spinor field of mass $\neq 0$, invariance under proper Lorentz transformation requires that

Heisenberg, Rochester Conference 1960

It has been emphasized that non-conservation of isospin in the electromagnetic forces must be due to an asymmetry of the ground state "world". The "world" possesses a very big total isospin, and this ground state is therefore highly degenerate.

.... here we have come to a very general mathematical problem, and we should now get accustomed to those problems in which the lowest state is degenerate. There are many different problems in physics that belong to this class. If one develops a new mathematical theory to deal with this class of problems, in the near future we may make great progress in the theory of elementary particles.

SSB in superconductivity

J. Bardeen, L. N. Cooper, J. R. Schrieffer, Phys. Rev. **108**, *1175 (1957)*

Electrons near the Fermi surface are described by the following equation

$$\begin{aligned} E\psi_{p,+} &= \epsilon_p \psi_{p,+} + \phi \psi^{\dagger}_{-p,-} \\ E\psi^{\dagger}_{-p,-} &= -\epsilon_p \psi^{\dagger}_{-p,-} + \phi \psi_{p,+} \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{\epsilon_p^2 + \phi^2}$$

Here, $\psi_{p,+}$ and $\psi^{\dagger}_{-p,-}$ are the wavefunctions for an electron and a hole of momentum p and spin $+$

Similarity with the Dirac equation

In the Weyl representation, the Dirac equations reads

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_2 + m\psi_1 \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{p^2 + m^2}$$

Here, ψ_1 and ψ_2 are the eigenstates of the chirality operator γ_5

Zero mass boson in superconductivity

P. W. Anderson, Phys. Rev. **112** *1900 (1958); Bogoliubov, Tolmachev, Shirkov, " A New Method in Theory of Superconductivity" (Academy of Sciences of USSR 1958); G. Ricayzen, Phys. Rev.* **115** *795 (1959); Y. Nambu, Phys. Rev.* **117***, 648 (1960)*

Approximate expressions for the charge density and the current associated to a quasi-particle in a BCS superconductor

$$\begin{aligned} \rho(x,t) &\simeq \rho_0 + \frac{1}{\alpha^2}\partial_t f \\ \boldsymbol{j}(x,t) &\simeq \boldsymbol{j}_0 - \boldsymbol{\nabla} f \end{aligned}$$

where $\rho_0 = e\Psi^\dagger \sigma_3 Z \Psi$ and $\boldsymbol{j}_0 = e\Psi^\dagger(\boldsymbol{p}/m)Y\Psi$ with Y, Z and α constants and f satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{\alpha^2}{\partial_t}^2\right) f \simeq -2e\Psi^\dagger\sigma_2\phi\Psi$$

Here, $\Psi^\dagger = (\psi_1^\dagger, \psi_2)$

Plasmons

The Fourier transform of the wave equation for f gives

$$\tilde{f} \propto \frac{1}{q_0^2 - \alpha^2 q^2}$$

The pole at $q_0^2 = \alpha^2 q^2$ describes the excitation spectrum of the zero-mass boson.

Due to the Coulomb force, this spectrum is shifted to the plasma frequency $e^2 n$, where n is the number of electrons per unit volume. In this way the electromagnetic field acquires a mass. This is the essence of what will be known later as the Higgs mechanism.

The Meissner effect and the Landau-Ginzburg equation for the vector potential

In a superconductor a magnetic field penetrates only slightly. For the case of an infinite slab bounded by the $x = 0$ plane we have

$$\mathbf{H}(\mathbf{x}) = \exp\left[-\mu x\right]\mathbf{H}(\mathbf{0}) \tag{5}$$

with $\mu = e^2 n$ that is the mass of the plasmon. In the Landau-Ginzburg theory

$$\frac{d^2}{dx^2}\mathbf{A} - e^2 n\mathbf{A} = 0 \tag{6}$$

Analogies

Electromagnetic current		Axial current
	$\Longleftrightarrow$	
$\bar{\psi}\gamma_\mu\psi$		$\bar{\psi}\gamma_5\gamma_\mu\psi$

The axial current is the analog of the electromagnetic current in BCS theory. In the hypothesis of exact conservation, the matrix elements of the axial current between nucleon states of four-momentum p and p' have the form

$$\Gamma^A_\mu(p',p) = \left(i\gamma_5\gamma_\mu - 2m\gamma_5 q_\mu/q^2\right) F(q^2) \qquad q = p' - p$$

Conservation is compatible with a finite nucleon mass m provided there exists a massless pseudoscalar particle, the Nambu-Goldstone boson.

The Goldstone theorem

J. Goldstone, Nuovo Cimento **19***, 154 (1961); J. Goldstone, A. Salam, S. Weinberg, Phys. Rev.* **127***, 965 (1962)*

Whenever the original Lagrangian has a continuous symmetry group, which does not leave the ground state invariant, massless bosons appear in the spectrum of the theory.

physical system	broken symmetry	massless bosons
ferromagnets	rotational invariance	spin waves
crystals	translational invariance	phonons

Nambu's conjecture

Y. Nambu, Phys. Rev. Lett. **4**, *380 (1960)*

In Nature, the axial current is only approximately conserved. Nambu's hypothesis was that a small violation of axial current conservation gives a mass to the massless boson, which is then identified with the π meson, and renormalizes the axial vector part of the β-decay constant. Under this hypothesis, one can write

$$\Gamma_\mu^A(p', p) \simeq \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2 + m_\pi^2} \right) F(q^2) \qquad q = p' - p$$

This expression implies a relationship between the pion nucleon coupling constant G_π, the pion decay coupling g_π and the axial current β-decay constant g_A

$$2mg_A \simeq \sqrt{2} G_\pi g_\pi$$

This is the Goldberger–Treiman relation

An encouraging calculation

Y. Nambu, GJL, Phys. Rev. **124**, *246 (1961), Appendix*

It was experimentally known that the ratio between the axial vector and vector β-decay constants $R = g_A/g_V$ was slightly greater than 1 and about 1.25. The following two hypotheses were then natural:

1. under strict axial current conservation there is no renormalization of g_A;
2. the violation of the conservation gives rise to the finite pion mass as well as to the ratio $R > 1$ so that there is some relation between these quantities.

Under these assumptions a perturbative calculation of the convergent difference of renormalization effects for $\mu_\pi \neq 0$ and $\mu_\pi = 0$ gives

$$R \simeq 1 + \frac{G_\pi^2}{16\pi^2}\frac{\mu^2}{m^2}\ln\frac{m^2}{\mu^2} \simeq 1.24$$

Typical graphs considered in the evaluation of $R = g_A/g_V$

2. NJL MODEL

The NJL model

Y. Nambu, GJL, Phys. Rev. **122**, *345 (1961)*

The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

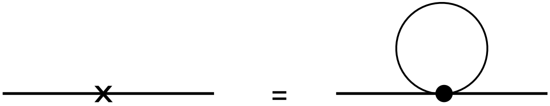
It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

By the Fierz transformation the non-linear term is equivalent to

$$-g\left[(\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2\right]$$

Mean field approximation



$$m = -\frac{g_0 m i}{2\pi^4} \int \frac{d^4 p}{p^2 - m^2 - i\varepsilon} \, F(p, \Lambda)$$

The spectrum of the NJL model

Mass equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off

Spectrum of bound states

nucleon number	mass μ	spin-parity	spectroscopic notation
0	0	0^-	$^1S_0$
0	$2m$	0^+	$^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	$^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	$^1S_0$

The effective action

GJL, Nuovo Cimento **34**, *1790 (1964)*

Let $G[J]$ be the generator of the vacuum expectation values (in statistical mechanics G is the free energy in the presence of an external field J)

$$\frac{\delta G}{\delta J} = \langle \Phi \rangle = \phi$$

The effective action is the dual functional $\Gamma[\phi]$ defined by the Legendre transformation

$$\frac{\delta \Gamma}{\delta \phi} = -J$$

The vacuum of the theory is defined by the variational principle

$$\frac{\delta\Gamma}{\delta\phi} = 0$$

$\Gamma[\phi]$ is the generator of the one-particle irreducible amplitudes and can be constructed by simple diagrammatic rules. Its general form is

$$\Gamma[\phi] = L_{\mathsf{cl}}[\phi] + \hbar Q[\phi]$$

3. VECTOR MESONS

SSB and the mass of gauge vector mesons

P. W. Anderson, Phys. Rev. **130**, 439 (1963)
F. Englert, R. Brout, Phys. Rev. Lett. **13**, 321 (1964)
P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964)

A simple example (Englert, Brout). Consider a complex scalar field $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$ interacting with an abelian gauge field A_μ

$$H_{\text{int}} = ieA_\mu\varphi^\dagger \overset{\leftrightarrow}{\partial_\mu} \varphi - e^2\varphi^\dagger\varphi A_\mu A_\mu$$

If the vacuum expectation value of φ is $\neq 0$, e.g. $\langle\varphi\rangle = \langle\varphi_1\rangle/\sqrt{2}$, the polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory is

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2\langle\varphi_1\rangle^2 \left[g_{\mu\nu} - \left(q_\mu q_\nu/q^2\right)\right]$$

Therefore the A_μ field acquires a mass $\mu^2 = e^2\langle\varphi_1\rangle^2$ and gauge invariance is preserved, $q_\mu\Pi_{\mu\nu} = 0$.

Electroweak unification

S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interaction. What could be more natural than to unite these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and the electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum.

The NJL model as a low-energy effective theory of QCD

e.g. T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 221 (1994)

The NJL model has been reinterpreted in terms of quark variables. One is interested in the low energy degrees of freedom on a scale smaller than some cut-off $\Lambda \sim 1$ Gev. The short distance dynamics above Λ is dictated by perturbative QCD and is treated as a small perturbation. Confinement is also treated as a small perturbation. The total Lagrangian is then

$$L_{\mathsf{QCD}} \simeq L_{\mathsf{NJL}} + L_{\mathsf{KMT}} + \varepsilon \left(L_{\mathsf{conf}} + L_{\mathsf{OGE}} \right)$$

where the Kobayashi–Maskawa–'t Hooft term

$$L_{\mathsf{KMT}} = g_D \det_{i,j} \left[\bar{q}_i (1 - \gamma_5) q_j + \text{h.c.} \right]$$

mimics the axial anomaly and L_{OGE} is the one gluon exchange potential.

4. NON-EQUILIBRIUM

SSB in non-equilibrium

SSB has been studied so far mainly as an equilibrium phenomenon typical of systems with infinitely many degrees of freedom. It was discovered however some time ago that out of equilibrium SSB can take place through mechanisms not available in equilibrium: currents are flowing through the system and their dynamics is crucial.

Stationary states are the obvious generalization of equilibrium states but the conditions under which SSB takes place in nonequilibrium are different from equilibrium. In stationary nonequilibrium states SSB may be possible even when it is not permitted in equilibrium.

A toy model exhibiting SSB of CP

V. Popkov, M. R. Evans, David Mukamel, Journal of Physics A: Mathematical and Theoretical **41** *(2008)*

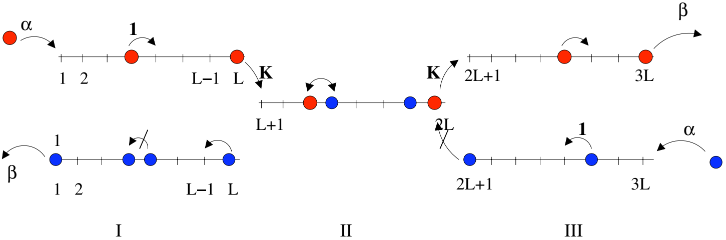


Figure 1. The bridge model with two junctions. Positively (negatively) charged particles hop to the right (left). The model is invariant with respect to left–right reflection and charge inversion. Section II is the bridge. It contains positive and negative particles and holes. Sections I and III comprise parallel segments each containing pluses and holes or minuses and holes.

Summarizing the dynamics, during a time interval dt three types of exchange events can take place between two adjacent sites

$$+0 \to 0+ \,, \quad 0- \to -0 \,, \quad +- \to -+ \,, \tag{7}$$

with probability dt. The last one takes place only on the bridge. At the left of the access lane of plus particles we have

$$0 \to + \,, \tag{8}$$

with probability αdt. At the right end of the exit lane of plus particles

$$+ \to 0 \,, \tag{9}$$

with probability βdt, and similarly for minus particles after reflection.

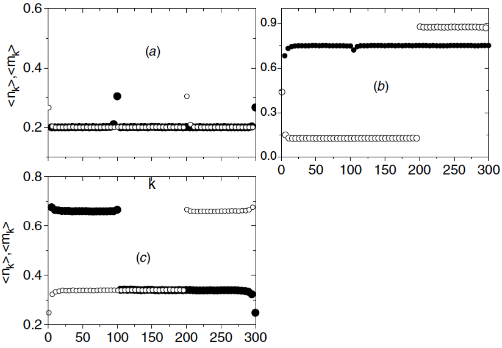


Figure 3. Average density profiles for pluses and minuses, from Monte Carlo simulations, in the LDS1 phase (panel (*a*)), in the SSB phase with pluses majority (panel (*b*)) and in the LDS2 phase (panel (*c*)). Pluses (minuses) correspond to closed (open) circles. The system of 300 sites was equilibrated and then, averaging over 10^6 Monte Carlo steps was done. Parameters: (*a*) $\alpha = 0.2$, $\beta = 0.4$; (*b*) $\alpha = 0.2$, $\beta = 0.25$; (*c*) $a = 0.8$, $\beta = 0.9$.

Why is nonequilibrium SSB interesting?

GJL, Progr. Theor. Phys. **124**, *731 (2010)*

There are facts in the world around us that so far have eluded a really satisfactory explanation.

At the planetary scale we know that in living matter left-handed chiral molecules are the rule.

At the cosmic scale matter is widespread and we do not see regions with antimatter.

Explanations have been proposed invoking initial small fluctuations which are amplified over a long nonequilibrium evolution to reach the present state.

Baryogenesis

M. Shaposhnikov, J. Phys.: Conf. Ser. 171 (2009), 012005.

Baryogenesis gives a possible answer to the following question: Why there is no antimatter in the Universe? A (qualitative) solution to this problem is known already for quite some time: the Universe is charge asymmetric because it is expanding (the existence of arrow of time, in Sakharov's wording), baryon number is not conserved and the discrete CP-symmetry is broken. If all these three conditions are satisfied, it is guaranteed that some excess of baryons over anti-baryons will be generated in the course of the Universe evolution. However, to get the sign and the magnitude of the baryon asymmetry of the Universe (BAU) one has to understand the precise mechanism of baryon (B) and lepton (L) number non-conservation, to know exactly how the arrow of time is realized and what is the relevant source of CP-violation.

A non-equilibrium scenario

The universe is not in equilibrium.

The idea is to view baryogenesis as the outcome of a nonequilibrium phase transition.

This means that if the nonequibrium conditions are such that an approximately stationary phase is an SSB phase, depending on the initial conditions the system of interest will relax to one of the degenerate states avoiding the difficulties of reconstructing a history with many uncertainties.