QCD and **SUPERCOMPUTERS**

rather LATTICE QCD and SUPERCOMPUTERS

Dipartimento di Fisica



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<u>PLAN OF THE TALK</u> 1) General introduction

2) From simple to complex: the hadron spectrum

- 3) Euclidean matrix elements: decay constants and form factors
- 4) Increasing precision: isospin breaking and radiative corrections
- 5) Increasing complexity: Long distance corrections to ε and ε'/ε
- 5) Conclusion & Outlook

Many thanks to V. Lubicz, F. Sanfilippo, S. Simula, L. Vittorio

THE QCD LAGRANGIAN

$$\mathcal{L} = -\frac{1}{2} Tr[G_{\mu\nu}G^{\mu\nu}] + \sum_{f} \bar{q}_{f} (\not D - m_{f}) q_{f}$$

 $+ \theta \, Tr[G_{\mu
u} ilde{G}^{\mu
u}]$

Simple Lagrangian but very complicated dynamics

Asymptotic freedom and infrared slavery



What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires \geq 104 numbers at every point of spacetime

- $ightarrow \infty$ number of numbers in our continuous spacetime
- \rightarrow must temporarily "simplify" the theory to be able to calculate



Ken Wilson invents lattice QCD (1974): construct a theory on a spacetime that is a finite cubic lattice of points, which reduces to QCD when the mesh of the lattice is infinitely fine

- $\rightarrow\,$ number of numbers to describe a state of the system becomes finite
 - \rightarrow solve the problem with a computer
- → repeat calculation for larger and finer lattices
 → obtains predictions relevant for our continuous universe



(KEK)

All the physical information can be extracted from the Green function

$$Z^{-1}\int [d \phi] \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) e^{i S(\phi)}$$

On a finite volume (L) and with a finite lattice spacing (a) this is now an integral on L^4 real variables which can be performed with

Important sampling techniques

This requires the use of a mesh in a Euclidean space-time (see later)



At large time distances the lightest (one particle) states dominate :

$$G(t,\vec{q}) = \sum_{n} \langle 0|\phi^{\dagger}|n\rangle \langle n|\phi|0\rangle \frac{e^{-E_{n}t}}{2E_{n}} \rightarrow \langle 0|\phi^{\dagger}|\vec{q}\rangle \langle \vec{q}|\phi|0\rangle \frac{e^{-E_{q}t}}{2E_{q}}$$



Continuum limit





a/ ξ = m a ~1 The size of the object is comparable to the lattice spacing



a/ $\xi \ll 1$ i.e. m a -> 0 The size of the object is much larger than the lattice spacing

Similar to a $\sum_{n} \rightarrow \int dx$



Bálint Joó – OLCF, Oak Ridge National Laboratory Lattice 2021 Virtual Meeting (MIT) July 30, 2021

Introduction – a brief history

- I recall at Lattice 2006 in Tuscon, first hearing about the idea of LQCD calculations on GPUs in a talk by Daniel Nogradi.
 - Programming was done using OpenGL.
 - I thought: "This looks fun, but it will never catch on!"
- I had to eat my words as I gave a talk about GPU accelerated computing in Squaw Valley at Lattice 2011 – 10 years ago.
- 2012-2018 was an era of 'friendly competition' between NVIDIA GPUs and Intel Knights
 - The Knights fought well, but were discontinued
 - Remaining fighting Knights are getting close to retirement
- OLCF Summit exceeded 1.88 ExaOps (in 32 and 16-bit precisions) on a Genomics Machine Learning Application in 2018, kicking off the Exascale era



Egri et. al. "Lattice QCD as a video game" Comput. Phys. Commun. 177:631-639,2007 arXiv;hep-lat/0611022





JLab 9g cluster (2009)

JLab 10g cluster (2010)





OLCF Titan (2012)

JLab 12k/m cluster (2012)





JLab 16p and 18p KNL Cluster (2016-...)

NERSC Cori KNL (2016)



OLCF Summit 2017

VALUE ADDITION

Other Noteworthy Systems for LQCD

- Supercomputer Fugaku, RIKEN CCS Japan
 - #1 Top 500 list: 537PF (Rpeak), 442 PF (Rmax)
 - Please see talk by Yoshifumi Nakamura!
- Summit at OLCF, U.S.A
 - #2 Top 500 list: 200PF (Rpeak), 148.6 PF (RMax)
- Sunway TaihuLight, National Supercomputing Center, Wuxi, China
 - #4 on on Top 500 list: 125.4 PF(Rpeak) 93PF(Rmax)
- Tianhe-2A, National Super Computer Center, Guangzhou, China
 - #7 on the Top 500 list; 100.6 PF (Rpeak) 61PF (Rmax)
- JUWELS Booster, Forschungszentrum Jülich, Germany
 - #8 on Top500 List: 71PF (Rpeak), 44 PF (Rmax)
 - 4 NVIDIA A100 GPUs, switched directly to HCAs
 - Infiniband Network in DragonFly+ configuration.



Supercomputer Fugaku



OLCF Summit



TaihuLight





US Exascale and Pre-Exascale Systems: ALCF Aurora

- Aurora will be the New HPE/Cray "Shasta" system at Argonne Leadership Computing Facility
 - >1 Exaflop peak (DP) performance
- Accelerators will be Intel Xe architecture based "Ponte Vecchio" GPUs
- Unified Memory Architecture
 - Across GPU and CPU
- Low Latency, High Bandwidth all-to-all connectivity within Node
- HPE/Cray Slingshot interconnect
 - 8 fabric endpoints per node
- Programming Models
 - MPI, Intel oneAPI DPC++ (based on SYCL), OpenMP-5offload, ...
- <u>https://alcf.anl.gov/aurora</u>

COMPUTING RATIO





Open slide master to edit

Physics Reach (Mainly Heavy Flavor Physics) many slides from Lattice Conferences

- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited





Light Hadron Masses

[Dürr et al (Budapest-Marseille-Wuppertal collaboration (BMWc)), Science 322 (2008) 1224]



Hadron Masses & Isospin Violation



Strong + Higgs + Electromagnetism = Experiment

Laurent Lellouch University of Southampton, 25 July 2016

Further Hadron Masses



- ightarrow QCD mass generation mechanism checked at few % level
- \rightarrow Impressive validation of nonperturbative QCD

Hadron Masses & Isospin Violation



LATTICE QCD, we can compute:

 α_{s} (i.e. the strong interaction scale), hadron and Quark Masses

Leptonic decay constants: f_{π} , f_{K} , f_{D} , $f_{D_{S}}$, f_{B} , $f_{B_{S}}$, f_{ρ}

Electromagnetic form factors : $F_{\pi}(Q^2)$, $G_M(Q^2)$, ...

Semileptonic form factors (hadron β decays): f^{+,0}(Q²), A_{0,..3}(Q²), V(Q²) for K -> π , D -> K, K^{*}, π , ρ , B -> D, D^{*}, π , ρ B -> K^{*} γ The Isgur-Wise function

B-parameters:

$$\langle K^{0} | Q^{\Delta S=2} | \overline{K^{0}} \rangle$$
 and $\langle B^{0} | Q^{\Delta B=2} | \overline{B}^{0} \rangle$

Weak decays :

$$\langle \pi | Q^{\Delta S=1} | K \rangle$$
 and $\langle \pi \pi | Q^{\Delta S=1} | K \rangle$

Major fields of investigation

- QCD thermodynamics
- Hadron spectrum
 Hadronic matrix elements $(K \rightarrow \pi\pi)$, structure functions, etc. see below)

EW

QCD

• Strong interacting Higgs Models • Strong interacting chiral models

- Surface dynamics
- Quantum gravity

Flavour, CP Violation and New Physics Physics Motivations:

Flavour phenomenology plays a fundamental role in indirect searches of New Physics (NP):

- looks for deviation from the SM whatever the origin is;
- needs good theoretical control of the SM contribution only.

the path leading to NP@ the TeV scale is much narrower after the results from LHC

CKM The Wolfenstein Parametrization

<mark>V_{td}</mark> λ ~ 0.2 η ~ 0.2	A ~ 0. ρ ~ 0.	8 Sin θ_{12} = Sin θ_{23} = Sin θ_{13} e ⁻	λ Α λ ² ^{i γ} = λ ³ (ρ-i η)
Αλ ³ × (1-ρ-iη)	-A λ ²	1	
- λ	1 - 1/2 λ ²	A λ^2	+ Ο(λ ⁴)
1 - 1/2 λ ²	λ	A λ ³ (ρ - i η)	V _{ub}

The Standard Triangle of the Standard Model

y triangle:





The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavour physics requires the control of hadronic effects for which lattice QCD simulations are essential.

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

$$Q^{EXP} = \sum_{i} C^{i}_{SM}(M_{W}, m_{t}, \alpha_{s}) \langle F | \hat{O}_{i} | I \rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_{\beta}, \alpha_{s}) \langle F | \hat{O}_{i'} | I \rangle$$

Leptonic (π ,*K*,*D*,*B*)





Semileptonic (K,D,B)⁺/

OZ /

 K^+

 O_a^W

connected diag.





(some) Radiative and Rare long distance effects (also $K \rightarrow \pi l^+ l^-$)



Non-leptonic but only below the inelastic threshold (may be also 3 body decays) $B \rightarrow \pi\pi, K\pi, etc. No !$



Neutral meson mixing (local)



meson mixing + short distance contributions to $B \rightarrow K^{(*)} l^+ l^-$



2021 results

 $\overline{\rho} = 0.156 \pm 0.012 \ \overline{\eta} = 0.350 \pm 0.010$



CKM matrix is the dominant source of flavour mixing and CP violation

PROGRESS SINCE 1988



LATTICE PARAMETERS (2015)

It does not make sense to improve the precision on B_K if we do not control <u>long distance effects</u>; Similarly for f_{π} or f_K <u>without radiative corrections</u>

	Lattice	Prediction	Pull
\hat{B}_K	0.766 ± 0.010	0.84 ± 0.07	0.9
	1.3~%	8.3~%	
$\overline{f_{B_s}}$	0.226 ± 0.005	0.2256 ± 0.0039	0.0
	2.2~%	2.7~%	
f_{B_s}/f_{B_d}	1.204 ± 0.016	1.197 ± 0.056	0.0
	1.3~%	0.4~%	
B_s	0.875 ± 0.040	0.875 ± 0.030	0.0
	1.3~%	0.4~%	
B_s/B_d	1.03 ± 0.08	1.096 ± 0.062	0.7
101	7.8 %	5.7 %	

Do we still care? Tensions and Unknowns

- 1)A``classical'' example B -> τv
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) V_{cb} I, B mixing and ϵ_{K}
- 4)D-mixing
- 5)R(D) and R(D*)
- 6)B -> K* ll
- 7)Physics BSM?

TENSIONS LEPTON FLAVOR UNIVERSALITY?

two critical issues in semileptonic $B \to D^{(*)} \ell \nu_{\ell}$ decays

* exclusive/inclusive | V_{cb} | puzzle:

 $|V_{cb}|(BGL) \cdot 10^3 = 39.08 \ (91)$ exclusive (FLAG '19): <u>inclusive</u> (HFLAV '19): $|V_{cb}| \cdot 10^3 = 42.00$ (65) $|V_{cb}|(CLN) \cdot 10^3 = 39.41$ (60) $|V_{ch}| \cdot 10^3 = 42.16$ (50) differences of ~ 2.6σ (Bordone et al. 2107.00604) $R(D^*)$ $\Delta \chi^2 = 1.0$ contours HFLAV average * $R(D^{(*)})$ anomalies: 0.4 LHCb15 BaBar12
$$\begin{split} & \frac{\mathscr{B}(B \to D\tau\nu_{\tau})}{\mathscr{B}(B \to D\ell\nu_{\ell})} \\ & \frac{\mathscr{B}(B \to D^*\tau\nu_{\tau})}{\mathscr{B}(B \to D^*\ell\nu_{\ell})} \end{split}$$
0.35 R(D)3σ_-LHCb18 $\ell=e,\mu$ 0.3 $R(D^*)$ = H Belle19 Belle15 0.25 Belle17 **HFLAV** differences of ~ 3.1σ between exp.'s and SM + Average of SM predictions R(D) = 0.299 ± 0.003 0.2 Spring 2019 $R(D^*) = 0.258 \pm 0.005$ $P(\chi^2) = 27\%$ 0.2 0.3 0.4 0.5



* the Dispersion Matrix approach is an attractive tool to implement unitarity and lattice QCD calculations in the analysis of exclusive semileptonic decays of hadrons. The main features are:

- it does not rely on any assumption about the momentum dependence of the hadronic form factors
- it can be based entirely on first principles using lattice determinations both of the relevant form factors and of the dispersive bounds (the susceptibilities) from appropriate 2-point and 3-point (Euclidean) correlation functions
- it allows to implement unitarity and kinematical constraints in a rigorous and parameterization-independent way
- it predicts band of values that are equivalent to the infinite number of BGL (or BCL) fits satisfying unitarity and kinematical constraints and reproducing exactly a given set of data points
- it can be applied to any exclusive semileptonic decay of hadrons
- * we have applied the DM approach to $D \to K\ell\nu_{\ell}$ decays (2105.02497), to $B \to D^{(*)}\ell\nu_{\ell}$ decays (2105.08674, 2109.15248), to $B \to \pi\ell\nu_{\ell}$ and $B_s \to K\ell\nu_{\ell}$ decays (in preparation) and to $B_s \to D_s^{(*)}\ell\nu_{\ell}$ decays (in progress)

* results for <i>B</i>	$\rightarrow D^{0}$	$^{(*)}\ell\nu_{\ell}$ decays:					
10^{-103}	_	41.0 ± 1.2	$(\boldsymbol{B} \rightarrow \boldsymbol{D})$			DM	experiment
$ V_{cb} _{DM} \cdot 10^{-5}$	=	41.0 ± 1.2	$(B \rightarrow D)$	$ V_{cb} _{incl.} \cdot 10^3 = 42.16 \pm 0.50$	R(D)	0.289 (8)	0.340 (27)
	=	41.3 ± 1.7	$(B \rightarrow D^*)$		R(D*)	0.269 (8)	0.295 (11)
			differen	$\cos < 1\sigma$	-		+

differences of $\approx 1.6\sigma$

* results for $B \to \pi \ell \nu_{\ell}$ decays: $|V_{ub}|_{DM} \cdot 10^3 = 3.88 \pm 0.32$ consistent with $|V_{ub}|_{incl.} \cdot 10^3 = 4.10 \pm 0.28$

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking and em effects cannot be neglected anymore:

 f_{π} = 130.2(0.8) MeV ε =0.6% f_{K} = 155.7(0.3) MeV ε =0.2% f_{K}/f_{π} = 1.1932(19) ε =0.16% F^{Kπ}(0) =0.9698(17) ε =0.18%

A remark on useful and useless precision of lattice calculations:

1) ε_K and long distance charm contributions 2) isospin breaking and electromagnetic corrections to f_K and f_{π} leptonic decays of PS mesons

extraction of CKM matrix elements

$$\Gamma(PS^+ \to \ell^+ \nu_{\ell}) = \frac{G_F^2}{8\pi} |V_{q_1q_2}|^2 m_{\ell}^2 \left(1 - \frac{m_{\ell}^2}{m_{PS^+}^2}\right) M_{PS^+} f_{PS}^2 S_{ew} \left(1 + \delta R_{IB}^{PS} + \delta R_{QED}^{PS}\right)$$
universal electroweak correction ($\simeq 1.032$)

 f_{PS} : leptonic decay constant in isoQCD ($m_u = m_d, e_f = 0$)

$$\delta R_{IB}^{PS}$$
: strong isospin breaking correction $\propto O[(m_d - m_u)/\Lambda_{QCD}] \simeq O(1\%)$

$$\delta R_{QED}^{PS}$$
: QED correction $\propto O(\alpha_{em}) \simeq O(1\%)$

* lattice determinations of f_{PS} have reached an accuracy below the percent level

 $\frac{f_K}{f_{\pi}}$: relative error of $\simeq 0.15\%$ FLAG-4 [EPJC '20]

need of determining δR_{IB}^{PS} and δR_{QED}^{PS} on the lattice

* the infrared (IR) problem: only $\Gamma(\Delta E_{\gamma}) = \Gamma_0 + \Gamma_1(\Delta E_{\gamma})$ is IR finite [Block&Nordsiek '37] Γ_n : n photons in the final state

RM123+Soton strategy: $\Gamma(\Delta E_{\gamma}) = \lim_{V \to \infty} \left[\Gamma_0 - \Gamma_0^{pt} \right] + \lim_{V \to \infty} \left[\Gamma_0^{pt} + \Gamma_1(\Delta E_{\gamma}) \right]$ pt = point-like IR finite IR finite PRD '15 arXiv:1502.00257 (master formula) (FVEs) $\lim_{V \to \infty} \left[\Gamma_0 - \Gamma_0^{pt} \right]$ on the lattice PRD '17 arXiv:1611.08497 (FVEs) $\lim_{W \to \infty} \left[\Gamma_0 - \Gamma_0^{pt} \right]$ on the lattice PRL '18 arXiv:1711.06537 (π and K) $\lim_{W_{\gamma} \to 0} \left[\Gamma_0^{pt} + \Gamma_1^{pt}(\Delta E_{\gamma}) \right]$ within the pt approximation (small ΔE_{γ})

From f_K/f_{π} and $f^{K\pi}(0)$ we can extract V_{us}/V_{ud} and V_{us}

Figure 10: The plot compares the information for $|V_{ud}|$, $|V_{us}|$ obtained on the lattice for $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ with $|V_{ud}|$ extracted from nuclear β transitions Eqs. (71) and (72). The dotted line indicates the correlation between $|V_{ud}|$ and $|V_{us}|$ that follows if the CKM-matrix is unitary. For the $N_f = 2$ results see the 2016 edition [3].

Real photon emission

KLOE experiment $K \rightarrow e \nu_e \gamma$

[EPJC '09]

FIG. 1. Left panel: comparison of the KLOE experimental data $\Delta R^{\exp,i}$ [9] (red circles) with the theoretical predictions $\Delta R^{\text{th},i}$, (blue squares) evaluated with the vector and axial form factors of Ref. [8] given in Eqs. (13)–(17), for the 5 bins (see Table IV). The green diamonds correspond to the prediction of ChPT at order $O(e^2 p^4)$, based on the vector and axial form factors given in Eq. (53). Right panel: comparison of the form-factor $F^+(x_{\gamma})$ extracted by the KLOE collaboration in Ref. [9] and the theoretical prediction from Eqs. (13)–(17). The shaded areas represent uncertainties at the level of 1 standard deviation.

***** good consistency *****

B meson real photon emissions

Factorization at leading power in an expansion of the decay amplitude in Λ_{QCD}/E_{γ} and Λ_{QCD}/mb has been established to all orders in the strong coupling α_s . In this approximation, the branching fraction depends only on the leading-twist B-meson light-cone distribution amplitude (LCDA) $\phi_{+}(\omega, \mu)$

More precisely, it is proportional to $1/\lambda_B$, the most important LCDA parameter in exclusive decays, is uncertain by a large factor ranging from 200 MeV favoured by non-leptonic decays to 460 MeV from QCD sum rules.

The radiative leptonic decay has therefore been suggested as a measurement of λ_B

Figure 1. Leading contribution to $B \to \gamma \ell \nu_{\ell}$.

For large photon energies the form factors can be written as [9]

$$F_V(E_{\gamma}) = \frac{e_u f_B m_B}{2E_{\gamma} \lambda_B(\mu)} R(E_{\gamma}, \mu) + \xi(E_{\gamma}) + \Delta \xi(E_{\gamma}),$$

$$F_A(E_{\gamma}) = \frac{e_u f_B m_B}{2E_{\gamma} \lambda_B(\mu)} R(E_{\gamma}, \mu) + \xi(E_{\gamma}) - \Delta \xi(E_{\gamma}).$$
(2.7)

The first term is equal in both expressions and represents the leading-power contribution in the heavy-quark expansion (HQE). It originates only from photon emission from the light spectator quark in B meson (Fig. 1). In the above, f_B is the decay constant of Bmeson, and the quantity λ_B is the first inverse moment of the B-meson LCDA,

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \,\phi_+(\omega,\mu)\,. \tag{2.8}$$

Further applications in decays of heavy neutral B mesons: Virtual corrections (some questions still open)

Enhanced electromagnetic correction to the rare B-meson decay $B_{s,d} \rightarrow \mu^+ \mu^-$

Martin Beneke,¹ Christoph Bobeth,^{1,2} and Robert Szafron¹

Further applications in decays of heavy neutral B mesons: real corrections (some questions still open) $\mathbb{P}^{0} \rightarrow + - \mathbb{P}^{0} \rightarrow + -$

 $B^0_s
ightarrow \mu^+ \mu^- \gamma$ from $B^0_s
ightarrow \mu^+ \mu^-$

Francesco Dettori^{*a*}, Diego Guadagnoli^{*b*} and Méril Reboud^{*b,c*}

Figure 3: Dimuon invariant mass distribution from LHCb's measurement of $\mathcal{B}(B_s^0 \to \mu^+ \mu^-)$ [52] overlayed with the contribution expected from $B_s^0 \to \mu^+ \mu^- \gamma$ decays (ISR only). Assumes flat efficiency versus $m_{\mu^+\mu^-}$. The line denoted as $B_s^0 \to \mu^+ \mu^- \gamma$ NP' refers to the V - A case with $\delta C_9 = -12\% C_9^{\text{SM}}$ (see also Fig. 2). The two filled curves are not stacked onto each other.

Particle(s) from weak vertex with momenta q

• **FCNC** Qb= Qq (need long distance in addition) :

 $\mathsf{H}^{\mathsf{weak}} \sim \mathsf{O}_{9,10} : B_{d,s} \to \ell^+ \ell^- \gamma$

flavoured

axion

 $\mathsf{H}^{\mathsf{weak}} \sim \mathsf{O}_7 : B_{d,s} \to \ell^+ \ell^- \gamma$

 $\mathsf{H}^{\mathsf{weak}} \sim \bar{q} \gamma_{\mu} b_L \partial^{\mu} a$: $B_{d,s} \rightarrow \ell^+ \ell^- a$

$$F(q^2, k^2)$$

 $F(q^2) = F(q^2,0)$ Bobeth's talk

 $F^*(k^2) = F(0,k^2)$

 $F(m_a^2, k^2) \rightarrow F^*(k^2)$ Ziegler's talk

or dark photon, scalar DM, ...

• **FCCC** Qb ≠ Qq :

$$\bigvee \bigvee_{\nu}^{\ell^+} \qquad \mathsf{H}^{\mathsf{weak}} \sim V_{ub} \, \bar{u} \gamma_{\mu} b_L \ell \gamma^{\mu} \nu_L : B_u \to \ell^+ \nu \gamma$$

• Physics: helicity suppression of $B \to f_i \bar{f}_j$ relieved in radiative decay!

Roman Zwicky@ Tenerife —

Status of Lattice Calculations of gA

- $\hfill\square$ The neutron lifetime and g_A (neutron decay) are used to probe the limits of the Standard Model
- $\label{eq:constraint} \square \ \ We \ should \ have \ a \ (meaningful) \ Standard \ Model \ prediction \ for \ g_A \ \ LQCD \ (lattice \ QCD)$
- To gain confidence in the application of LQCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest, such as g_A
- □ In order for the theoretical uncertainty on g_A to match the larger uncertainty in the neutron lifetime measurements, we must determine g_A with < 0.2% uncertainty is this crazy?

nucleon axial coupling from LQCD

- To gain confidence in the application of Lattice QCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest
- □ g_A was supposed to be a good benchmark calculation for single nucleon structure - but it proved to have significant systematic challenges, preventing results with the precision anticipated
- □ FLAG 2019 has included single nucleon quantities in their averaging for the first time
- Notice one result is significantly more precise than the others

0.81%	1.35 - model average 1.30 -	$g_A^{LOCD}(\epsilon_{\pi}, a = 0)$ $\Phi g_A^{PDG} = 1.2723(23)$
0.81%	1.30 -	$\Phi g_A = 1.2723(23)$
0.9107	0	
polation 0.31%	1.25 -	T T
0.12%	5	
0.15%		
0.03%	1.15 - $g_A(\epsilon_\pi, a \simeq 0.$ $g_A(\epsilon_\pi, a \simeq 0.$	$\begin{array}{ccc} 15 \text{ fm} \\ 12 \text{ fm} \end{array} \stackrel{\bullet}{=} a \simeq 0.15 \text{ fm} \\ \bullet a \simeq 0.12 \text{ fm} \end{array}$
tion 0.43%	$1.10 - g_{A}(\epsilon_{\pi}, a \simeq 0.$	$\frac{1}{100} \text{ fm} = a \simeq 0.09 \text{ fm}$
0.99%	0.00 0.05 0.10 0.15	0.20 0.25 0.30
	tion 0.43% 0.99%	$\frac{1.15}{0.03\%}$ $\frac{1.15}{0.00}$ $\frac{1.15}{0.00}$ $\frac{1.15}{0.99\%}$ 1.10 $\frac{1.10}{0.00}$ $\frac{1.10}{0.05}$ $\frac{1.10}{0.10}$ $\frac{1.10}{0.15}$ $\epsilon_{\pi} = m_{\pi}/t$

 $g_A^{\text{QCD}} = 1.2711(103)^s (39)^{\chi} (15)^a (19)^V (04)^I (55)^M$

□ More precise results at the physical pion mass will improve the three largest uncertainties:
□ statistical (s), extrapolation (χ) and model selection (M) NOTE, a12m130 has 2.3% uncertainty
□ Following our existing strategy, we anticipate getting to 0.5% by the end of this year
□ Getting below (or maybe to 0.5%) will require a 4th lattice spacing as well (~0.06fm)
□ Adding a FV study on additional pion mass points will improve the FV uncertainty
□ The isospin uncertainty seems unnecessary...

• Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathscr{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | \mathscr{H}_W | \alpha \rangle \langle \alpha | \mathscr{H}_W | K^0 \rangle}{m_K - E_{\alpha}} = 3.483(6) \times 10^{-12} \,\mathrm{MeV}.$$

• The above correlation function gives $(T = t_B - t_A + 1)$

$$C_{4}(t_{A}, t_{B}; t_{i}, t_{f}) = |Z_{K}|^{2} e^{-m_{K}(t_{f}-t_{i})} \sum_{n} \frac{\langle \bar{K}^{0} | \mathscr{H}_{W} | n \rangle \langle n | \mathscr{H}_{W} | K^{0} \rangle}{(m_{K}-E_{n})^{2}} \times \left\{ e^{(M_{K}-E_{n})T} - (m_{K}-E_{n})T - 1 \right\}.$$

From the coefficient of T we can therefore obtain

$$\Delta m_K^{\rm FV} \equiv 2 \sum_n \frac{\langle K^0 | \mathscr{H}_W | n \rangle \langle n | \mathscr{H}_W | K^0 \rangle}{(m_K - E_n)} \,.$$

physical terms

Chris Sachrajda

0

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$
 $\left(\frac{\text{Re }}{\text{Re }}\right)$

 $(\epsilon' \epsilon)_{exp} = (16.6 \pm 2.3) \cdot 10^{-4}$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2}\right)_{\text{exp}} = 22.4$$

= **31.0** \pm **6.6**

1

Results for $\operatorname{Re}[A_0]$, $\operatorname{Im}[A_0]$ and $\operatorname{Re}[\epsilon'/\epsilon]$

Xu Feng Lattice 2017

[RBC-UKQCD, PRL115 (2015) 212001]

- Determine the $K \rightarrow \pi \pi (I = 0)$ amplitude A_0
 - Lattice results

 $Re[A_0] = 4.66(1.00)_{stat}(1.26)_{syst} \times 10^{-7} GeV$ $Im[A_0] = -1.90(1.23)_{stat}(1.08)_{syst} \times 10^{-11} GeV$

Experimental measurement

 $Re[A_0] = 3.3201(18) \times 10^{-7} GeV$ $Im[A_0]$ is unknown

• Determine the direct *CP* violation $\operatorname{Re}[\epsilon'/\epsilon]$

 $Re[\epsilon'/\epsilon] = 0.14(52)_{stat}(46)_{syst} \times 10^{-3}$ Lattice $Re[\epsilon'/\epsilon] = 1.66(23) \times 10^{-3}$ Experiment

Phase of final state interaction smaller than the experimental value

2.1 σ deviation \Rightarrow require more accurate lattice results

Four dominant contributions to ϵ'/ϵ in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

Assumes that ReA_0 and ReA_2 ($\Delta I=1/2$ Rule) fully described by SM (includes isospin breaking corrections)

 ϵ / ϵ from RBC-UKQCD

Calculate all contributions directly (no isospin breaking corrections)

$$\left[-\left(6.5\pm3.2\right)+25.3\cdot\mathsf{B}_{6}^{(1/2)}+\left(1.2\pm0.8\right)-10.2\cdot\mathsf{B}_{8}^{(3/2)}\right]$$

$\Delta I = 1/2 \quad K \rightarrow \pi \pi$ (Qi Liu)

- Code 50 different contractions
- For each of 400 configurations invert with source at each of 32 times.
- Use Ran Zhou's deflation code

(3.2
$$\sigma$$
) $\varepsilon'/\varepsilon = (2.2 \pm 3.8) \cdot 10^{-4}$
Anatomy of $\varepsilon'/\varepsilon = -4$ new flavour anomaly?
AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx
RBC-UKQCD
 $\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$
RBC-QCD values
 $B_6^{(1/2)} = 0.57 \pm 0.15$
 $B_8^{(3/2)} = 0.76 \pm 0.05$

$$\varepsilon'/\varepsilon = (6.3\pm2.5)\cdot10^{-4}$$

large N bounds (AJB, Gérard) $B_6^{(1/2)} = B_8^{(3/2)} = 0.76$

$$\varepsilon'/\varepsilon = (9.1 \pm 3.3) \cdot 10^{-4}$$

large N bounds (AJB, Gérard) $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$

exp:
$$\varepsilon'/\varepsilon = (16.6 \pm 3.3) \cdot 10^{-4}$$

Systematic error budget

- Primary systematic errors of 2015 work:
 - Finite lattice spacing: 12%
 - Wilson coefficients: 12%
 - Renormalization (mostly PT matching): 15%
 - Excited-state: \leq 5% but now known to be significantly underestimated
 - Lellouch-Luscher factor (derivative of $\pi\pi$ phase shift wrt. energy): 11%
- In our new work we have used step-scaling to raise the renormalization scale from $1.53 \rightarrow 4.00 \text{ GeV}$: $15\% \rightarrow 5\%$
- 3 operators have dramatically improved understanding of $\pi\pi$ system: Lellouch-Luscher factor $11\% \rightarrow 1.5\%$
- Detailed analysis shows no evidence of remaining excited-state contamination: Excited state error now negligible!
- Still single lattice spacing: Discretization error unchanged.
- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by higher μ: Wilson coeff error unchanged.

Christopher Kelly (RBC & UKQCD collaborations) Lattice2021, MIT, USA

Final result for ε'

• Combining our new result for $Im(A_0)$ and our 2015 result for $Im(A_2)$, and again using expt. for the real parts, we find

$$\operatorname{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \operatorname{Re}\left\{\frac{i\omega e^{i(\delta_{2}-\delta_{0})}}{\sqrt{2}\varepsilon} \left[\frac{\operatorname{Im}A_{2}}{\operatorname{Re}A_{2}} - \frac{\operatorname{Im}A_{0}}{\operatorname{Re}A_{0}}\right]\right\}$$
$$= 0.00217(26)(62)(50)$$
$$\overset{\bullet}{\underset{\text{stat}}} \overset{\bullet}{\underset{\text{sys}}} \overset{\bullet}{\underset{\text{IB}}} + \operatorname{EM}$$

Consistent with experimental result:

 $\operatorname{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$

A second group should do this calculation!!

THANKS FOR YOUR ATTENTION

