Touschek & Statistical Mechanics

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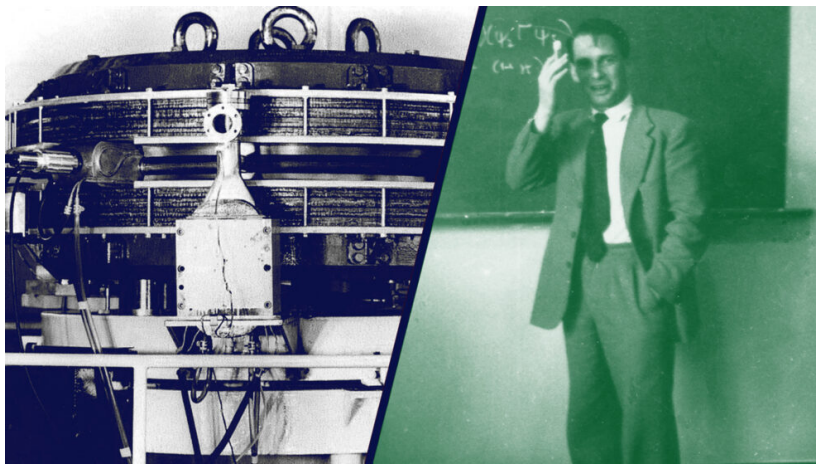


Bruno Touschek, my mentor

- Like many others here, I was a Touschek student
- I graduated in 1966 under his supervision defending the thesis

  "$e^+e^- \to \mu^+\mu^- + \gamma$ annihilation and the Bloch–Nordsieck method"
  - from which my first paper in collaboration with Mario originated

IL NUOVO CIMENTO Vol. L A, N. 1 1° Luglio 1967

A Note on the Infra-Red Divergence.

M. Greco and G. Rossi
Laboratori Nazionali del CNEN - Frascati

(ricevuto il 24 Gennaio 1967)

Summary. — In this paper we show how the infra-red divergence can be eliminated in the matrix element, provided that physically true final states for an experiment involving creation and destruction of charged particles are used.

based on the abelian coherent states formalism of GPPS
  - extended to the non-abelian case of QCD parton processes
- From 1967 I was "assistente volontario di Meccanica Statistica"

Touschek' teaching and lecturing activity

A wide, dedicated and highly valued teaching activity

Touschek at the blackboard
with cigarette and chalk

1. A course on "Meccanica Statistica" (IV year)
2. "Metodi Matematici della Fisica" (III year)
3. Lectures on "Renormalization" (Scuola di Perfezionamento)
4. "Sull'insegnamento della teoria dei quanti" (Lincei)
5. Notes on the Laser effect (original notes are in my possession)
6. . . .

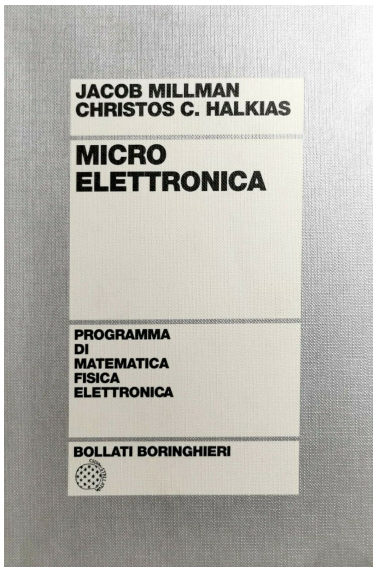
I'll focus on (1) on the development of which I was directly involved

- as a student (a.a. 1964-65)
- as a co-author of the book (1967-1970)

The book

BORINGHIERI, Torino,1970
Programma di Matematica, Fisica, Elettronica

The BORINGHIERI-BOLLATI program



The birth of the book - I

A long and twisted story

- The material grew slowly from the lectures Touschek delivered to IV year students in Physics at “La Sapienza” from 1959 to 1968
- A first set of notes were collected by M. Gambarelli that appeared as “dispense” in 1965
- I followed as a student Touschek course in 1964-65 writing afresh a second set of notes
- With the idea of publishing a book, Touschek decided to produce a first manuscript in English
- He did so systematically, lecture by lecture in the a.a. 1967-68
- I took notes of these lectures in Italian, later published by “La Goliardica” in 1969

The birth of the book - II

At this point we had two versions

- a collection of lecture notes in English produced by Touschek
- a manuscript in Italian assembled by me out of Touschek lectures

The plan was to reorganize the whole material in a book

- first in Italian for Boringhieri
- then possibly translate it into English for Wiley, AP, . . . (?)

It took two more years of work to finalize the book as it is now

- thanks to long sessions in Touschek' apartment
- where the Italian version elaborated by me was scrutinized and revised mainly for what was the presentation and development of the arguments supporting/proving theorems and physical results

The birth of the book - III

- Long sessions in Touschek' apartment either from 10 in the morning till noon or later in the afternoon after 3 p.m.
- Irrespective of the time of my arrival, I was always finding on the desk "un fiasco di Chianti" and 2 (!) glasses
- Chianti, Touschek used to say, was the "ideal magic potion to make the brain work smoothly and brilliantly"
- Naturally after a while my brain wasn't so much focused and brilliant!
- I soon learned that I should never empty my glass

The environment

- Discussions were not only about Physics. Touschek was keen to speak about politics, science in general, sociology & everyday life
- His point of view was always unexpectedly original
- Humor and some disenchanted cynicism were the colours of Touschek arguments
- All the time I spent working on this project was for me not simply a guide for my carrier as a Physicist, but also a school of formation as a person

The content of the book

- PARTE PRIMA: STATICA STATISTICA
  1. Meccanica statistica e termodinamica dell'oscillatore armonico
  2. Teoria dell'*ensemble* di Gibbs
  3. Termodinamica covariante
  4. Termodinamica di un gas ideale di particelle identiche
  5. Gas degenere e imperfetto
  6. Sistemi in cui il numero di particelle non é costante
- PARTE SECONDA: DINAMICA STATISTICA
  1. Proprietá degli stati di non-equilibrio
  2. I fondamenti microscopici della *master equation*
  3. Applicazioni della *master equation*
  4. Teoria del trasporto
- APPENDICI
  1. A. Teoria quantistica del conteggio
  2. B. Teorema adiabatico
  3. C. Un esperimento statistico

A few highlights

- Il metodo del punto di sella

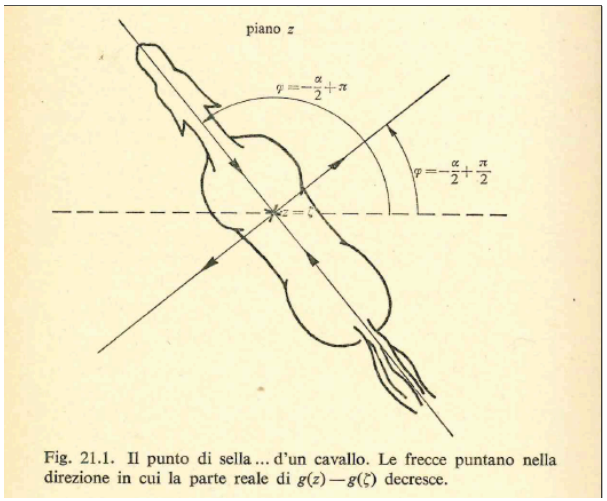


Fig. 21.1. Il punto di sella ... d'un cavallo. Le frecce puntano nella direzione in cui la parte reale di $g(z) - g(\zeta)$ decresce.

- Termodinamica covariante
  - A covariant formulation of thermodynamics
  - How does temperature transform relativistically?
- La clessidra
  - $N \to \infty$ number of sand grains
  - The periodic statistical clock
- Un esperimento statistico
  - Reversibility vs. irreversibility
  - Fluctuations
  - The second principle of Thermodynamics

Covariant Thermodynamics, T Lorentz transformations

How does temperature transform under a Lorentz transformation?

- We seem to have a problem ($c = 1$)
  - If T is defined from the gas state equation $RT = pV$
    - T transforms like a length, as p is an invariant
      $T(v) = T(0)[1 - v^2]^{1/2}$
  - If T is defined from the second law $dS = \delta Q/T$
    - T transforms like an energy, as dS is an invariant
      $T(v) = T(0)[1 - v^2]^{-1/2}$
- The solution
  - A relativistically covariant Thermodynamics can be constructed
    - Compute occupation numbers in the Gibbs *ensemble*
    - $\sum_n a_n = N\,, \quad \sum_n a_n p_n^\mu = P^\mu \implies \ln a_n + \lambda + \beta_\mu p_n^\mu = 0$
    - (λ, β_μ) Lagrange multipliers for (number, 4-momentum) conservation
    - $a_n = \frac{N}{Z} e^{-\beta_\mu p_n^\mu}$ with $Z = \sum_n e^{-\beta_\mu p_n^\mu}$ & $P_\mu = -N \frac{\partial \ln Z}{\partial \beta^\mu}$
  - T transformation properties depend on its definition for $|v| \neq 0$
    - Indeed one finds $\beta_\mu = u_\mu / k_B T(0)$ with $u_\mu = [1 - v^2]^{-1/2}(1, \vec{v})$
    - Transformation properties depend on how one measures T
      i.e. which component of β^μ is employed to define T

Master equation: hourglass & periodic statistical clock

- The hourglass
  - $p_s(t) =$ probability of having s grains in the lower part at time t
  - $\lambda =$ transition frequency (of a grain falling per time unit)

$$\dot{p}_0(t) = -\lambda p_0(t) \qquad p_0(0) = 1$$
$$\dot{p}_{s+1}(t) = \lambda[p_s(t) - p_{s+1}(t)]\,, \qquad p_{s+1}(0) = 0\,, \quad s = 0, 1, \ldots$$

$$p_s(t) = \frac{(\lambda t)^s}{s!} e^{-\lambda t}\,, \quad s = 0, 1, \ldots$$
$$\langle s \rangle = \lambda t\,, \qquad \frac{\sigma}{\langle s \rangle} = \frac{1}{\sqrt{\langle s \rangle}}$$

- The periodic statistical clock
  - $s = 0, 1, , \ldots, N-1$, mod(N), labels the states of the clock
  - $p_s(t) =$ probability to be in the state s at time t

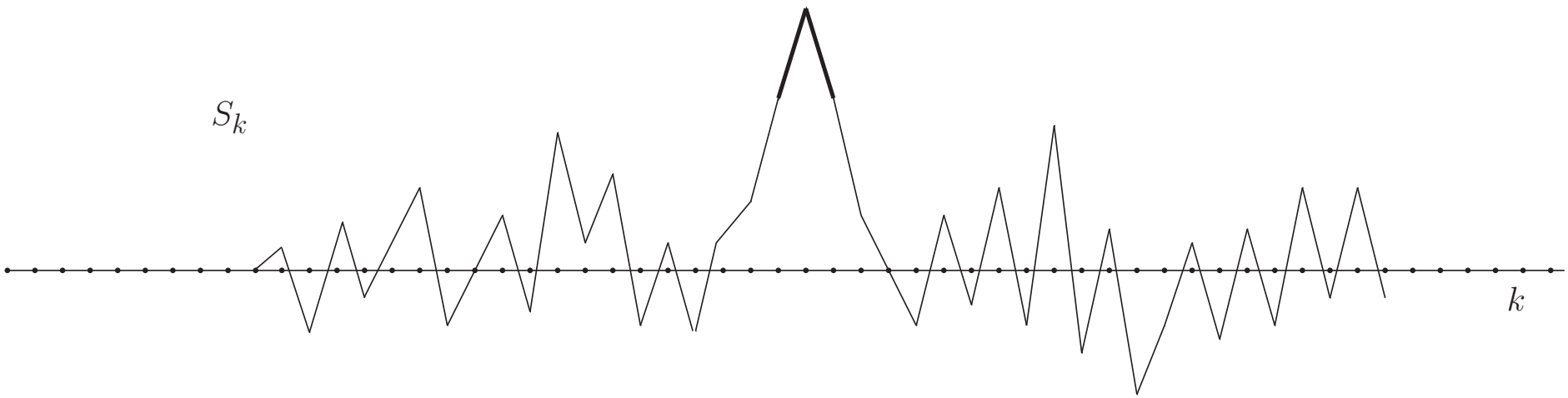
$$\dot{p}_{s+1}(t) = \lambda[p_s(t) - p_{s+1}(t)]\,, \qquad p_s(t) = p_{s+N}(t)\,, \quad s = 0, 1, \ldots, N-1$$

  - It can be solved by normal modes decomposition
  - System visits periodically every $T = N/\lambda$ the initial state $p_s(0) = \delta_{s,0}$

Micro-reversibility vs. macro-irreversibility - I

A statistical exercise

- Extract numbers, x_n, uniformly $\in [-1/2, 1/2]$
- Construct the random variable $S_k = \frac{1}{100} \sum_{n=k}^{k+100} x_n$
- $\langle S \rangle = 0$, $\langle (S)^2 \rangle = 1/12$
- A typical "$k \sim$ time" behaviour of S_k



Micro-reversibility vs. macro-irreversibility - II

- Micro-reversibility
  - micro-reversibility
    - equal positive and negative slopes around local maxima and minima
    - looking at the k-history, one cannot tell which way time is running
- Macro-irreversibility
  - Macro-physics looks irreversible because
    - one is (almost) always starting from a large fluctuation
    - the system moves away from this very low probability state
    - thus it is as if one is always at the maximum of a fluctuation
- The probability of a fluctuation
  - one can compute $P[S > \overline{S}]$
  - surprisingly we found that it does not fit with actual data
  - CRAY random numbers were not sufficiently random!
- Annoyed by this situation, Touschek proposed an algorithm to "randomize random" numbers

Conclusions & Outlook

- A great scientist, a brilliant teacher and amazing person
- Touschek was for me of great inspiration
- Interacting with him was really a fantastic human and scientific adventure
- A regret for Touschek (and for me) was (and still is) that he didn't arrive to publish an English version of the book.
- Is it too late today?

Thanks for your attention