Radiative one-meson and $\eta \pi \pi$ τ decays and prospects in related theoretical computations

Pablo Roig LPT (CNRS), Orsay (France)

Liverpool, 18-19/09/10

SUMMARY:

- Hadron decays of the τ lepton
- Theoretical setting: χPT, Large N_c , RχT
  - $\tau^- \rightarrow (\pi/K)^- \gamma \nu_\tau$
  - $\tau^- \rightarrow \eta\pi^-\pi^0 \gamma \nu_\tau$
- Future plans

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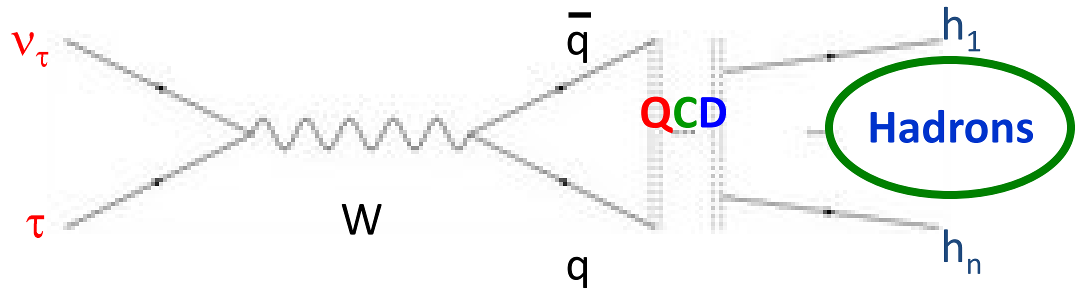
Hadron decays of the τ lepton :



See talks by M. Jamin and A. Pich

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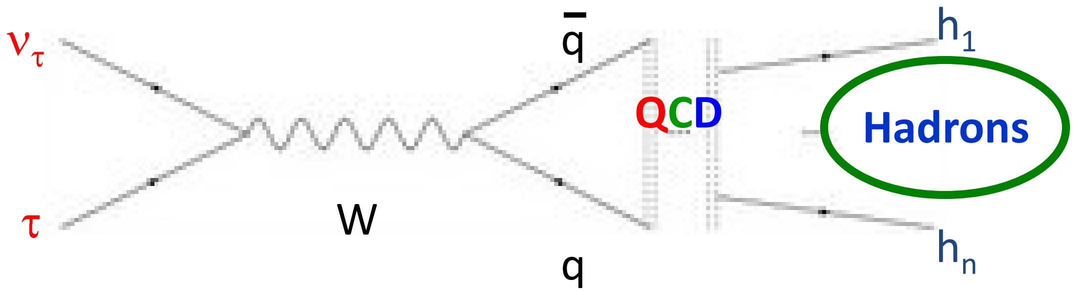

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$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

T_μ= <Hadrons|(V-A)$_\mu$ $e^{iS_{QCD}}$|0> = Σ_i(Lorentz Structure)^i $F_i(Q^2,s_j)$

$$d\Gamma = \frac{G_F^{\ 2}}{4M_\tau^{\ 2}} |V_{CKM}|^2 \, d\Phi^{(n+1)} \, L_{\mu\nu}T^\mu T^{\nu *}$$

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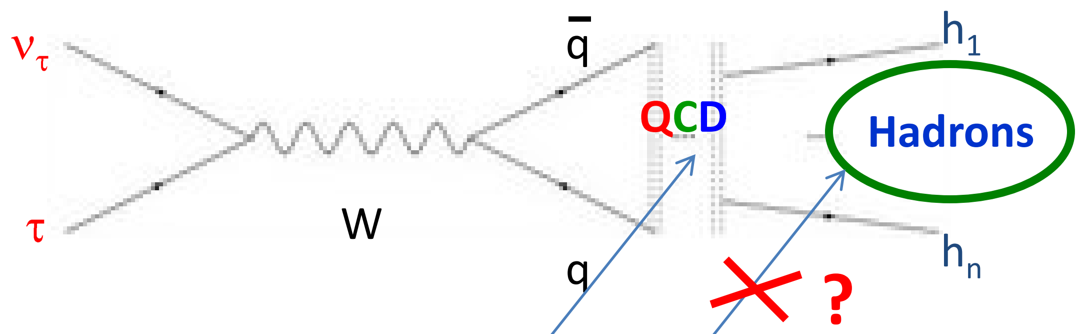

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$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}\,(q_i, G_\mu{}^a; g_s, m_i)$$

$$\mathrm{M} = \frac{G_F}{\sqrt{2}} V_{CKM}\,\overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

$$T_\mu = \langle \mathrm{Hadrons}|(V\text{-}A)_\mu\, e^{iS_{QCD}}|0\rangle = \Sigma_i(\text{Lorentz Structure})^i\, F_i(Q^2, s_j)$$

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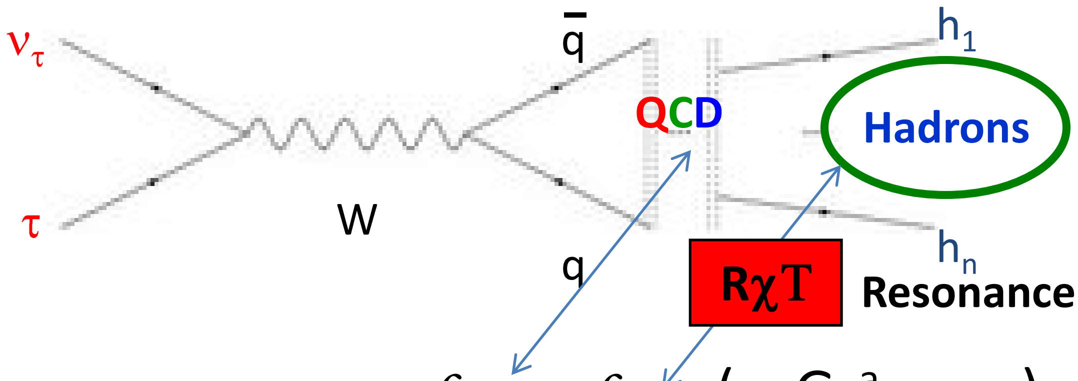

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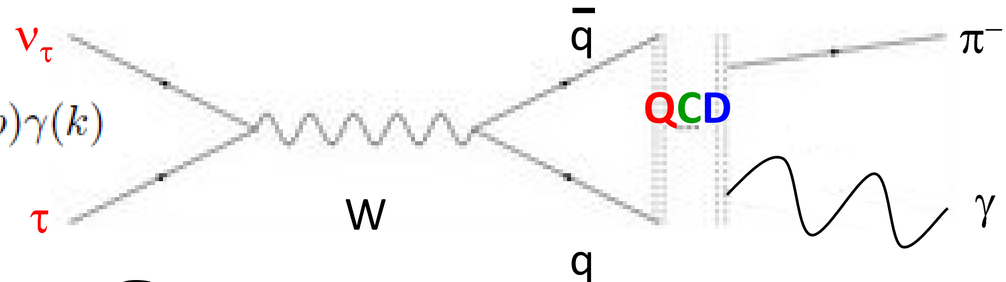
Hadron decays of the τ lepton :

$$\tau^-(s) \longrightarrow \nu_\tau(q)\pi^-(p)\gamma(k)$$



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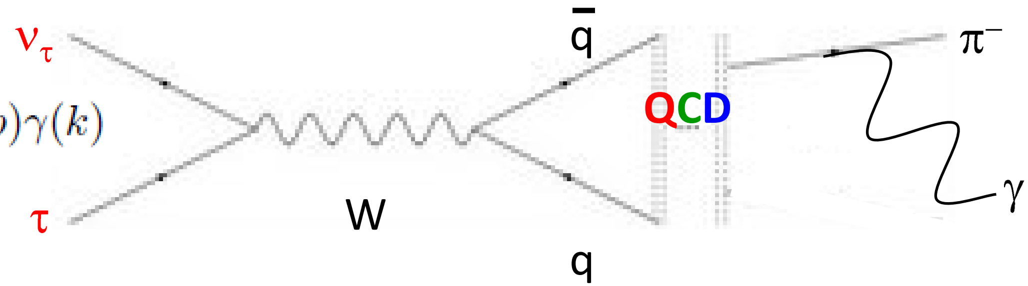

$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

Structure independent

$$i\mathcal{M}_{\mathrm{IB}_{\tau+\pi}} = G_F\, V_{ud}\, e\, F_\pi\, m_\tau\, \epsilon^\nu(k)\overline{u}_{\nu_\tau}(q)\,(1+\gamma_5)\left(\frac{s_\nu}{s\cdot k} - \frac{p_\nu}{p\cdot k} - \frac{\not k\gamma_\nu}{2s\cdot k}\right)u_\tau(s)$$

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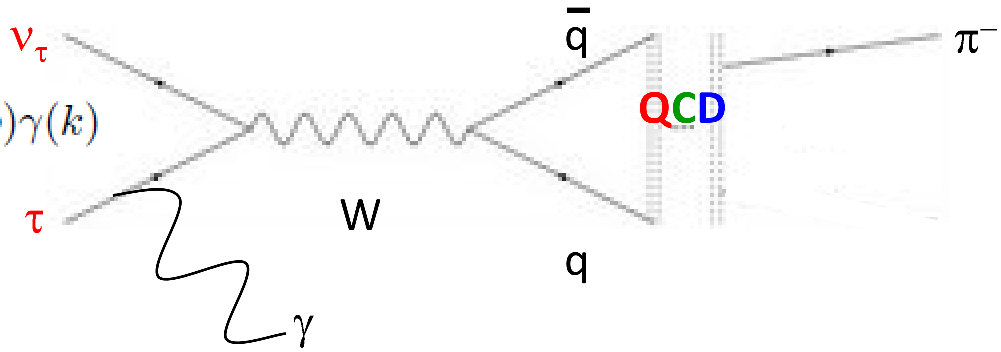

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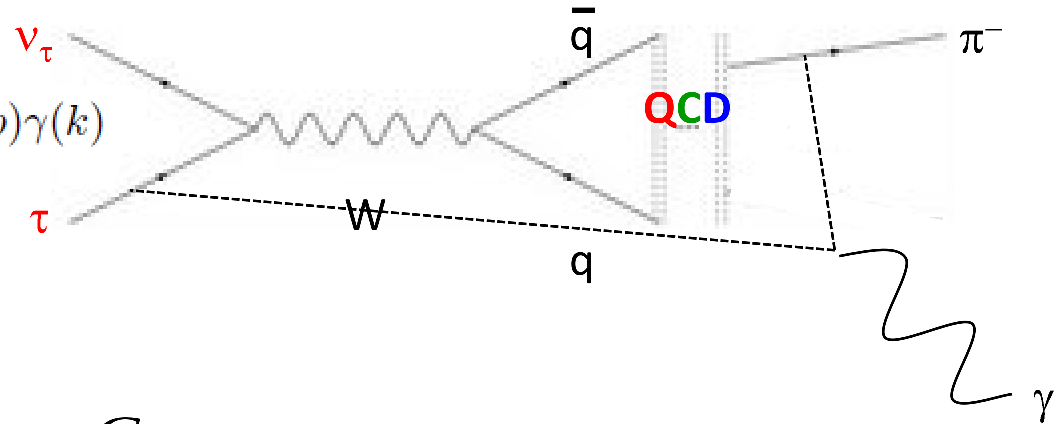

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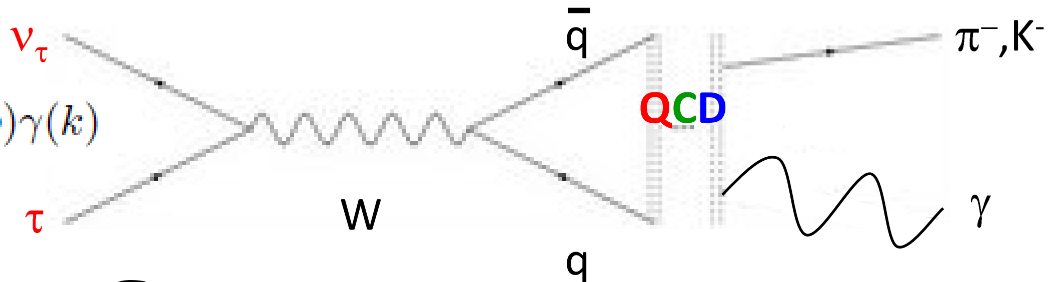

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Structure dependent

$$i\mathcal{M}_{\mathrm{IB_V}} = iG_F\, V_{ud}\, e\, \overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\, u_\tau(s)\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^\nu(k)\, k^\alpha p^\beta\, F_V^\pi(t)$$

$$i\mathcal{M}_{\mathrm{IB_A}} = G_F\, V_{ud}\, e\, \overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\, u_\tau(s)\,\epsilon^\nu(k)\,\left[(t-m_\pi^2)g_{\mu\nu} - 2k_\mu p_\nu\right]\, F_A^\pi(t)$$

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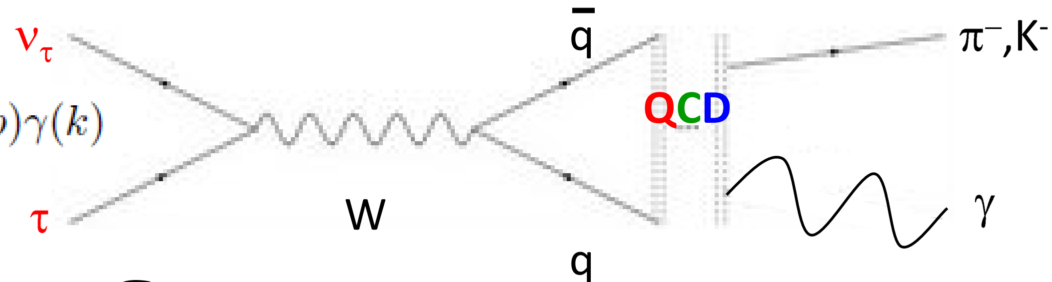

$$\text{M} \;= \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

Structure independent

$$i\mathcal{M}_{\mathrm{IB}_{\tau+\pi}} = G_F\, V_{ud}\, e\, F_\pi\, m_\tau\, \epsilon^\nu(k)\overline{u}_{\nu_\tau}(q)\,(1+\gamma_5)\left(\frac{s_\nu}{s\cdot k} - \frac{p_\nu}{p\cdot k} - \frac{k\!\!\!/\gamma_\nu}{2s\cdot k}\right) u_\tau(s)$$

Structure dependent

$$i\mathcal{M}_{\mathrm{IB_V}} = iG_F\, V_{ud}\, e\, \overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\,u_\tau(s)\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^\nu(k)\,k^\alpha p^\beta\, F_V^\pi(t)$$

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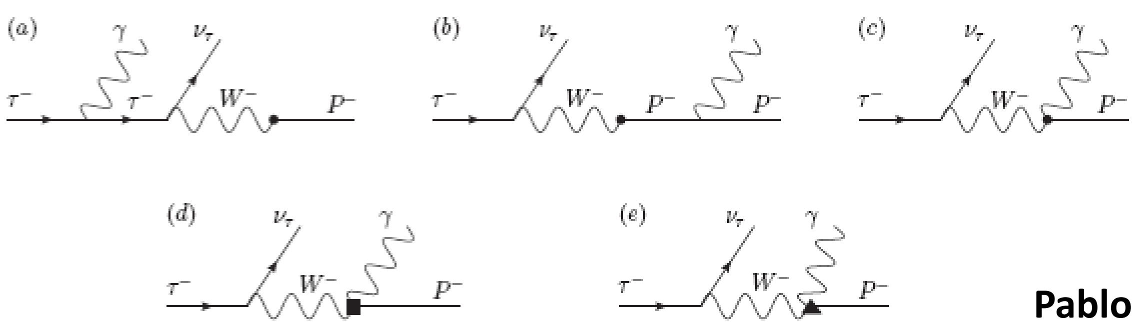

Prospects in Hadron τ decays

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Hadron decays of the τ lepton :

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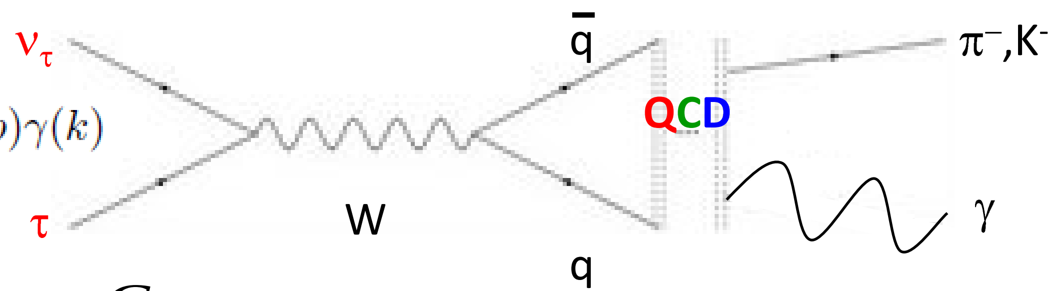

$$\mathrm{M} = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

Structure independent

$$i\mathcal{M}_{\mathrm{IB}_{\tau+\pi}} = G_F\, V_{ud}\, e\, F_\pi\, m_\tau\, \epsilon^\nu(k)\overline{u}_{\nu_\tau}(q)\,(1+\gamma_5)\left(\frac{s_\nu}{s\cdot k} - \frac{p_\nu}{p\cdot k} - \frac{k\gamma_\nu}{2s\cdot k}\right) u_\tau(s)$$

Structure dependent

$$i\mathcal{M}_{\mathrm{IB_V}} = iG_F\, V_{ud}\, e\, \overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\,u_\tau(s)\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^\nu(k)\,k^\alpha p^\beta F_V^\pi(t)$$

$$i\mathcal{M}_{\mathrm{IB_A}} = G_F\, V_{ud}\, e\, \overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\,u_\tau(s)\,\epsilon^\nu(k)\,\left[(t-m_\pi^2)g_{\mu\nu} - 2k_\mu p_\nu\right] F_A^\pi(t)$$

$$\frac{d^2\Gamma}{dx\,dy} = \frac{m_\tau}{256\pi^3}\overline{|\mathcal{M}|^2}$$

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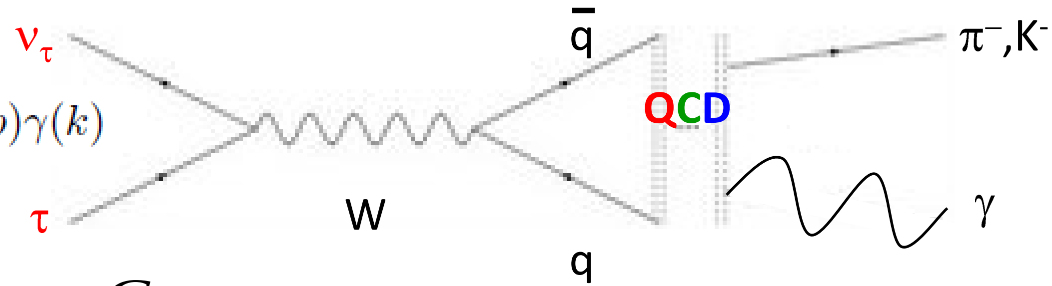

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Structure independent

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Structure dependent

$$i\mathcal{M}_{\mathrm{IB}_\mathrm{V}} = iG_F\, V_{ud}\, e\,\overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\,u_\tau(s)\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^\nu(k)\,k^\alpha p^\beta F_V^\pi(t)$$

$$i\mathcal{M}_{\mathrm{IB}_\mathrm{A}} = G_F\, V_{ud}\, e\,\overline{u}_{\nu_\tau}(q)\,\gamma^\mu(1-\gamma_5)\,u_\tau(s)\,\epsilon^\nu(k)\,\left[(t-m_\pi^2)g_{\mu\nu} - 2k_\mu p_\nu\right] F_A^\pi(t)$$

$$\frac{d^2\Gamma}{dx\,dy} = \frac{m_\tau}{256\pi^3}\overline{|\mathcal{M}|^2}$$

$$x := \frac{2s\cdot k}{m_\tau^2} \qquad y := \frac{2s\cdot p}{m_\tau^2} \qquad E_\gamma = \frac{m_\tau}{2}x \qquad E_\pi = \frac{m_\tau}{2}y$$

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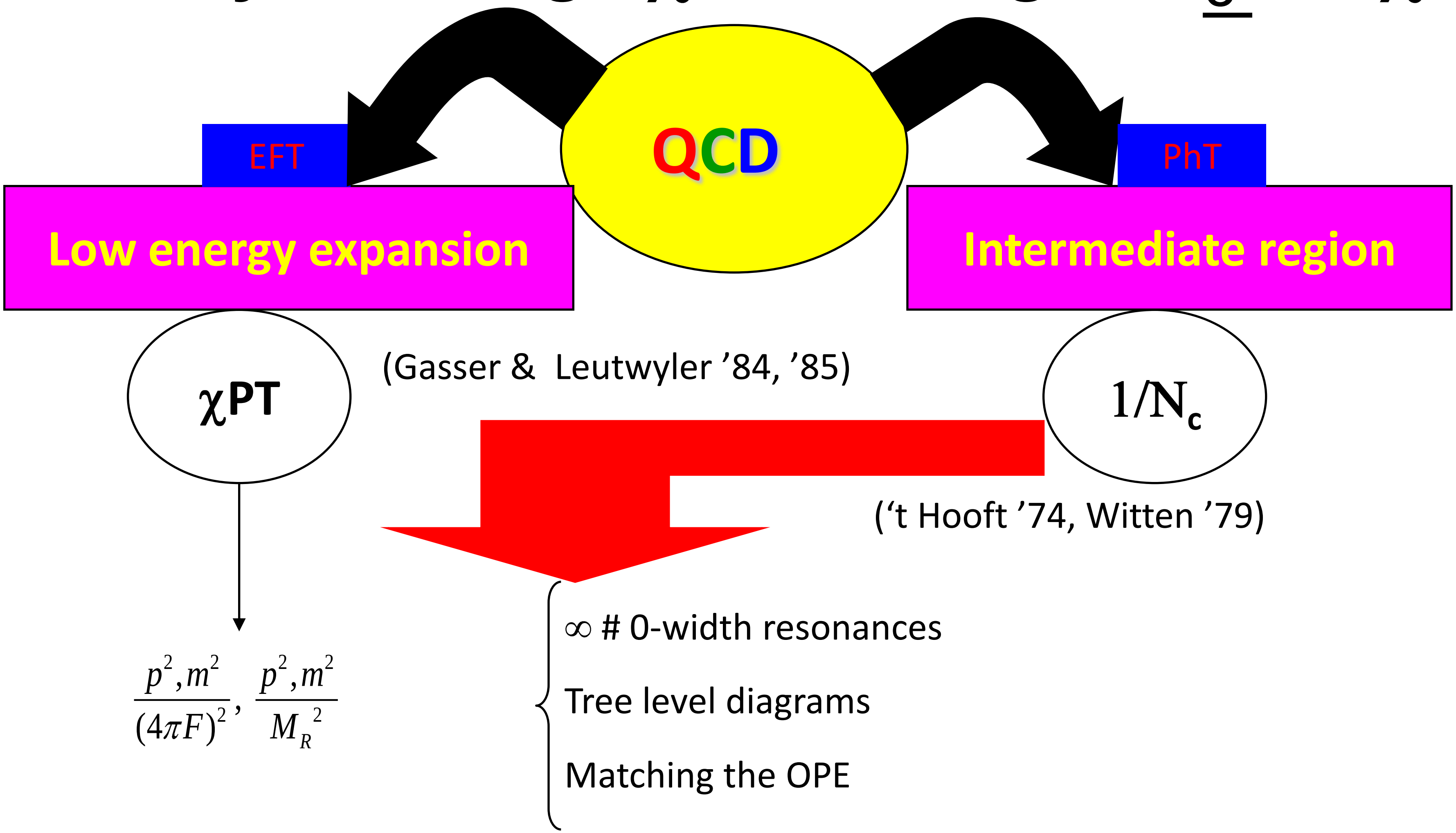
Theory setting: χPT, Large N_c, RχT



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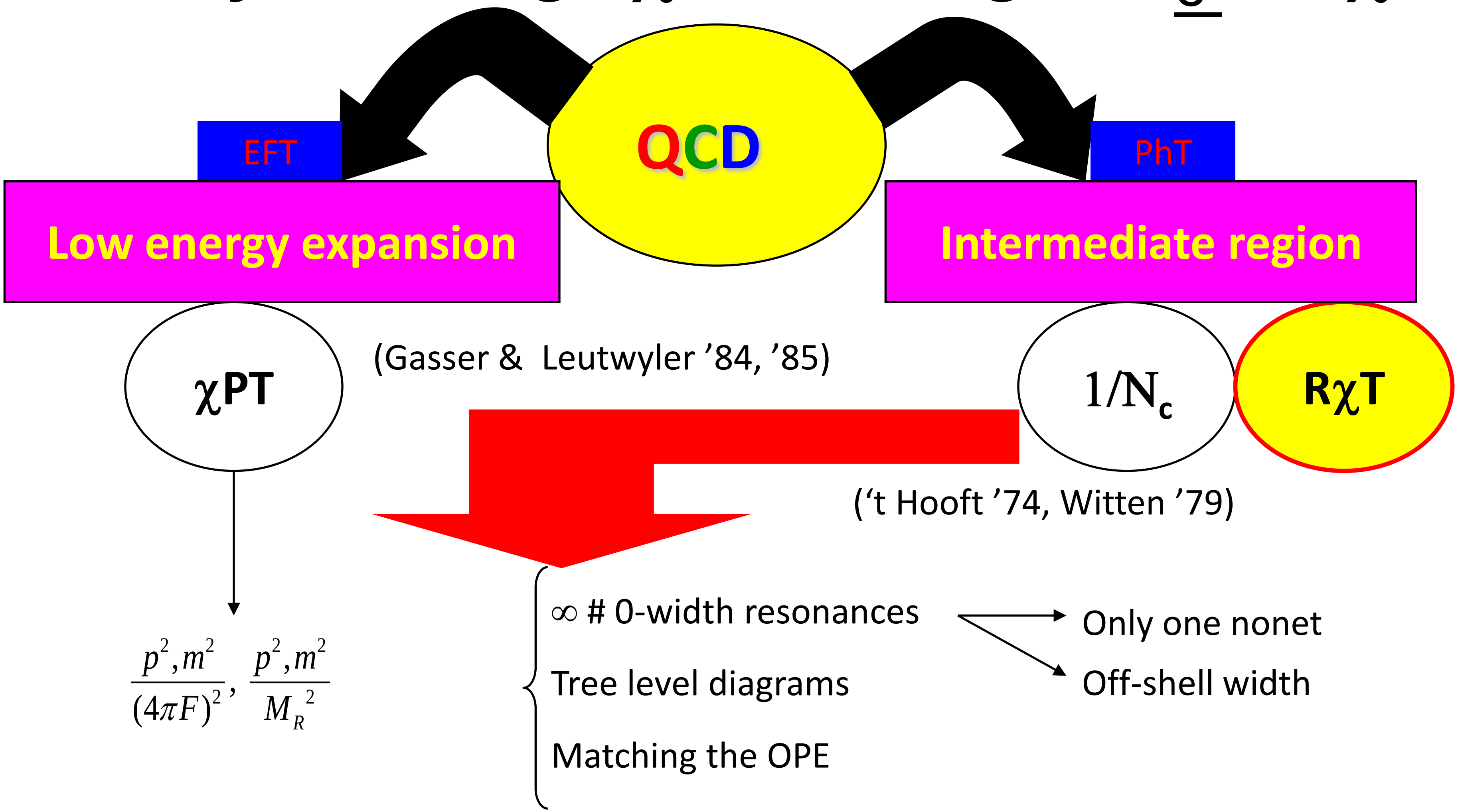

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$\tau^- \rightarrow \pi^- \gamma \nu_\tau$

Hadronic contributions

Axial form factor

Vector form factor

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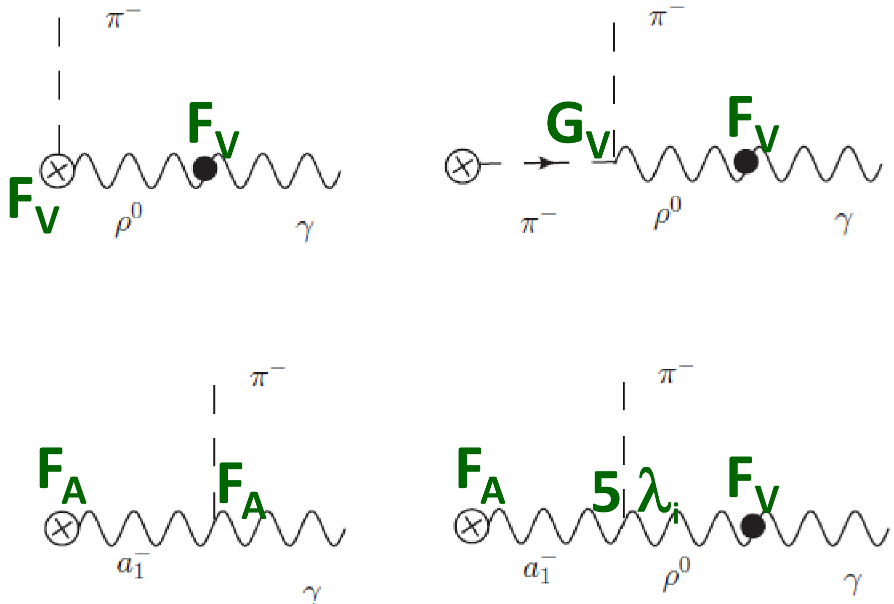
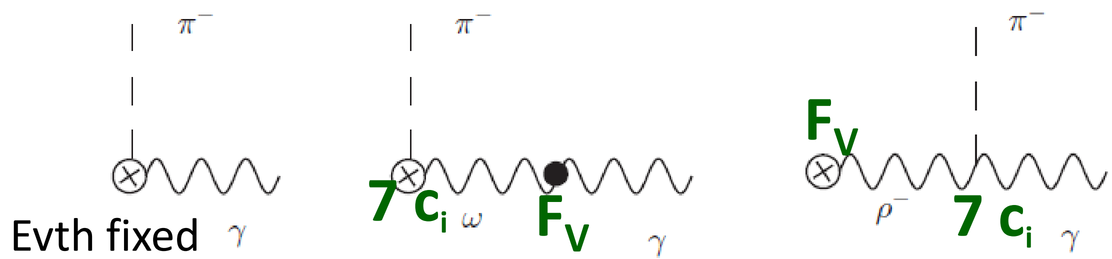
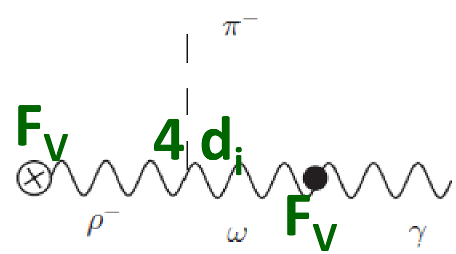
Hadronic contributions $\tau^- \to \pi^- \gamma \nu_\tau$

Axial form factor



Vector form factor





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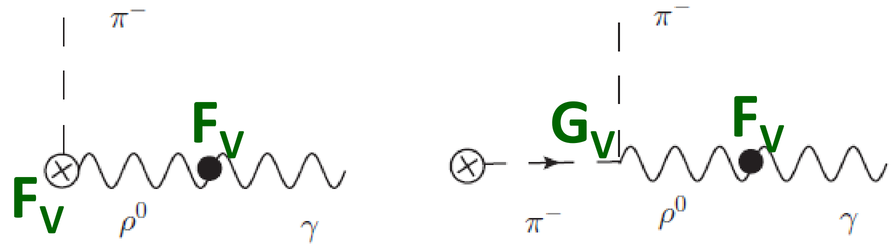

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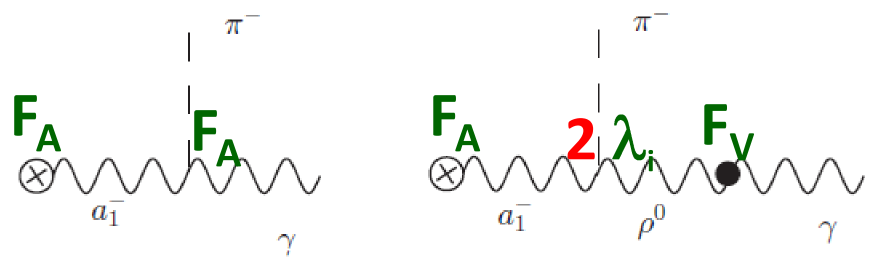
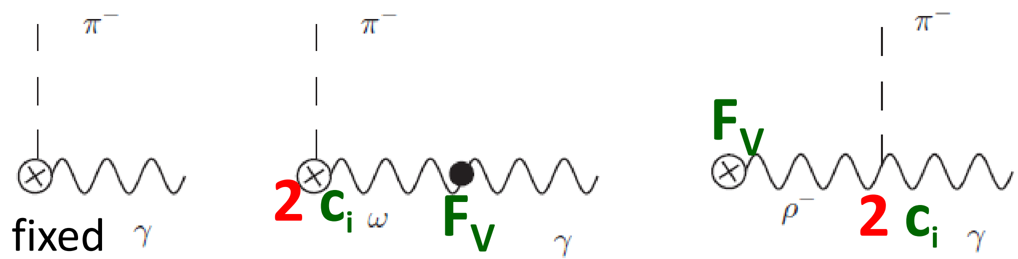
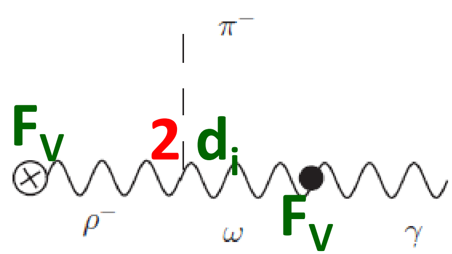


Vector form factor



(One c_i contributes to both diags)

19 → 10



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The program (for hadronic τ decays)

- After evaluating the matrix elements, we require the short-distance QCD constraints. This reduces the number of independent couplings and renders RχT predictive.
- Then we perform a phenomenological analysis using all the available information at hand.
- For the previous step a faithful description of the off-shell width of the broadest resonances is mandatory. **(Phys.Rev.D62:054014,2000; Phys.Lett.B685:158-164,2010)**

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High-energy QCD constraints on $\tau^- \to \pi^- \gamma \nu_\tau$

(more details in backup slides)

- If one subtraction is assumed, no conditions on **axial** form factor.

(Decker, Finkemeier '93)

- If no subtraction is assumed in the **axial** form factor, the results are **consistent** with those in $\tau^- \to (PPP)^- \nu_\tau$

(Phys.Rev.D81:034031,2010; Phys.Lett.B685:158-164,2010)

$$F_V^P(t \to -\infty) = \frac{F}{t}$$ (B rodsky, Lepage ' 7 9, ' 81)

- In the **VFF** the results are **consistent** with those in $\tau^- \to (PPP)^- \nu_\tau$

(Phys.Rev.D81:034031,2010)

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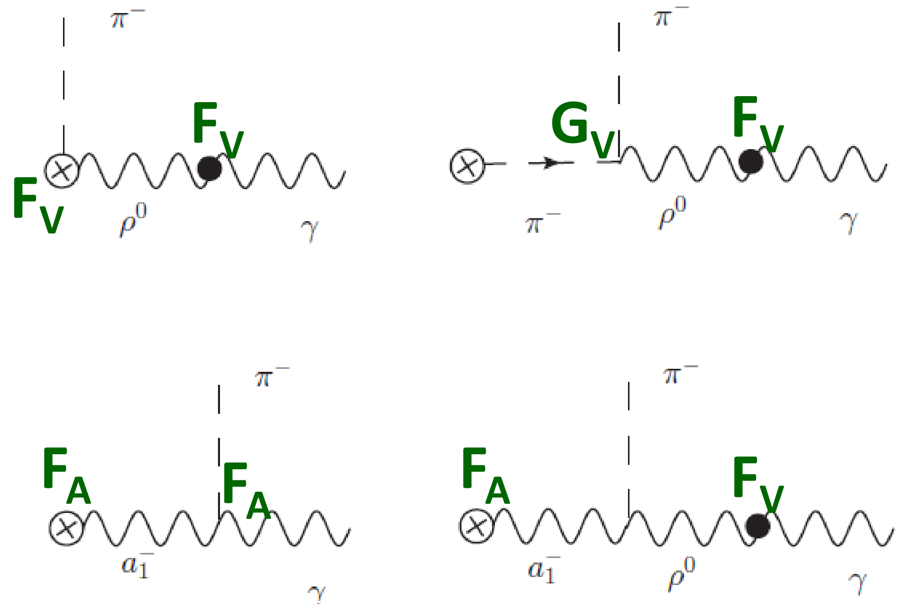
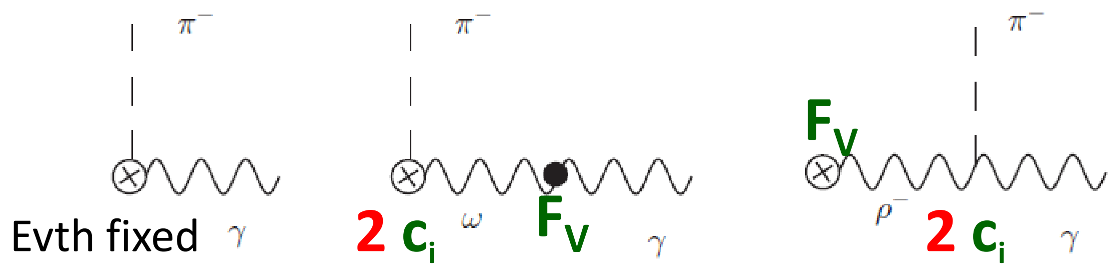
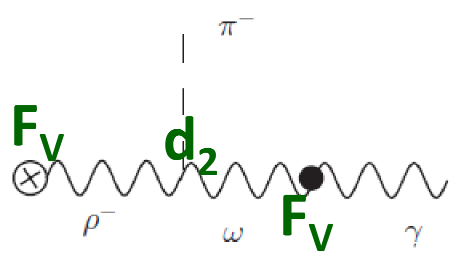
$\tau^- \rightarrow \pi^- \gamma \nu_\tau$

Axial form factor



Vector form factor





$19 \rightarrow 10 \rightarrow 2$

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The program (for $\tau^- \rightarrow \pi^- \gamma \nu_\tau$)

- Short-distance **QCD** constraints required to the participating axial-vector and vector form-factors: 10 unknowns $\rightarrow$ 2 free couplings (isospin breaking).
- These 2 unknowns can be predicted using **QCD** high-energy conditions for the VVP Green Function (**JHEP 0307:003,2003**)
- Since this mode has not been measured yet there are no experimental constraints but we can give a parameter-free prediction to be tested with the discovery data.

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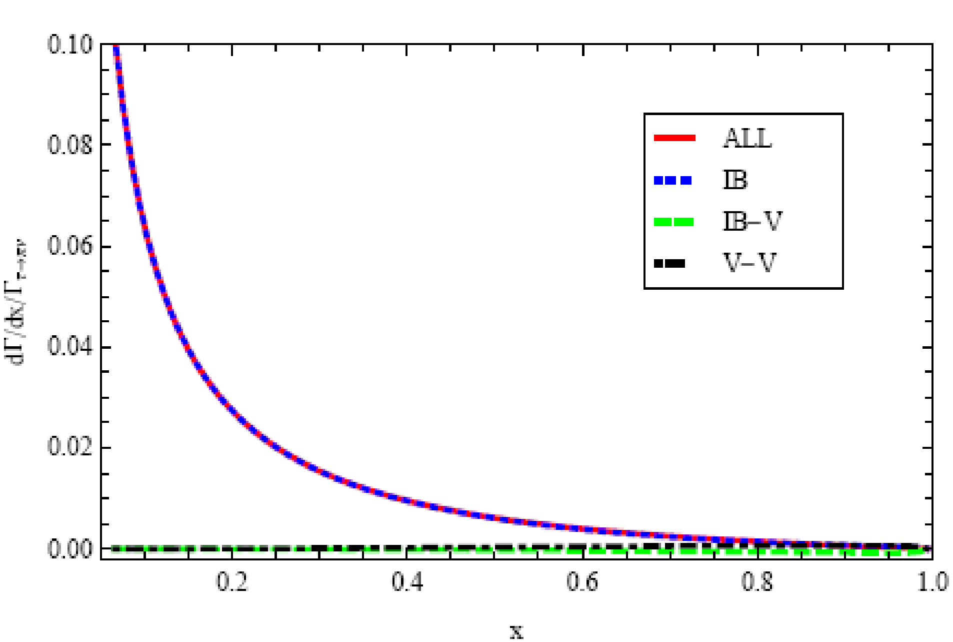
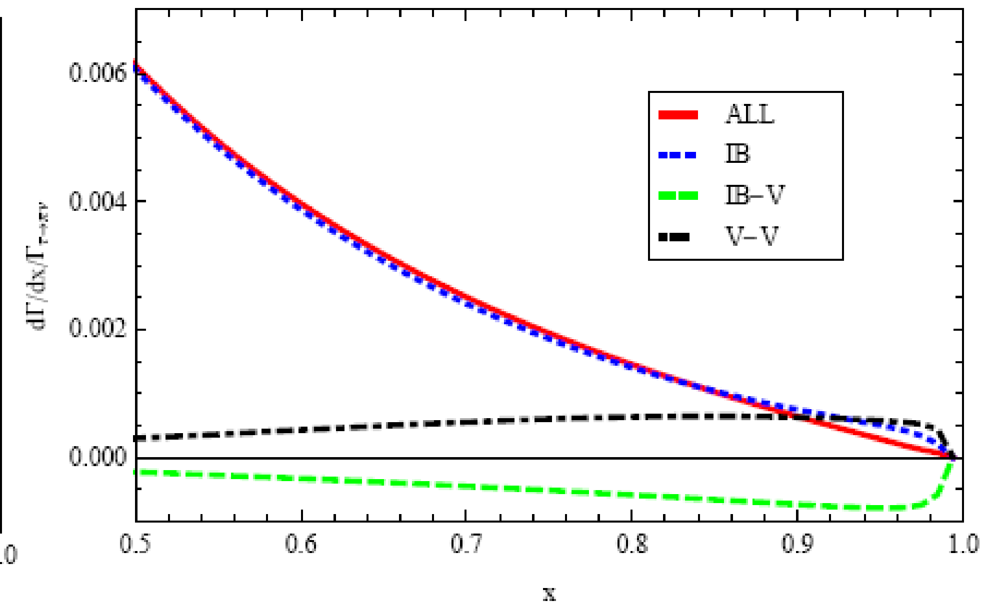
$\tau^- \to \pi^- \gamma \, \nu_\tau$

Model independent prediction: Only WZW for the VFF





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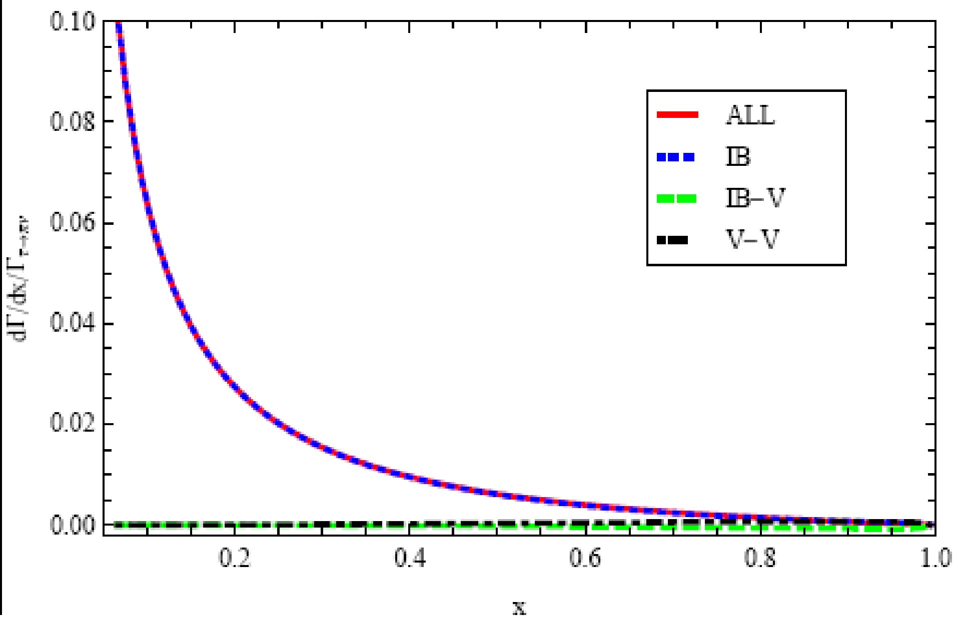

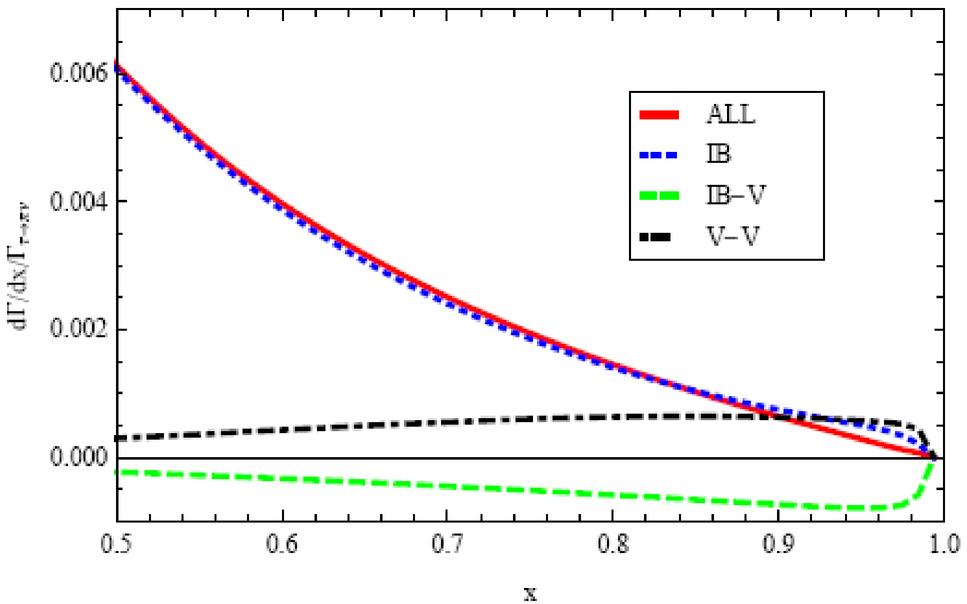

$\Gamma(\tau^- \to \pi^- \gamma \nu_\tau)$ = 3.18 2 · 10^{-15} GeV

$\Gamma(\tau^- \to \pi^- \gamma \nu_\tau)$ = 3.615 · 10^{-16} GeV

For any reasonable cut on E_γ, this decay should have already been discovered by the heavy-flavour factories

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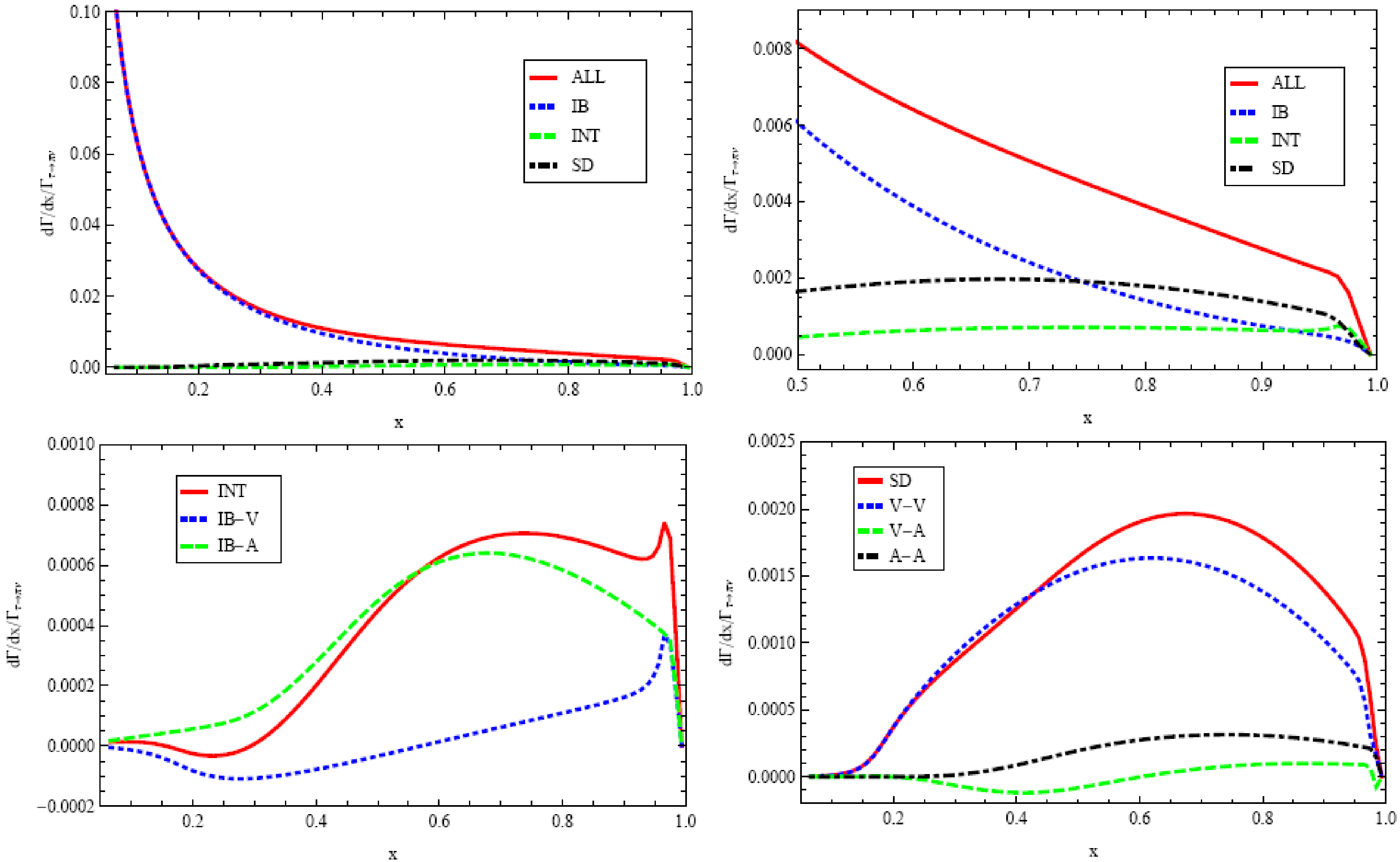
$\tau^- \to \pi^- \gamma \nu_\tau$

All contributions



Prospects in Hadron τ decays

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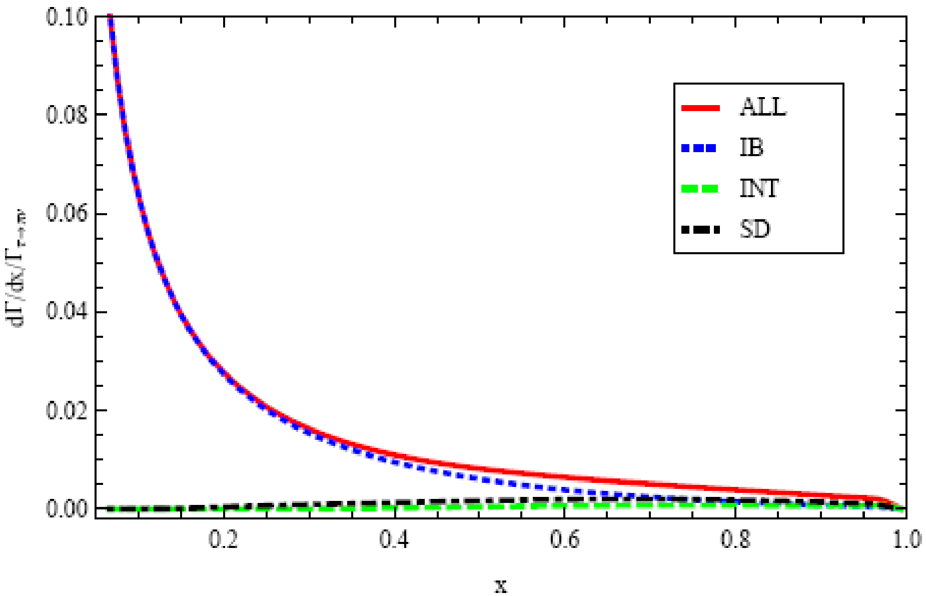

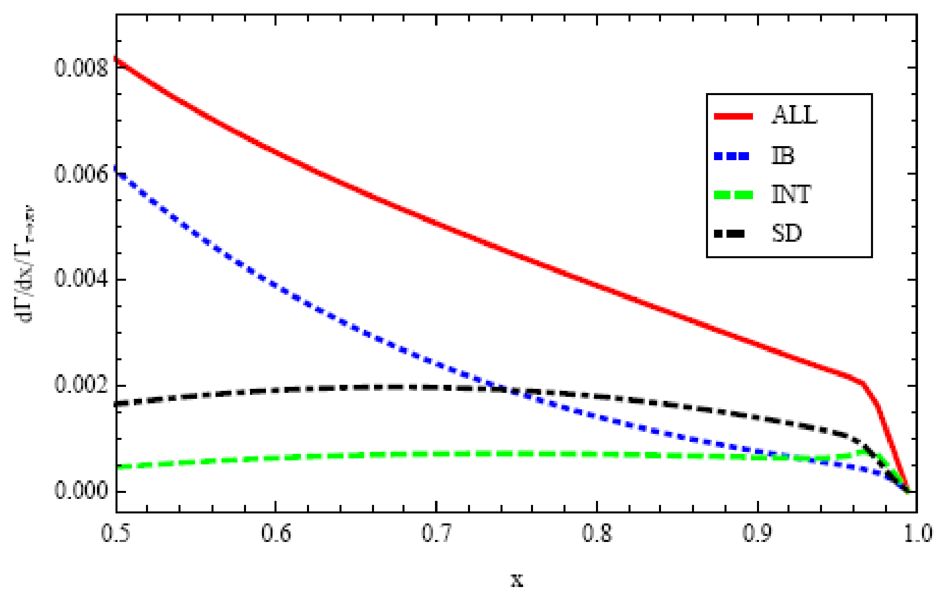

$\Gamma(\tau^- \to \pi^- \gamma \nu_\tau) = 3.304 \cdot 10^{-14}$ GeV

$\Gamma(\tau^- \to \pi^- \gamma \nu_\tau) = 6.116 \cdot 10^{-15}$ GeV

$\Gamma(\tau^- \to \pi^- \nu_\tau) = 2.471 \cdot 10^{-13}$ GeV

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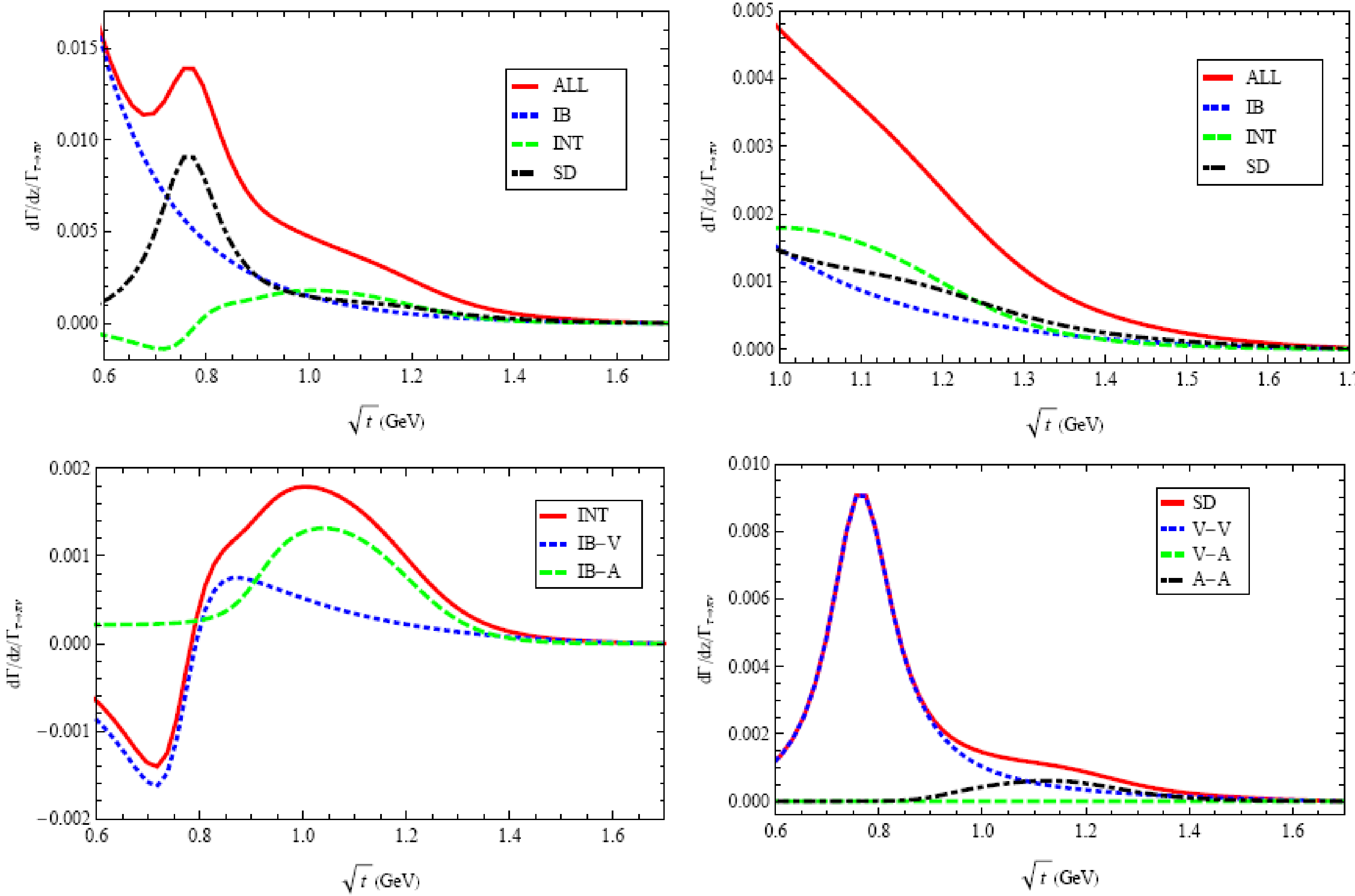
$\tau^- \to \pi^- \gamma \nu_\tau$

$$t := (p_\tau - q)^2 = (k+p)^2 = M_\tau^2(x+y-1)$$

All contributions



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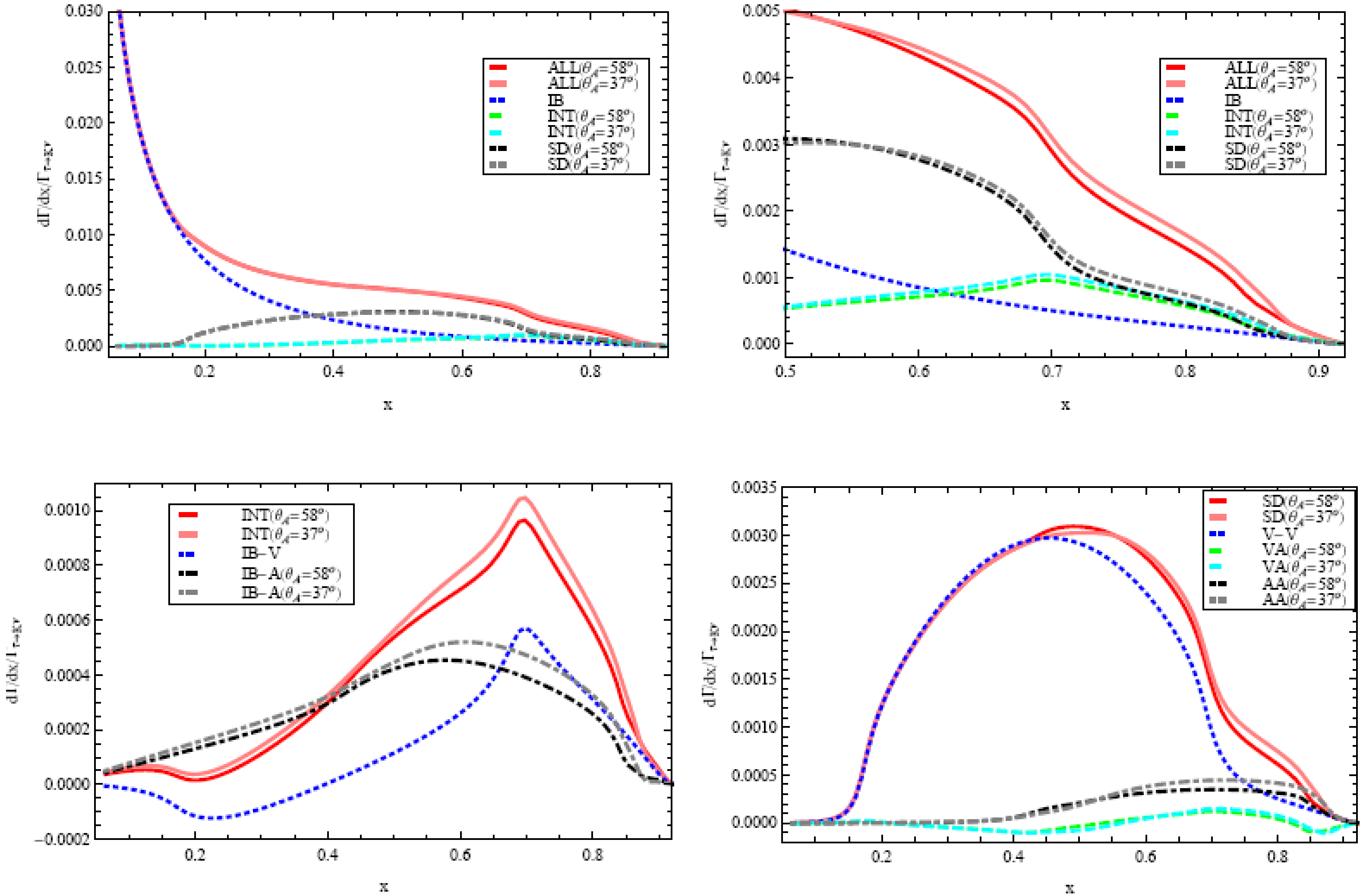
$\tau^- \to K^- \gamma \nu_\tau$

All contributions



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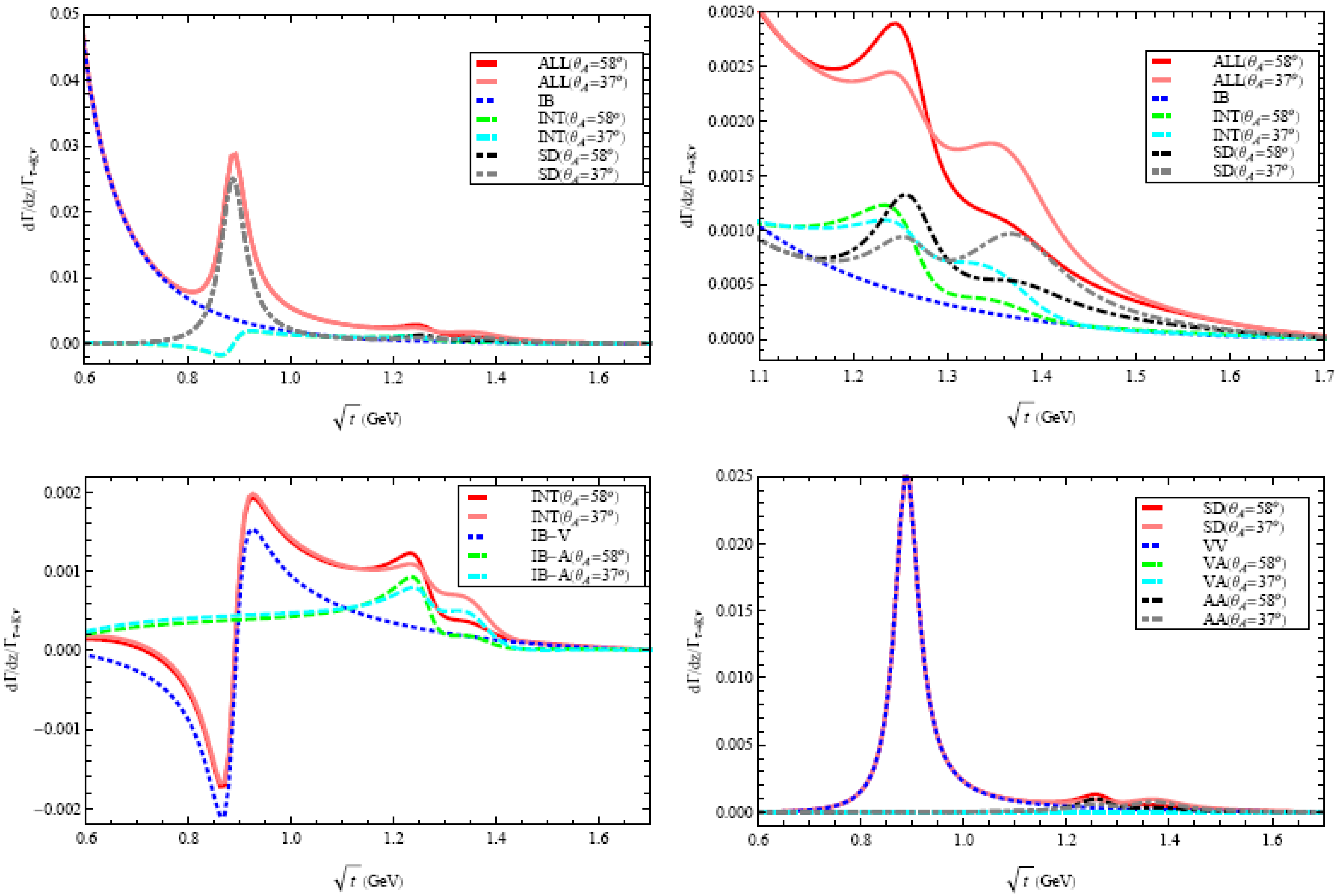
$\tau^- \rightarrow K^- \gamma \nu_\tau$

$$t := (p_\tau - q)^2 = (k+p)^2 = M_\tau^2(x+y-1)$$

All contributions



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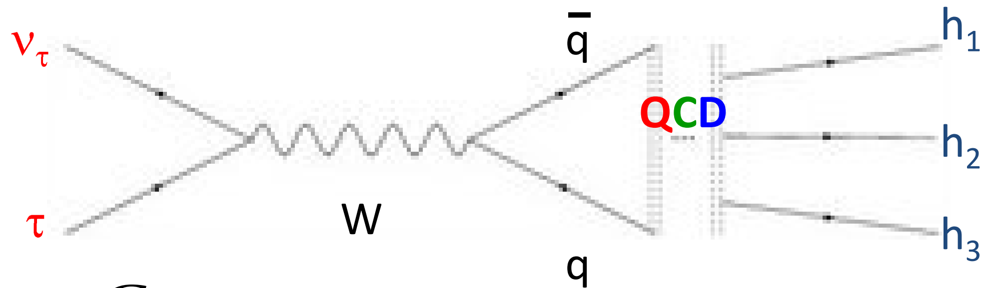
Hadron decays of the τ lepton :



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

$$\tau^- \to h_1(p_1)h_2(p_2)h_3(p_3)\nu_\tau$$

$$(p_1+p_2+p_3)^\mu = Q^\mu,\ V_{\substack{1\mu \\ 2\mu}} = \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}\right)(p_{\substack{2 \\ 3}} - p_1)^\nu$$

$$T_\mu = V_{1\mu}F_1 + V_{2\mu}F_2 + Q_\mu F_P + \underbrace{i\varepsilon_{\mu\nu\rho\sigma}p_1^{\ \nu} p_2^{\ \rho} p_3^{\ \sigma}}_{V_{3\mu}} F_V$$

$$\frac{d\Gamma}{dQ^2} = \frac{G_F^{\ 2}|V_{CKM}|^2}{128(2\pi)^5 M_\tau^{\ 3}} \int ds\,dt\, f\left(I_{0^-}, I_{1^+}, I_{1^-}\right)$$

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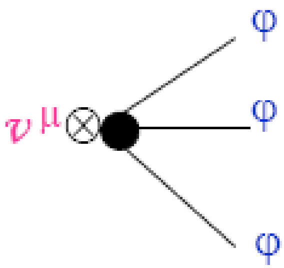
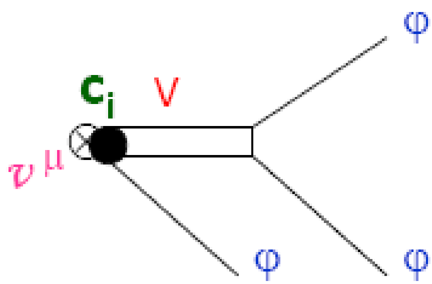
$\tau^- \to \eta\pi^-\pi^0\nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)

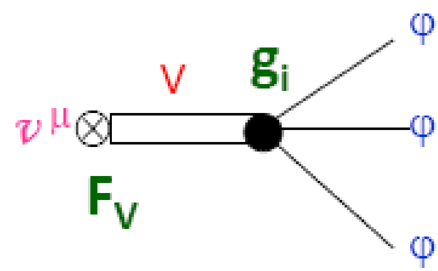
Only Vector form factor

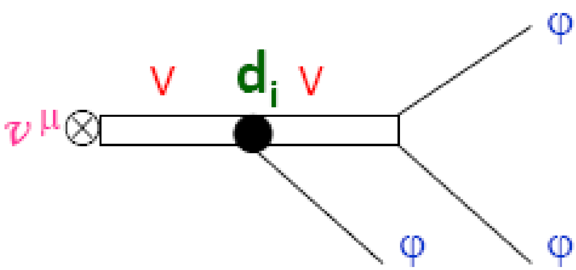


χPT at LO



RχT, 1R





RχT, 2R

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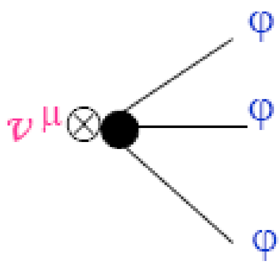
$\tau^- \to \eta\pi^-\pi^0\nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)

Only Vector form factor



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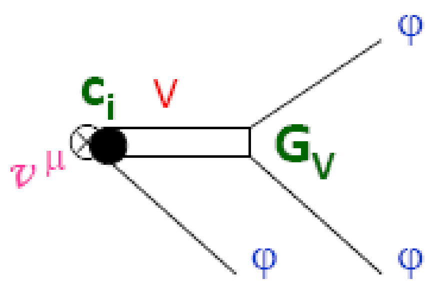
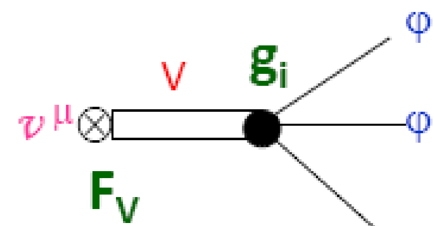
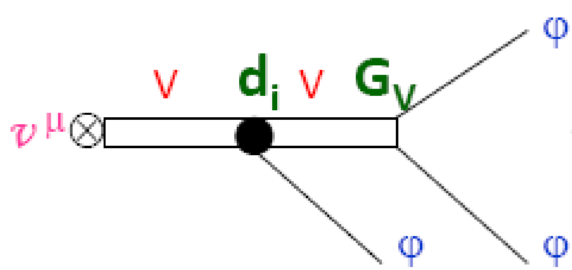


7 c_i



5 g_i



4 d_i

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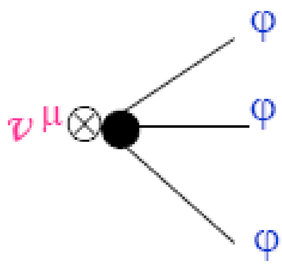
$\tau^- \to \eta\pi^- \pi^0 \nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)

Only Vector form factor



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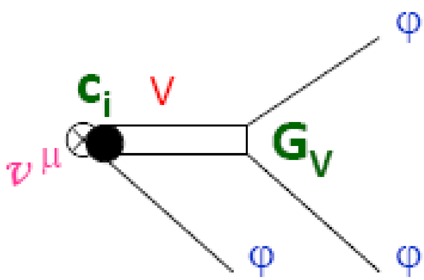
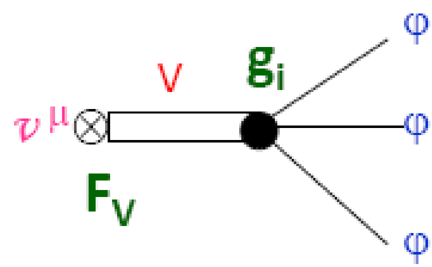
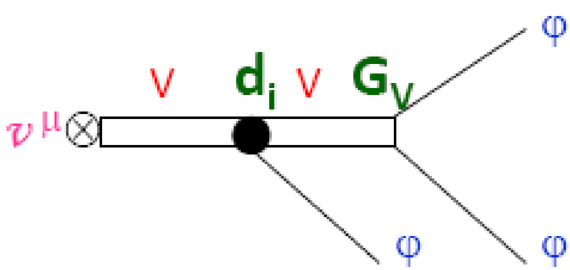


$4\ c_i$



$4\ g_i$



$3\ d_i$

Computation

$18 \to 13$

Prospects in Hadron τ decays

Pablo Roig

LPT (CNRS), Orsay (France)

High-energy QCD constraints

$\tau^- \to (PPP)^- \nu_\tau$ (D. Gómez Dumm, A. Pich, J. Portolés, P.R)

$$\Im m\, \Pi_{V,A}(q^2) \xrightarrow[q^2\to\infty]{} \frac{N_C}{12\,\pi}$$ (Floratos, Narison and De Rafael '79)

$$W_A = -(V_1^\mu F_1 + V_2^\mu F_2)(V_{1\mu} F_1 + V_{2\mu} F_2)^*$$

↓

$$\lim_{Q^2\to\infty} \int_0^{Q^2} ds \int_0^{Q^2-s} dt \frac{W_A}{(Q^2)^2} = 0$$

And analogously for the vector form factor with $W_B = (F_3 V_{3\mu})(F_3 V_{3\mu})^*$

$v^\mu V \varphi$

$$c_1 - c_2 + c_5 = 0 \qquad c_1 - c_2 - c_5 + 2c_6 = -\frac{N_C}{96\pi^2}\frac{M_V F_V}{\sqrt{2}\, F^2}$$

$V\, V\, \varphi$

$$d_3 = -\frac{N_C}{192\pi^2}\frac{M_V^2}{F^2}$$

$V\, \varphi\varphi\varphi$

$$g_2 = \frac{N_C}{192\pi^2}\frac{M_V}{\sqrt{2}\, F_V} \qquad g_1 + 2g_2 - g_3 = 0$$

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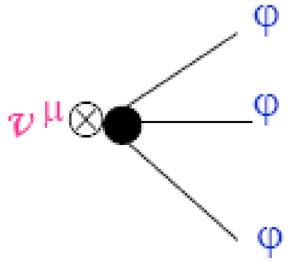
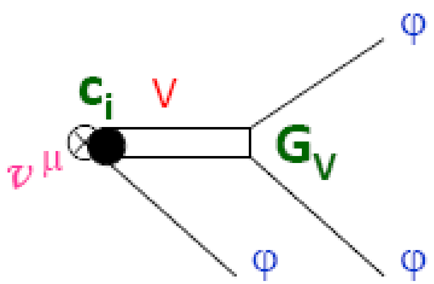
$\tau^- \to \eta \pi^- \pi^0 \nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)

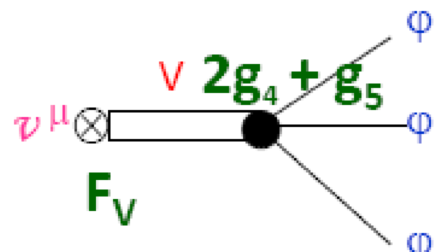
Only Vector form factor

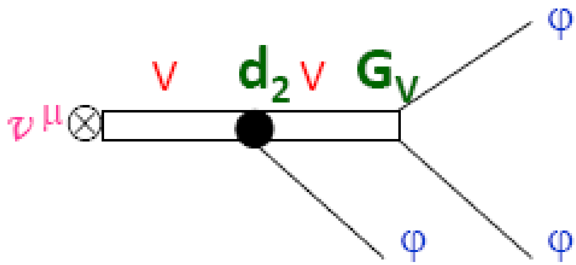


Evth fixed



$2c_i$





d_2

Computation

18 → 13 → 4

Imposing high-energy QCD constraints

Pablo Roig

LPT (CNRS), Orsay (France)

The program

- Short-distance QCD constraints ✓
- Then we perform a phenomenological analysis using all the available information at hand.

(Only BR for $\tau^- \to \eta\pi^- \pi^0 \nu_\tau$). Use of **<VVP>** (Ruiz-Femenía, Pich and Portolés'03) constraints (2 relations).

$2g_4 + g_5$ from $\omega \to 3\pi$ **c_4** from $\tau^- \to (KK\pi)^- \nu_\tau$ (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.2640**)

- For the previous step a faithful description of the off-shell width of the broadest resonances is mandatory. (Gómez-Dumm, Pich, Portolés **arXiv: hep-ph/0003320**) (Gómez-Dumm, Pich, Portolés **arXiv: 0312183**) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)

This way, the BR reported by PDG is obtained in $\tau^- \to \eta\pi^- \pi^0 \nu_\tau$ for natural values of the remaining 2 free parameters. ✓

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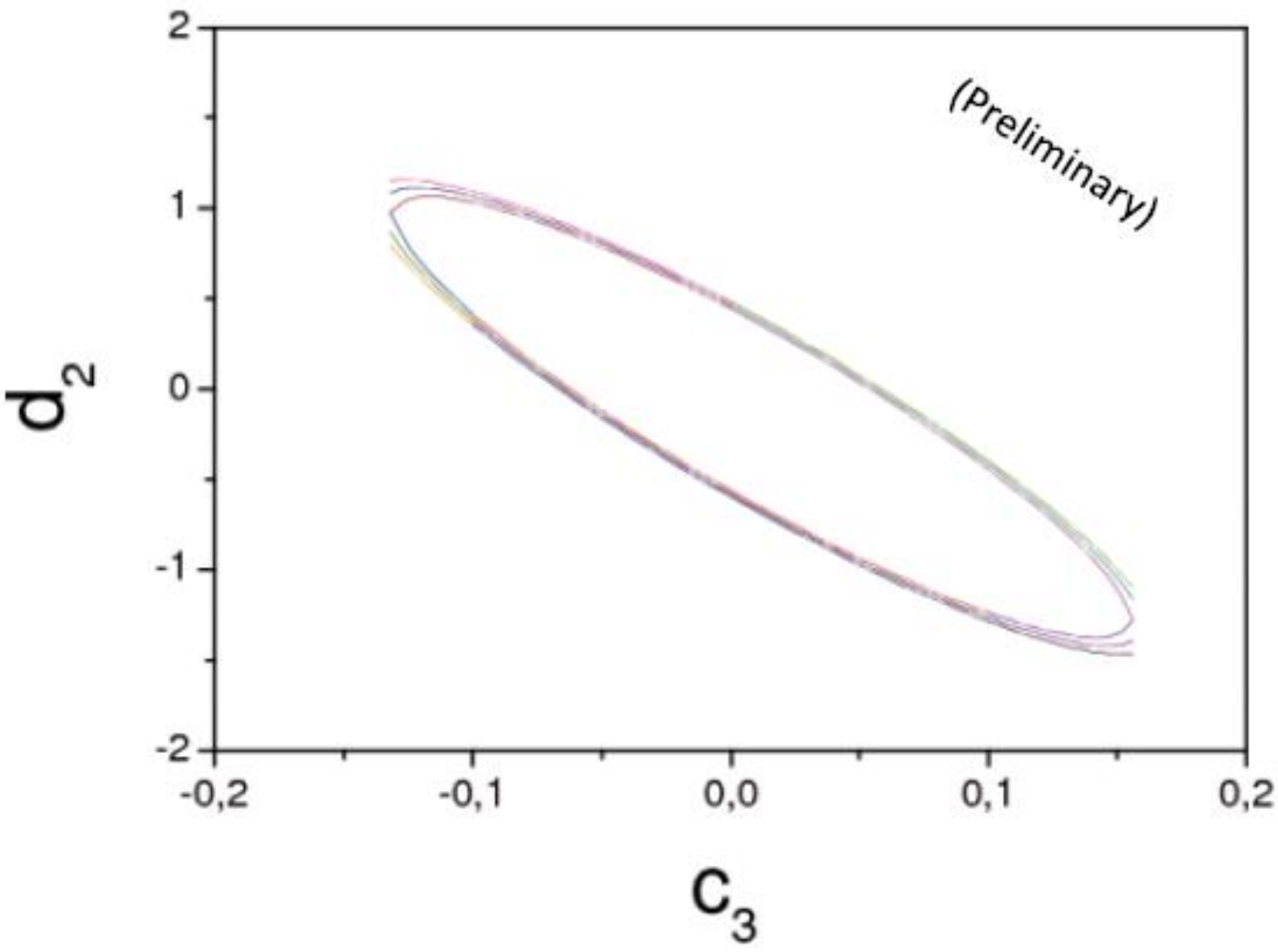
$\tau^- \to \eta\pi^- \pi^0 \nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)



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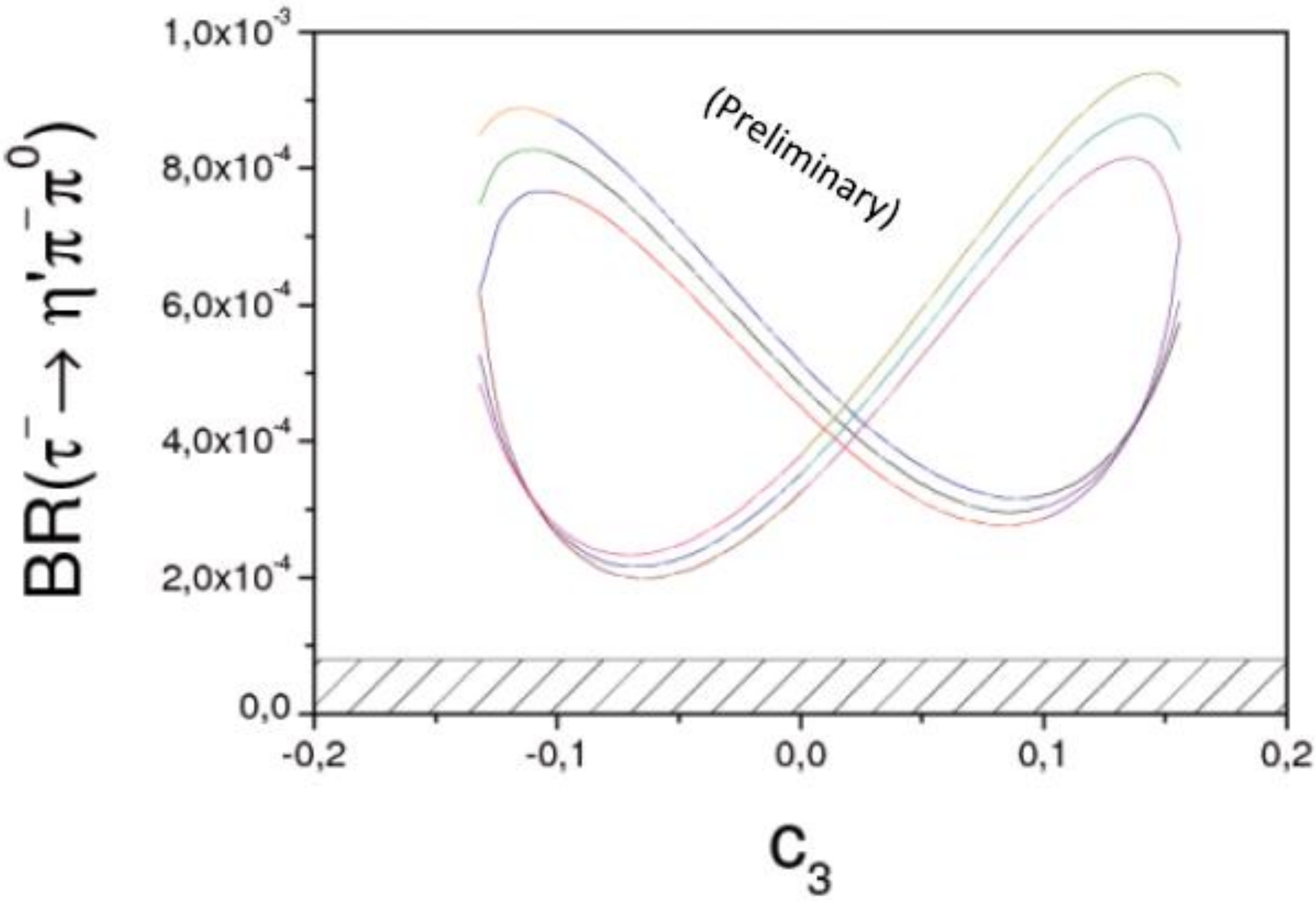
$\tau^- \to \eta' \pi^- \pi^0 \nu_\tau$ (D. Gómez Dumm, A. Pich, P.R)



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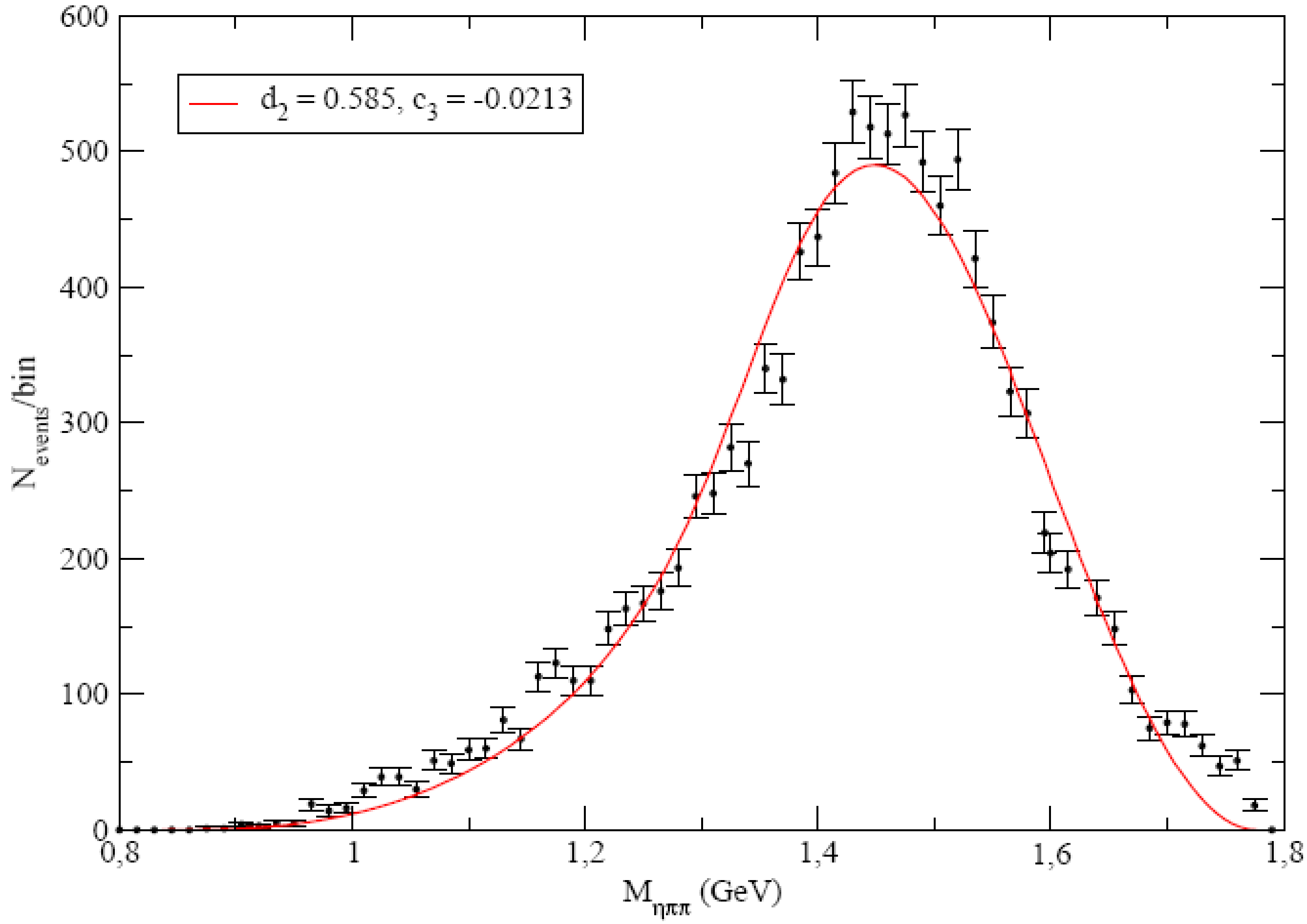
$\tau \to \eta\pi^-\pi^0$ (Collaboration with D. Gómez Dumm, Pich)



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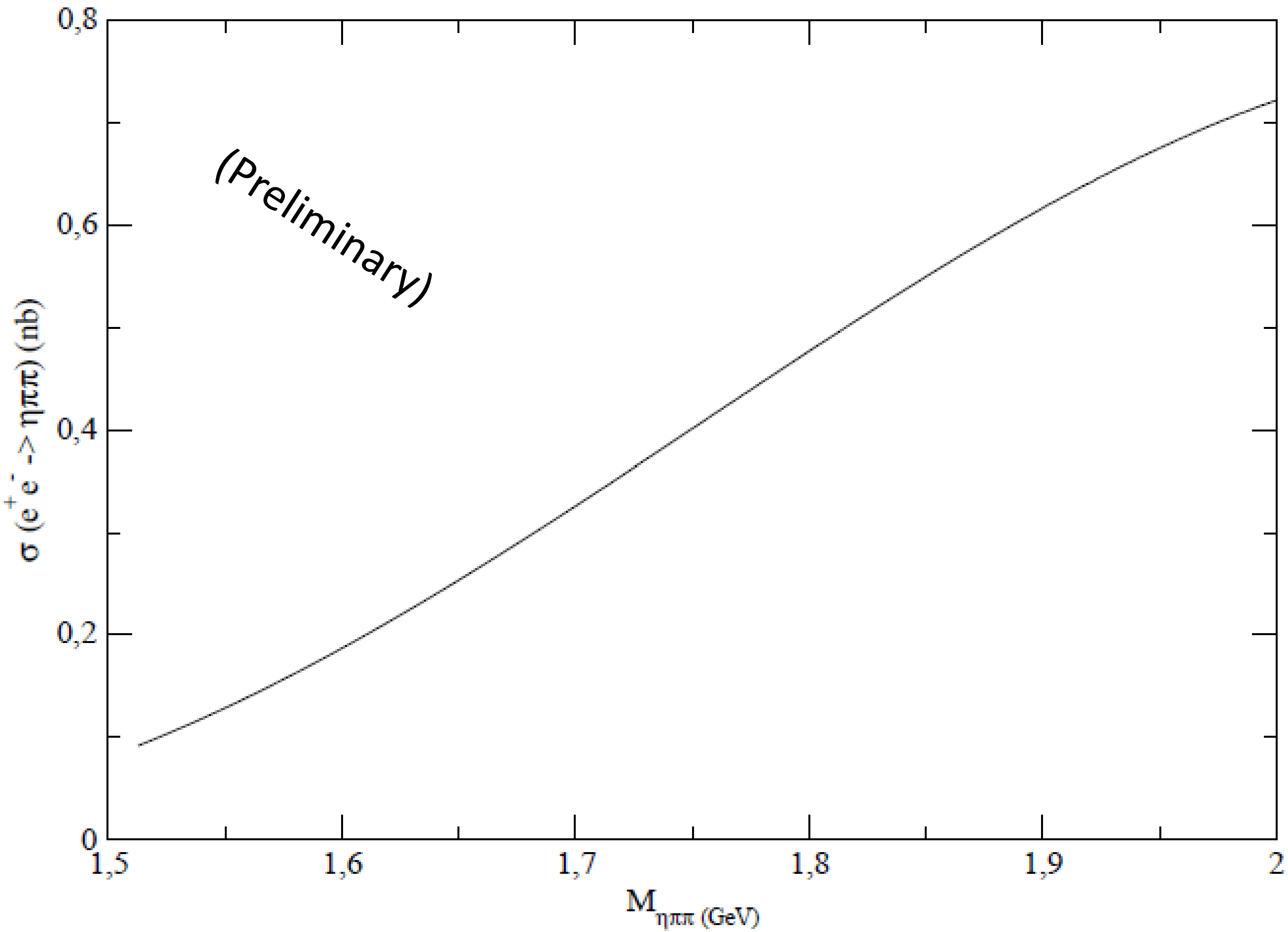
$e^+e^- \to \eta\pi\pi$ (Collaboration with D. Gómez Dumm, Pich)

Using **CVC**, one can relate the form factors appearing in τ decays to those in e^+e^- scattering into the same hadron states.



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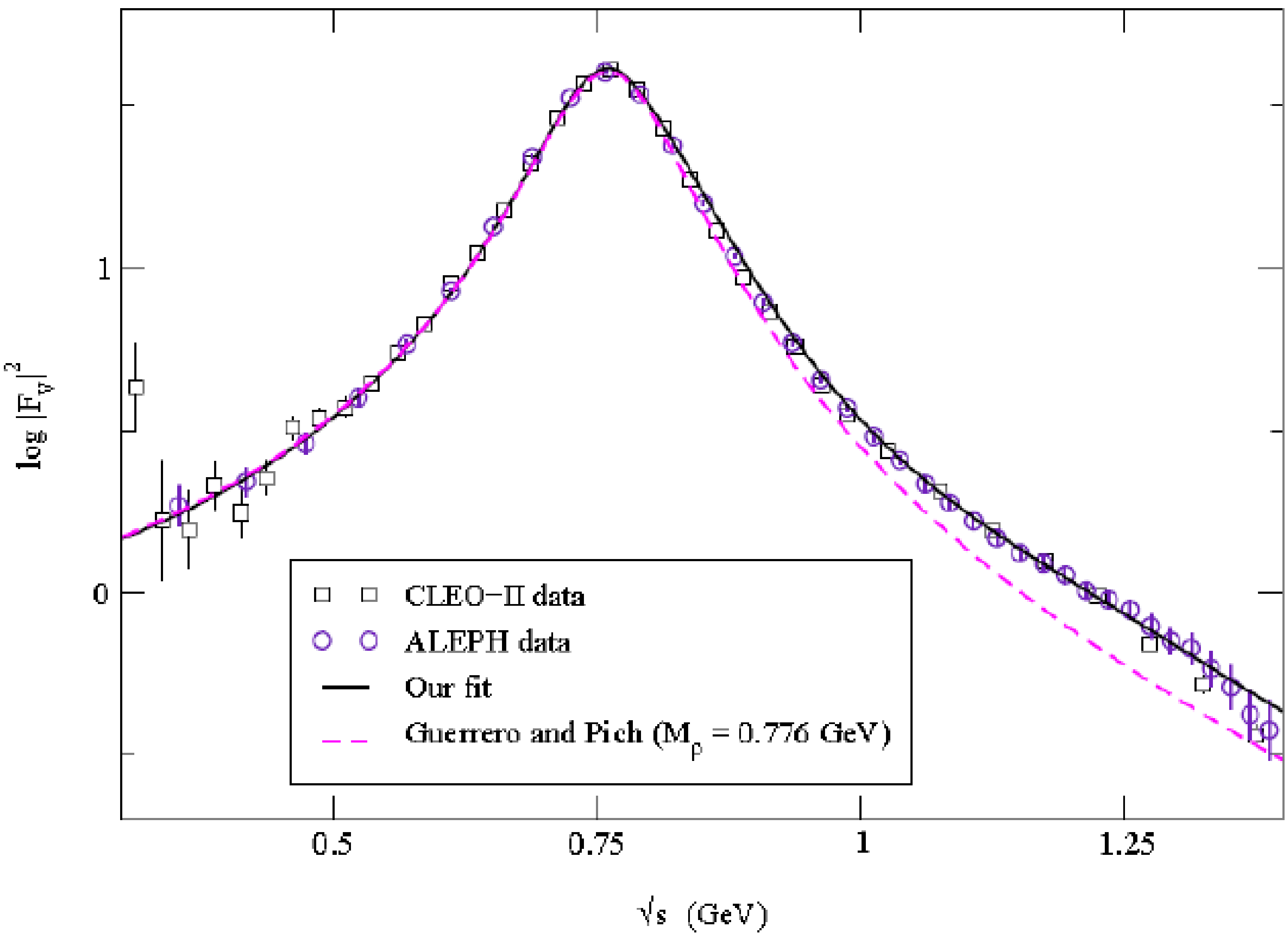
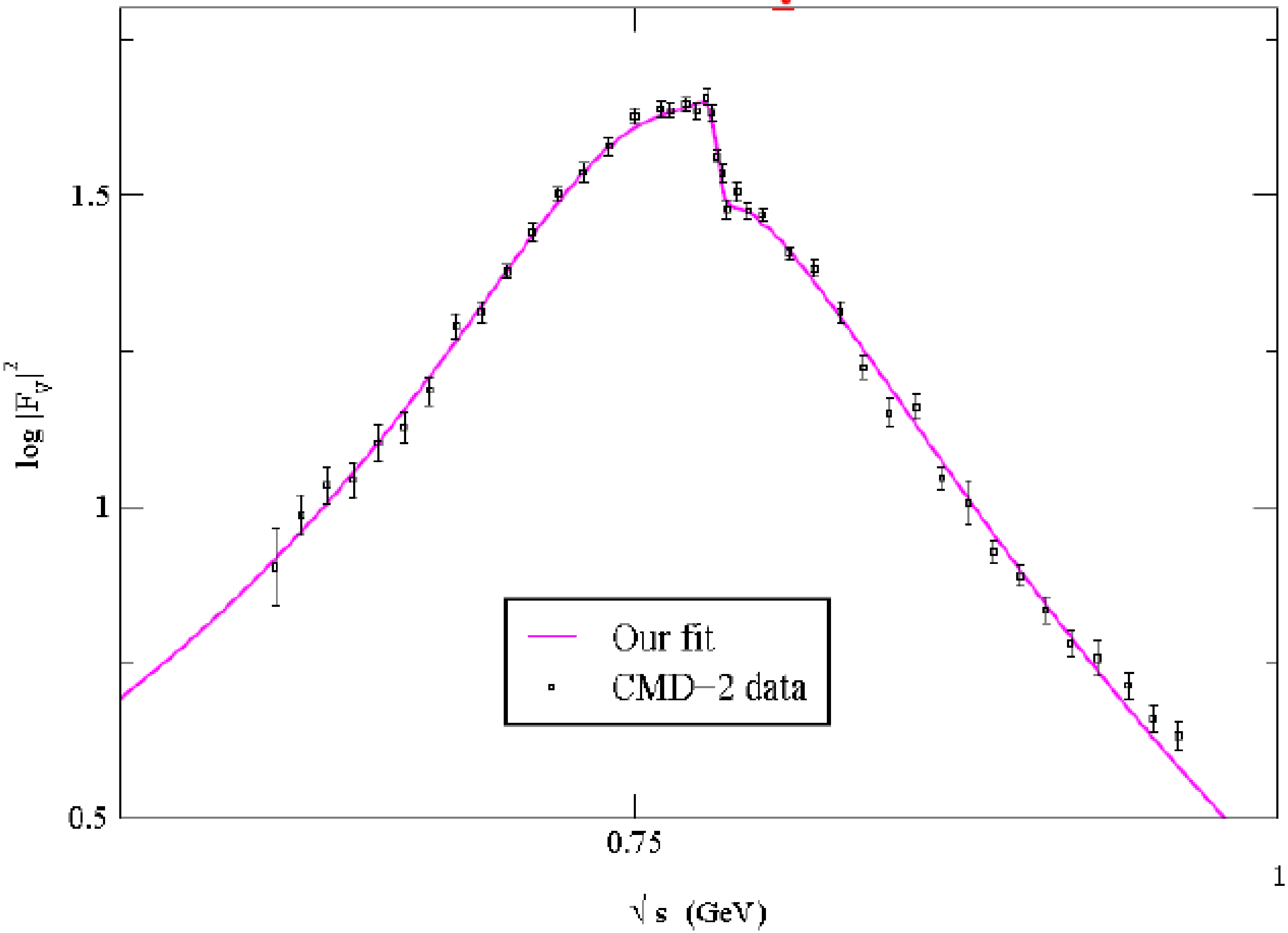
(Guerrero, Pich '97) $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (Pich, Portolés '01, '03)



$\tau^- \to \pi^- \pi^0 \nu_\tau$

(Pich, Portolés '01, '03)



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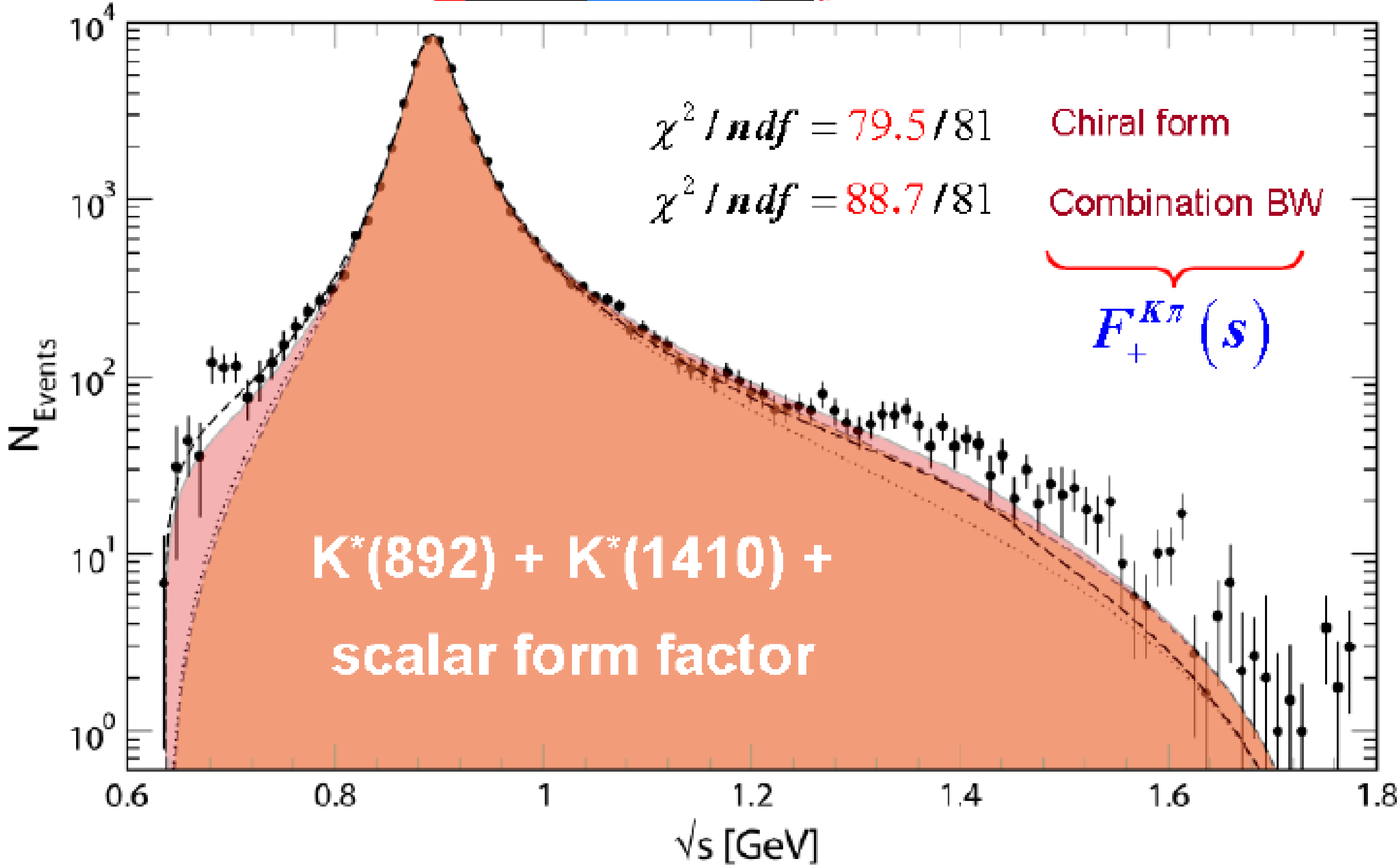
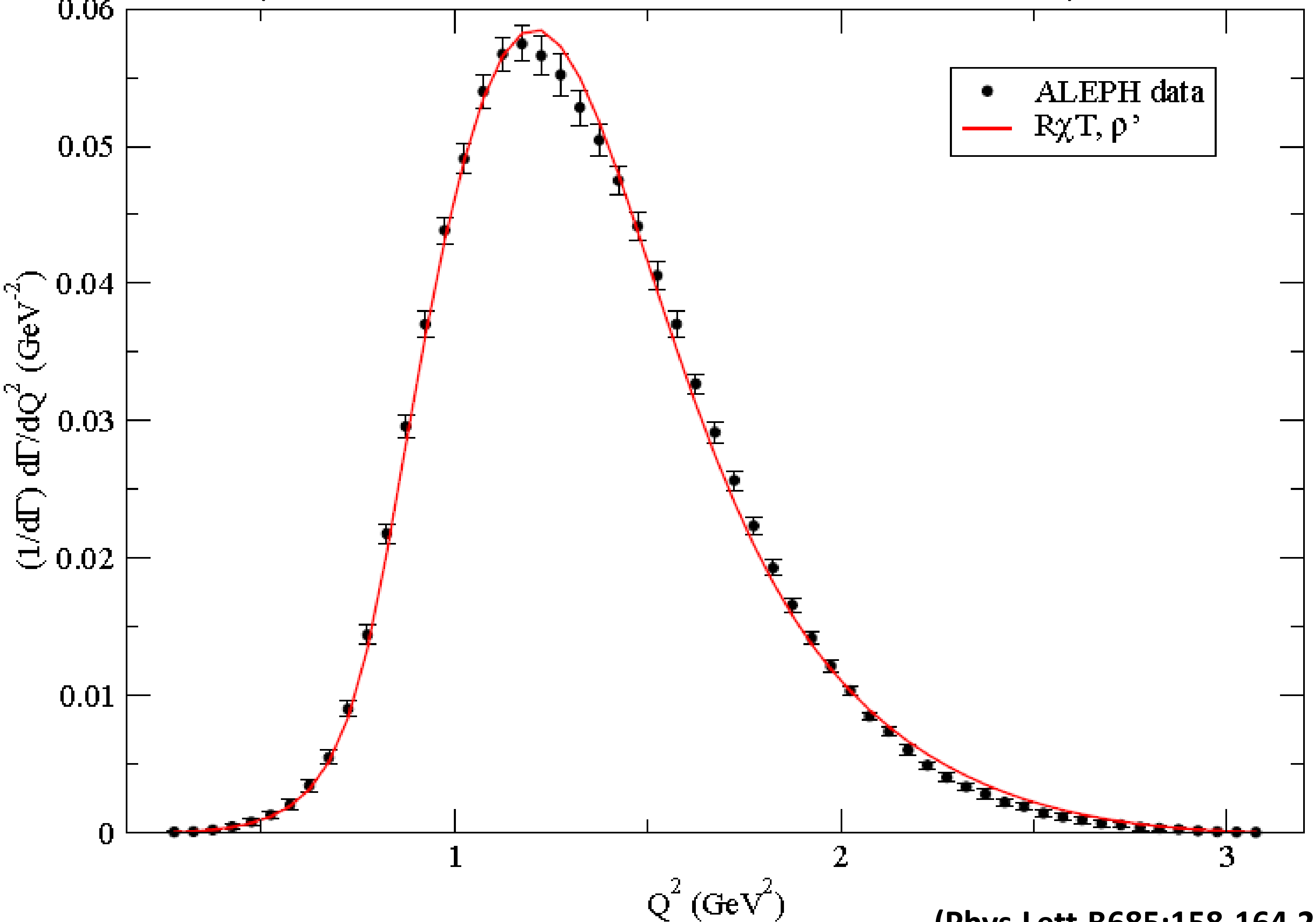
$\tau^- \rightarrow (K\pi)^- \nu_\tau$ (Jamin, Pich & Portolés, 08)



$\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)



(Phys.Lett.B685:158-164,2010)

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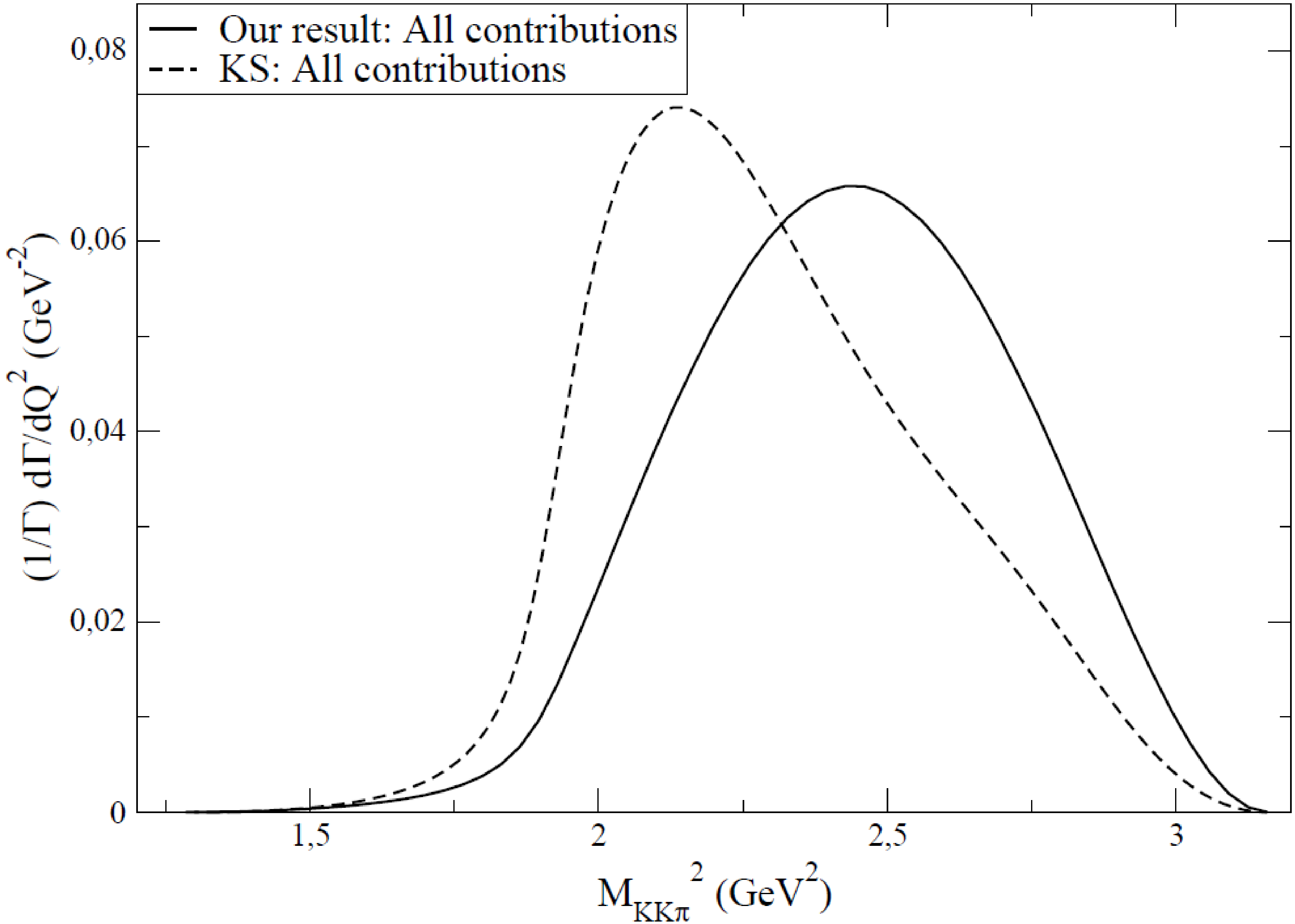
(**Phys.Rev.D81:034031,2010**)

$\tau \to K^+ K^- \pi^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.2640**)



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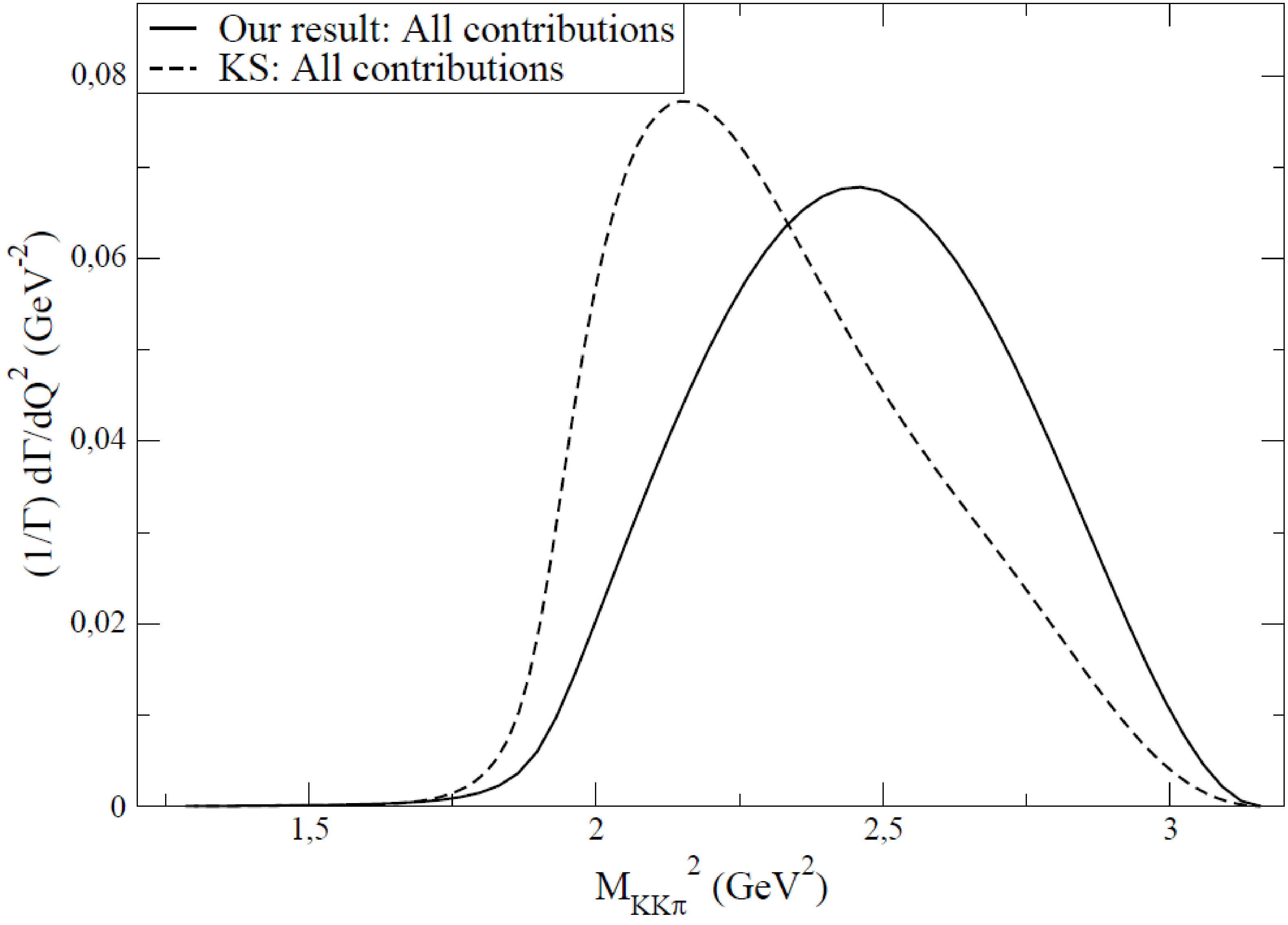
(**Phys.Rev.D81:034031,2010**)

$\tau \to K^- K^0 \pi^0 \nu_\tau$ (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.2640**)



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Prospects in Hadron τ decays

SUMMARY AND FUTURE PLANS

Pablo Roig

LPT (CNRS), Orsay (France)

SUMMARY

- Resonance Chiral Theory is a convenient framework to study hadron decays of the tau based on some properties of QCD: its chiral limit, its large-N_C limit and its known asymptotic behaviour.
- We have applied it to the study of the $\tau^- \to (\pi/K)^- \gamma \nu_\tau$ and $\eta\pi\pi$ decays and checked the consistency of the whole procedure with previous results in other $\tau^- \to (PPP)^- \nu_\tau$ processes.
- Our results (also $\pi\pi$ and $K\pi$) are being implemented in **TAUOLA** providing the experimental community a theory based tool to analyze these decays.

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FUTURE PLANS

- We are studying some remaining relevant modes $\tau^- \rightarrow$ **(PP/PPP)**$^-$ ν_τ: $K\eta$, $K\eta$ (Boito, Escribano, Roig), $K\pi\pi$ (Gómez-Dumm, Jamin, Pich, Roig). We plan to study KKK and KK decay channels.
- It is not likely that we are able to tackle the $\pi\pi\pi\pi$ and **K**$\pi\pi\pi$ decays in the near future.
- The extremely important decay channel $\pi\pi$ may be revisited in the light of new data in order to obtain a more accurate theory input for the **TAUOLA**.

BACKUP SLIDES

Short-distance QCD constraints

• **Axial** form factor: No subtraction is assumed and the results are **consistent** with those in $\tau^- \to (PPP)^- \nu_\tau$. **(Phys.Rev.D81:034031,2010; Phys.Lett.B685:158-164,2010)**

Restrictions found in $\tau^- \to \pi^- \gamma \nu_\tau$: $F_V = \sqrt{3}F$ $\quad F_V G_V = F^2$ $\quad F = \sqrt{3}G_V$ $\quad \lambda'' = \frac{2G_V - F_V}{2\sqrt{2}F_A}$

+ Weinberg Sum Rules: $F_V^2 - F_A^2 = F^2$ $\qquad M_V^2 F_V^2 - M_A^2 F_A^2 = 0$

(Phys.Rev.Lett.18:507-509,1967)

+ Relations for <VAP>: $\lambda' = \frac{M_A}{2\sqrt{2}M_V}$ $\qquad 4\lambda_0 = \lambda' + \lambda''$

(Phys.Lett.B596:96-106,2004)

$$F_A^2 = 2F^2 \quad , \quad M_A = \frac{6\pi F}{\sqrt{N_C}}$$

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Short-distance QCD constraints

$$F_V^P(t \to -\infty) = \frac{F}{t}$$

• **Vector** form factor: Brodsky Lepage behaviour demanded. The results are **consistent** with those in $\tau^- \to (PPP)^- \nu_\tau$. **(Phys.Rev.D81:034031,2010)**

Restrictions found in $\tau^- \to \pi^- \gamma \nu_\tau$: $c_1 - c_2 + c_5 = 0$ $\quad c_5 - c_6 = \frac{N_C M_V}{32\sqrt{2}\pi^2 F_V} + \frac{F_V}{\sqrt{2}M_V} d_3$

$$F = \frac{M_V \sqrt{N_C}}{2\sqrt{6}\pi}$$

+ Weinberg Sum Rules: $F_V^2 - F_A^2 = F^2$ $\qquad M_V^2 F_V^2 - M_A^2 F_A^2 = 0$
(Phys.Rev.Lett.18:507-509,1967)

+ Relations for <VVP>: $c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$
(**JHEP 0307:003,2003**)

$$d_3 = -\frac{N_C M_V^2}{192\pi^2 F^2}$$

In agreement with (**Phys.Rev.D81:034031,2010)**
5 % deviation with respect to relation in (**JHEP 0307:003,2003**): $d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2}$

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Axial form factor and a_1: $\tau^- \to (3\pi)^- \nu_\tau$

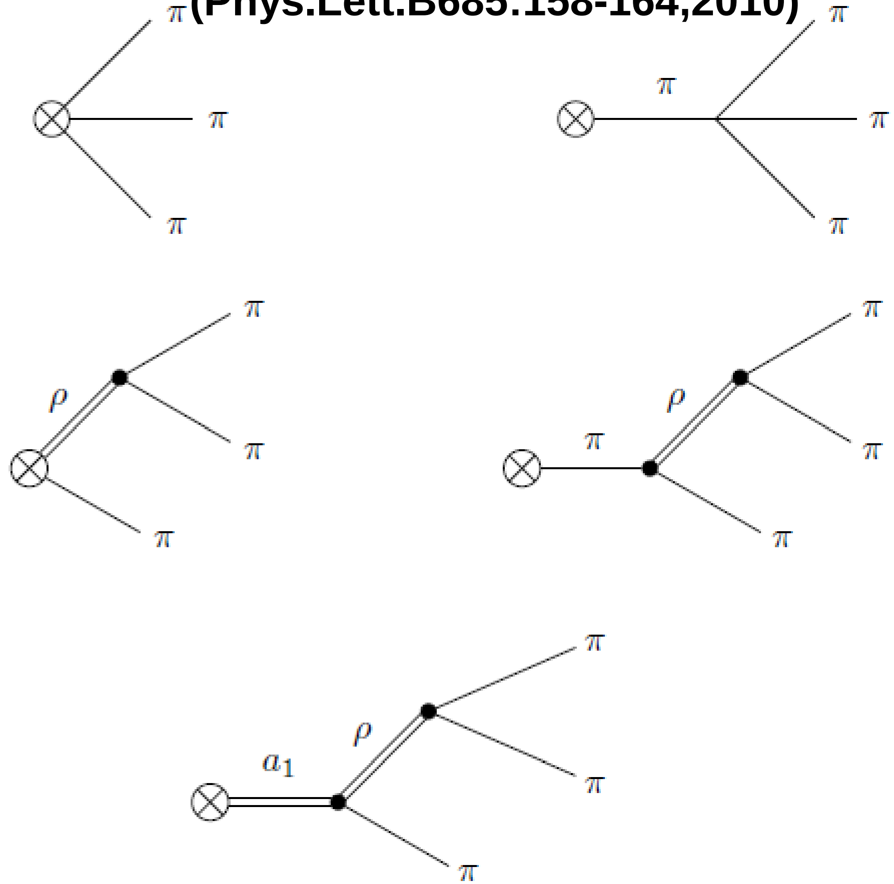
(Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)

(Phys.Lett.B685:158-164,2010)

χPT $\mathcal{O}(p^2)$

RχT, 1R

RχT, 2R



Axial form factor and a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)
(Phys.Lett.B685:158-164,2010)

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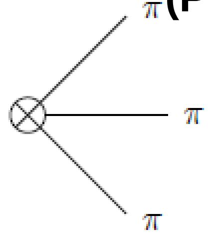
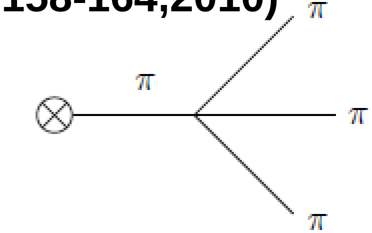
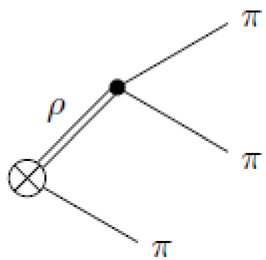


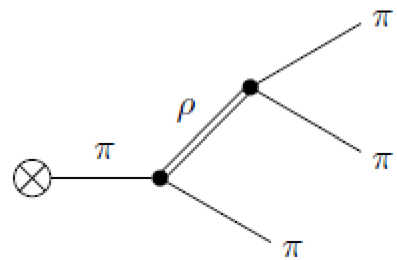
F



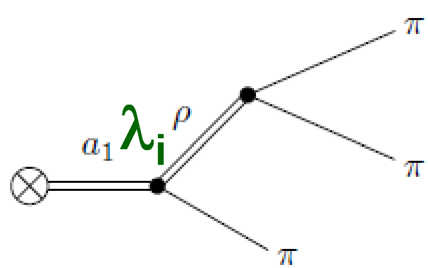
RχT, 1R



G_V, F_V



RχT, 2R



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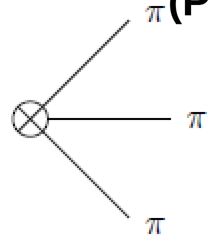
Axial form factor and a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)
(Phys.Lett.B685:158-164,2010)

χPT $\mathcal{O}(p^2)$



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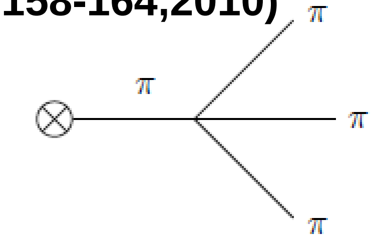
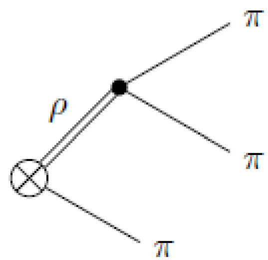
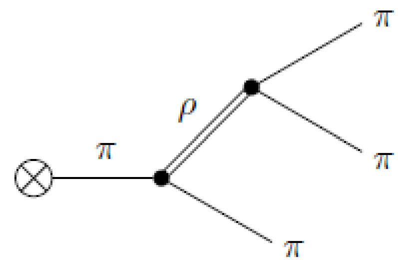


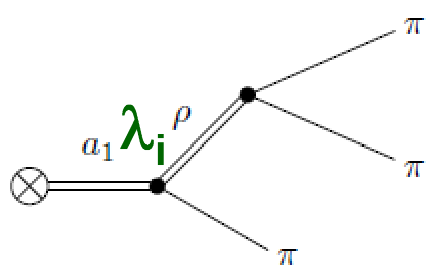
RχT, 1R



G_V, F_V



RχT, 2R



7 unknown couplings

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Axial form factor and a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)

(Phys.Lett.B685:158-164,2010)

7 unknown couplings

↓

Brodsky-Lepage behaviour demanded to the Form Factors (**7-6 = 1** coupling).

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Axial form factor and a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)
(Phys.Lett.B685:158-164,2010)

7 unknown couplings

↓

Brodsky-Lepage behaviour demanded to the Form Factors (**7-6 = 1** coupling).

We have **improved** the off-shell description of the **a_1 width** by including all cuts corresponding to 3π and $KK\pi$ intermediate states in the A-A correlator.

The value of this coupling that provides a pretty **accurate description of ALEPH data** is **consistent with** the prediction from **<VAP>** (Cirigliano, Ecker, Eidemüller, Pich, Portolés '04).

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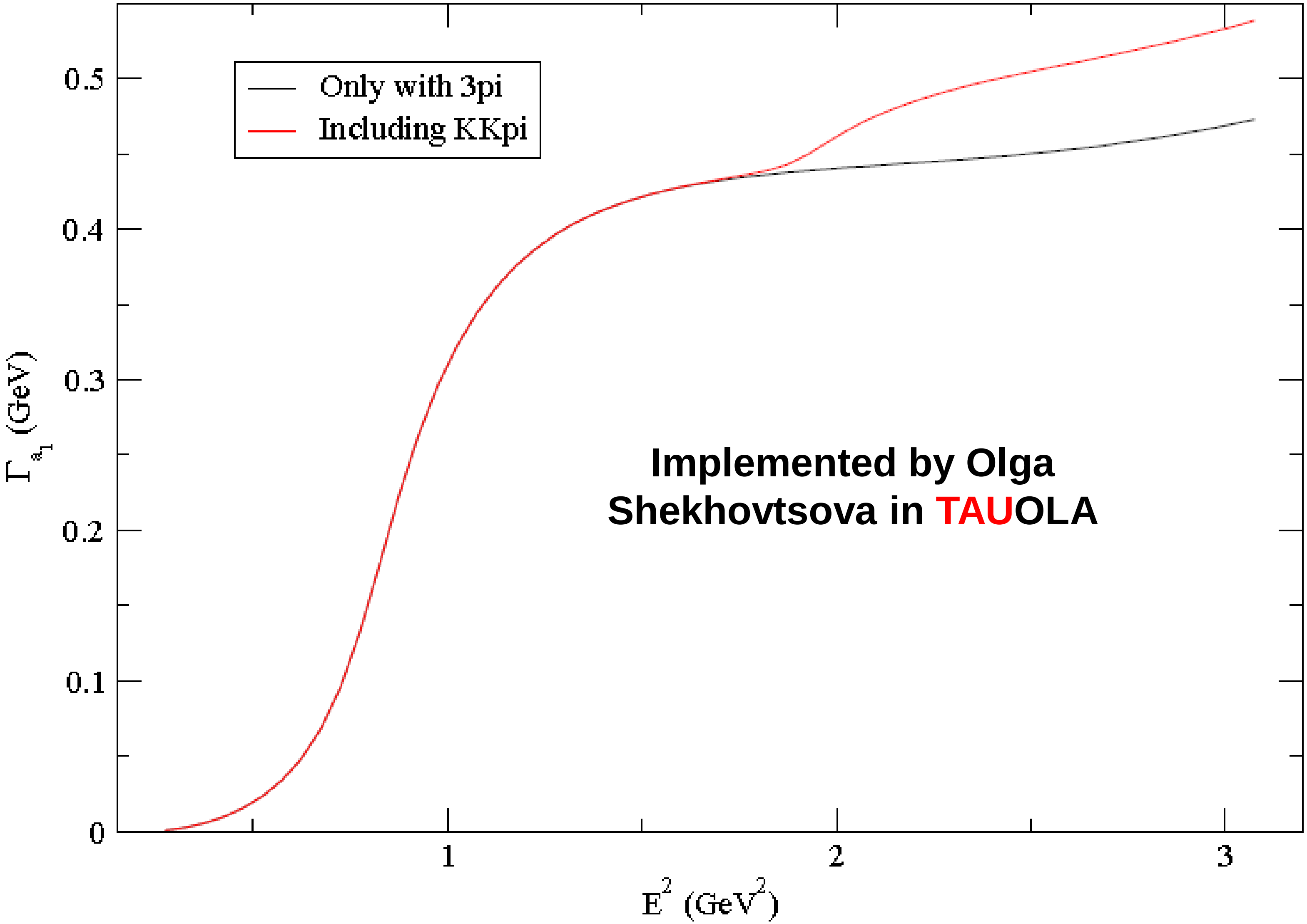
Axial-FF and the a_1: Γ_{a_1} (in TAUOLA)

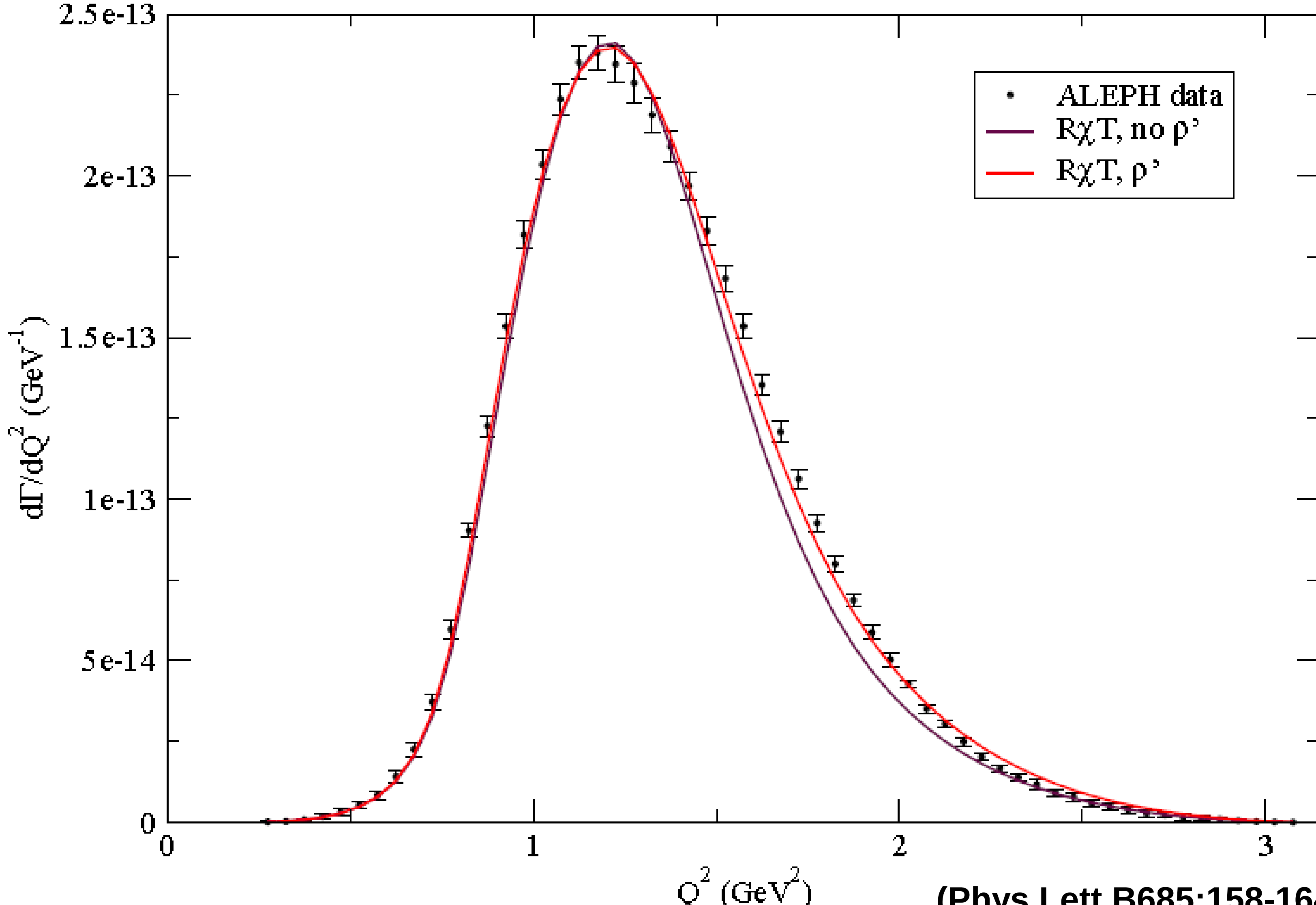
(R. , Shekhovtsova, Was in progress)



Implemented by Olga Shekhovtsova in TAUOLA

Axial-FF and the a_1 a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)



(Phys.Lett.B685:158-164,2010)

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χPT: The low-energy EFT of QCD

(Gasser & Leutwyler '84, '85)

$$\phi(x)=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

Goldstone Bosons

$$SU(3)_L \otimes SU(3)_R \to SU(3)_V$$

$$u(x)=\exp\left(\frac{i\phi(x)}{\sqrt{2}F}\right),\quad u_\mu = i\left[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right]$$

$$\chi = 2B_0(s+ip),\quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi u$$

$$f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu} u$$

$$X \to h(g,\Phi)\, X\, h(g,\Phi)^\dagger$$

$$\mathcal{L}^{(2)}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$\mathcal{L}^{(4)}_\chi = L_1\langle u_\mu u^\mu\rangle^2 + \ldots + L_4\langle u_\mu u^\mu\rangle\,\langle\chi_+\rangle + \ldots + L_7\langle\chi_-\rangle^2 + \ldots - iL_9\langle f_+^{\mu\nu}u_\mu u_\nu\rangle + \ldots$$

$\mathcal{L}^{(4)}_{\chi,\,WZW}$ in the odd-intrinsic parity sector

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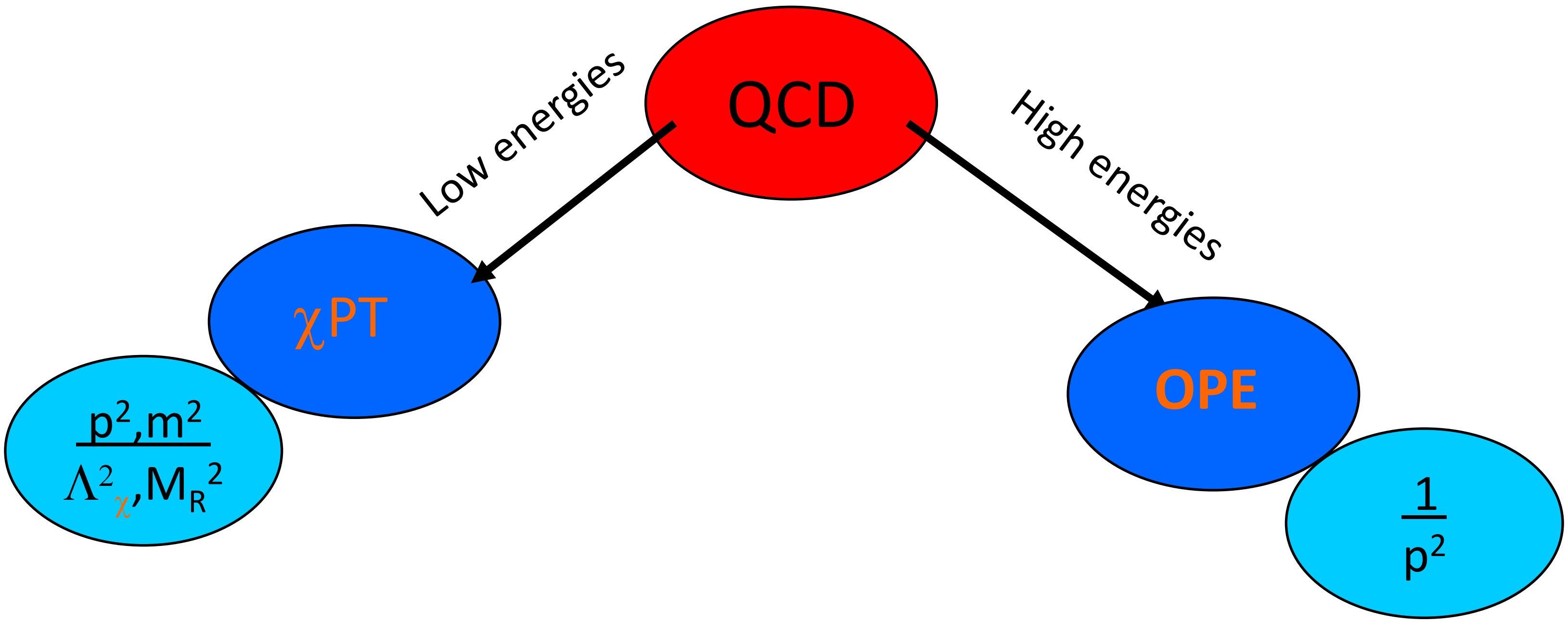
RχT matching to the OPE allows it to reproduce QCD high-energy behaviour:



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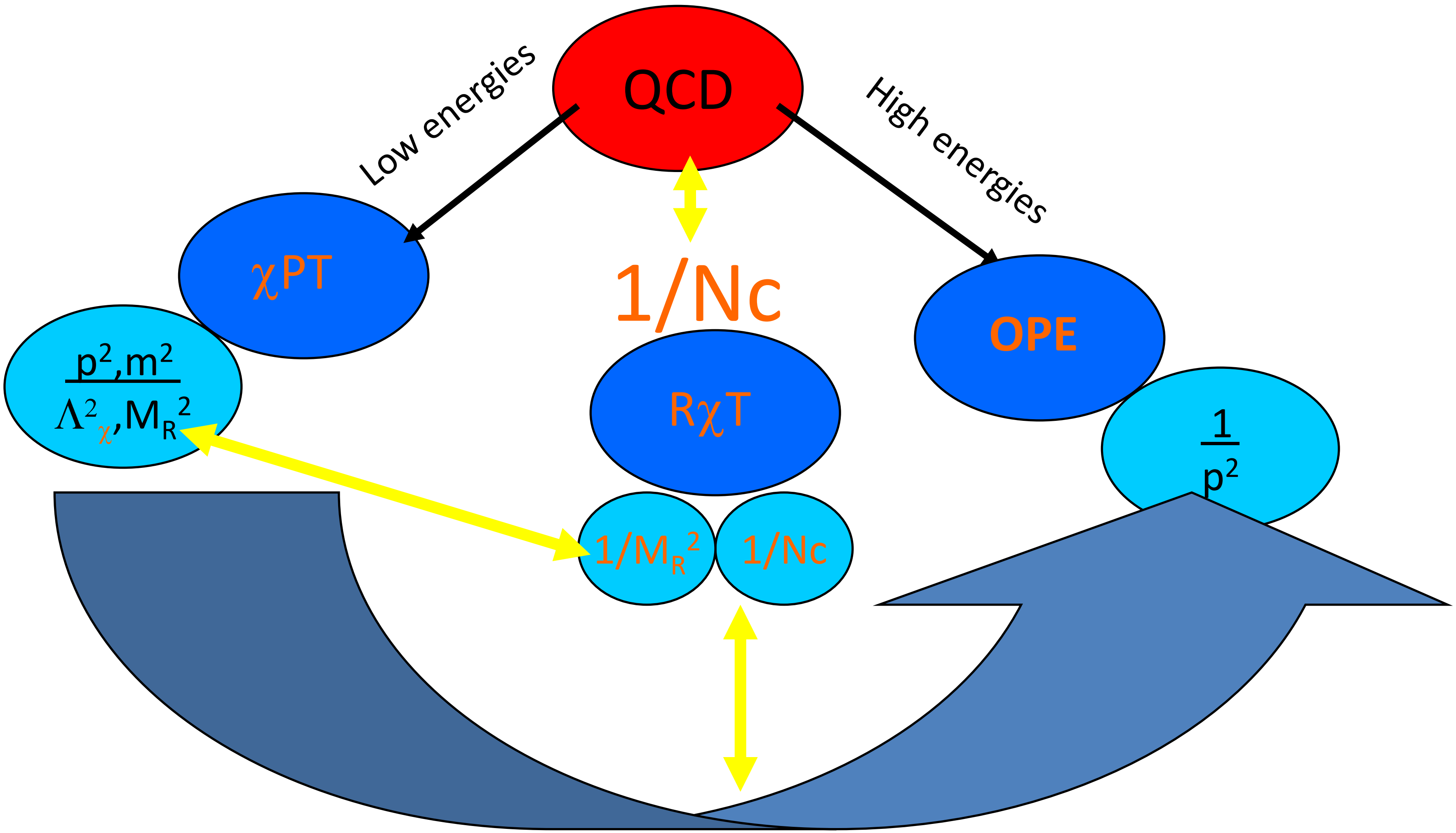
RχT matching to the OPE allows it to reproduce QCD high-energy behaviour:



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Resonances+ Goldstone Bosons

TOOLS : RχT

(Ecker, Gasser, Pich, De Rafael '89)

(Ecker, Gasser, Leutwyler, Pich, De Rafael '89)

$$\mathcal{L}^{(P_I=+)}_{R\chi T} = \mathcal{L}^{(2)}_{\chi} + \mathcal{L}^{kin}_{V,A} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VAP};$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$

$$V_{\mu\nu}(x) = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}_{\mu\nu}$$

Antisymmetric tensor formalism

$$\mathcal{L}_{VAP} = \sum_{i=1}^{5} \lambda_i O^i(V_{\mu\nu}, A^{\mu\nu}, \phi) = \lambda_1 \langle [V_{\mu\nu}, A^{\mu\nu}]\chi_-\rangle + \ldots$$

(Gómez Dumm, Pich, Portolés '04)

VMD

$$\mathcal{L}^{(P_I=-)}_{R\chi T} = \mathcal{L}^{(4)}_{\chi(WZW)} + \mathcal{L}_{VJP} + \mathcal{L}_{VVP} + \mathcal{L}_{VPPP};$$

$$\mathcal{L}_{VJP} = \sum_{i=1}^{7} \frac{c_i}{M_V} O^i(V_{\mu\nu}, j^\nu, \partial^\mu \phi) = \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle \{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\} u^\sigma\rangle + \ldots$$

(Ruiz-Femenía, Pich, Portolés '03)

$$\mathcal{L}_{VVP} = \sum_{i=1}^{5} d_i O^i(V_{\mu\nu}, V_{\rho\sigma}, \phi) = d_1 \varepsilon_{\mu\nu\rho\sigma}\langle \{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots$$

$$\mathcal{L}_{VPPP} = \sum_{i=1}^{5} \frac{g_i}{M_V} O^i(V_{\mu\nu}, \phi) = \frac{g_4}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle \{V^{\mu\nu}, u^\alpha u^\beta\}\chi_-\rangle + \ldots$$

(Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.2640**)

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Resonances+ Goldstone Bosons

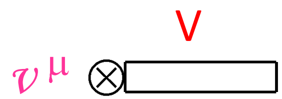
TOOLS : RχT

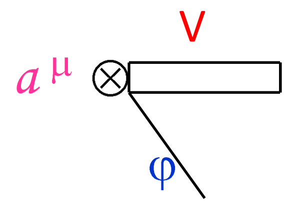
(Ecker, Gasser, Pich, De Rafael ‘89)

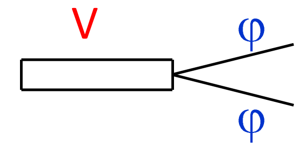
(Ecker, Gasser, Leutwyler, Pich, De Rafael ‘89) ,…

$$\mathcal{L}^{(P_I=+)}{}_{R\chi T} = \mathcal{L}^{(2)}{}_{\chi} + \mathcal{L}^{kin}{}_{V,A} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VAP};$$

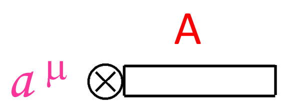
$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$



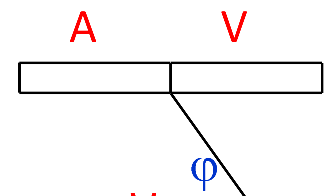




$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$

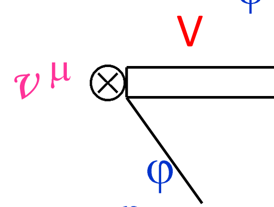


$$\mathcal{L}_{VAP} = \sum_{i=1}^{5}\lambda_i O^i(V_{\mu\nu}, A^{\mu\nu}, \phi) = \lambda_1\langle[V_{\mu\nu}, A^{\mu\nu}]\chi_-\rangle + \ldots$$

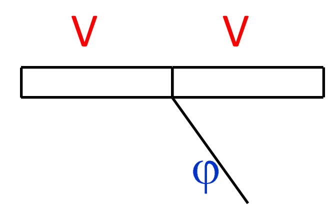


$$\mathcal{L}^{(P_I=-)}{}_{R\chi T} = \mathcal{L}^{(4)}{}_{\chi(WZW)} + \mathcal{L}_{VJP} + \mathcal{L}_{VVP} + \mathcal{L}_{VPPP};$$

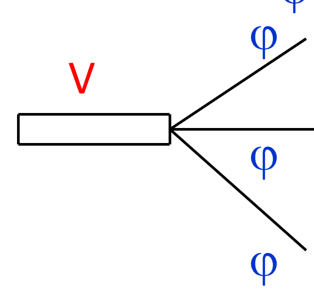
$$\mathcal{L}_{VJP} = \sum_{i=1}^{7}\frac{c_i}{M_V} O^i(V_{\mu\nu}, j^\nu, \partial^\mu\phi) = \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\}u^\sigma\rangle + \ldots$$



$$\mathcal{L}_{VVP} = \sum_{i=1}^{5} d_i O^i(V_{\mu\nu}, V_{\rho\sigma}, \phi) = d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots$$



$$\mathcal{L}_{VPPP} = \sum_{i=1}^{5}\frac{g_i}{M_V} O^i(V_{\mu\nu}, \phi) = \frac{g_4}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle\{V^{\mu\nu}, u^\alpha u^\beta\}\chi_-\rangle + \ldots$$



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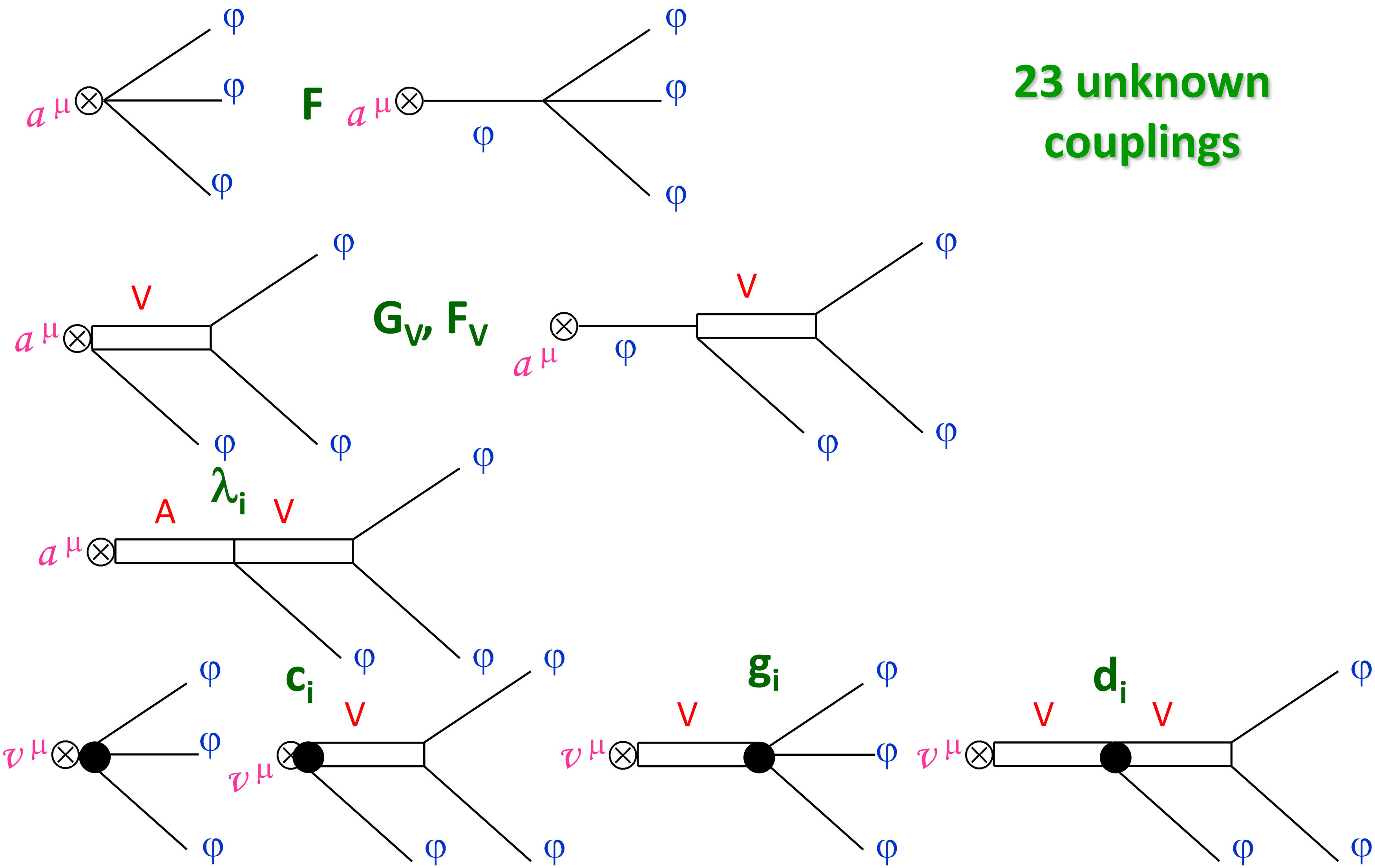
RχT APPLIED



23 unknown couplings

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The axial-form factor and the a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '00) (Gómez-Dumm, Pich, Portolés, R. **arXiv:0911.4436**)

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F^2}\left[\sigma^3{}_\pi \Theta(s-4m_\pi{}^2) + \frac{1}{2}\sigma^3{}_K \Theta(s-4m_K{}^2)\right]$$

$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}{}^{3\pi}(Q^2) + \Gamma_{a_1}{}^{K\bar{K}\pi}(Q^2) + \Gamma_{a_1}{}^{(K\pi)^0 K^0}(Q^2),$$

$$\Gamma_{a_1}{}^{3\pi}(Q^2) = \frac{1}{48(2\pi)^3 M_{a_1}}\left(\frac{Q^2}{M_{a_1}{}^2}\right)\iint ds\,dt\ \ F_1{}' V_{1\mu} + F_2{}' V_{2\mu}\ \cdot$$

$$F_1{}'^{\dagger} V_{1\mu} + F_2{}'^{\dagger} V_{2\mu}\ ,\quad F_i{}' = F_i \frac{M_{a_1}{}^2 - Q^2}{\sqrt{2} F_A Q^2}$$

(Phys.Rev.D62:054014,2000) (Phys.Lett.B685:158-164,2010)

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