Final State Radiation in Chiral Effective Theory

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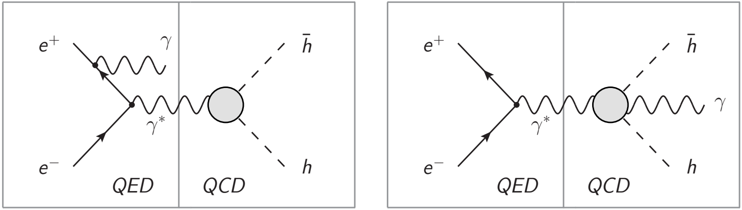
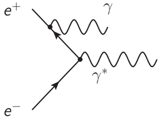
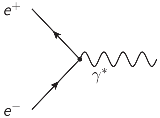
Process

$$e^+ + e^- \to \text{Hadrons} + \gamma$$



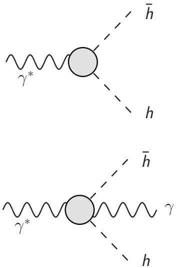
QED-Part





- Well understood theory
- Loop corrections easy to calculate and renormalize

QCD-Part

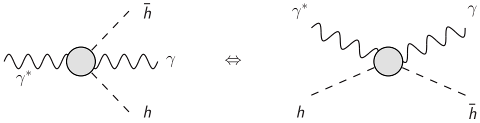


- More complex theory
- Perturbative approach to QCD only for high energies possible
- Therefore: for low energies alternative concept necessary

Alternative

Low energy theory of strong interactions:
Chiral Perturbation Theory (χPT)

- **Effective field theory** (EFT, due to confinement no quarks or gluons): implements hadronic d.o.f.
- In principle: FSR $=$ VCS



Outline

1. Basic Concepts of χPT

- Based on so-called chiral symmetry of QCD: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- In nature: spontaneously broken down to $\mathrm{SU}(2)_V$ $\rightarrow$ massless Goldstone-Bosons (GB)
- GB: pions ($\approx .14\,\mathrm{GeV} \ll \Lambda$) as effective d.o.f.
- EFT: based on the **most general Lagrangian** compatible with all symmetries of the underlying theory $\rightarrow$ infinite number of terms and coupling constants

J. Goldstone, Nuovo Cim. **19**, 154 (1961)
J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. **127**, 965 (1962)
J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984)

1. Basic Concepts of χPT

- Important: scheme for ordering terms due to their predicted relevance (so-called **power-counting**):

Weinberg's power-counting

Chiral order $\mathcal{O}(q^D)$ defined by

$$\mathcal{M}(tq, t^2 m_{\mathcal{Q}}) = t^D \mathcal{M}(q, m_{\mathcal{Q}})$$

- Physical quantities in terms of two expansions:
  - Loops: $\hbar$
  - Chiral orders: $(q/\Lambda)^D \propto \mathcal{O}(q^D)$

S. Weinberg, Physica A **96**, 327 (1979)

1. Parametrization of the Pion Fields

- Group-theoretical considerations imply

$$U = \exp\left(\frac{\mathrm{i}\phi}{F}\right), \quad \phi = \sum_{k=1}^{3} \tau^k \pi^k = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

as dynamical d.o.f. (but other parametrizations feasible)

- Extension to $\mathrm{SU}(3)$ with K and η:

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

S. Scherer, Adv. Nucl. Phys. **27**, 277 (2003)

1. Pionic Lagrangian

Building blocks

$$U = \exp\left(\frac{\mathrm{i}\phi}{F}\right) \propto \boldsymbol{\mathcal{O}(q^0)}, \quad F = .092\,\mathrm{GeV}$$

$$\mathrm{D}_\mu U = \partial_\mu U - \tfrac{1}{2}\mathrm{i}e\mathcal{A}_\mu\left[U, \tau^3\right] \propto \boldsymbol{\mathcal{O}(q^1)}$$

$$f^{\mu\nu} = -\tfrac{1}{2}e\tau^3\left(\partial^\mu\mathcal{A}^\nu - \partial^\nu\mathcal{A}^\mu\right) \propto \boldsymbol{\mathcal{O}(q^2)}$$

Pionic VCS-Lagrangian ($\mathcal{O}(q^4)$)

$$\mathcal{L}_\pi^{(4)} = \tfrac{1}{4}F^2\mathrm{Tr}\left\{\mathrm{D}_\mu U\left(\mathrm{D}^\mu U\right)^\dagger\right\} + \tfrac{1}{4}F^2M^2\mathrm{Tr}\left\{U^\dagger + U\right\}$$
$$+ l_5\mathrm{Tr}\left\{f_{\mu\nu}Uf^{\mu\nu}U^\dagger\right\} + \tfrac{\mathrm{i}}{2}l_6\mathrm{Tr}\left\{f_{\mu\nu}\left[\mathrm{D}^\mu U, \left(D^\nu U\right)^\dagger\right]\right\}$$

J. Gasser, H. Leutwyler, Annals. Phys. **158**, 142 (1984)

2. Precision

In terms of precision: basic χPT not sufficient – means of improvement:

A: Inclusion of resonances (widens range of applicability)

B: Higher order calculations (improve precision):

- Increase in chiral orders
- One or higher loop calculations
- Usage of well suited renormalization scheme

χPT + Resonances $\rightarrow$ χEFT

2.A: Resonances

- Mass of lightest resonance **not explicitly** included sets upper limit for applicability (due to propagator)
- Obvious strategy: implement resonances as new effective d.o.f.

2.A: Implementation of Resonances

Consistent method for including resonances:

- Choose suitable parametrization of resonance fields (e.g. consider d.o.f.; comparatively simple)
- Write down most general lagrangian consistent with all symmetries (impossible)
- Construct suitable power counting for choosing relevant terms (mandatory)
- **However:** since these are unstable particles an adapted renormalization scheme has to be used

2.A: Rho-Meson

Generic first step: explicit inclusion of the rho-meson

- Vector-isovector (i.e. six d.o.f.)
- Decays into two pions ($\approx$ 100%, PDG)
- Mass $= m_\rho - \frac{\mathrm{i}}{2}\Gamma_\rho = (.770 - .075\mathrm{i})\,\mathrm{GeV}$

Because of vector-isovectorial character: more complex building blocks and boundary constraints necessary

2.A: Rho-Meson

Building blocks

$$u = \sqrt{U} \propto \mathcal{O}(q^0)$$

$$\rho_\mu = \tfrac{1}{2}\rho_\mu^a \tau^a \propto \mathcal{O}(q^0)$$

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - \mathrm{i}g\left[\rho_\mu, \rho_\nu\right] \propto \mathcal{O}(q^0)$$

$$\Gamma_\mu = \tfrac{1}{2}\left(u^\dagger\left(\partial_\mu - \tfrac{\mathrm{i}e}{2}\mathcal{A}_\mu \tau^3\right)u + u\left(\partial_\mu - \tfrac{\mathrm{i}e}{2}\mathcal{A}_\mu \tau^3\right)u^\dagger\right) \propto \mathcal{O}(q^1)$$

$$\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + \left[\Gamma_\mu, \Gamma_\nu\right] \propto \mathcal{O}(q^2)$$

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u \propto \mathcal{O}(q^2)$$

$$f_\pm^{\mu\nu} = u f^{\mu\nu} u^\dagger \pm u^\dagger f^{\mu\nu} u \propto \mathcal{O}(q^2), \quad f^{\mu\nu} = -{}^{e}/_{2}\tau^3 F^{\mu\nu}$$

2.A: Rho-Meson

Lowest order VCS-Lagrangian

containing pions and rho-mesons

$$\mathcal{L}_{\pi\rho} = \left(m^2 + \frac{c_x}{4}\mathrm{Tr}\{\chi_+\}\right)\mathrm{Tr}\left\{\left(\rho_\mu - \frac{\mathrm{i}}{g}\Gamma_\mu\right)\left(\rho^\mu - \frac{\mathrm{i}}{g}\Gamma^\mu\right)\right\}$$
$$-\frac{1}{2}\mathrm{Tr}\{\rho_{\mu\nu}\rho^{\mu\nu}\} + \mathrm{i}d_x\mathrm{Tr}\{\rho_{\mu\nu}\Gamma^{\mu\nu}\} + f_x\mathrm{Tr}\{\rho_{\mu\nu}f_+^{\mu\nu}\}$$
$$+\mathrm{i}u_x\mathrm{Tr}\left\{\left(\rho_\mu - \frac{\mathrm{i}}{g}\Gamma_\mu\right)\left(\rho_\nu - \frac{\mathrm{i}}{g}\Gamma_\nu\right)f_+^{\mu\nu}\right\}$$

D. Djukanovic, J. Gegelia, A. Keller, S. Scherer, Phys. Lett. B**680**, 235 (2009)

2.B: Higher Order Calculations

Increase in precision can be achieved by NLO calculations:

- Higher chiral orders $\mathcal{O}(q^D)$
- One or more loops

But: additional loops imply larger D
$\rightarrow$ Consideration of one loop more important than increase in chiral orders

2.B: Loop Calculations

Loop calculations in QFTs give rise to divergencies i.e. useless predictions

Therefore:
Consistent scheme for abolishing these infinities essential:

1. Regularization (makes the infinities manifestly visible); e.g. Dim. Reg., Cut-Off, etc.
2. **Renormalization** (removes their contributions to physical quantities)

2.B: Renormalization Procedure

Parameters and fields of lagrangians are a-priori so-called bare (i.e. infinite) quantities:

1. Split bare into renormalized constants and counter terms
2. Define renormalized quantities by matching observables with experimental data (fixing the renormalization point)
3. Rewrite all relevant quantities in terms of renormalized parameters

Assume $\mathcal{L} = \mathcal{L}(g^0, \Phi^0)$ and **real** observable σ:

$$\sigma = f^0(g^0) \;\Rightarrow\; g^R := \phi(\sigma) \;\Rightarrow\; g^0 = \varphi(g^R) \;\Rightarrow\; \sigma = f^R(g^R)$$

2.B: Complex-Mass-Scheme

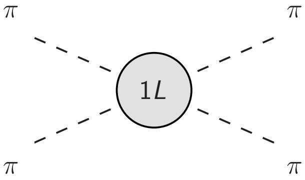
- Because of decaying resonances as effective d.o.f.: **Complex-Mass-Scheme** (CMS)
- Renormalization point: complex mass of unstable particle (i.e.: renormalized = complex mass)
- Although complex coupling constant: CMS does not wreck unitarity:

Consider **complex** quantity ω and **real** observable σ:

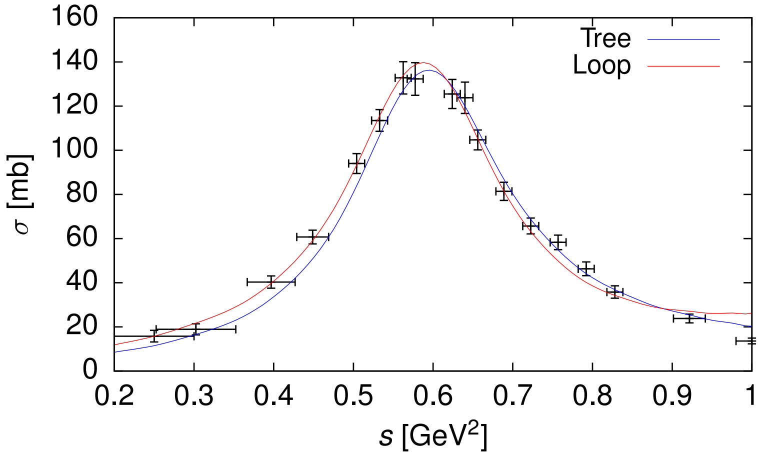
$$\omega = \tilde{f}^R(g^R) \quad \Leftrightarrow \quad g^R = (\tilde{f}^R)^{-1}(\omega)$$
$$\sigma = f^R(g^R) = \left(f^R \circ (\tilde{f}^R)^{-1}\right)(\omega) = F^R(\omega)$$

2.B: Example: $\pi\pi$-Scattering

$\pi\pi$-scattering using χEFT – i.e. pions and rho-mesons – up to one loop order:



2.B: $\pi\pi$-Scattering



Data taken from: S. D. Protopopescu et. al., Phys. Rev. D**7**, 1279 (1973)

3. Perspectives

- Pion structure in χEFT using CMS
- Electromagnetic pion form factor:
  ρ-ω-mixing, isospin symmetry breaking
- Calculation of FSR/VCS in χEFT

  FSR: $g-2$
  VCS: e.m. polarizabilities

4. Summary

- χEFT provides powerful framework for low energy FSR/VCS calculations
- Inclusion of additional d.o.f. like ϕ mesons etc. consistently possible
- $SU(3)$-formulation enables description of final states containing kaons and eta
- **Major drawback:** LECs not determined by QCD, need fixing by experiment or Lattice-QCD

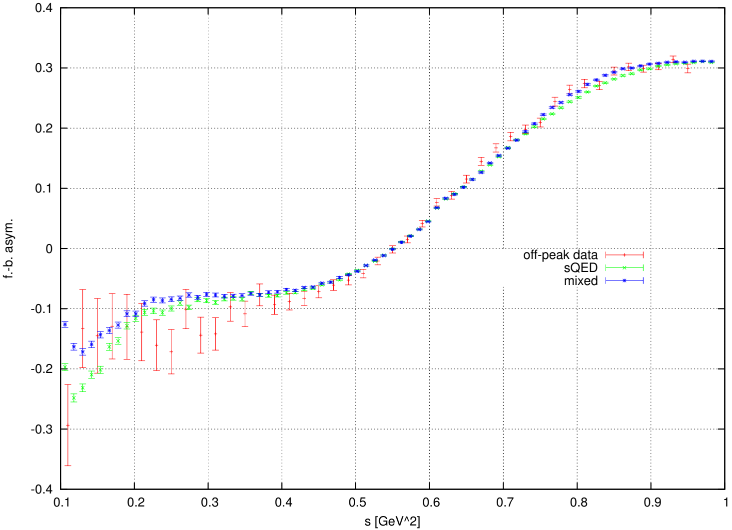
Spare Parts

Consider

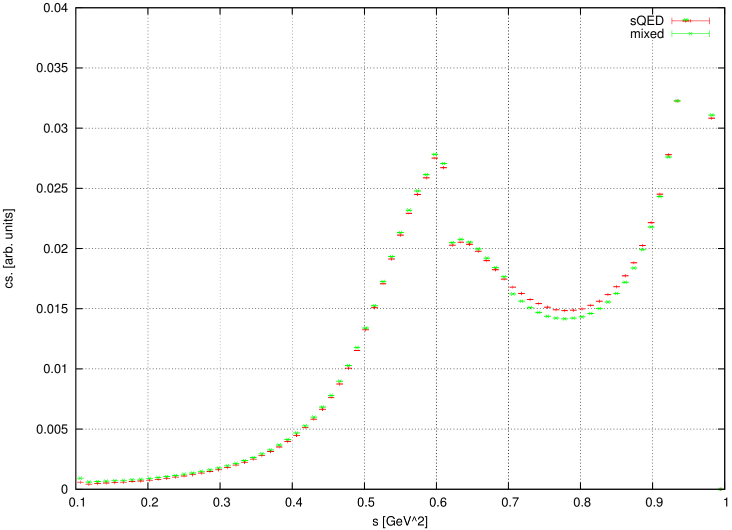
$$e^+ + e^- \to \pi^+ + \pi^- + \gamma$$

- Modify final state contributions by constant phases
- Generate events and calculate forward-backward asymmetries
- Check impact of this phases on the cross-section

Spare Parts


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
f.-b. asym.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
s [GeV^2]
off-peak data
sQED
mixed

Spare Parts


sQED
mixed
cs. [arb. units]
s [GeV^2]
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0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1