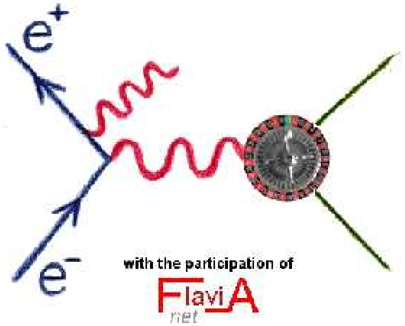
PHOKHARA 7.0 Monte Carlo generator.

H. CZYŻ, IF, UŚ, Katowice

MonteCarLow WG Meeting, Liverpool 2010



Based on:

H. Czyż, J. H. Kühn and A. Wapienik,
"Four-pion production in tau decays and e+e- annihilation:
an update,"
Phys. Rev. D **77** (2008) 114005

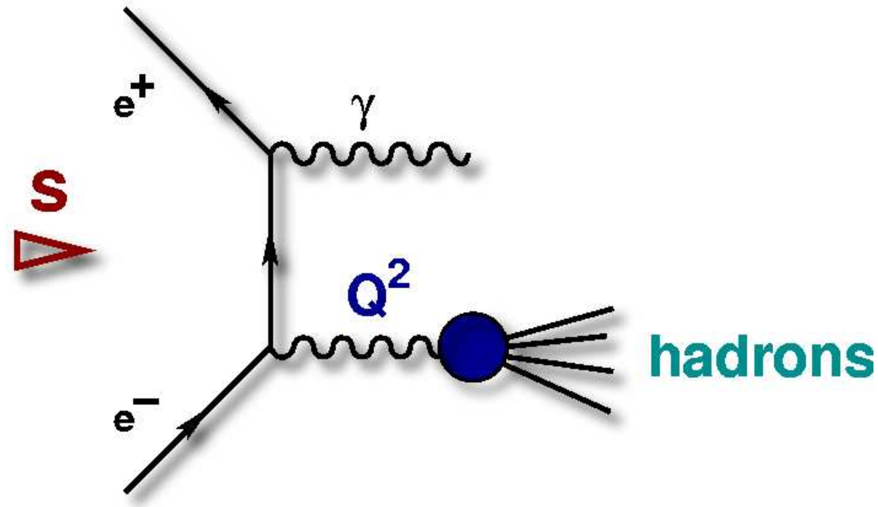
H. Czyż, J. H. Kühn
"Strong and Electromagnetic J/psi and psi(2S) Decays
into Pion and Kaon Pairs,"
Phys. Rev. D **80** (2009) 034035

H. Czyż, A. Grzelińska and J. H. Kühn
'Narrow resonances studies with the radiative return method,"
Phys. Rev. D **81** (2010) 094014

THE RADIATIVE RETURN METHOD

$$d\sigma(e^+e^- \to \text{hadrons} + \gamma(ISR)) =$$

$$H(Q^2, \theta_\gamma)\; d\sigma(e^+e^- \to \text{hadrons})(s = Q^2)$$



- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO,...)

High precision measurement of the hadronic cross-section at meson-factories

MC generators needed

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

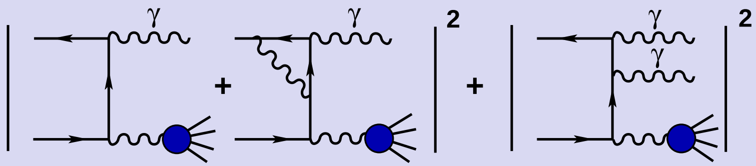
- ISR at LO + Structure Function

[Czyż, Kühn,2000]

H.C., A. Grzelińska, J. H. Kühn, E. Nowak-Kubat, G. Rodrigo, A. Wapienik

PHOKHARA 7.0: $\pi^+\pi^-$, $\mu^+\mu^-$, 4π, $\bar{N}N$, 3π, KK, $\Lambda(\to \cdots)\bar{\Lambda}(\to \cdots)$, $J/\psi, \psi(2S)$

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at NLO:$\pi^+\pi^-$, $\mu^+\mu^-$,K^+K^-
- tagged or untagged photons
- Modular structure

http://ific.uv.es/~rodrigo/phokhara/

Isospin relations: 4π

$$\langle \pi^+\pi^-\pi_1^0\pi_2^0|J_\mu^3|0\rangle = J_\mu(p_1,p_2,p^+,p^-)$$

$$\langle \pi_1^+\pi_2^+\pi_1^-\pi_2^-|J_\mu^3|0\rangle =$$
$$J_\mu(p_2^+,p_2^-,p_1^+,p_1^-) \quad + J_\mu(p_1^+,p_2^-,p_2^+,p_1^-)$$
$$+ J_\mu(p_2^+,p_1^-,p_1^+,p_2^-) \quad + J_\mu(p_1^+,p_1^-,p_2^+,p_2^-)$$

$$\langle \pi^-\pi_1^0\pi_2^0\pi_3^0|J_\mu^-|0\rangle =$$
$$J_\mu(p_2,p_3,p^-,p_1) + J_\mu(p_1,p_3,p^-,p_2) + J_\mu(p_1,p_2,p^-,p_3)$$

$$\langle \pi_1^-\pi_2^-\pi^+\pi^0|J_\mu^-|0\rangle =$$
$$J_\mu(p^+,p_2,p_1,p^0) \quad + J_\mu(p^+,p_1,p_2,p^0)$$

J. H. Kühn (1999)

Isospin relations: 4π

$$\int J_\mu^{em} (J_\nu^{em})^* \; d\bar{\Phi}_n(Q; q_1, \ldots, q_n)$$
$$= \frac{1}{6\pi}\left(Q_\mu Q_\nu - g_{\mu\nu} Q^2\right) \; R(Q^2)$$

$$R(Q^2) = \sigma(e^+e^- \to hadrons)(Q^2)/\sigma_{point}$$

Isospin relations: 4π

$$\frac{d\Gamma_{\tau\to\nu+hadrons}}{dQ^2}$$

$$= 2\ \Gamma_e \frac{|V_{ud}|^2 S_{EW}}{m_\tau^2}\left(1-\frac{Q^2}{m_\tau^2}\right)^2\left(1+2\frac{Q^2}{m_\tau^2}\right) R^\tau\left(Q^2\right)$$

$$\int\ J_\mu^- J_\nu^{-*}\ \ d\bar{\Phi}_n(Q;q_1,\dots,q_n) =$$

$$\frac{1}{3\pi}\left(Q_\mu Q_\nu - g_{\mu\nu}Q^2\right)\ R^\tau(Q^2)$$

Isospin relations: 4π

$$R^{\tau}\,(-\;0\;0\;0) = \frac{1}{2}\,R\,(+\;+\;-\;-)$$

$$R^{\tau}\,(-\;-\;+\;0) = \frac{1}{2}\,R\,(+\;+\;-\;-) + R\,(+\;-\;0\;0)$$

Isospin relations: 4π; exp. situation

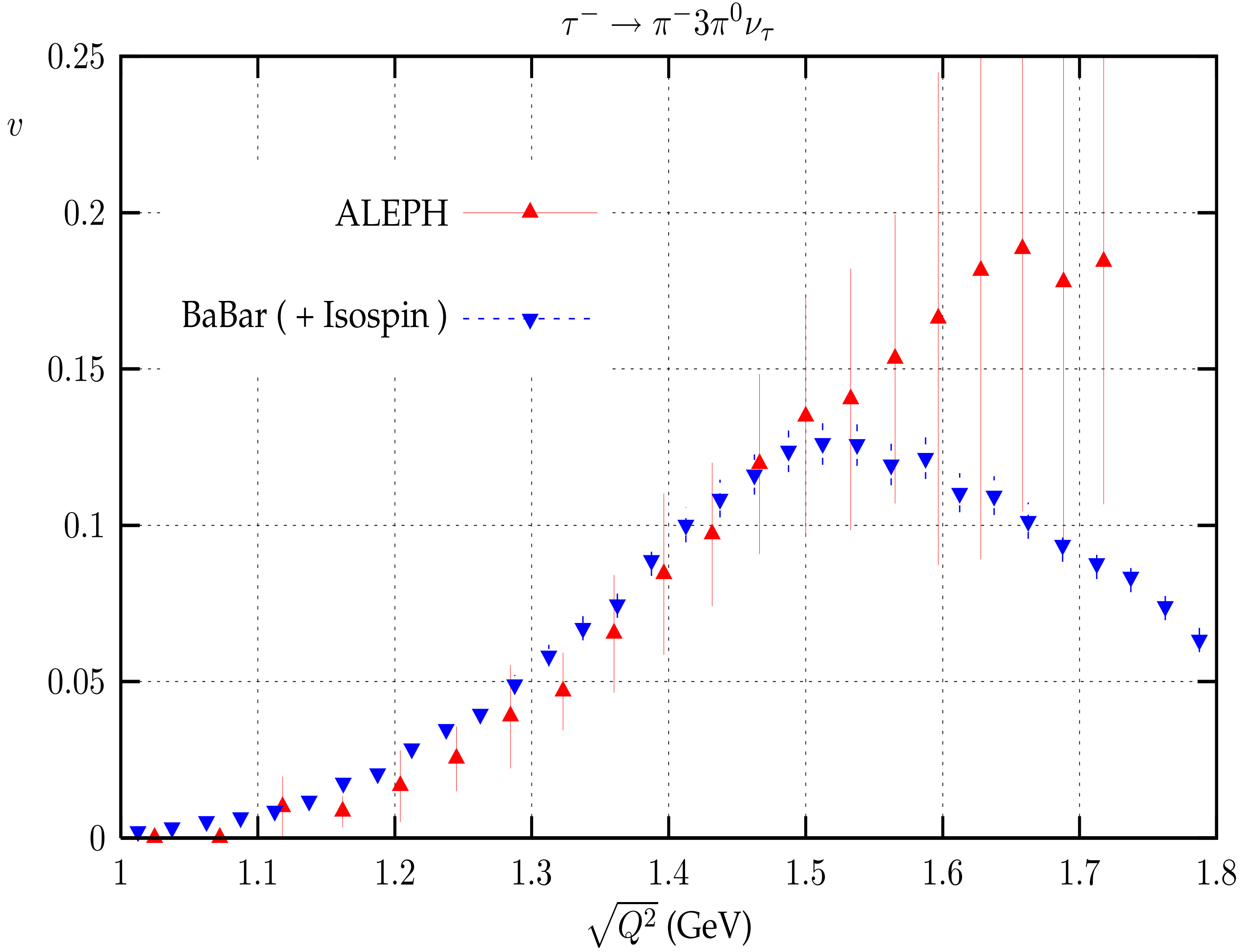
$e^+e^- \to 2\pi^+2\pi^-$: BaBar, CMD2, SND

$e^+e^- \to 2\pi^0\pi^+\pi^-$: BaBar(preliminary), CMD2, SND

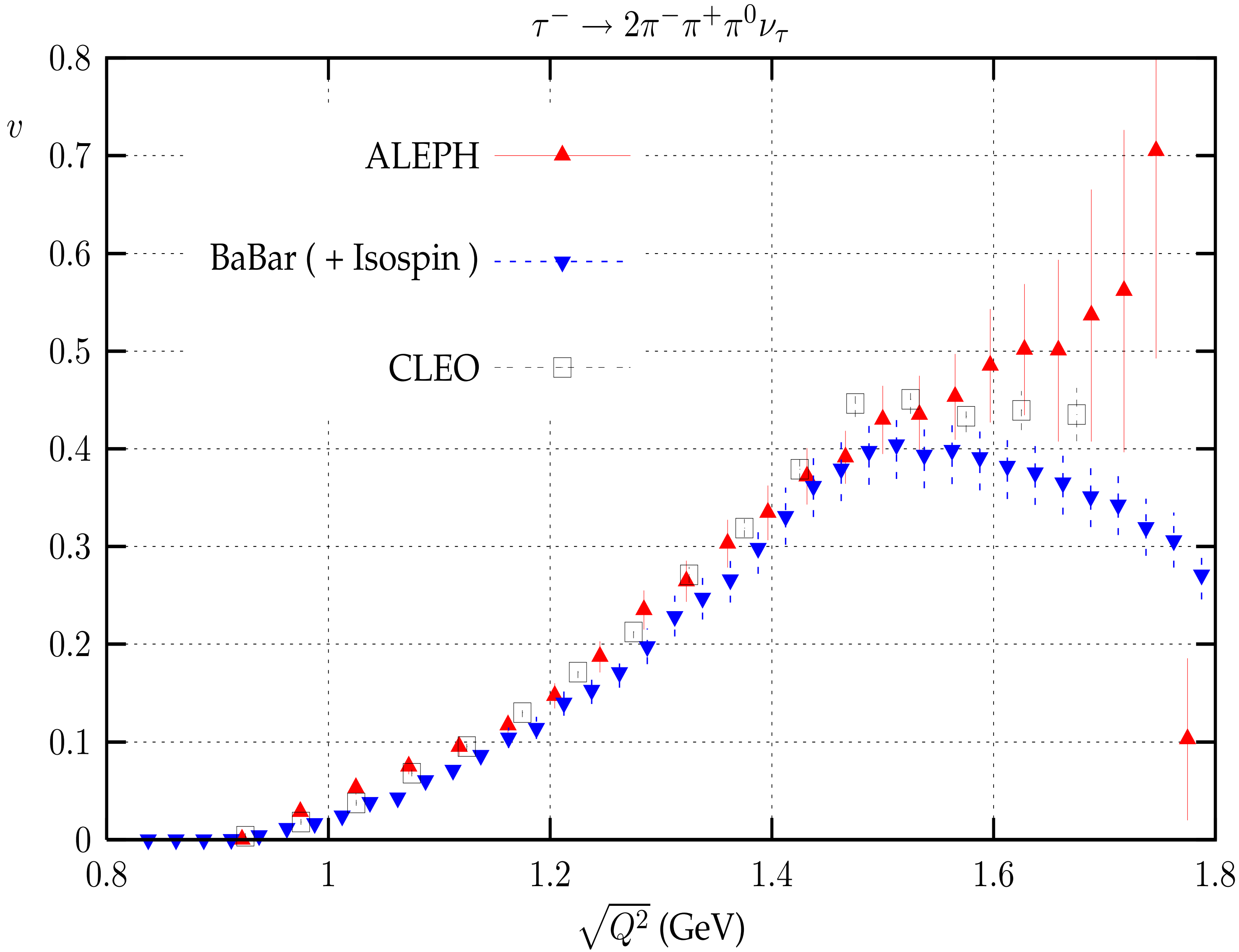
$\tau^- \to \nu 3\pi^0\pi^-$: ALEPH

$\tau^- \to \nu 2\pi^-\pi^+\pi^0$: ALEPH, CLEO

Isospin relations: 4π; exp. situation



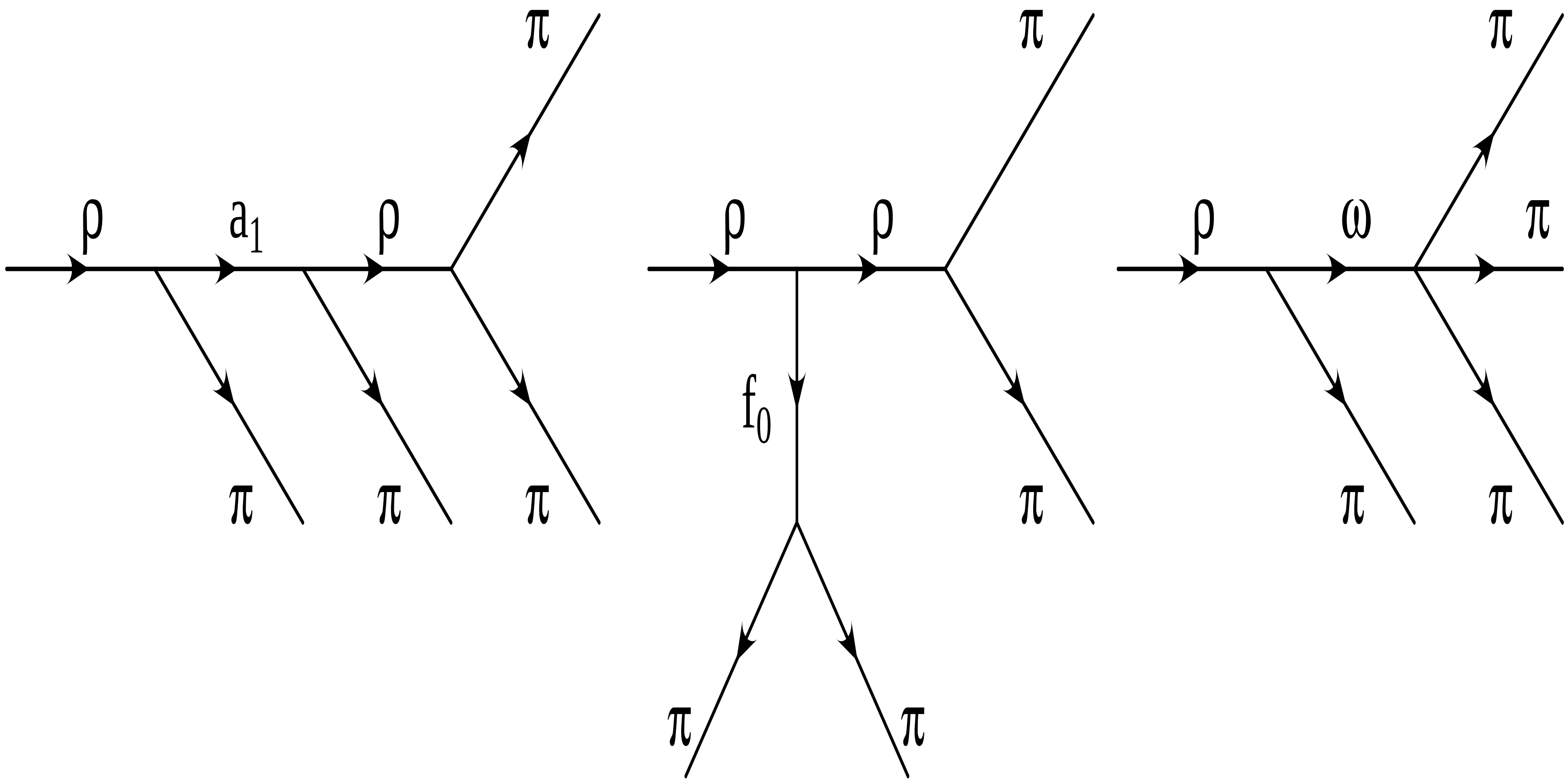
Isospin relations: 4π; exp. situation



4π: exp. situation

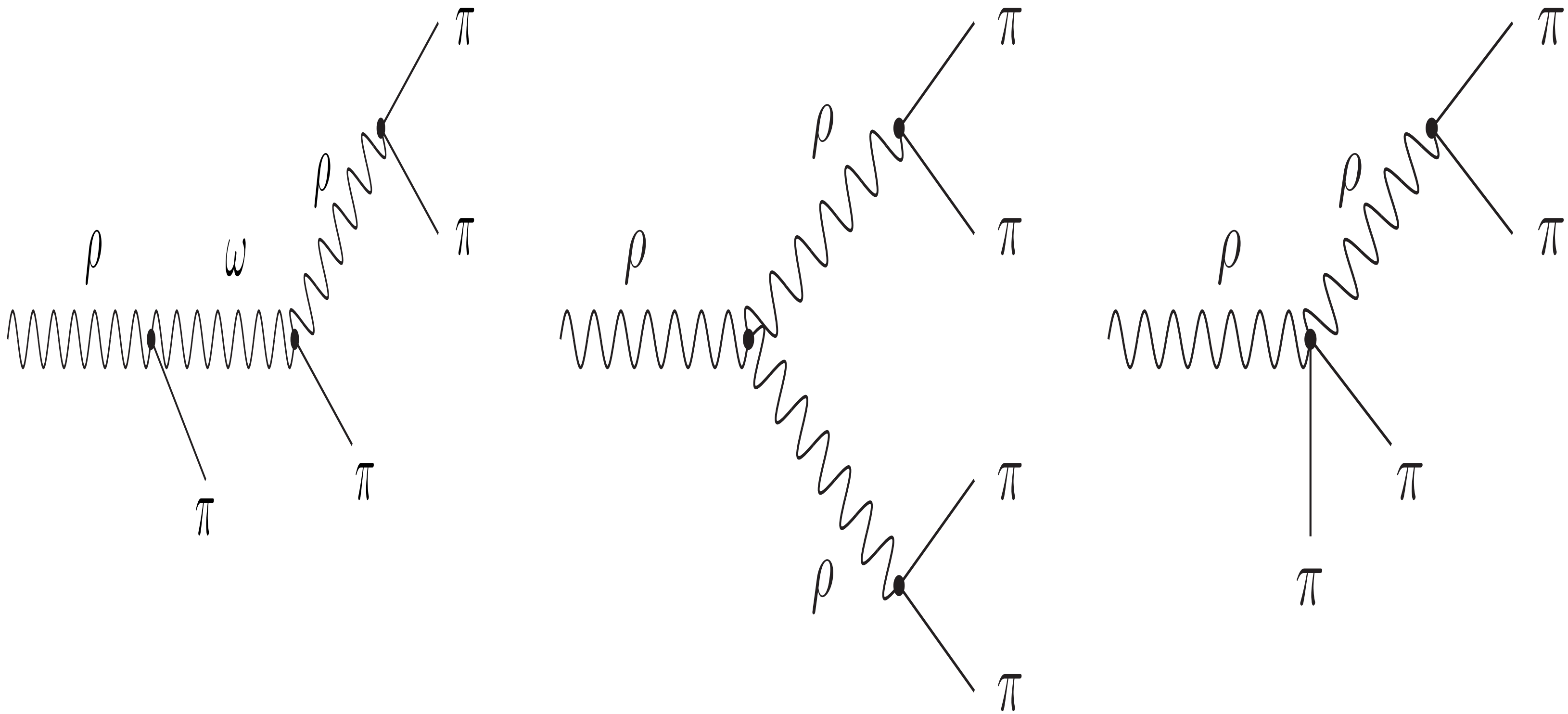
- $\pi\omega(\rightarrow \pi^+\pi^-\pi^0)$: CLEO, BaBar(prel.)
- $\rho \rightarrow \rho(\rightarrow \pi\pi)\rho(\rightarrow \pi\pi)$: BaBar(prel.)

The model



H.C., J.H. Kühn (2000)

The model



H.C., J.H. Kühn, A. Wapienik (2008)

H.C., A. Grzelińska, J.H. Kühn, G. Rodrigo(2006)

Comparing with τ data

	$\mathrm{Br}(\tau- \rightarrow \nu_\tau 2\pi^-\pi^+\pi^0)$
PDG06	$(4.46 \pm 0.06)\%$
model	$(4.12 \pm 0.21)\%$
BaBar (CVC)	$(3.98 \pm 0.30)\%$

	$\mathrm{Br}(\tau- \rightarrow \nu_\tau \pi^-\omega(\pi^-\pi^+\pi^0))$
PDG06	$(1.77 \pm 0.1)\%$
model	$(1.60 \pm 0.13)\%$
BaBar (CVC)	$(1.57 \pm 0.31)\%$

Comparing with τ data

	$\mathrm{Br}(\tau- \to \nu_\tau \pi^- 3\pi^0)$
PDG06	$(1.04\pm 0.08)\%$
model	$(1.06 \pm 0.09)\%$
BaBar (CVC)	$(1.02\pm 0.05)\%$

Experimental data: PDG09 + ...

	J/ψ	$\psi(2S)$
M [MeV]	3096.916±0.011	3686.09±0.04
Γ_{ee} [keV]	5.55±0.14±0.02	2.38±0.04
$\mathcal{B}(e^+e^-)[\%]$	5.94±0.06	0.752±0.017
$\mathcal{B}(K^+K^-)[10^{-5}]$	23.7±3.1	6.3±0.7
$\mathcal{B}(K^0_S K^0_L)[10^{-5}]$	14.6±2.6	5.4±0.5
$\mathcal{B}(\pi^+\pi^-)[10^{-5}]$	14.7±2.3	0.9±0.5[w.a. by CLEO]
$\sigma(K^+K^-)$[pb]	-	5.7±0.8 [CLEO]
$\sigma(K^0_S K^0_L)$[pb]	-	< 0.74 (90% C.L.) [CLEO]
$\sigma(\pi^+\pi^-)$[pb]	-	9.0±2.2 [CLEO]
$\Delta\alpha$	0.02117 [F.J.+...]	0.02219 [F.J.+...]

Contributions to $e^+e^- \to P\bar{P}$

$$\sigma\left(e^+e^- \to P\bar{P}\right) = \frac{\pi\alpha^2}{3s}|F_P|^2\,\beta^3$$

$$\times\left(\frac{1}{(1-\Delta\alpha)^2} + \sum_R\left\{\frac{9s}{\alpha^2}\frac{(\Gamma_e^R)^2}{(s-M_R^2)^2+\Gamma_R^2M_R^2}\right.\right.$$

$$\times\left[|1+c_P^R|^2 + \frac{2\alpha M_R}{3\sqrt{s}(1-\Delta\alpha)}\frac{\Gamma_R}{\Gamma_e^R}\mathrm{Im}(c_P^R)\right]$$

$$\left.\left.+\frac{6\sqrt{s}\Gamma_e^R}{\alpha(1-\Delta\alpha)}\frac{\left(1+\mathrm{Re}(c_P^R)\right)(s-M_R^2)}{(s-M_R^2)^2+\Gamma_R^2M_R^2}\right\}\right)$$

Decay width

One should not use

$$\Gamma(R \to P\bar{P}) = \Gamma^{QED} \times |1 + c_P^R|^2$$

but

$$\Gamma(R \to P\bar{P}) = \Gamma^{QED} \times \Big[|1 + c_P^R|^2$$

$$+ \frac{2\alpha M_R}{3\sqrt{s}(1-\Delta\alpha)} \frac{\Gamma_R}{\Gamma_e^R} \mathrm{Im}(c_P^R) \Big]$$

Pion form factor

$$|F_\pi|^2 = \frac{4\mathcal{B}(R\to\pi^+\pi^-)}{\beta_\pi^3\mathcal{B}(R\to e^+e^-)}$$

	J/ψ	$\psi(2S)$
$\lvert F_\pi\rvert^2[10^{-3}]$ above Eq.	10.0 ± 1.6	4.8 ± 2.73
$\lvert F_\pi\rvert^2[10^{-3}]$ off peak	-	5.92 ± 1.46

CLEO:Phys. Rev. Lett. 95,(2005) 261803

$$|F_\pi|^2(\sqrt{s} = 3.671\mathrm{GeV}) = (5.63 \pm 1.42)\cdot 10^{-3}$$

$\psi(2S) \to K^+K^-$

$$R_+ \equiv \frac{4\mathcal{B}(K^+K^-)}{\beta^3_{K^+}\mathcal{B}(e^+e^-)} = |F_{K^+}|^2 \left[|1 + c_+|^2 + r\,\mathrm{Im}c_+\right]$$

$$r = \frac{2\alpha}{3(1-\Delta\alpha)\mathcal{B}(e^+e^-)} = 0.663 \pm 0.015$$

$$R_+ = (3.75 \pm 0.43) \cdot 10^{-2}$$

$\psi(2S) \to K^+K^-$

$$S_+ - R_+\frac{\gamma^2}{4} = |F_{K^+}|^2\left[1 + \gamma(1 + \mathrm{Re}\,c_+)\right]$$

$$S_+ = \sigma(e^+e^- \to K^+K^-)(1 - \Delta\alpha)^2 / \left(\frac{\pi\alpha^2}{3s}\beta^3_{K^+}\right)$$

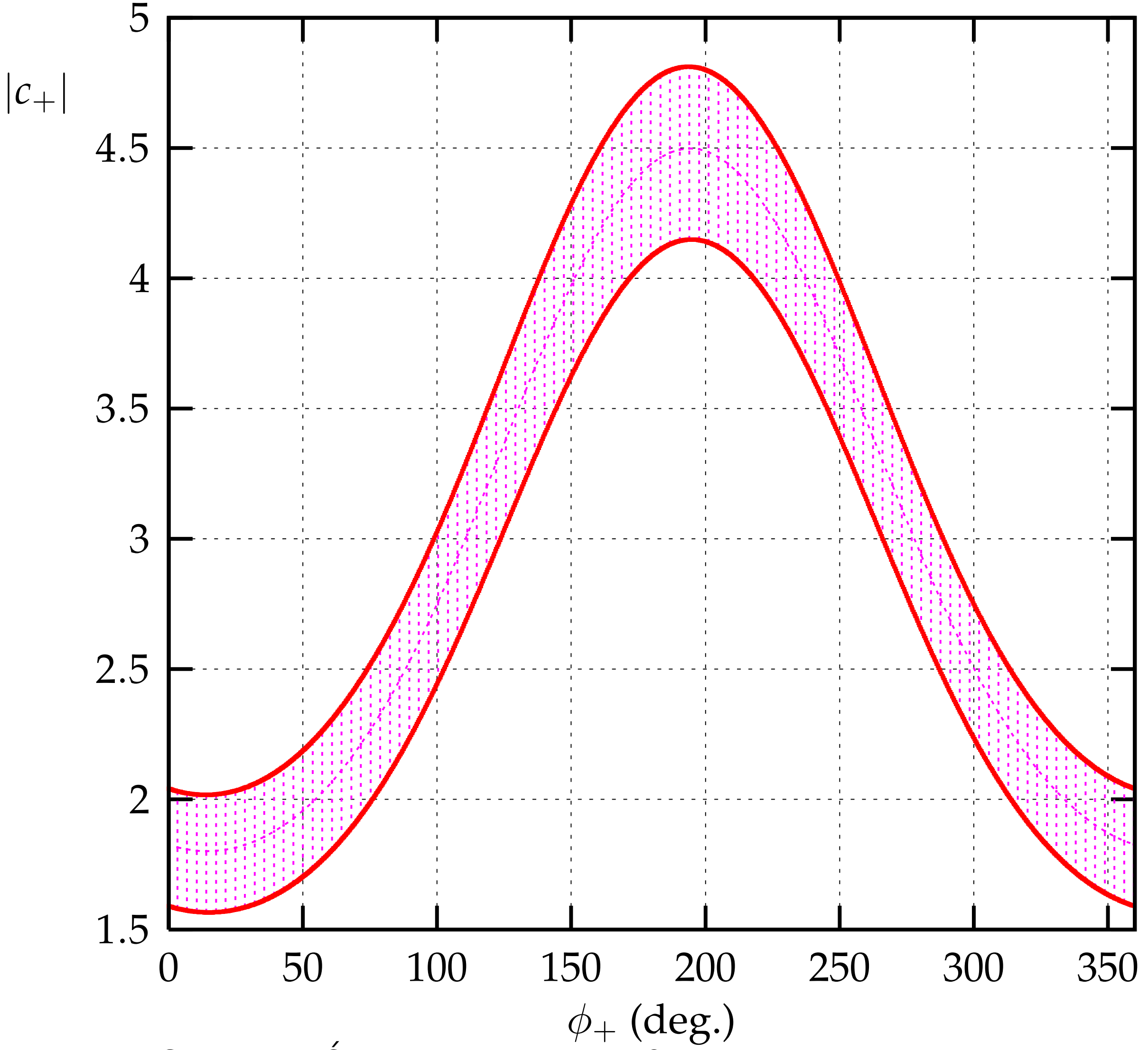
$$\gamma = \frac{\Gamma_e}{E - M_R}\frac{3(1-\Delta\alpha)}{\alpha}$$

CLEO: $\gamma = -0.063 \pm 0.001$

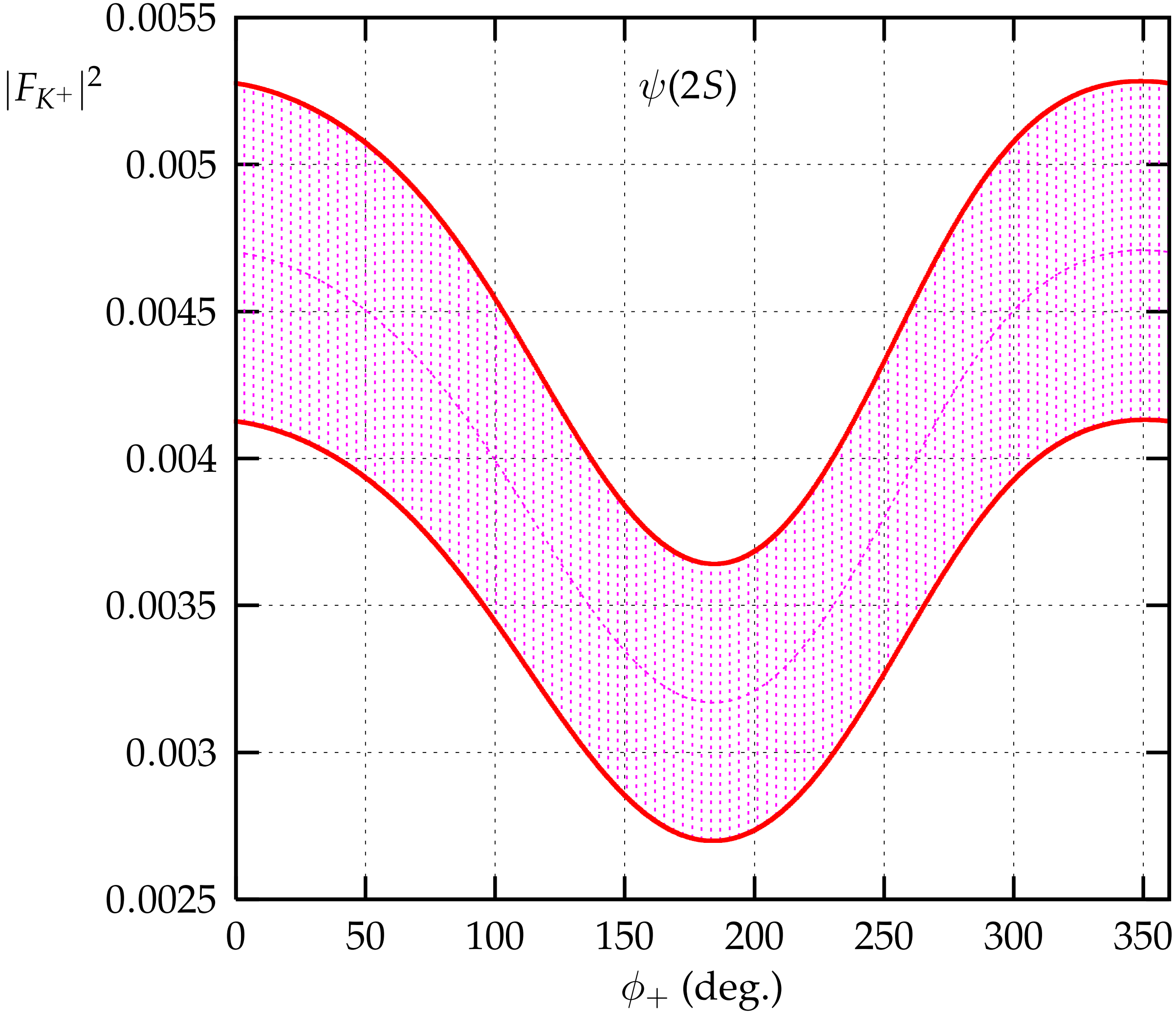
$\psi(2S) \to K^+K^-$

$$\frac{R_+}{S_+ - R_+\frac{\gamma^2}{4}} = \frac{1+2|c_+|\cos(\phi_+)+|c_+|^2+r|c_+|\sin(\phi_+)}{1+\gamma(1+|c_+|\cos(\phi_+))}$$

$\psi(2S) \to K^+K^-$



$\psi(2S) \to K^+K^-$


0.0055
0.005
0.0045
0.004
0.0035
0.003
0.0025
$|F_{K^+}|^2$
$\psi(2S)$
0
50
100
150
200
250
300
350
ϕ_+ (deg.)

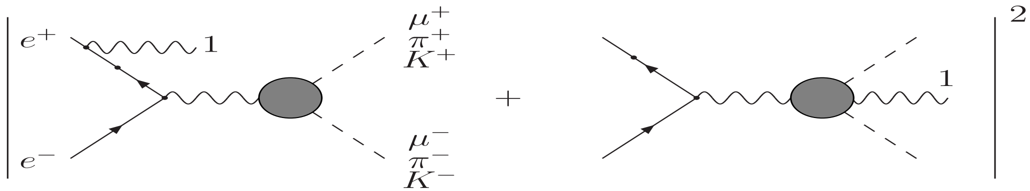
$$\psi(2S) \to K^+K^-$$

$$0.052 < |F_{K^+}| < 0.073$$

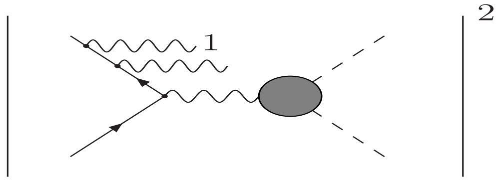
to be compared with CLEO

$$0.059 < |F_{K^+}| < 0.067$$

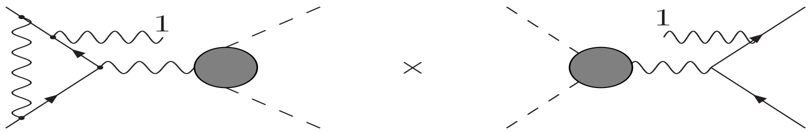
Contributing amplitudes



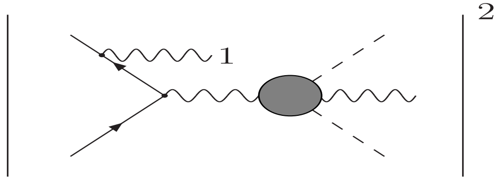
$|M_{\gamma_1,LOISR} + M_{\gamma_1,LOFSR}|^2$



$|M_{2\gamma,ISR}|^2$

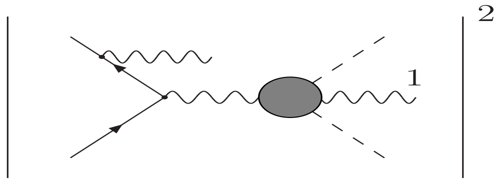


$2\ \mathrm{Re}(M_{\gamma_1,NLOISR} \times M^{\dagger}_{\gamma_1,LOISR})$

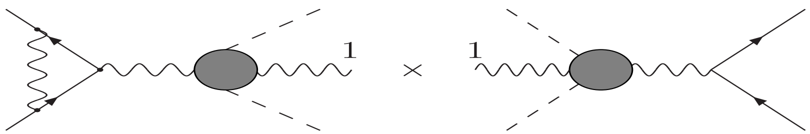


$|M_{\gamma_1,ISR\ ;\ \gamma,FSR}|^2$

$2\ \mathrm{Re}(M^{NLOFSR}_{\gamma_1,LOISR} \times M^{\dagger}_{\gamma_1,LOISR})$



$|M_{\gamma,ISR\ ;\ \gamma_1,FSR}|^2$



$2\ \mathrm{Re}(M^{NLOISR}_{\gamma_1,LOFSR} \times M^{\dagger}_{\gamma_1,LOFSR})$

The cross section

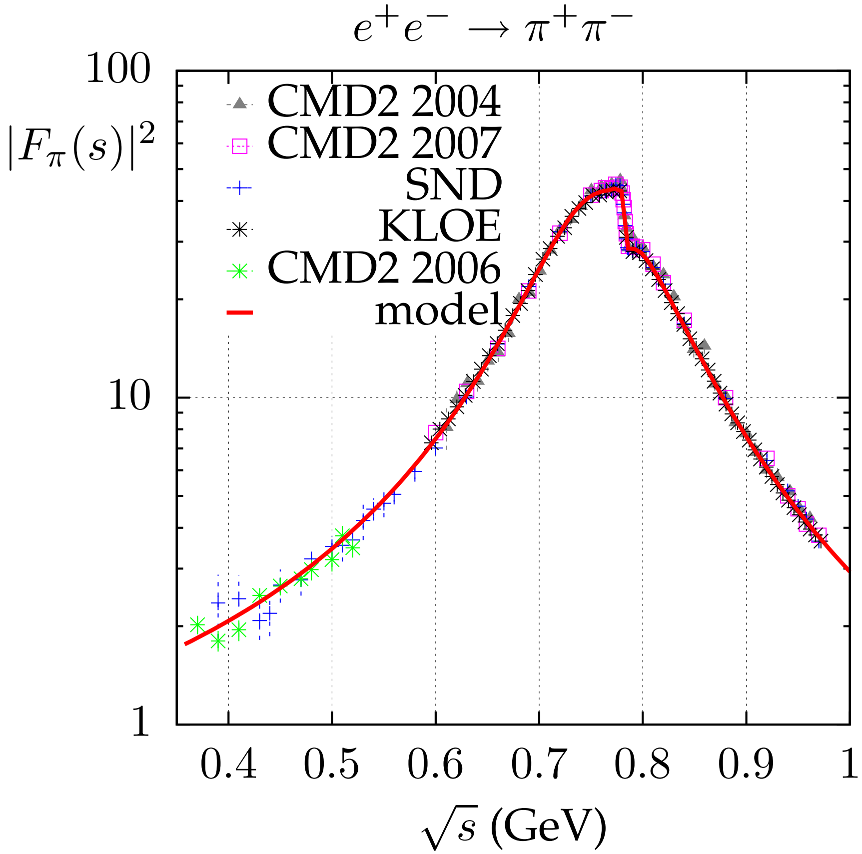
$$
\begin{aligned}
d\sigma = & \\
& |M_{\gamma_1,LOISR} \cdot C^{VP}_{R,P}(Q^2) + M_{\gamma_1,LOFSR} \cdot C^{VP}_{R,P}(s)|^2 d\Phi_1 \\
& + |M_{2\gamma,ISR} \cdot C^{VP}_{R,P}(Q^2)|^2 d\Phi_2 \\
& + 2\ \mathrm{Re}(M_{\gamma_1,NLOISR} \times M^{\dagger}_{\gamma_1,LOISR}) \cdot |C^{VP}_{R,P}(Q^2)|^2 d\Phi_1 \\
& + |M_{\gamma_1,ISR\ ;\ \gamma,FSR} \cdot C^{VP}_{R,P}((Q+k_\gamma)^2)|^2 d\Phi_2 \\
& + 2\ \mathrm{Re}(M^{NLOFSR}_{\gamma_1,LOISR} \times M^{\dagger}_{\gamma_1,LOISR}) \cdot |C^{VP}_{R,P}(Q^2)|^2 d\Phi_1 \\
& + |M_{\gamma,ISR\ ;\ \gamma_1,FSR} \cdot C^{VP}_{R,P}((Q+k_{\gamma_1})^2)|^2 d\Phi_2 \\
& + 2\ \mathrm{Re}(M^{NLOISR}_{\gamma_1,LOFSR} \times M^{\dagger}_{\gamma_1,LOFSR}) \cdot |C^{VP}_{R,P}(s)|^2 d\Phi_1 \ ,
\end{aligned}
$$

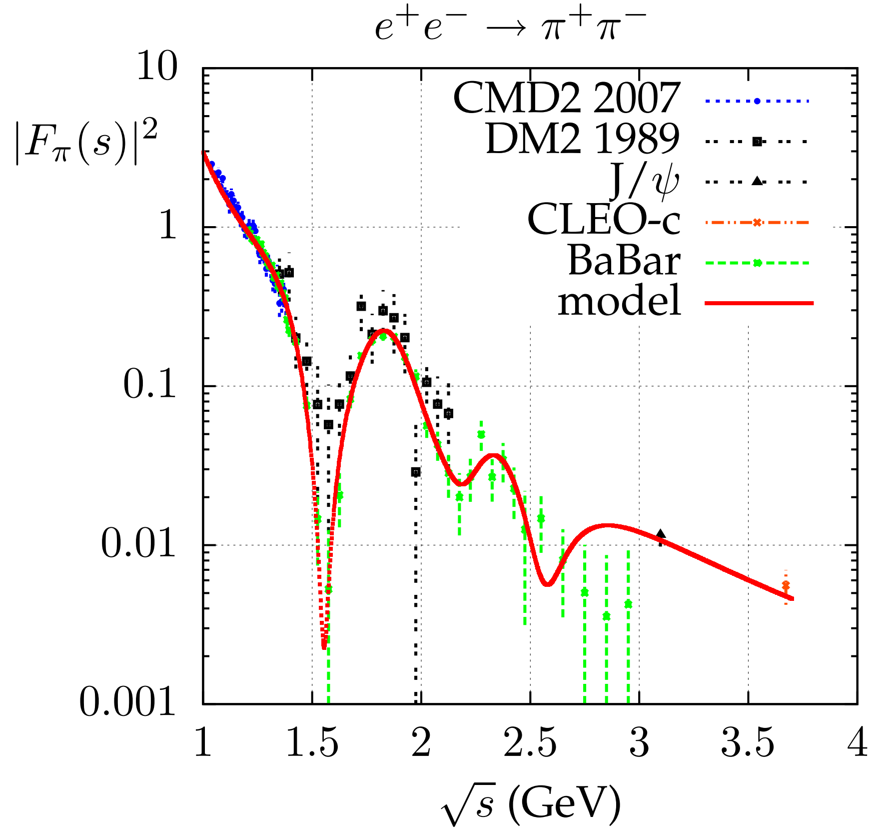
The cross section ...

$$C^{VP}_{R,P}(s) = \frac{1}{1-\Delta\alpha(s)} - \frac{3\Gamma^{\phi}_{e}}{\alpha m_{\phi}} BW_{\phi}(s)\delta_P$$
$$+ C_{J/\psi,P}(s) + C_{\psi(2S),P}(s) \ ,$$

$$C_{R,P}(s) = \frac{3\sqrt{s}}{\alpha}\frac{\Gamma^R_e(1+c^R_P)}{s-M^2_R+i\Gamma_R M_R} \ .$$

The pion form factor





$$\chi^2/d.o.f. = 271/270$$

C. Bruch, A. Khodjamirian and J.H. Kühn, Eur. Phys. J. C39(2005)41

H. C., A. Grzelińska and J.H. Kühn, Phys.Rev.D81:094014,2010

The pion form factor

$$F_\pi(s) = \left[\sum_{n=0}^{N} c^\pi_{\rho_n} BW_{\rho_n}(s)\right]_{fit} + \left[\sum_{n=(N+1)}^{\infty} c^\pi_{\rho_n} BW_{\rho_n}(s)\right]_{dQCD}$$

$$BW_{\rho_n}(s) = \frac{m^2_{\rho_n} + H(0)}{m^2_{\rho_n} - s + H(s) - i\sqrt{s}\,\Gamma_{\rho_n}(s)}$$

The pion form factor

ω contribution

$$c^\pi_{\rho_0} BW_{\rho_0}(s) \rightarrow \frac{c^\pi_{\rho_0} BW_{\rho_0}(s)}{1+c^\pi_\omega}(1+c^\pi_\omega BW_\omega)$$

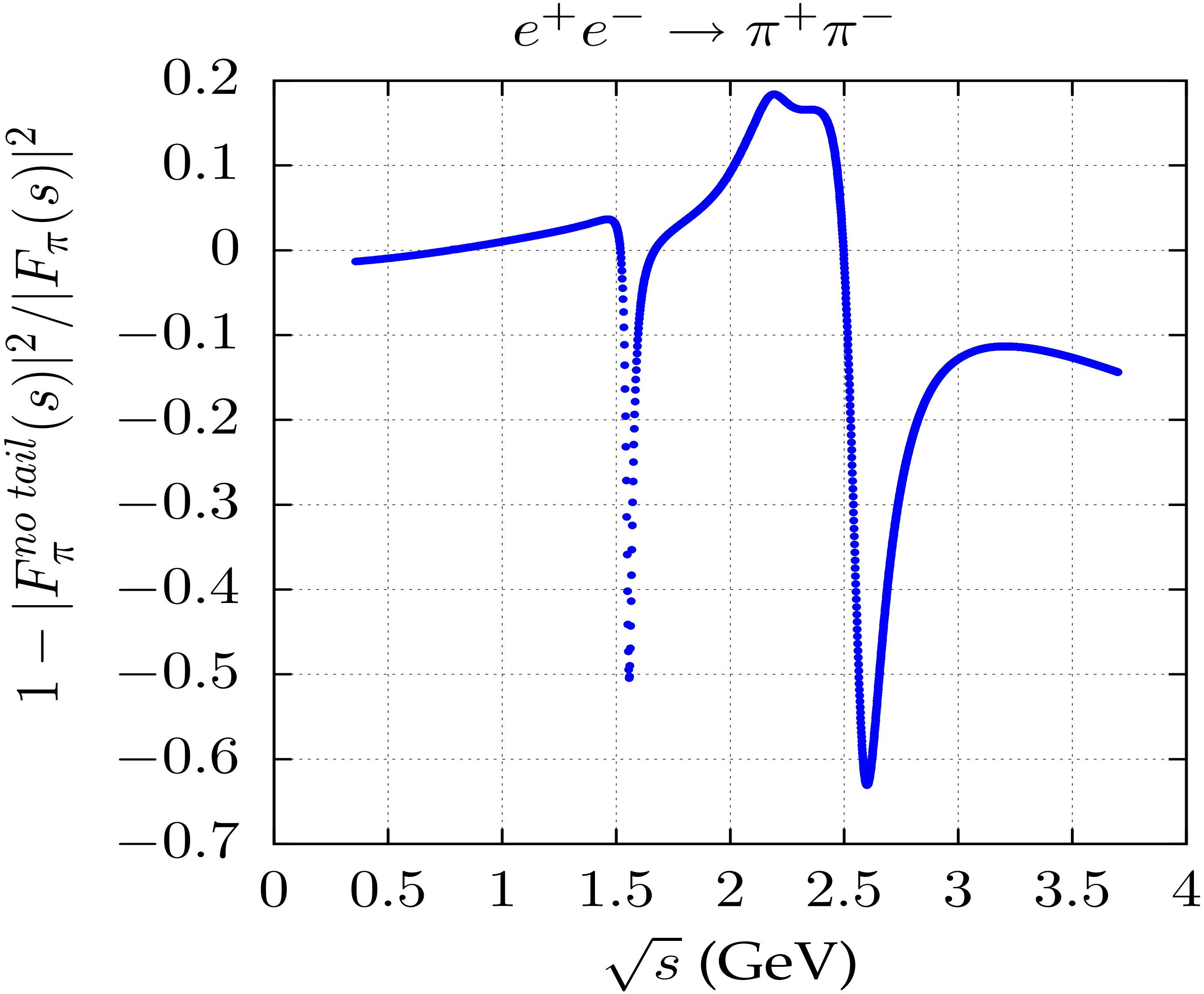
expansion parameters

$$c^\pi_{\rho_n} = \frac{(-1)^n \Gamma(\beta-1/2)}{\alpha' m^2_{\rho_n}\sqrt{\pi}\Gamma(n+1)\Gamma(\beta-1-n)} \, ,$$

$$\alpha' = 1/(2m^2_{\rho_0}), \quad m^2_{\rho_n} = m^2_{\rho_0}(1+2n)$$

$$f_n = F_n \left(\sum_{i=1}^{5} c^\pi_{\rho_i}\right) / \left(\sum_{i=1}^{5} F_i\right) \; n = 1,2,3,4,5$$

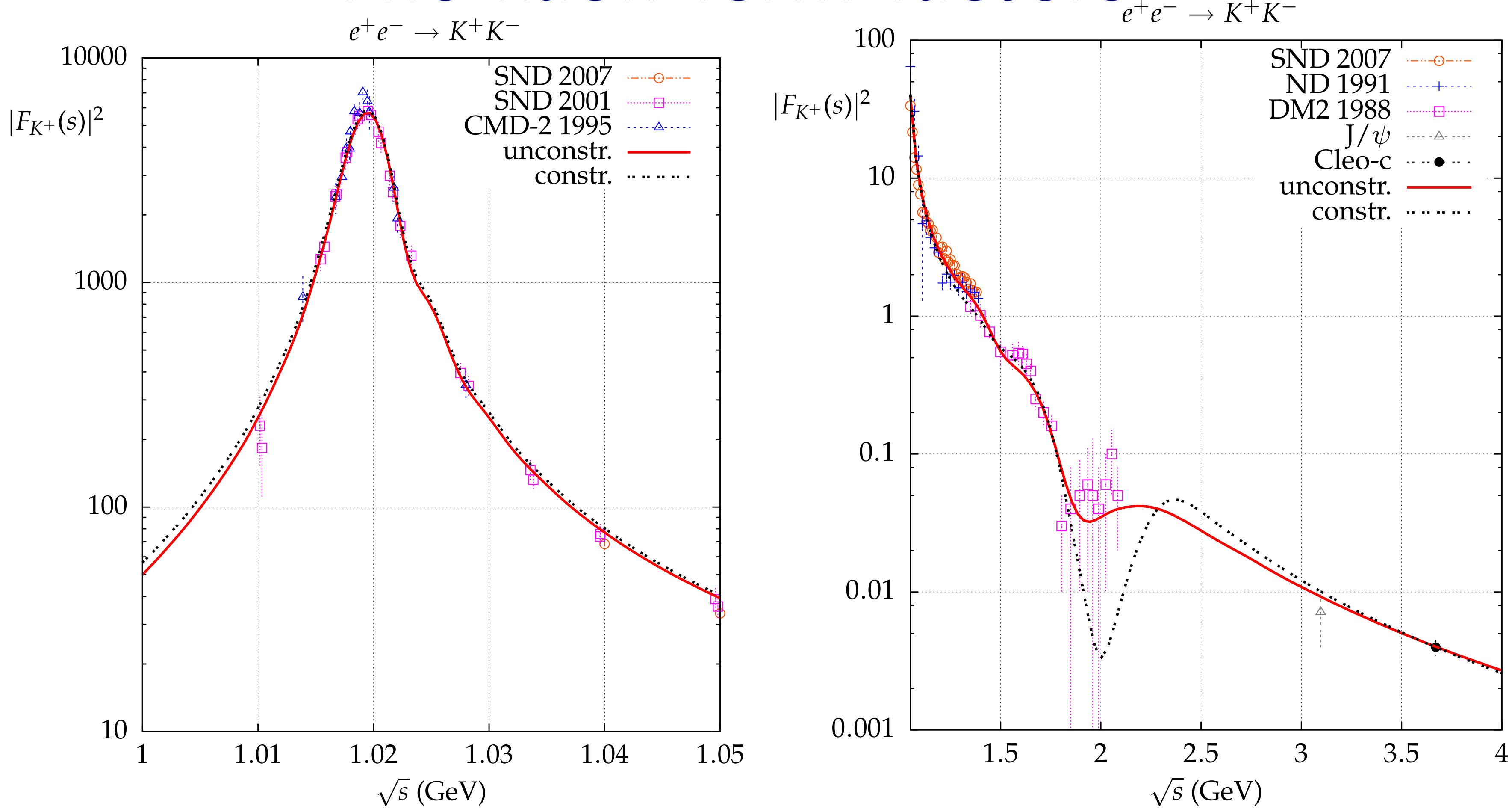
The pion form factor



The pion form factor

Parameter	model(fit)	PDG value [34]	model
m_{ρ_0}	773.37 ± 0.19	775.49 ± 0.34	input
Γ_{ρ_0}	147.1 ± 1.0	149.4 ± 1.0	input
m_{ω}	782.4 ± 0.5	782.41 ± 0.12	-
Γ_{ω}	8.33 ± 0.27	8.49 ± 0.08	-
m_{ρ_1}	1490 ± 11	1465 ± 25	1340
Γ_{ρ_1}	429 ± 27	400 ± 60	256
m_{ρ_2}	1870 ± 25	1720 ± 20	1730
Γ_{ρ_2}	357 ± 46	250 ± 100	330
m_{ρ_3}	2120 [22]	-	2047
Γ_{ρ_3}	300 [22]	-	391
m_{ρ_4}	model	-	2321
Γ_{ρ_4}	model	-	444
m_{ρ_5}	model	-	2567
Γ_{ρ_5}	model	-	491
β	2.148 ± 0.003	-	input
$\lvert c^{\pi}_{\omega}\rvert$	$(18.7 \pm 0.5) \cdot 10^{-4}$	-	-
$Arg(c^{\pi}_{\omega})$	0.106 ± 0.020	-	-

The kaon form factors

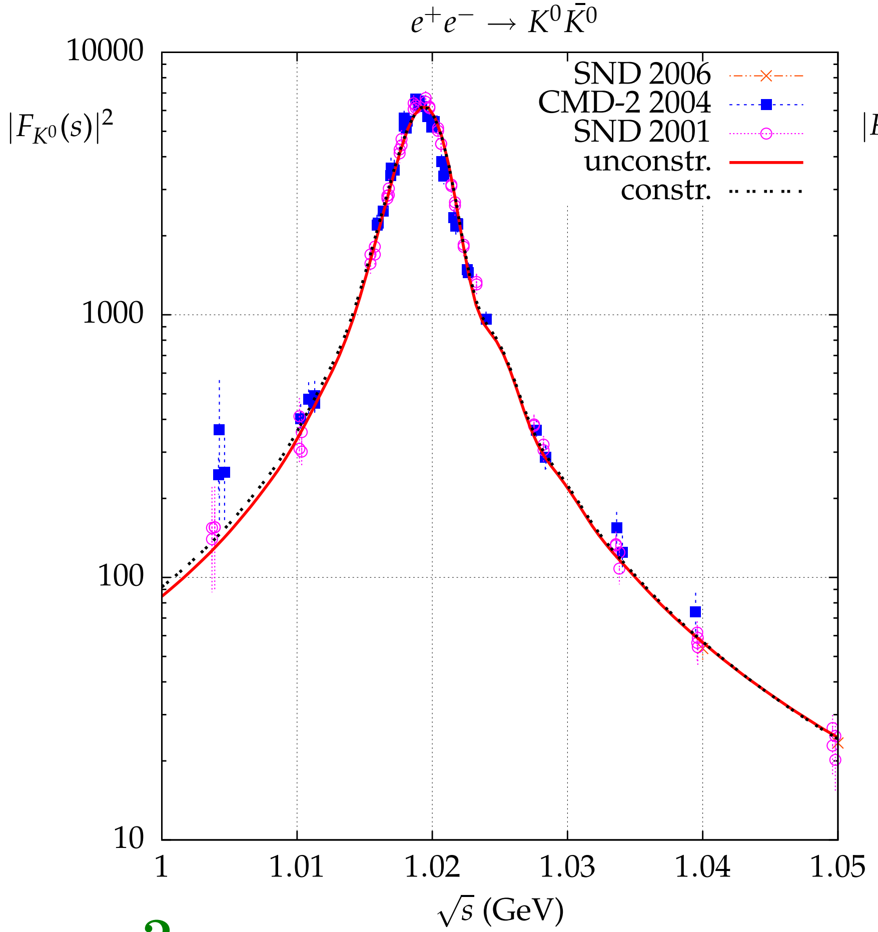


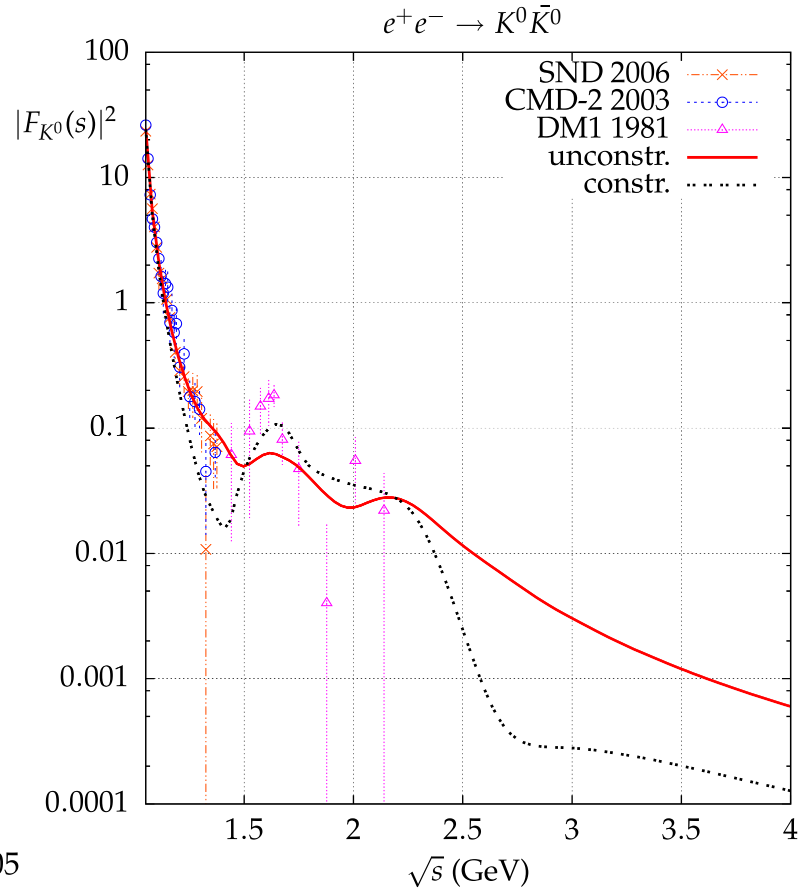
$$\chi^2/d.o.f. = 277/256(con)221/260(uncon)$$

C. Bruch, A. Khodjamirian and J.H. Kühn, Eur. Phys. J. C39(2005)41

H. C., A. Grzelińska and J.H. Kühn, Phys.Rev.D81:094014,2010

The kaon form factors





$$\chi^2/d.o.f. = 277/256(con)221/260(uncon)$$

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H. C., A. Grzelińska and J.H. Kühn, Phys.Rev.D81:094014,2010

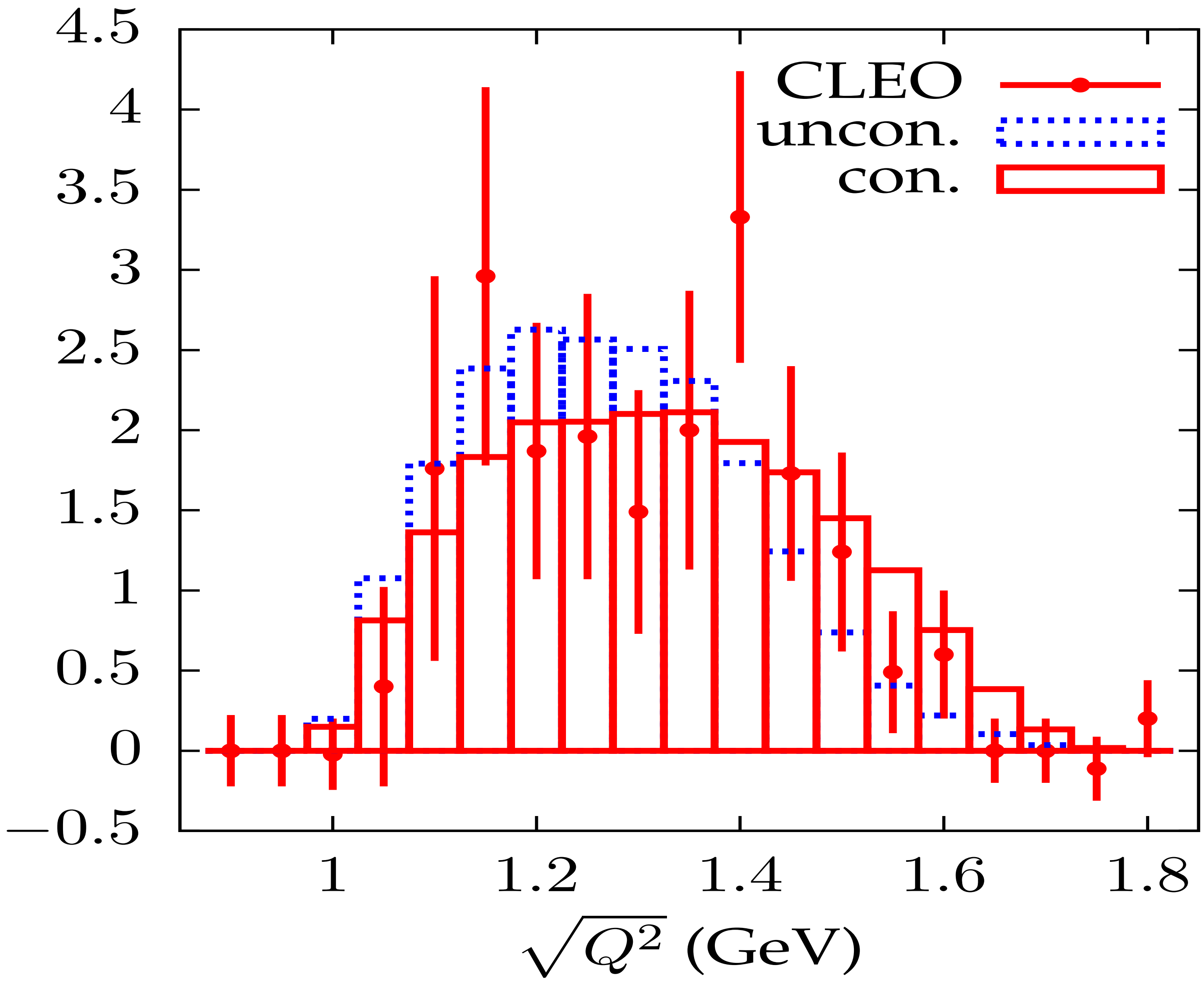
The kaon form factors

$$F_{K^+}(s) = \frac{1}{2}\left(\left[\sum_{n=0}^{N_\rho} c^K_{\rho_n} BW_{\rho_n}(s)\right]_{fit} + \left[\sum_{n=N_\rho+1}^{\infty} c^K_{\rho_n} BW_{\rho_n}(s)\right]_{dQCD}\right)$$

$$+\frac{1}{6}\left(\left[\sum_{n=0}^{N_\omega} c^K_{\omega_n} BW^c_{\omega_n}(s)\right]_{fit} + \left[\sum_{n=N_\omega+1}^{\infty} c^K_{\omega_n} BW^c_{\omega_n}(s)\right]_{dQCD}\right)$$

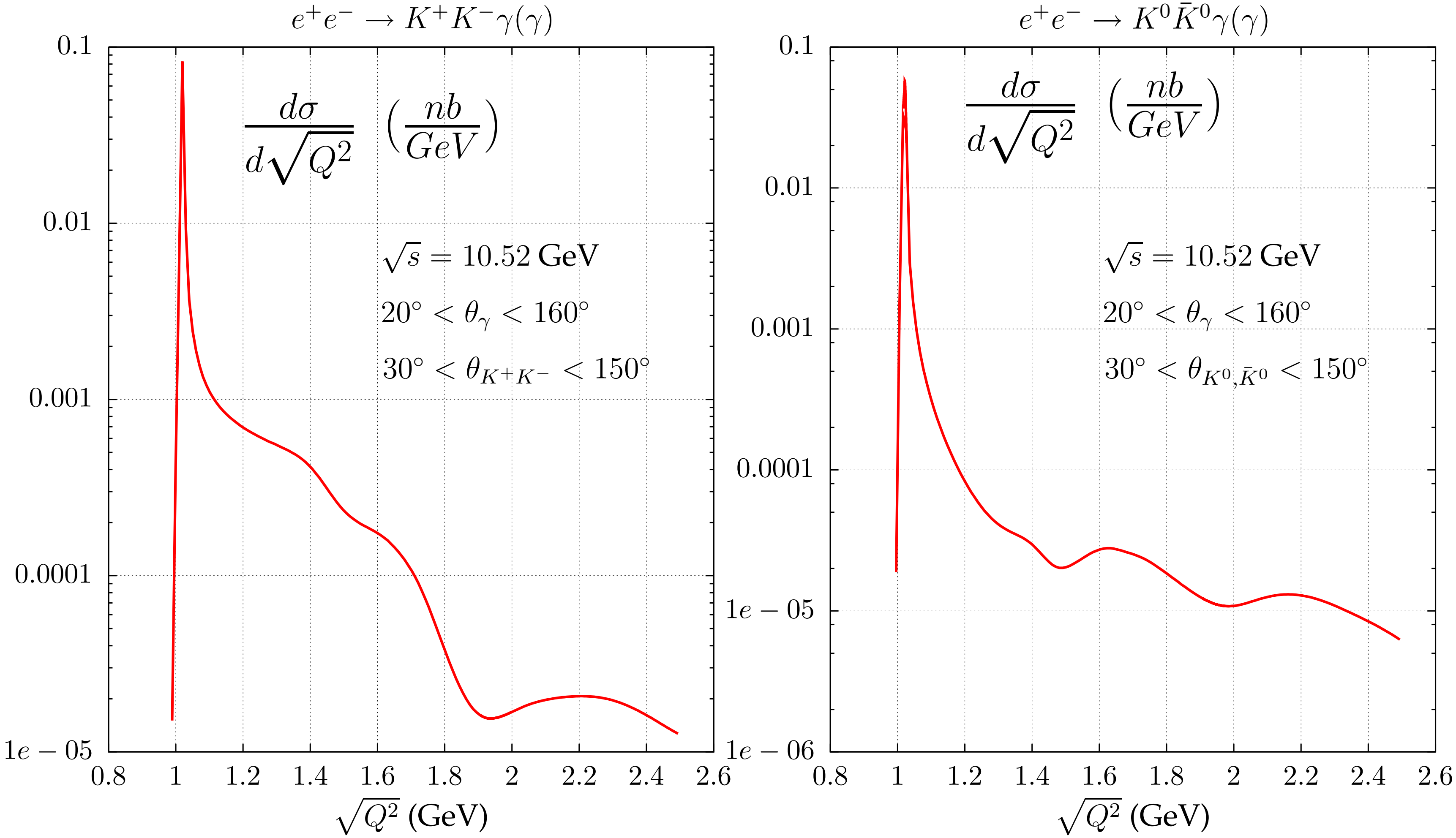
$$+\frac{1}{3}\left(\left[\sum_{n=0}^{N_\phi} c^K_{\phi_n} BW^K_{\phi_n}(s)\right]_{fit} + \left[\sum_{n=N_\phi+1}^{\infty} c^K_{\phi_n} BW^K_{\phi_n}(s)\right]_{dQCD}\right)$$

$\tau \to KK\nu$



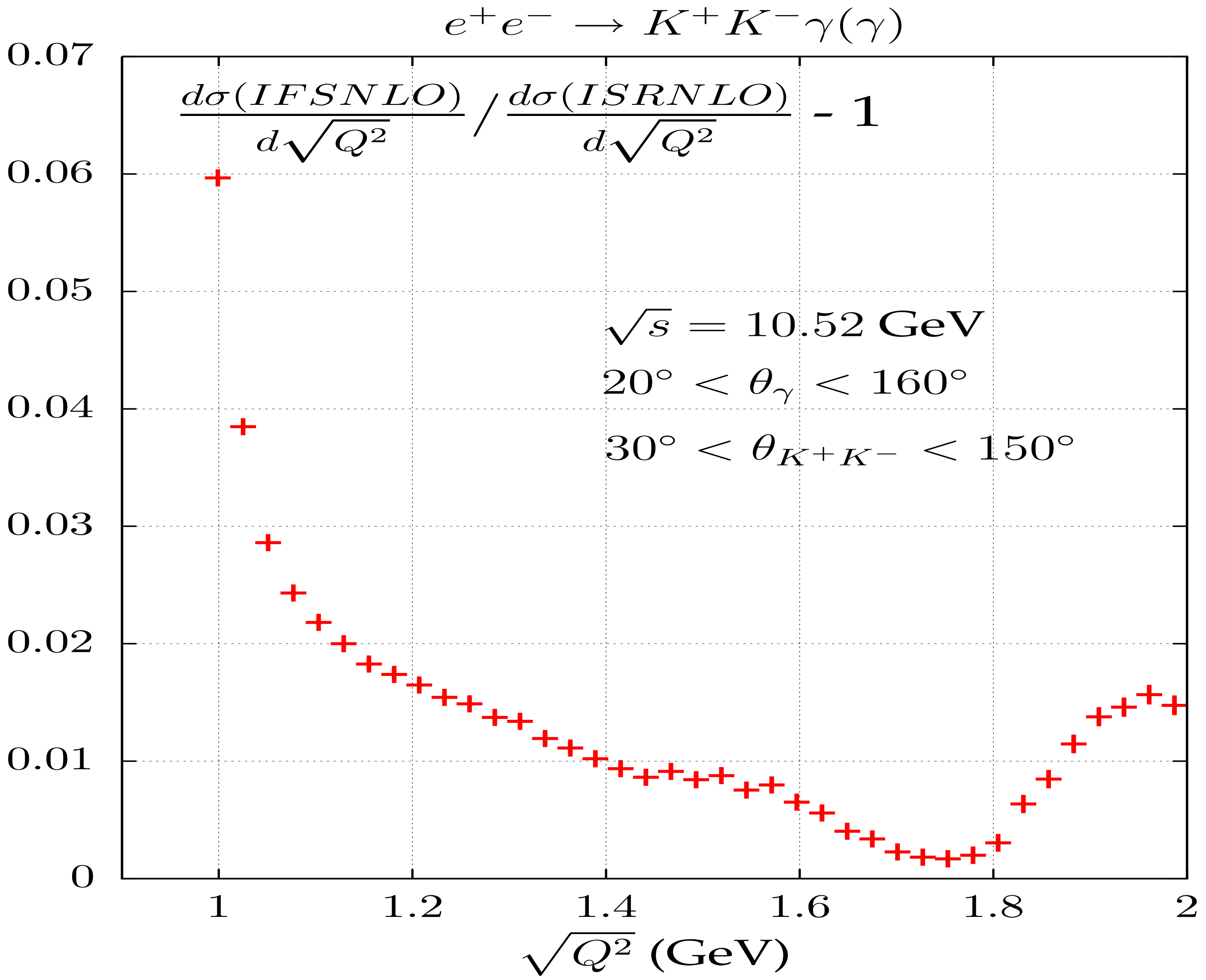
CLEO Phys. Rev. D 53, 6037 (1996)

PHOKHARA 7.0



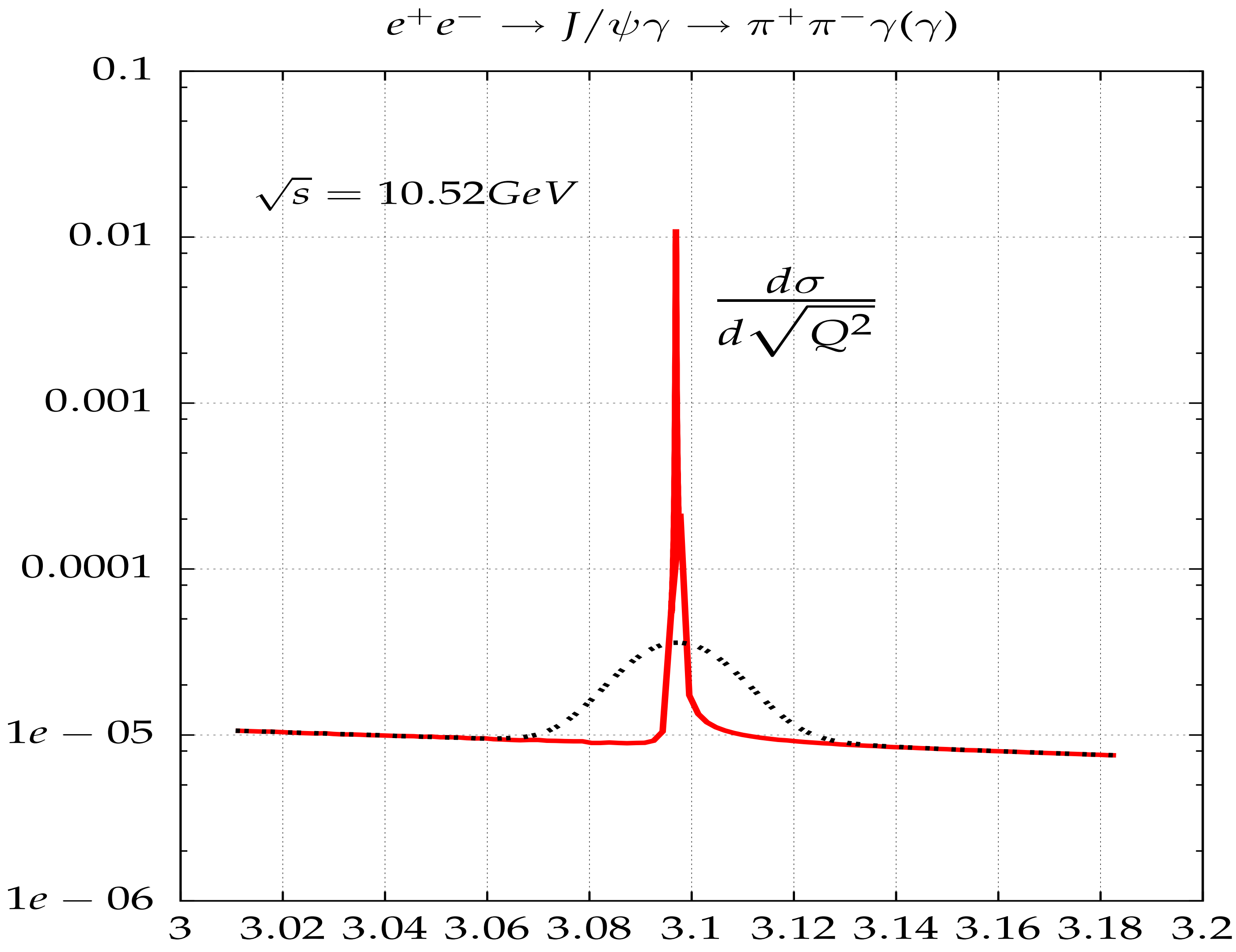
$e^+e^- \to K^+K^-\gamma(\gamma)$
$\frac{d\sigma}{d\sqrt{Q^2}}$ $\left(\frac{nb}{GeV}\right)$
$\sqrt{s} = 10.52$ GeV
$20^\circ < \theta_\gamma < 160^\circ$
$30^\circ < \theta_{K^+K^-} < 150^\circ$
0.1
0.01
0.001
0.0001
1e − 05
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
$\sqrt{Q^2}$ (GeV)
$e^+e^- \to K^0\bar{K}^0\gamma(\gamma)$
$\frac{d\sigma}{d\sqrt{Q^2}}$ $\left(\frac{nb}{GeV}\right)$
$\sqrt{s} = 10.52$ GeV
$20^\circ < \theta_\gamma < 160^\circ$
$30^\circ < \theta_{K^0,\bar{K}^0} < 150^\circ$
0.1
0.01
0.001
0.0001
1e − 05
1e − 06
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
$\sqrt{Q^2}$ (GeV)

PHOKHARA 7.0 - FSR

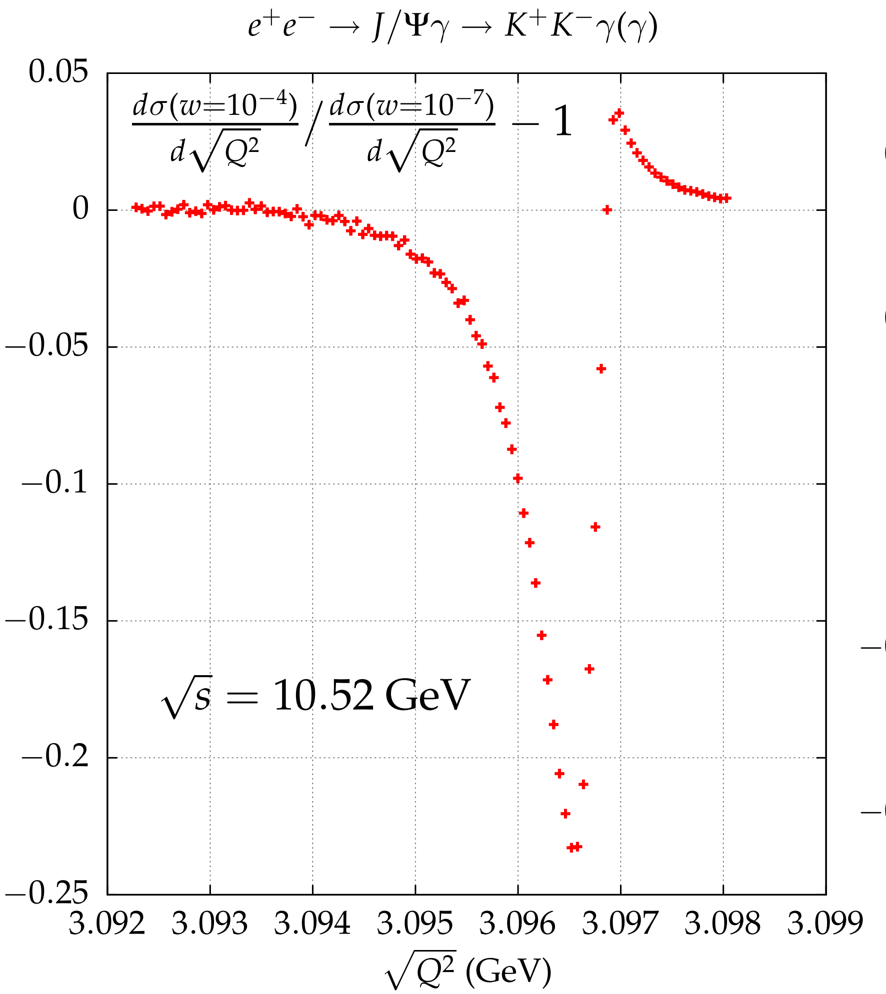


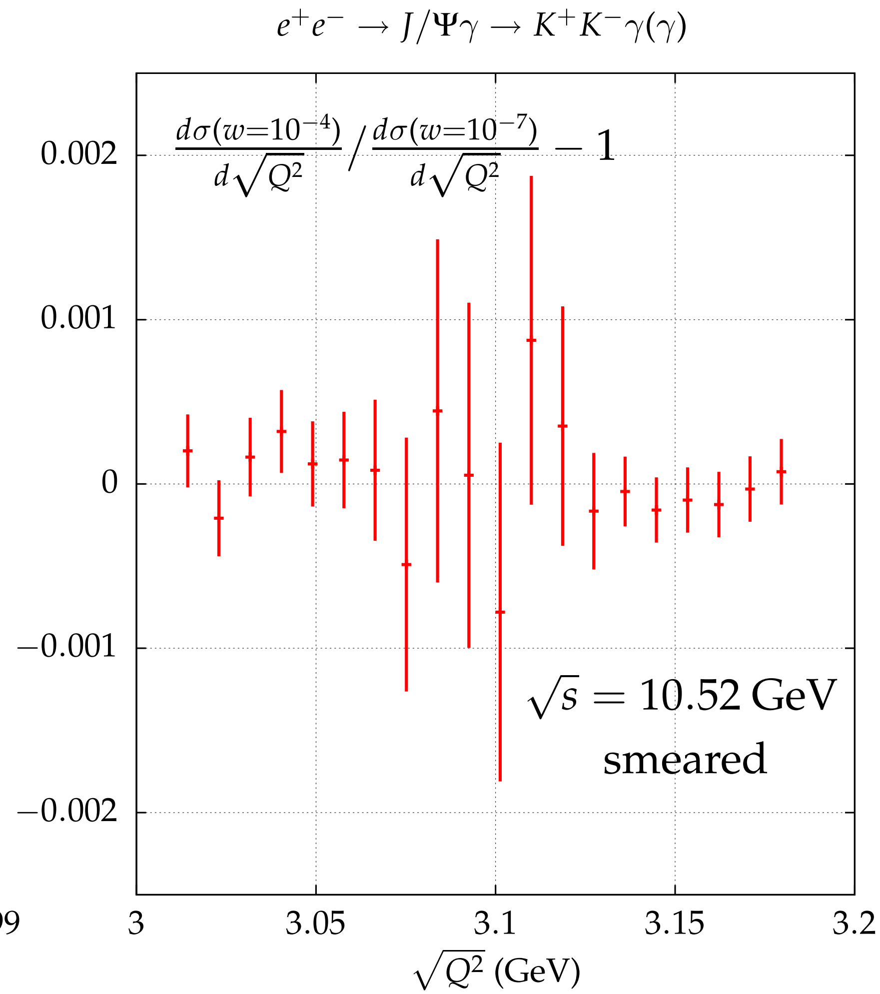
Energy resolution

$\Delta q = 14.5$ MeV

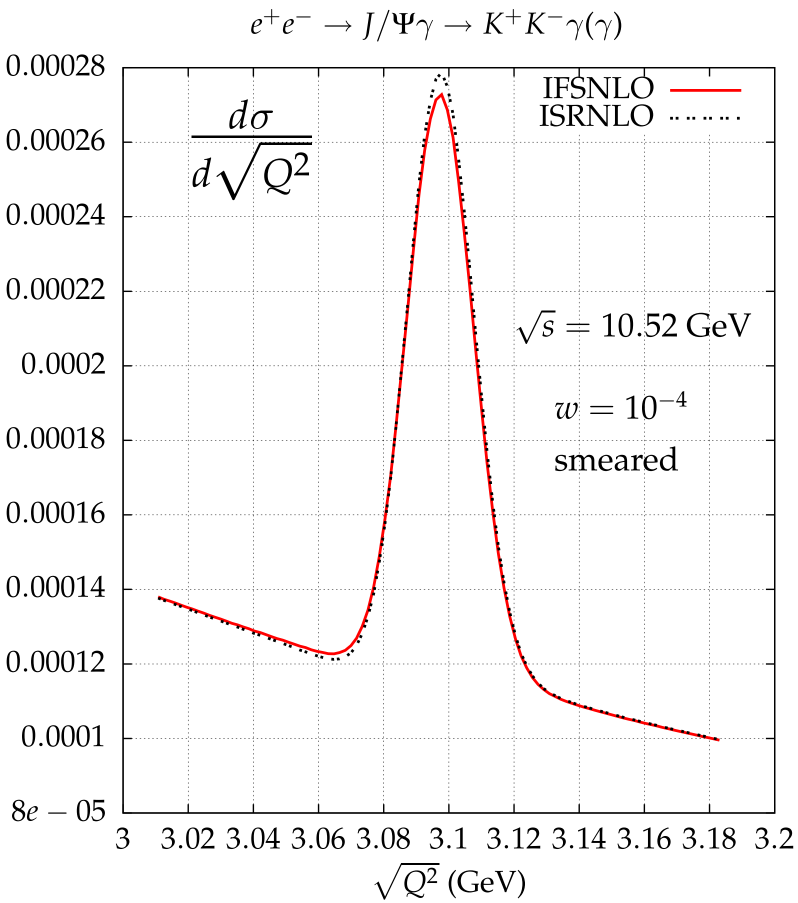


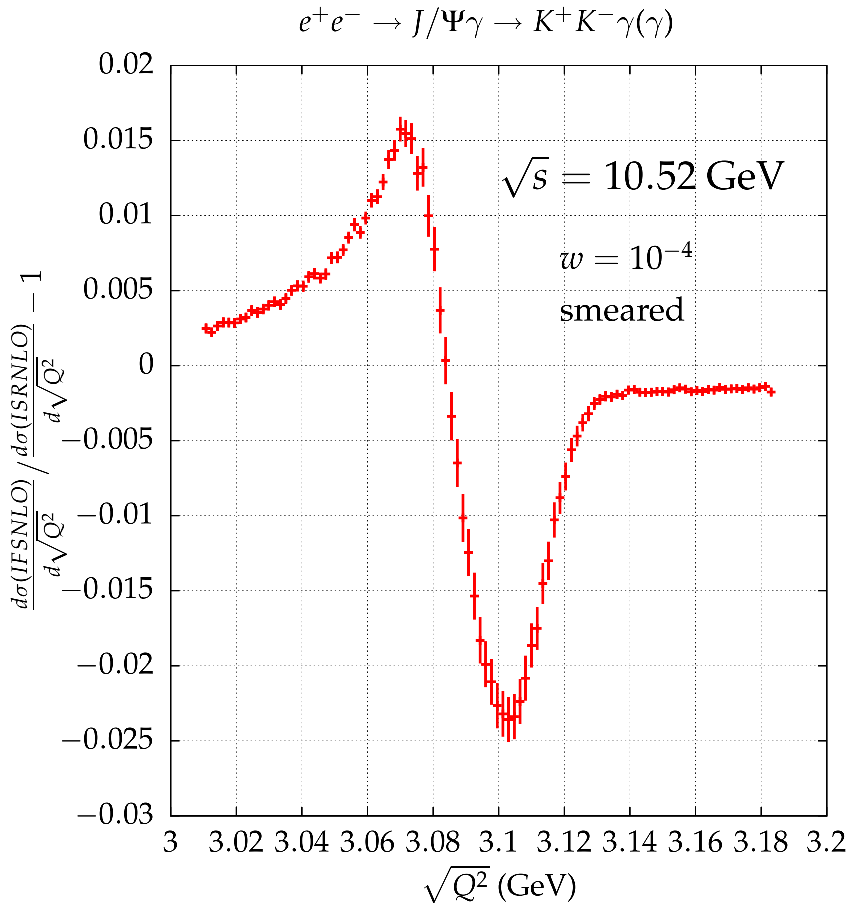
PHOKHARA 7.0 - FSR





PHOKHARA 7.0 - FSR





Summary and outlook

PHOKHARA 7.0 - fully tested

►**new 4 π hadronic current**

►**new pion and kaon form factors**

►J/ψ **and** $\psi(2S)$ **contributions included**

NLO FSR corrections important
at a few percent level

Left over 1-loop corrections to $e^+e^- \to \mu^+\mu^-\gamma$ **to be included soon**