Monte Carlo luminosity tools: status and perspectives

Guido Montagna

Dipartimento di Fisica Nucleare e Teorica, Università di Pavia
Istituto Nazionale Fisica Nucleare, Sezione di Pavia
guido.montagna@pv.infn.it

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in collaboration with the BabaYaga@NLO authors
and the contributors to the Luminosity Section of the WG Report
Eur. Phys. J. **C66** (2010) 585-686, arXiv:0912.0749 [hep-ph]

Typical theory of the MC generators

- ★ The most precise MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher–order leading logarithmic contributions
  [+ vacuum polarisation, using a data based routine for the calculation of the non–perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ contribution]
- ★ The methods used to account for multiple photon corrections are the (LEP/SLC borrowed) analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in the generators BabaYaga/BabaYaga@NLO] is a MC solution of the QED DGLAP equation for the electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The PS solution can be cast into the form

$$D(x, Q^2) \;=\; \Pi(Q^2) \sum_{n=0}^{\infty} \int \frac{\delta(x - x_1 \cdots x_n)}{n!} \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i)\; L\; dx_i \right]$$

  ★ $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$
  $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

- The accuracy is improved by matching exact NLO with higher-order leading log corrections

  G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 & Phys. Lett. **663** (2008) 209

  ★ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

Status of the luminosity generators

Generator	Processes	Theory	Accuracy	Web address
BHAGENF/BKQED	$e^+e^-/\gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$	1%	www.lnf.infn.it/~graziano/bhagenf/bhabha.html
BabaYaga v3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$\sim 0.5\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + PS	$\sim 0.1\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP1)	placzek.home.cern.ch/placzek/bhwide
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + SF	$< 0.2\%$	cmd.inp.nsk.su/~sibid

Sources of (possible) differences and theoretical uncertainty

- ⋆ "Technical precision": due to different details in the implementation of the *same* radiative corrections [e.g. different scales in higher–order collinear logs]. It can be estimated through *tuned comparisons* between the predictions of the different generators
- ⋆ Theoretical accuracy: due to *approximate or partially included* pieces of radiative corrections [e.g. exact NNLO photonic or pair corrections]. It can be evaluated through explicit comparisons with the exact perturbative calculations, if available
- At $\mathcal{O}(\alpha^2)$, infrared–enhanced photonic $\mathcal{O}(\alpha^2 L)$ most important NNLO sub–leading corrections taken into account through factorization of $\mathcal{O}(\alpha L) \times \mathcal{O}(\alpha)_{\mathrm{non-log}}$ contributions

G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. **B385** (1996) 348

Large–angle Bhabha: tuned comparisons & technical precision

Without vacuum polarisation, to compare QED corrections consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga people, Wang Ping and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

★ Agreement well below 0.1%! ★

At BaBar (cross sections in nb)

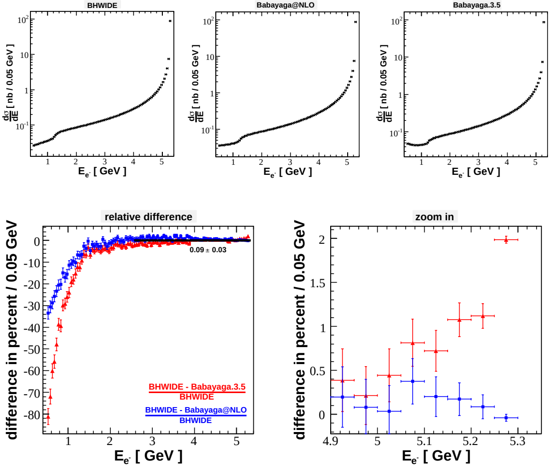
By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

★ Agreement at the $\sim$ 0.1% level! ★

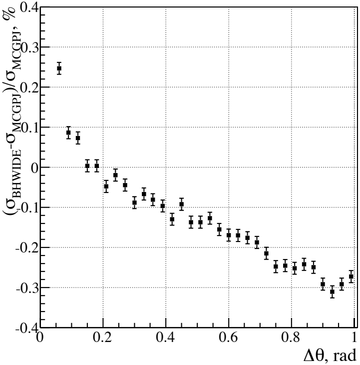
BabaYaga@NLO vs BHWIDE at BaBar

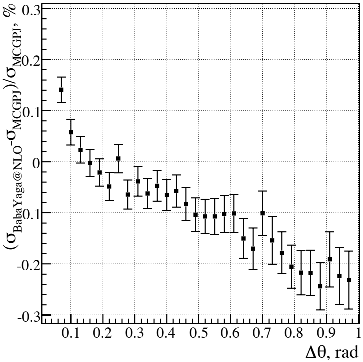
From the Luminosity Section of the WG Report
Using realistic luminosity cuts @



- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for distributions. Larger differences correspond to very hard photon emission and do not influence noticeably the luminosity measurement

From the Luminosity Section of the WG Report
With realistic selection cuts for luminosity @ CMD–2





- The three generators agree within 0.1% for the typical experimental acollinearity cut $\Delta\theta \sim 0.2 \div 0.3$ rad

★ Main conclusion from tuned comparisons: technical precision of the generators well under control, the small remaining differences being due to slightly different details in the calculation of the same theoretical ingredients [additive vs factorized formulations, different scales for higher–order leading log corrections]

The main question: how to establish the MC theoretical accuracy?

1. By comparing with the available NNLO calculations
2. Recent NNLO progress under study in comparison with MC results [more in the following] :
   - ★ Exact NNLO (real+virtual) pair corrections to Bhabha by a Desy–Zeuthen and Katowice collaboration
   - ★ Exact NLO SV corrections to single hard bremsstrahlung in $e^+e^-\gamma$, $\mu^+\mu^-\gamma$ by Actis, Mastrolia and Ossola

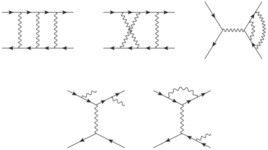
For example, the (approximate) BabaYaga@NLO cross section with NNLO photonic corrections can be cast into the form

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

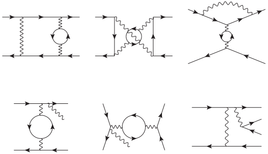
- $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$ $\longrightarrow$ compared with the corresponding available NNLO QED calculation
- $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ presently estimated relying on existing (partial) results
- $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

Some advances in NNLO Bhabha calculations

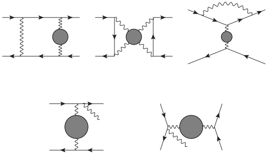
- **Photonic corrections** A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- **Electron loop corrections** R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280 / S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

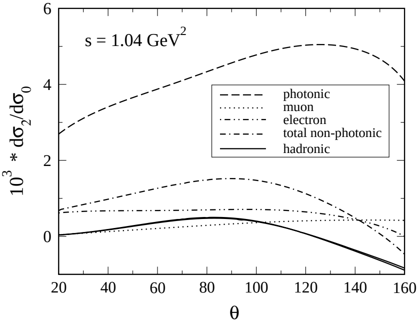


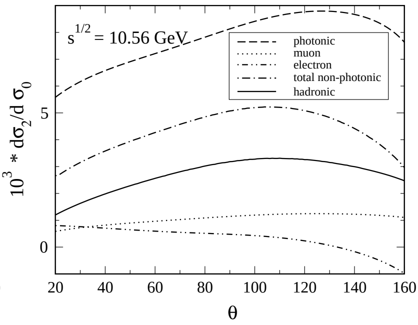
- **Heavy fermion and hadronic loops** R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601 / S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602 / J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



NNLO QED corrections: typical size at Φ and B factories

From the Luminosity Section of the WG Report





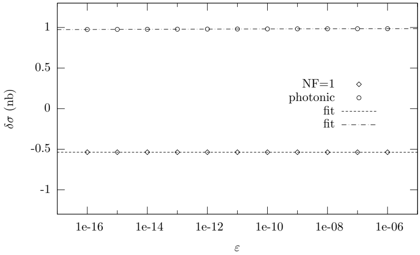
- NNLO QED corrections amount to some per mille and are dominated by photonic (dashed line) and electron loop (dashed–dotted) corrections
- The bulk [due to the reducible contributions] of such corrections is *effectively* incorporated in the most precise generators through the matching of NLO corrections with multiple photon contributions and the insertion of vacuum polarisation in the $\mathcal{O}(\alpha)$ diagrams.
  To what extent?

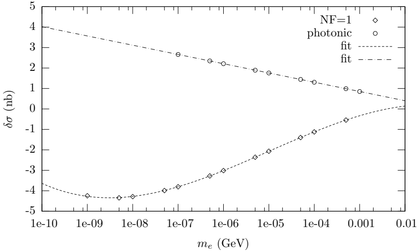
Comparison with NNLO calculation for $\sigma_{\mathrm{SV}}^{\alpha^2}$

Using realistic cuts for luminosity @

Comparison of $\sigma_{\mathrm{SV}}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)





★ differences are infrared safe, as expected

★ $\delta\sigma(\mathrm{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

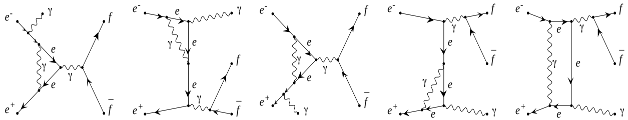
- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\mathrm{SV}}^{\alpha^2}(\mathrm{Penin}) - \sigma_{\mathrm{SV}}^{\alpha^2}(\mathrm{BabaYaga@NLO}) \; < \; 0.02\% \times \sigma_0$$

Uncertainty due to $e^+e^- \to e^+e^-\gamma$ at one loop

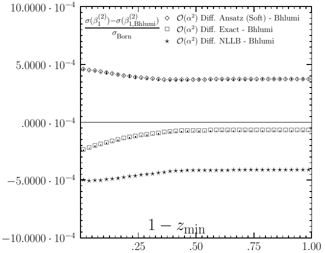
★ New! The exact perturbative calculation of $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ for full $s+t$ Bhabha scattering in QED appeared about one year ago ★ [Comparisons in progress]

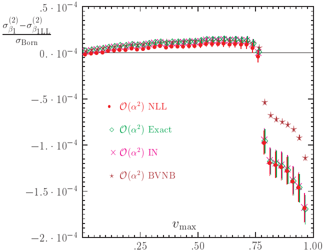
S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419, `arXiv:0909.1750`



- Using the results available for $t-$channel Bhabha scattering (left plot) and $s-$channel annihilation processes (right plot)

S. Jadach, M. Melles, B.F.L. Ward and S. Yost, PL **B377** (1996) 168 & PL **B450** (1999) 262
C. Glosser, S. Jadach, B.F.L. Ward and S. Yost, Phys. Lett. **B605** (2005) 123





the uncertainty of the most precise generators for one–loop corrections to single hard bremsstrahlung can be safely estimated to be $\sim 0.05\%$

A further important source of error: lepton and hadron pairs

In progress!: Preliminary results in the Luminosity Section of the WG Report

- A Desy–Zeuthen & Katowice collaboration (T. Riemann, H. Czyz, J. Gluza, M. Gunia and M. Worek) did a new, exact calculation of pair corrections, based on exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-, l = e, \mu, \tau), e^+e^-(\pi^+\pi^-)$
- Preliminary results: in comparison with the approximation of BabaYaga@NLO and using realistic luminosity cuts (cross sections in nb)

	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
Electron pair corrections				
KLOE	529.469	-1.794	-1.570	0.04
BaBar	6.744	-0.008	-0.008	0.00
Muon pair corrections				
KLOE	529.469	-0.241	-0.250	0.002
BaBar	6.744	-0.004	-0.003	0.015
Pion pair corrections				
KLOE	529.469	-0.186	in progress	–
BaBar	6.744	-0.003	in progress	–

⋆ The uncertainty due to lepton and hadron pair corrections is at the level of a few units in 10^{-4} [other comparisons in progress + paper in preparation] ⋆

Status of the MC theoretical accuracy

Main conclusion of the Luminosity Section of the WG Report

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ [in progress]	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$ [in progress]	$\sim$0.05	$\sim$0.1[1]	$\sim$0.02[2]
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12÷0.14	0.18	0.11÷0.12
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07÷0.08	0.11	0.06÷0.07

- For the experiments on top of and closely around the J/ψ resonance, the accuracy slightly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ routines [see next slide]
- ★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved about ten years ago for small–angle Bhabha luminosity monitoring at LEP/SLC

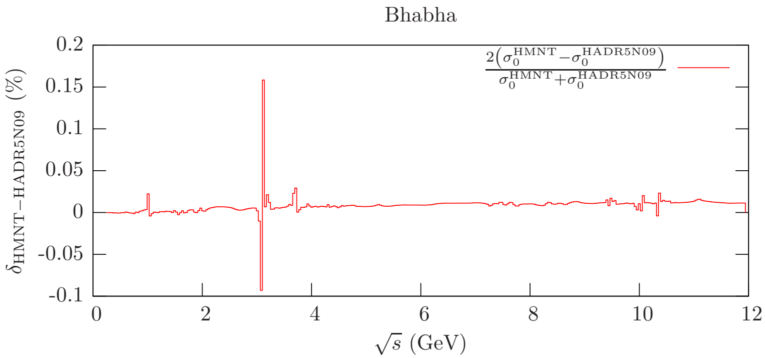
[1] Very preliminary, work in progress using realistic BES-III and CLEO-c luminosity cuts

[2] Preliminary and assuming BaBar cuts. Work in progress for BELLE event selection

Vacuum polarisation: HADR5N09 vs. HMNT for $e^+e^- \to e^+e^-$

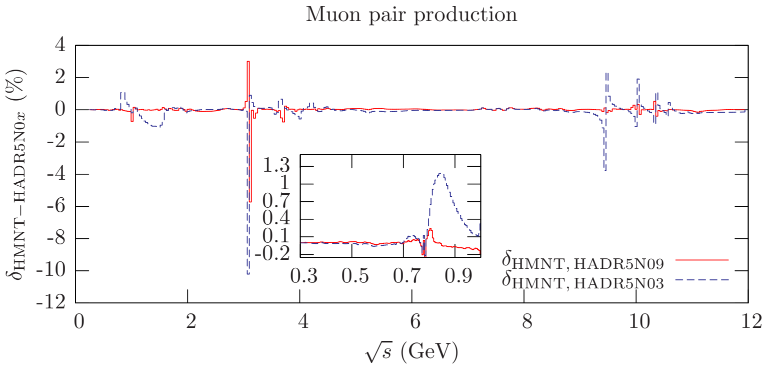
For a discussion see the Vacuum polarisation Section of the WG Report

HADR5N09, F. Jegerlehner, `http://www-com.physik.hu-berlin.de/ fjeger/hadr5n09.f`
Nucl. Phys. Proc. Suppl. **135** (2008) 181
HMNT: K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, Phys. Lett. **B649** (2007) 173
T. Teubner, K. Hagiwara, R. Liao, A.D. Martin and D. Nomura
Chinese Phys. **C34** (2010) 728, `arXiv:1001.5401`



- Bhabha largely dominated by t-channel (space-like) scattering
- The two parameterisations agree within 0.5 $\times 10^{-3}$ for all c.m. energies, at $\sim 0.1 - 0.2$ % around the J/ψ

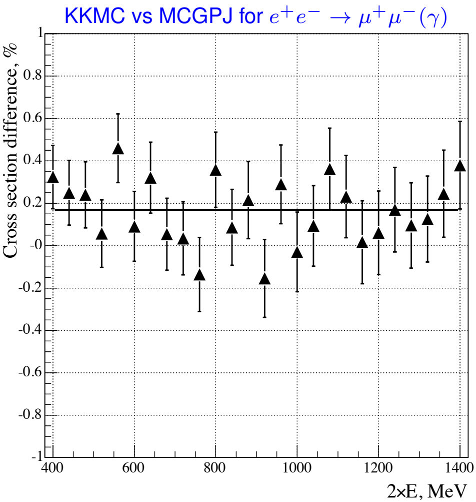
Vacuum polarisation: HADR5N09[03] vs. HMNT for $e^+e^- \to \mu^+\mu^-$



- Muon pair production mediated by s-channel (time-like) virtualities
- The different parameterisations can induce cross section differences at one – some per cent level around the very narrow resonances
- Note that J03 is the “undressing” parameterisation used by KLOE in the measurement of the pion form factor, while HMNT is practically equivalent to the VEPP-2M parameterisation used by CMD2/SND

Tuned comparisons for non–Bhabha luminosity processes

From R Section of the WG Report

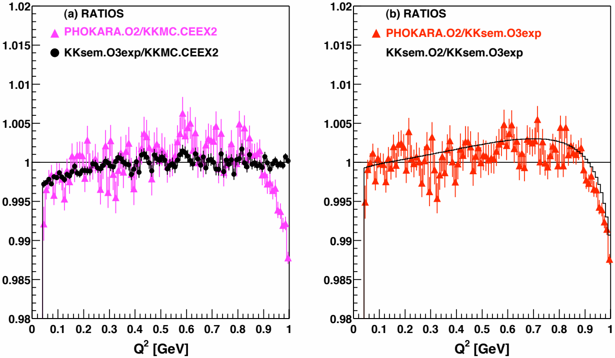


- Predictions of KKMC and MCGPJ agree on the average at the 0.2% level
- New and more detailed comparisons could be done and should be extended to cover $\gamma\gamma$ production (e.g. BabaYaga@NLO vs. MCGPJ)

Tuned comparisons for non–Bhabha luminosity processes

S. Jadach, Acta Phys. Pol. **B36** (2005) 2387

KKMC vs PHOKHARA for $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$: including initial–state radiation only, both in the signal and radiative corrections



- Predictions of KKMC and PHOKHARA for the muon pair spectrum $d\sigma/dQ^2$ in $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$ at $\sqrt{s} = 1.02$ GeV agree within 0.2% in the central region and differ at high Q^2 by $\sim 1\%$
- New comparisons involving BabaYaga@NLO and Actis-Mastrolia-Ossola?

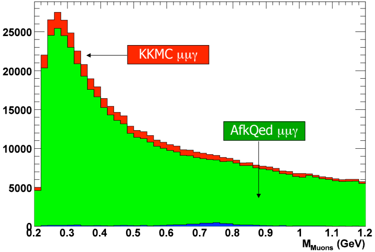
Conclusions & perspectives

- The WG activity led to a control of the theoretical error to the luminosity measurements down to $\sim$ 0.1%
- ⋆ Both exact $\mathcal{O}(\alpha)$ and multiple photon corrections are implemented in the most precise MC luminosity tools and are necessary ingredients for 0.1% theoretical accuracy [together with vacuum polarisation]
- ⋆ At least three generators for large–angle Bhabha scattering (BabaYaga@NLO, BHWIDE, MCGPJ) agree within 0.1% for integrated cross sections and $\sim 1\%$ (or better) for distributions
- Precision generators also available for $\gamma\gamma$ production (BabaYaga@NLO, MCGPJ) and $\mu^+\mu^-$ final states (BabaYaga@NLO, KKMC, MCGPJ)
- ⋆ NNLO QED calculations essential to assess the MC theoretical accuracy at the 0.1% level and to improve it below the one per mille
- ⋆ The present MC accuracy is robust and already sufficient for per mille luminosity measurements at meson factories ⋆
- ⋆ Work in progress and possible future activity
  - Theoretical accuracy: deeper analysis of the uncertainty due to pair corrections [in progress] and one–loop corrections to $e^+e^- \to e^+e^-\gamma$ [in progress]
  - Tuned comparisons: new generators' tests for $e^+e^- \to \gamma\gamma$ and $e^+e^- \to \mu^+\mu^-, \mu^+\mu^-\gamma$

Backup Slides

Tuned comparisons for non–Bhabha luminosity processes

Some discrepancy at BaBar between KKMC and AfkQED for muons invariant mass [see talk by N. Berger @ EPS HEP 2007]



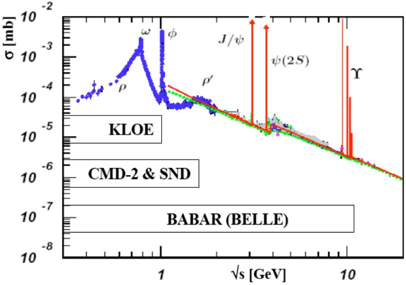
- Leading–order (w/o radiative corrections) predictions of BabaYaga@NLO and Dixon calculation of $e^+e^- \to \mu^+\mu^-\gamma$ at a B–factory $\sqrt{s} = 10.58\,\mathrm{GeV}$ with cuts: $M_{\mu\mu} \leq 2\,\mathrm{GeV}, \ |\cos\vartheta_\gamma| \leq 0.9$, no muon cuts

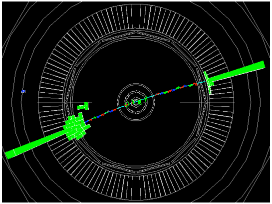
Thanks to Lance Dixon!

$M_{\mu\mu}$ (GeV)	σ_{LO} Dixon [pb]	σ_{LO} BabaYaga@NLO [pb]
$0.320 \div 0.480$	$2.88(1)$	$2.90(3)$
$0.480 \div 0.640$	$2.12(1)$	$2.11(1)$
$0.640 \div 0.800$	$1.66(1)$	$1.66(1)$
$0.800 \div 0.960$	$1.37(1)$	$1.37(1)$
$0.960 \div 1.120$	$1.17(1)$	$1.18(1)$

⋆ Excellent agreement! ⋆

Why precision luminosity generators?





Bhabha tracks @ the B–factory PEP-II

- Precision measurements of the hadronic cross section at low energies require a precise knowledge of the e^+e^- collider luminosity $\mathcal{L}$

$$\int \mathcal{L}\, dt = N_{\rm obs}/\sigma_{\rm th}$$

★ Precise knowledge of the luminosity needs normalization processes with clean topology, high statistics and calculable with high theoretical accuracy $\rightarrow$ wide–angle QED processes $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering), $e^+e^- \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow \mu^+\mu^-$, with typical experimental errors in the range few $0.1\% \div \mathcal{O}(1\%)$

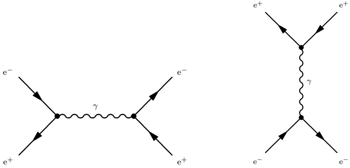
- High theoretical accuracy and comparison with data require precision Monte Carlo (MC) tools, including radiative corrections at the highest standard as possible

The luminosity monitoring QED processes

Using wide angles selection cuts, with typical experimental errors in the range few $0.1\% \div \mathcal{O}(1\%)$ [e.g. $\delta\mathcal{L}_{\rm exp}/\mathcal{L}_{\rm exp} = 0.3\%$ for Bhabha @ KLOE]

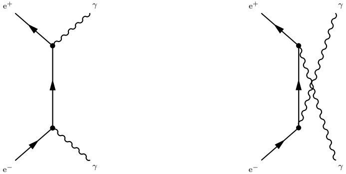
Homi J. Bhabha (1909-1966)
Proc. Roy. Soc. **A154** (1936) 195

- $e^+e^- \to e^+e^-$ (Bhabha scattering)
  [KLOE, CMD-2 and SND, BES, CLEO-c, BaBar]



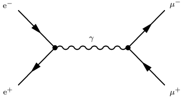
$$|M|^2 \propto \alpha^2 \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} + \frac{2u^2}{ts} \right)$$

- $e^+e^- \to \gamma\gamma$ [KLOE, CLEO-c, BaBar, BES-III]



$$|M|^2 \propto \alpha^2 \left(\frac{u}{t} + \frac{t}{u} \right)$$

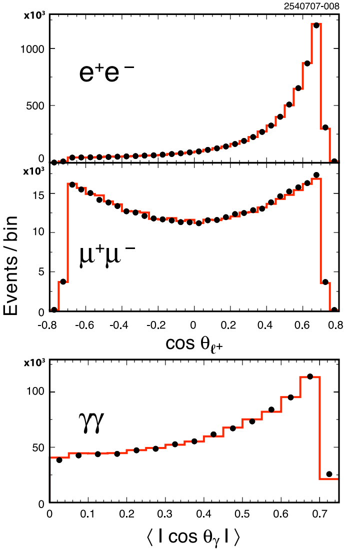
- $e^+e^- \to \mu^+\mu^-$ [CLEO-c, BaBar]



$$|M|^2 \propto \alpha^2 \frac{t^2+u^2}{s^2}$$

Experimental luminosity errors: from Φ to $B-$factories

S. Dobbs *et al.*, [CLEO–c Coll.], Phys. Rev. **D76** (2007) 112001

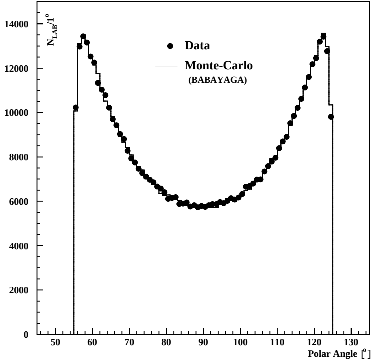


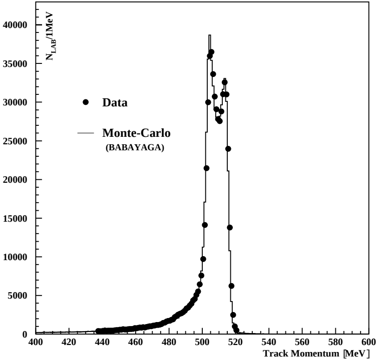
Using wide angles selection cuts

- Bhabha scattering
  - KLOE: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} = 0.3\%$
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1\%$
  - BES-III: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim \text{few } 0.1\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} = 0.7\%$
- $\gamma\gamma$ production
  - KLOE: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim \text{few } 0.1\%$
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1.5\%$
- $\mu^+\mu^-$ production
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 0.8\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 0.5\%$

KLOE Bhabha data vs BabaYaga v3.5/BabaYaga@NLO

F. Aloisio *et al.*, [KLOE Coll.], Phys. Lett. **B606** (2005) 12





$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.5\%\,({\rm th\ BabaYaga\,v3.5}) = 0.6\%\ [\text{as of 2006}]$$

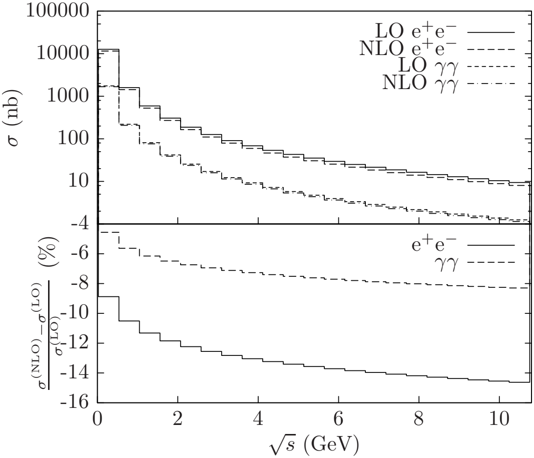
F. Ambrosino *et al.*, [KLOE Coll.], Eur. Phys. J. **C47** (2006) 589

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.1\%\,({\rm th\ BabaYaga@NLO}) = 0.3\%\ [\text{now!}]$$

F. Ambrosino *et al.*, [KLOE Coll.], `arXiv:0707.4078 [hep-ex]`

NLO corrections to e^+e^- and two–photon production

Bhabha and $\gamma\gamma$ production cross section as a function of the c.m. energy



- ⋆ NLO corrections range from several per cent from $\Phi-$factories to about 10–15% at the $B-$factories
- The corrections to $\gamma\gamma$ production are about one half of those to Bhabha, for comparable cuts

Matching NLO and higher–order corrections

C.M. Carloni Calame *et al.*, Nucl. Phys. **584** (2000) 459 & Nucl. Phys. Proc. Suppl. **131** (2004) 48
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 & Phys. Lett. **663** (2008) 209 [BabaYaga@NLO]

Exact NLO soft+virtual (SV) corrections and hard bremsstrahlung (H) matrix elements can be combined with the QED PS through a matching procedure

- $d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{LL}^{SV}(\varepsilon) + d\sigma_{LL}^{H}(\varepsilon)$
- $d\sigma_{\rm exact}^{\alpha} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 \equiv d\sigma_{\rm exact}^{SV}(\varepsilon) + d\sigma_{\rm exact}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

$$d\sigma_{\rm matched}^{\infty} = F_{SV} \, \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$$

in such a way that

★ $[\sigma_{\rm matched}^{\infty}]_{\mathcal{O}(\alpha)} = \sigma_{\rm exact}^{\alpha}$, avoiding double counting and preserving exponentiation of $\alpha^n L^n, n \geq 2$ leading logs

Large–angle Bhabha: size of the radiative corrections

for bare (w/o photon recombination) $e^{\pm}$ final–states

Event selection criteria: for $\phi-$ and B–factories

- a $\sqrt{s} = 1.02\,\mathrm{GeV},\ E^{\pm}_{\mathrm{min}} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\mathrm{max}} = 10^\circ$
- b $\sqrt{s} = 1.02\,\mathrm{GeV},\ E^{\pm}_{\mathrm{min}} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\mathrm{max}} = 10^\circ$
- c $\sqrt{s} = 10\,\mathrm{GeV},\ E^{\pm}_{\mathrm{min}} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\mathrm{max}} = 10^\circ$
- d $\sqrt{s} = 10\,\mathrm{GeV},\ E^{\pm}_{\mathrm{min}} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\mathrm{max}} = 10^\circ$

Relative corrections (in %)

setup	a.	b.	c.	d.
$\delta_\alpha^{\mathrm{exact}}$	-10.00	-12.52	-12.00	-14.43
$\delta_\alpha^{\mathrm{non-log}}$	-0.40	-0.65	-0.41	-0.70
δ_{HO}	0.39	0.93	0.80	1.64
$\delta_{\alpha^2 L}$	0.04	0.09	0.06	0.11
δ_{VP}	1.73	2.43	4.59	6.03

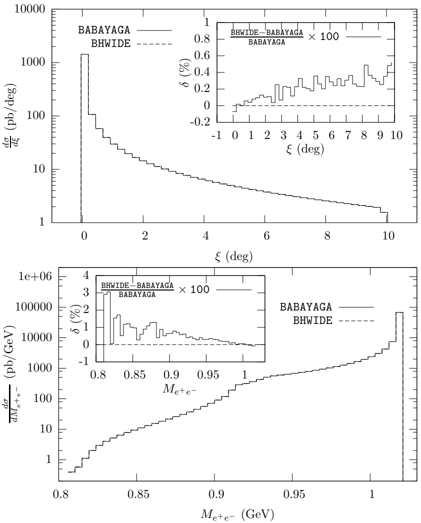
★ Both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarisation) necessary for 0.1% theoretical precision ★

- Vacuum polarisation included in both lowest–order and NLO diagrams with $\Delta\alpha^{(5)}_{\mathrm{had}}$ contribution through a parameterization routine (Jegerlehner, HMNT, ...), returning a data driven error estimate

F. Jegerlehner, Nucl. Phys. Proc. Suppl. **126/181-182** (2004/2008) 325/135
K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, PR **D69** (2004) 093003 and PL **B649** (2007) 173

BabaYaga@NLO vs BHWIDE at DAΦNE

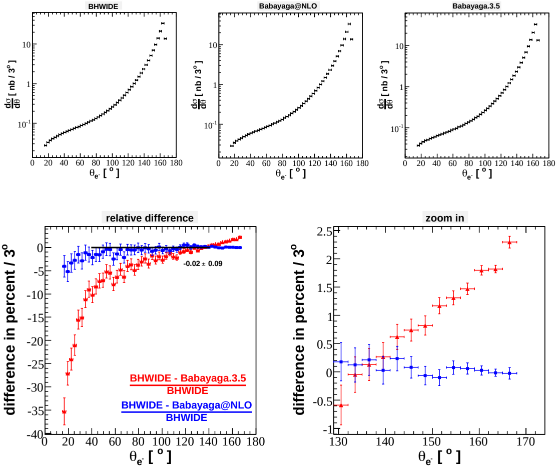
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227



- Agreement for distributions within a few 0.1%, a few % only in the dynamically suppressed hard tails

BabaYaga@NLO vs BHWIDE at BaBar

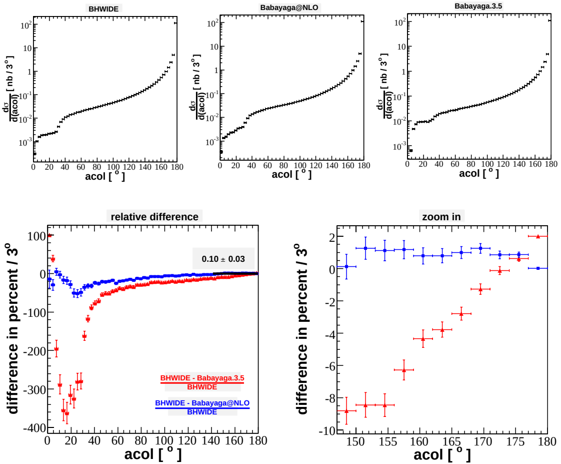
By A. Hafner and A. Denig
with realistic selection cuts for luminosity at BaBar



- `BabaYaga@NLO` and `BHWIDE` well agree (at a few per mille level) also for distributions

BabaYaga@NLO vs BHWIDE at BaBar

By A. Hafner and A. Denig
with realistic selection cuts for luminosity at BaBar

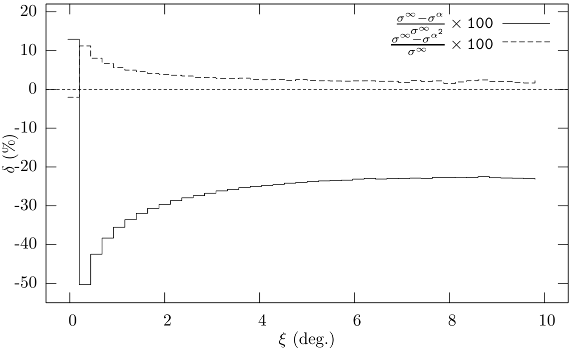


- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for distributions

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

Even with a complete two–loop generator at hand, resummation of leading logarithms beyond $\mathcal{O}(\alpha^2)$ could be neglected?

Bhabha cross section as a function of the acollinearity ξ @ DAΦNE



- Resummation beyond $\mathcal{O}(\alpha^2)$ important for precision predictions!

The $e^+e^- \to \gamma\gamma$ process: size of radiative corrections and accuracy

G. Balossini *et al.*, Phys. Lett. **B663** (2008) 209

Selection criteria – ϕ, τ–charm and B factories

a $\sqrt{s} = 1, 3, 10\,\mathrm{GeV}$, $E_{\text{min}} = 0.3\sqrt{s}$, $\vartheta_\gamma^{\text{min,max}} = 45^\circ \div 135^\circ$, $\xi_{\text{max}} = 10^\circ$

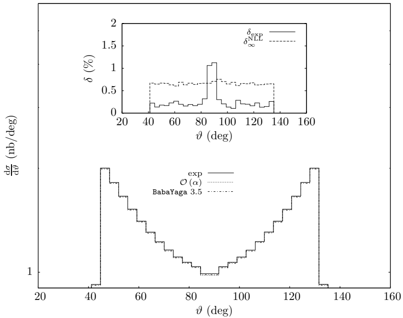
Cross sections (nb) & relative corrections (%)

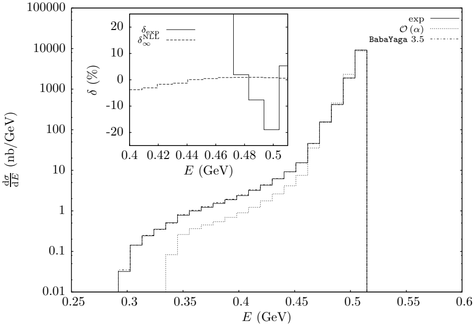
$\sqrt{s}$ (GeV)	1	3	10
$\sigma_{\rm Born}$	137.53	15.281	1.3753
$\sigma_\alpha^{\rm PS}$	128.55	14.111	1.2529
$\sigma_{\rm NLO}$	129.45	14.211	1.2620
$\sigma_{\rm exp}^{\rm PS}$	128.92	14.169	1.2597
$\sigma_{\rm matched}$	129.77	14.263	1.2685
δ_α	-5.87	-7.00	-8.24
δ_∞	-5.65	-6.66	-7.77
$\delta_\alpha^{\rm non\text{–}log}$	0.70	0.71	0.73
$\delta_{\rm HO}$	0.24	0.37	0.51

- Like for Bhabha, both exact $\mathcal{O}(\alpha)$ and higher–order corrections necessary for 0.1% theoretical precision in $\gamma\gamma$ production ⋆

⋆ Theoretical accuracy: $\sim 0.1\%$, also thanks to no contribution (and related $\Delta\alpha_{\rm had}^{(5)}$ uncertainty) due to vacuum polarisation correction

Angular and energy distribution of the most energetic photon





⋆ Interplay of NLO and multiple photon corrections also necessary for precise simulations of $\gamma\gamma$ differential cross sections

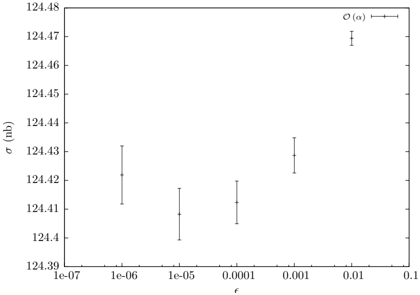
$e^+e^- \to \gamma\gamma(n\gamma)$ in BabaYaga@NLO: technical tests

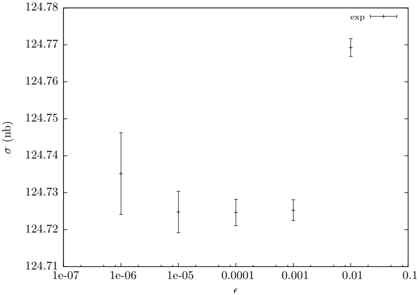
- Perfect agreement with BKQED for the $\mathcal{O}(\alpha)$ [NLO] corrections to the inclusive $e^+e^- \to \gamma\gamma(\gamma)$ cross section

F.A. Berends and R. Kleiss, Nucl. Phys. **B186** (1981) 22

$\sqrt{s}$(GeV)	6	10	20
$\delta_T^{\mathrm{BKQED}}$(%)	13.8	15.3	17.4
$\delta_T^{\mathrm{BabaYaga@NLO}}$(%)	13.81(1)	15.30(1)	17.51(10)

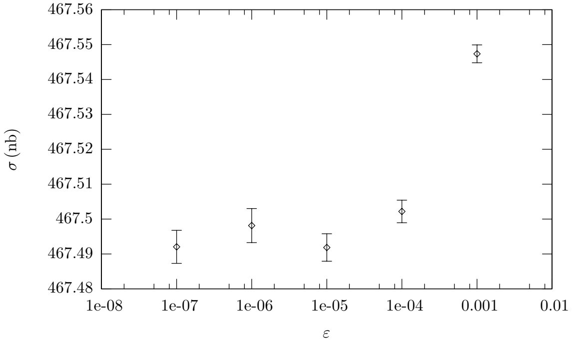
- Successful independence from the soft–hard photon separator ϵ, in the numerical limit $\epsilon \to 0$





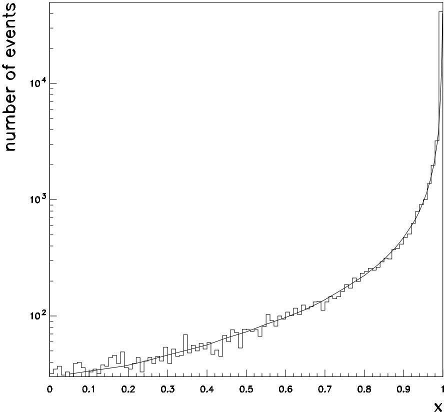
Technical test of BabaYaga@NLO: ϵ independence

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227



- Independence of the matched PS cross section from variations of the soft–hard separator ϵ successfully checked! [for large–angle Bhabha cross section @ DAΦNE]

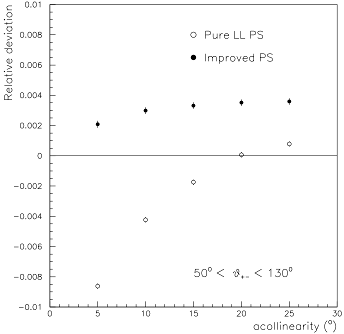
C.M. Carloni Calame *et al.*, Nucl. Phys. **B584** (2000) 459

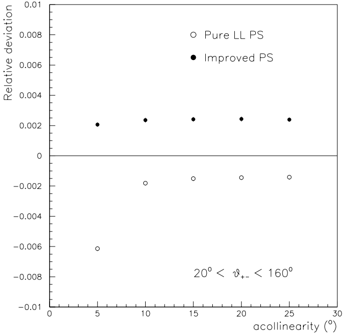


- Parton Shower reconstruction (histogram) of the x distribution of the electron Structure Function $D(x, Q^2)$ (solid line)

Theoretical accuracy of BabaYaga v3.5

C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16



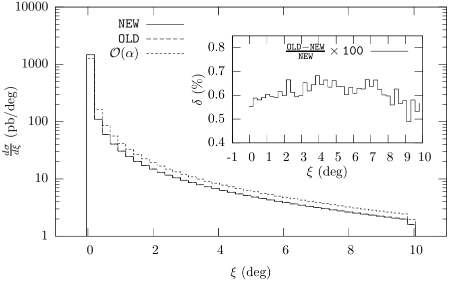


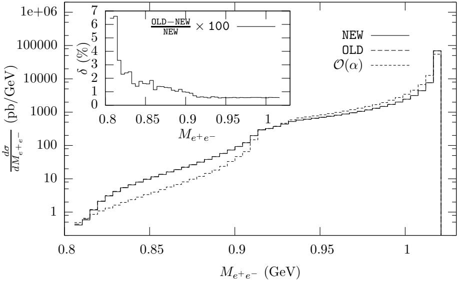
- Relative difference between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ Bhabha cross section, as a function of the acollinearity cut, for two angular acceptances at $\sqrt{s} = 1$ GeV

BabaYaga@NLO vs BabaYaga v3.5 at DAΦNE

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

$$\sqrt{s} = 1.02\,\text{GeV}, \;\; E^{\pm}_{\text{min}} = 0.408\,\text{GeV}, \;\; \vartheta_{\mp} = 55^\circ \div 125^\circ, \;\; \xi_{\text{max}} = 10^\circ$$

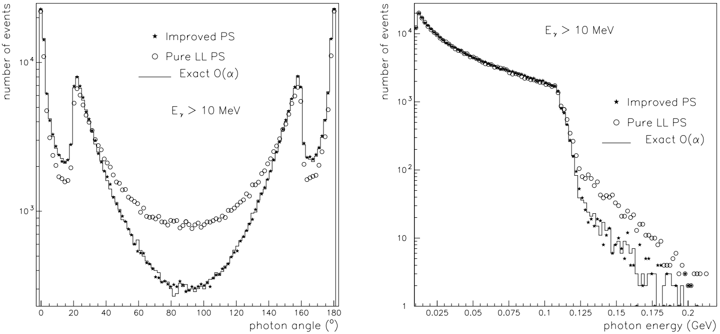




- BabaYaga@NLO differs from BabaYaga v3.5 at ~ 0.5 % level in the statistically dominant regions for luminosity monitoring at the Φ–factories, due to $\mathcal{O}(\alpha)$ non–log contributions
- Higher–order [beyond $\mathcal{O}(\alpha)$] leading log corrections amount to several per cent on distributions and are essential for precision luminosity studies

Improved PS algorithm in BabaYaga v3.5

C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16



- Comparison between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ matrix element for the angular and energy photon distributions