# Status and Comparison of VP Compilations

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II. Status as of our WG report. News since then

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# I. Introduction. $\Delta\alpha_{\rm had}$

- Why Vacuum Polarisation / running $\alpha$ corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

— Corrections for data used as input for $g-2$: 'undressed' $\sigma^0_{\rm had}$

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\,\sigma^0_{\rm had}(s)K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

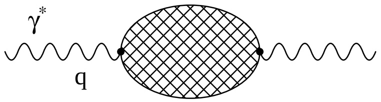
— Determination of $\alpha_s$ and quark masses from total hadronic cross section $R_{\rm had}$ at low energies and of resonance parameters.

— Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.

— $\alpha(M_Z^2)$ a fundamental parameter at the $Z$ scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.

$\longrightarrow$ Ingredient in MC generators for many processes.

- Photon Vacuum Polarisation (VP) a quantum effect which leads to the running of the renormalised (effective) QED coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$.
- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi\ =$ 


Full photon propagator $\sim\ 1\ +\ \Pi\ +\ \Pi\cdot\Pi\ +\ \Pi\cdot\Pi\cdot\Pi\ +\ \ldots$

$$\rightsquigarrow \qquad \alpha(q^2)\ =\ \frac{\alpha}{1-\mathrm{Re}\Pi(q^2)}$$

- Effect from both leptonic and hadronic loops;
  - leptonic VP calculable in Perturbation Theory,
  - hadronic VP receives contributions from non-perturbative sector

  $\rightsquigarrow$ calculation via dispersion integral using experimental $\sigma_{\rm had}(e^+e^- \to hadrons)$:

$$\alpha(q^2)\ =\ \alpha\,/\,\left(1-\Delta\alpha_{\rm lep}(q^2)-\Delta\alpha_{\rm had}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\rm had}(q^2) \;=\; -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\rm had}(s)\,\mathrm{d}s}{s-q^2}\,, \quad \sigma_{\rm had}(s) = \frac{\sigma^0_{\rm had}(s)}{|1-\Pi|^2}$$

[$\rightarrow$ $\sigma^0$ requires ‘undressing’, e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections $\sigma_{\rm had}$ contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$, including the Imaginary part, $\Pi = e^2(P+iA)$.

- However, (formally) the Imaginary part is suppressed by $e^2$ w.r.t. the Real part:

$$|1+e^2(P+iA)+e^4(P+iA)^2+\ldots|^2 \;=\; 1+e^2\,2P+e^4\,(3P^2-A^2)+e^6\,4P(P^2-A^2)+\ldots$$

To account for $\mathrm{Im}\Pi$ we can use the summed form: $\dfrac{1}{|1-e^2(P+iA)|^2} \;\equiv\; \dfrac{1}{|1-\Pi|^2}$

- Note:

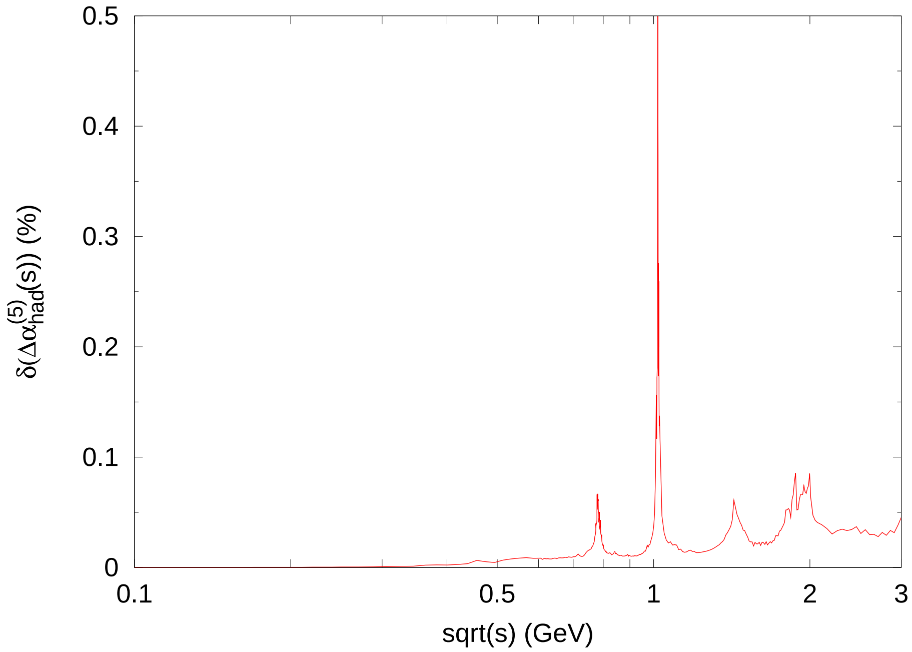
  — At narrow resonance energies, if $|\Pi| \sim 1$, the summation breaks down

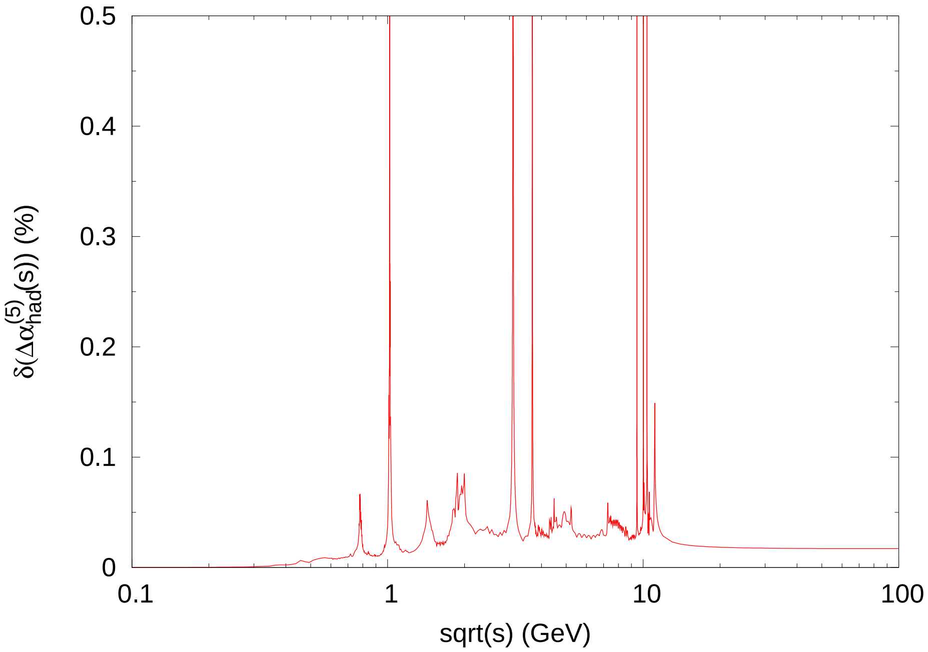
  $\rightsquigarrow$ need other formulation, e.g. Breit-Wigner resonance propagator.

  — Summation of bubbles covers only the class of graphs (factorisable blobs); does *not* cover other diagrams (e.g. interference with ISR).

- **Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$**

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):




$\longrightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances
where the bubble summation is not well justified.

Enough in the long term? Need for more work in resonance regions.

## Narrow resonances

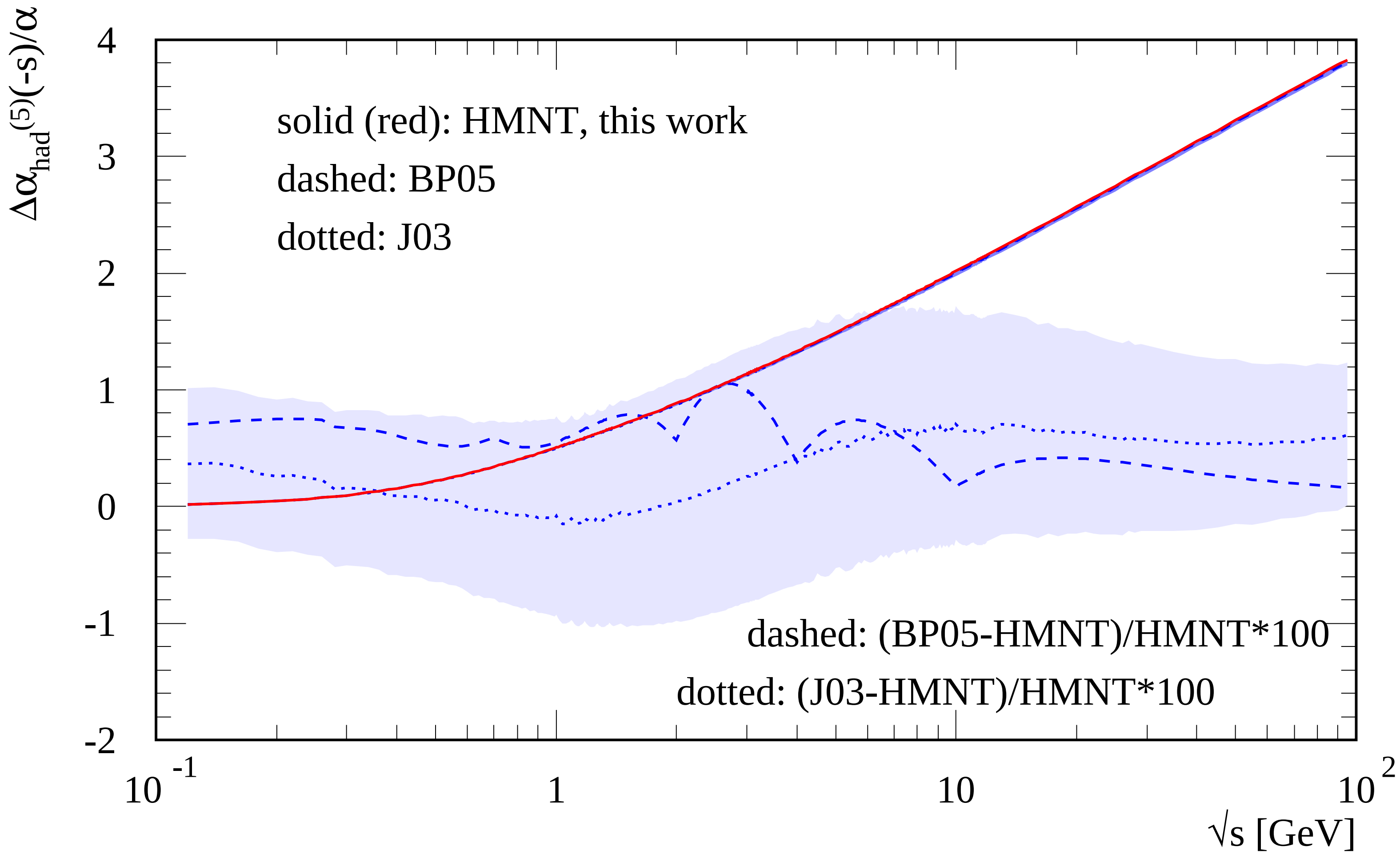
- For the $\omega$ and $\phi$ resonances the data is suitable for direct integration, avoiding parametrisation ambiguities/uncertainties.
  The same is true for the higher charm excitations, $\psi(3770, 4040, 4160, 4415)$.
- For $J/\psi$, $\psi'$ and the $\Upsilon$ family $(1-6S)$ one can easily calculate their contributions to ($g-2$ and) $\Delta\alpha$ through the Narrow width approximation or via a Breit-Wigner parametrisation. Can do better for open $b\bar{b}$ resonance region..
- However close to resonance the summation in an effective coupling breaks down, signalled by a very large correction.
- Also need to take care of properly undressing the electronic widths $\Gamma_{ee}$:
  - Using the dressed width would be inconsistent and introduce sizeable effects (a few percent), undressing via the smooth spacelike running $\alpha$ comes closer numerically but is not fully correct.
  - Use of NR undressing formula

$$\Gamma_{ee}^{V,\,0} \;=\; \frac{\left[\alpha/\alpha_{\mathrm{no\,V}}(M_V^2)\right]^2}{1+3\alpha/(4\pi)}\,\Gamma_{ee}^{V}$$

## II. Status as of our WG report. News since then

- Three parametrisations available:
  - CMD-2 Novosibirsk (Fedor Ignatov's thesis and web-page)
  - Fred Jegerlehner's `hadr5n.f` version 2003 from his web-page
    $\longrightarrow$ for EPJC published version one figure replaced by newer (Feb. 2010) version `hadr5n09.f`
  - HMNT (now HLMNT, from authors upon request)
- Still no dedicated recent publications for codes :(
- In addition: Burckhardt/Pietrzyk (BP05, from their web-page), 'easy' spacelike only
- Davier *et al.*? (Use their own code, but so far not made available)

- Comparison of Spacelike $\Delta\alpha^{(5)}_{\rm had}(-s)/\alpha$ (smooth $\alpha(q^2 < 0)$)




— Differences between parametrisations clearly visible but within error band (of HLMNT)

— Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.

— Encourage use of more accurate recent tabulations; $\boldsymbol{\Delta\alpha(M_Z^2)}$

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine still available from his web-page)

$$\alpha(s) = \alpha \,/\, \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

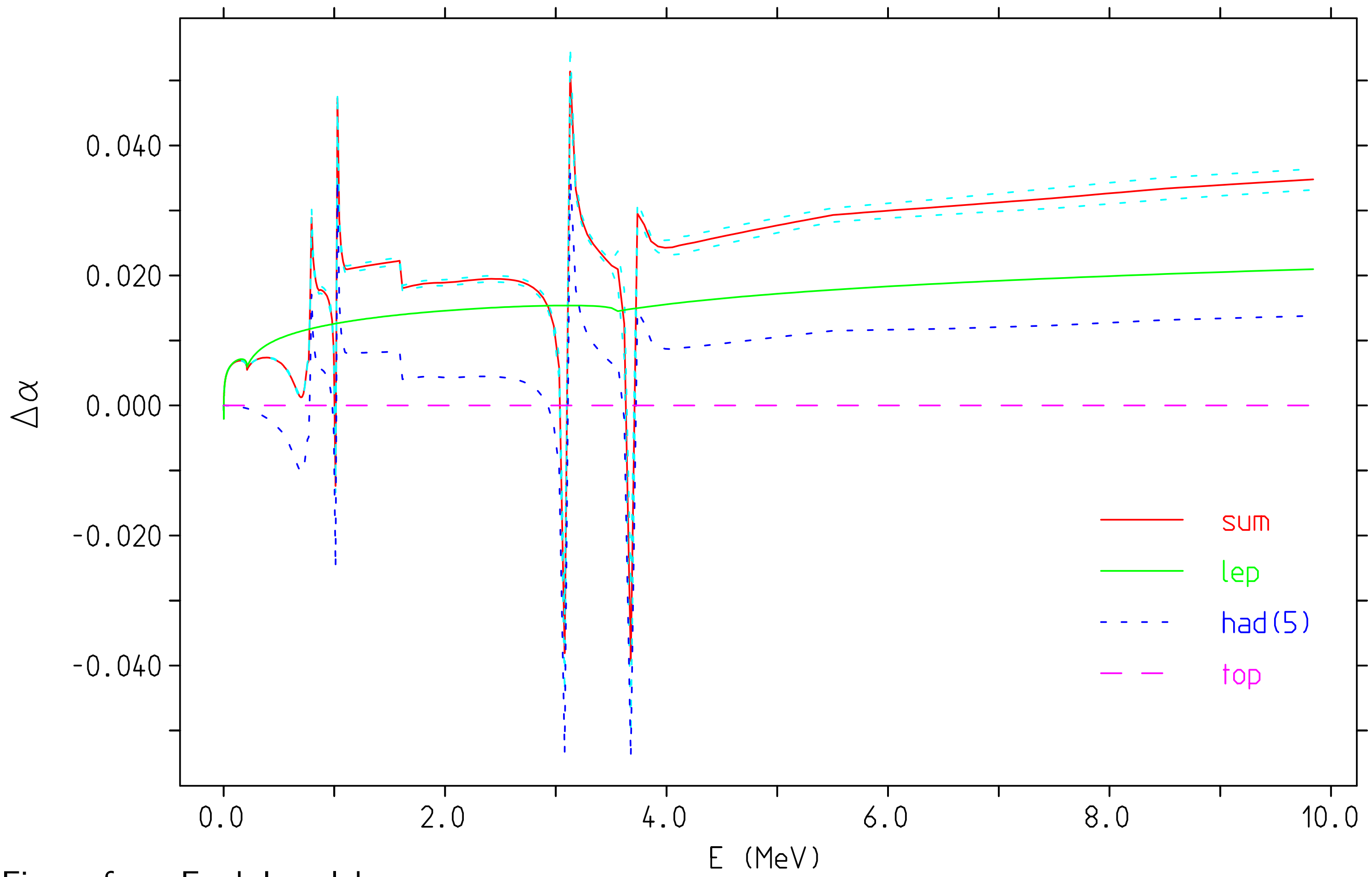
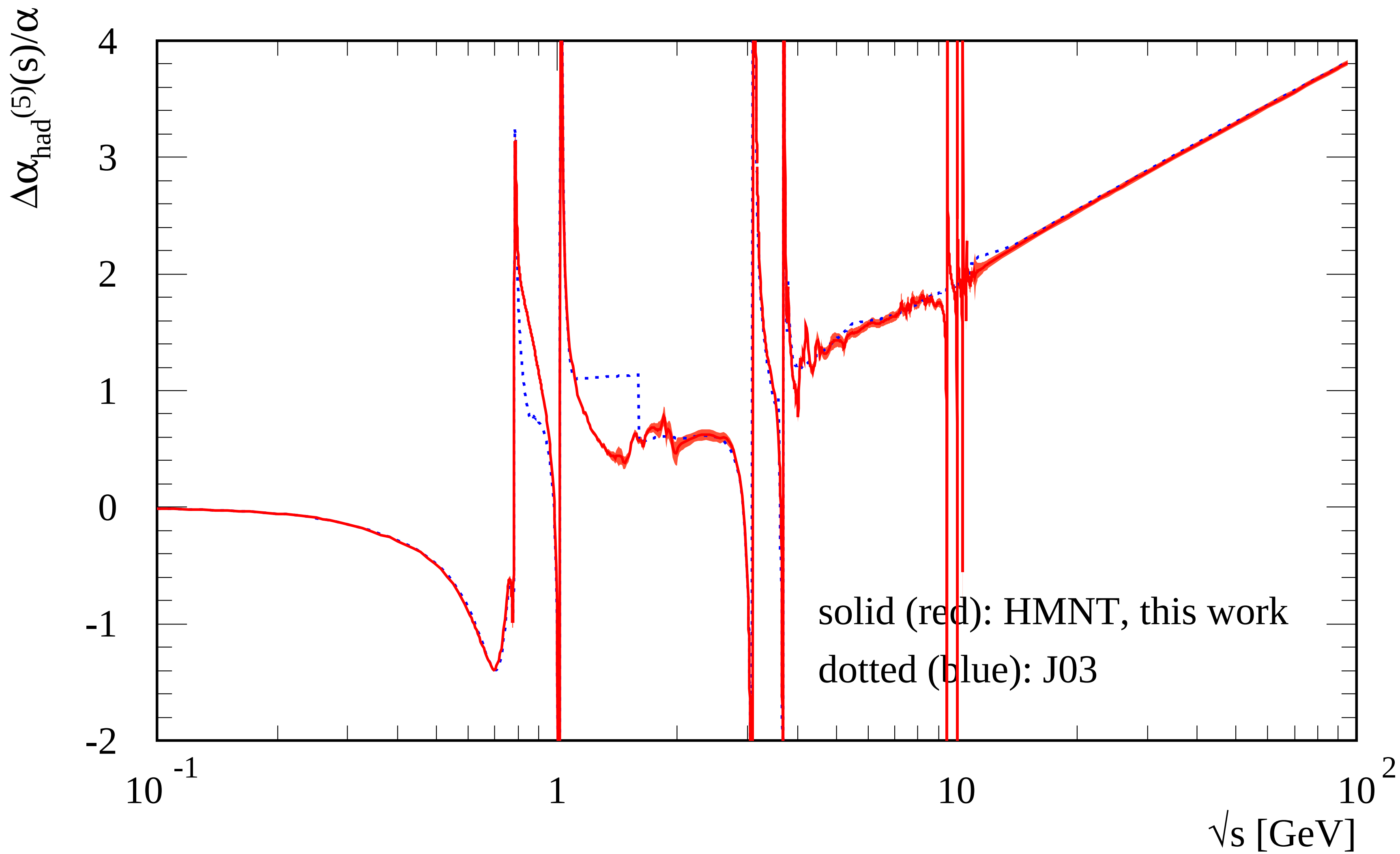



Figure from Fred Jegerlehner

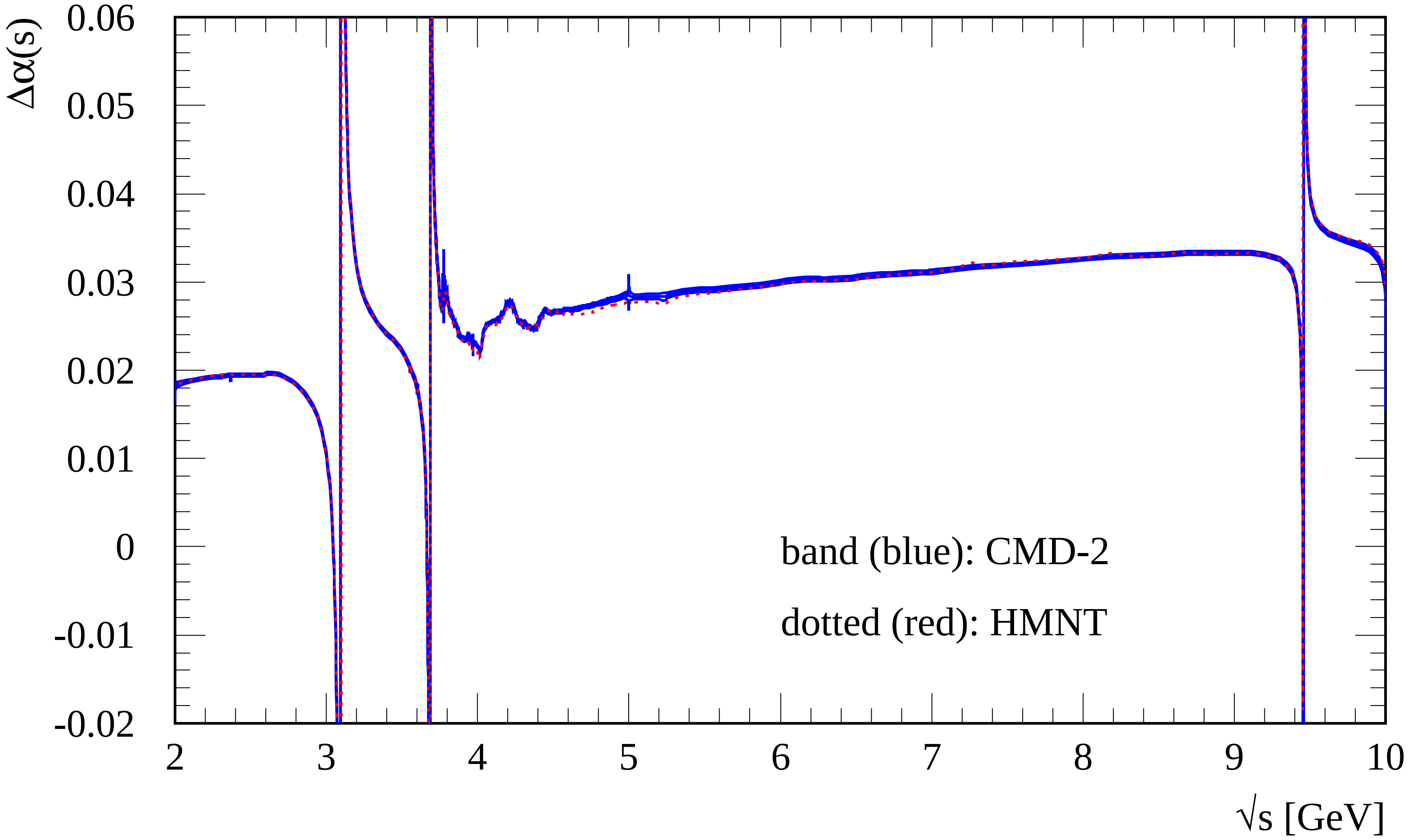
Timelike $\alpha(q^2 > 0)$ follows resonance structure:



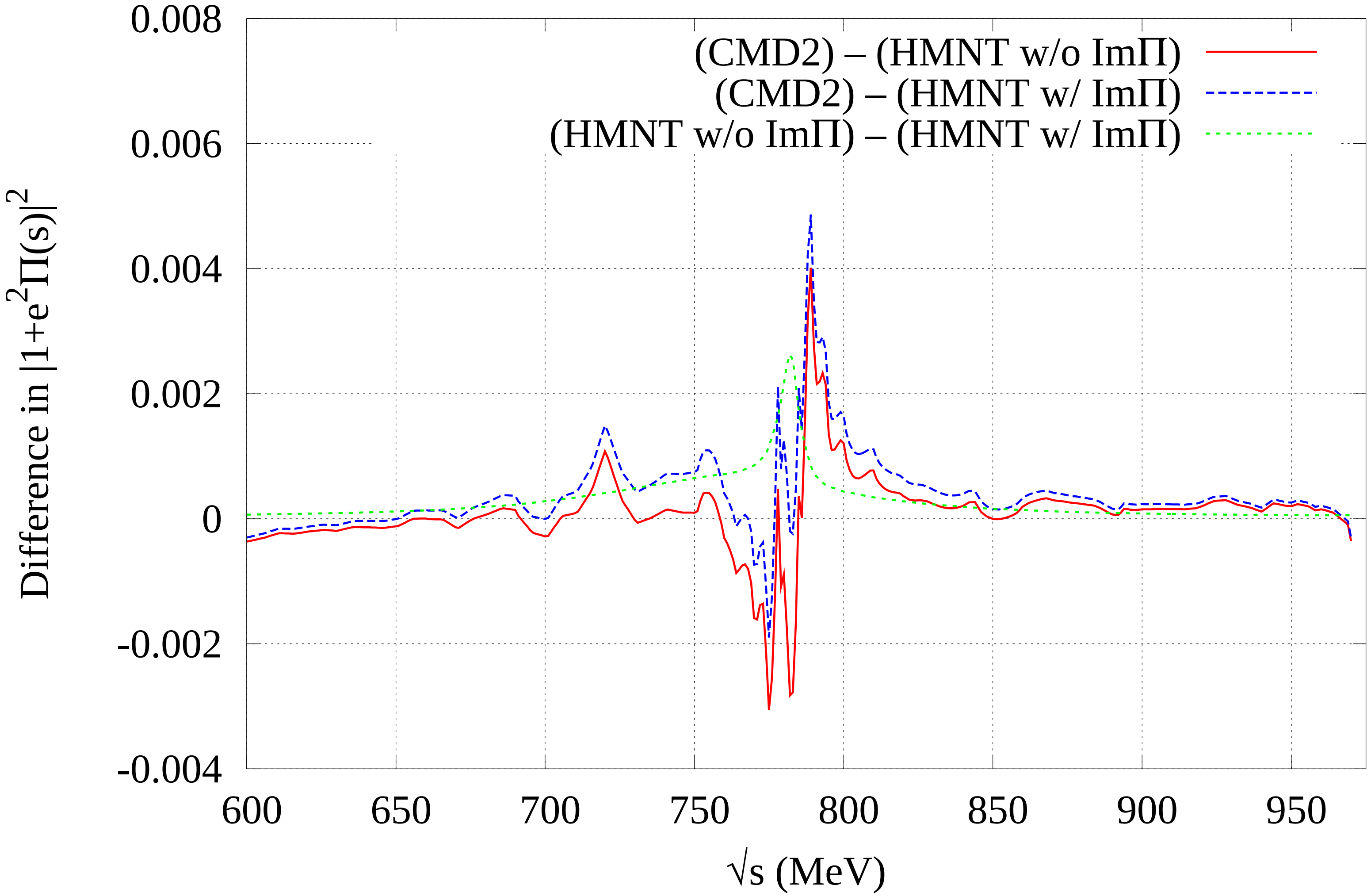

— Step below $1.4$ GeV was a feature of unfortunate grid in much used 2003 version

— Difference below 1 GeV also not expected from data and other analyses;
different in new version, see below

- HLMNT compared to Novosibirsk   Timelike, $\Delta\alpha(q^2)$



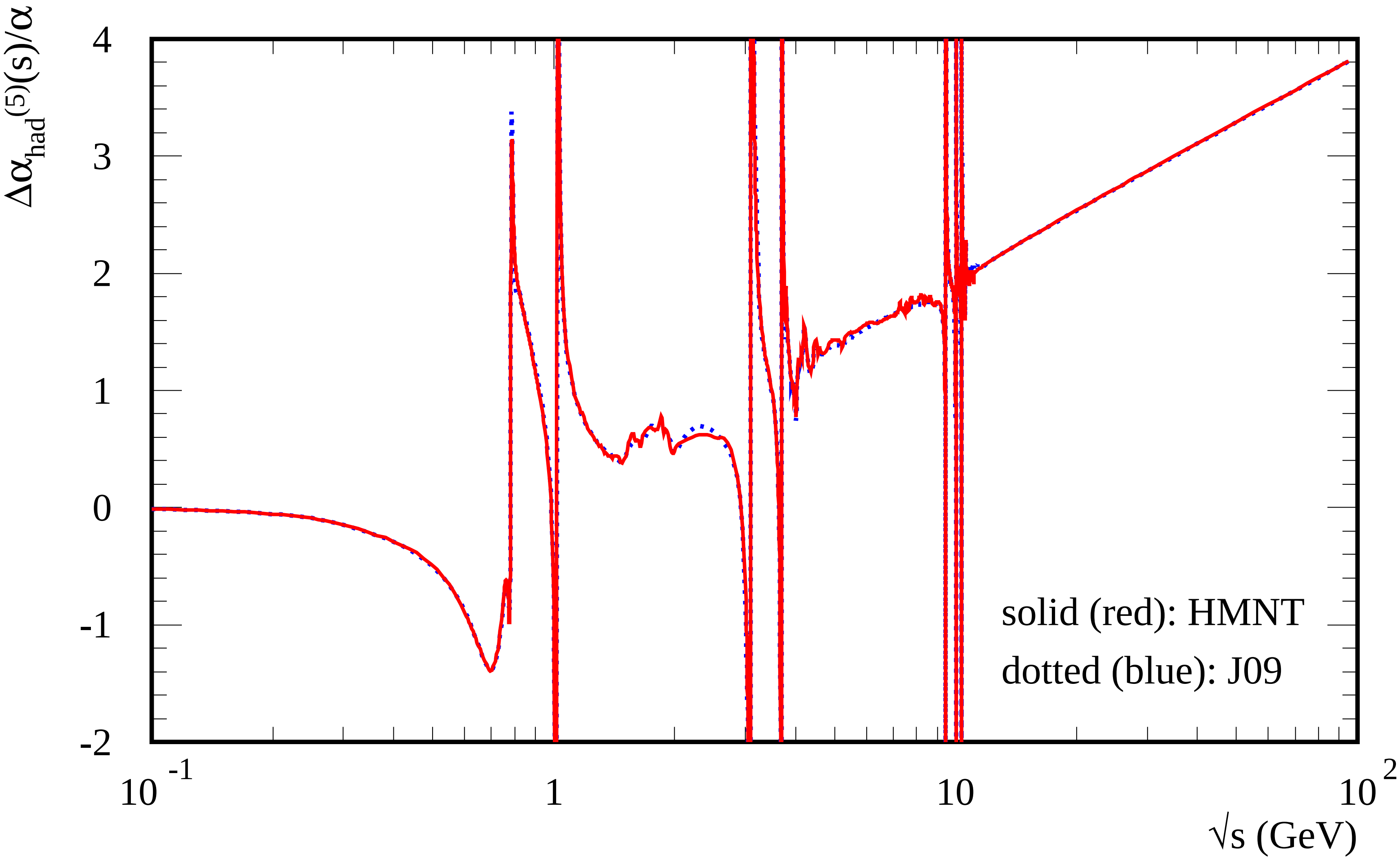
- HLMNT compared to Novosibirsk    Timelike, differences in $|1 - \Pi(s)|^2$




$\longrightarrow$ Differences of about one per-mille in the 'undressing' factor, up to -3/+5 per-mille in the $\rho - \omega$ interference regime, but likely to cancel at least partly in applications.

$\longrightarrow$ As expected small negative contribution from $\mathrm{Im}\Pi$.

- HLMNT compared to Fred Jegerlehner's new version




– As expected main 'puzzles' now gone.

– For smaller differences see below.

- Features of HLMNT code

– Latest version is `VP_HLMNT_v2_0`, version 2.0, 15 July 2010

– Simple set of (standard) Fortran routines; completely standalone, no libs needed; all explanations in comment-headers

– Gives separately real and imaginary part ($\Delta\alpha(s)$ and $R(s)$)

– Tabulation/interpolation of hadronic part, for both space- and time-like region, including errors; no input data files or `rhad` installation needed

– Leptonic part coded analytically; all special function included (partly with custom made expansions)

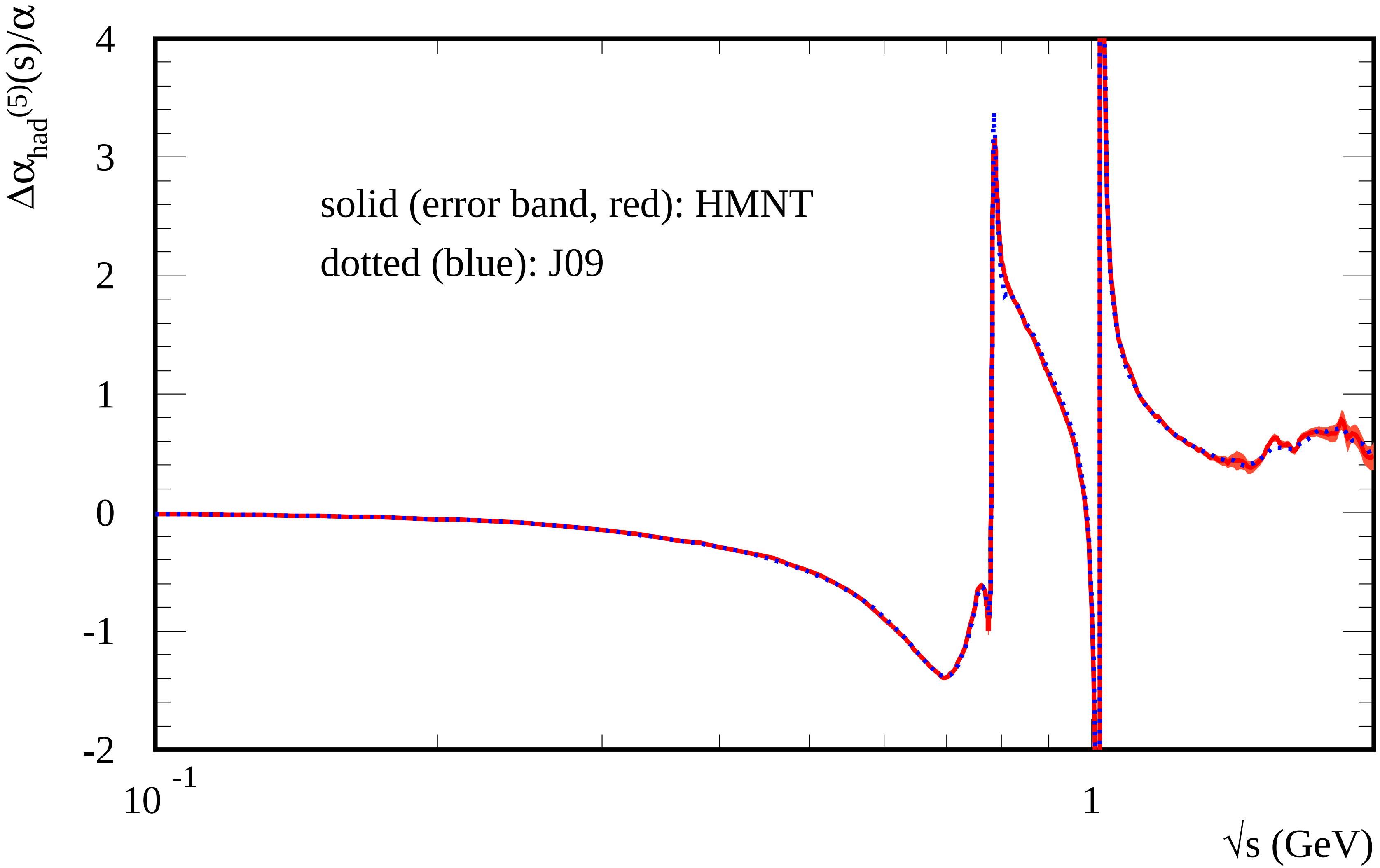
– top contribution in the same way

→ Flag to include or exclude very narrow resonances $\boldsymbol{J/\psi}$, $\boldsymbol{\psi'}$, $\boldsymbol{\Upsilon(1-6\,S)}$
[$\phi$ always included via integral over final state data ($3\pi$, $KK$)]

# III. More comparisons

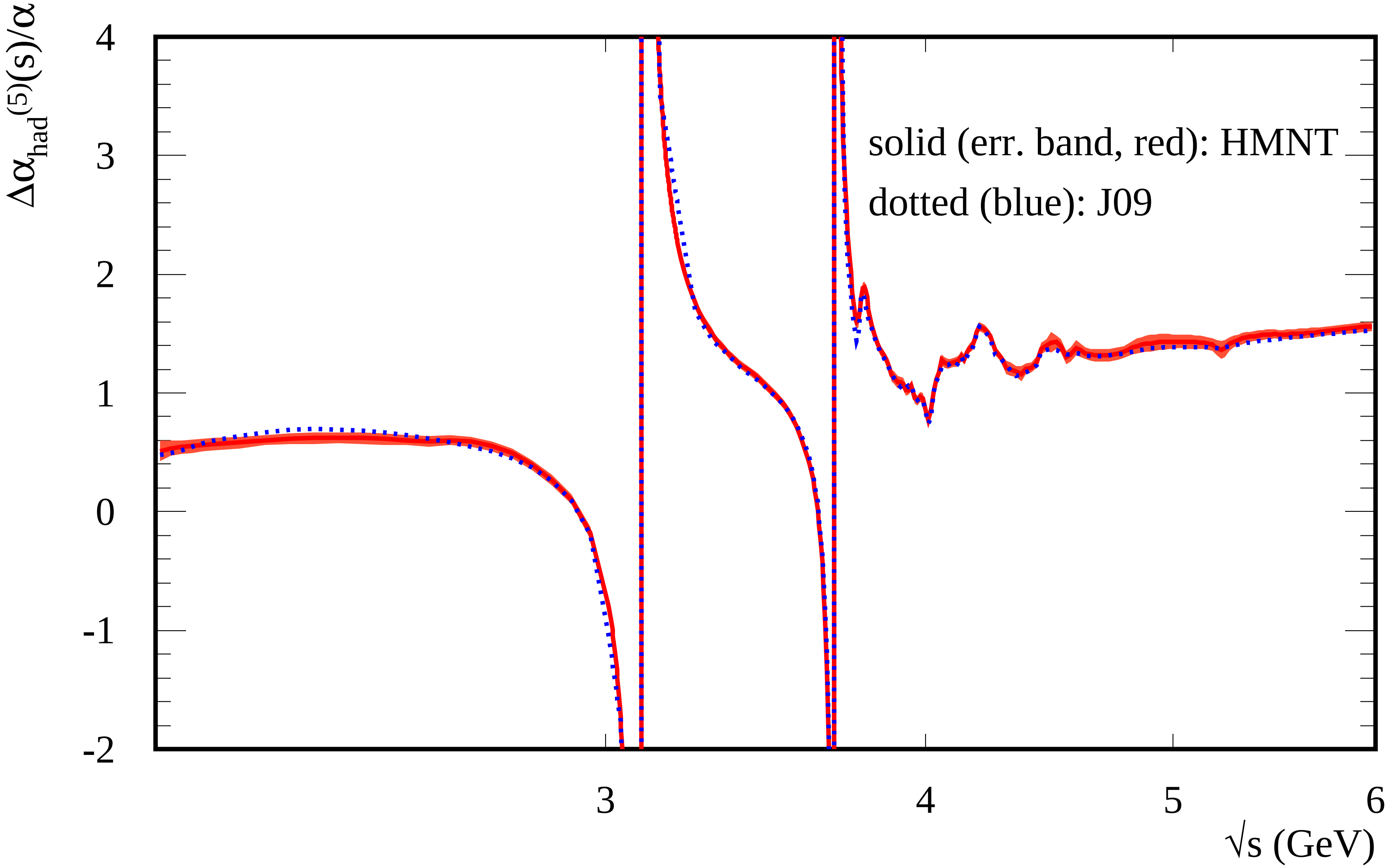
- HLMNT compared to Fred Jegerlehner's new version: Detailed look

Low energies: $\rho$ and $\phi$



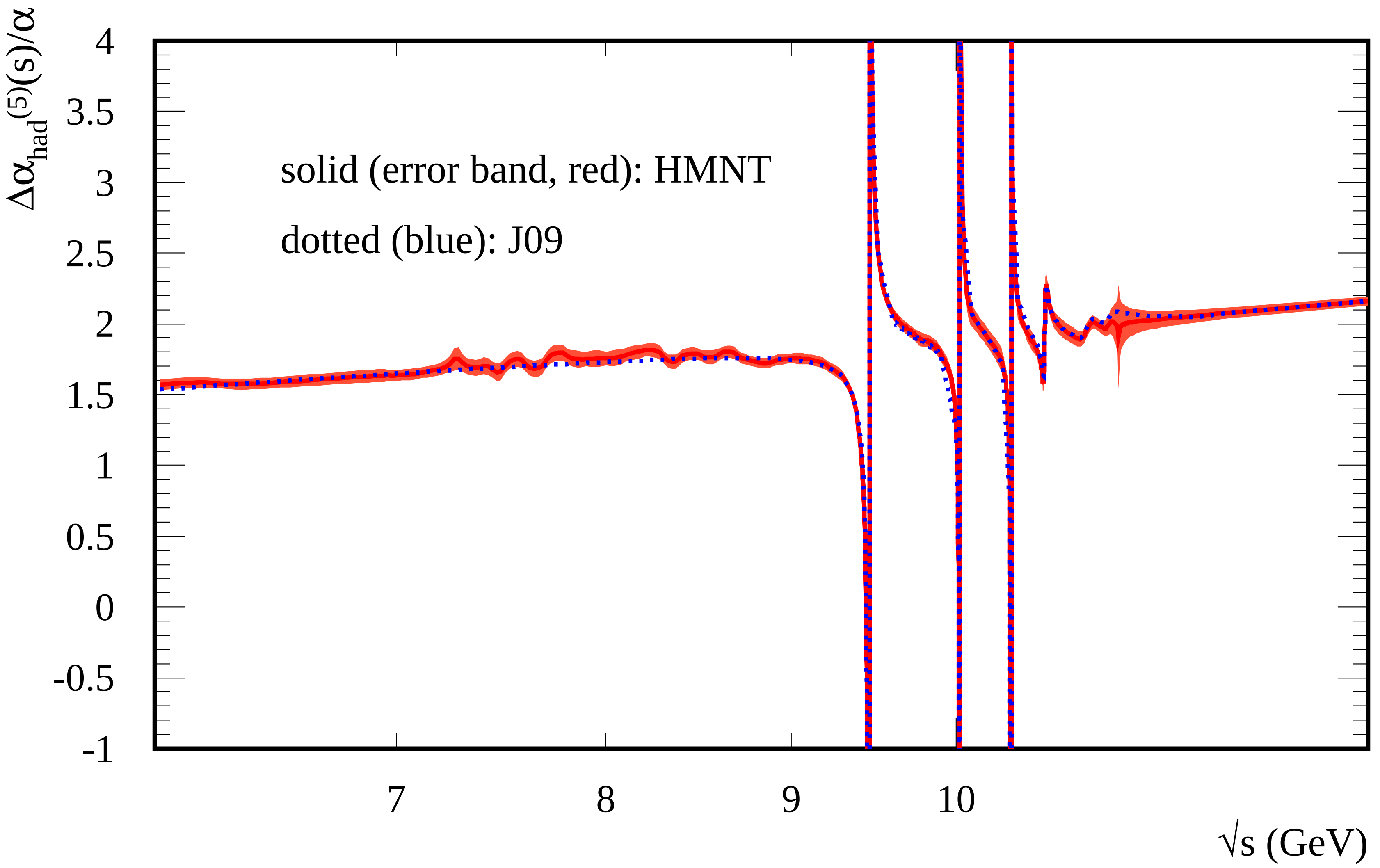
- HLMNT compared to Fred Jegerlehner's new version: Detailed look

Medium energies: continuum and charm



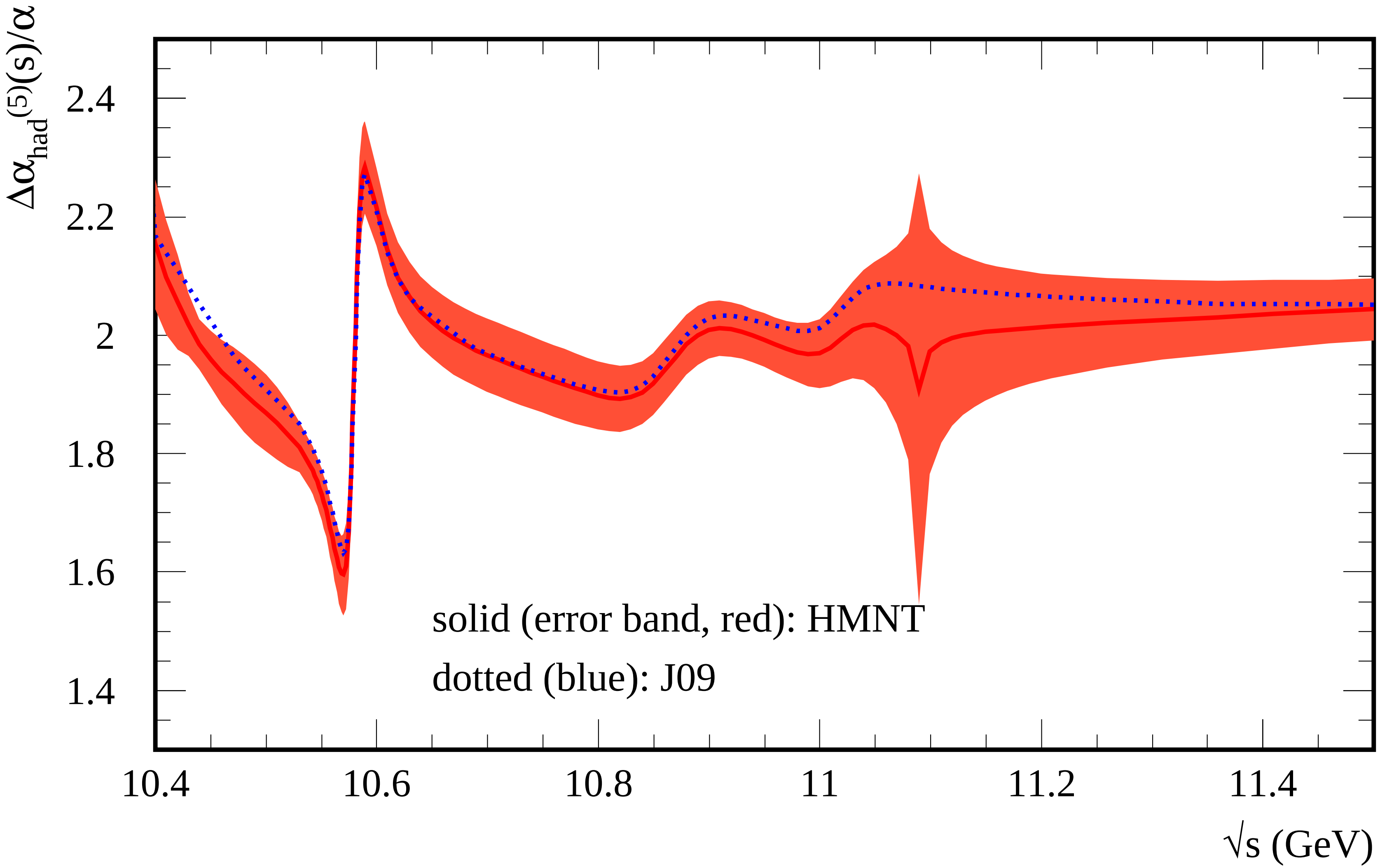
- HLMNT compared to Fred Jegerlehner's new version: Detailed look

Higher energy continuum; bottom



- HLMNT compared to Fred Jegerlehner's new version: Detailed look

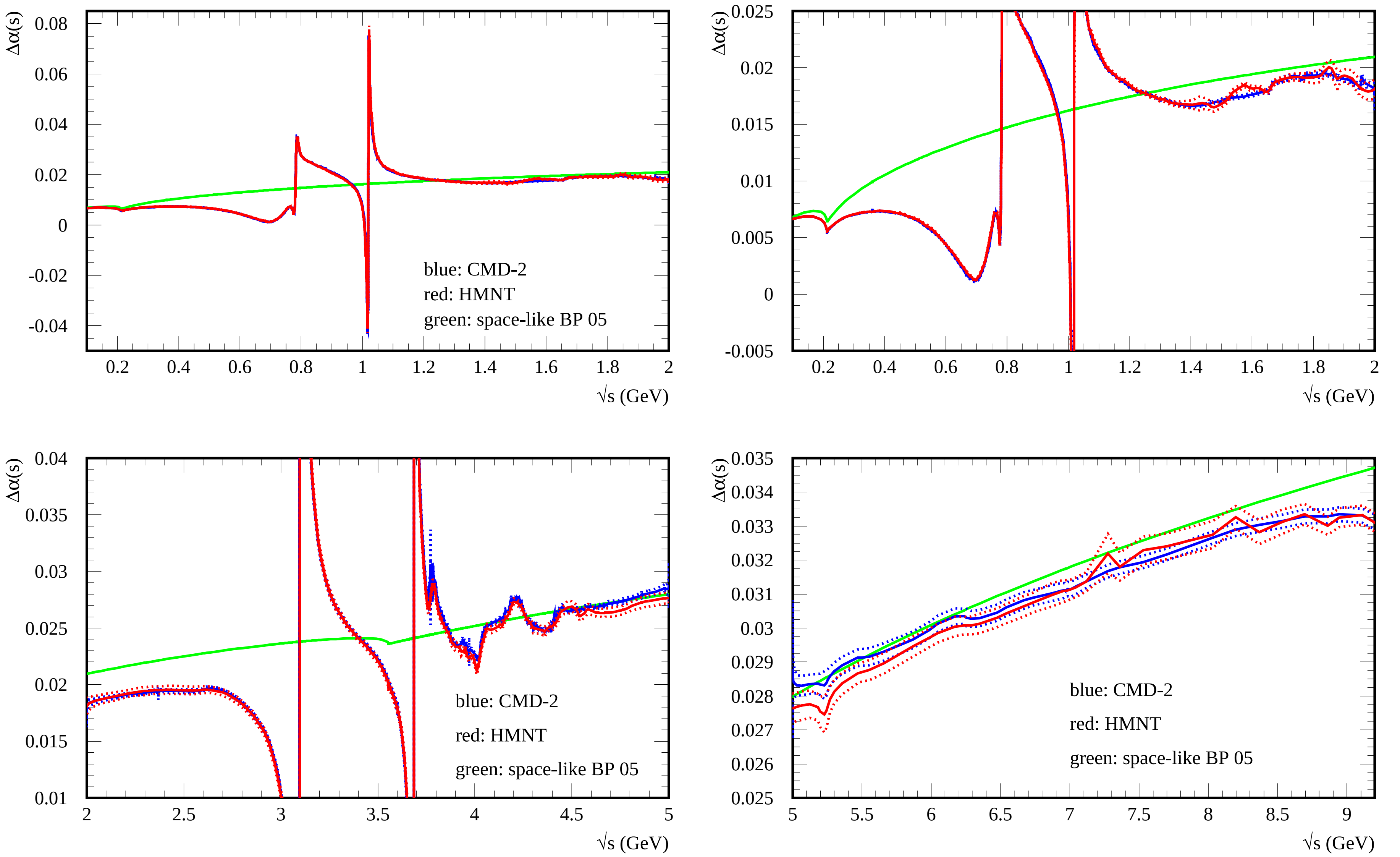
Details of higher $\Upsilon(4,5,6\,S)$ [10580, 10860, 11020] / open bottom region



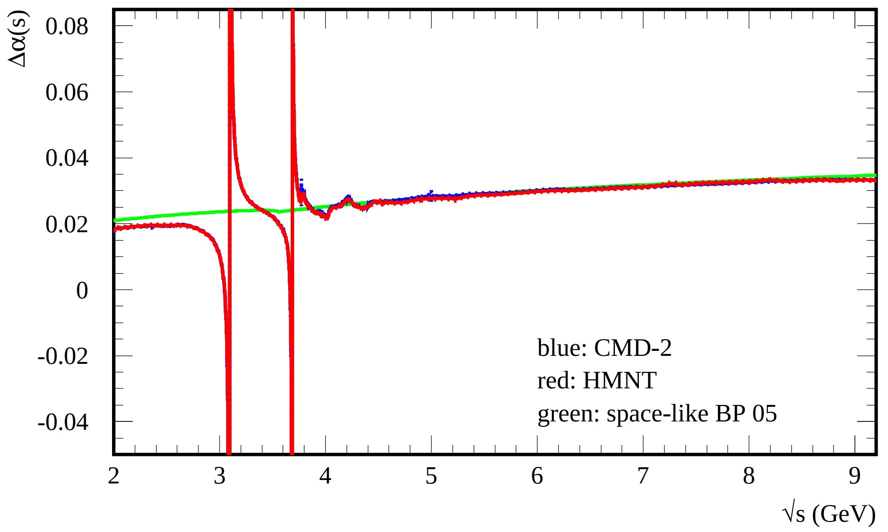

$\longrightarrow$ HLMNT still to include BaBar's $R_{b\bar{b}}$ data; ISR unfolding.. work in progress

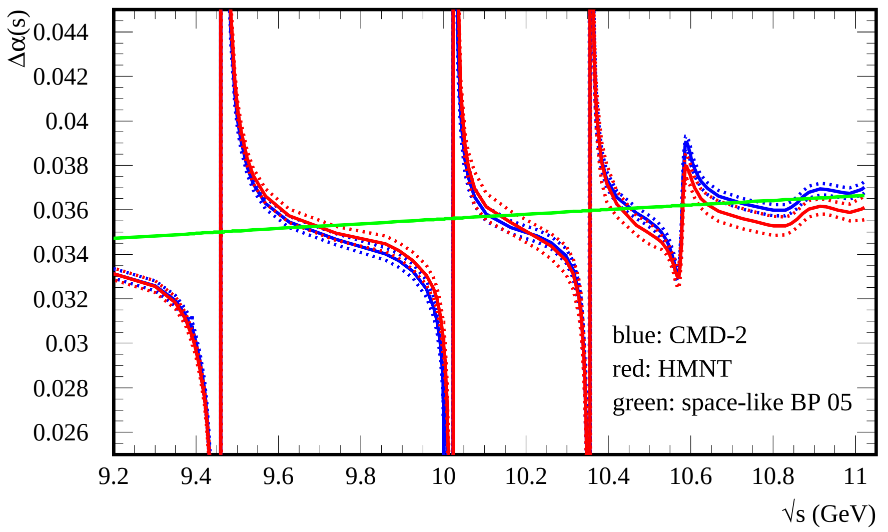
— expected to smooth and improve region above 11 GeV

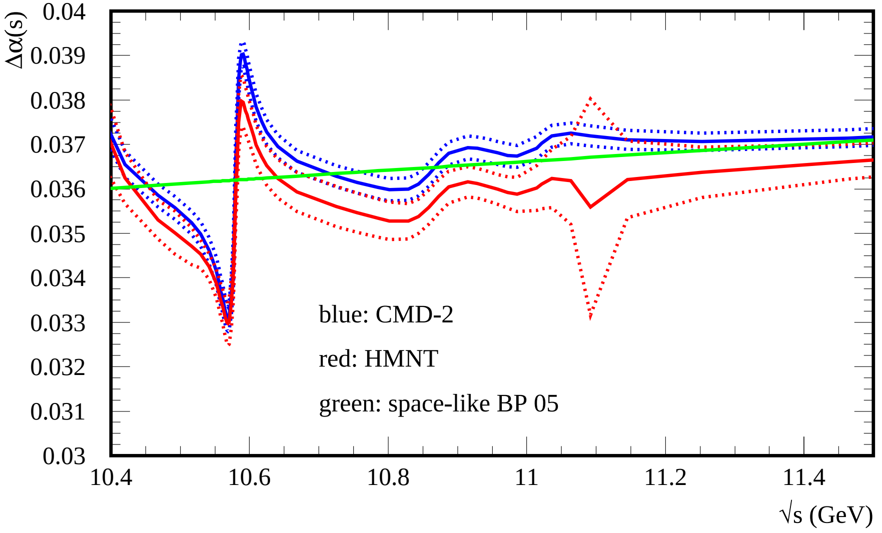
- HLMNT compared to CMD-2's routine: Detailed looks



- HLMNT compared to CMD-2's routine: and two more zooms



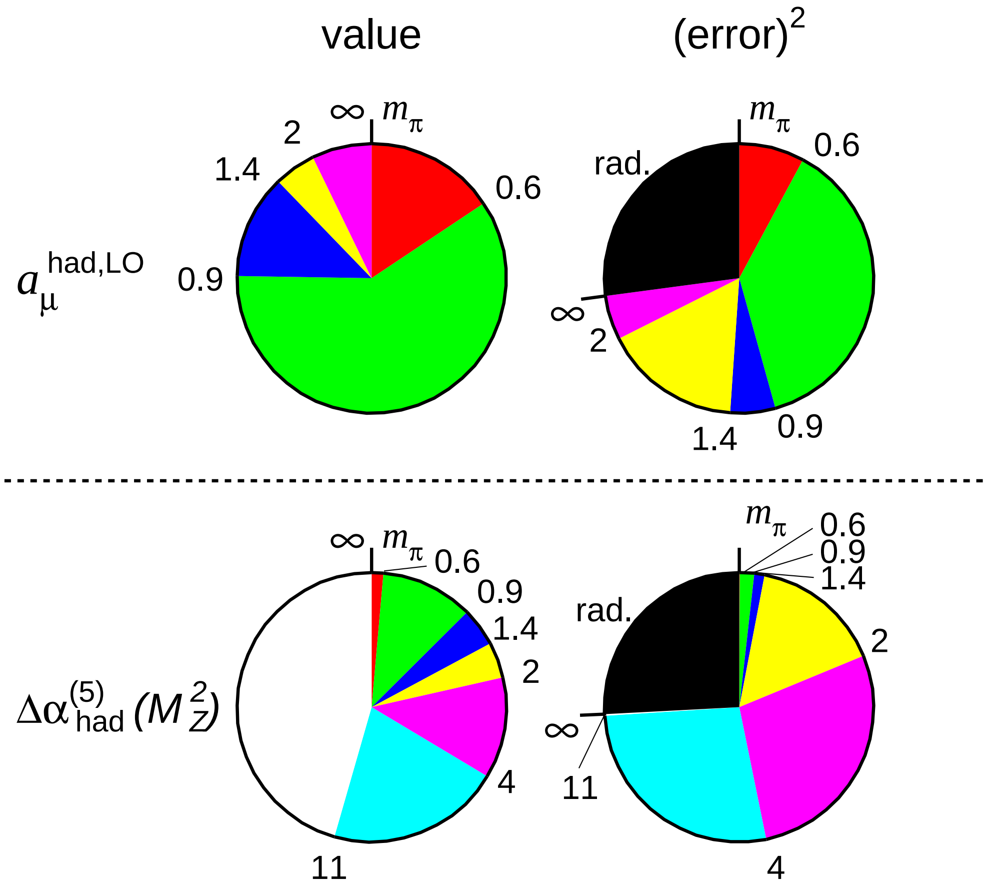
# IV. Outlook

**Where is improvement needed most urgently?** Had. VP the biggest error in $a_\mu^{\rm SM}$, $\alpha(M_Z^2)$

Pie diagrams of contributions to $a_\mu$ and $\alpha(M_Z)$ and their errors$^2$: enjoy!

## Prospects for squeezing the error!

- More 'Radiative Return' in progress at KLOE.
- Further prospects with DA$\Phi$NE-2.
- Big improvement envisaged with CMD-3 and SND at VEPP2000.
- At higher energies, BES-III at BEPCII in Beijing is on; opportunities for BELLE!?



Extras:

## Approximations/Accuracy:

- Leptonic:
  - Leading and next-to-leading order known analytically; lepton masses the only *tiny* uncertainty.
  - NNLO available as expansion in the lepton mass, i.e. in $m_\ell^2/q^2$ Steinhauser

  $\rightsquigarrow$ no limitations from this sector.

- Hadronic:
  - 'All-order' using experimental data and dispersion integral for low energies

    $\hookrightarrow$ stat. $+$ sys. uncertainties from input data
  - Non-resonant 'continuum' contributions can be evaluated by perturbative QCD; especially above well above (charm and) bottom thresholds.
  - Strongly suppressed top quark contribution added using pQCD.

▶ Uncertainties in running $\alpha_{\mathrm{QED}}(q^2)$ / VP dominated by hadronic contributions at low (to medium) $q^2$ (see discussion below).

# Hadronic Contributions via the Dispersion Integral

- For compilation done and used by the Novosibirsk group see e.g. the excellent talks by Gennadiy Fedotovich & Fedor Ignatov at Beijing meeting Oct. 2008.
- For Fred Jegerlehner's results see e.g. his Nucl. Phys. Proc. Suppl. 181-182 (2008) 135 and references therein.
- HLMNT use their data compilation for $g-2$ also for their own $\Delta\alpha(q^2)$ and $R(q^2)$ routines, for details and Refs see

  Hagiwara+Martin+Nomura+T: PRD 69(2004)093003; PLB 649(2007)173.

  – Data compilation uses most of the available data, with the leading hadronic channels $2\pi$, $3\pi$, $KK$, $4\pi$, but altogether sum of $\sim 24$ exclusive channels and inclusive data for $\sqrt{s}$ above $1.43-2$ GeV to get total $\sigma^0_{\rm had}$ with high precision.
  – Some subleading channels via isospin symmetry. Chiral PT for relevant thresholds.
  – Data driven, i.e. use of *state-of-the-art* perturbative QCD only above $\sim 11.09$ GeV.
  – Note: by using pQCD in a wider range one could improve the error at the expense of a more TH-driven approach.

– Data combination by non-linear $\chi^2_{min}$ fit which takes into account correlations through systematic errors; fit of one renormalization factor for each set (within/governed by systematics).

– Radiative corrections ('VP undressing') (re-)done as required in each set; where no reliable information is available an additional error due to radiative corrections has been assigned.

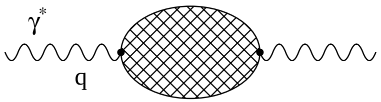
[HLMNT make no attempt at having a 'dressed' VP compilation.]

- Once the data are corrected for VP (and FSR) and suitably combined and continued in the perturbative regime, the numerical dispersion integral is straightforward (but has to take into account the Principal Value description).

- Narrow Resonances $J/\psi$, $\psi'$ and the $\Upsilon$ family are added separately.

- The error estimate comes through combined statistical, systematic and parametric ($\alpha_s$, quark masses, renormalisation scale in case of pQCD, resonance parameters for NR) uncertainties.

# The 'running coupling' $\alpha_{\rm QED}(q^2)$ and the Higgs mass




- Vacuum polarisation leads to the 'running' of $\alpha$ from $\alpha(q^2 = 0) = 1/137.035999084(51)$ to $\alpha(q^2 = M_Z^2) \sim 1/129$

- $\alpha(s) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}(s)\right)$

- Use of a dispersion relation:
$\Delta\alpha_{\rm had}^{(5)} = -\frac{\alpha s}{3\pi} P \int_{s_{\rm th}}^{\infty} \frac{R_{\rm had}(s')\, ds'}{s'(s'-s)}$

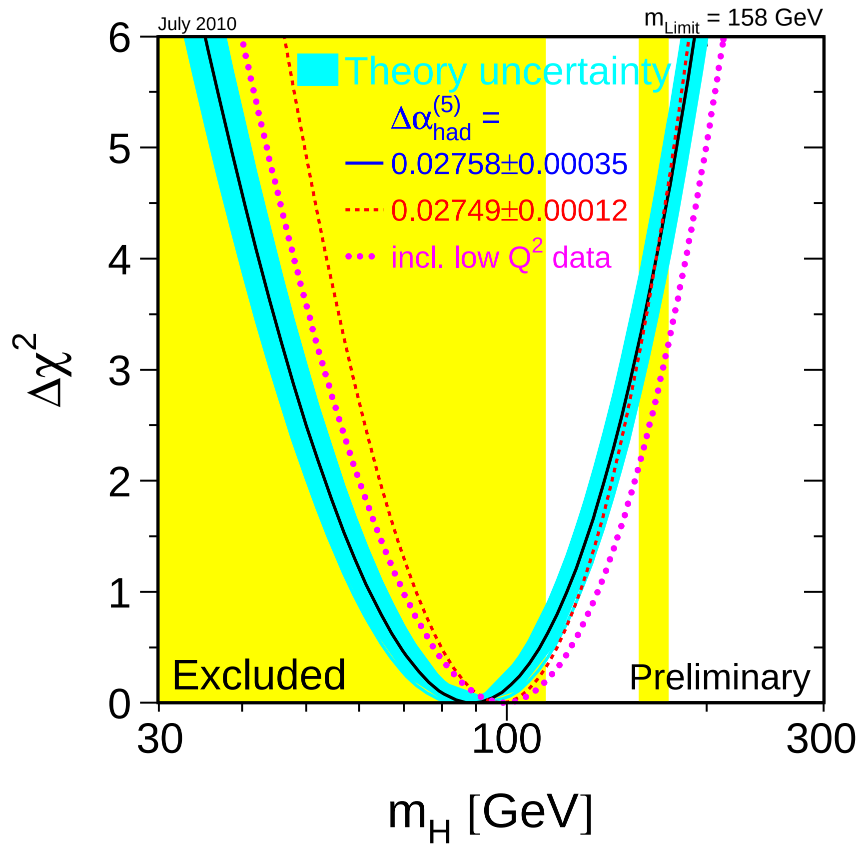
▶ HLMNT-routine for $\alpha(q^2)$ and $R_{\rm had}$ available

- Hadronic uncertainties $\rightsquigarrow$ $\alpha$ is the least well known Electro-Weak SM parameter of $[G_\mu, M_Z$ and $\alpha(M_Z^2)]$ !

- We find: $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.02759 \pm 0.00015$

i.e. $\alpha(M_Z^2)^{-1} = 128.953 \pm 0.020$ (HLMNT 10 prel.)

Fit of the SM Higgs mass: LEP EWWG




- $M_H = 89\,^{+35}_{-26}$ GeV ($m_t = (173.3 \pm 1.1)$ GeV) ($M_H < 158$ GeV (95% CL), $< 185$ GeV incl. direct limit $M_H < 114$ GeV.)

- $M_H$ moves further down with new $\Delta\alpha$.

- What about $\boldsymbol{\Delta\alpha(M_Z^2)}$?

$\longrightarrow$ With the same data compilation of $\sigma^0_{\rm had}$ as for $g-2$ HLMNT find:

$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02760 \pm 0.00015}$ (HLMNT 09 prelim.)

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.947 \pm 0.020$ [HMNT '06: $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$]

Earlier compilations:

| Group | $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ | remarks |
|---|---|---|
| Burkhardt+Pietrzyk '05 | $0.02758 \pm 0.00035$ | data driven |
| Troconiz+Yndurain '05 | $0.02749 \pm 0.00012$ | pQCD |
| Kühn+Steinhauser '98 | $0.02775 \pm 0.00017$ | pQCD |
| Jegerlehner '08 | $0.027594 \pm 0.000219$ | data driven/pQCD |
| ($M_0 = 2.5$ GeV) | $0.027515 \pm 0.000149$ | Adler fct, pQCD |
| HMNT '06 | $0.02768 \pm 0.00022$ | data driven |

Adler function: $D(-s) \;=\; \dfrac{3\pi}{\alpha} s \dfrac{\mathrm{d}}{\mathrm{d}s} \Delta\alpha(s) \;=\; -(12\pi^2) s \dfrac{\mathrm{d}\Pi(s)}{\mathrm{d}s}$

allows use of pQCD and minimizes dependence on data.