NNLO corrections to the Bhabha scattering cross section

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Aim of the work

- creation of the virtual (determined by the package bha_nnlo_hf) and real corrections (Monte Carlo generators PHOKHARA, BHAGHEN–1 and HELAC–PHEGAS) at NNLO for Bhabha scattering
- discussion of the numerical resultsat the energies and with realistic cuts used at the Φ factory Dafne, at the B factories PEP-II and at KEK and at the charm/τ factory BEPC II, Beijing
- comparison complete calculations with approximate ones realized in the MC generator BabaYaga

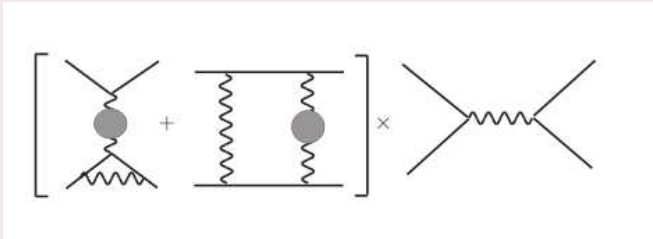
Collaboration: C. Carloni Calame, H. Czyż, J. Gluza, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek

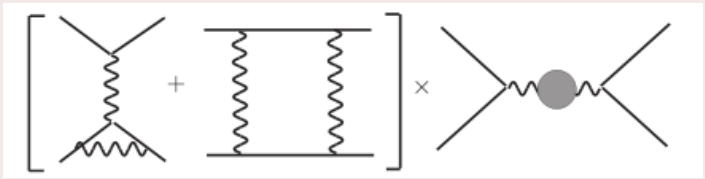
The massive NNLO corrections

The complete NNLO $N_f = 1, 2$ corrections to Bhabha scattering consist of three parts:

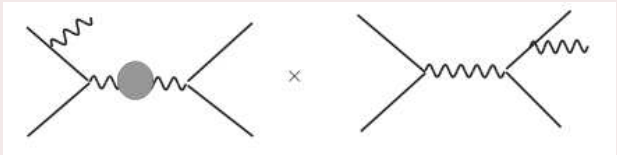
$$\begin{aligned} \frac{d\sigma_{N_f}^{\mathrm{NNLO}}}{d\Omega} &= \frac{d\sigma_{virt}^{\mathrm{NNLO}}}{d\Omega} + \frac{d\sigma_{\gamma}^{\mathrm{NLO}}}{d\Omega} + \frac{d\sigma_{real}^{\mathrm{LO}}}{d\Omega} \\ &= \frac{d\sigma_{e^+e^-}}{d\Omega} + \frac{d\sigma_{\mu^+\mu^-}}{d\Omega} + \frac{d\sigma_{\tau^+\tau^-}}{d\Omega} + \frac{d\sigma_{had}}{d\Omega}. \end{aligned} \tag{1}$$

- the $\sigma_{virt}^{\mathrm{NNLO}}$ consists of virtual two-loop corrections $\sigma_{2L}^{\mathrm{NNLO}}$ and loop-by-loop corrections $\sigma_{1L1L}^{\mathrm{NNLO}}$





- contributions with real photon emission $\sigma_{\gamma}^{\mathrm{NLO}} = \sigma_{\gamma,soft}^{\mathrm{NLO}}(\omega) + \sigma_{\gamma,hard}^{\mathrm{NLO}}(\omega)$



- contributions with real pair or hadron emission $\sigma_{real}^{\mathrm{LO}} = \sigma_{e^+e^-(e^+e^-)}^{\mathrm{LO}} + \sigma_{e^+e^-(f^+f^-)}^{\mathrm{LO}} + \sigma_{e^+e^-(hadrons)}^{\mathrm{LO}}$

,,Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data". EPJC(2010)66

Table: The NNLO lepton and pion pair corrections to the Bhabha scattering Born cross section σ_B.All cross sections are given in nanobarns.

Electron pair corrections					
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}
KLOE	529.469	9.502	-11.567	-2.065	0.271
BaBar	6.744	0.246	-0.271	-0.025	0.017
Muon pair corrections					
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}
KLOE	529.469	1.494	-1.736	-0.241	–
BaBar	6.744	0.091	-0.095	-0.004	0.0005
Tau pair corrections					
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}
KLOE	529.469	0.020	-0.023	-0.003	–
BaBar	6.744	0.016	-0.017	-0.0007	$< 10^{-7}$
Pion pair corrections					
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}
KLOE	529.469	1.174	-1.360	-0.186	–
BaBar	6.744	0.062	-0.065	-0.003	0.00003

The calculations of hard photonic corrections σ_h^{NLO} with real hard photon radiation are based at BHAGHEN-1PH Monte Carlo generator (M. Caffo, H. Czyz, Comput.Phys.Commun. 100 (1997) 99-118) with additional vacum polarisation. This part contains e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $\Pi^+\Pi^-$ pairs inside the loop.

The vacuum polarisation function:

$$\Pi(q^2) = -\frac{q^2}{\pi}\int_{4m^2}^{\mathrm{inf}} \frac{dz}{z}\frac{Im\Pi(z)}{q^2 - z + i\epsilon} \tag{2}$$

It can be rewrite into the form:

$$\Pi(q^2) = \frac{\alpha q^2}{3\pi}\int_{4m^2}^{\mathrm{inf}} \frac{dz}{z}\frac{R(z)}{q^2 - z + i\epsilon} \tag{3}$$

Pions

For the vacuum polarisation with pion productions form factor for numerical calculations was taked from PHOKARA Monte Carlo generator.

$$R(z) = \frac{\beta^3(z)}{4}|F_\pi(z)|^2 \tag{4}$$

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{z}} \tag{5}$$

Leptons

For leptons inside the vacuum polaristion loop the integral (3) was calculated analitycally.

Cuts dependence study for different experiments

1.Φ factories KLOE/DAΦNE (Frascati)

(a) $\sqrt{s} = 1.02$ GeV
(b) $E_{min} = 0.4$ GeV
(c) For $\theta\pm$ two selections have to be checked
i. wider selection $20^\circ < \theta\pm < 160^\circ$
ii. tighter selection $55^\circ < \theta\pm < 125^\circ$
(d) ζ_{max}=4,5,6,7,8,...,14°

2. B-factories BABAR/PEP-II (SLAC) & BELLE/KEKB (KEK)

(a) $\sqrt{s} = 10.56$ GeV
(b) $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$
or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$
(c) For $|cos(\theta\pm)|$ the following selections have to be checked
i. $|cos(\theta\pm)| < 0.70$ and $|cos(\theta+)| < 0.65$ or $|cos(\theta-)| < 0.65$
ii. $|cos(\theta\pm)| < 0.65$ and $|cos(\theta+)| < 0.60$ or $|cos(\theta-)| < 0.60$
iii. $|cos(\theta\pm)| < 0.60$ and $|cos(\theta+)| < 0.55$ or $|cos(\theta-)| < 0.55$
(d) ζ^{3d}_{max}= 20,22,24,...,40°

Cuts dependence study for different experiments

3. BES-III experiment at BEPCII (Beijing)

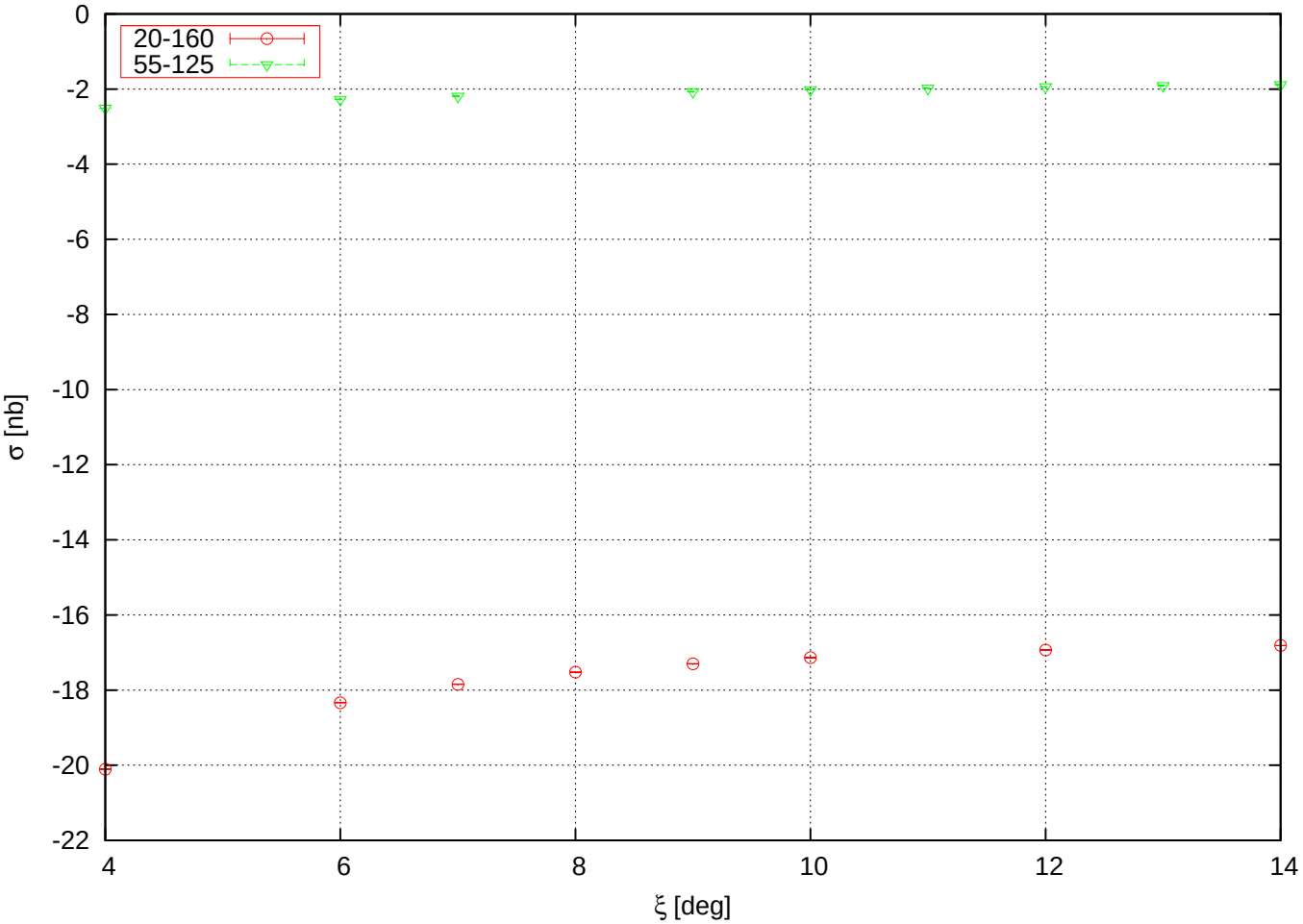
(a) four different values have been chosen for our studies
i.$\sqrt{s} = 2$GeV
ii.$\sqrt{s} = 3.097$ GeV
iii.$\sqrt{s} = 3.686$ GeV
iv.$\sqrt{s} = 5$GeV
(b) $E_{min} > 0.25\sqrt{s}$
(c) $cos(\theta+)cos(\theta+) < 0.02$
(d) For $|cos(\theta\pm)|$ the following selections have to be checked
i. $|cos(\theta\pm)| < 0.80$
ii. $|cos(\theta\pm)| < 0.75$
iii. $|cos(\theta\pm)| < 0.70$
(d) ζ_{max}= 5,6,7,...,15°

Where:

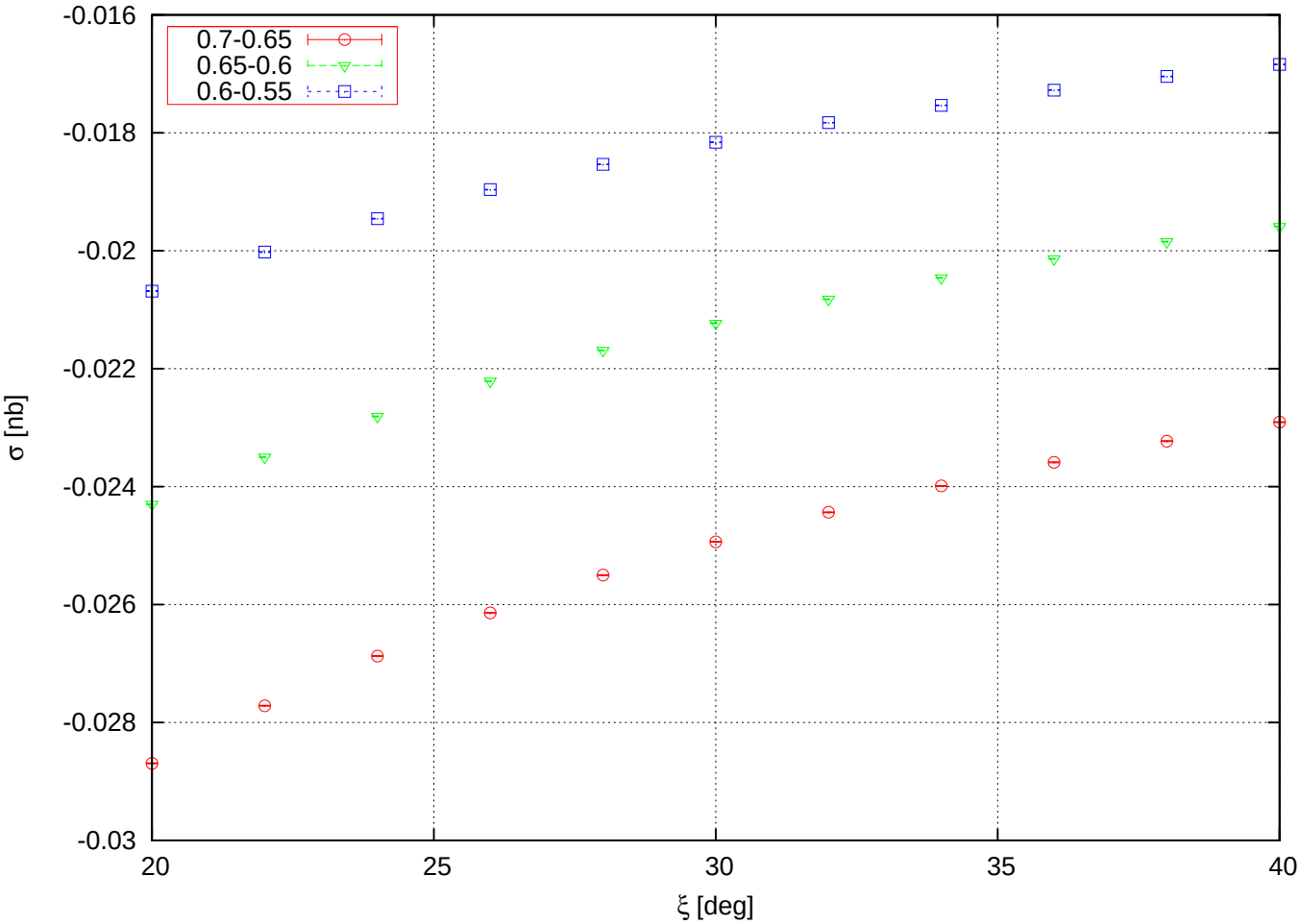
$$\zeta = |\theta_+ + \theta_- - 180| \tag{6}$$

$$\zeta^{3d} = |\arccos s(\vec{p}_+\vec{p}_-/|\vec{p}_+|/|\vec{p}_-|)180/\pi - 180| \tag{7}$$

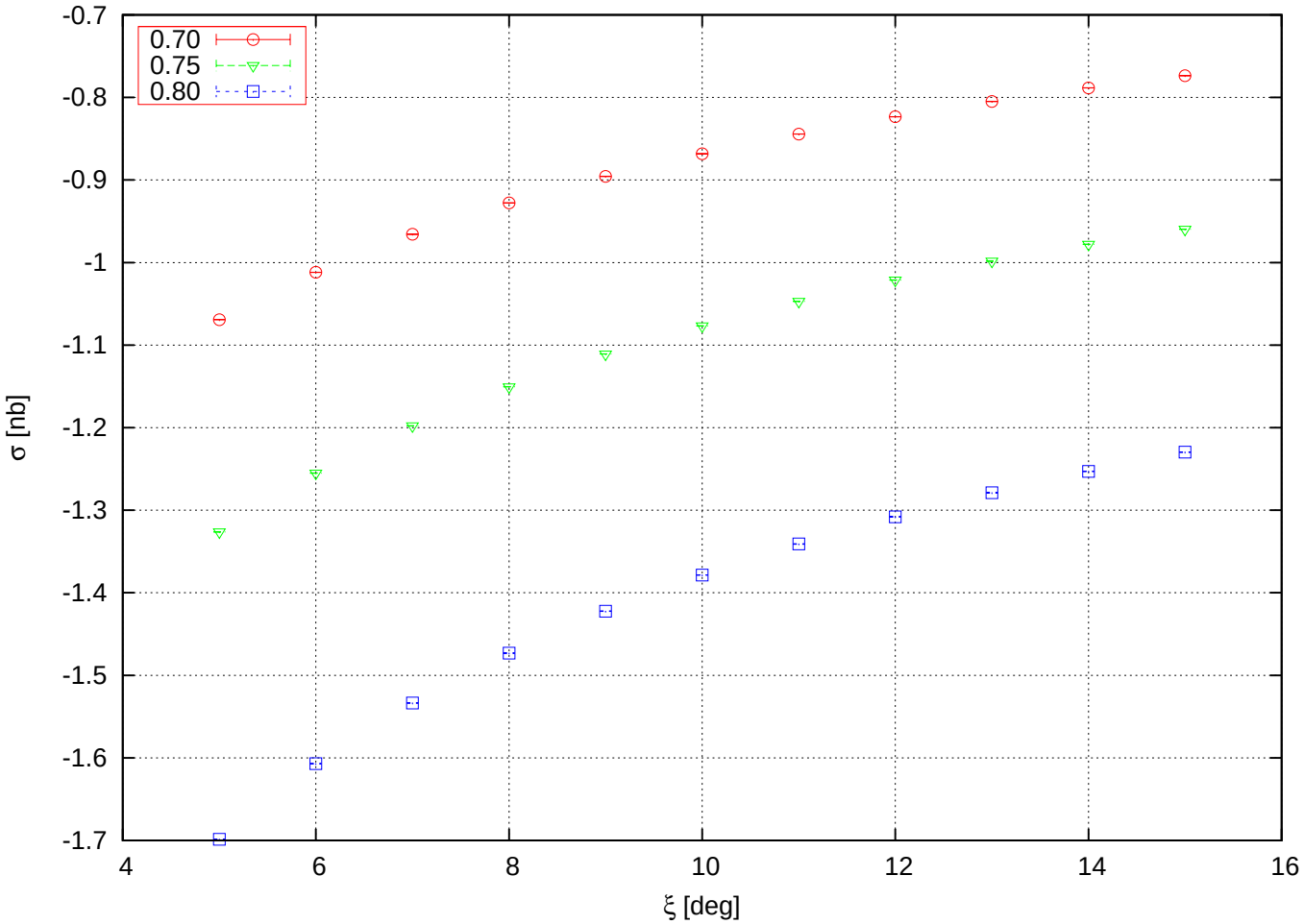
KLOE, e^+e^-, $\frac{\omega}{E_{beam}} = 10^{-4}$



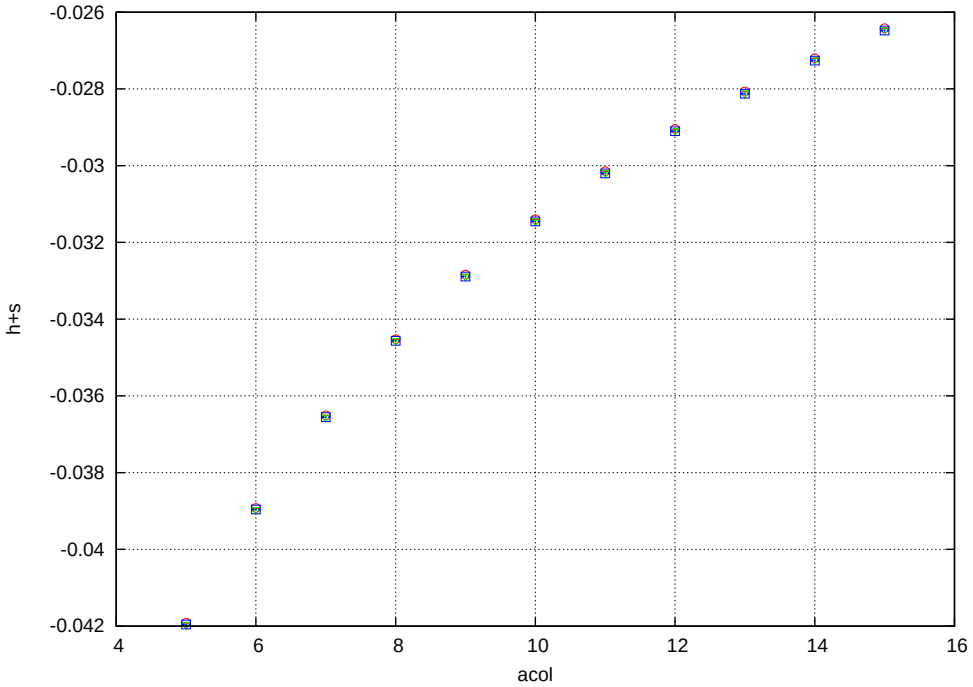
BABAR, $e^+e^-, \frac{\omega}{E_{beam}} = 10^{-4}$



BES, $\sqrt{s} = 2GeV$, e^+e^-, $\frac{\omega}{E_{beam}} = 10^{-4}$

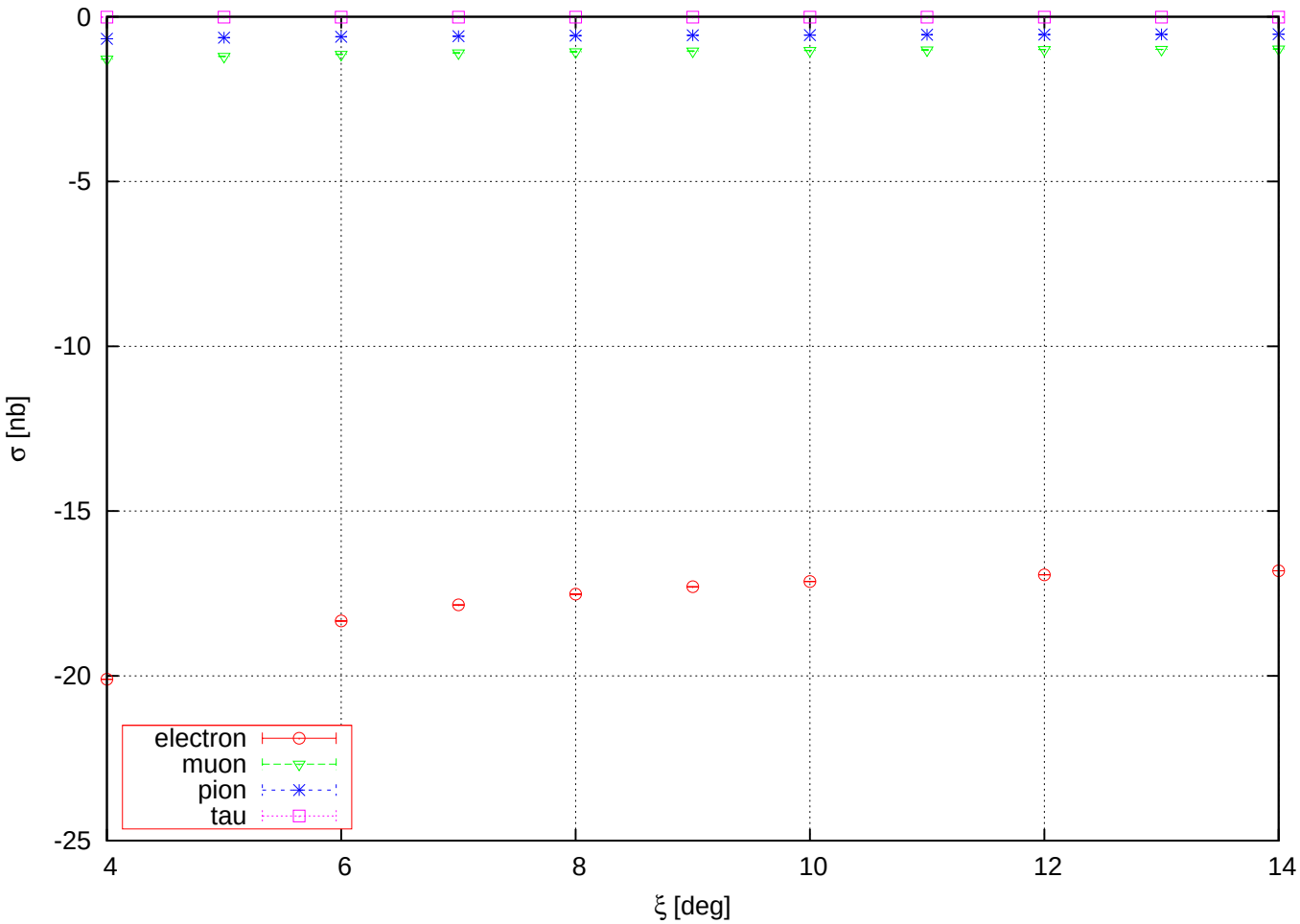


$\sigma_{v+s} + \sigma_h$, BES, pions, $\sqrt{5}GeV$, $|cos(\theta\pm)| < 0.70$, $\frac{\omega}{E_{beam}} = 10^{-5}, 10^{-4}, 10^{-3}$

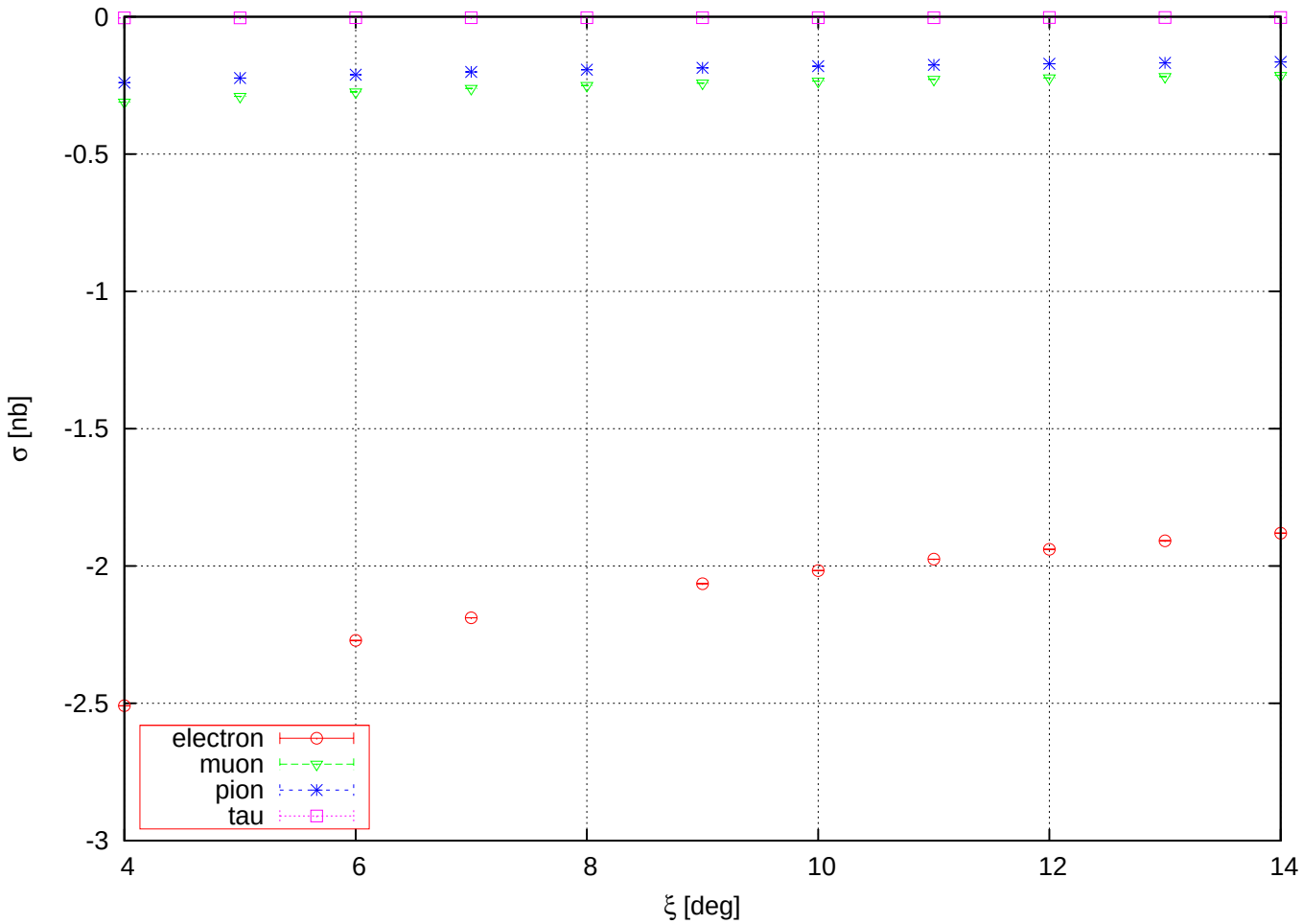


The sum $\sigma_{v+s}^{\mathrm{NNLO}} + \sigma_h^{\mathrm{NNLO}}$ is independent of ω.

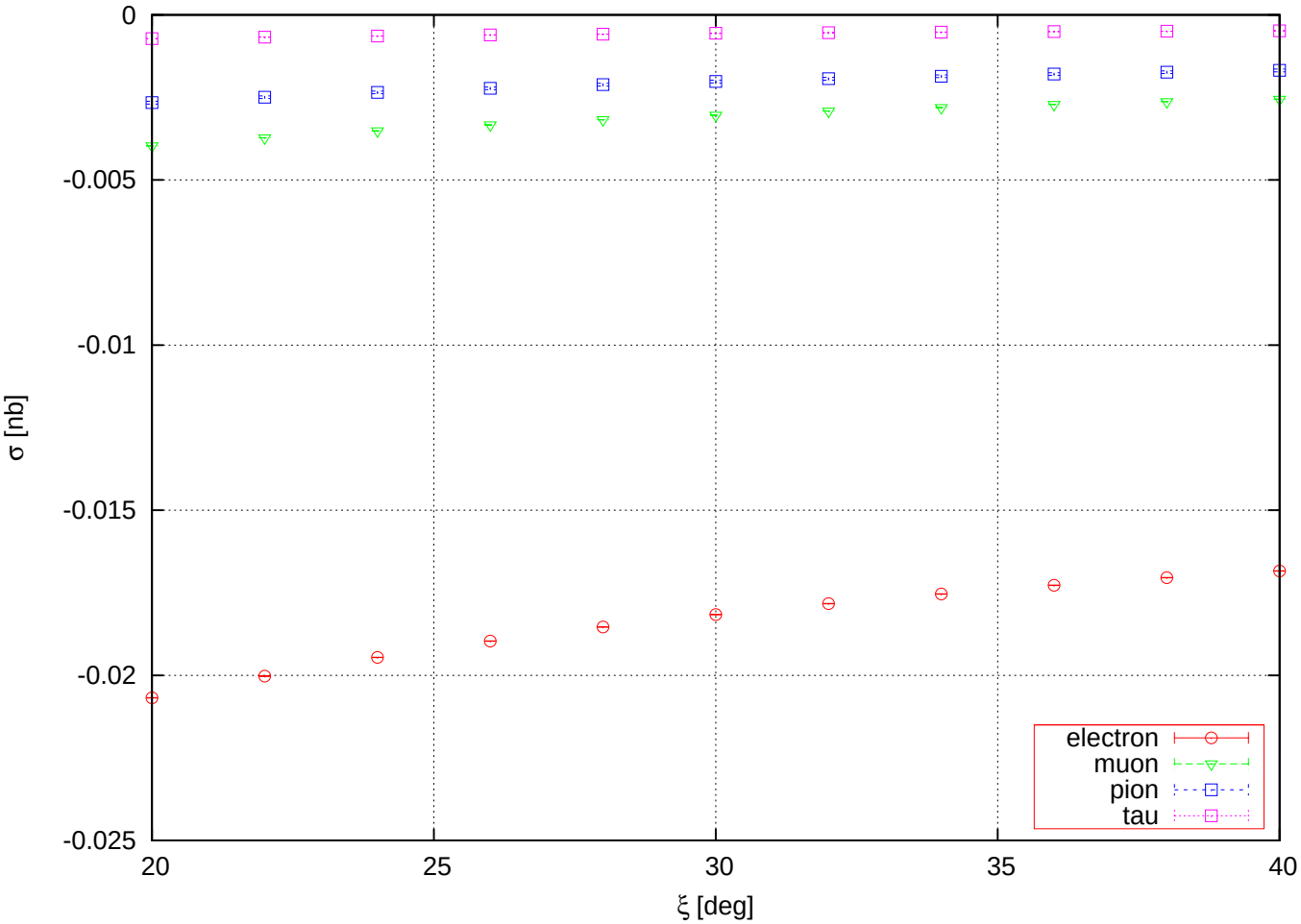
$\sigma_{v+s} + \sigma_h$,KLOE, $\frac{\omega}{E_{beam}} = 10^{-4}$,$20° < \theta\pm < 160$ °



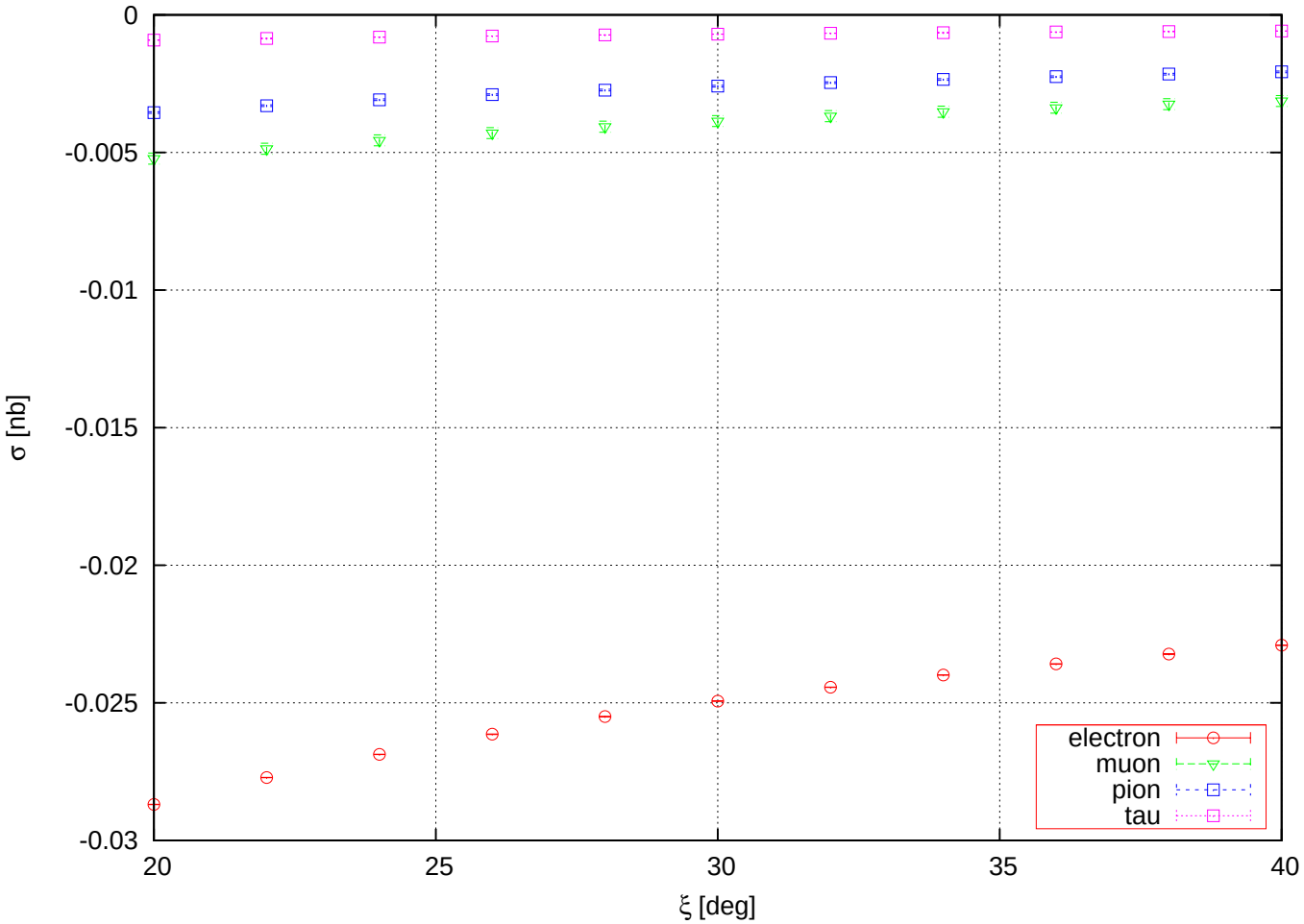
$\sigma_{v+s} + \sigma_h$,KLOE, $\frac{\omega}{E_{beam}} = 10^{-4}$,$55^\circ < \theta\pm < 125^\circ$



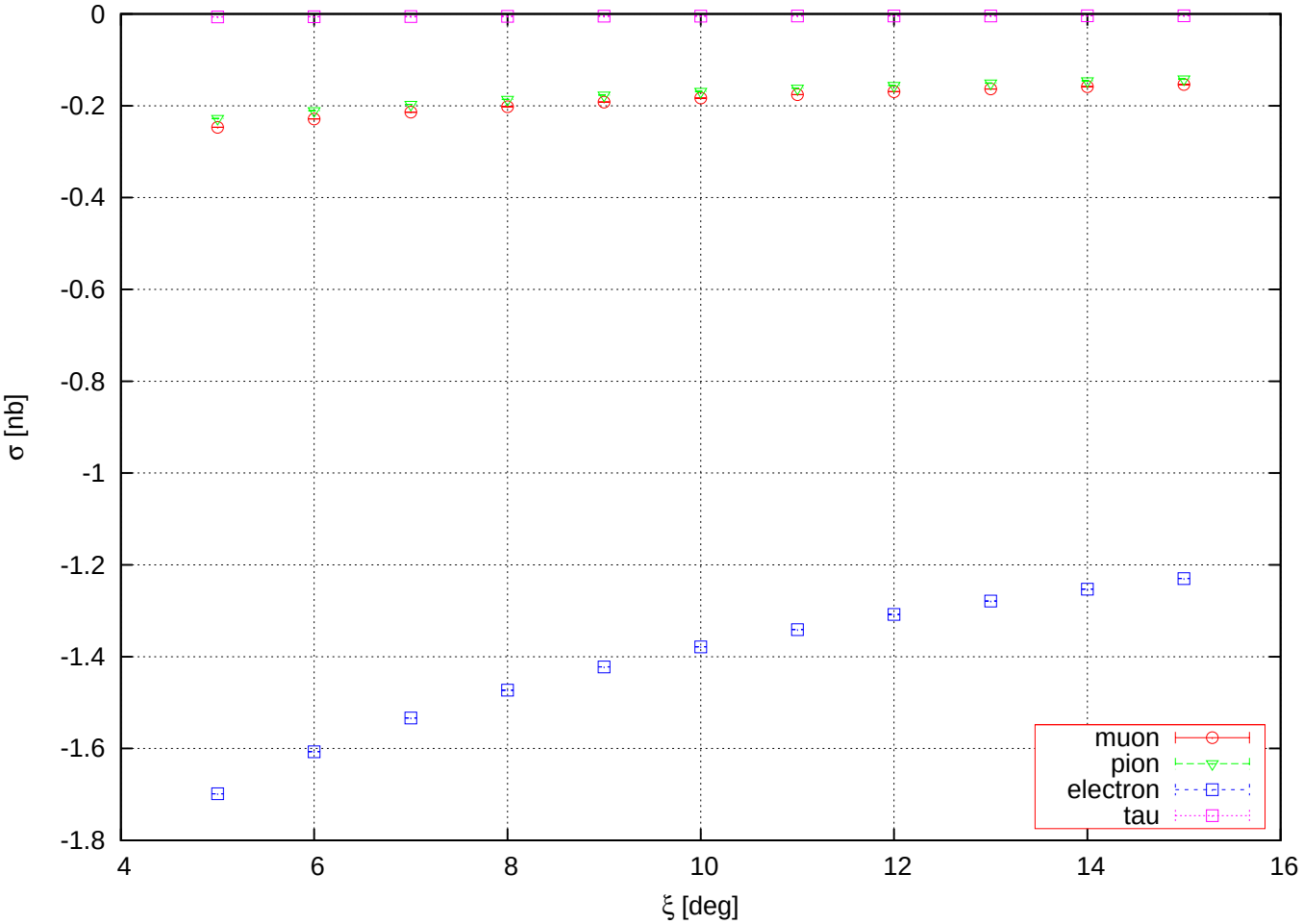
$\sigma_{v+s} + \sigma_h$,BABAR, $\frac{\omega}{E_{beam}} = 10^{-4}$, $|cos(\theta\pm)| < 0.60$ and $|cos(\theta+)| < 0.55$ or $|cos(\theta-)| < 0.55$



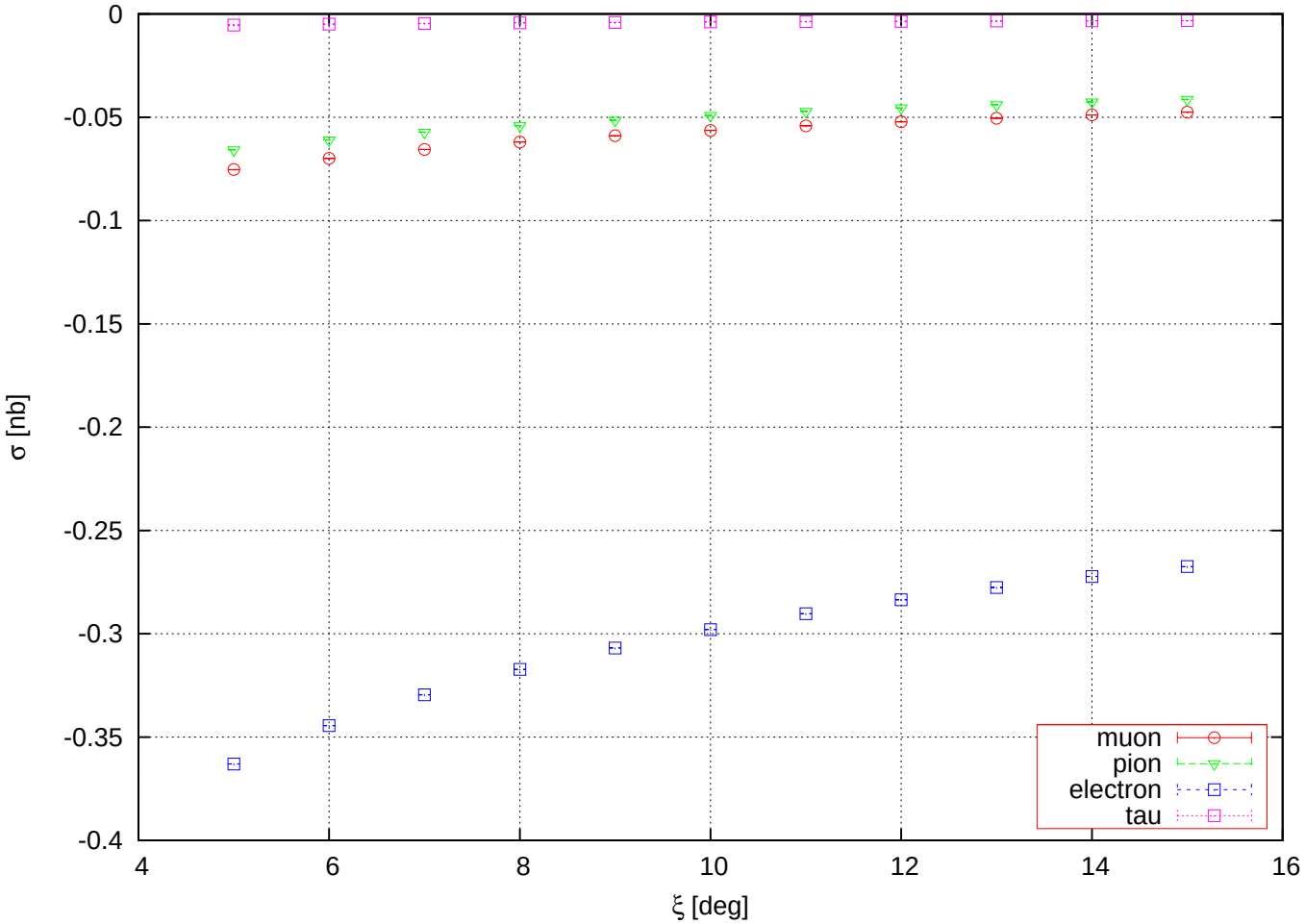
$\sigma_{v+s} + \sigma_h$,BABAR, $\frac{\omega}{E_{beam}} = 10^{-4}$, $|cos(\theta\pm)| < 0.70$ and $|cos(\theta+)| < 0.65$ or $|cos(\theta-)| < 0.65$



$\sigma_{v+s} + \sigma_h$, BES, $\sqrt{s} = 2\text{GeV}$, $\frac{\omega}{E_{beam}} = 10^{-4}$, $|cos(\theta\pm)| < 0.80$



$\sigma_{v+s} + \sigma_h$, BES, $\sqrt{s} = 5\text{GeV}$, $\frac{\omega}{E_{beam}} = 10^{-4}$, $|cos(\theta\pm)| < 0.80$



Conclusions

- calculations of corrections: $\sigma_{virt}^{\mathrm{NNLO}}$ and $\sigma_{\gamma}^{\mathrm{NLO}}$ were made and added together;
- indepedndence from ω between this corrections was checked;
- other calculations are in progress.