Direct Evaluation of the Static Force from the Lattice QCD

Viljami Leino

Based on:

Nora Brambilla[1], V.L.[1], Owe Philipsen[2], Christian Reisinger[23], Antonio Vairo[1], and Marc Wagner[23] hep-lat/2106.01794

Nora Brambilla[1], V.L.[1], Julian Mayer-Steudte[1], and Antonio Vairo[1]: In preparation

Affiliations: [1]Technische Universität München,
[2]Goethe-Universität Frankfurt am Main, [3]Helmholtz Research Academy Hesse for FAIR

HaSp-Strong 2020
16.09.2021

Static Energy $E_0(r)$

- The QCD static energy of a quark-antiquark pair, $E(r)$ is a physical observable

- Given by the Wilson loop

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}\,, \qquad W_{r\times T} = P\left\{ \exp\left(i \oint_{r\times T} dz_\mu\, gA_\mu \right) \right\}$$

- Can be described by perturbation theory and measured on lattice

- For $r\Lambda_{\mathrm{QCD}} \ll 1$ both descriptions should agree

Use of $E_0(r)$ for measuring α_s

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E(r) = \Lambda_s - C_F \frac{\alpha_s}{r} \Bigg\{ 1 + \frac{\alpha_s}{4\pi} \tilde{a}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \tilde{a}_2 + \left(\frac{\alpha_s}{4\pi}\right)^3 \left[a_3^{\mathrm{L}} \log \frac{C_A \alpha_s}{2} + \tilde{a}_3 \right] + \left(\frac{\alpha_s}{4\pi}\right)^4 \left[a_4^{\mathrm{L2}} \log^2 \frac{C_A \alpha_s}{2} + a_4^{\mathrm{L}} \log \frac{C_A \alpha_s}{2} + \tilde{a}_4 \right] \Bigg\}$$

- Ultrasoft contributions can be understood in pNRQCD

$$E(r) = \Lambda_s + V_s(r, \nu, \mu_{us}) + \delta_{us}(r, \nu, \mu_{us})$$

- Can be used to extract α_s by equating lattice results to perturbative evaluated results
- No lattice to $\overline{\mathrm{MS}}$ scheme change required

[1] For review of perturbative results, see: X. Tormo Mod. Phys. Lett. A28 (2013)

Issues with $E_0(r)$

$$E_0(r) = \Lambda_{\rm s} - \frac{C_{\rm F}\alpha_{\rm s}}{r}\left(1 + \#\alpha_{\rm s} + \#\alpha_{\rm s}^2 + \#\alpha_{\rm s}^3 + \#\alpha_{\rm s}^3 \ln \alpha_{\rm s} + \ldots\right)$$

- The perturbative expansion affected by a renormalon ambiguity of order Λ in PT side
- On lattice: Linear UV divergence
- The slope of the potential not affected
- All interesting physics is in the slope
  - Do arbitrary shift
  - Take a derivative

    $\Rightarrow$ Static Force: $F(r) = \partial_r E_0(r)$
    - Take numerical derivative of lattice data (noisy)
    - Integrate PT result:

$$E_0(r) = \int_{r^*}^{r} dr' F(r', \alpha_{\rm s}(1/r')) + \text{const}$$

Static force $F(r)$

- Alternatively define directly[1]:

$$F(r) = -\lim_{T\to\infty} \frac{i}{\langle \mathrm{Tr}(W_{r\times T})\rangle} \left\langle \mathrm{Tr}\left(P\left\{ \exp\left(i \oint_{r\times T} \mathrm{d}z_\mu \, gA_\mu \right) \hat{\mathrm{r}} \cdot g\mathrm{E}(\mathrm{r}, t^*) \right) \right\} \right\rangle$$
$$= \frac{\langle \mathrm{Tr}\{\mathrm{PW_{R\times T}} gE_j(r, t^*)\}}{\mathrm{Tr}\{\mathrm{PW_{R\times T}}\}}$$

- Chromoelectric field E inserted to Wilson loop
- Works also for Polyakov loops
- The insertion location t^* arbitrary
  - On lattice, reduce boundary terms and choose $t^* = T/2$
- Can be used to extract α_s without the usual renormalon issues
- Also useful for scale setting

[1] A. Vairo Mod. Phys. Lett. A 31 (2016) & EPJ Web Conf. 126 (2016), Brambilla *et.al.*PRD63 (2001)

Renormalization of $F(r)$

- On continuum perturbation theory, no renormalization needed
- On lattice E has finite size and Different discretizations
- The self energy contributions of E converge slowly to continuum[1]
  $\rightarrow$ need renormalization Z_E
- Earlier literature has different ways to deal with this
  - Perturbative calculation (needs higher loops, hard)
  - Huntley-Michael procedure (non-perturbative, perfect only 1-loop)
  - Tadpole improvement (non-perturbative, perfect only tree-level)
  - We do something different
- This is also problem on many other observables

[1] See e.g. Lepage *et.al.*PRD48 (1993), G. Bali Phys. Rept. 343 (2001), and many others ...

Alternative approach to Z_E

- Static force can be measured as numerical derivative of static potential
- Numerical derivative should agree with direct measurement of the force
- We can define Z_E from our simulations by comparing the numerical derivative (does not need Z_E) and directly measured force (needs Z_E)
- The renormalization constant can be expected to have small r-dependence
- In that case, we don't need Z_E at all, we can just calculate $F(r)/F(r')$ for arbitrary r'

[1] See e.g. Lepage *et.al.*PRD48 (1993), G. Bali Phys. Rept. 343 (2001), and many others ...

Simulation details

- Use Wilson gauge action, pure gauge
- Heath bath with overrelaxation
- 3 Ensembles A: a=0.06fm, B: a=0.05fm, C: a=0.04fm
- Scale setting with[1]:

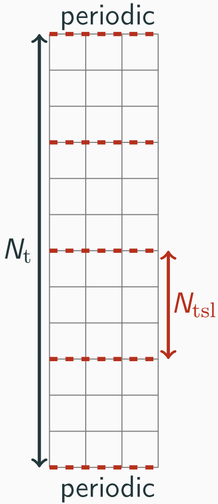
  $\ln(a/r_0) = -1.6804 - 1.7331\,(\beta - 6) + 0.7849\,(\beta - 6)^2 - 0.4428\,(\beta - 6)^3$
- Tree-level improve the force[1]:

$$r_{\mathrm{I}} = \sqrt{\frac{2a}{4\pi\,[G(r+a) - G(r-a)]}} \qquad G(r) = \frac{1}{a}\int_{-\pi}^{\pi} \frac{\mathrm{d}k^3}{(2\pi)^3} \frac{\cos(rk_3/a)}{4\sum_j \sin(k_j/2)}$$

- Note that the tree level imrpovement is same for discrete symmetric derivative of static potential and for direct force measurement
- Multilevel & Wilson loops: APE-smearing for spatial links $\alpha_{\mathrm{APE}} = 0.5, N_{\mathrm{APE}} = 50$

[1] S. Necco & R. Sommer. Nucl. Phys. B622 (2002)

Algorithm 1: Multilevel algorithm

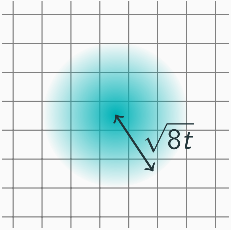


- Algorithm for quenched simulations
  - Cannot be generalized to un-quenched
- Improves signal of noisy observables
- Idea: Divide the lattice to temporal slices of size N_{tsl}
- Update each sub-lattice independently keeping boundaries fixed
- Average over different boundary configurations
  - \+ Allows reaching better statistics with less configurations
- Spatial Wilson lines located at the boundaries

Lüscher and Weisz JHEP09 (2001)

Algorithm 2: Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$



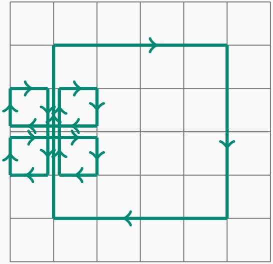
- Evolve gauge along fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}

+ Automatically renormalizes gauge invariant observables
+ Can be used un-quenched (This work: quenched)

- Generally needs zero flowtime limit

Lüscher JHEP 1008, 071 (2010)

Measured operators

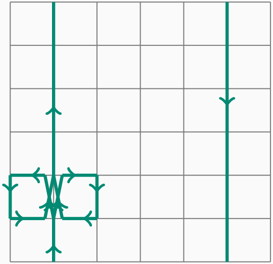


Wilson loop with Clover

$$E_i = \frac{1}{2iga^2}\left(\Pi_{i0} - \Pi_{i0}^\dagger\right)$$

$$\Pi_{\mu\nu} = \frac{1}{4}\left(P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu}\right)$$

Used with multilevel and flow



Polyakov loop with Butterfly

$$E_i = \frac{1}{2}\left(F_{0i} + F_{-i0}\right)$$

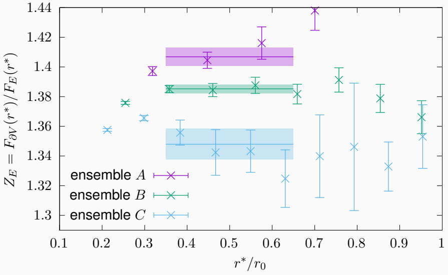
$$F_{\mu\nu} = \frac{1}{2iga^2}\left(P_{\mu,\nu} - P_{\mu,\nu}^\dagger\right)$$

$$P_{\mu,\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

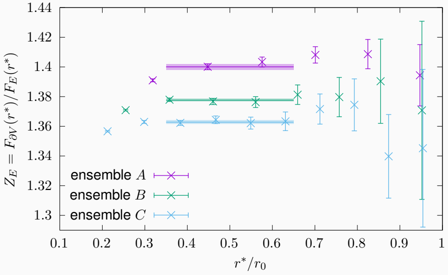
Only used with multilevel

For discretizations, see e.g. Bali *et.al.*PRD56 (1997) or Koma & Koma Nucl. Phys. B769 (2007)

Renormalization constant $Z_\mathrm{E} = \partial_r E(r)/F_E(r)$



Wilson loop

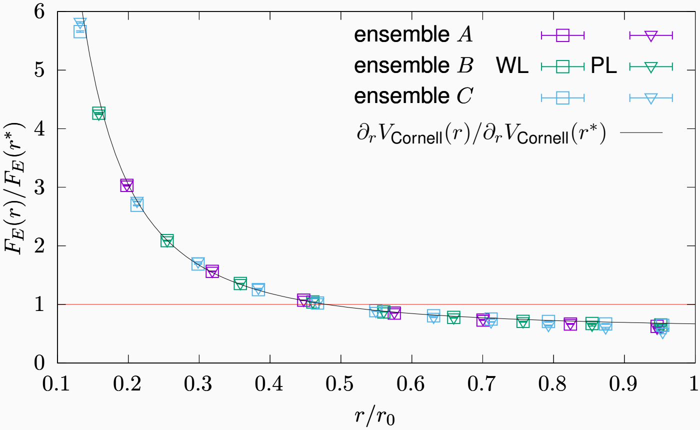


Polyakov loop

Ensemble	a in fm	Z_E from Wilson loops	Z_E from Polyakov loops
A	0.060	1.4068(63)	1.4001(20)
B	0.048	1.3853(30)	1.3776(10)
C	0.040	1.348(11)	1.3628(13)

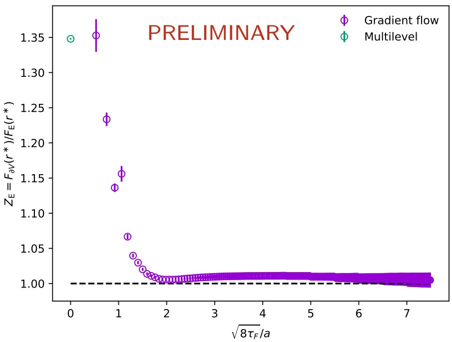
- Force from numerical derivative of E_0 differs from force from F_E
- Nonperturbative Z_E. Very little r-dependence

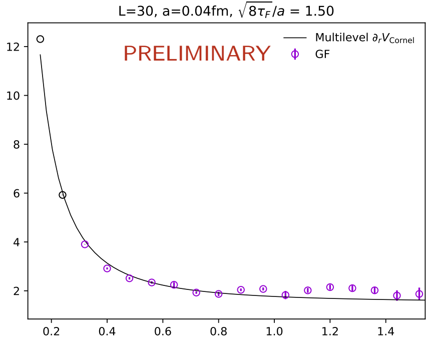
Multilevel result



- Remove Z_E by dividing with measurement at $r^* = 0.48r_0$
- Proof of concept:
  - Both derivative of potential and direct force agree
  - Both Wilson loop and Polyakov loops agree

Gradient flow results





- Gradient flow automatically renormalizes the force at finite flowtime
  $\rightarrow$ No need for Z_{E}
- Divide with the leading flow time dependence for potential
- Early GF results indicate a good agreement to multilevel results
- The continuum and zero flowtime limits still need to be done

Conclusions

- Proof of concept: Static force can be measured directly from lattice by inserting chromoelectric field to a Wilson loop.
- Issue with self energy of chromoelectric field can be solved by:
    - Dividing the force with force at fixed separation r^*
    - Using gradient flow
- This work can be expanded in future to many operators appearing in NREFTs

Conclusions

- Proof of concept: Static force can be measured directly from lattice by inserting chromoelectric field to a Wilson loop.
- Issue with self energy of chromoelectric field can be solved by:
  - Dividing the force with force at fixed separation r^*
  - Using gradient flow
- This work can be expanded in future to many operators appearing in NREFTs

Thank you!