arXiv: 2109.03759 [hep-ph]

The physical Riemann surfaces of the ratio G_E^Λ/G_M^Λ

ALESSIO MANGONI SIMONE PACETTI EGLE TOMASI-GUSTAFSSON

Istituto Nazionale di Fisica Nucleare - Sezione di Perugia

Agenda

- The Λ baryons
- Available data
- Theoretical aspects
- The G_E^Λ/G_M^Λ ratio
- The model
- Results
- Conclusions

The first exploration of the physical Riemann surfaces of the ratio G_E^Λ/G_M^Λ

Alessio Mangoni
INFN Sezione di Perugia, I-06100, Perugia, Italy

Simone Pacetti
INFN Sezione di Perugia and Università di Perugia, I-06100, Perugia, Italy

Egle Tomasi-Gustafsson
DPhN, IRFU, CEA, Université Paris-Saclay, 91191 Gif-sur-Yvette Cedex, France

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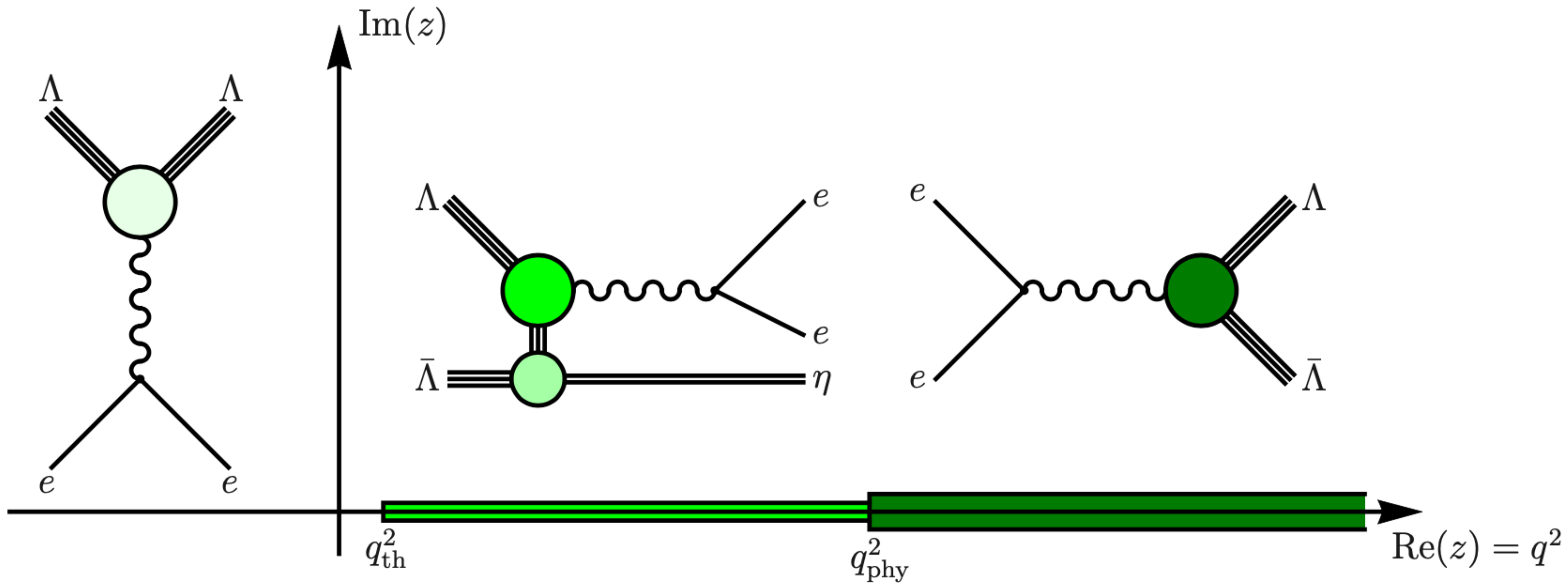

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The Λ baryons

G_E^Λ G_M^Λ

Λ form factors (FFs)

Lorentz scalar functions of the four-momentum transferred squared q^2

Analytic functions with a branch cut along the positive real axis, for $q^2 \geq q^2_{\rm th}$ with $q^2_{\rm th} = (2M_\pi + M_\pi^0)^2$

FFs are real for real values of $q^2 \leq q^2_{\rm th}$
(thanks to the Schwarz reflection principle)

In the space-like region ($q^2 \leq 0$):
$G_E^\Lambda, G_M^\Lambda \in \mathbb{R}$
data can be extracted studying the scattering process $e^-\Lambda \to e^-\Lambda$

In the time-like region where $q^2 \geq q^2_{\rm phys} = (2M_\Lambda)^2$:
$G_E^\Lambda, G_M^\Lambda \in \mathbb{C}$
data can be extracted studying the annihilation processes $e^+e^- \to \Lambda\bar{\Lambda}$ and $\Lambda\bar{\Lambda} \to e^+e^-$

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The Λ baryons

Scattering and annihilation experiments are hindered due to the impossibility to obtain stable beams or targets of Λ baryons

The weak decay $\Lambda \to p\pi^-$ can be used to obtain information about the polarization of the Λ baryon

the knowledge of the sinus does not provide information on the determination of the relative phase $\arg\left(\frac{G_E^\Lambda}{G_M^\Lambda}\right)$

The component of the polarization vector orthogonal to the scattering plane xz of the spin-1/2 baryon B in a generic annihilation process $e^+e^- \to B\overline{B}$ can be written as

$$\mathcal{P}_y = -\frac{\sqrt{\frac{q^2}{4M_B^2}}\frac{|G_E^\Lambda|}{|G_M^\Lambda|}\sin(2\theta)\sin\left(\arg\left(\frac{G_E^\Lambda}{G_M^\Lambda}\right)\right)}{\frac{q^2}{4M_B^2}(1+\cos^2(\theta))+\frac{|G_E^\Lambda|^2}{|G_M^\Lambda|^2}\sin^2(\theta)}$$

q is the momentum transferred, M_B is the baryon mass and θ is the scattering angle in the CM reference frame

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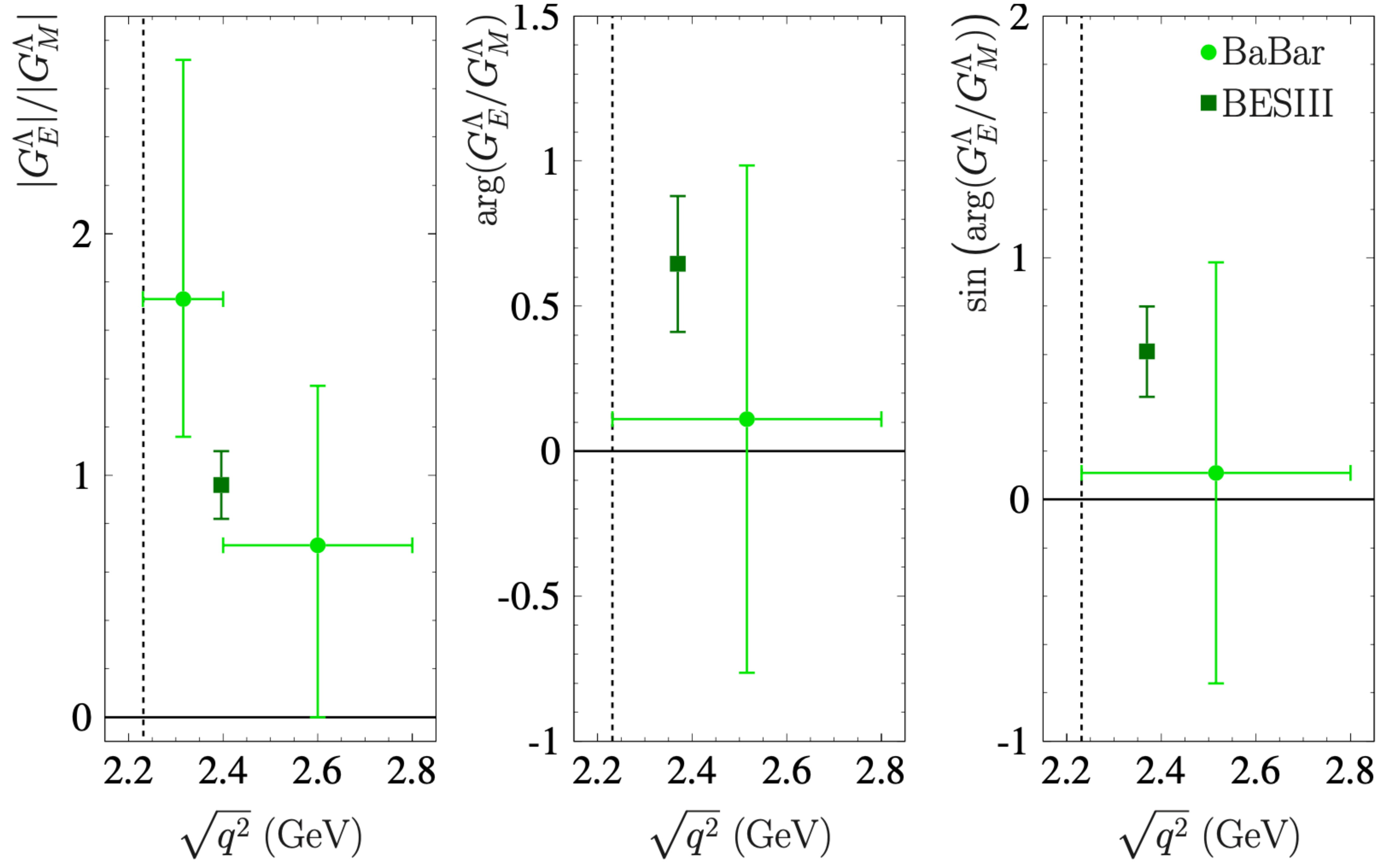
Available data

Total available data at present:
5 data points

Actually we have only two sets of data
from BESIII (2019) and BaBar (2006) experiments

- 1 datum on modulus of $G_E^\Lambda / G_M^\Lambda$
- 1 datum on phase of $G_E^\Lambda / G_M^\Lambda$

- 2 data on modulus of $G_E^\Lambda / G_M^\Lambda$
- 1 datum on phase of $G_E^\Lambda / G_M^\Lambda$



Data for the ratio:

$$\{q_j^2, |R_j|, \delta|R_j|\}_{j=1}^{M}$$

$$M = 3$$

Data for the phase:

$$\{q_k^2, \sin(\phi_k), \delta\sin(\phi_k)\}_{k=1}^{P}$$

$$P = 2$$

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Theoretical aspects

$R(z)$: analytic multivalued function, defined in the domain D and with real brunch cut (x_0, ∞)

Dispersion relations for real and imaginary part

$$R(z) \in \mathbb{R}, \forall z \in D \cap \mathbb{R}$$

$$R(z) = o(1/\ln(|z|)) \qquad z \to \infty$$

$$R(z) \propto_{z \to x_0} (z - x_0)^{\alpha}, \quad \mathrm{Re}(\alpha) > -1$$

$$f(z) = \frac{1}{\pi} \int_{x_0}^{\infty} \frac{\mathrm{Im}(f(x))}{x - z} dx$$

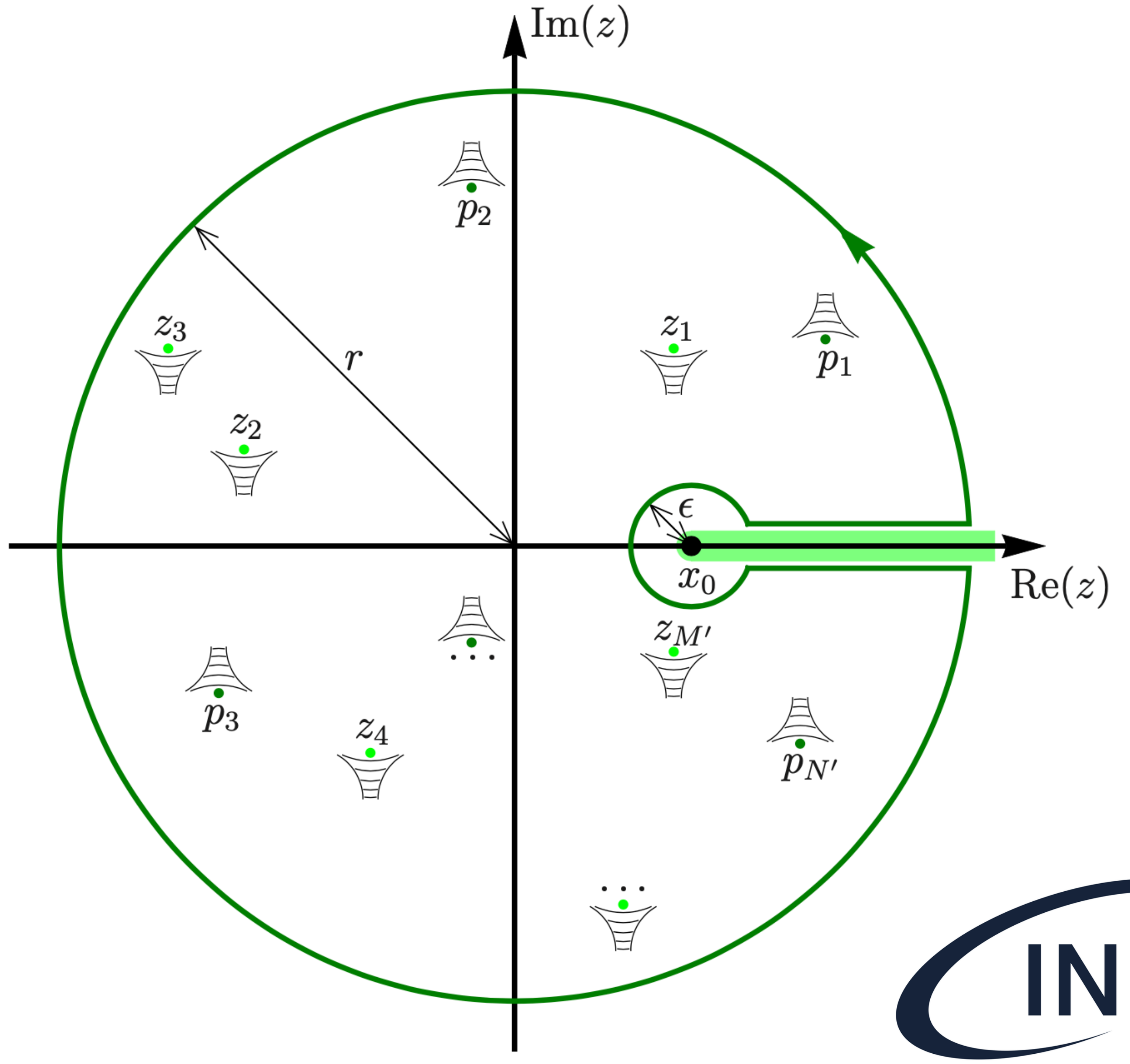
One-subtracted DR at x_1, when $R(z) = \mathcal{O}(1) \qquad z \to \infty$

$$f(z) = f(x_1) + \frac{z - x_1}{\pi} \int_{x_0}^{\infty} \frac{\mathrm{Im}(f(x))}{(x - x_1)(x - z)} dx$$

$$x_0 > x_1 \in \mathbb{R}$$

Levinson's theorem

$$\arg(R(\infty)) - \arg(R(x_0)) = \pi(M - N)$$



M: # of zeros
N: # of poles

The G_E^Λ/G_M^Λ ratio

G_E^Λ and G_M^Λ form factors → Analyticity domain $D = \{z \in \mathbb{C} : z \notin (q_{\rm th}^2, \infty)\}$ → $R = \frac{G_E^\Lambda}{G_M^\Lambda}$ ratio

Multivalued meromorphic function with:

- Brunch cut $(q_{\rm th}^2, \infty)$
- A set of isolated poles
- Schwarz reflection principle: $R^*(q^2) = R(q^{2*})$
- Same asymptotic behavior in space-like and time-like regions

$R(q^2) = \mathcal{O}(1) \quad |q^2| \to \infty$

Under the hypothesis that G_M^Λ has no zeros

- **Domain** $D = \{z \in \mathbb{C} : z \notin (q_{\rm th}^2, \infty)\}$
- **Same zeros as G_E^Λ**
- **Has at least 1 zero in the origin, being $G_E^\Lambda(0) = Q_\Lambda = 0$**
- **Fulfills the requirement of the dispersion relation**

The model

Parametrization for the imaginary part of the ratio

$R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$ in terms of Chebyshev polynomials T_j

$$\mathrm{Im}\left[R(q^2)\right] \equiv Y\left(q^2; \vec{C}, q^2_{\mathrm{asy}}\right)$$

$$Y\left(q^2; \vec{C}, q^2_{\mathrm{asy}}\right) = \begin{cases} \sum_{j=0}^{N} C_j T_j\left[x(q^2)\right] & q^2_{\mathrm{th}} < q^2 < q^2_{\mathrm{asy}} \\ 0 & q^2 \geq q^2_{\mathrm{asy}} \end{cases}$$

$$x(q^2) = 2\frac{q^2 - q^2_{\mathrm{th}}}{q^2_{\mathrm{asy}} - q^2_{\mathrm{th}}} - 1$$

$$\mathrm{Re}[R(q^2)] = \frac{q^2}{\pi}\mathrm{Pr}\int_{q^2_{\mathrm{th}}}^{q^2_{\mathrm{asy}}} \frac{\mathrm{Im}[R(s)]}{s(s-q^2)}ds$$

$$\begin{cases} R(q^2_{\mathrm{th}}) \in \mathbb{R} & \Rightarrow\ Y\left(q^2_{\mathrm{th}}; \vec{C}, q^2_{\mathrm{asy}}\right) = 0 \\ R(q^2_{\mathrm{phy}}) \in \mathbb{R} & \Rightarrow\ Y\left(q^2_{\mathrm{phy}}; \vec{C}, q^2_{\mathrm{asy}}\right) = 0 \\ R(q^2 \geq q^2_{\mathrm{asy}}) \in \mathbb{R} & \Rightarrow\ Y\left(q^2 \geq q^2_{\mathrm{asy}}; \vec{C}, q^2_{\mathrm{asy}}\right) = 0 \end{cases}$$

Theoretical constraints

$N+2$ **free parameters:**
$\vec{C} = \{C_0, C_1, \ldots, C_N\}$ and the asymptotic threshold $q^2_{\mathrm{asy}} \in (q^2_{\mathrm{phy}}, \infty)$

$N-1$ **independent parameters:** $\{C_3, C_4, \ldots, C_N, q^2_{\mathrm{asy}}\}$
(degree of freedom for the parametrization)

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The model

$$R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$$

Pr stands for Principal Value

$$X(q^2) = \mathrm{Re}[R(q^2)] = \frac{q^2}{\pi}\mathrm{Pr}\int_{q_{th}^2}^{q_{asy}^2} \frac{\mathrm{Im}[R(s)]}{s(s-q^2)}ds$$

Constraints for $\mathrm{Re}[R(q^2)]$

$$\chi_{\mathrm{phy}}^2 = \left(1 - X(q_{\mathrm{phy}}^2)\right)^2$$

$$\chi_{\mathrm{asy}}^2 = \left(1 - X(q_{\mathrm{asy}}^2)^2\right)^2$$

with weights τ_{phy}, τ_{asy}

χ^2 definition

$$\chi^2\left(\vec{C}, q_{\mathrm{asy}}^2\right) = \chi_{|R|}^2 + \chi_{\phi}^2 + \tau_{\mathrm{phy}}\chi_{\mathrm{phy}}^2 + \tau_{\mathrm{asy}}\chi_{\mathrm{asy}}^2 + \tau_{\mathrm{curv}}\chi_{\mathrm{curv}}^2$$

$$\chi_{|R|}^2 = \sum_{j=1}^{M}\left(\frac{\sqrt{X\left(q_j^2\right)^2 - Y\left(q_j^2\right)^2} - |R_j|}{\delta|R_j|}\right)^2 \qquad \chi_{\phi}^2 = \sum_{k=1}^{P}\left(\frac{\sin\left(\arctan\left(Y\left(q_k^2\right)/X\left(q_k^2\right)\right)\right) - \sin(\phi_k)}{\delta\sin(\phi_k)}\right)^2$$

$\tau_{\mathrm{curv}}\chi_{\mathrm{curv}}^2$ is a contribution to stabilize the solution, with $\chi_{\mathrm{curv}}^2 = \int_{q_{\mathrm{th}}^2}^{q_{\mathrm{asy}}^2}\left|\frac{d^2Y(s)}{ds^2}\right|^2 ds$

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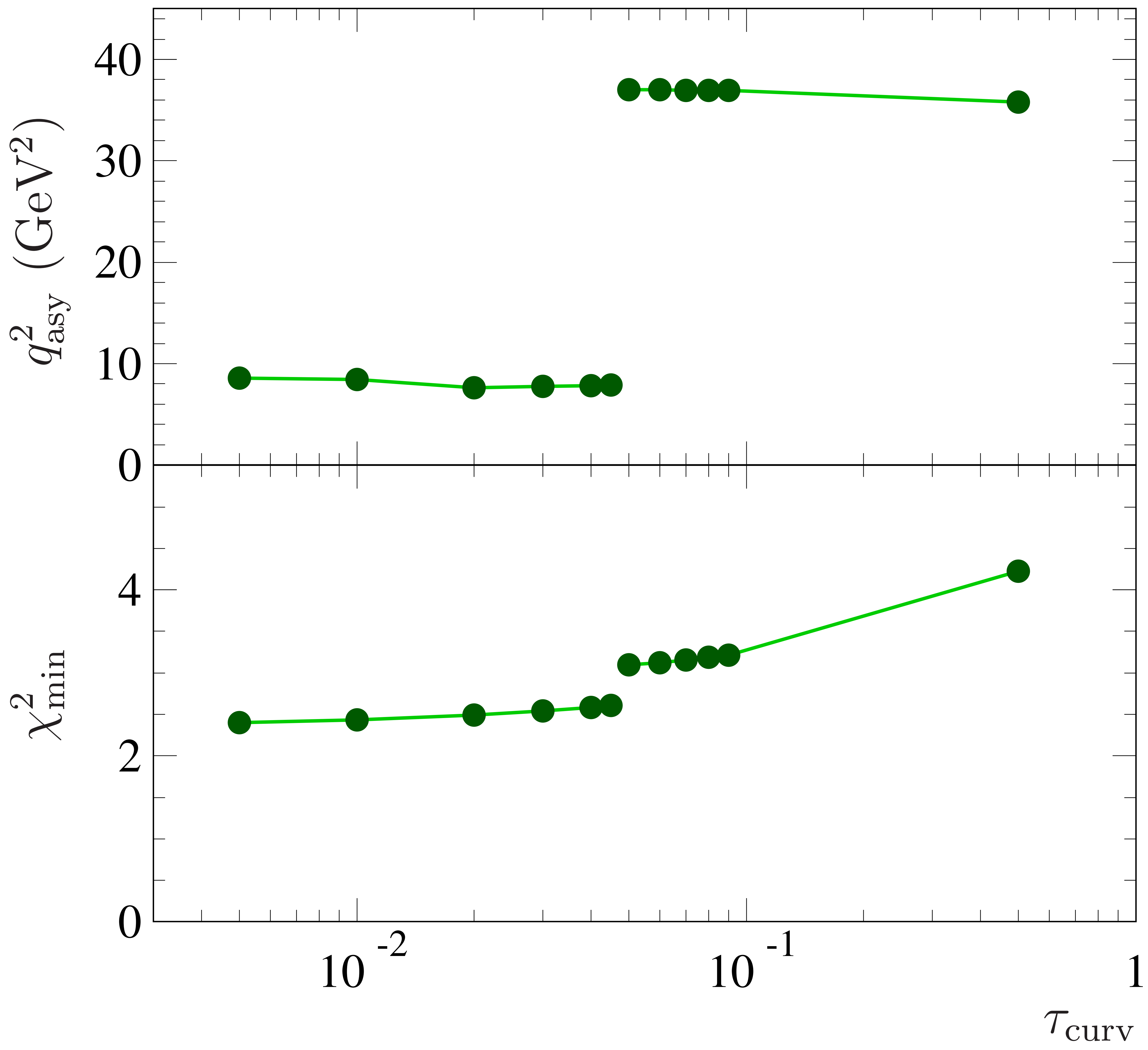
The model

We have to choose N and $\tau_{\rm curv}$ balancing the increase of the total curvature as N increases and the suppression of the oscillations as $\tau_{\rm curv}$ increases

Best values of $q^2_{\rm asy}$ and the corresponding χ^2 minima for $N = 5$ as a function of $\tau_{\rm curv}$



Final values chosen:

$N = 5$ and $\tau_{\rm curv} = 0.05$

Results

The ratio $R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$ **is real** for $q^2 = q_{\rm th}^2$ and $q^2 = q_{\rm asy}^2$

$$N_{\rm th} = \frac{1}{\pi}\arg[R(q_{\rm th}^2)]$$

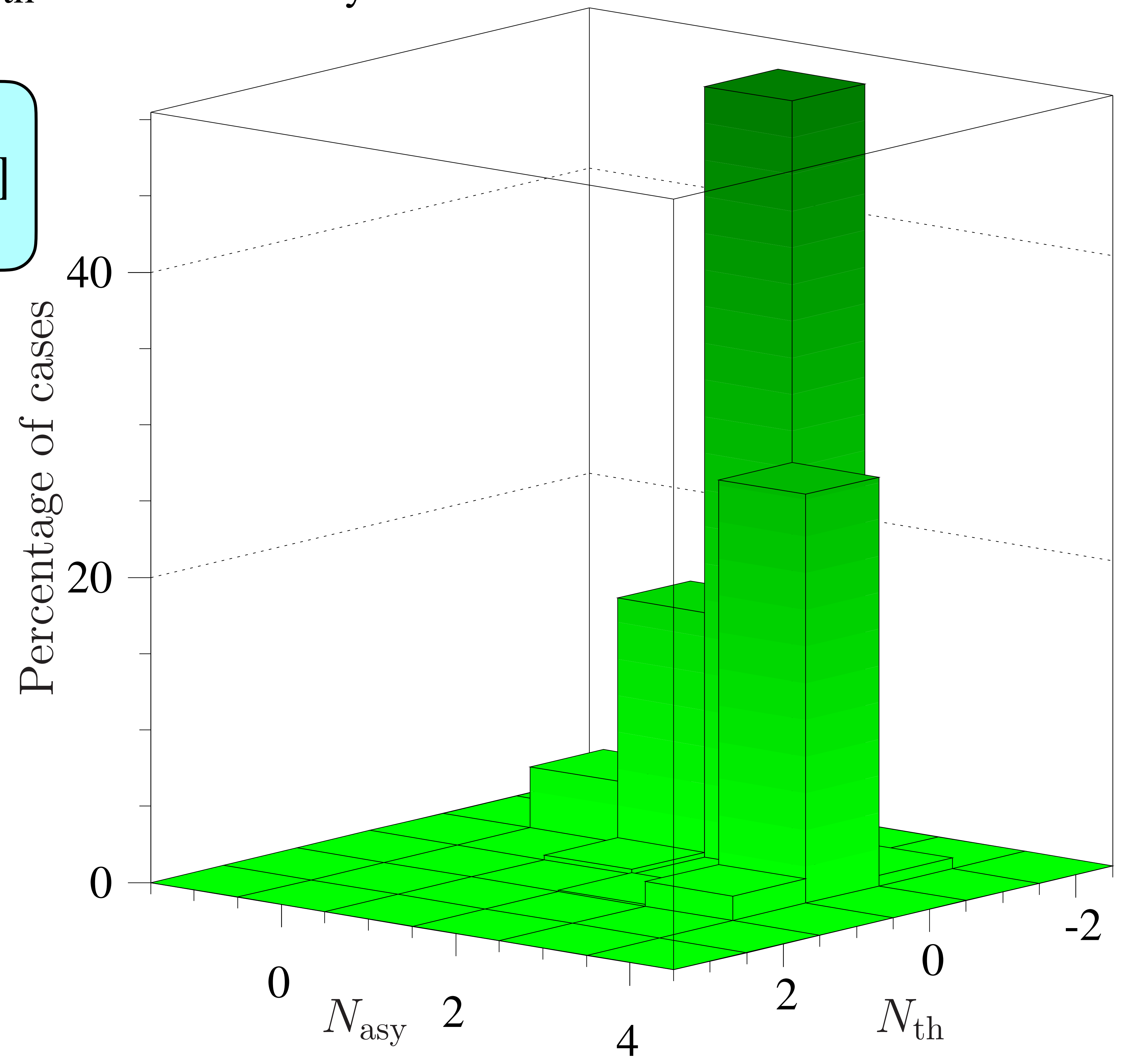
$N_{\rm th} \in \mathbb{N}$ $N_{\rm asy} \in \mathbb{N}$

$$N_{\rm asy} = \frac{1}{\pi}\arg[R(q_{\rm asy}^2)]$$

Using the available data we obtain the results shown in table, where the probability is given by a Monte Carlo procedure

$N_{\rm th}$	$N_{\rm asy}$	%	Visual percentage
-1	0	4.0	
-1	1	16.0	
-1	2	50.5	
-1	3	0.7	
0	1	0.3	
0	3	26.8	
1	2	0.1	
1	3	1.6	

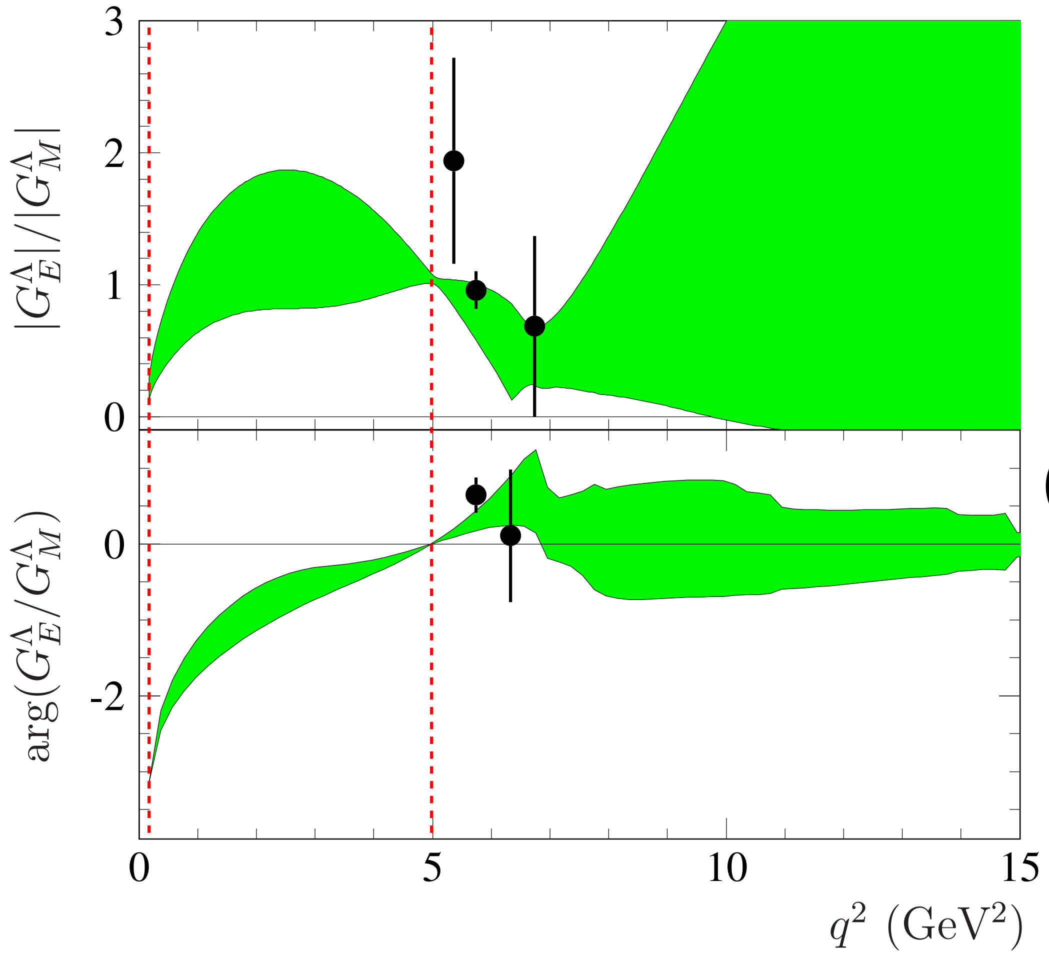
6 relevant cases



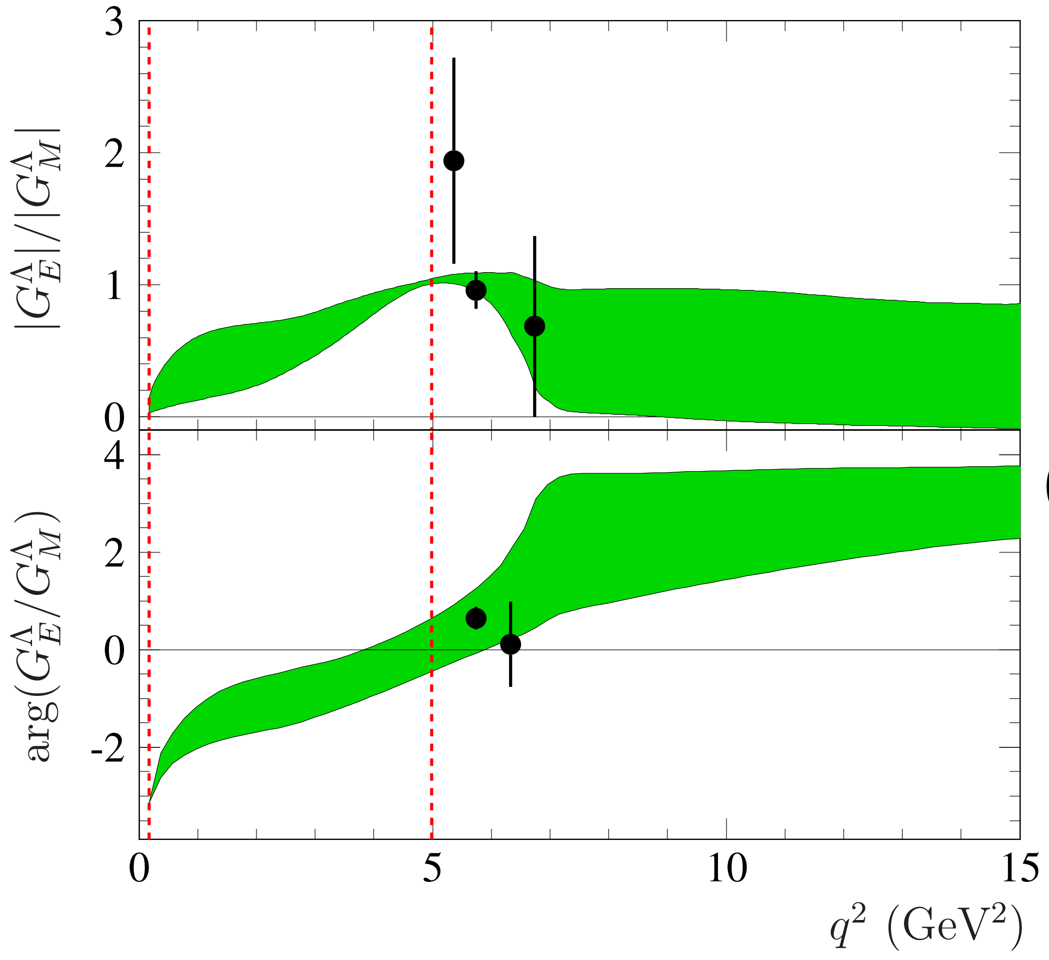
Results

Modulus and phase of R for the six relevant cases

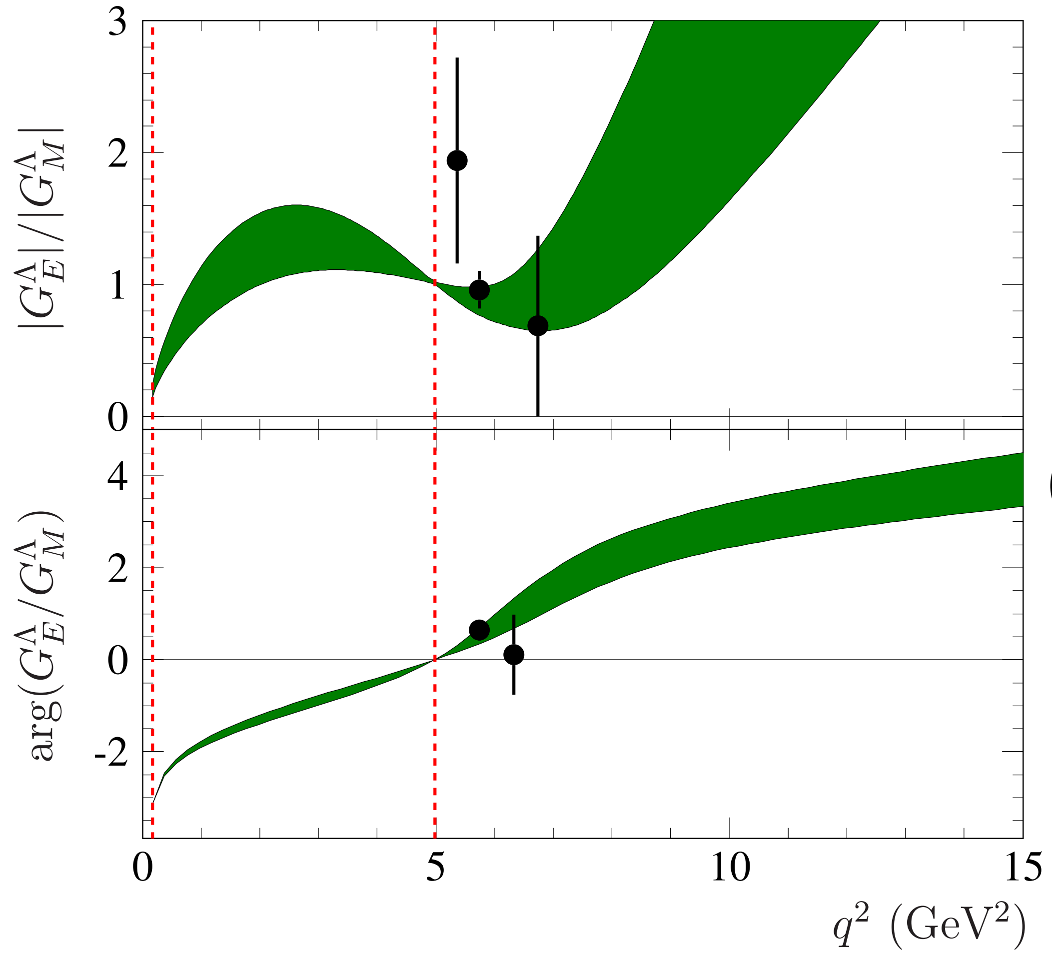
$$(N_{\text{th}}, N_{\text{asy}}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$$



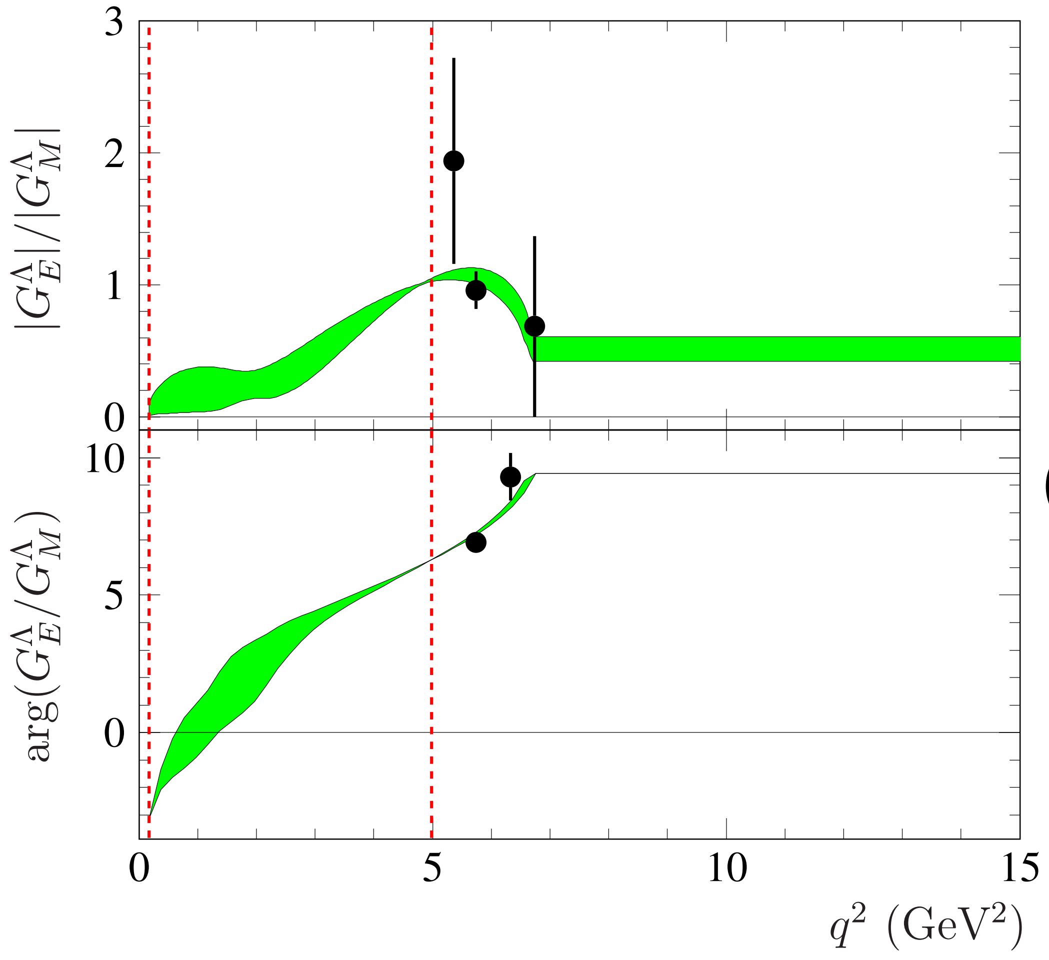
(−1,0)
4 %



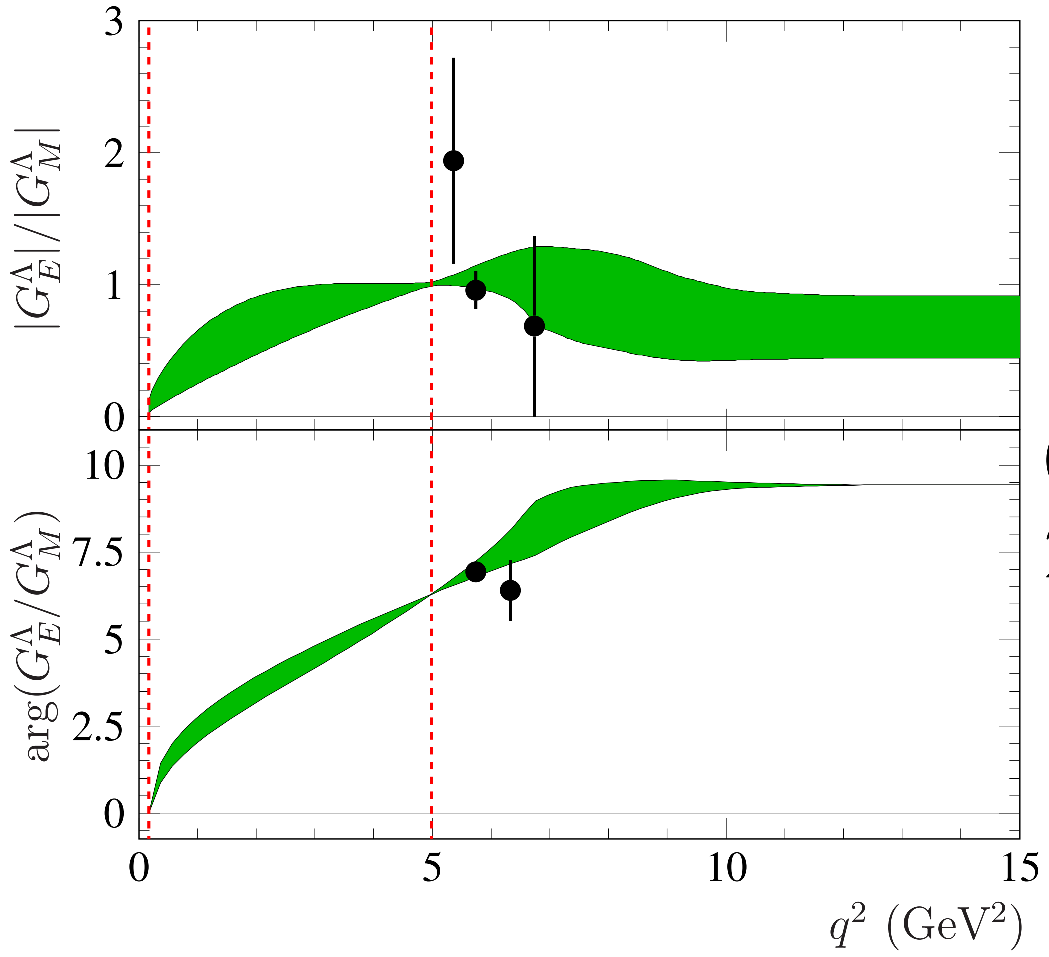
(−1,1)
16 %



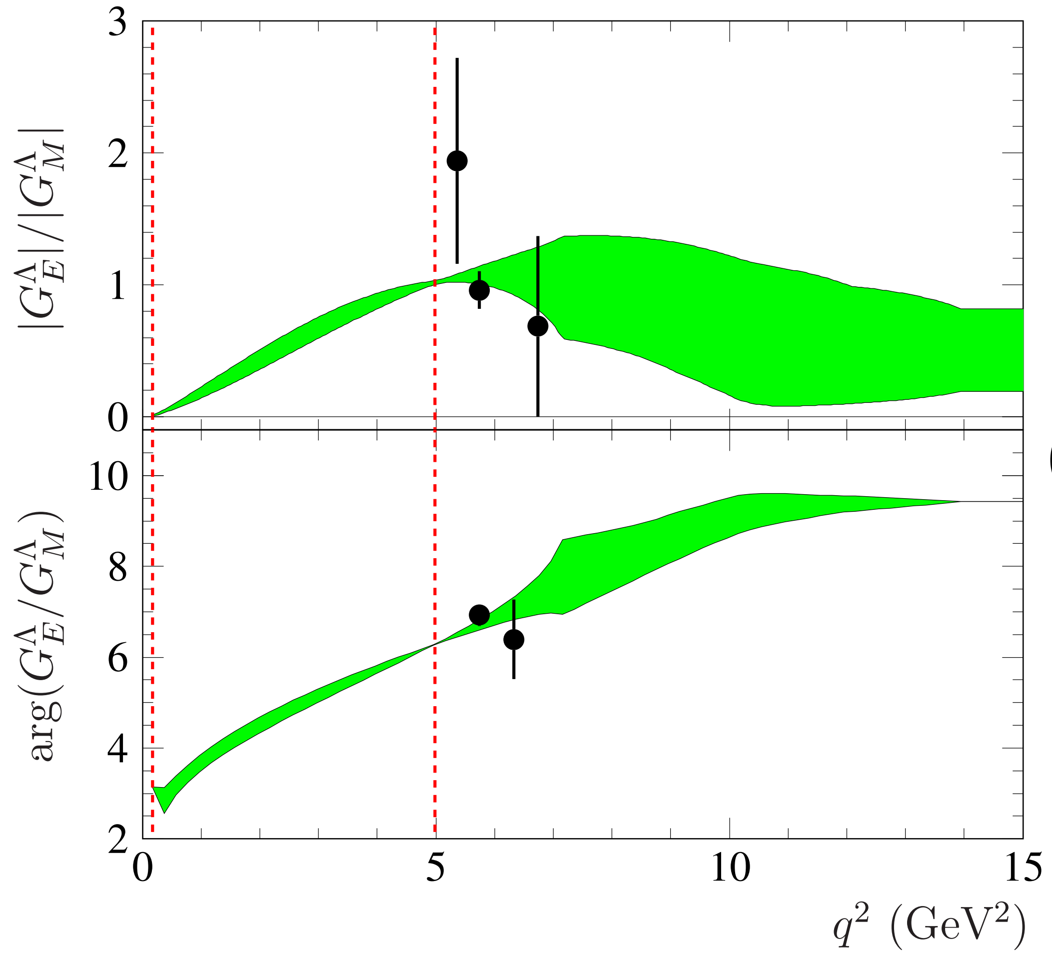
(−1,2)
50.5 %



(−1,3)
0.7 %



(0,3)
26.8 %



(1,3)
1.6 %

Results

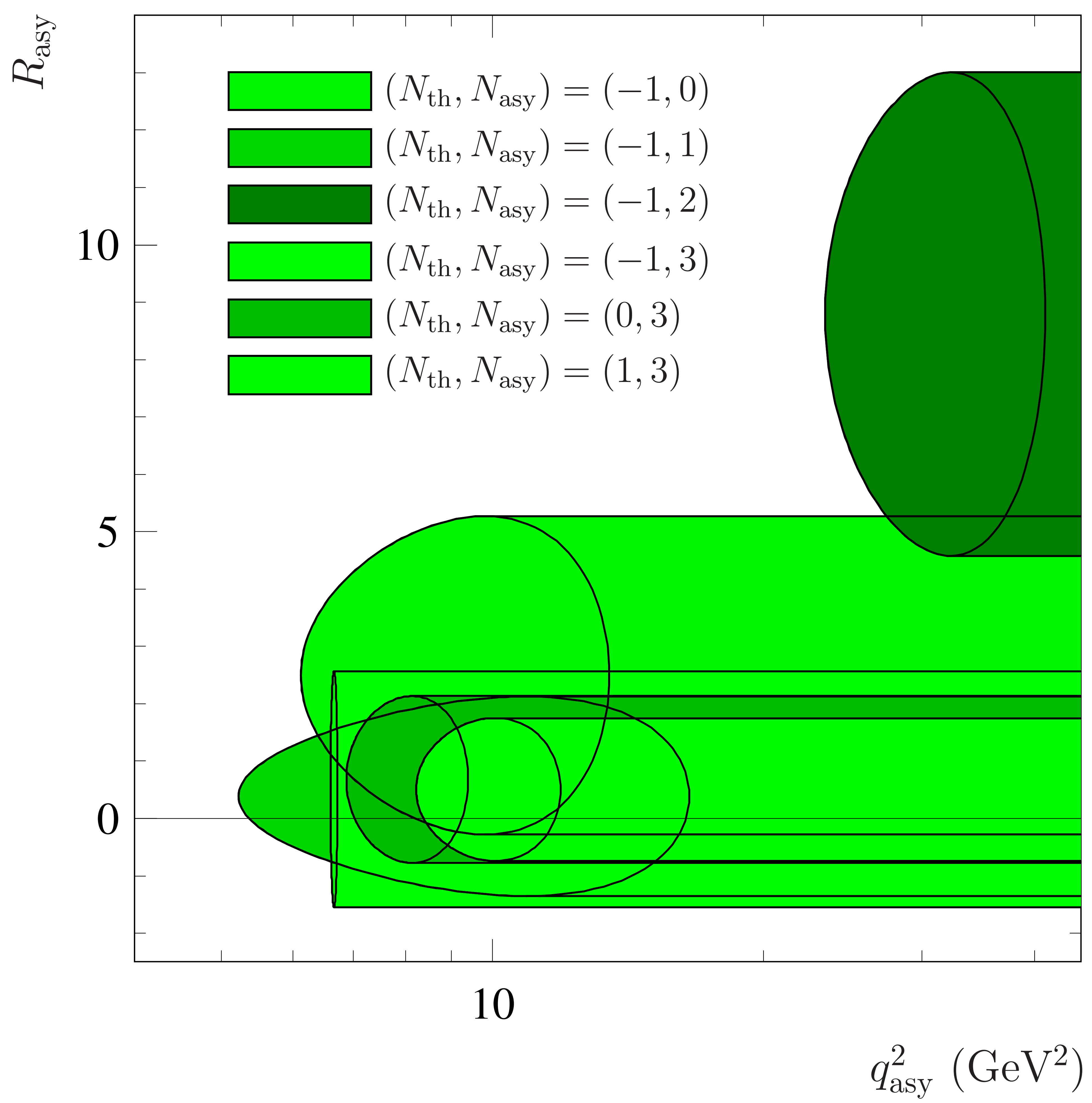
Asymptotic behavior for the six relevant cases

$(N_{th}, N_{asy}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$

We obtain the asymptotic threshold $(q^2_{asy} \pm \delta q^2_{asy})$ and the corresponding values of the modulus of the ratio $(R_{asy} \pm \delta R_{asy})$

$$R_{asy} = |R(q^2_{asy})|$$

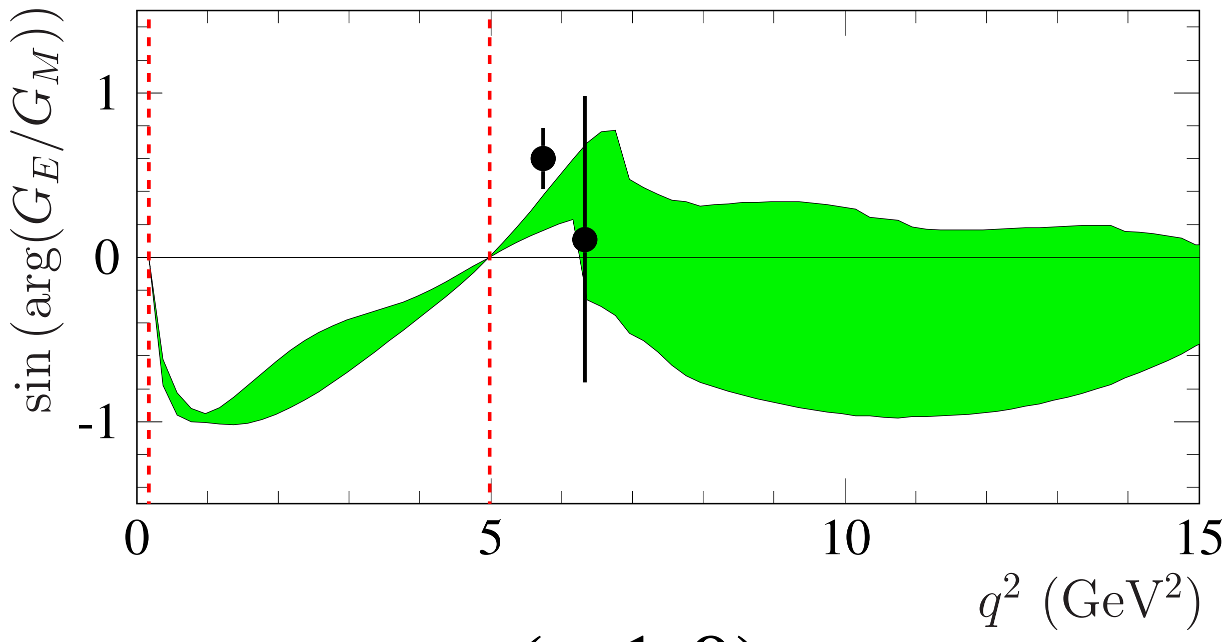
The knowledge of $|R(q^2)|$ at higher time-like q^2 could gives hints to choose or neglect the case $(-1, -2)$



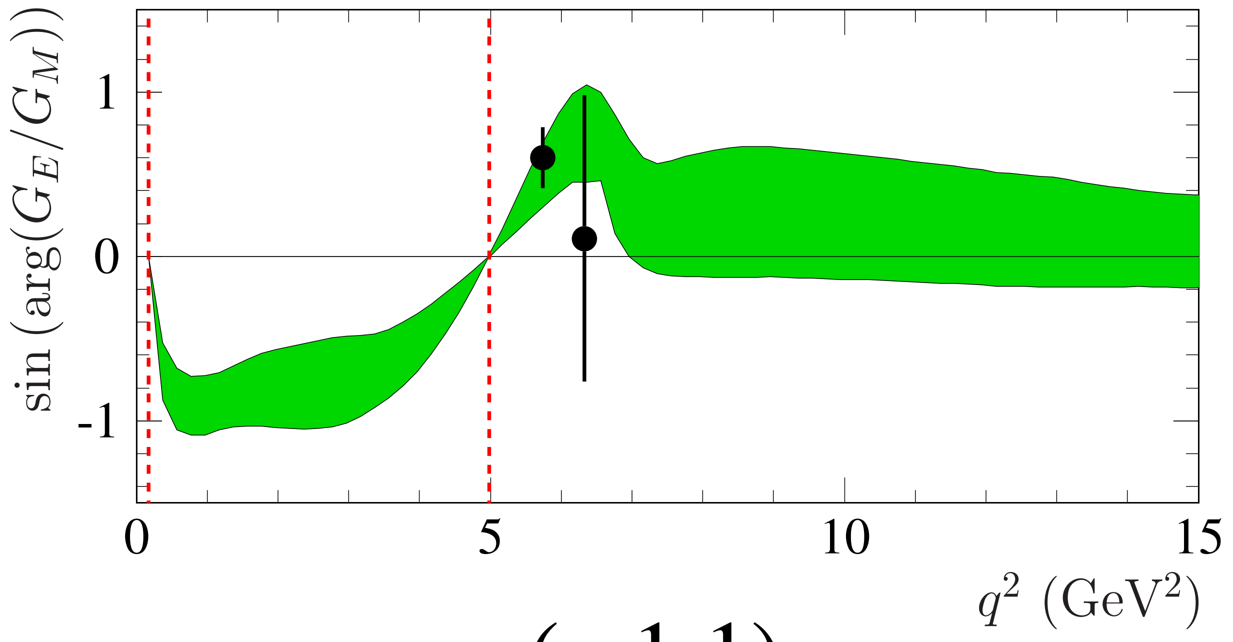
Results

Sinus of the phase of R for the six relevant cases

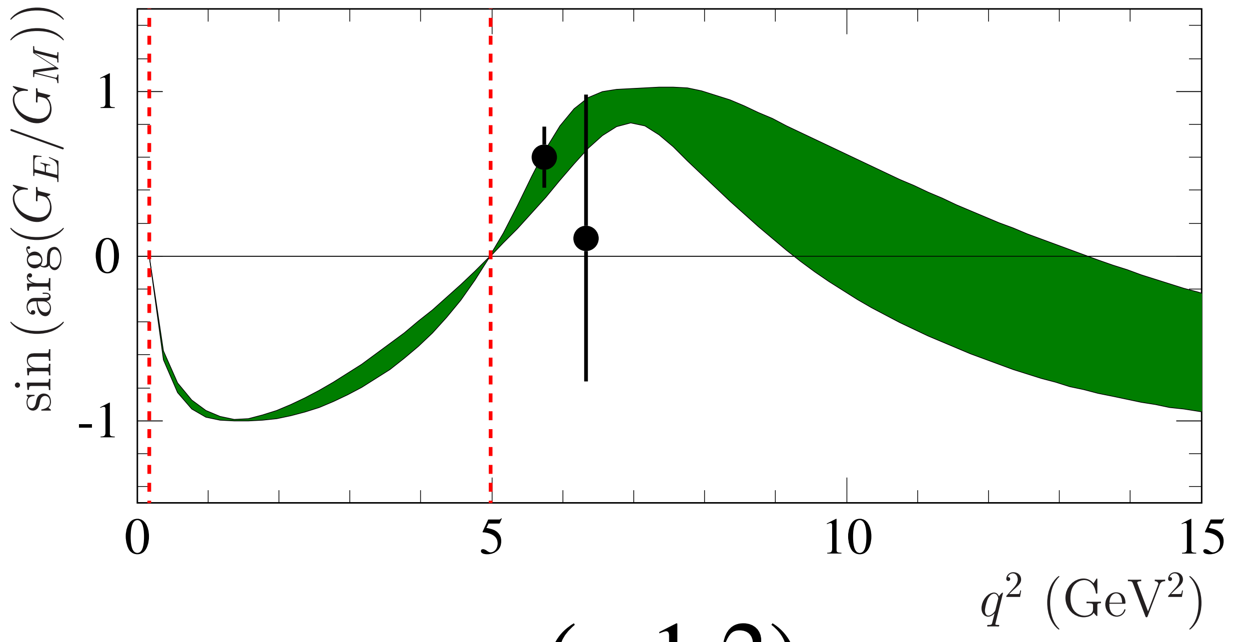
$(N_{\text{th}}, N_{\text{asy}}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$



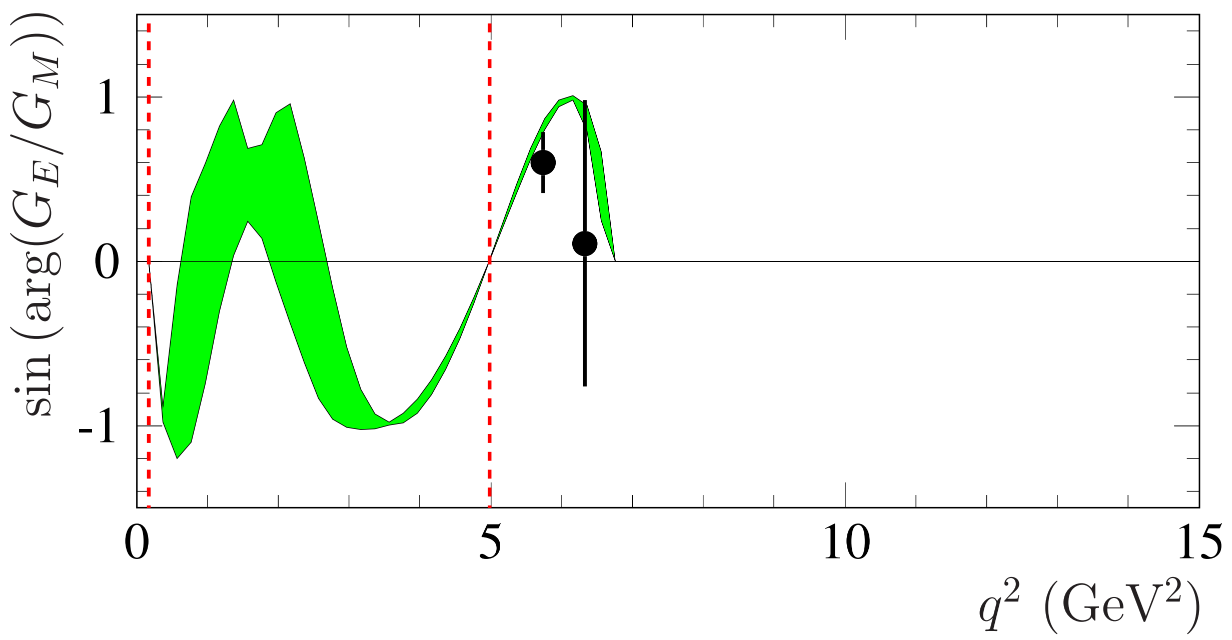
$(-1,0)$
4 %



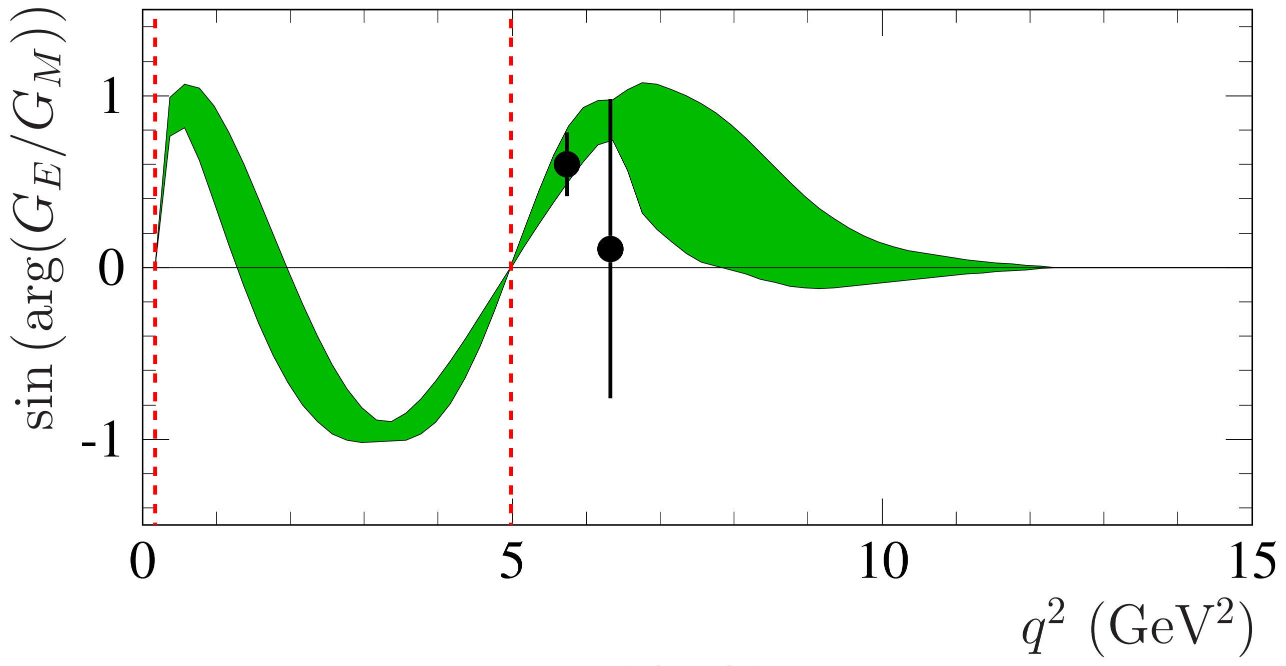
$(-1,1)$
16 %



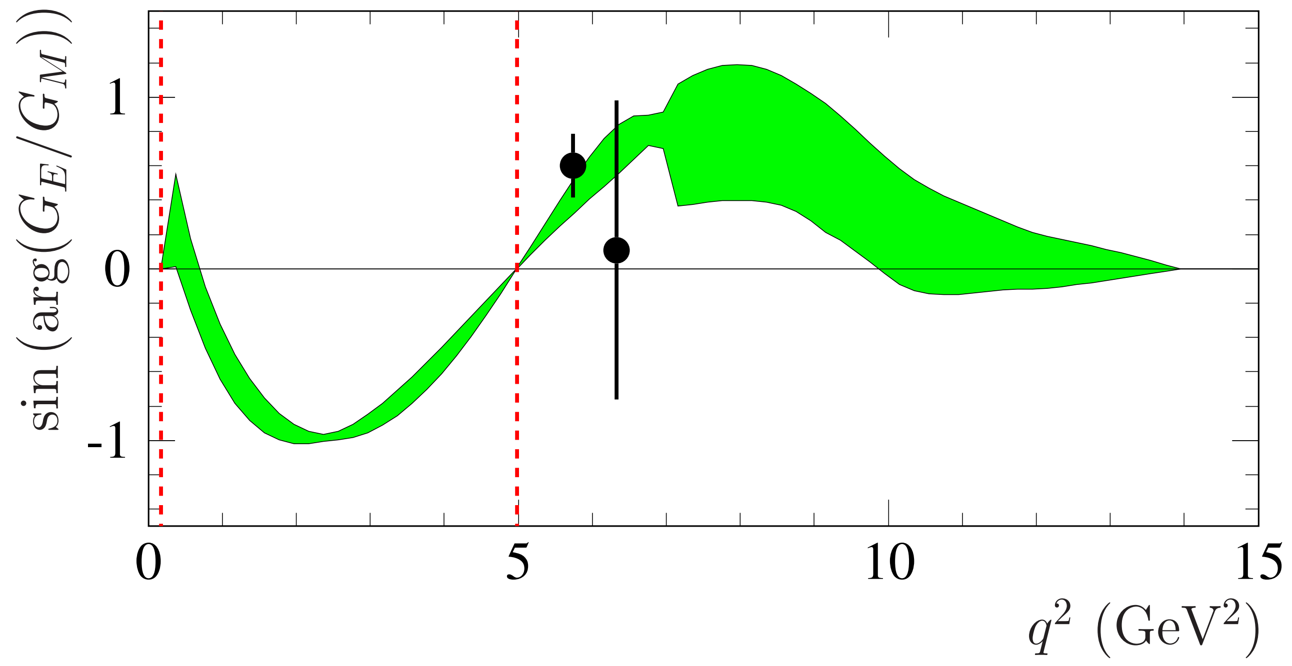
$(-1,2)$
50.5 %



$(-1,3)$
0.7 %



$(0,3)$
26.8 %



$(1,3)$
1.6 %

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Results

The complete knowledge of the ratio $R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$ can be used to calculate the the so-called charge root-mean square radius $\langle r_E \rangle$

$$\langle r_E \rangle^2 = 6 \frac{dG_E^\Lambda(q^2)}{dq^2}\bigg|_{q^2=0}$$

For the Λ baryon, being $G_E^\Lambda(0) = 0$ and $G_M^\Lambda(0) = \mu = (-0.613 \pm 0.004)\mu_N \neq 0$

$$\begin{aligned} \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0} &= \frac{1}{G_M(q^2)}\left(\frac{dG_E(q^2)}{dq^2} - \frac{G_E(q^2)}{G_M(q^2)}\frac{dG_M(q^2)}{dq^2}\right)\bigg|_{q^2=0}, \\ &= \left(\frac{1}{G_M(q^2)}\frac{dG_E(q^2)}{dq^2}\right)\bigg|_{q^2=0} = \frac{1}{\mu}\frac{\langle r_E \rangle^2}{6} \end{aligned}$$

$$\langle r_E \rangle^2 = 6\mu \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0}$$

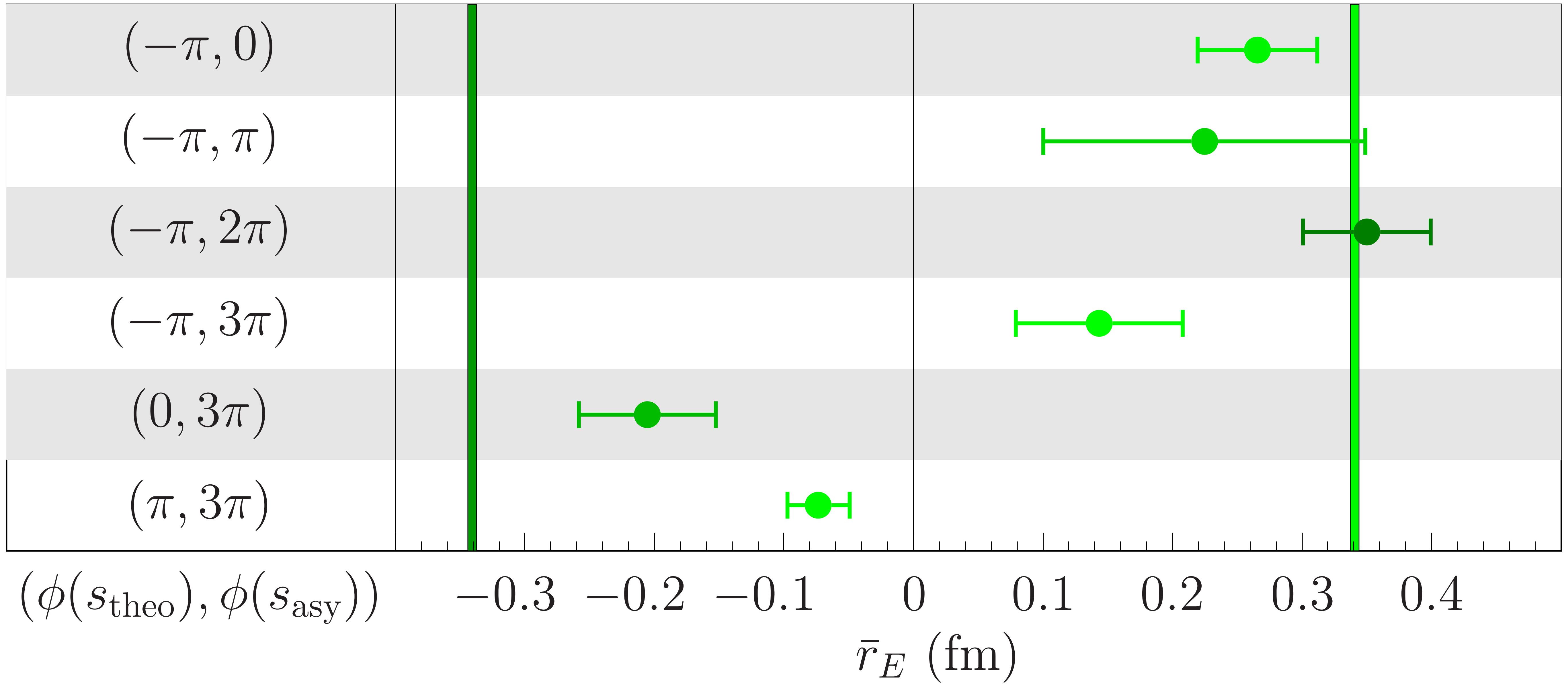
Results

Using the following expression from our model

$$\left.\frac{dR(q^2)}{dq^2}\right|_{q^2=0} = \frac{1}{\pi}\int_{q^2_{\rm th}}^{\infty}\frac{{\rm Im}\,(R(s))}{s^2}ds = \frac{1}{\pi\Delta q^2}\sum_{j=0}^{N} C_j\int_{-1}^{1}\frac{T_j\,(x)}{\left(x+1+q^2_{\rm th}/\Delta q^2\right)^2}dx$$

$$\Delta q^2 = (q^2_{\rm asy} - q^2_{\rm th})/2$$

$$\langle r_E^{\Lambda}\rangle^2 = \frac{6\mu_{\Lambda}}{\pi\Delta q^2}\sum_{j=0}^{N} C_j\int_{-1}^{1}\frac{T_j\,(x)}{\left(x+1+q^2_{\rm th}/\Delta q^2\right)^2}dx$$



the symmetric vertical green bars indicate the negative normalized neutron charge radius and its reflection

Conclusions

- We propose a phenomenological model based on first principles, as analyticity, to study the Λ baryon electromagnetic FFs, G_E^Λ and G_M^Λ and their ratio $R = G_E^\Lambda / G_M^\Lambda$
- Thanks to the Levinson's theorem, in our case, the difference $(N_{\rm asy} - N_{\rm th})$ gives the total number of zeros for R and, hence, for G_E^Λ, assuming $G_M^\Lambda \neq 0$

$$N_{\rm th,asy} = \frac{1}{\pi} \arg[R(q^2_{\rm th,asy})]$$

- We determine, **for the first time**, the complex structure of the ratio knowing the experimental values of its modulus and phase
- Despite the few available data, the model allows to select the following acceptable six cases (with a probability $> 0.5\,\%$): $(N_{\rm th}, N_{\rm asy}) = (-1{,}0), (-1{,}1), (-1{,}2), (-1{,}3), (0{,}3), (1{,}3)$
- The model gives information about the number of space-like zeros and the determination of the phase that is not directly accessible by experiments (the sinus of the phase is insensitive to its determination)

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