Heavy Quark Momentum Diffusion Coefficient from the Lattice

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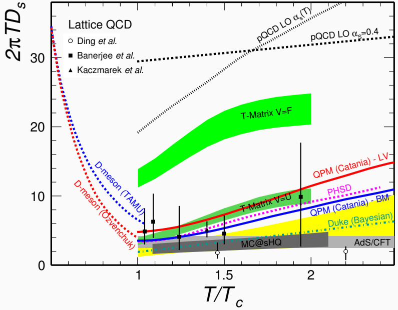
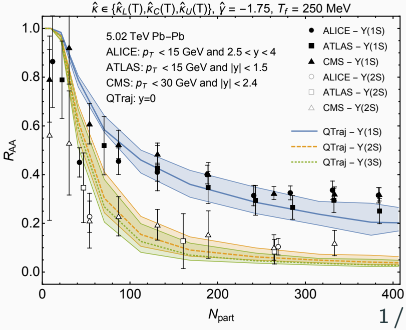
Motivation for the Diffusion Coefficient

- Nuclear modification factor R_{AA} and the elliptic flow ν_2 described by spatial diffusion coefficient D_x
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}





UP: X. Dong CIPANP (2018)
DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, JHEP 05 (2021) 136

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  $\rightarrow$ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t - t')$$

- Heavy quark momentum diffusion coefficient κ related to many interesting phenomena

Such as: Spatial diffusion coefficient $D_{\mathrm{s}} = 2T^2/\kappa$,
Drag coefficient $\eta_{\mathrm{D}} = \kappa/(2MT)$,
Heavy quark relaxation time $\tau_{\mathrm{Q}} = \eta_{\mathrm{D}}^{-1}$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Quarkonium in medium

- Quarkonium in QGP (environment energy scale πT)

$$M \gg \frac{1}{a_0} \gg \pi T \gg E\,, \quad \tau_R \gg \tau_E \sim 1/\pi$$

- HQ mass M, Bohr radius a_0, binding energy E, correlation time τ_E
- Quarkonium in fireball can be described by Limbland equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm}\left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\}\right)$$

- All terms depend on two free parameters κ and γ
- κ turns out to be the heavy quark diffusion coefficient and related to thermal width $\Gamma(1S) = 3a_0^2\kappa$
- γ is correction to the heavy quark-antiquark potential and related to mass shift $\delta M(1S) = 2a_0^2\gamma/3$
- Unquenched lattice measurements of $\Gamma(1S)$ and $\delta M(1S)$ available

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018)

κ from perturbation theory

- From kinetic theory we can derive

$$\kappa^{\rm LO} = \frac{g^4 C_{\rm F}}{12\pi^3} \int_0^\infty q^2 {\rm d}q \int_0^{2q} \frac{p^3 {\rm d}p}{(p^2 + \Pi_{00}^2)^2} \times N_{\rm c}\, n_{\rm B}(q)(1+n_{\rm B}(q)) \left(2 - \frac{p^2}{q^2} + \frac{p^4}{4q^4}\right)$$

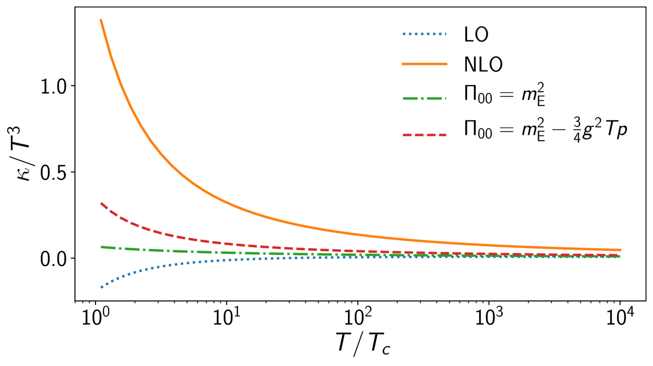
- Solution depends on assumptions
  - Is $m_{\rm E} \ll T$
  - How to expand the temporal gluon self-energy $\Pi_{00} \simeq m_{\rm E}$
- Alternatively from Kubo formula:

$$\frac{\kappa}{T^3} = \frac{g^4 C_f N_c}{18\pi} \left[\left(\ln \frac{2T}{m_{\rm E}} + \xi \right) + \frac{m_{\rm E}}{T} C \right], \quad \xi \simeq -0.64718$$

- Truncated LO: $C = 0$, NLO: $C \simeq 2.3302$
- $N_f = 0$ assumed here

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

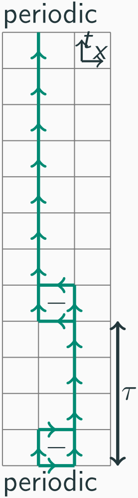
κ from perturbation theory



- Clearly $m_{\mathrm{E}} \ll T$ is too strict assumption on small T
- Huge perturbative variation
  $\Rightarrow$ needs non-perturbative measurements
- Also huge scale dependence trough $m_{\mathrm{E}} = g(\mu) T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy quark diffusion from lattice: Euclidean Correlator



- Traditional approach uses HQ curent-current correlators:
  Problem: Transport peak at zero
- HQEFT inspired Euclidean correlator is peak free

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,0)]\rangle}$$

- Chromoelectric field E needs discretization
- On Lattice E has non-physical self-energy contribution
  $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$
  (Christensen and Laine PLB02 (2016))
- To get momentum diffusion coefficient κ, a spectral function $\rho(\omega)$ needs to be reversed:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega, T)K(\omega,\tau T)\,, \qquad K(\omega,\tau T) = \frac{\cosh\left(\frac{\omega}{T}\left(\tau T - \frac{1}{2}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho(\omega)}{\omega}$$

Caron-Huot *et.al.*JHEP04 (2009) 053

Heavy quark diffusion from lattice: Spectral function

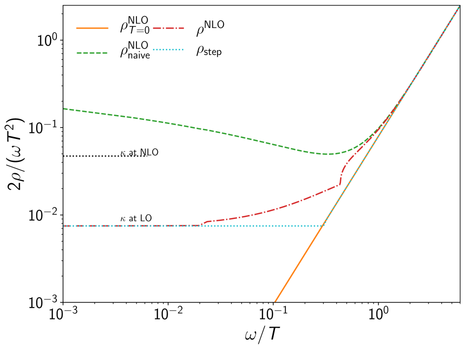
- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- Compare to ansatz: Trivial IR behavior ($\omega \ll T$)

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

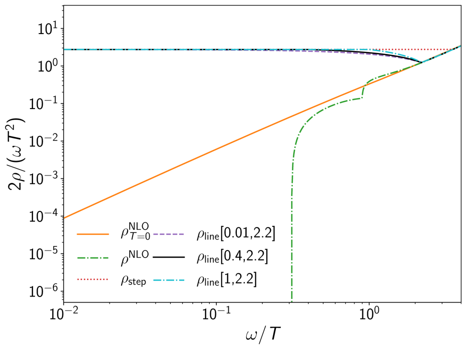
- Perturbative behavior (N)LO in UV ($\omega \gg T$):
- NLO $\rho(\omega)$ known from (Burnier *et.al.*JHEP08 (2010))
- Full HTL resummed NLO ρ over corrects and gives negative κ at small T
- Naive QCD NLO $\rho(\omega)$ diverges logarithmically:

$$\begin{aligned}
\rho_{\mathrm{QCD,naive}}(\omega) \quad = \quad & \frac{g^2 C_F \omega^3}{6\pi}\left\{1 + \frac{g^2}{(4\pi)^2}\left[N_{\mathrm{c}}\left(\frac{11}{3}\ln\frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3}\right)\right]\right\} \\
& + \frac{g^2 C_F}{6\pi}\frac{g^2}{2\pi^2}\left\{N_{\mathrm{c}}\int_0^\infty \mathrm{d}q\, n_{\mathrm{B}}(q)\left[(q^2 + 2\omega^2)\ln\left|\frac{q+\omega}{q-\omega}\right| + q\omega\left(\ln\frac{|q^2-\omega^2|}{\omega^2} - 1\right)\right.\right. \\
& \left.\left. + \frac{\omega^4}{q}\mathbb{P}\left(\frac{1}{q+\omega}\ln\frac{q+\omega}{\omega} + \frac{1}{q-\omega}\ln\frac{\omega}{|q-\omega|}\right)\right]\right\},
\end{aligned}$$

Spectral function behavior



$T = 10\,T_c$



$T = 1.1\,T_c$

- NLO spectral function works only at very high temperatures
- Try different models for $\omega \sim T$ behavior
- Instead of inverting integral equation, compare to ansatz

Lattice parameters

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$	T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$	T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
	12×48^3	6.407	1350		12×48^3	7.193	1579		12×48^3	8.211	1807
1.1	16×48^3	6.621	2623	3	16×48^3	7.432	1553	10	16×48^3	8.458	2769
	20×48^3	6.795	2035		20×48^3	7.620	1401		20×48^3	8.651	2073
	24×48^3	6.940	2535		24×48^3	7.774	1663		24×48^3	8.808	2423
	12×48^3	6.639	1801		12×48^3	7.774	1587		12×48^3	14.194	1039
1.5	16×48^3	6.872	2778	6	16×48^3	8.019	1556	10^4	16×48^3	14.443	1157
	20×48^3	7.044	2081		20×48^3	8.211	1258		20×48^3	14.635	1139
	24×48^3	7.192	2496		24×48^3	8.367	1430		24×48^3	14.792	1375
2.2	12×48^3	6.940	1535	2×10^4	12×48^3	14.792	1948				

- Quenched multilevel simulations

  Code from: Banerjee *et.al.* PRD85 (2012)

- 4 sublattices with 2000 updates
- Temperatures between $1.1\,T_c - 10^4\,T_c$
- Scale setting with

  (Francis *et.al.*PRD91 (2015))

- Other lattice results

  Meyer NJP13 (2011),

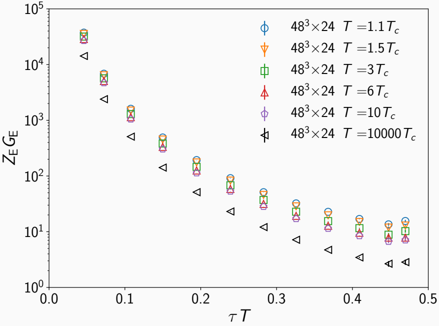
  Ding *et.al.*JPG38 (2011),

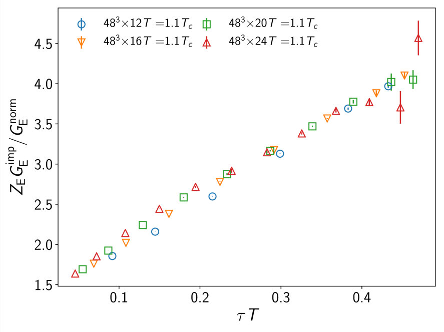
  Banerjee *et.al.* PRD85 (2012),

  Francis *et.al.* PRD92 (2015)

  Altenkort *et.al.* PRD103 (2021)

Lattice correlator



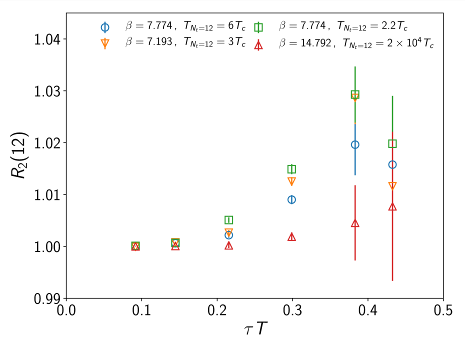


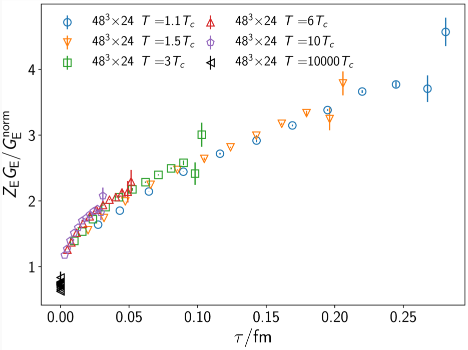
- Normalize lattice data with the LO Perturbative result:

$$G_{\mathrm{E}}^{\mathrm{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- Perform tree-level improvement by matching lattice and continuum perturbation theories

Caron-Huot *et.al.*JHEP04 (2009), Francis *et.al.*PoSLattice (2011)

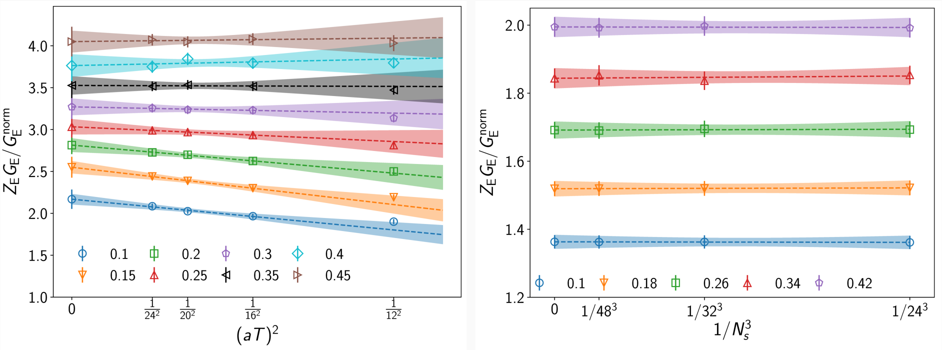




$$R_2(N_t) = \frac{G_\mathrm{E}(N_t,\beta)}{G_\mathrm{E}^\mathrm{norm}(N_t)} \Big/ \frac{G_\mathrm{E}(2N_t,\beta)}{G_\mathrm{E}^\mathrm{norm}(2N_t)} \,.$$

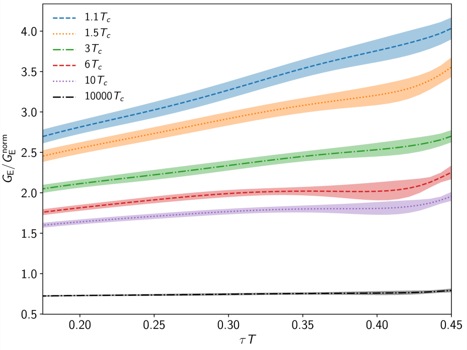
- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$, then grow

Continuum limit and finite size effects

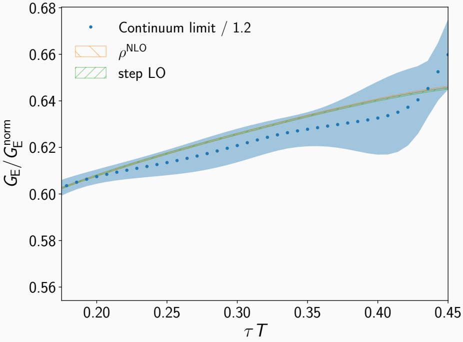


- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- $\chi^2/\mathrm{d.o.f.} < 5$ for $\tau T > 0.20$ when using 3 largest lattices (< 10 with $N_t = 12$)
- Finite size effects are negligible

Normalization of Continuum limit



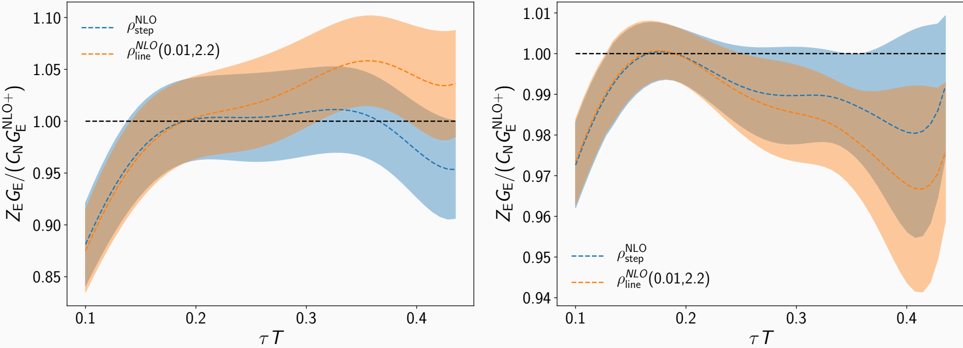
$T = 1.1\,T_c$



$T = 10^4\,T_c$

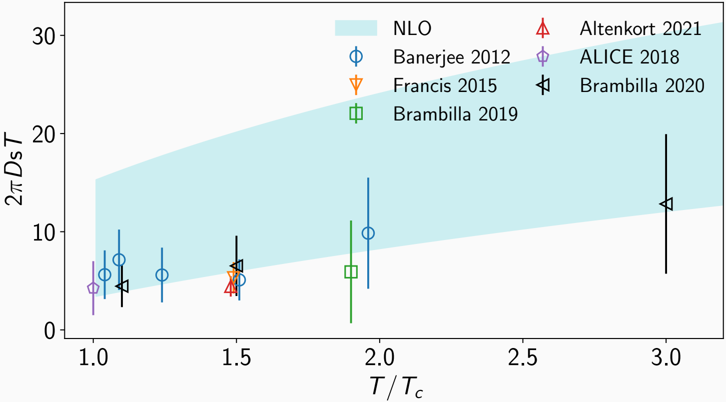
- Data needs additional normalization, do this at $\tau T = 0.19$
- Great agreement to perturbation theory at very high temperatures

κ extraction



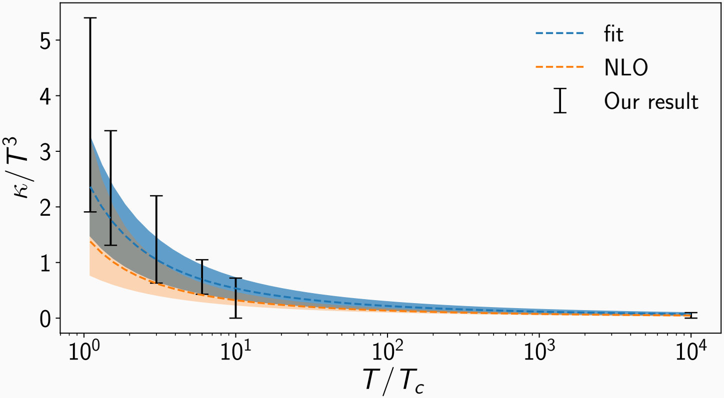
- Take continuum limit of the lattice data
- Normalize with different models for spectral function
- Extract κ as all values that normalize to unity in $0.19 \leq \tau T \leq 0.45$

Lattice results for D_s



- On low temperature close to T_{c}, agreement with other results, including ALICE

Lattice results for κ



- Unprecedented temperature range: $T = 1.1 - 10^4\,T_\mathrm{c}$

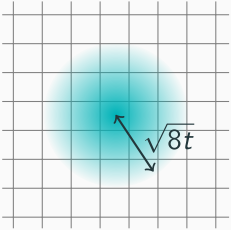
$$\frac{\kappa^{\mathrm{NLO}}}{T^3} = \frac{g^4 C_\mathrm{F} N_\mathrm{c}}{18\pi}\left[\ln\frac{2T}{m_\mathrm{E}} + \xi + C\frac{m_\mathrm{E}}{T}\right].$$

- Can fit temperature dependence $C = 3.81(1.33)$

Brambilla *et.al.* PRD102 (2020) 7, 074503 (hep-lat/2007.10078)

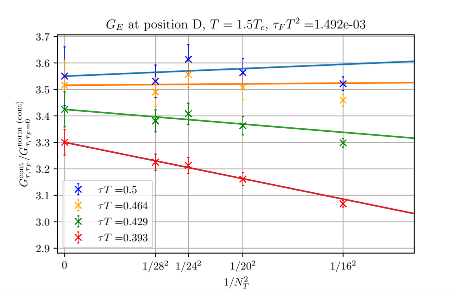
Alternative approach: Gradient flow

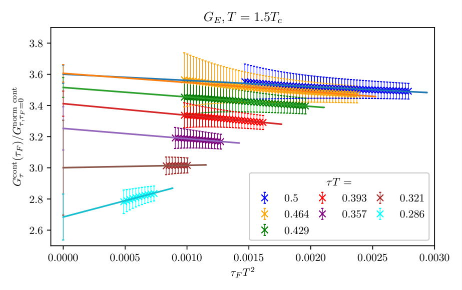
$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{the original gauge field}$$



- Evolve gauge along fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}
- Automatically renormalizes gauge invariant observables
- Zero flowtime limit $\mathcal{O}(x, t) \xrightarrow{t \to 0} \sum_j c_j(t) \mathcal{O}_j^R(x)$

Procedure





- Measure $G_{\mathrm{E}}(\tau_F, \tau)$, τ_F: flowtime, τ: E-field separation
- Take continuum limit
- Take zero flowtime limit.

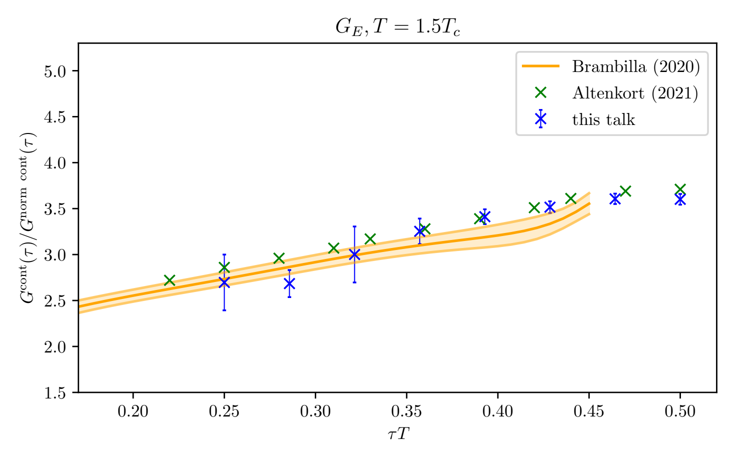
  Must be taken before solving $\rho(\omega)$: (Altenkort *et.al.* PRD103 (2021))

  Limit flow regime:

$$a \lesssim \sqrt{8\tau_F} \lesssim \frac{\tau - a}{3}$$

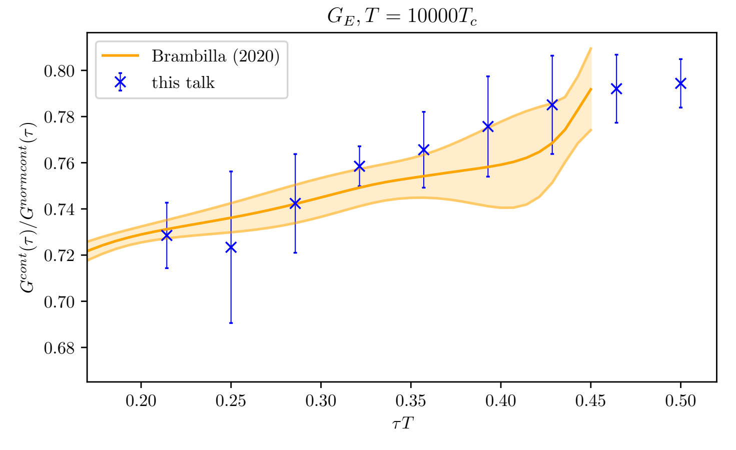
- Find κ trough $\rho(\omega)$. (In this talk we focus on Euclidean correlators)

G_{E} at $1.5\,T_{\mathrm{c}}$



- After continuum and zero flowtime limits, we replicate the previous studies:
  - Brambilla *et.al.* PRD102 (2020) Previous multilevel
  - Altenkort *et.al.* PRD103 (2021) Previous Gradient flow

G_{E} at $10^4\, T_{\mathrm{c}}$



- We see agreement with our previous multilevel result at large τ
- Small separations, larger lattices needed for continuum limit (in progress)

Mass-suppressed effects to HQ diffusion

- Considering full Lorentz force:

$$F(t) = \dot{p} = q\left(E + v \times B)\right)(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_{\rm tot} \simeq \kappa_{\rm E} + \frac{2}{3}\langle \mathrm{v}^2 \rangle \kappa_{\rm B}$$

- $\kappa_{\rm B}$ related to correlation of chromo magnetic fields:

$$G_{\rm B}(\tau) = \sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\ [U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\, U(1/T,0)\rangle}$$

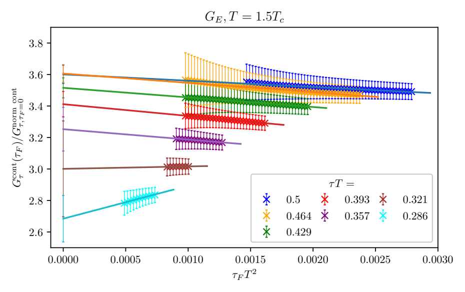
$$G_{\rm B}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\rm B}(\omega,T)K(\omega,\tau T)\ , \qquad \kappa_{\rm B} = \lim_{\omega\to 0}\frac{2T\rho_{\rm B}(\omega)}{\omega}$$

- Same tree level expansion as $G_{\rm E}$, NLO has divergence:

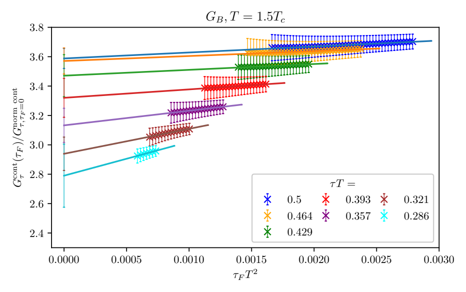
$$\rho_{\rm B} = \frac{g^2 C_f \omega^3}{6\pi}\left[1 - \frac{g^2 C_A}{(4\pi)^2}\frac{2}{\varepsilon} + (\mathrm{finite})\right] + \mathcal{O}(g^6)$$

A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Flowtime dependence of G_E and G_B at $1.5\,T_\mathrm{c}$



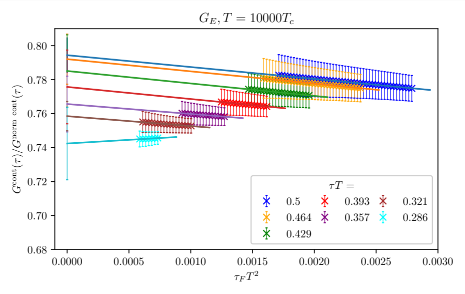
G_E



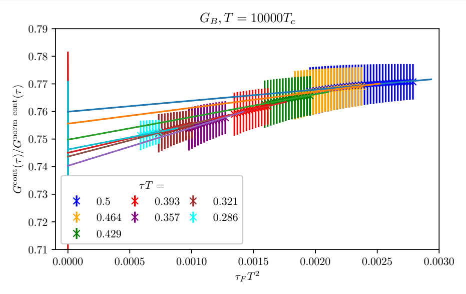
G_B

- We observe different small flow time scaling between G_E and G_B
- Possible indication of divergence or $\log(\tau_F)$ contributions

Flowtime dependence of G_{E} and G_{B} at $10^4\,T_{\mathrm{c}}$



G_{E}



G_{B}

- Similar story at higher temperatures

Conclusions

- Prior Study: Measured κ_{E} at wide range of temperatures with multilevel
- Now: Measured G_{E} with gradient flow.
- At $1.5\,T_{\mathrm{c}}$ we replicate the existing results, promising results at large temperatures
- Preliminary results on G_{B}.
- Possible indication of divergent contribution to zero flowtime limit

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Thank you!