

Inclusive Decays of Heavy Quark Hybrids

STRONG 2020 Online Workshop

Sept 14, 2021

Abhishek Mohapatra (TU Munich)

Nora Brambilla (TU Munich)

Wai Kin Lai (South China Normal university)

Antonio Vairo (TU Munich)

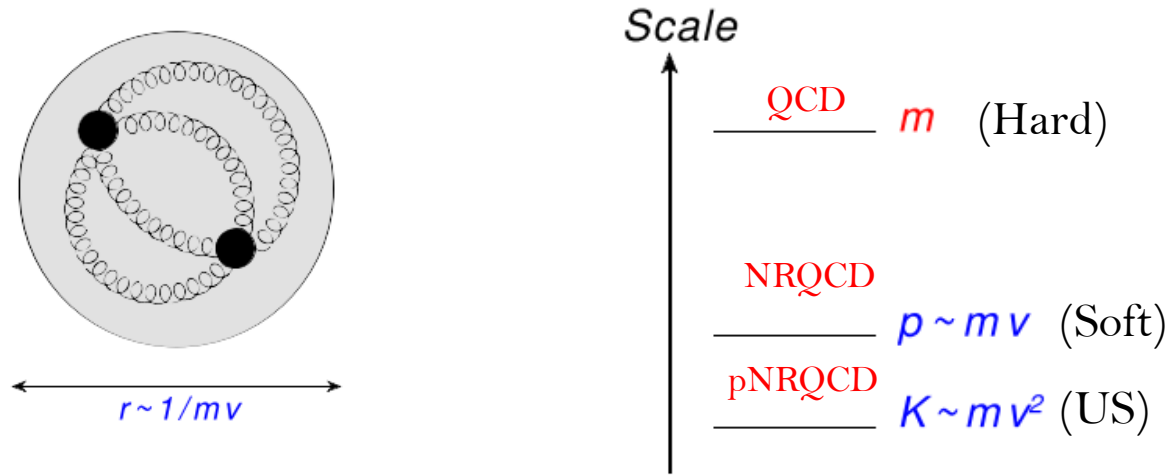


Outline

- **Introduction to X Y Z mesons**
- **EFT for Quarkonium Hybrids**
 - **BO-EFT effective theory**
 - **Quarkonium Hybrid Spectrum**
- **Inclusive Decay Rates for hybrid to quarkonium**
- **Summary and Outlook**

Introduction

- Quarkonium: Color singlet bound state of $Q\bar{Q}$ ($Q = c, b$).

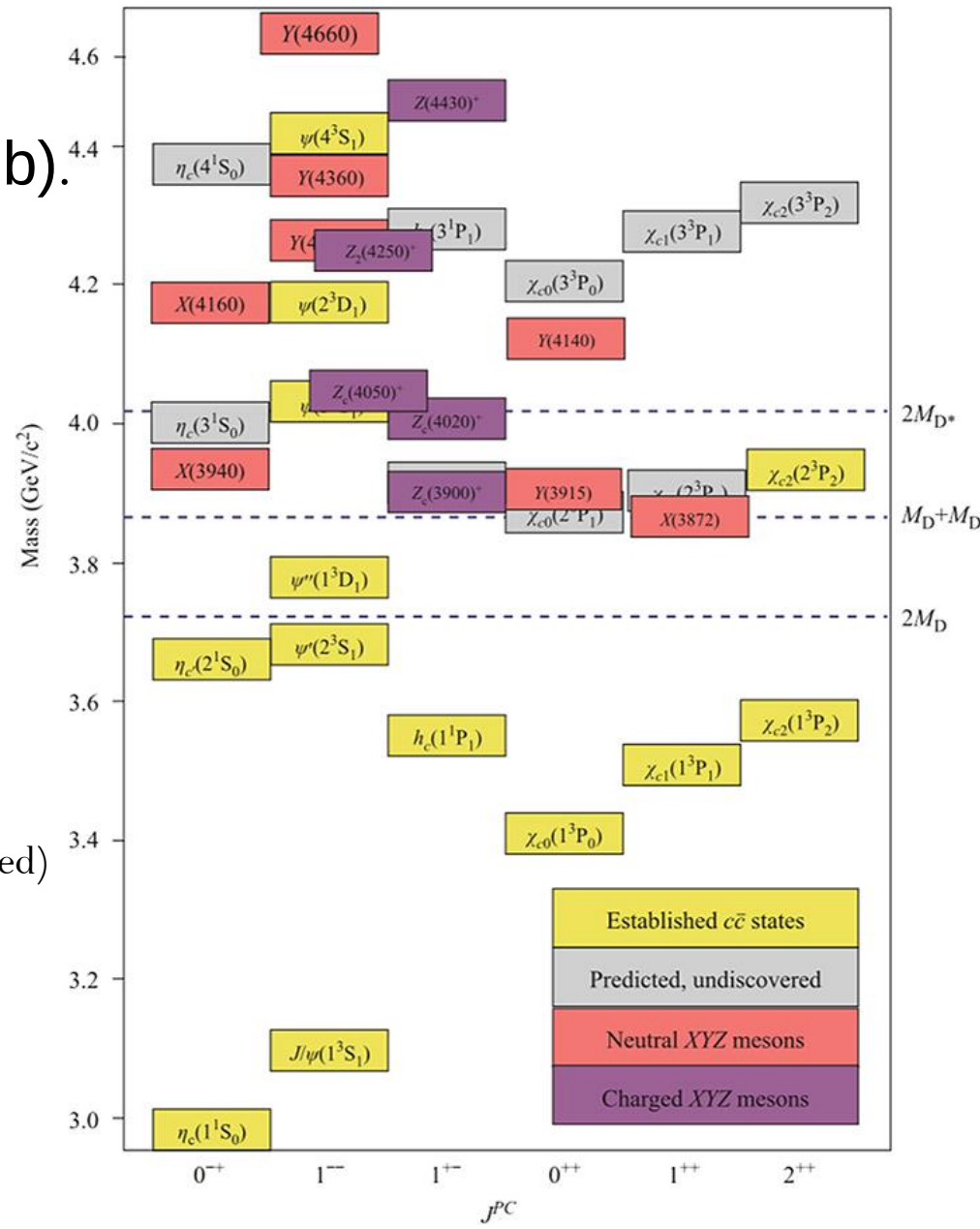


- Hierarchy of Energy Scales in $Q\bar{Q}$:

$$m \gg mv \gg mv^2, \Lambda_{QCD} \quad (\text{perturbative dynamics: Weakly Coupled})$$

$$m \gg mv, \Lambda_{QCD} \gg mv^2 \quad (\text{nonperturbative dynamics: Strongly Coupled})$$

- pNRQCD: Relevant EFT for Quarkonium.



Bodwin, Braaten & Lepage (1994), Mehen and Fleming, Phys. Rev. D73, 034502 (2005)
 Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005) Luke, Manohar & Rothstein (2000)

Introduction

- Quark Model:

Mesons: quark-antiquark states Baryons: 3-quark states

- QCD spectrum also allows for more complex structures called as **Exotics**.

- Exotic states: XYZ mesons

- ✓ Quarkonium-like states that don't fit traditional $Q\bar{Q}$ spectrum.
- ✓ In some cases exotic quantum numbers (charged Z_c and Z_b states)

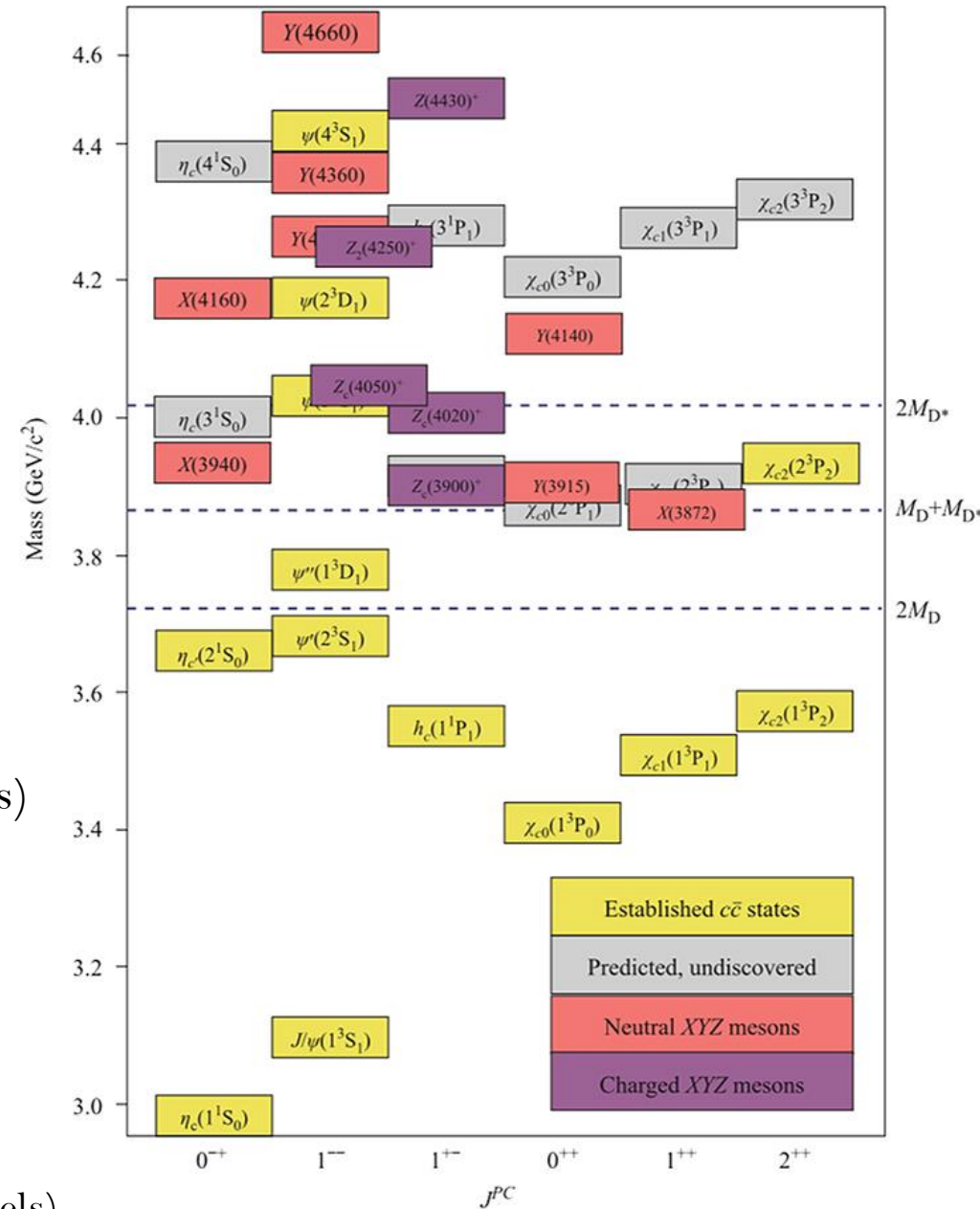
For review see Brambilla et al. *Phys. Reports.* 873 (2020)

- $X(3872)$: First exotic state discovered in 2003 by Belle.

Phys. Rev. Lett. 91, 262001 (2003)

- Several **new** heavy quark exotic states have been discovered since 2003 (masses & decay rates measured in various channels).

PDG 2021



Front. Phys. 10 101401 (2015)

Quarkonium hybrids: EFT

- Hybrids ($Q\bar{Q}g$): Color singlet combination of color octet $Q\bar{Q}$ + gluonic excitations.

- Separation of scales in hybrids:

$$m \gg mv \gg \Lambda_{QCD} \gg mv^2$$

❖ light d.o.f: Λ_{QCD}

❖ Relative separation between heavy quarks: $r \sim 1/mv$

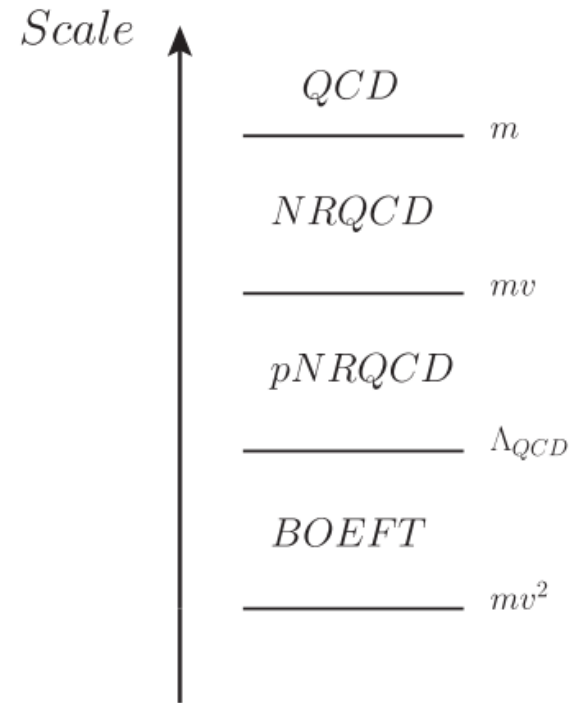
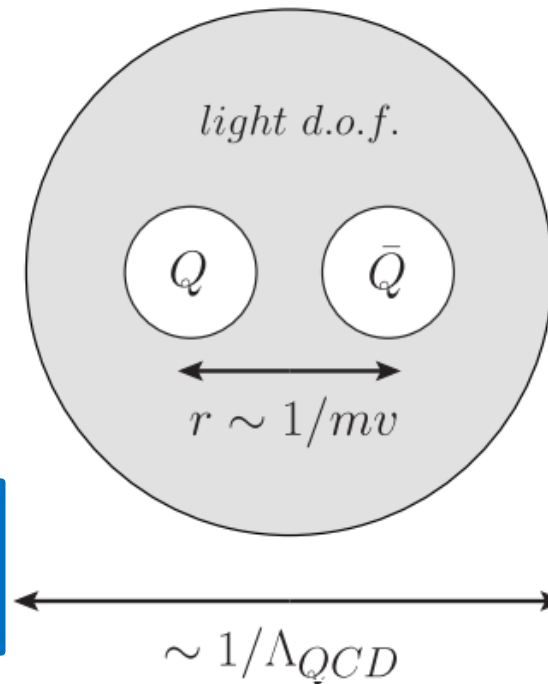
❖ Heavy Quark K.E scale: mv^2

- Time-scale for dynamics of $Q\bar{Q}$: $\sim \frac{1}{mv^2} \gg \frac{1}{\Lambda_{QCD}}$

Born-Oppenheimer Approximation

Braaten, Langmack, Smith

Phys. Rev. D. 90, 014044 (2014)



- Appropriate EFT framework for Hybrids: **Born-Oppenheimer EFT (BOEFT)**

$$QCD \rightarrow NRQCD \rightarrow pNRQCD \rightarrow BOEFT$$

Brambilla, Krein, Castellà, Vairo Phys. Rev. D. 97, (2018)

Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92, (2015)

R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Quarkonium hybrids: BOEFT

- Static limit ($m \rightarrow \infty$): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- K : angular momentum of light d.o.f.
 $\lambda = \hat{\mathbf{r}} \cdot \mathbf{K} = 0, \pm 1, \pm 2, \pm 3, \dots$
 $\Lambda = |\lambda| = 0, 1, 2, 3, \dots$ ($\Sigma, \Pi, \Delta, \Phi, \dots$)
- Eigenvalue of CP : $\eta = +1$ (g), -1 (u)
- σ : eigenvalue of reflection about a plane containing $\hat{\mathbf{r}}$ (only for Σ states)

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.
- For $r \rightarrow 0$: static energies are degenerate. Characterized by $O(3) \times C$ symmetry group.

Labelled by: $(K^{PC}, \Lambda_{\eta}^{\sigma})$

Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92, (2015)

Gluonic static energies

M. Foster and C. Michael, Phys. Rev. D59 (1999)

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

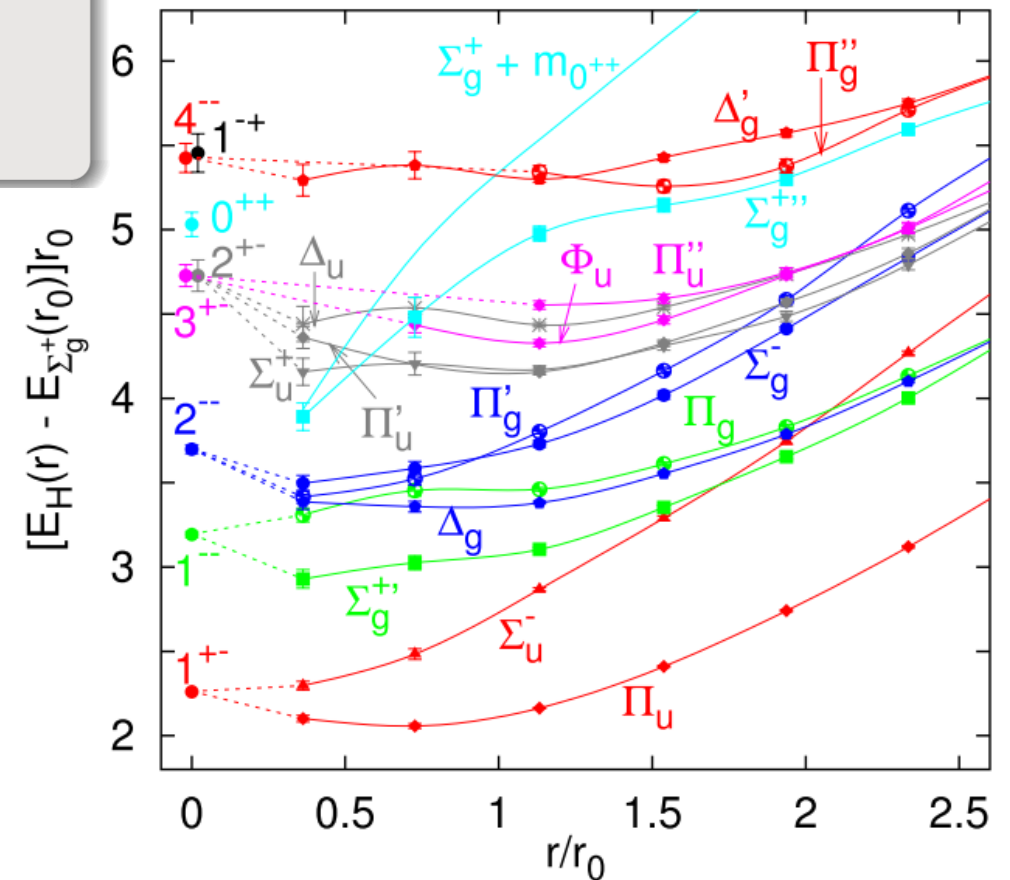


Fig from G. S. Bali and A. Pineda, Phys. Rev. D69 (2004)

Quarkonium hybrids: BOEFT

- Static limit ($m \rightarrow \infty$): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- $\mathbf{K}$: angular momentum of light d.o.f.
 $\lambda = \hat{\mathbf{r}} \cdot \mathbf{K} = 0, \pm 1, \pm 2, \pm 3, \dots$
 $\Lambda = |\lambda| = 0, 1, 2, 3, \dots$ ($\Sigma, \Pi, \Delta, \Phi, \dots$)
- Eigenvalue of CP : $\eta = +1$ (g), -1 (u)
- σ : eigenvalue of reflection about a plane containing $\hat{\mathbf{r}}$ (only for Σ states)

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.
- For $r \rightarrow 0$: static energies are degenerate. Characterized by $O(3) \times C$ symmetry group.

Labelled by: $(K^{PC}, \Lambda_{\eta}^{\sigma})$

Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92, (2015)

Focus on these two for low lying hybrids

Gluonic operators characterizing Hybrids in Wilson loop

Λ_{η}^{σ}	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\mathbf{r}} \cdot \mathbf{B}, \hat{\mathbf{r}} \cdot (\mathbf{D} \times \mathbf{E})$
Π_u	1^{+-}	$\hat{\mathbf{r}} \times \mathbf{B}, \hat{\mathbf{r}} \times (\mathbf{D} \times \mathbf{E})$
$\Sigma_g^{+'}$	1^{--}	$\hat{\mathbf{r}} \cdot \mathbf{E}, \hat{\mathbf{r}} \cdot (\mathbf{D} \times \mathbf{B})$
Π_g	1^{--}	$\hat{\mathbf{r}} \times \mathbf{E}, \hat{\mathbf{r}} \times (\mathbf{D} \times \mathbf{B})$
Σ_g^-	2^{--}	$(\hat{\mathbf{r}} \cdot \mathbf{D})(\hat{\mathbf{r}} \cdot \mathbf{B})$
Π_g'	2^{--}	$\hat{\mathbf{r}} \times ((\hat{\mathbf{r}} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\hat{\mathbf{r}} \cdot \mathbf{B}))$
Δ_g	2^{--}	$(\hat{\mathbf{r}} \times \mathbf{D})^i (\hat{\mathbf{r}} \times \mathbf{B})^j + (\hat{\mathbf{r}} \times \mathbf{D})^j (\hat{\mathbf{r}} \times \mathbf{B})^i$
Σ_u^+	2^{+-}	$(\hat{\mathbf{r}} \cdot \mathbf{D})(\hat{\mathbf{r}} \cdot \mathbf{E})$
Π_u'	2^{+-}	$\hat{\mathbf{r}} \times ((\hat{\mathbf{r}} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\hat{\mathbf{r}} \cdot \mathbf{E}))$
Δ_u	2^{+-}	$(\hat{\mathbf{r}} \times \mathbf{D})^i (\hat{\mathbf{r}} \times \mathbf{E})^j + (\hat{\mathbf{r}} \times \mathbf{D})^j (\hat{\mathbf{r}} \times \mathbf{E})^i$

Quarkonium hybrids: BOEFT

- BOEFT d.o.f involve color singlet fields $\hat{\Psi}_{\kappa\lambda}(\mathbf{r}, \mathbf{R}, t) \propto P_{\kappa\lambda}^i O^{a\dagger}(\mathbf{r}, \mathbf{R}, t) G_{\kappa}^{ia}(\mathbf{R}, t)$
 - $O^{a\dagger}(\mathbf{r}, \mathbf{R}, t) G_{\kappa}^{ia}(\mathbf{R}, t)$: Gluelump operator. Eigenvector of NRQCD Hamiltonian in ($m \rightarrow \infty$):

$$H^{(0)} O^{a\dagger}(\mathbf{r}, \mathbf{R}, t) G_{\kappa}^{ia}(\mathbf{R}, t) |0\rangle = (V_0(r) + \Lambda_{\kappa}) O^{a\dagger}(\mathbf{r}, \mathbf{R}, t) G_{\kappa}^{ia}(\mathbf{R}, t) |0\rangle$$

Λ_{κ} : Gluelump energy

- $P_{\kappa\lambda}^i$: Projection operators of light d.o.f along heavy quark-antiquark axis.
- BOEFT Lagrangian:

$$L_{\text{BOEFT}} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \hat{\Psi}_{\kappa\lambda}^{\dagger}(\mathbf{r}, \mathbf{R}, t) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P_{\kappa\lambda}^{i\dagger} \frac{\nabla_r^2}{m} P_{\kappa\lambda'}^i \right\} \hat{\Psi}_{\kappa\lambda'}(\mathbf{r}, \mathbf{R}, t) + \dots$$

- Schrödinger Eq: Dynamics of $Q\bar{Q}$ at scale $mv^2 \ll \Lambda_{\text{QCD}}$

Schrödinger equation

$$\left[-P_{\kappa\lambda}^{i\dagger} \frac{\nabla_r^2}{m} P_{\kappa\lambda'}^i + V_{\kappa\lambda\lambda'}(r) \right] \Psi_{\kappa\lambda'}^n(\mathbf{r}) = E_n \Psi_{\kappa\lambda}^n(\mathbf{r})$$

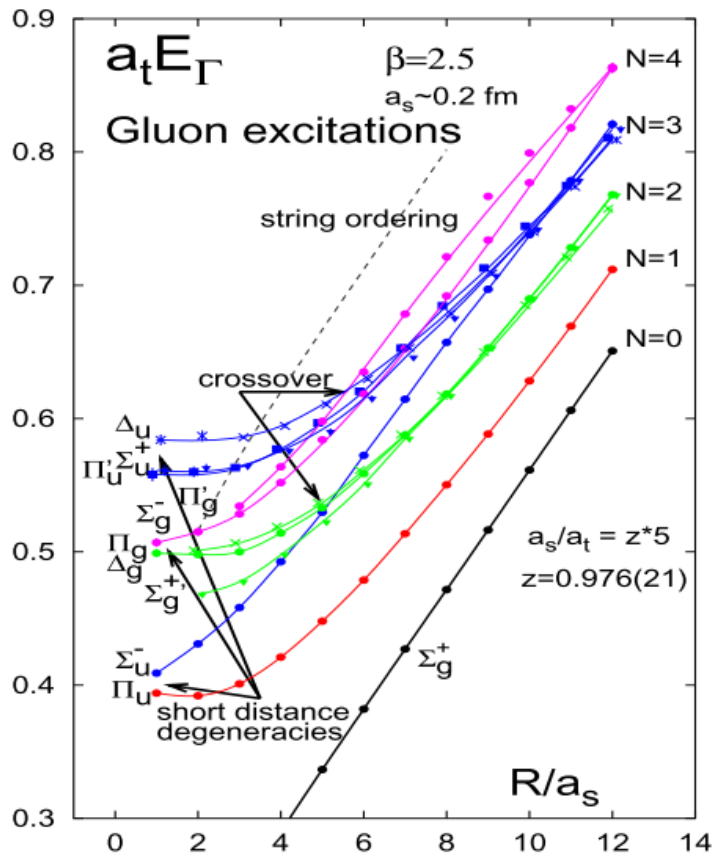
Hybrid wf

- Coupled Eq. due to projection operators. Mixes Σ_u and Π_u states.

Quarkonium hybrids: Spectrum

- Lattice potentials for solving the Schrödinger Eq:

Gluonic Static energies from lattice:



Potential after fitting lattice data:

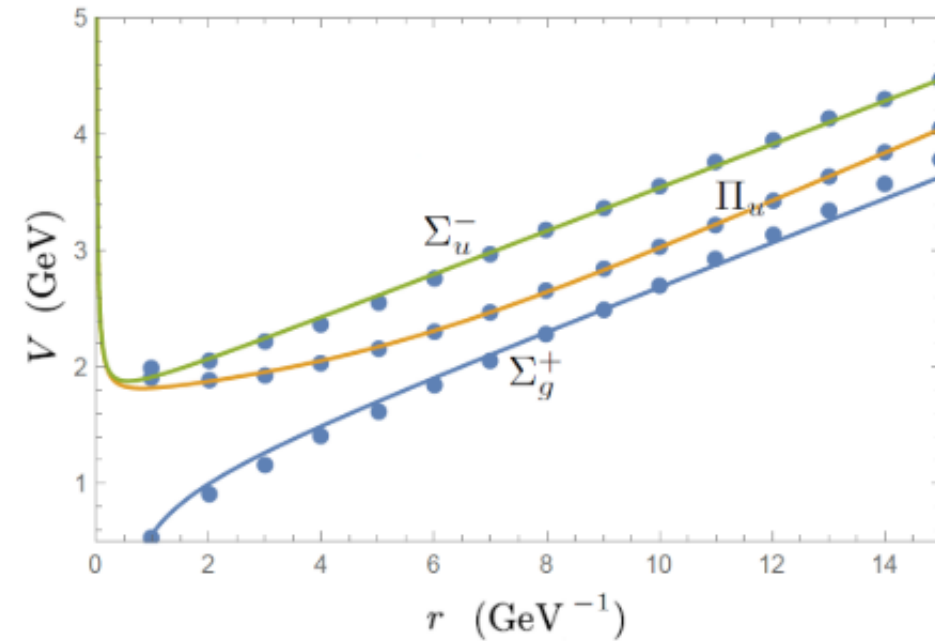


Fig from R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Quarkonium hybrids: Spectrum

- Results for Hybrids from [Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92, \(2015\)](#)

$$m_c = 1.477 \text{ GeV} \quad m_b = 4.863 \text{ GeV}$$

multiplet	J^{PC}	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
		m_H	$\langle 1/r \rangle$	E_{kin}	P_{Π}	m_H	$\langle 1/r \rangle$	E_{kin}	P_{Π}	m_H	$\langle 1/r \rangle$	E_{kin}	P_{Π}
H_1	$\{1^{--}, (0, 1, 2)^{-+}\}$	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H'_1		4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	$\{1^{++}, (0, 1, 2)^{+-}\}$	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H'_2		4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	$\{0^{++}, 1^{+-}\}$	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	$\{2^{++}, (1, 2, 3)^{+-}\}$	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	$\{2^{--}, (1, 2, 3)^{-+}\}$	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	$\{3^{--}, (2, 3, 4)^{-+}\}$	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	$\{3^{++}, (2, 3, 4)^{+-}\}$	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Other notation of hybrid states

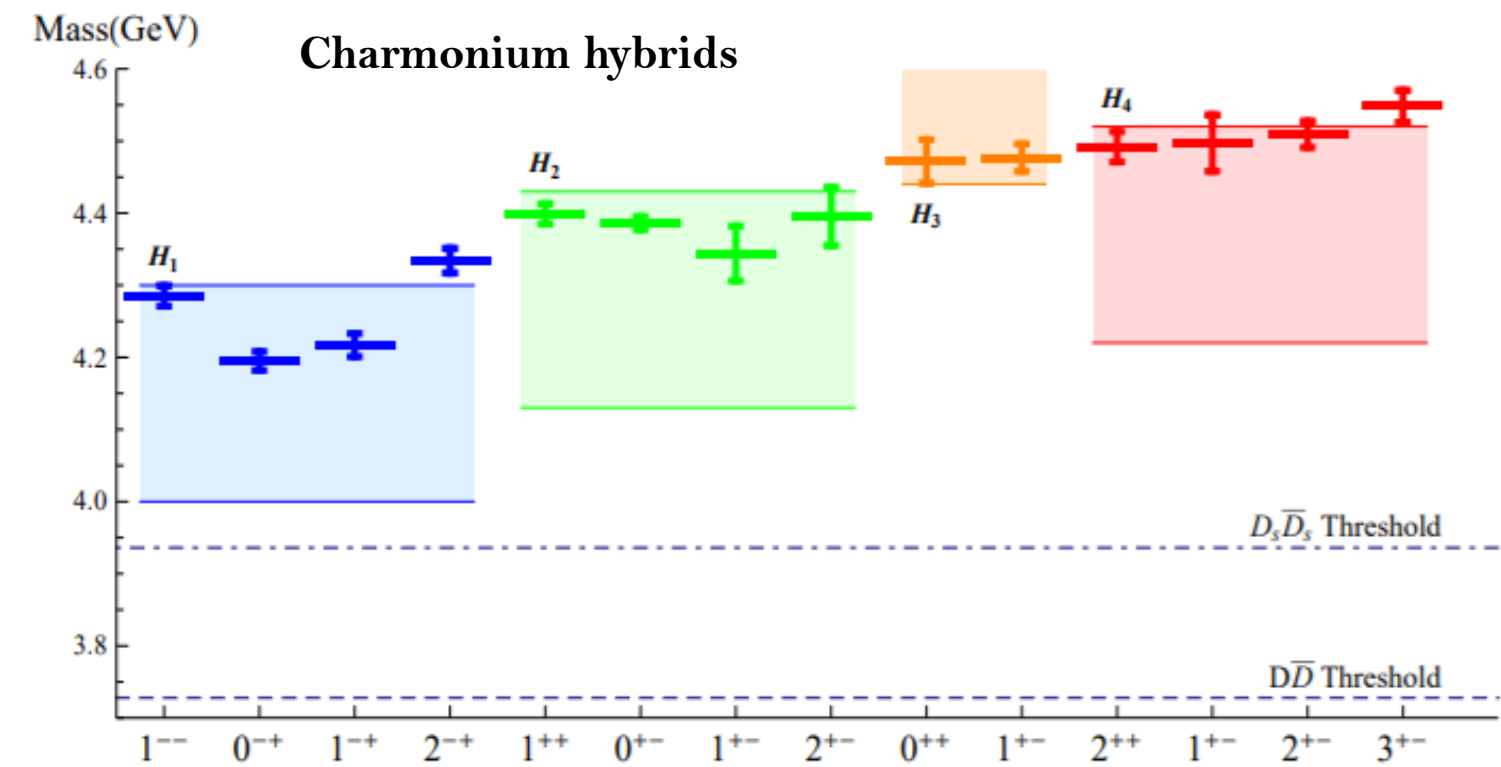
	l	$J^{PC} \{s=0, s=1\}$	$E_n^{(0)}$
$N(s/d)_1 \rightarrow$	H_1	1 $\{1^{--}, (0, 1, 2)^{-+}\}$	Σ_u^-, Π_u
$Np_1 \rightarrow$	H_2	1 $\{1^{++}, (0, 1, 2)^{+-}\}$	Π_u
$Np_0 \rightarrow$	H_3	0 $\{0^{++}, 1^{+-}\}$	Σ_u^-
$N(p/f)_2 \rightarrow$	H_4	2 $\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^-, Π_u
$Nd_2 \rightarrow$	H_5	2 $\{2^{--}, (1, 2, 3)^{-+}\}$	Π_u

[Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 \(2014\)](#)

[R. Oncala, J. Soto, Phys. Rev. D96 \(2017\)](#)

Quarkonium hybrids: Spectrum

- Comparison with lattice results:



Other notation of hybrid states

		l	$J^{PC}\{s=0,s=1\}$	$E_n^{(0)}$
$N(s/d)_1 \longrightarrow$	H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
$Np_1 \longrightarrow$	H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
$Np_0 \longrightarrow$	H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
$N(p/f)_2 \longrightarrow$	H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
$Nd_2 \longrightarrow$	H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

- Λ - doubling: opposite parity states non-degenerate. Also confirmed by lattice

Lattice data from
Hadron Spectrum collaboration JHEP 1207 (2012) 126

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)
Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)
R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Inclusive Decays

- Dozens of XYZ states have been discovered (mass and decay rates measured) but physics still unknown.
- Several theoretical models for exotic states but no general consensus.
- Most of the exotic states discovered from decays to quarkonium. So, decays might provide information on the structure of XYZ.
- Consider the process: $H_m \rightarrow Q_n + X$; H_m : low-lying hybrid, Q_n : low-lying quarkonium.
 - ✓ $\Delta \equiv m_H - m_Q \gtrsim 1 \text{ GeV}$. For low-lying states, we observe that $\Delta \gg \Lambda_{\text{QCD}} \gg mv^2$
 - ✓ Hierarchy of Scales: $mv \gg \Delta \gg \Lambda_{\text{QCD}} \gg mv^2$
- Start with pNRQCD effective theory and obtain BOEFT by matching: Integrate out modes of scale $\sim \Delta$ and $\sim \Lambda_{\text{QCD}}$.

Inclusive Decays

- pNRQCD Lagrangian:

Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3 R \left\{ \int d^3 r \left(\text{Tr} \left[S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right] \right. \right. \\ \left. + g \text{Tr} \left[S^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger \mathbf{r} \cdot \mathbf{E} S + \frac{1}{2} O^\dagger \mathbf{r} \cdot \{\mathbf{E}, O\} \right] + \frac{g}{4m} \text{Tr} \left[O^\dagger \mathbf{L}_{Q\bar{Q}} \cdot [\mathbf{B}, O] \right] \right. \\ \left. \left. + \frac{g^c F}{m} \text{Tr} \left[S^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} O + O^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} S + O^\dagger \mathbf{S}_1 \cdot \mathbf{B} O - O^\dagger \mathbf{S}_2 O \cdot \mathbf{B} \right] - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a} \right\}$$

- BOEFT:

BOEFT Hamiltonian

$$H_{\text{BOEFT}} = \int d^3 x \int d^3 R \text{Tr} \left[H^{i\dagger} \left(h_o \delta^{ij} + V_{\text{soft}}^{ij} \right) H^j \right]$$

Potential term in BOEFT

- Decays are computed from local imaginary terms in the BOEFT Lagrangian.
- Imaginary term in V_{soft}^{ij} from 1-loop diagram in pNRQCD and then matching to BOEFT.

Inclusive Decays

- pNRQCD Lagrangian:

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

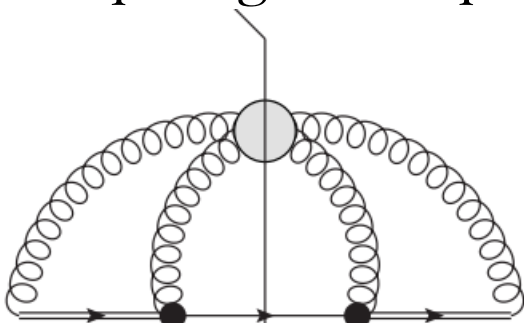
Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \left\{ \int d^3r \left(\text{Tr} \left[S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right] \right. \right. \\ + g \text{Tr} \left[S^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger \mathbf{r} \cdot \mathbf{E} S + \frac{1}{2} O^\dagger \mathbf{r} \cdot \{ \mathbf{E}, O \} \right] + \frac{g}{4m} \text{Tr} \left[O^\dagger \mathbf{L}_{Q\bar{Q}} \cdot [\mathbf{B}, O] \right] \\ \left. \left. + \frac{g c_F}{m} \text{Tr} \left[S^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} O + O^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} S + O^\dagger \mathbf{S}_1 \cdot \mathbf{B} O - O^\dagger \mathbf{S}_2 \cdot \mathbf{B} O \right] - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a} \right\}$$

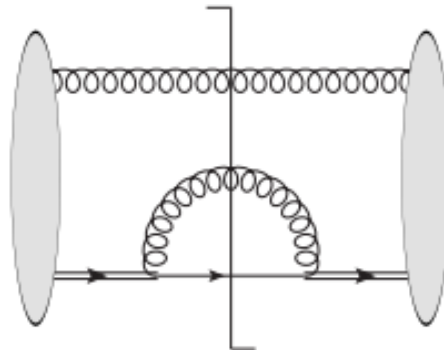
- Spin preserving decays

- Spin flipping decays

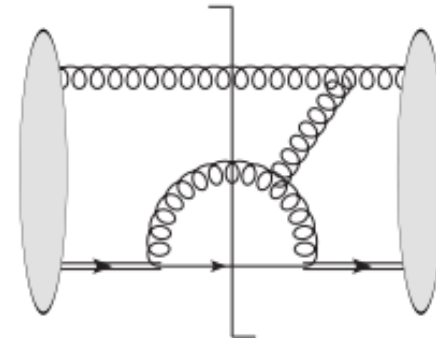
- 1-loop diagram in pNRQCD contributing to $\text{Im } V_{soft}^{ij}$ in BOEFT:



=



+



Black dot: vertex of pNRQCD

Spectator gluon approximation

example of α_s correction

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Spin-preserving inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' f_{mq}^i(E) g_{qn}^j(E) \times g_{q'n}^{j\dagger}(E') f_{mq'}^{i\dagger}(E') (\Lambda + E/2 + E'/2 - E_n^s)^3$$

Overlap functions:

$$f_{mq}^i(E) = \left[\int d^3r \Psi_m^{i\dagger}(\mathbf{r}) \Phi_{E,q}^o(\mathbf{r}) \right]$$

$$g_{qn}^j(E) = \left[\int d^3r \Phi_{E,q}^{o\dagger}(\mathbf{r}) r^j \Phi_n^s(\mathbf{r}) \right]$$

$$h_{nn'} = \int d^3r \Phi_{n'}^{s\dagger}(\mathbf{r}) \Phi_n^Q(\mathbf{r})$$

- Double integral over the energies E, E' of the octet wave function $\Phi_{E,q}^o$.
- Depends on overlap function of singlet w.f Φ_n^s and quarkonium w.f Φ_n^Q .
- Last factor $(\Lambda + E/2 + E'/2 - E_n^s)^3 \sim \Delta^3$ parametrically. ($\Delta \equiv m_H - m_Q$)
- Above result different from the one in [R. Oncala, J. Soto, Phys. Rev. D96, 014004 \(2017\)](#).

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo

(in progress)

Spin-preserving inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' f_{mq}^i(E) g_{qn}^j(E) \\ \times g_{q'n}^{j\dagger}(E') f_{mq'}^{i\dagger}(E') (\Lambda + E/2 + E'/2 - E_n^s)^3$$

Assumption:

$f_{mq}^i(E) \neq 0$ only for $E_m \approx E + \Lambda$

$h_{nn'} \approx 1$ and $E_m^Q \approx E_m^s$



Spin-preserving inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} (E_m - E_n^Q)^3 T^{ij} (T^{ij})^*$$

$$T^{ij} \equiv \int d^3r \Psi_m^{i\dagger}(\mathbf{r}) r^j \Phi_n^Q(\mathbf{r})$$

- Above result looks similar to the one in [R. Oncala, J. Soto, Phys. Rev. D96, 014004 \(2017\)](#). In general has **tensor structure T^{ij}** that agrees with [J. Castellà, E. Passemar, arXiv:2104.03975](#).

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

Spin-preserving inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} (E_m - E_n^Q)^3 T^{ij} (T^{ij})^*$$

$$T^{ij} \equiv \int d^3r \Psi_m^{i\dagger}(\mathbf{r}) r^j \Phi_n^Q(\mathbf{r})$$

- R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017): only **diagonal elements** T^{ii} are considered

Inclusive decay rate for $H_m \rightarrow Q_n + X$ computed by Oncala and Soto

$$\Gamma^{\text{Oncala}}(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} (E_m - E_n^Q)^3 T^{ii} (T^{jj})^*$$

- T^{ii} leads to selection rule: ~~Hybrids such as $Np_1 (H_2)$ where $L=J$ don't decay to quarkonium.~~
- T^{ij} : allows for the decay of $Np_1 (H_2)$ hybrid decays.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Spin conserving decay rates for **Charm hybrids**:

$NL_J \rightarrow N'L'$	ΔE (GeV)	$\alpha_s(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV ⁻¹)	Γ (MeV)
$1p_0 \rightarrow 1s$	1.418	0.307	1.187	274.48
$1p_0 \rightarrow 2s$	0.808	0.408	1.748	146.23
$2p_0 \rightarrow 1s$	1.852	0.276	0.312	38.0
$2p_0 \rightarrow 2s$	1.242	0.326	1.633	370.8
$3p_0 \rightarrow 1s$	2.231	0.259	0.144	13.23
$3p_0 \rightarrow 2s$	1.620	0.290	0.388	41.2
$1p_1 \rightarrow 1s$	1.078	0.349	1.886	346.0
$2p_1 \rightarrow 1s$	1.444	0.305	0.862	151.48
$2p_1 \rightarrow 2s$	0.834	0.401	1.908	187.90
$3p_1 \rightarrow 1s$	1.796	0.279	0.447	71.73
$3p_1 \rightarrow 2s$	1.186	0.334	0.942	109.62
$2(s/d)_1 \rightarrow 1p$	0.861	0.394	2.322	301.56
$4(s/d)_1 \rightarrow 1p$	1.224	0.329	3.667	1802.13

Decays not
allowed in
R. Onocala, J. Soto,
Phys. Rev. D96,
014004 (2017)

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Spin conserving decay rates for **Bottom hybrids**:

$NL_J \rightarrow N'L'$	ΔE (GeV)	$\alpha_s(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV $^{-1}$)	Γ (MeV)
$1p_0 \rightarrow 1s$	1.569	0.294	0.520	68.20
$1p_0 \rightarrow 2s$	1.002	0.363	1.270	130.86
$2p_0 \rightarrow 1s$	1.857	0.276	0.264	27.26
$2p_0 \rightarrow 2s$	1.290	0.321	0.583	51.96
$2p_0 \rightarrow 3s$	0.943	0.375	1.910	254.63
$3p_0 \rightarrow 1s$	2.109	0.264	0.162	14.37
$3p_0 \rightarrow 2s$	1.542	0.296	0.290	20.35
$4p_0 \rightarrow 1s$	2.338	0.254	0.111	8.88
$4p_0 \rightarrow 2s$	1.770	0.281	0.170	10.06
$4p_0 \rightarrow 3s$	1.423	0.307	0.329	21.31
$1p_1 \rightarrow 1s$	1.320	0.317	0.721	84.29
$2p_1 \rightarrow 1s$	1.528	0.297	0.546	70.35
$2p_1 \rightarrow 2s$	0.961	0.371	0.059	0.25
$3p_1 \rightarrow 1s$	1.732	0.283	0.395	51.00
$3p_1 \rightarrow 2s$	1.165	0.336	0.417	20.54
$2(s/d)_1 \rightarrow 1p$	0.978	0.367	0.890	60.51
$3(s/d)_1 \rightarrow 1p$	1.176	0.335	1.073	141.05
$3(s/d)_1 \rightarrow 2p$	0.819	0.405	0.131	0.85
$4(s/d)_1 \rightarrow 1p$	1.248	0.326	3.056	1312.72
$4(s/d)_1 \rightarrow 2p$	0.890	0.386	4.484	1218.65
$5(s/d)_1 \rightarrow 1p$	1.376	0.311	0.467	39.28
$5(s/d)_1 \rightarrow 2p$	1.019	0.360	1.011	86.36

Decays not
allowed in
R. Onocala, J. Soto,
Phys. Rev. D96,
014004 (2017)



Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Result for **spin-flipping** decays due to $\mathbf{S} \cdot \mathbf{B}$ term:

Spin-flipping inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F c_F^2}{3N_c m_Q^2} T^{ij} (T^{ij})^* (E_m - E_n^Q)^3$$

$$T^{ij} \equiv \int d^3r \Psi_m^{i\dagger}(\mathbf{r}) S^j \Phi_n^Q(\mathbf{r})$$

- Spin-flipping decays implies $|S_H = 1\rangle \longrightarrow |S_Q = 0\rangle$ & $|S_H = 0\rangle \longrightarrow |S_Q = 1\rangle$
 - Above result agrees with [J. Castellà, E. Passemar, arXiv:2104.03975](#).
- $Q_m \rightarrow Q_n + X$ spin-flipping decays: Decay rate suppressed by additional $(\mathbf{r} \cdot \mathbf{E})^2 \sim v^2$ vertex factor.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Spin flipping decay rates for **Charm hybrids**:

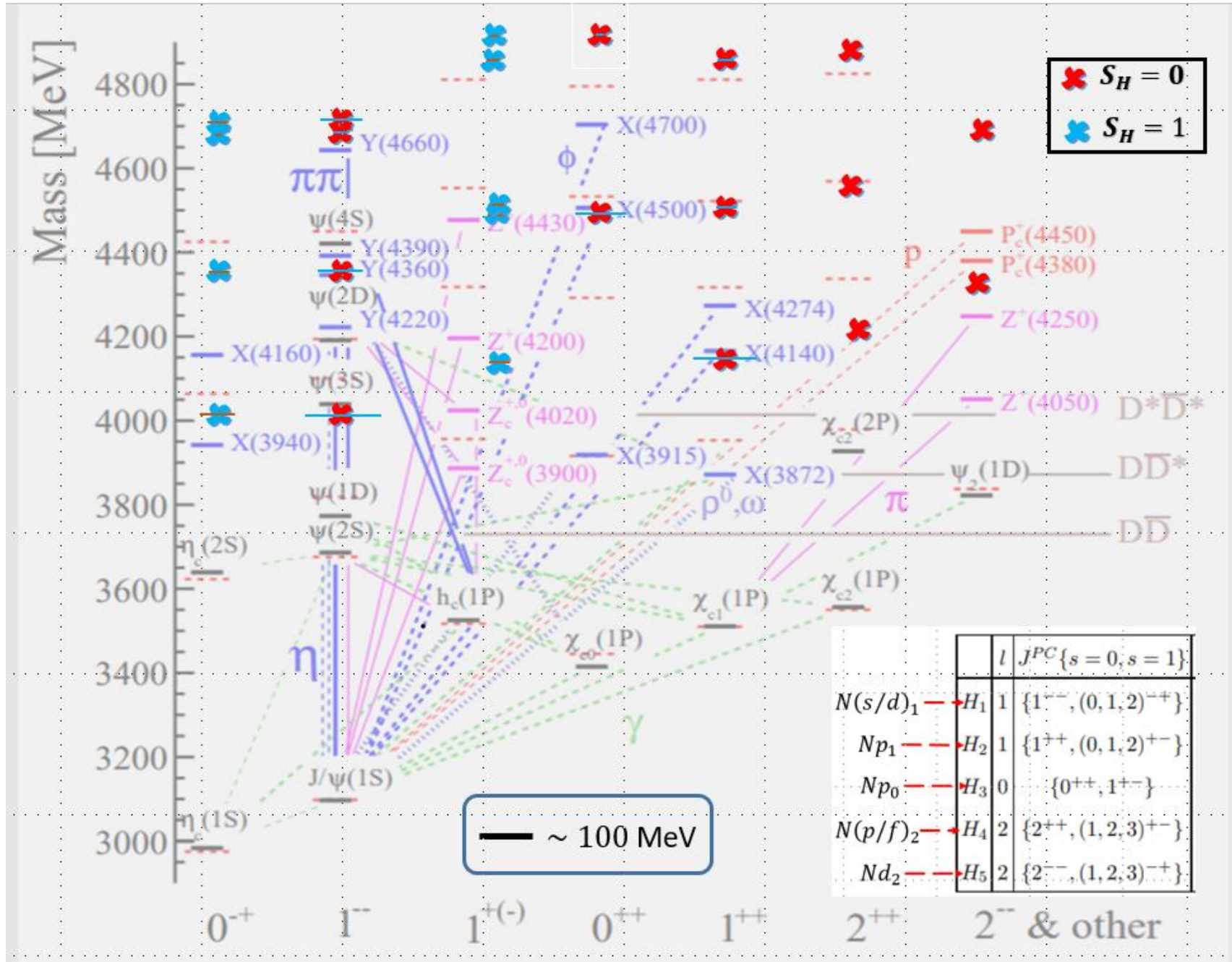
$NL_J \rightarrow N' L'$	ΔE (GeV)	$\alpha_S (\Delta E)$	$\Gamma(\text{MeV})$ ($1 \rightarrow 0$)	$\Gamma(\text{MeV})$ ($0 \rightarrow 1$)
$1p_0 \rightarrow 1p$	0.993	0.365	35.60	106.81
$2p_0 \rightarrow 1p$	1.426	0.307	1.44	4.31
$2p_0 \rightarrow 2p$	0.952	0.373	31.34	94.03
$3p_0 \rightarrow 1p$	1.804	0.279	0.41	1.22
$3p_0 \rightarrow 2p$	1.330	0.316	1.52	4.56
$1p_1 \rightarrow 1p$	0.653	0.467	46.57	139.72
$2p_1 \rightarrow 1p$	1.018	0.360	14.51	43.53
$2p_1 \rightarrow 2p$	0.544	0.531	26.13	78.39
$3p_1 \rightarrow 1p$	1.369	0.312	5.69	17.07
$3p_1 \rightarrow 2p$	0.895	0.385	10.97	32.90
$1(s/d)_1 \rightarrow 1s$	0.944	0.374	74.98	224.95
$2(s/d)_1 \rightarrow 1s$	1.288	0.321	32.51	97.53
$2(s/d)_1 \rightarrow 2s$	0.678	0.455	18.86	56.57
$3(s/d)_1 \rightarrow 1s$	1.624	0.290	7.73	23.18
$3(s/d)_1 \rightarrow 2s$	1.014	0.360	1.04	3.11
$4(s/d)_1 \rightarrow 1s$	1.650	0.288	7.35	22.06
$4(s/d)_1 \rightarrow 2s$	1.040	0.356	38.86	116.57

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Spin flipping decay rates for **Bottom hybrids**:

$NL_J \rightarrow N'L'$	ΔE (GeV)	$\alpha_S(\Delta E)$	$\Gamma(\text{MeV})$ ($1 \rightarrow 0$)	$\Gamma(\text{MeV})$ ($0 \rightarrow 1$)
$1p_0 \rightarrow 1p$	1.103	0.346	3.94	11.81
$2p_0 \rightarrow 1p$	1.391	0.310	0.54	1.61
$2p_0 \rightarrow 2p$	1.033	0.357	2.95	8.84
$3p_0 \rightarrow 1p$	1.643	0.289	0.19	0.56
$3p_0 \rightarrow 2p$	1.286	0.321	0.50	1.49
$4p_0 \rightarrow 1p$	1.872	0.275	0.09	0.28
$4p_0 \rightarrow 2p$	1.515	0.299	0.19	0.57
$4p_0 \rightarrow 3p$	1.226	0.328	0.45	1.35
$1p_1 \rightarrow 1p$	0.853	0.396	6.32	18.96
$2p_1 \rightarrow 1p$	1.062	0.352	3.22	9.67
$2p_1 \rightarrow 2p$	0.704	0.444	1.99	5.97
$3p_1 \rightarrow 1p$	1.266	0.323	1.62	4.85
$3p_1 \rightarrow 2p$	0.909	0.382	2.16	6.47
$1(s/d)_1 \rightarrow 1s$	1.247	0.326	8.6	25.86
$2(s/d)_1 \rightarrow 1s$	1.288	0.305	5.95	17.84
$2(s/d)_1 \rightarrow 2s$	0.876	0.390	0.56	1.68
$3(s/d)_1 \rightarrow 1s$	1.642	0.289	3.56	10.67
$3(s/d)_1 \rightarrow 2s$	1.075	0.350	0.95	2.84
$4(s/d)_1 \rightarrow 1s$	1.713	0.284	0.50	1.51
$4(s/d)_1 \rightarrow 2s$	1.146	0.339	2.56	7.68
$4(s/d)_1 \rightarrow 3s$	0.799	0.411	0.37	1.12



Charm Hybrids: Spin flipping Decays

Background Fig from

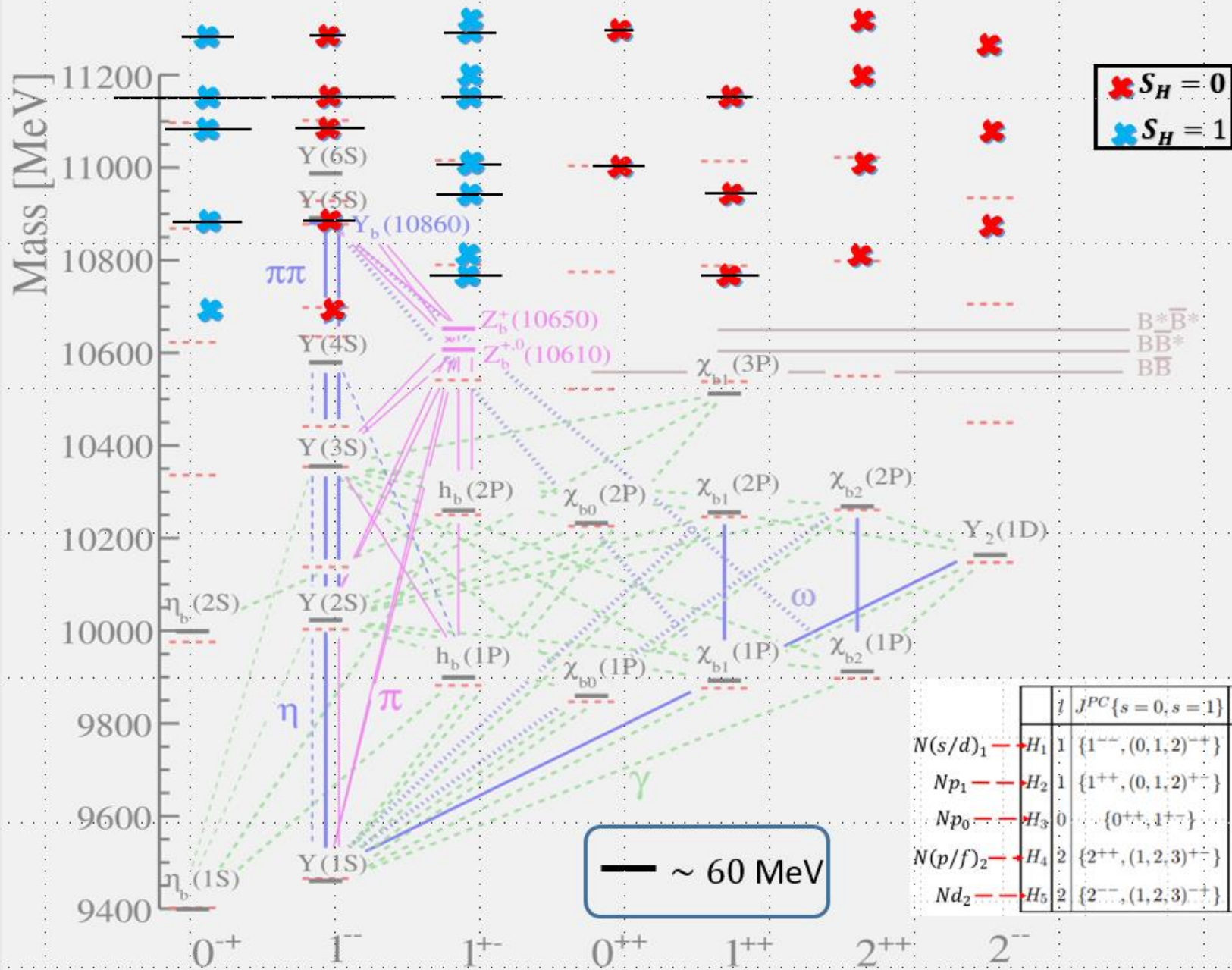
S.L. Olsen, T. Skwarnicki, D. Zieminska

Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now
for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)



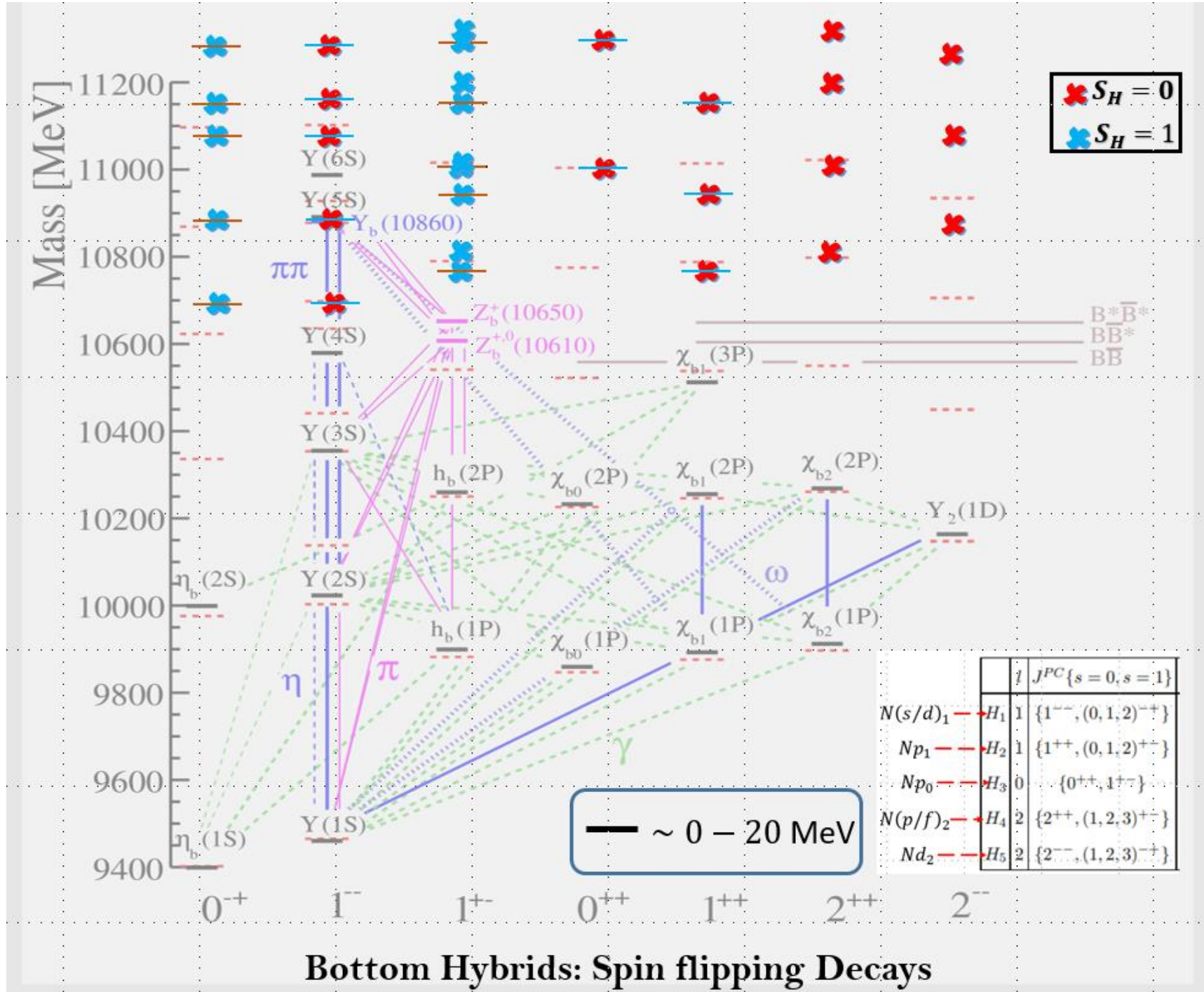
Bottom Hybrids: Spin conserving Decays

Background Fig from
S.L. Olsen, T. Skwarnicki, D. Zieminska
Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now
for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)



Background Fig from
S.L. Olsen, T. Skwarnicki, D. Zieminska
Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now
for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Summary/Outlook

- BOEFT provides a model-independent & systematic way to study heavy quark hybrids (exotic) and decays.
- General formula for the H_m inclusive decay based on overlap functions:

Inclusive decay rate for $H_m \rightarrow Q_n + X$

$$\Gamma(H_m \rightarrow Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' f_{mq}^i(E) g_{qn}^j(E) \\ \times g_{q'n}^{j\dagger}(E') f_{mq'}^{i\dagger}(E') (\Lambda + E/2 + E'/2 - E_n^s)^3$$

- Computed preliminary results on decay rates for spin-flipping and spin preserving decays.

- ←-----→
- Future work includes:

- Computing decay rates using the formulae based on overlap functions.
- Quantifying errors in the decay rates & comparing with the PDG data for observed exotic states.
- Exclusive decays: $H \rightarrow Q\pi\pi$ and include effect of mixing with excited quarkonia $Q' \rightarrow Q + X$.
- Extending this analysis to study tetraquarks.

Thank you!!