NN Models

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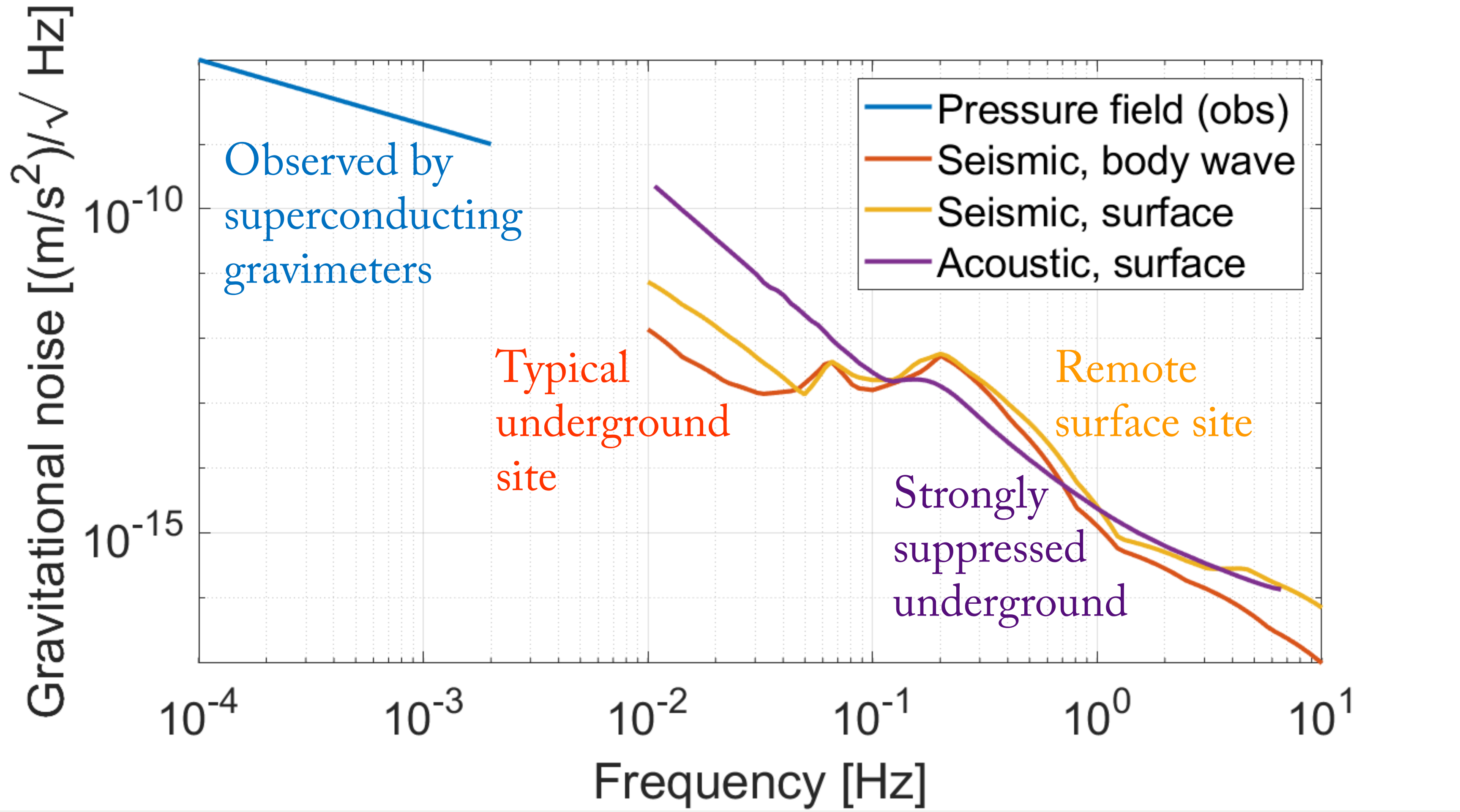
Terrestrial gravity fluctuations



Simplest analytical models

Method

- Calculate the gravitational perturbation caused by a single plane wave
- Average over propagation directions

$$\delta\phi_{\text{bulk}}(\mathbf{r}_0, t) = G\rho_0 \int_{\mathscr{V}} \mathrm{d}V \frac{\nabla \cdot \xi(\mathbf{r}, t)}{|\mathbf{r} - \mathbf{r}_0|}$$

$$\delta\phi_{\text{surf}}(\mathbf{r}_0, t) = -G\rho_0 \int \mathrm{d}S \frac{\mathbf{n}(\mathbf{r}) \cdot \xi(\mathbf{r}, t)}{|\mathbf{r} - \mathbf{r}_0|}$$

Body-wave NN

$$\begin{aligned} C_{\text{NN}} &= \left(\frac{4}{3}\pi G\rho_0\right)^2 \left[4\langle(\vec{e}_{\text{tm}} \cdot \vec{\xi}^{\,\text{P}}(\vec{r}_0, \omega))^2\rangle + \langle(\vec{e}_{\text{tm}} \cdot \vec{\xi}^{\,\text{S}}(\vec{r}_0, \omega))^2\rangle\right] \\ &= \left(\frac{4}{3}\pi G\rho_0\right)^2 S(\xi; \omega)(3p + 1) \end{aligned}$$

Surface and Rayleigh-wave NN

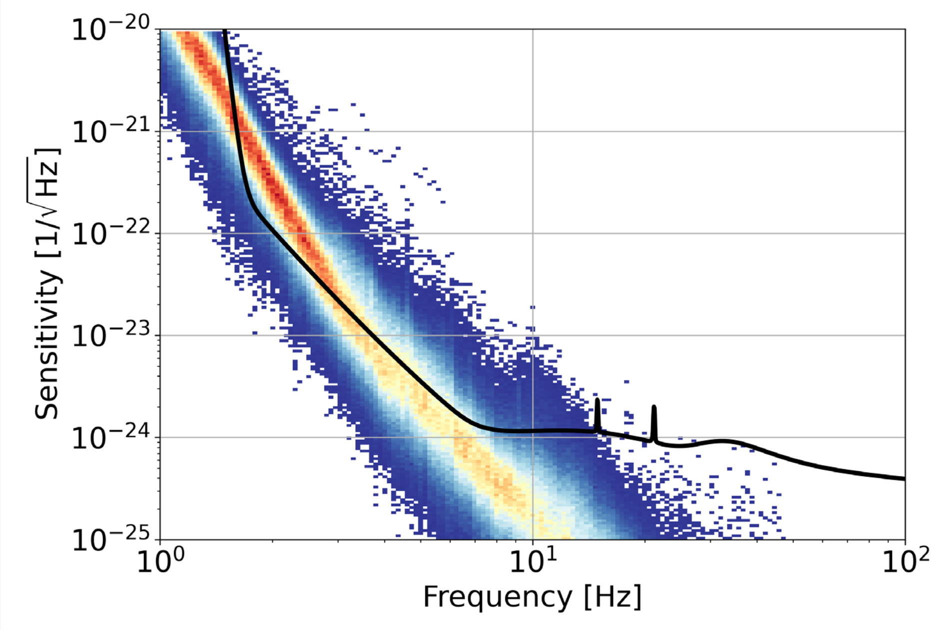
$$X_{\text{NN}}(f) = \frac{1}{\sqrt{2}} 2\pi\gamma G\rho_0 \frac{\xi(f)}{(2\pi f)^2} \exp(-2\pi h/\lambda)$$

Simplest analytical models

Assumptions

- Isotropy of seismic fields
- Homogeneity of seismic fields
- Small caverns (<< $\lambda/(2\pi)$)
- Geology
  - Homogeneous (for body waves)
  - Stratified (for Rayleigh waves)
- Flat surfaces

Example: Sos Enattos



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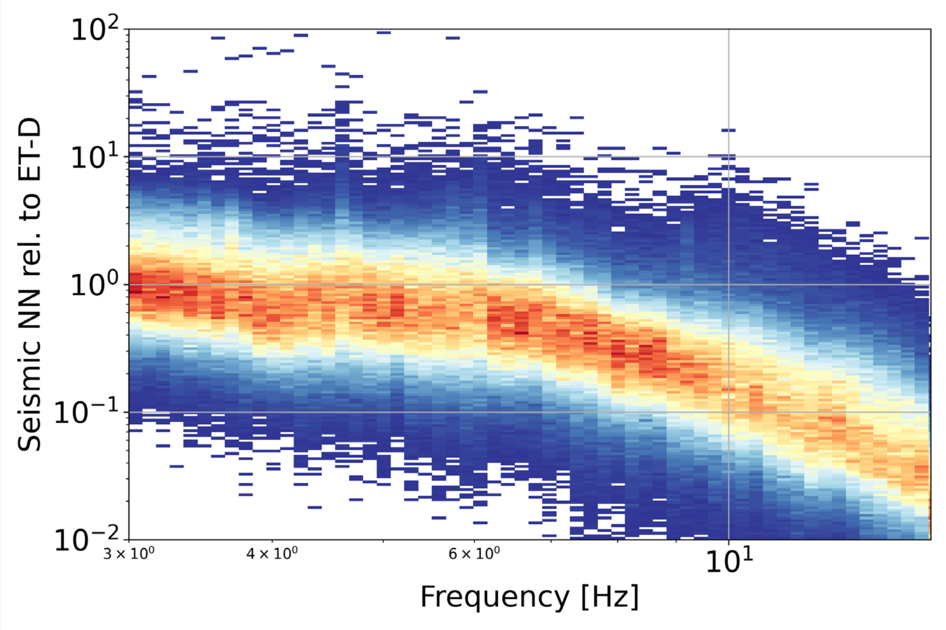


Simplest numerical models

Dipole equation (oscillating mass elements)

$$\delta\mathbf{a}(\mathbf{r}_0, t) = -G\int \mathrm{d}V\,\rho(\mathbf{r})(\xi(\mathbf{r}, t)\cdot\nabla_0)\cdot\frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|^3}$$

$$= G\int \mathrm{d}V\,\rho(\mathbf{r})\frac{1}{|\mathbf{r}-\mathbf{r}_0|^3}\left(\xi(\mathbf{r}, t) - 3(\mathbf{e}_{rr_0}\cdot\xi(\mathbf{r}, t))\mathbf{e}_{rr_0}\right)$$

Method

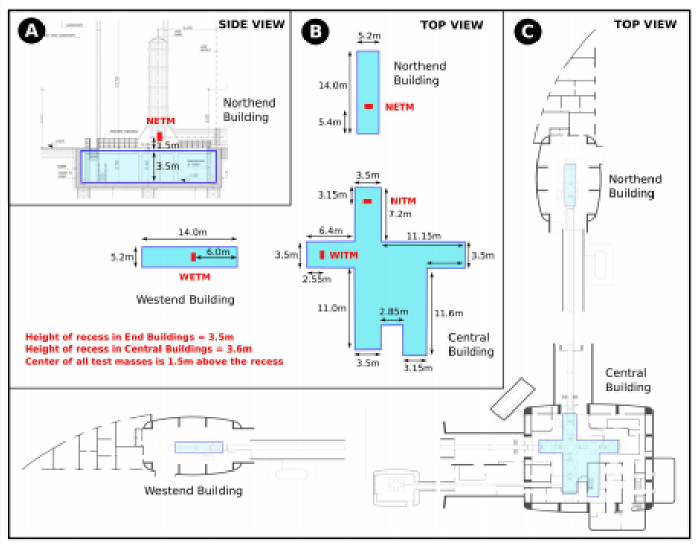
- Inject waves of known analytical form into a finite-element model
- Numerically integrate the associated gravity perturbation using the dipole equation
- Numerically challenging since one typically sums over a large number of finite elements, and their individual contributions can partially cancel

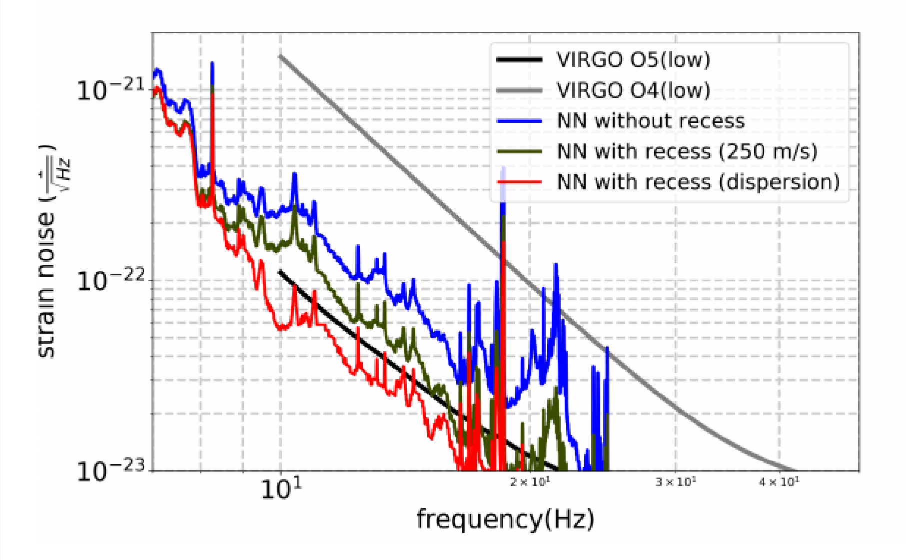
Simplest numerical models

Improvement

- Arbitrary distributions of seismic sources
- Arbitrary ground density
  - ➢ However, analytical form of seismic waves likely inconsistent with density model
- Arbitrary topography
  - ➢ However, analytical form of seismic waves likely inconsistent with topography
- Waves not necessarily plane

Example: Virgo NN estimate





Singha et al

Numerical models based on seismic correlations

Example: surface NN, homogeneous model

$$S(\delta a_x;\omega) = (2\pi G\rho_0\gamma(\nu))^2\,\frac{1}{2\pi}\int \mathrm{d}^2\varrho\left[\frac{x^2}{\varrho^2}\frac{2h}{((2h)^2+\varrho^2)^{3/2}} + \frac{y^2-x^2}{\varrho^4}\left(1-\frac{2h}{((2h)^2+\varrho^2)^{1/2}}\right)\right]C(\xi_z;\varrho,\omega)$$

Seismic correlations

Method

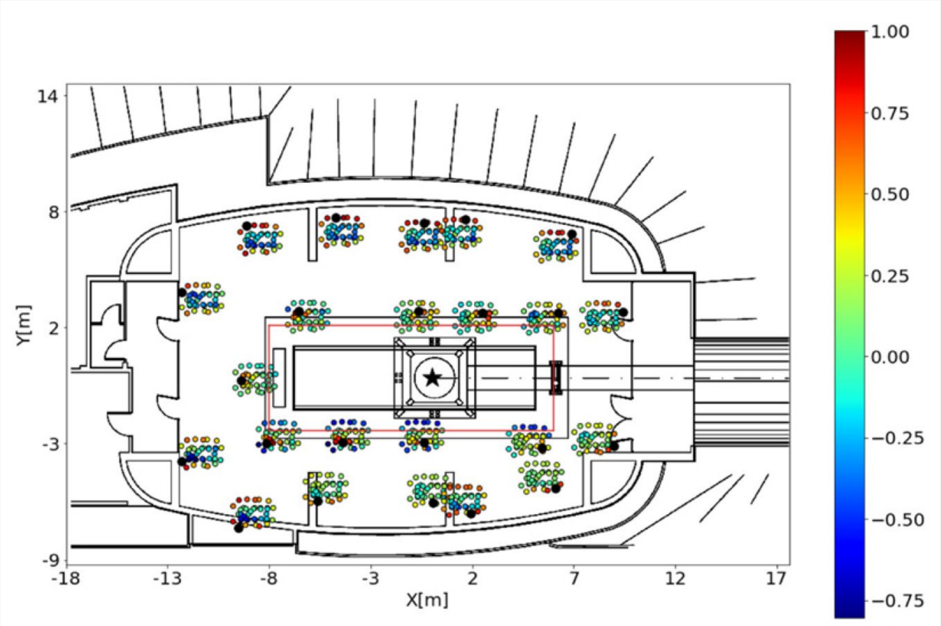
- Use observed seismic correlations
- Integrate numerically over surface area using an appropriate kernel
- Numerical errors are still important to consider

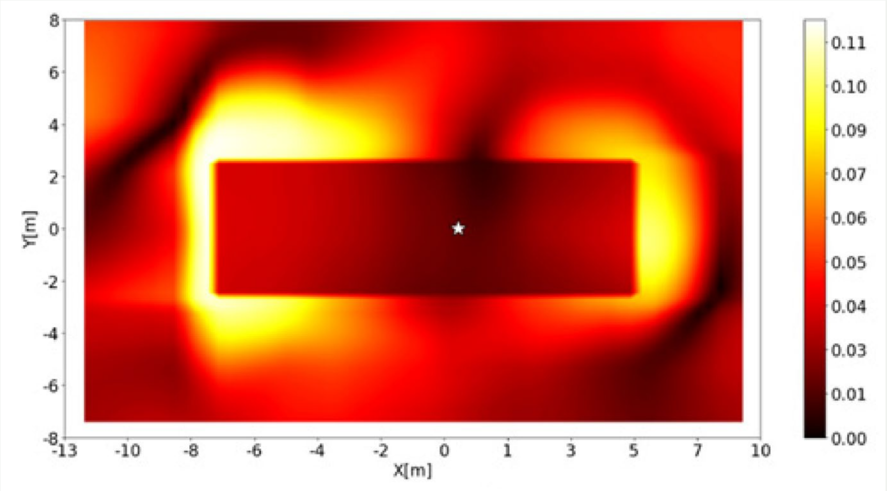
Numerical models based on seismic correlations

Improvement

- Works for inhomogeneous fields
- Does not require analytical models of seismic waves

Example: Virgo NNC optimization

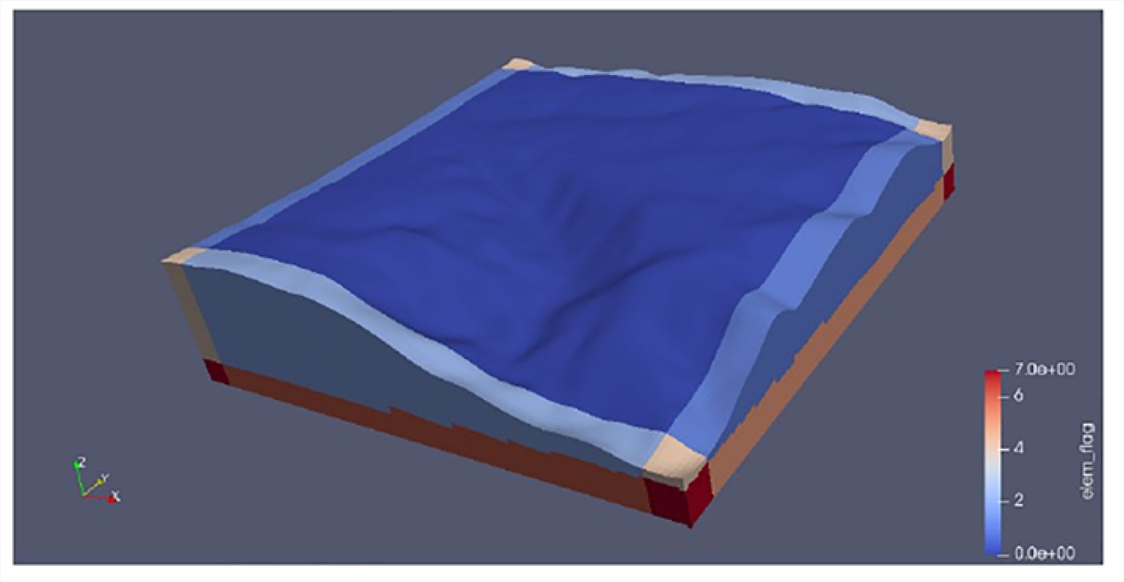




Badaracco et al

Dynamical finite-element simulations

Example: Sardinia (A3) model



Method

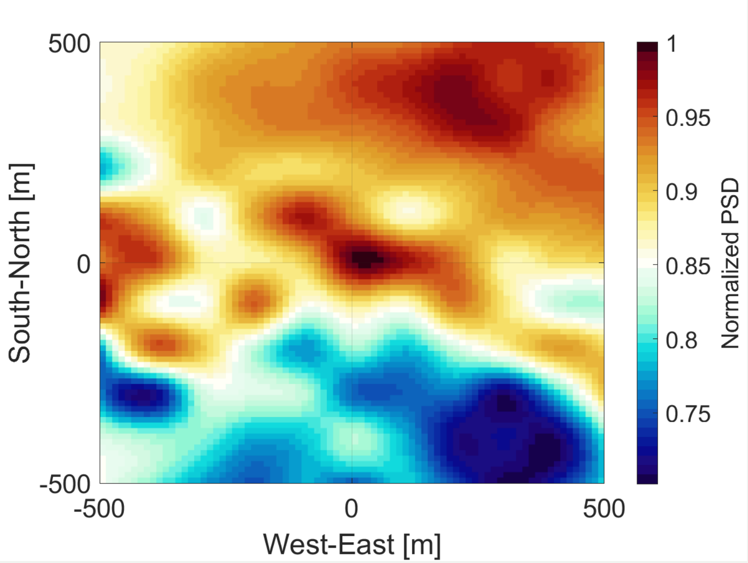
- Implement arbitrary inhomogeneous models
- Leave the calculation of the wavefield to Comsol, ANSYS, SPECFEM3D,...
- SPECFEM3D can directly solve for seismic correlations
- Numerically extremely demanding and prone to systematic errors

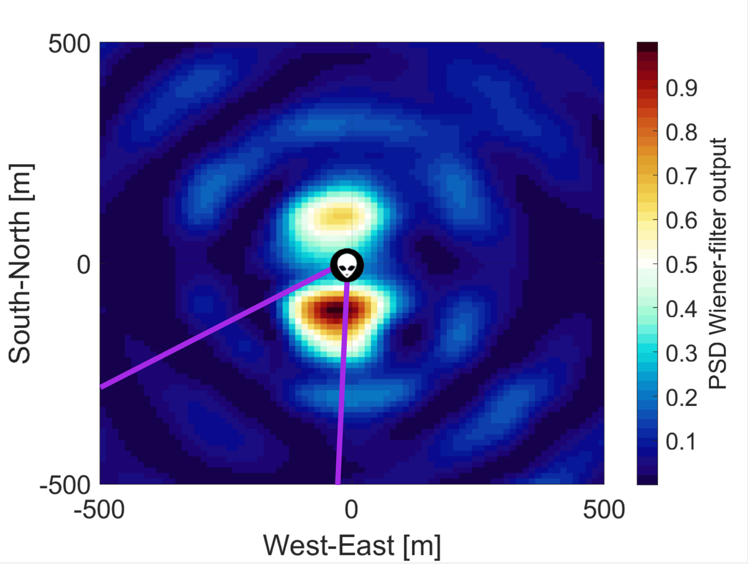
Dynamical finite-element simulations

Improvement

- Any scenario can in principle be modelled

Example: Seismic PSD and gravitational coupling at A3

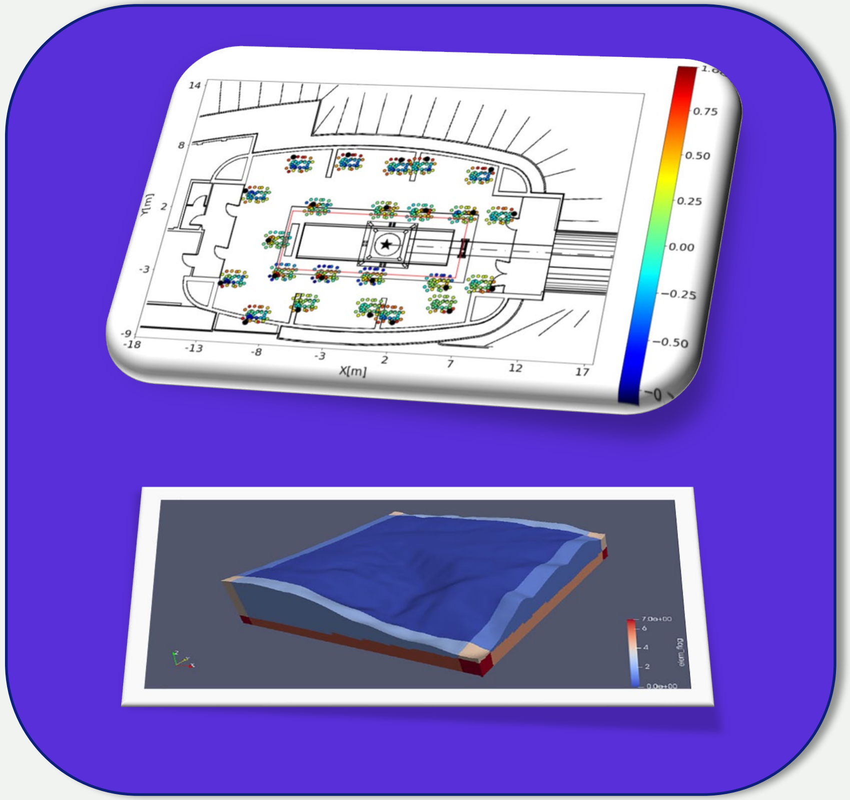




Andric/Harms

Bayesian NN estimation

Combine models and measurements



Method

- Measure seismic correlations
- Model seismic correlations, e.g., with SPECFEM3D
- Set up surrogate model of seismic correlations (e.g., using GPR)
- Numerically integrate surrogate model with appropriate kernel to obtain Newtonian noise

Limitations

- Numerical errors
- Limited understanding of the physics
- Incomplete measurements

Summary

	Seismic fields	Geology	Source distribution	Connection to seismic measurements
Analytical models	hom&iso	hom	iso	Temporal spectrum, seismic speed
Simple numerical models	hom	arbitrary (potentially inconsistent with seismic field)	arbitrary	Temporal and spatial spectra
Correlation models	arbitrary	arbitrary	arbitrary	Two-point spatial correlations
Dynamical FEM	arbitrary	arbitrary	arbitrary	Qualitative
Bayesian methods	arbitrary	arbitrary	arbitrary	Two-point spatial correlations