Rare K Decays

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Rome

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Goal

The Goal

- find New Physics (**NP**)
- understand the NP

Kaon FCNC processes $\nearrow$ suppressed within SM
Kaon FCNC processes $\searrow$ sensitive to NP

- $K_L \to \pi^0 e^+ e^-$
  $K_L \to \pi^0 \mu^+ \mu^-$

$\sim 40\%$ SD

How precise?
Are long distance effects under control?

- $K^+ \to \pi^+ \nu\bar{\nu}$
  $K_L \to \pi^0 \nu\bar{\nu}$

$\sim 95\%$ SD

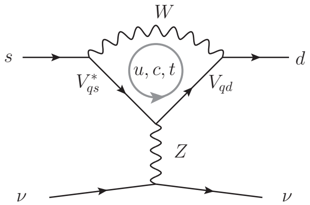
What can rare K decays tell us about NP?

General SM Structure

FCNC processes

loop-induced within SM

$K \to \pi\nu\bar{\nu}$ in the parton picture



u,c,t-Loop

CKM-Factors

non-perturbative parts

- charm-loop $\propto \frac{m_c^2}{M_W^2}$
- top-loop $\propto \frac{m_t^2}{M_W^2}$

quadratic suppression of lower scales

Hard Quadratic GIM

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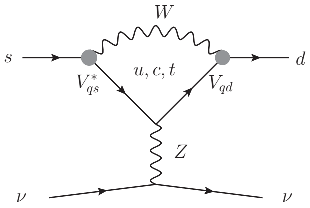

u,c,t-Loop

CKM-Factors

non-perturbative parts

- $\mathcal{A}_{q-\text{contrib.}} \propto V_{qs}^* V_{qd} \equiv \lambda_q$
- $\lambda_u + \lambda_c + \lambda_t = 0$
- parametrised by $\lambda \approx \sin\theta_{\text{Cabibbo}}$

CPC :	$\lambda_t \sim \lambda^5$	$\lambda_c \sim \boldsymbol{\lambda}$
$\cancel{CP}$:	$\text{Im}\lambda_t \sim \lambda^5$	$\text{Im}\lambda_c \sim \lambda^5$

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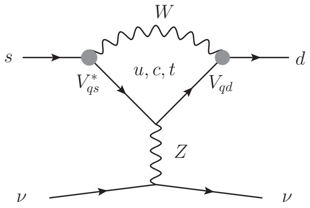


u,c,t-Loop

CKM-Factors

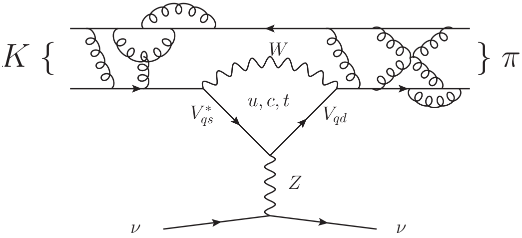
non-perturbative parts

GIM mechanism
+
CKM structure

$\Longrightarrow$ charm pollution

FCNC processes

loop-induced within SM



the involved reality

u,c,t-Loop

CKM-Factors

non-perturbative parts

Separate scales (OPE)

- Lattice calculations
- Chiral perturbation theory (χPT)
- Data , which should not be sensitive to SD

Motivations

Master-Amplitude in SM

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \quad V^i_{\text{CKM}} \quad F_i(\text{high energy parameters})$$

valid for B,D,K -Decays

F_i	:	Loop Functions (SD)	:	different in BSM
V^i_{CKM}	:	CKM factors	:	change in MFV
$\langle Q_i \rangle$	:	Matrix elements of operators (LD)		

Additional operators can be generated by NP

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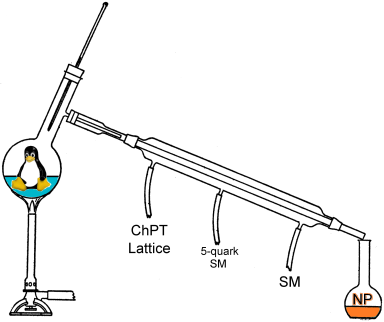
valid for B,D,K -Decays

NP may hide everywhere



tower of **EFTs** essential for the NP distillation

Motivations for K-Physics

Master-Amplitude

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \; \boxed{V^i_{\text{CKM}}} \; F_i$$

$\boxed{F_i}$:

B-Physics: $b \to s \quad \propto \lambda^2$

$b \to d \quad \propto \lambda^3$

K-Physics: $s \to d \quad \propto \lambda^5$ (top)

Extreme Suppression

$\Downarrow$

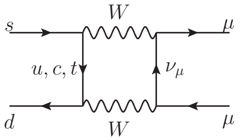
High Sensitivity to deviations from MFV

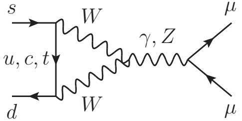
1. Contribution : direct $\not{CP}$

DCPV

$$\mathcal{H}_{\text{eff}}^{V,A} = -\frac{G_F \alpha}{\sqrt{2}} \lambda_t \left[\boxed{y_{7V}} \, (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu l) + \boxed{y_{7A}} \, (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu \gamma_5 l) \right] + h.c.$$

y_{7V}, y_{7A}





- effective coupling for vector- and axial-vector operators
- perturbative calculation
- + γ-Penguin $\longrightarrow$ logarithmic GIM

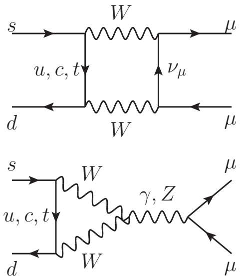
$\to$ NLO QCD (Buras, Lautenbacher, Misiak, Münz 94)

$\approx \pm 5\%$ uncertainty

1. Contribution : direct $\cancel{CP}$ DCPV

$$\mathcal{H}_{\text{eff}}^{V,A} = -\frac{G_F \alpha}{\sqrt{2}} \lambda_t \left[y_{7V} \; (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu l) + y_{7A} \; (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu \gamma_5 l) \right] + h.c.$$

Matrix Elements

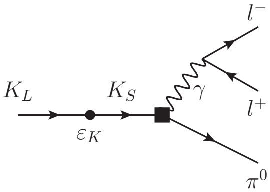


- $\langle \pi^0 e^+ e^- | Q_A | K^0(P) \rangle$ suppressed by m_e
- extracted from $K \to \pi l \nu$ (K_{l3}) decays including isospin breaking effects

few ‰ uncertainty

(Mescia, Smith 07)

2. Contribution : indirect $\not{C}\not{P}$ **ICPV**



- extract with χPT
- $K_L \to \varepsilon_K K_1[\to \pi^0 l^+ l^-]$
- 2 relevant parameters

$\to \varepsilon_K$ (see talk by J. Brod)
$\to |a_S|$ (main theory error, only absolute value known from $K_L \to l^+ l^-$)

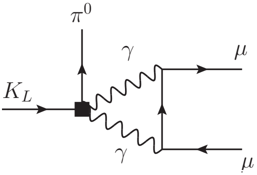
$\approx 20\%$ uncertainty

(Buchalla, D'Ambrosio, Isidori 03; Friot, Greynat, De Rafael 04)

- possible to extract sign from lepton asymmetry

(Mescia, Smith, Trine 06)

3. Contribution : CP conserving CPC



- NOT divergent as in $K_L \to \mu^+\mu^-$
- NO amplitude saturation
- helicity suppressed (0 for e-mode)
- 30% uncertainty

(Isidori, Smith, Unterdorfer 04)

DCPV : ICPV : CPC compete

$$\mathrm{Br}^{theo}(K_L \to \pi^0 e^+ e^-) = 3.54^{+0.98}_{-0.85} \quad (1.56^{+0.62}_{-0.49}) \quad \times 10^{-11}$$
$$\mathrm{Br}^{theo}(K_L \to \pi^0 \mu^+ \mu^-) = 1.41^{+0.28}_{-0.26} \quad (0.95^{+0.22}_{-0.21}) \quad \times 10^{-11}$$

(Mescia, Smith, Trine, 06)

$$\mathrm{Br}^{exp}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \qquad \text{[KTeV 04]}$$
$$\mathrm{Br}^{exp}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10} \qquad \text{[KTeV 04]}$$

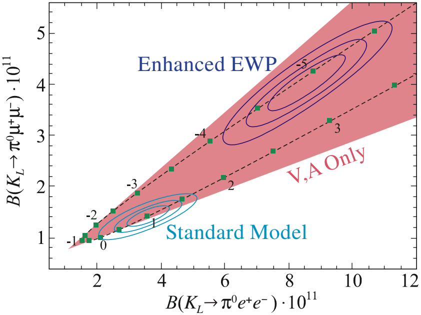
What can we do with $K_L \to \pi^0 l^+ l^-$?

$\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-)/\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ -plane

ideal for discriminating NP models

(Isidori, Smith, Unterdorfer 04; Mescia, Smith,Trine 06)

Model Independent Analysis



(Mescia, Smith,Trine 06)

1. Scenario

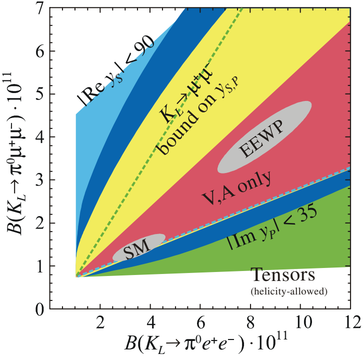
- NP changes only y_{7A}, y_{7V}
  (SD Wilson Coefficients)
- No new operators
- e^+e^- VS $\mu^+\mu^-$
  $\to$ different phase-space
  $\to$ $K_L \to \gamma\gamma$ contribution

$\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-)/\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ -plane

ideal for discriminating NP models

(Isidori, Smith, Unterdorfer 04; Mescia, Smith,Trine 06)

Model Independent Analysis



(Mescia, Smith,Trine 06)

2. Scenario

- New Operators generated
- Scalar- Pseudoscalar Operators

  $(\bar{s}d)(\bar{l}l)$

  $(\bar{s}d)(\bar{l}\gamma_5 l)$
- Tensor- Pseudotensor- Operators

  $(\bar{s}\sigma_{\mu\nu}d)(\bar{l}\sigma^{\mu\nu}l)$

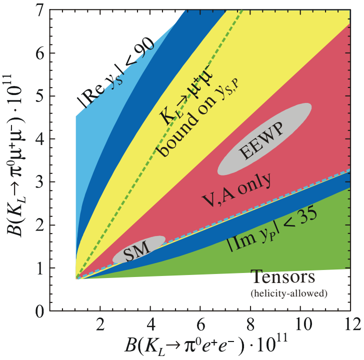
  $(\bar{s}\sigma_{\mu\nu}d)(\bar{l}\sigma^{\mu\nu}\gamma_5 l)$
- $\underbrace{\text{With}}$ & $\underbrace{\text{Without}}$ Helicity Suppression

$\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-)/\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ **-plane**

ideal for discriminating NP models

(Isidori, Smith, Unterdorfer 04; Mescia, Smith,Trine 06)

Model Independent Analysis



(Mescia, Smith,Trine 06)

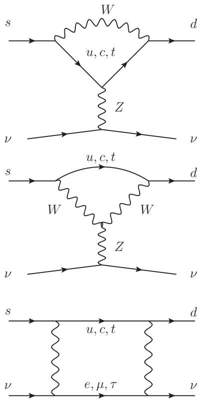
2. Scenario

Pick your favourite Model:

- MSSM large $\tan\beta$
- SUSY without R-parity
- leptoquarks
- ...

disentangled on the $\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-)/\mathcal{B}(K_L \to \pi^0 e^+ e^-)$

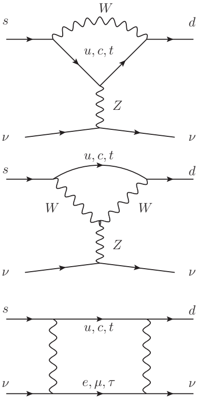
$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{q=u,c,t} \lambda_q \; X(x_q) \; (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu \nu_{lL}) \; + h.c.$$



- 2 modes: $K^+ \to \pi^+\nu\bar{\nu}$ some charm pollution
  $K_L \to \pi^0\nu\bar{\nu}$ only top contribution
  $\cancel{CP}$
- one dominant dim-6 operator
- no γ Penguin
- no $\gamma\gamma$ contribution

$K \to \pi\nu\bar{\nu}$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{q=u,c,t} \lambda_q \; X(x_q) \; (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL} \gamma^\mu \nu_{lL}) + h.c.$$



Matrix Elements

- extracted from K_{l3} decays
- isospin breaking effects included

(Mescia, Smith 07)

- κ_ν^+ and κ_ν^L
- only 8% and 21% of total theory error!

NLO perturbative calculations relevant

Why EW corrections on X_t?

$$\mathcal{H}_{\text{eff}} = \frac{2}{\pi\sqrt{2}} \boxed{G_F \frac{\alpha}{\sin^2\theta_W}} \lambda_t \, X_t\Big(\boxed{\frac{m_t^2}{M_W^2}}\Big) + \ldots \text{charm} \ldots + h.c.$$

$$X_t = \underbrace{X^{(0)}}_{\text{LO}} + \underbrace{\frac{\alpha_s}{4\pi} X^{(1)}}_{\text{NLO QCD}} + \underbrace{\frac{\alpha_s^2}{(4\pi)^2} X^{(2)}}_{\text{NNLO QCD}} + \underbrace{\frac{\alpha_s^3}{(4\pi)^3} X^{(3)}}_{\text{NNNLO QCD}} + \ldots$$

LO EW parameters still NOT fixed

- ambiguity from EW renormalisation scheme (on-shell VS $\overline{MS}$)

$$X_t = \underbrace{X^{(0)}}_{\text{LO}} + \underbrace{\frac{\alpha}{4\pi} X^{(EW)}}_{\text{NLO EW}} + \ldots$$

- only fixed after calculating $X^{(EW)}$

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Table 10.2: Notations used to indicate the various schemes discussed in the text. Each definition of $\sin^2\theta_W$ leads to values that differ by small factors depending on m_t and M_H. Approximate values are also given for illustration.

Scheme	Notation	Value
On-shell	s_W^2	0.2233
NOV	$s_{M_Z}^2$	0.2311
$\overline{\text{MS}}$	$\hat{s}_Z^2$	0.2313
$\overline{\text{MS}}$ ND	$\hat{s}_{\text{ND}}^2$	0.2315
Effective angle	$\bar{s}_f^2$	0.2316

◦ ambi $\overline{MS}$)

◦ only fixed after calculating $X^{(EW)}$ (PDG 10)

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$$\mathcal{H}_{\text{eff}} = \frac{2}{\pi\sqrt{2}} \boxed{G_F \frac{\alpha}{\sin^2\theta_W}} \; \lambda_t \; X_t\Big(\boxed{\frac{m_t^2}{M_W^2}}\Big) + \ldots \text{charm} \ldots + h.c.$$

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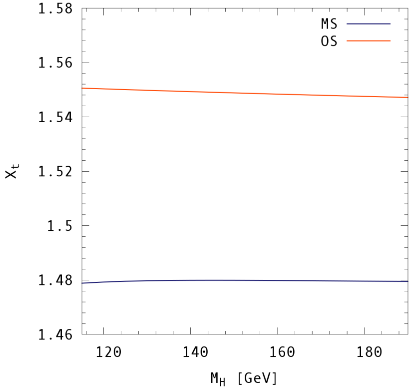
EW corrections



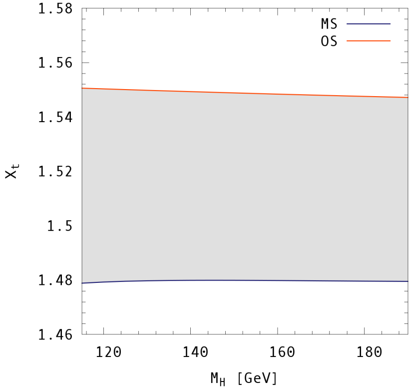
on-shell

$\overline{MS}$

- $X^{(EW)}$ known for large-m_t limit
- large-m_t known to be a bad approximation

(Buchalla, Buras 98)

- $\pm 2\%$ uncertainty in X_t scales to $\pm 4\%$ uncertainty in Branching Ratios



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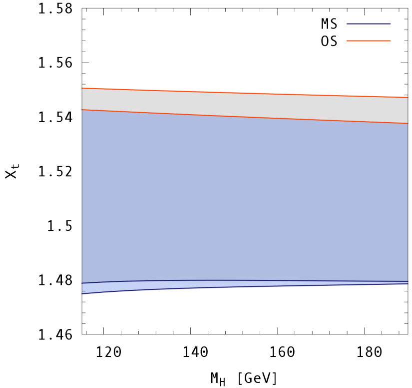

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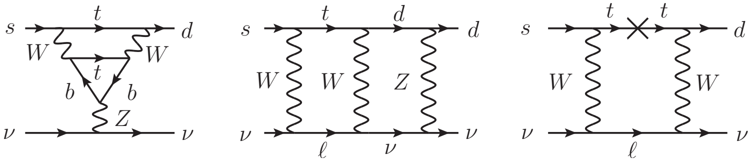


Calculation

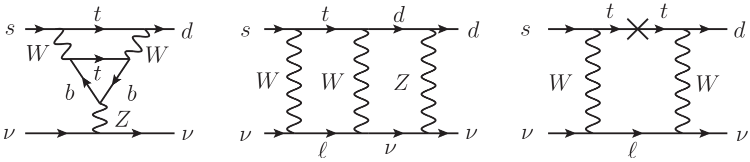
- full two-loop calculation $\mathcal{O}(1000)$ diagrams
- matching on 5-quark effective theory at μ_t
- $\overline{MS}$ renormalisation for calculation

Details

- HV-scheme for diagrams with anomalies

Checks

- 2 independent calculations (FORM, MATHEMATICA)
- reproduced large-m_t limit



Error Estimation

3 schemes

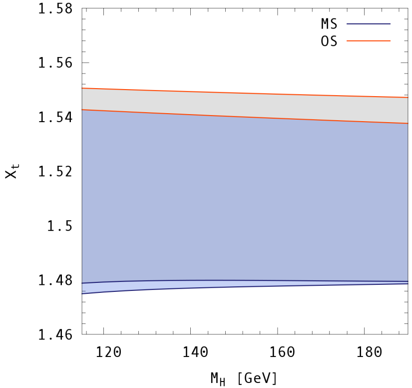
on-shell: G_F, M_Z, M_t, M_H, and α

$\overline{MS}$**:** g_1, g_2, v, λ, and y_t

one-loop running and fit for initial conditions

$\to$ introduces residual M_H dependence

mix: on-shell masses, but $\overline{MS}$ couplings

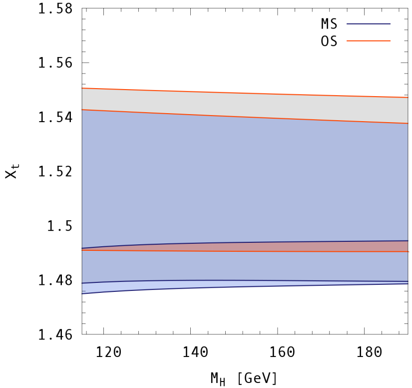


on-shell

$\overline{MS}$

completely fixes the renormalisation scheme dependence
$\pm 2\% \to \pm 0.3\%$

(arXiv:1009.0947v1 (hep-ph))



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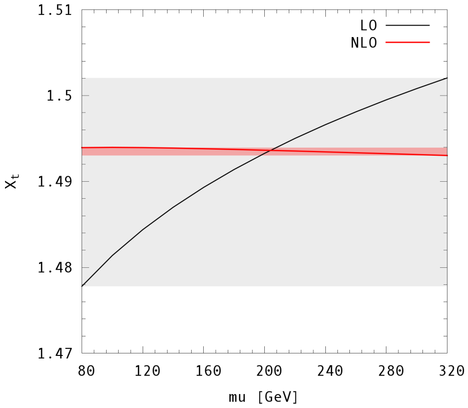


removed the remaining scale dependence

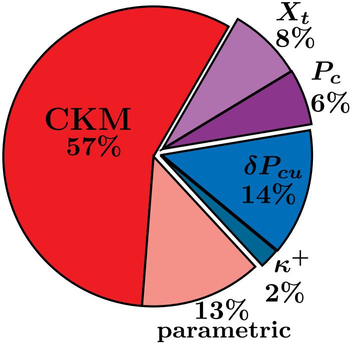
Loop-functions are universal:

Correction applies also to: $B \to X_{d,s}\nu\bar{\nu}$

Branching Ratios

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \kappa_\nu^+ \left| \lambda_t X_t + \mathrm{Re}\lambda_c \left(P_c + \delta P_{c,u} \right) \right|^2$$

Long Distance



- κ_ν^+ with isospin corrections
- QED radiative corrections

  (Mescia, Smith 07)
- $\delta P_{c,u}$: dim-8 operators below μ_c

  (Falk, Lewandowski, Petrov 01)
- $\delta P_{c,u}$: light quark contributions

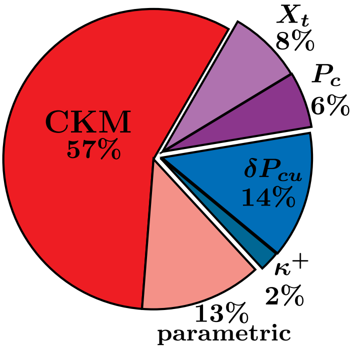
  (Isidori, Mescia, Smith 05)
- Improvement by lattice possible!

  (Isidori, Martinelli, Turchetti 06)

Branching Ratios

$$\text{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \kappa_\nu^+ \left| \lambda_t \boxed{X_t} + \text{Re}\lambda_c \left(\boxed{P_c} + \delta P_{c,u} \right) \right|^2$$

Short Distance

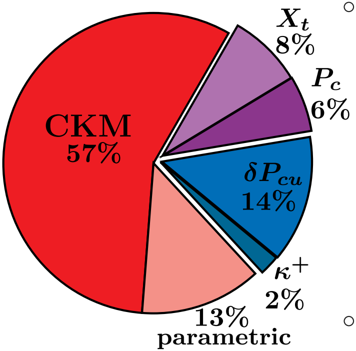


- P_c : NNLO QCD
  (Buras, Gorbahn, Haisch, Nierste 05)
- P_c : NLO EW
  (Brod, Gorbahn 08)
- X_t : NLO QCD
  (Misiak, Urban; Buchalla, Buras 99)
- X_t : NLO EW
  (Brod, Gorbahn, ES 10)

Branching Ratios

$$\text{Br}\left(K^+ \to \pi^+ \nu\bar\nu\right) = \kappa_\nu^+ \left| \lambda_t X_t + \text{Re}\lambda_c \left(P_c + \delta P_{c,u} \right) \right|^2$$

Numbers



- 7 events at E787/949

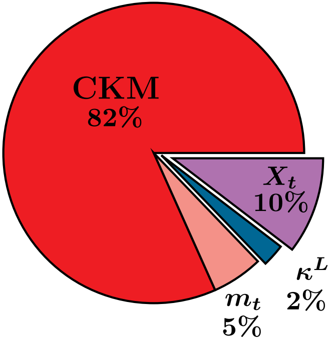
$$\text{Br}^{exp} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

$$\text{Br}^{theo} = (7.81^{+0.80}_{-0.71} \pm 0.29) \times 10^{-11}$$

(Brod, Gorbahn, ES 10)

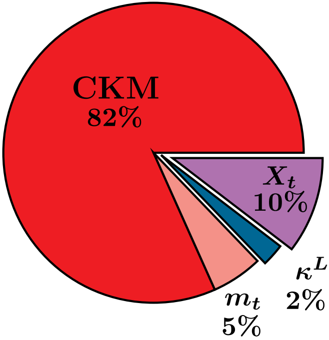
- NA62@CERN aims at $\mathcal{O}(100)$ events!

$$\mathrm{Br}\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \kappa_\nu^L \left(1 - \sqrt{2}|\epsilon_K| \frac{1+P_c(X)/A^2 X_t - \rho}{\eta}\right) \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2$$



- CPC contribution negligible (Buchalla, Isidori 98)
- small 1% effect from ICPV (Buchalla, Buras 96)
- $\not{CP}$: only top contribution
- very clean and sensitive to SD

$$\text{Br}\left(K_L \to \pi^0 \nu\bar\nu\right) = \kappa_\nu^L \left(1 - \sqrt{2}|\epsilon_K|\frac{1+P_c(X)/A^2 X_t - \rho}{\eta}\right)\left(\frac{\text{Im}\lambda_t}{\lambda^5} X_t\right)^2$$



Numbers

- upper bound from E391a

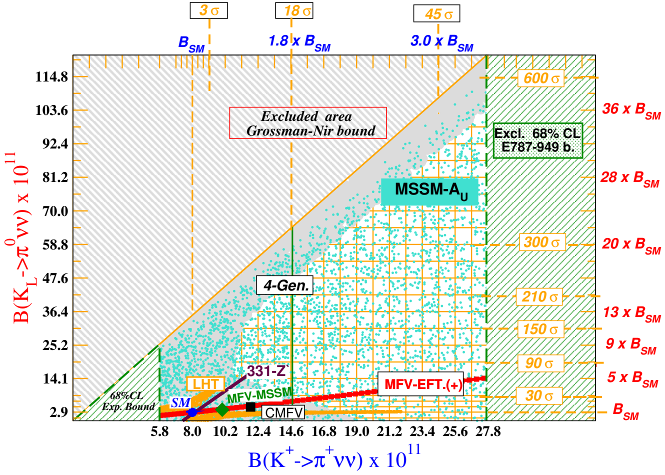
$$\text{Br}^{exp} < 6.7 \times 10^{-8}$$

$$\text{Br}^{theo} = (2.43^{+0.40}_{-0.37} \pm 0.06) \times 10^{-11}$$

(Brod, Gorbahn, ES 10)

- KOTO@JPARC $\mathcal{O}(100)$ events ?

The $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$-plane



(http://www.lnf.infn.it/wg/vus/content/Krare.html)

Conclusions

Even after 60 years of intensive Kaon studies (and 2 Nobel prices)

Kaons are still exciting!

Together with ε_K, rare K decays:

- probe high energy regimes
- provide stringent bounds on NP models
- discriminate between models BSM (e.g. $K_L \to \pi^0 l^+ l^-$)
- can measure NP parameters (e.g. $K \to \pi\nu\bar{\nu}$)

Interesting times are ahead!