A model for neutrino masses and mixing based on the non-abelian discrete symmetry A4

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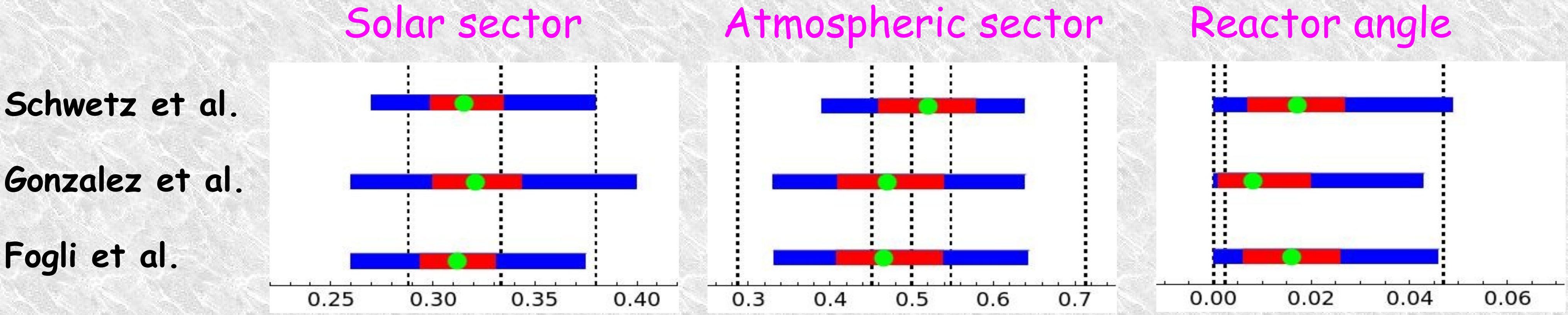
Talk based on G.Altarelli & DM, J.Phys.G36:085005,2009

Introduction

Neutrino physics is entering a “precision era”



Courtesy by S. Morisi

$\sin^2(\theta_{12}) = 1/3$ $\sin^2(\theta_{23}) = 1/2$ $\sin^2(\theta_{13}) = 0$

Tri-bimaximal mixing

Introduction

In the standard parameterization of the mixing matrix

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\theta_{13} = 0$

$\theta_{23} = 45^o$

$\theta_{12} \sim 33^o$

This is the starting point of our discussion:

We assume that this agreement is not an accidental fact

TBM and discrete symmetries

As for the quark case, the neutrino mixing matrix U_{PMNS} is given by the product of two different rotation matrices

$$U_{PMNS} = U^{+}_{l} \; U_{\nu}$$

from charged leptons

from neutrino sector

If $U_{\nu} = U_{TBM}$

$$M_{\nu} = U_{TBM} \; M^{diag}_{\nu} \; U^{T}_{TBM} \longrightarrow M_{\nu} = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

This is the most general structure of M_{ν} diagonalized by TBM

This is a symmetric matrix with $a_{11} + a_{12} = a_{22} + a_{23}$

TBM and discrete symmetries

Then if you want to get a TBM neutrino mixing, your model must be able to produce this kind of mass matrix

The group A_4 is a good candidate

1 - A_4 is the group of even permutations of 4 objects → 12 elements

2 – as usual, to generate all the group elements we need to identify " generators of the group" and their action on the elements of the group

these are called S and T

↓

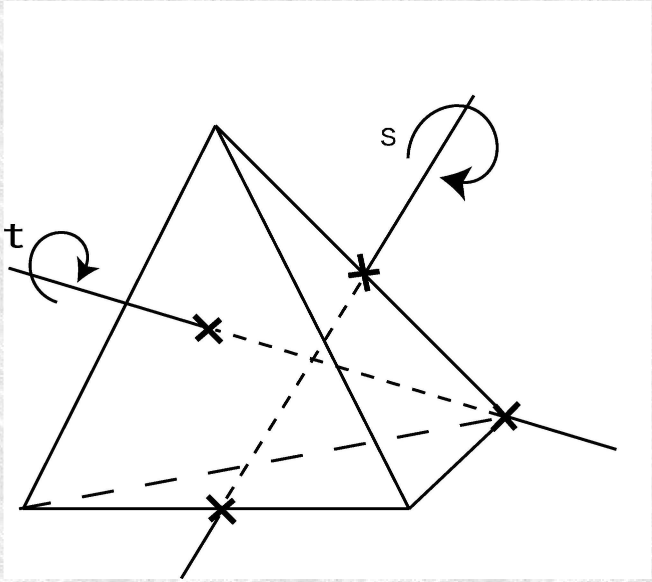
One possible "representation": $S^2=T^3=(ST)^3=1$

3 – they act as follows: (1234) → (4321) under S

(1234) → (2314) under T

TBM and discrete symmetries

The group A_4 is also the symmetry of a tetrahedron



The 12 elements are obtained considering all possible permutations of 1234.
They belong to 4 conjugacy classes... $given \ a \ of \ G, \{g^{-1} a g, \forall g \in G\}$

A_4 has 4 irreducible representations

- three singlets 1, 1' and 1''
- 1 triplet 3

TBM and discrete symmetries

A representation of the group

"T-diagonal representation"

1 : $S=1 \quad T=1$

$1'$: $S=1 \quad T=e^{i 4 \pi/3} = \omega^2$

$1''$: $S=1 \quad T=e^{i2 \pi/3} = \omega$

3_1 :

$$T=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad S=\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Table of multiplication

$$1_1 \otimes any = any$$

$$1' \otimes 1' = 1'' \quad 1' \otimes 1'' = 1 \quad 1'' \otimes 1'' = 1'$$

$$3 \otimes 3 = 1 + 1' + 1'' + 3_S + 3_A$$

TBM and discrete symmetries

The TBM is derived considering that:

A_4 is a symmetry of the Nature at a very high energy scale Λ

The symmetry is spontaneously broken by a set of scalar multiplets Φ (FLAVONS) with VEV aligned in some particular directions

The preserved subgroups in the charged and neutrino sectors must be different otherwise

$$U_l = U_\nu \quad \Rightarrow \quad U_{PMNS} = U^{+}_{l} U_\nu = I$$

↓

Different symmetry breaking patterns needed !!!

corresponding to some subgroup of A4

The model: $A_4 \times Z_4$

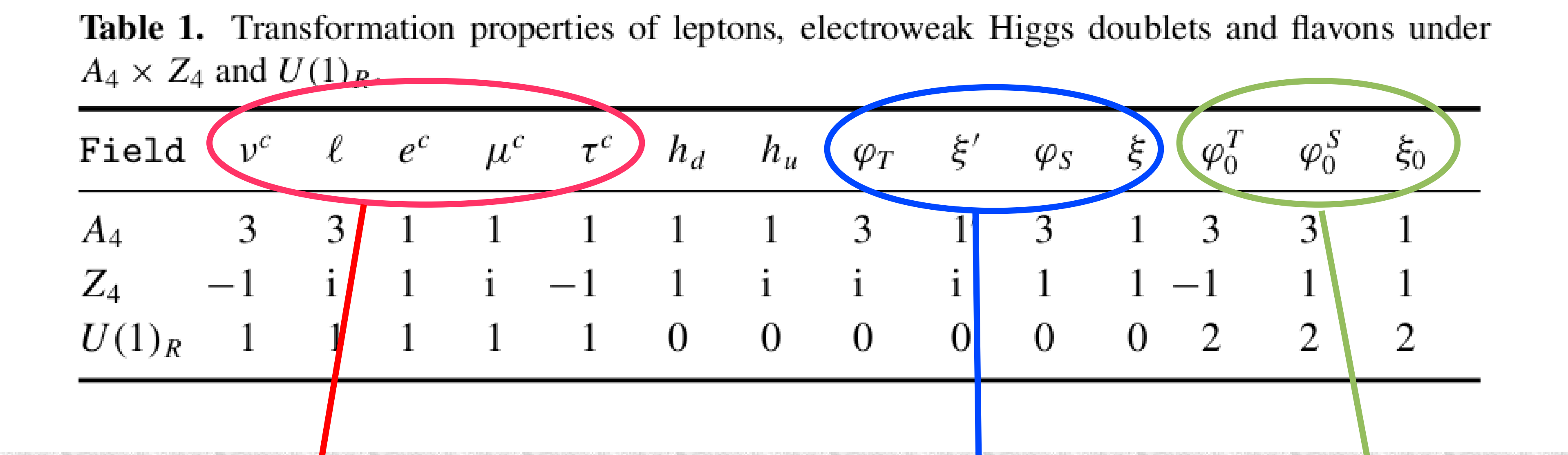


Table 1. Transformation properties of leptons, electroweak Higgs doublets and flavons under $A_4 \times Z_4$ and $U(1)_R$.

Field	ν^c	ℓ	e^c	μ^c	τ^c	h_d	h_u	φ_T	ξ'	φ_S	ξ	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	1	1	1	3	1	3	1	3	3	1
Z_4	-1	i	1	i	-1	1	i	i	i	1	1	-1	1	1
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	2	2	2

Leptons, including right-handed neutrinos

Auxiliary fields for the flavon alignment

Sector responsible for the A4 breaking:

- ϕ_S and ξ act in the neutrino sector
- ϕ_T and ξ' in the charged lepton sector

Z_4 maintains the neutrino and charged lepton sectors separated at leading order

The charged lepton sector

$$\frac{y_\tau}{\Lambda}\tau^c(\ell\varphi_T)h_d$$

τ entry

$$\frac{y_\mu}{\Lambda^2}\mu^c(\ell\varphi_T\varphi_T)h_d+\frac{y'_\mu}{\Lambda^2}\mu^c(\ell\varphi_T)''\xi'h_d$$

μ entries

electrons

$$+\frac{y_e}{\Lambda^3}e^c(\ell\varphi_T\varphi_T)''\xi'h_d+\frac{y'_e}{\Lambda^3}e^c(\ell\varphi_T)'\xi'^2h_d+\frac{y''_e}{\Lambda^3}e^c(\ell\varphi_T)'(\varphi_T\varphi_T)''h_d$$

$$+\frac{y'''_e}{\Lambda^3}e^c(\ell\varphi_T)''(\varphi_T\varphi_T)'h_d+\frac{y^{\mathrm{iv}}_e}{\Lambda^3}e^c(\ell\varphi_T)_1(\varphi_T\varphi_T)_1h_d+\cdots$$

- the hierarchy is garanteed by flavon insertion
- the diagonal form of the mass matrix by Z4

The charged lepton sector

The A4 symmetry is completely broken by

$$\langle\phi_T\rangle = v_T(0,1,0) \quad \text{and} \quad \langle\xi'\rangle = u'$$

and gives

$$m_\ell = \begin{pmatrix} \frac{v_T v_d}{\Lambda^3}\left(2y_e v_T u' + y_e' u'^2 + y_e'' v_T^2\right) & 0 & 0 \\ 0 & \frac{v_T v_d}{\Lambda^2}(2y_\mu v_T + y_\mu' u') & 0 \\ 0 & 0 & \frac{y_\tau v_d v_T}{\Lambda} \end{pmatrix}$$

If we assume that both vevs v_T and u' have the same order of magnitude then the charged lepton hierarchy is reproduced with

$$\frac{v_T}{\Lambda} = \frac{u'}{\Lambda} = \epsilon = \lambda_c^2$$

The neutrino sector

we assume that neutrino masses are generated by the See-Saw mechanism

$$M^{\nu} = -m_D^T m_M^{-1} m_D$$

Simmetry breaking: $\langle\phi_S\rangle = v_S(1,1,1)$ and $\langle\xi\rangle = u$ (Z2)

$$w_\nu = y_\nu(\nu^c\ell)h_u + \underbrace{(M + a\xi)\nu^c\nu^c + b\nu^c\nu^c\varphi_S}_{\text{Majorana mass term}}$$

Dirac mass term

$$m_D = y_\nu v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Majorana mass term

$$m_M = \begin{pmatrix} M + au + 2bv_S & -bv_S & -bv_S \\ -bv_S & 2bv_S & M + au - bv_S \\ -bv_S & M + au - bv_S & 2bv_S \end{pmatrix}$$

$$m = \begin{pmatrix} x & y & y \\ y & x + v & y - v \\ y & y - v & x + v \end{pmatrix}$$

Vacuum alignment

Any serious flavour model must derive the vacuum expectation values of the flavon fields from general principles

To achieve this aim, one introduces new Standard Model singlet fields, DRIVING FIELDS with well defined transformation properties under $A_4 \times Z_4$

- we build the most general superpotential W allowed by the symmetries of the theory and derive the scalar potential in the usual way

$$V = \Sigma \left| \frac{\partial W}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$$

ϕ_i are driving fields components

Since m_i are expected to be smaller than the mass scales in W, we can neglect this term and then working in the SUSY limit

$$V = 0 \quad \Rightarrow \quad \left| \frac{\partial W}{\partial \phi_i} \right| = 0$$

We get a set of equations for the flavon field components; the solution of the system of equations are the VEV configurations

Light neutruno spectrum

We parametrize the Lagrangian parameters in the following way:

$$w_\nu = y_\nu(\nu^c\ell)h_u + (M + a\xi)\nu^c\nu^c + b\nu^c\nu^c\varphi_S$$

$$A = M + au = |A|\,\mathrm{e}^{i\phi_A}, \qquad B = 3bv_S = |B|\,\mathrm{e}^{i\phi_B}$$

$$\alpha = \frac{|B|}{|A|}, \qquad \phi = \phi_B - \phi_A.$$

A is a measure of the Majorana mass scale

The LO light masses are:

$$m_1 = -\frac{v_u^2 y_\nu^2}{|A|\,\mathrm{e}^{i\phi_A}}\left(\frac{1}{1+\alpha\,\mathrm{e}^{i\phi}}\right)$$

$$m_2 = -\frac{v_u^2 y_\nu^2}{|A|\,\mathrm{e}^{i\phi_A}}$$

$$m_3 = \frac{v_u^2 y_\nu^2}{|A|\,\mathrm{e}^{i\phi_A}}\left(\frac{1}{1-\alpha\,\mathrm{e}^{i\phi}}\right).$$

Notice the tipical A4 sum rule

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

Light neutruno spectrum- Normal Hierarchy

It is easy to compute the ratio r:

$$r = \frac{\Delta m^2_{\mathrm{sol}}}{\left|\Delta m^2_{\mathrm{atm}}\right|} = \frac{(1+\alpha^2 - 2\alpha\cos\phi)(\alpha + 2\cos\phi)}{4|\cos\phi|}$$

A small r~1/30 is obtained for

- $\cos\phi \sim \alpha \sim 1$ Normal Hierarchy
- $\cos\phi = -\alpha/2 + \delta\alpha$ Inverted Hierarchy

For Normal Hierarchy

$$|m_1|^2 = \frac{1}{3}\Delta m^2_{atm} r \qquad |m_2|^2 = \frac{4}{3}\Delta m^2_{atm} r \qquad |m_3|^2 = \left(1+\frac{r}{3}\right)\Delta m^2_{atm}$$

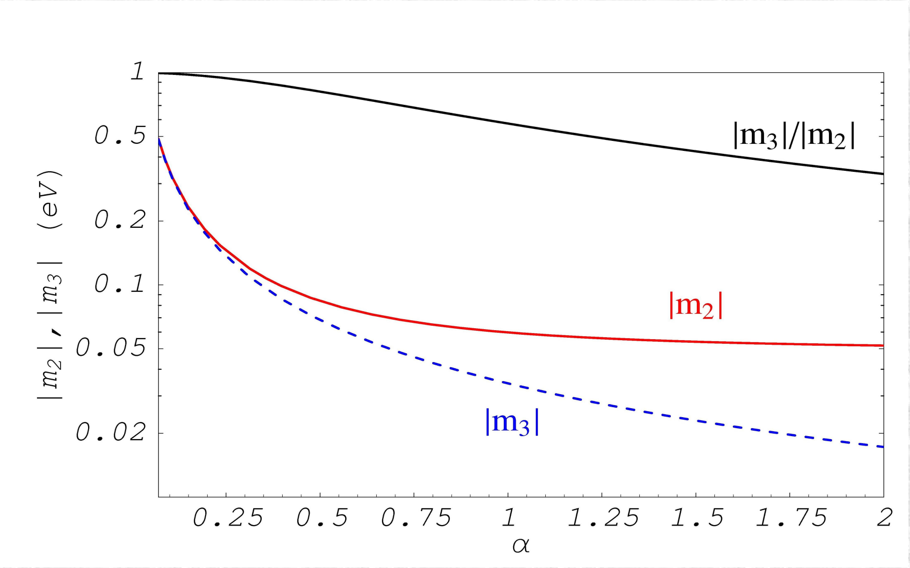
$$|m_{ee}|^2 = \left|m_1 U^2_{e1} + m_2 U^2_{e2}\right| = \frac{16}{27}\,\Delta m^2_{atm}\; r \sim (0.007\,eV)^2$$

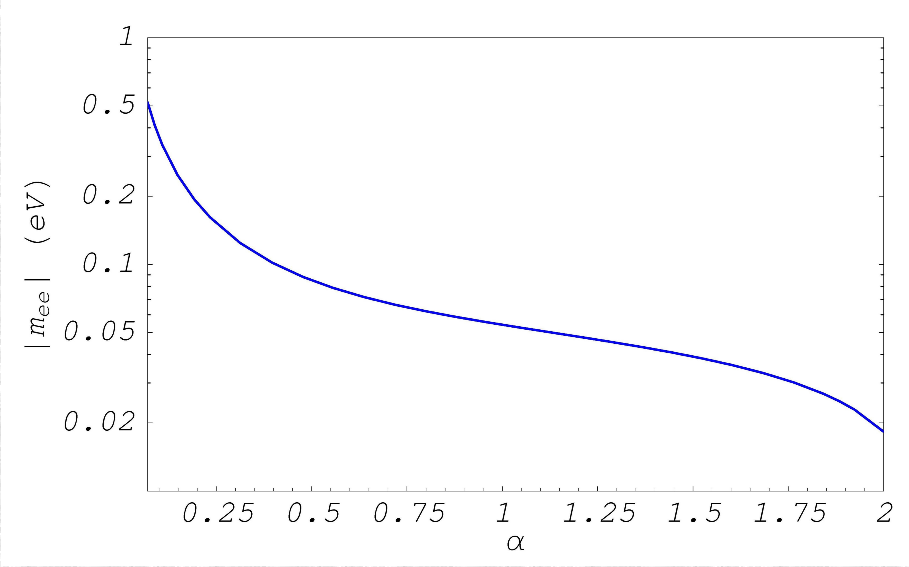
Light neutruno spectrum- Inverted Hierarchy

The results depend on α=|B/A|

$$|m_2|^2 = |m_1|^2 + |\Delta m^2_{\rm atm}| r = |\Delta m^2_{\rm atm}| \left(\frac{1+2\alpha^2}{2\alpha^2}\right)\left[1 + r\left(1 + \frac{1}{(1+2\alpha^2)^2}\right)\right] + \cdots$$
$$|m_3|^2 = |m_1|^2 - |\Delta m^2_{\rm atm}|.$$

$$|m_{ee}|^2 = |\Delta m^2_{\rm atm}| \left(\frac{1+2\alpha^2}{2\alpha^2}\right)\left(1 - \frac{2}{9}\alpha^2\right)$$



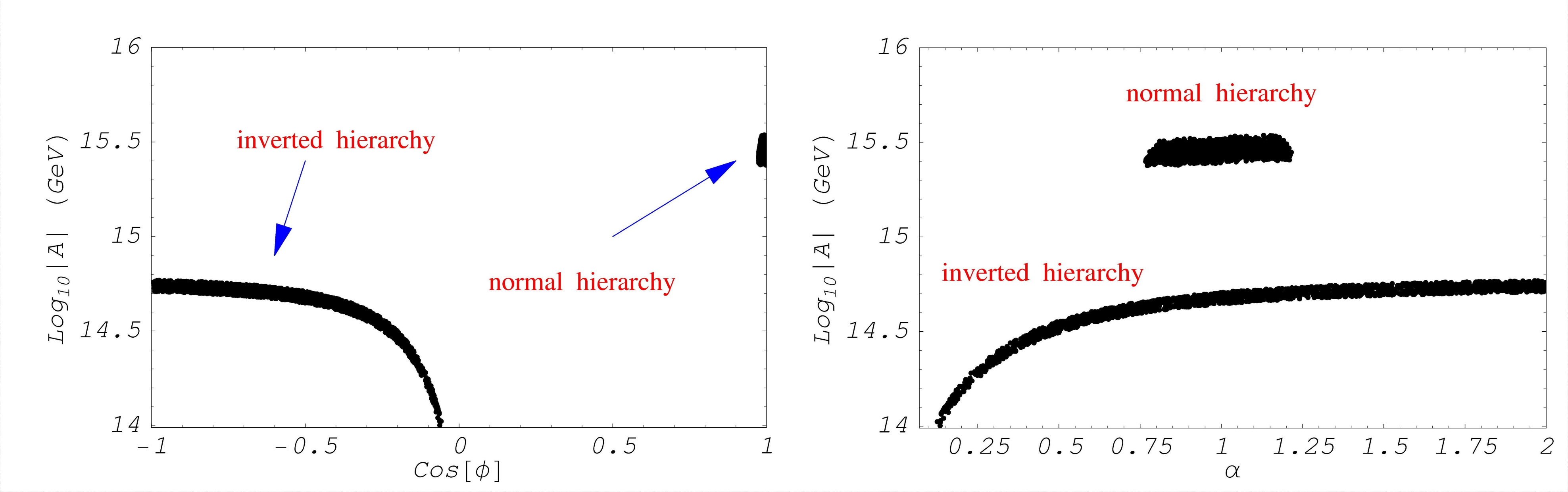


Lower limit on α ~ 0.07 from the requirement that $|m_i|$< 0.5 eV

Majorana neutrino masses

$$M_1 = |A| e^{\phi_A}(1+\alpha e^{\phi}) \qquad M_2 = |A| e^{\phi_A} \qquad M_3 = |A| e^{\phi_A}(-1+\alpha e^{\phi})$$

We explored the allowed parameter space under the restrictions that the experimental values of Δ_{sol} and r are fullfilled at 3 sigma level



M~10^{14}-10^{15} GeV

Beyond the LO approximation

At the next level of approximation the superpotential is corrected by operators of higher dimensions whose contributions are suppressed by at least one power of VEVs/Λ

NLO correction could affect any sector of the theory

Charged lepton and neutrino masses are modified by

Corrections to the vacuum alignment

$$\langle \Phi \rangle \rightarrow \langle \Phi \rangle^{LO} + \delta \Phi \quad where \quad \delta \Phi \sim \frac{\langle \Phi \rangle^{LO}}{\Lambda}$$

Higher order operators (NLO) of O(1/Λ) compared to the LO lagrangian

$$W^{tot} = W^{LO}(\langle \Phi \rangle^{LO}) + W^{LO}(\delta \Phi) + W^{NLO}(\langle \Phi \rangle^{LO})$$

Beyond the LO approximation

The results of such a procedure are:

$$\langle \varphi_S \rangle = v_S \left(1+\delta v_S, 1+\delta v_S, 1+\delta v_S\right)^T \qquad \langle \varphi_T \rangle = v_S \left(1+\delta v_{T1}, 1+\delta v_{T2}, 1+\delta v_{T3}\right)^T$$

Charged leptons

LO

NLO

$$m_\ell = \varepsilon v_d \begin{pmatrix} a_1\varepsilon^2 & 0 & 0 \\ 0 & b_2\varepsilon & 0 \\ 0 & 0 & c_3 \end{pmatrix} \quad \longrightarrow \quad m_\ell = \varepsilon v_d \begin{pmatrix} a_1\varepsilon^2 & a_2\varepsilon^2\varepsilon' & a_3\varepsilon^2\varepsilon' \\ b_1\varepsilon\varepsilon' & b_2\varepsilon & b_3\varepsilon\varepsilon' \\ c_1\varepsilon' & c_2\varepsilon' & c_3 \end{pmatrix}$$

The diagonalizing matrix is:

$$U_\ell = \begin{pmatrix} 1 & \left(\frac{b_1}{b_2}\varepsilon'\right)^* & \left(\frac{c_1}{c_3}\varepsilon'\right)^* \\ -\frac{b_1}{b_2}\varepsilon' & 1 & \left(\frac{c_2}{c_3}\varepsilon'\right)^* \\ -\frac{c_1}{c_3}\varepsilon' & -\frac{c_2}{c_3}\varepsilon' & 1 \end{pmatrix}$$

Beyond the LO approximation

Neutrinos

$$m_\nu = -m_D^T m_M^{-1} m_D = \underbrace{(m_\nu)_{TB}}_{\text{LO}} + \underbrace{v_S \varepsilon' \begin{pmatrix} A & B & C \\ B & D & F \\ C & F & E \end{pmatrix}}_{\text{NLO}}$$

The diagonalizing U_ν has the same strucure as in the charged lepton sector

$$U_{PMNS} = U^+_l U_\nu \quad \left\} \begin{aligned} & \sin\theta_{13} = |U_{e3}| \sim O(\lambda_C^2) \\ & \sin^2\theta_{12} = \frac{|U_{e2}|}{\sqrt{1-|U_{e3}|^2}} \sim \frac{1}{3} + O(\lambda_C^2) \\ & \sin^2\theta_{23} = \frac{|U_{\mu 3}|}{\sqrt{1-|U_{e3}|^2}} \sim \frac{1}{2} + O(\lambda_C^2) \end{aligned} \right.$$

Conclusions

- TBM is a good description of neutrino experimental data

- Models for neutrino masses and mixing based on non-abelian discrete symmetries are quite good in reproducing this pattern

- We presented an example based on A_4, able to generate NLO corrections to TBM compatible with the data

- The model allows for both mass spectra and is also compatible with leptogenesis

Backup slides

Announcement...

Workshop on FLAvour SYmmetries FLASY2011

Valencia-IFIC, 11-14 July 2011

Main goal:

- summarize the theoretical status of flavor symmetries, bringing together researchers in the field to stimulate discussions and new collaborations
- Interesting subjects: possible new physics scenarios to be tested at the LHC, as well as in future neutrino, cosmology experiments and dark matter searches.
- Talks mainly given by young researchers and PhD students

Official web page: soon available

Official e-mail address: flasy2011@ific.uv.es

Organizing Committee :

M. Hirsch (Valencia), D.Meloni (Wuerzburg), S.Morisi (Valencia),
S. Pastor (Valencia), E. Peinado (Valencia), J.W.F. Valle (Valencia).

Leptogenesis

Just an estimate....

The total CP asymmetries in the decay of a RH neutrino are defined as

$$\epsilon_i = \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}$$

At one loop:

$$\boxed{\epsilon_i = \frac{1}{8\pi(\tilde{Y}\tilde{Y}^+)_{ii}} \Sigma_{j\neq i} \Im(\tilde{Y}\tilde{Y}^+)^2_{ij} f\left(\frac{|M_j{}^2|}{|M_i{}^2|}\right)}$$

"hat" matrices are Yukawa matrices in the basis

where the Majorana mass matrix is diagonal

For susy theories:

$$f(x) = -\sqrt{x}\left[\frac{2}{x-1} + \log\left(\frac{1+x}{x}\right)\right]$$

Leptogenesis

In the basis where the Majorana mass matrix is diagonal:

$$v_d \tilde{Y} = v_d U^T_{TBM} Y_\nu \qquad Y = \frac{m_D}{v_d}$$

$$\tilde{Y}\tilde{Y}^+ = U^T_{TBM}\left(Y_\nu Y^+_\nu\right) U^*_{TBM} = U^T_{TBM}\left(diagonal\, matrix\right) U^*_{TBM} = I$$

NLO are crucial !

E.E.Jenkins and A.V.Manohar,
Phys. Lett. B 668, 210 (2008)

$$\delta(\tilde{Y}\tilde{Y}^+) = U^T_{TBM}\,\delta(Y_\nu Y^+_\nu)\,U^*_{TBM}$$

↓

$$\epsilon_1 \sim \epsilon_3 \sim \epsilon^3 \sim \lambda_C^6 \sim 10^{-5}$$

$$\epsilon_2 = 0$$

RGE evolution of the mixing parameters

The value of r forces the Majorana masses to be O(10^{15}) GeV

- The running of the angle θ_{12} from $M_M=10^{15}$ GeV down to $M_S=10^{3}$ GeV can be non-negligible for nearly degenerate mass spectrum
- The evolution is large for $|m_1|$ sufficiently close to $|m_2|$

In our model:

$$\frac{|m_2|}{|m_1|} = 1+\alpha^2+2\alpha\cos\phi \xrightarrow{\quad} \alpha\sim 0 \quad \alpha+2\cos\phi\sim 0$$

Inverted Hierarchy !

In the case of CP conserved neutrino mixing matrix and in the limit of vanishing electron and muon Yukawas the evolution is dictated by

P.H.Chankowski and S.Pokorski,
Int J.Mod.Phys.A 17, 575 (2002)

$$\boxed{\frac{d}{dt}s_{12}=\frac{1}{2}s_{12}(1-s_{12}^2)\,A_{21}\,y_\tau^2}$$

$$A_{21}=\frac{|m_1+m_2|^2}{\Delta m_{sol}^2}$$

$$s_{12}=\sin\theta_{12} \qquad t=\frac{1}{16\pi^2}\log\frac{M_M}{M_S}$$

RGE evolution of the mixing parameters

We find the solution in terms of the deviation from the TBM value $\lambda = 1/3 - s^2_{12}$

$$\lambda \;=\; \frac{2}{9} A_{21} \frac{y_\tau^2}{16\pi^2} \log \frac{M_M}{M_S}$$

Since λ cannot be larger than 0.05 and $y_\tau \sim 10^{-4}\,(1 + \tan^2 \beta)$ we can avoid a running of θ_{12} beyond the experimental limit if

$$A_{21}(1 + \tan^2 \beta) \;\leq\; 10^4$$

- $A_{21} \sim 10^4$ for $a \sim 0.07$

evolution non-negligible

- $A_{21} \sim 150$ for $a \sim 1$

evolution neglected for $\tan\beta < 10$

- $A_{21} \sim 2$ for $a \sim 2$

evolution negligible

$$A_{21} = \frac{|m_1 + m_2|^2}{\Delta m^2_{\text{sol}}} \sim \frac{1 + 2\alpha^2}{2\alpha^2 r}\left(4 - \alpha^2 + \frac{4\alpha^2 r}{1 + 2\alpha^2}\right)$$

More on the RGE evolution

$$\frac{d}{dt} s_{12} = \frac{1}{2} s_{12} (1 - s_{12}^2) A_{21} y_\tau^2$$

$$A_{21} = \frac{|m_1 + m_2|^2}{\Delta m_{sol}^2} = \cos^2\beta \frac{|m_1| + |m_2|}{|m_1| - |m_2|} + \sin^2\beta \frac{|m_2| - |m_1|}{|m_1| + |m_2|}$$

β being ½ of the m1 and m2 phase shift

$$A_{21} = \cos^2\beta \frac{|m_1| + \sqrt{\Delta_{sol} + |m_1|^2}}{|m_1| - \sqrt{\Delta_{sol} + |m_1|^2}} + \sin^2\beta \frac{\sqrt{\Delta_{sol} + |m_1|^2} - |m_1|}{|m_1| + \sqrt{\Delta_{sol} + |m_1|^2}}$$

Degeneracy means $m_1 >> \Delta_{sol}$ so that $A_{21} \rightarrow$ infinity

Vacuum alignment

$$\frac{\partial W_d}{\partial \varphi^S_{01}} = 2g_1(\varphi^2_{S_1} - \varphi_{S_2}\varphi_{S_3}) + (M + g_2\xi)\varphi_{S_1} = 0$$

$$\frac{\partial W_d}{\partial \varphi^S_{02}} = 2g_1(\varphi^2_{S_2} - \varphi_{S_1}\varphi_{S_3}) + (M + g_2\xi)\varphi_{S_3} = 0$$

$$\frac{\partial W_d}{\partial \varphi^S_{03}} = 2g_1(\varphi^2_{S_3} - \varphi_{S_1}\varphi_{S_2}) + (M + g_2\xi)\varphi_{S_2} = 0$$

$$\frac{\partial W_d}{\partial \xi_0} = g_3(\varphi^2_{S_1} + 2\varphi_{S_2}\varphi_{S_3}) + M\xi + g_4\xi + M_0^2 = 0$$

$$\langle \varphi_S \rangle = v_S (1, 1, 1)^T$$

$$\langle \xi \rangle = u \neq 0$$

with

$$v_S^2 = \frac{1}{3g_2^2 g_3}\left[g_2(M M_\xi - g_2 M_0^2) - g_4 M^2\right]$$

$$u = -\frac{M}{g_2}$$

We expect a common order of magnitude for the VEVs

$$\frac{v_T}{\Lambda} \sim \frac{u'}{\Lambda} \sim \epsilon$$

and

$$\frac{v_S}{\Lambda} \sim \frac{u}{\Lambda} \sim \epsilon'$$

Vacuum alignment

the superpotential

neutrinos

charged leptons

$$W_d = M\varphi_0^S\varphi_S + g_1(\varphi_0^S\varphi_S\varphi_S) + g_2(\varphi_0^S\varphi_S\xi)$$
$$+ g_3(\xi_0\varphi_S\varphi_S) + g_4\xi_0\xi^2 + M_\xi\xi_0\xi + M_0^2\xi_0 + h_1\xi'(\varphi_0^T\varphi_T)'' + h_2(\varphi_0^T\varphi_T\varphi_T)$$

$$\left.\begin{aligned}
\frac{\partial W_d}{\partial\varphi_{01}^T} &= 2h_2(\varphi_{T_1}^2 - \varphi_{T_2}\varphi_{T_3}) + h_1(\xi'\varphi_{T_3}) = 0\\
\frac{\partial W_d}{\partial\varphi_{02}^T} &= 2h_2(\varphi_{T_2}^2 - \varphi_{T_1}\varphi_{T_3}) + h_1(\xi'\varphi_{T_2}) = 0\\
\frac{\partial W_d}{\partial\varphi_{03}^T} &= 2h_2(\varphi_{T_3}^2 - \varphi_{T_1}\varphi_{T_2}) + h_1(\xi'\varphi_{T_1}) = 0
\end{aligned}\right\}$$

$$\langle\varphi_T\rangle = v_T(0,1,0)^T$$

$$\langle\xi'\rangle = u' \neq 0$$

$$v_T = -\frac{h_1 u'}{2h_2}$$

The model building

Large interest in recent years to find models for TBM (no GUT models in the list...)

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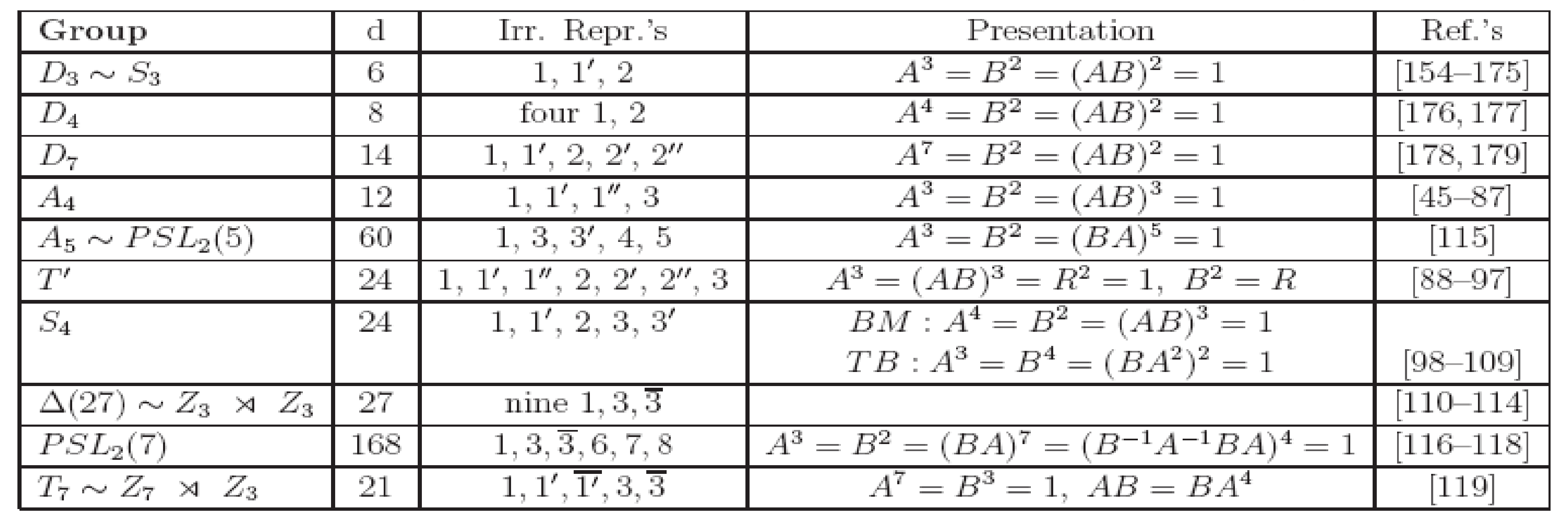
And many others...



Group	d	Irr. Repr.'s	Presentation	Ref.'s
$D_3 \sim S_3$	6	$1, 1', 2$	$A^3 = B^2 = (AB)^2 = 1$	[154–175]
D_4	8	four $1, 2$	$A^4 = B^2 = (AB)^2 = 1$	[176, 177]
D_7	14	$1, 1', 2, 2', 2''$	$A^7 = B^2 = (AB)^2 = 1$	[178, 179]
A_4	12	$1, 1', 1'', 3$	$A^3 = B^2 = (AB)^3 = 1$	[45–87]
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$	$A^3 = B^2 = (BA)^5 = 1$	[115]
T'	24	$1, 1', 1'', 2, 2', 2'', 3$	$A^3 = (AB)^3 = R^2 = 1, \ B^2 = R$	[88–97]
S_4	24	$1, 1', 2, 3, 3'$	$BM : A^4 = B^2 = (AB)^3 = 1$ $TB : A^3 = B^4 = (BA^2)^2 = 1$	[98–109]
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	nine $1, 3, \overline{3}$		[110–114]
$PSL_2(7)$	168	$1, 3, \overline{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$	[116–118]
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \overline{1'}, 3, \overline{3}$	$A^7 = B^3 = 1, \ AB = BA^4$	[119]

From Altarelli&Feruglio 2010

Effective operators for neutrino masses

Up to 1 flavon insertion

$$\begin{aligned} W_\nu^{\text{eff}} &= \frac{C}{\Lambda}\,(\ell h_u \ell h_u) + \frac{D}{\Lambda^2}\,(\ell h_u \ell h_u)\,\varphi_S + \frac{E}{\Lambda^2}\,(\ell h_u \ell h_u)\,\xi \\ &= \frac{C'}{\Lambda}\,(\ell h_u \ell h_u) + \frac{D}{\Lambda^2}\,(\ell h_u \ell h_u)\,\varphi_S. \end{aligned}$$

The corresponding mass matrix

$$m_{\text{eff}} = \frac{v_u^2}{\Lambda} \begin{pmatrix} C' + 2D\frac{v_S}{\Lambda} & -D\frac{v_S}{\Lambda} & -D\frac{v_S}{\Lambda} \\ -D\frac{v_S}{\Lambda} & 2D\frac{v_S}{\Lambda} & C' - D\frac{v_S}{\Lambda} \\ -D\frac{v_S}{\Lambda}' & C' - D\frac{v_S}{\Lambda} & 2D\frac{v_S}{\Lambda} \end{pmatrix}$$

Light neutrino masses

$$\begin{aligned} m_1 &= v_u^2\left(\frac{-y_\nu^2}{M + au + 3bv_S} + \frac{C'}{\Lambda} + 3D\frac{v_S}{\Lambda^2}\right) \\ m_2 &= v_u^2\left(\frac{-y_\nu^2}{M + au} + \frac{C'}{\Lambda}\right) \\ m_3 &= v_u^2\left(\frac{y_\nu^2}{M + au - 3bv_S} - \frac{C'}{\Lambda} + 3D\frac{v_S}{\Lambda^2}\right). \end{aligned}$$

It is natural to assume that *C ~ D ~ O(1)*. Then the *D* term is subleading and the importanceof the *C* term depends on the relative size of *M* and . We know that *M* must be considerably smaller than Λ. Then the contribution of the effective operators in equation (45) is suppressed at a level comparable to NLO corrections.