A unique $\mathbb{Z}_4^R$ symmetry for the MSSM

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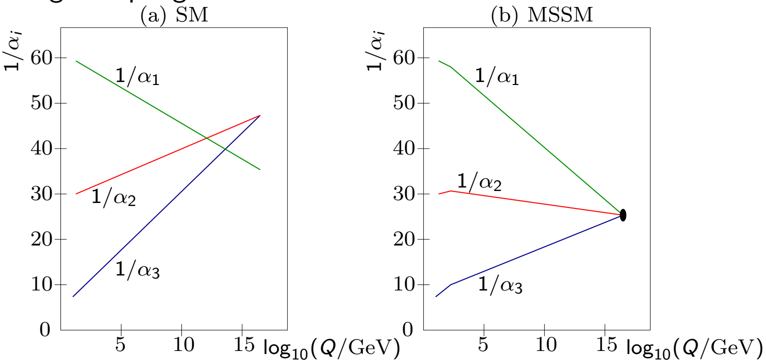
Based on:
H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. V.:
arXiv:1009.0905 [hep-ph], to appear in PLB
and follow-up paper

From the Standard Model to the MSSM

- Supersymmetry: squarks and sleptons
- Introduced as a solution to hierarchy problem
- Gauge coupling unification



- Dark matter candidate (LSP) ...

Problems of the MSSM

- μ-problem:

$$\mathcal{W} \supset \mu \bar{H} H$$

  $\mu \sim m_{3/2}$ needed, but $\mu \sim M_P$ expected
- Proton decay via operators, like $\bar{U}\bar{D}\bar{D}$ and $QQQL$
- CP and flavor problems

$\Rightarrow$ expect new (discrete) symmetries

Matter-parity (= R parity)

Dimopoulos, Georgi/ Dimopoulos, Raby, Wilczek

- $\mathbb{Z}_2$ with charges:

matter	1
Higgs	0

- Allowed couplings: $\mathcal{W} \supset Q\bar{U}H + Q\bar{D}\bar{H} + L\bar{E}\bar{H} + \mu\bar{H}H$
- Forbidden couplings: $LH + LL\bar{E} + Q\bar{D}L + \bar{U}\bar{D}\bar{D}$
- LSP is stable $\Rightarrow$ dark matter candidate
- Still problematic: $\mathcal{W} \supset \mu\bar{H}H + QQQL + \bar{U}\bar{U}\bar{D}\bar{E}$

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Proton-Hexality

- Anomaly-free $\mathbb{Z}_6$ with charges: Dreiner, Luhn, Thormeier 2005

Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	$\bar{H}$	H
0	1	-1	-2	1	1	-1

- Proton-Hexality $=$ Matter-parity $\times$ Baryon-triality Ibánez, Ross

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Förste, Nilles, Ramos-Sánchez, P.V. 2010

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Discrete Symmetries of the MSSM

- Extend the MSSM by a discrete symmetry that
  - Allows the Yukawa couplings
  - Suppresses proton decay
  - Solves μ-problem ($\mu = 0$ at the perturbative level)
  - Symmetry should be anomaly free!
  - Allow for discrete Green-Schwarz mechanism (inspired by string theory)

Araki, Kobayashi, Kubo, Ramos-Sánchez, Ratz and P.V. 2008

- Discrete anomalies without being embedded in gauge symmetry! Example: heterotic orbifolds

Green-Schwarz mechanism

- Symmetry generator α is anomalous if action invariant but path integral measure not, i.e.

$$D\Psi D\bar{\Psi} \mapsto e^{i\int \mathrm{d}^4x A(\alpha)} D\Psi D\bar{\Psi}$$

where

$$A(\alpha) = \frac{1}{32\pi^2}\mathrm{Tr}\left(\alpha F\tilde{F}\right) - \frac{1}{384\pi^2} R\tilde{R}\,\mathrm{Tr}\left(\alpha\right)$$

and $A(\alpha) \neq 0$.

Álvarez-Gaumé, Witten/ Álvarez-Gaumé, Ginsparg

- Note: $\int \mathrm{d}^4x A(\alpha) = 2\pi \frac{\text{integer}}{N}$ for $\mathbb{Z}_N$,
i.e. phase can be non-trivial for discrete transformation α

Green-Schwarz mechanism

- For example, under U(1) gauge transformation Λ

$$V \mapsto V + \frac{i}{2}\left(\Lambda - \Lambda^\dagger\right) \qquad \text{vector superfield}$$
$$\Phi^{(f)} \mapsto e^{-iq^{(f)}\Lambda}\Phi^{(f)} \qquad \text{chiral superfields}$$

- From $A(\alpha) \supset \text{Tr}\left(\alpha F\tilde{F}\right) \neq 0 \quad \Rightarrow$

$$\sum_f {q^{(f)}}^3 \neq 0 \quad \text{and} \quad \sum_{r^{(f)}} \ell\left(r^{(f)}\right) q^{(f)} \neq 0$$

$\Rightarrow$ Anomaly!

Green-Schwarz mechanism

- Add Dilaton superfield S

$$L \supset -\int \mathrm{d}^4\theta \ln\left(S + S^\dagger - \delta_{\mathrm{GS}} V\right) + \int \mathrm{d}^2\theta \frac{S}{4} W_\alpha W^\alpha + \text{h.c.}$$

with transformation property

$$S \mapsto S + \frac{i}{2}\delta_{\mathrm{GS}}\Lambda \quad \text{with} \quad \delta_{\mathrm{GS}} = \frac{1}{6\pi^2}\sum_f {q^{(f)}}^3$$

- Second term compensates non-invariance of path integral measure

$\Rightarrow$ Anomaly cancelled!

Discrete Green-Schwarz mechanism

- α is $\mathbb{Z}_N$ transformation
- Chiral superfields transform

$$\Phi^{(f)} \mapsto e^{-2\pi i q^{(f)} \frac{1}{N}} \Phi^{(f)}$$

- Assume $\int \mathrm{d}^4 x A(\alpha) = 2\pi \frac{\text{integer}}{N}$
- Again, add Dilaton superfield S with discrete shift trafo

$$S \mapsto S + \frac{i}{2} \Delta_{\mathrm{GS}} \quad \text{with} \quad \Delta_{\mathrm{GS}} = \frac{1}{\pi N} A_{G-G-\mathbb{Z}_N}$$

$\Rightarrow$ Anomaly cancelled!

$\mathbb{Z}_N$ (R) symmetry

- Matter superfields with charges $q^{(f)}$
  $\Rightarrow$ fermion components with charges $q^{(f)} - R$,
  with $R = 0$ or 1 for non-R or R symmetry
- Superpotential $\mathcal{W}$ has charge $2R$
- Anomaly coefficients for gauge group G

$$\boxed{A_{G-G-\mathbb{Z}_N} = \sum_{r^{(f)}} \ell\left(r^{(f)}\right)\left(q^{(f)} - R\right) + \ell\,(\text{adj})\,R}$$

- Anomaly cancellation

$$\boxed{A_{G-G-\mathbb{Z}_N} = \rho \bmod \eta \quad \text{where} \quad \eta = \begin{cases} N & \text{for } N \text{ odd,} \\ N/2 & \text{for } N \text{ even.} \end{cases}}$$

$\rho \neq 0$ Green-Schwarz anomaly cancellation

SU(5) universal charges

Consider simple situation:

- SU(5) universal charges for matter $q_{\mathbf{10}_i}$ and $q_{\overline{\mathbf{5}}_i}$
- MSSM $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ anomaly coefficients

$$
\begin{aligned}
A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 3R \\
A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 2R \\
& \quad + \frac{1}{2}\left(q_H + q_{\bar{H}} - 2R\right)
\end{aligned}
$$

SU(5) universal charges

- Anomaly cancellation
  (Allowing for the Green-Schwarz mechanism)

$$\begin{aligned} A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} - A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} &= 0 \bmod \eta \\ \Rightarrow \qquad q_H + q_{\bar{H}} &= 4R \bmod 2\eta \end{aligned}$$

- In contrast, μ-term allowed if

$$q_H + q_{\bar{H}} = 2R \bmod N \ .$$

- For non-R symmetry ($R = 0$): In the SU(5) case, anomaly cancellation generically implies the $\mu-$problem

SU(5) universal charges for $\mathbb{Z}_N^R$

Demand:

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg and P. V. 2010

- $\mathbb{Z}_N^R$-charges for matter are SU(5) universal
- $G - G - \mathbb{Z}_N^R$ anomalies are universal for $G \in$ SM
- Yukawa couplings and Weinberg operator are allowed

Result: N divides 24 and:

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_H	$q_{\bar{H}}$	ρ
4	1	1	0	0	1
6	5	3	4	0	0
8	1	5	0	4	1
12	5	9	4	0	3
24	5	9	16	12	9
3	2	0	1	0	0
6	2	0	4	0	0

(last two cases allow $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}} \Rightarrow$ Proton decay)

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SO(10) universal charges for $\mathbb{Z}_N^R$

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg and P. V. 2010

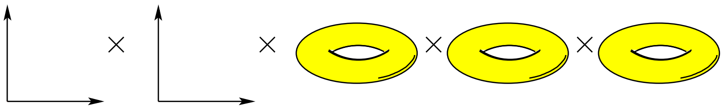
good $\mathbb{Z}_4^R$ of the MSSM is unique! good

- $\mathbb{Z}_4^R$ with SO(10) universal R-charges for matter
  Babu, Gogoladze, Wang
- It is: unique, forbidds Proton decay, allows Yukawas and neutrino masses and forbidds the μ-term
- $\rho = 1 \Rightarrow$ Green-Schwarz mechanism to cancel the anomaly
- Non-perturbative effects break $\mathbb{Z}_4^R$ to $\mathbb{Z}_2^R$
- Hence: dim. 5 Proton decay and μ-term at the non-pert. level
- Dangerous? See at explicit (string) model with $\mathbb{Z}_4^R$

String realization

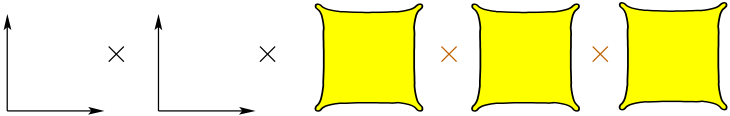
Part 2: String realization

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification



- 10d heterotic string on torus $T^6 = T^2 \times T^2 \times T^2$
- divided by discrete group $\mathbb{Z}_2 \times \mathbb{Z}_2$
- Closed strings:
  - Untwisted: gravity, gauge group and some matter
  - Twisted: matter
- Construct models using the C++ orbifolder

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification



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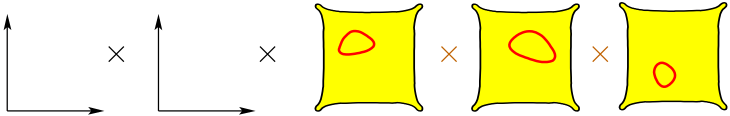

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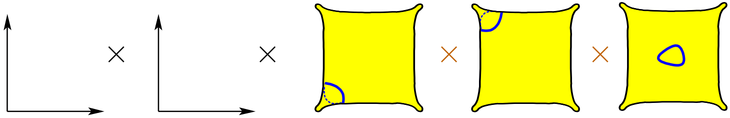

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The C++ orbifolder

Nilles, Ramos-Sánchez, P. V., Wingerter: to appear 2011

The C++ orbifolder

- automatically constructs inequivalent string models,
- checks anomaly freedom (including Green-Schwarz mechanism),
- identifies string MSSM candidates and
- offers linux-style command system to analyze the model in detail.

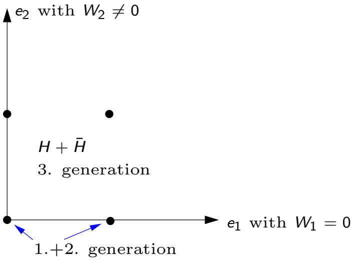
$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification and $\mathbb{Z}_4^R$

The model:

- Heterotic string on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold
- $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group
- 3 generations of quarks and leptons
- Plus vecor-like exotics (decouple) $\Rightarrow$ MSSM
- D_4 family symmetry $\Rightarrow$ ameliorates flavor problem
- Heavy top and non-trivial Yukawa patterns (a la Froggatt-Nielsen)
- $\mathbb{Z}_4^R \Rightarrow$ no proton decay and $\mu = 0$
- SU(6) GUT in 6d

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification and $\mathbb{Z}_4^R$

6d intermediate GUT:



- SU(6) GUT
- D_4 family symmetry, geometrical origin ($W_1 = 0$)
- 1. + 2. generation form D_4 doublet
- 3. generation in the 6d bulk

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification and $\mathbb{Z}_4^R$

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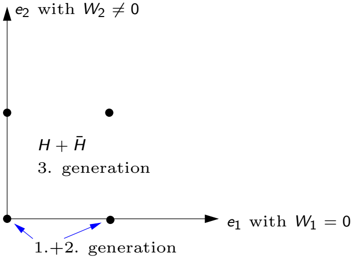

- Also: Higgs-pair in the 6d bulk $\Rightarrow$ $\mu \sim \langle \mathcal{W} \rangle$
  Kappl, Nilles, Ramos-Sánchez, Ratz, Schmidt-Hoberg and P.V. / Brümmer, Kappl, Ratz and Schmidt-Hoberg
- From Lorentz symmetry in $T^2/\mathbb{Z}_2 \Rightarrow \mathbb{Z}_4^R$
- MSSM $\mathbb{Z}_4^R$ with GS mechanism
  $\Rightarrow$ hidden sector gaugino condensate breaks $\mathbb{Z}_4^R$ to $\mathbb{Z}_2^R$ and $\langle \mathcal{W} \rangle \sim m_{3/2}$.

Conclusion

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- Anomaly-free $\mathbb{Z}_N$ (R) symmetries of the MSSM, allowing for discrete Green-Schwarz mechanism
- SU(5) universal charges for matter;
  Avoid Proton decay and μ-problem
  $\Rightarrow \mathbb{Z}_N^R$-symmetry with N divides 24
- SO(10) universal charges for matter $\Rightarrow$ unique $\mathbb{Z}_4^R$
- $\mathbb{Z}_4^R$ arises naturally in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds
- String realization $\mu \sim \langle \mathcal{W} \rangle \sim m_{3/2}$