

NEW ASPECTS OF SYMMETRY BREAKING IN GRAND UNIFIED THEORIES

DISCRETE 2010

Dec 6 2010 - Sapienza Università di Roma

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Based on 0912.1796 and 1011.1821 [hep-ph]

THE GRAND UNIFICATION PROGRAM

After **36 years** from the first proposal by Georgi and Glashow, GUTs still attract a lot of attention for their intrinsic predictivity of spectacular phenomena

- Matter instability
- GUT Monopoles

and their potential for understanding our low-energy world through nontrivial correlations among different SM sectors

- In many extensions of the SM, gauge couplings seem to unify in a narrow window still allowed by proton decay limits and a consistent QFT description
- The Yukawa sector supports simultaneously both quark and lepton textures
- Predictive link between the tiny neutrino masses and the heavy GUT states

Still no consensus on which is the the **minimal** theory to be falsified the day after the discovery of proton decay

THE CONCEPT OF MINIMALITY

The concept of minimality admits a certain number of interpretations

- Minimum rank of the group
- Higgs sector dimensionality
- Naturalness of the D-T splitting
- Complexity of the gauge unification pattern
- ...
- Predictivity: i.e. the number of independent couplings

SO(10) GUTs usually score better than SU(5) models

- **More predictive** (SM matter falls into three 16_F rep.'s)
- Natural relief from the troubles with the simplest SU(5) models

Sticking to the SO(10) case minimality is essentially equivalent to the **complexity of the Higgs sector**

THE MINIMAL SO(10) HIGGS SECTOR

Just by group theoretical arguments we require the following rep.'s in order to break SO(10) to the SM

- **Rank reduction:** 16_H or 126_H
breaks B-L giving mass to neutrinos but leaves an SU(5) little group
- **Further SU(5) breaking:** 45_H or 54_H or 210_H
the adjoint 45_H can admit little groups different from $SU(5) \otimes U(1)$

However since the early 1980's it has been observed that the vacuum dynamics aligns the adjoint along the $SU(5) \otimes U(1)$ direction

- non-SUSY: approximate alignment \longrightarrow clashes with unification constraints
[Yasue (1981), Anastaze, Derendinger, Buccella (1983), Babu, Ma (1985)]
- SUSY: exact alignment \longrightarrow little group is SU(5) [Buccella, Derendinger, Savoy, Ferrara (1981)]

The subject of this talk is to provide ways out to these two issues ...

INTERMEDIATE SCALES IN THE NON-SUSY SO(10)

SUSY not mandatory for unification after we trade naturalness for predictivity

The unification ansatz in non-SUSY SO(10) **predicts** the existence of intermediate scales in the range $10^{10\div 14}$ GeV (ideal for neutrino masses and leptogenesis)

$$(100 \text{ GeV})^2 / M_{\text{seesaw}} \gtrsim \sqrt{\Delta m_{\text{atm}}^2} \Rightarrow M_{\text{seesaw}} \lesssim 10^{14} \text{ GeV}$$

The breaking of non-SUSY SO(10) to the SM can be **minimally** achieved with a pair of Higgs multiplets only: $45_H \oplus 16_H$ (or 126_H)

$$SO(10) \xrightarrow[\omega_Y \subset \langle 45_H \rangle]{M_G} 3_c 2_L 2_R 1_{B-L} \xrightarrow[\omega_R \subset \langle 45_H \rangle]{M_I} 3_c 2_L 1_R 1_{B-L} \xrightarrow[\chi_R \subset \langle 16_H \rangle \text{ or } \langle 126_H \rangle]{M_{B-L}} 3_c 2_L 1_Y$$

$$SO(10) \xrightarrow[\omega_R \subset \langle 45_H \rangle]{M_G} 4_C 2_L 1_R \xrightarrow[\omega_Y \subset \langle 45_H \rangle]{M_I} 3_c 2_L 1_R 1_{B-L} \xrightarrow[\chi_R \subset \langle 16_H \rangle \text{ or } \langle 126_H \rangle]{M_{B-L}} 3_c 2_L 1_Y$$

where $M_I \ll M_G$ by unification constraints

TREE LEVEL POTENTIAL

Very minimal potential analyzed long ago [Bucella, Ruegg, Savoy (1980)]

$$V_0 = V_{45_H} + V_{16_H} + V_{45_H 16_H}$$

$$V_{45_H} = -\mu^2 \text{Tr } 45_H^2 + a_1 (\text{Tr } 45_H^2)^2 + a_2 \text{Tr } 45_H^4$$

$$V_{16_H} = -\nu^2 16_H^\dagger 16_H + \lambda_1 (16_H^\dagger 16_H)^2 + \lambda_2 (16_H \Gamma 16_H)(16_H^\dagger \Gamma 16_H^\dagger)$$

$$V_{45_H 16_H} = \alpha (16_H^\dagger 16_H) \text{Tr } 45_H^2 + \beta 16_H^\dagger 45_H^2 16_H + \tau 16_H^\dagger 45_H 16_H$$

From the positivity of the scalar states $(1,3,0)$ and $(8,1,0) \subset 45_H$

[Yasue (1981), Anastaze, Derendinger, Bucella (1983), Babu, Ma (1985)]

$$M^2(1, 3, 0) = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)$$

$$M^2(8, 1, 0) = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y)$$



$$a_2 < 0,$$

$$-2 < \omega_Y / \omega_R < -\frac{1}{2}$$

The only possibility allowed by gauge coupling unification requires a splitting between ω_Y and ω_R of at least four orders of magnitude!

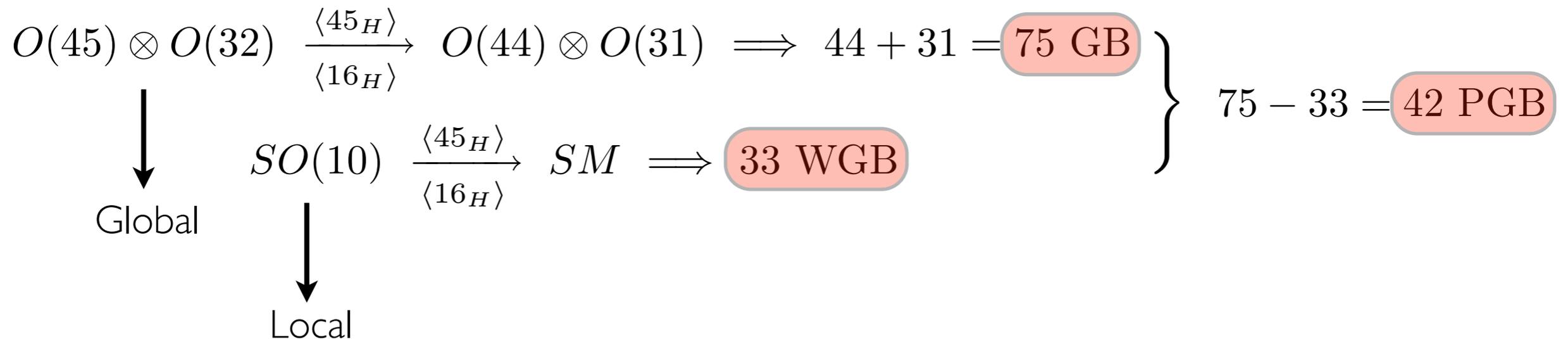
- This is the origin of the common knowledge that non-SUSY SO(10) GUTs with just the adjoint driving the GUT breaking are not phenomenologically viable

A TREE LEVEL ACCIDENT

Why the masses of the states (1,3,0) and (8,1,0) are so tightly correlated?"

Enhanced global symmetries in a trivial limit of the potential $a_2 = \lambda_2 = \beta = \tau = 0$

$$V_{\text{moduli}} = -\mu^2 \text{Tr } 45_H^2 + a_1 (\text{Tr } 45_H^2)^2 - \nu^2 16_H^\dagger 16_H + \lambda_1 (16_H^\dagger 16_H)^2 + \alpha (16_H^\dagger 16_H) \text{Tr } 45_H^2$$



- The states (1,3,0) and (8,1,0) belong to this set of PGB
- Nothing would prevent the explicit breaking couplings λ_2 β and τ to enter at the quantum level !

Second law of progress in theoretical physics: [S.Weinberg (1983)]

“Do not trust arguments based on the lowest order of perturbation theory”

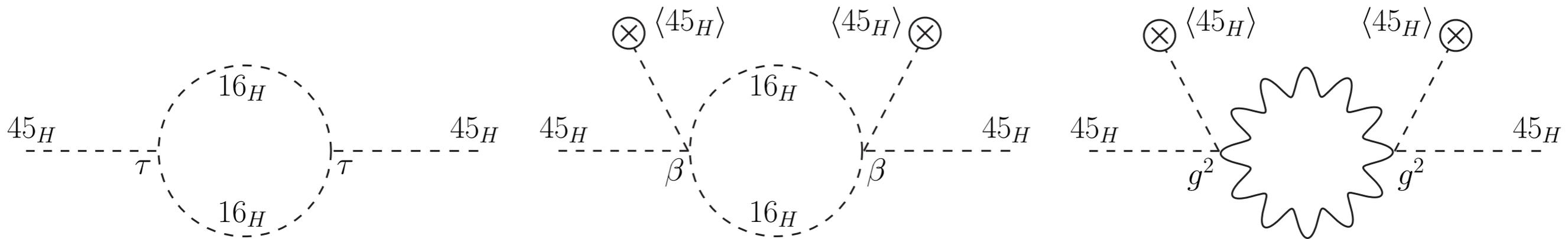
REVIVING THE MINIMAL SO(10) GUT

Explicit computation of the one-loop PGB masses using Effective-Potential methods

[Coleman, E. Weinberg (1973)]

$$M^2(1, 3, 0) = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R) + \frac{1}{4\pi^2} [\tau^2 + \beta^2(2\omega_R^2 - \omega_R\omega_Y + 2\omega_Y^2) + g^4(16\omega_R^2 + \omega_Y\omega_R + 19\omega_Y^2)] + \text{Log's}(\mu)$$

$$M^2(8, 1, 0) = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y) + \frac{1}{4\pi^2} [\tau^2 + \beta^2(\omega_R^2 - \omega_R\omega_Y + 3\omega_Y^2) + g^4(13\omega_R^2 + \omega_Y\omega_R + 22\omega_Y^2)] + \text{Log's}(\mu)$$



An hierarchy between ω_Y and ω_R (as required by unification), while keeping the scalar states positive (minimum condition), is now possible just by taking $|a_2| < 10^{-2}$

Holds for **any** non-SUSY SO(10) model with one adjoint triggering the GUT breaking

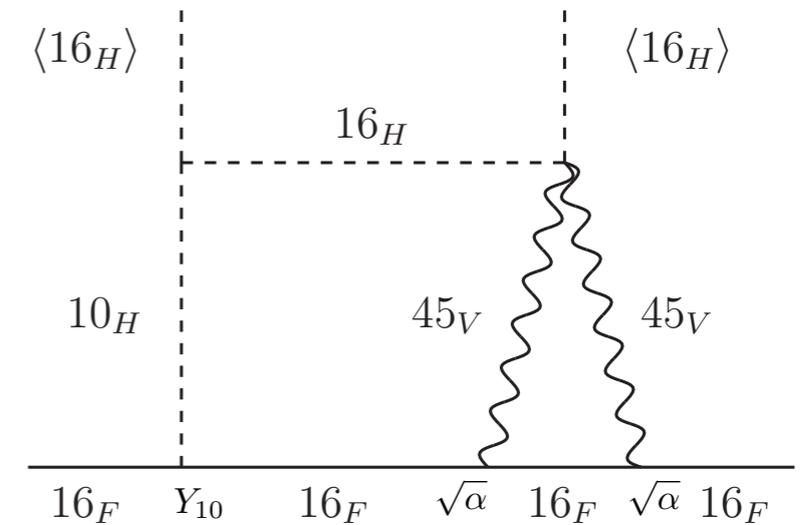
WHAT ABOUT NEUTRINOS ? (NON-SUSY)

The simplest scenario featuring the Higgs scalars in $10_H \oplus 16_H \oplus 45_H$ is likely to fail when addressing the absolute neutrino mass scale

RH neutrino mass M_N (entering type-I seesaw)

- Radiative seesaw
$$M_N \sim \left(\frac{\alpha}{\pi}\right) Y_{10} \frac{M_{B-L}^2}{M_G}$$

[Witten (1980); Bajc, Senjanovic (2005)]



- New Physics at the Planck scale
$$M_N \sim Y_P \frac{M_{B-L}^2}{M_P} \subset \frac{Y_P}{M_P} 16_F 16_F 16_H^* 16_H^*$$

Since $M_{B-L} \ll M_G$, M_N undershoots the natural range suggested by the the seesaw mechanism

Considering 126_H in place of 16_H the absolute neutrino mass scale would be fine

$$16_F 16_F 126_H^* \supset M_N \sim \langle 126_H^* \rangle \sim M_{B-L}$$

Work in progress...

WHAT ABOUT NEUTRINOS ? (SUSY)

Invoking TeV-scale SUSY

- The gauge running prefers M_{B-L} in the proximity of M_G
- The D=5 operator $16_F^2 \overline{16}_H^2 / M_P$ can naturally reproduce the desired range for M_N

Higgs sectors based on rep.'s up to the adjoint are therefore very interesting !

The superpotential does not support a renormalizable breaking to a SUSY-SM

[Buccella, Derendinger, Savoy, Ferrara (1981)]

$$W = 45_H^2 + 16_H \overline{16}_H + 16_H 45_H \overline{16}_H$$

- D-Flatness: $|\langle 16_H \rangle| = |\langle \overline{16}_H \rangle|$

- F-Flatness: $\langle 45_H \rangle \propto \langle 16_H \overline{16}_H \rangle$

The little group is SU(5) due to the alignment of the adjoint along the spinors

New physics at the Planck scale, parametrized by effective M_P -suppressed operators, allows for the adjoint misalignment

[Babu, Barr (1995)]

THE GUT SCALE LITTLE HIERARCHY

The hierarchy induced in the Higgs spectrum by $M_G / M_P \approx 10^{-2}$ factors splits the GUT-scale thresholds over several orders of magnitude

- Fast **proton decay** via neutrino mass operators

$$W_Y \supset \frac{1}{M_P} 16_F g 16_F 16_H 16_H + \frac{1}{M_P} 16_F f 16_F \bar{16}_H \bar{16}_H \supset \frac{\langle 16_H \rangle}{M_P} (Q g L \bar{T} + Q f Q T)$$

$$M_T \approx M_G \quad \longrightarrow \quad \Gamma^{-1}(\bar{\nu} K^+) \sim (0.6 - 3) \times 10^{33} \text{ yrs} \quad [\text{Babu, Pati, Wilczek (2000)}]$$

$$M_T \approx \frac{M_G}{M_P} M_G \quad \longrightarrow \quad \Gamma^{-1}(\bar{\nu} K^+)^{\text{NR}} = \left(\frac{M_G}{M_P} \right)^2 \Gamma^{-1}(\bar{\nu} K^+) \ll \Gamma^{-1}(\bar{\nu} K^+)^{\text{exp}} > 0.670 \times 10^{33} \text{ yrs}$$

- Upset of the **one-step unification** pattern favoured by the MSSM

Unification could be preserved close to the Planck scale

- Unwelcome for **RH neutrino masses** $M_N \approx M_G^2 / M_P$

Is it possible to overcome the SU(5) lock at the renormalizable level while keeping only spinorial and adjoint rep.'s? \longrightarrow Flipped embeddings ...

DIGRESSION: FLIPPED EMBEDDINGS

$$G \supset H \otimes U(1)_X \supset [K \otimes U(1)_Z] \otimes U(1)_X \quad G \text{ Simple} \quad H \quad K \quad \text{Simple or Semi-simple}$$

- Standard breaking chain $G \rightarrow H \rightarrow K \otimes U(1)_Z$
- **Flipped** breaking chain $G \rightarrow H \otimes U(1)_X \rightarrow K \otimes U(1)_{\tilde{Z}} \quad \tilde{Z} = \alpha Z + \beta X, \quad \beta \neq 0$

Requiring that the rep.'s of G decompose in submultiplets with the same quantum numbers under both $K \otimes U(1)_Z$ and $K \otimes U(1)_{\tilde{Z}}$ fixes α and β [Barr (1989)]

Basic example: Flipped SU(5) [De Rujula, Georgi, Glashow (1980); Barr (1982)]

$$G \equiv SO(10) \quad H \equiv SU(5) \quad K \equiv SU(3)_C \otimes SU(2)_L \quad \tilde{Z} \equiv Y \quad \alpha = -\frac{1}{5}, \quad \beta = +\frac{1}{5}$$

$$16 \equiv \left\{ \begin{array}{l} (u^c \oplus \ell)_{\bar{5}_{-3}} \\ (d^c \oplus Q \oplus \nu^c)_{10_{+1}} \\ (e^c)_{1_{+5}} \end{array} \right.$$

- Flipping: corresponds to a π rotation in the $SU(2)_R$ space !
- Notice the SM-singlet in the 10 of Flipped SU(5)

HYPERCHARGE EMBEDDINGS IN $SO(10) \otimes U(1)_X$

$$SO(10) \otimes U(1)_X \supset SU(5) \otimes U(1)_Z \otimes U(1)_X \supset SU(3)_C \otimes SU(2)_L \otimes \underbrace{U(1)_{Y'} \otimes U(1)_Z \otimes U(1)_X}_{U(1)_Y}$$

$$Y = \alpha Y' + \beta Z + \gamma X$$

Given the anomaly-free X charge matter assignment $(X_{16}, X_{10}, X_1) = (+1, -2, +4)$ there are **only three solutions** which accommodate the SM quantum numbers over a $16 \oplus 10 \oplus 1$ matter representation

$$\alpha = 1 \quad \beta = 0 \quad \gamma = 0$$

- Standard

$$\alpha = -\frac{1}{5} \quad \beta = \frac{1}{5} \quad \gamma = 0$$

- Flipped SU(5)

[De Rujula, Georgi, Glashow (1980); Barr (1982)]

$$\alpha = -\frac{1}{5} \quad \beta = -\frac{1}{20} \quad \gamma = \frac{1}{4}$$

- Flipped SO(10)

[Kephart, Nakagawa (1984); Rzos, Tamvakis (1988)]

- The active role of the $U(1)_X$ generator in the SM hypercharge identification gives the opportunity of breaking the gauge symmetry at the renormalizable level and by means of only rep.'s up to the adjoint

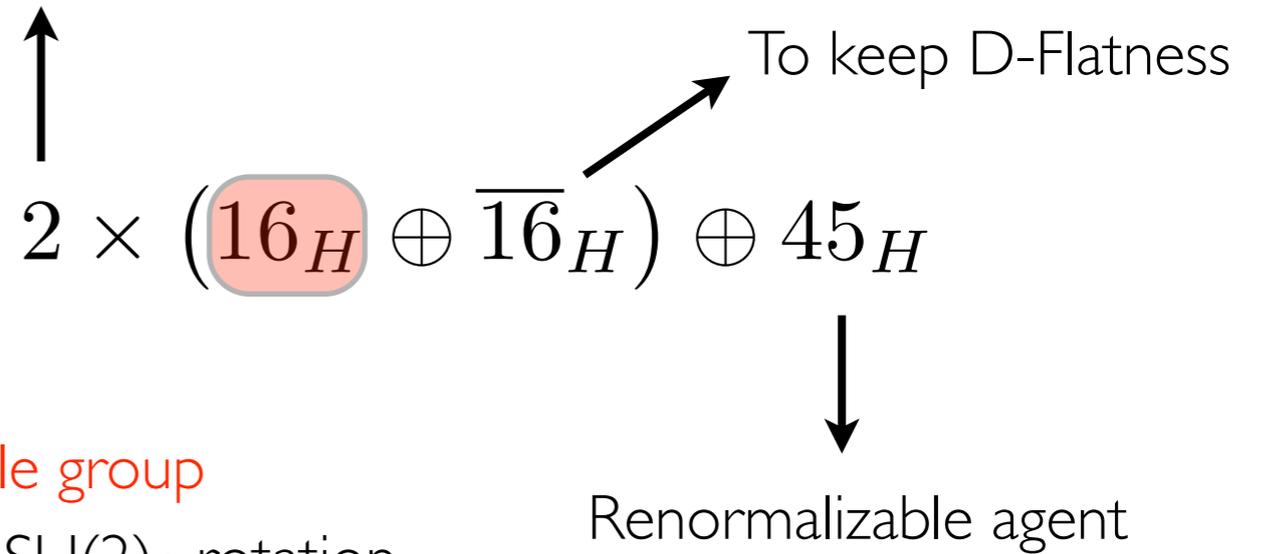
MINIMAL FLIPPED $SO(10) \otimes U(1)$ SUSY HIGGS SECTOR

- Spinor decomposition

$SO(10)$	$SO(10)_f$
$(\bar{3}, 1; +\frac{1}{3})_{\bar{5}}$	$(\bar{3}, 1; +\frac{1}{3})_{\bar{5}}$
$(1, 2; -\frac{1}{2})_{\bar{5}}$	$(1, 2; +\frac{1}{2})_{\bar{5}}$
$(3, 2; +\frac{1}{6})_{10}$	$(3, 2; +\frac{1}{6})_{10}$
$(\bar{3}, 1; -\frac{2}{3})_{10}$	$(\bar{3}, 1; +\frac{1}{3})_{10}$
$(1, 1; +1)_{10}$	$(1, 1; 0)_{10} \equiv e$
$(1, 1; 0)_1$	$(1, 1; 0)_1 \equiv \nu$

- Preserve a SM little group
- Connected by an $SU(2)_R$ rotation

Full rank reduction



Renormalizable superpotential $W_H = \frac{\mu}{2} \text{Tr } 45^2 + \rho_{ij} 16_i \bar{16}_j + \tau_{ij} 16_i 45 \bar{16}_j \quad i, j = 1, 2$

Imposing D- and F-flatness at the GUT scale:

- A nontrivial vacuum requires ρ and τ to be hermitian
- The inspection of the gauge spectrum reveals the following little group

SM for misaligned $16 \oplus \bar{16}$ pairs

TOWARDS A REALISTIC FLAVOR

The flipped $SO(10)$ embedding offers also the possibility of triggering the EW symmetry breaking **without a 10_H**

Simplified Yukawa sector with just one pair of Higgs spinors + Z_2 matter parity

$$W_Y = Y_U 16_F 10_F 16_H + \frac{1}{M_P} [Y_E 10_F 1_F \overline{16}_H \overline{16}_H + Y_D 16_F 16_F \overline{16}_H \overline{16}_H]$$

	$SO(10)$	$SO(10)_f$
16_F	$(D^c \oplus L)_{\overline{5}} \oplus (U^c \oplus Q \oplus E^c)_{10} \oplus (N^c)_1$	$(D^c \oplus \Lambda^c)_{\overline{5}} \oplus (\Delta^c \oplus Q \oplus S)_{10} \oplus (N^c)_1$
10_F	$(\Delta \oplus \Lambda^c)_5 \oplus (\Delta^c \oplus \Lambda)_{\overline{5}}$	$(\Delta \oplus L)_5 \oplus (U^c \oplus \Lambda)_{\overline{5}}$
1_F	$(S)_1$	$(E^c)_1$
$\langle 16_H \rangle$	$(0 \oplus \langle H_d \rangle)_{\overline{5}} \oplus (0 \oplus 0 \oplus 0)_{10} \oplus (\nu_H)_1$	$(0 \oplus \langle H_u \rangle)_{\overline{5}} \oplus (0 \oplus 0 \oplus e_H)_{10} \oplus (\nu_H)_1$
$\langle \overline{16}_H \rangle$	$(0 \oplus \langle H_u \rangle)_5 \oplus (0 \oplus 0 \oplus 0)_{\overline{10}} \oplus (\nu_H)_1$	$(0 \oplus \langle H_d \rangle)_5 \oplus (0 \oplus 0 \oplus e_H)_{\overline{10}} \oplus (\nu_H)_1$

$$Q = (U, D) \quad L = (N, E) \quad \Lambda = (\Lambda^0, \Lambda^-) \quad \Lambda^c = (\Lambda^{c+}, \Lambda^{c0}) \quad \langle H_u \rangle = (0, v_u) \quad \langle H_d \rangle = (v_d, 0)$$

- The SM fermions span necessarily over a reducible $16 \oplus 10 \oplus 1$ matter rep.
- All the fermions, **but the up-quarks**, need Planck-suppressed contributions
- The top/bottom hierarchy is due to an $M_f/M_P \sim 10^{-2}$ factor \longrightarrow **$\tan \beta \sim 1$**

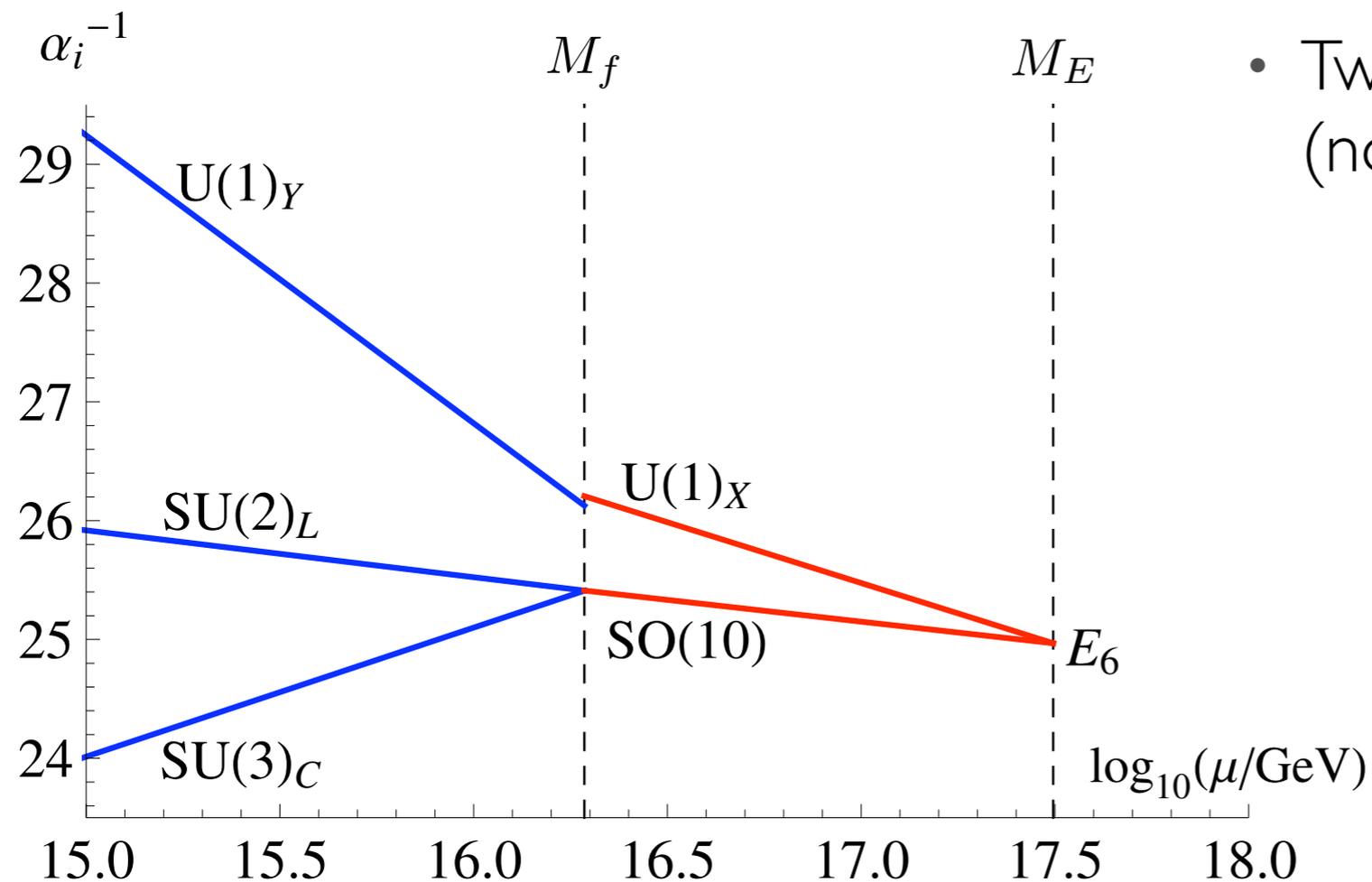
A UNIFIED E_6 SCENARIO

The flipped $SO(10) \otimes U(1)$ model can be naturally embedded in an E_6 GUT with one 78_H and two pairs of $27_H \oplus \overline{27}_H$ in the Higgs sector

$$W_H = \frac{\mu}{2} \text{Tr } 78^2 + \rho_{ij} 27_i \overline{27}_j + \tau_{ij} 27_i 78 \overline{27}_j + \alpha_{ijk} 27_i 27_j 27_k + \beta_{ijk} \overline{27}_i \overline{27}_j \overline{27}_k \quad i, j = 1, 2$$

SUSY vacuum

- The little group is $SU(5)$ for misaligned $27_H \oplus \overline{27}_H$ pairs
- Needs effective adjoint interactions near the Planck scale in order to reach the SM



- Two-loop MSSM with TeV scale SUSY (no GUT-scale thresholds)

CONCLUSIONS

A longstanding result claims that non-SUSY $SO(10)$ GUTs with just the adjoint triggering the GUT breaking can not provide a successful gauge unification

- We argued that this result is an artifact of the tree-level potential and showed that **quantum corrections** have a dramatic impact
- A model featuring $10_H \oplus 126_H \oplus 45_H$ in the Higgs sector has all the ingredients to be a viable minimal non-SUSY $SO(10)$ candidate

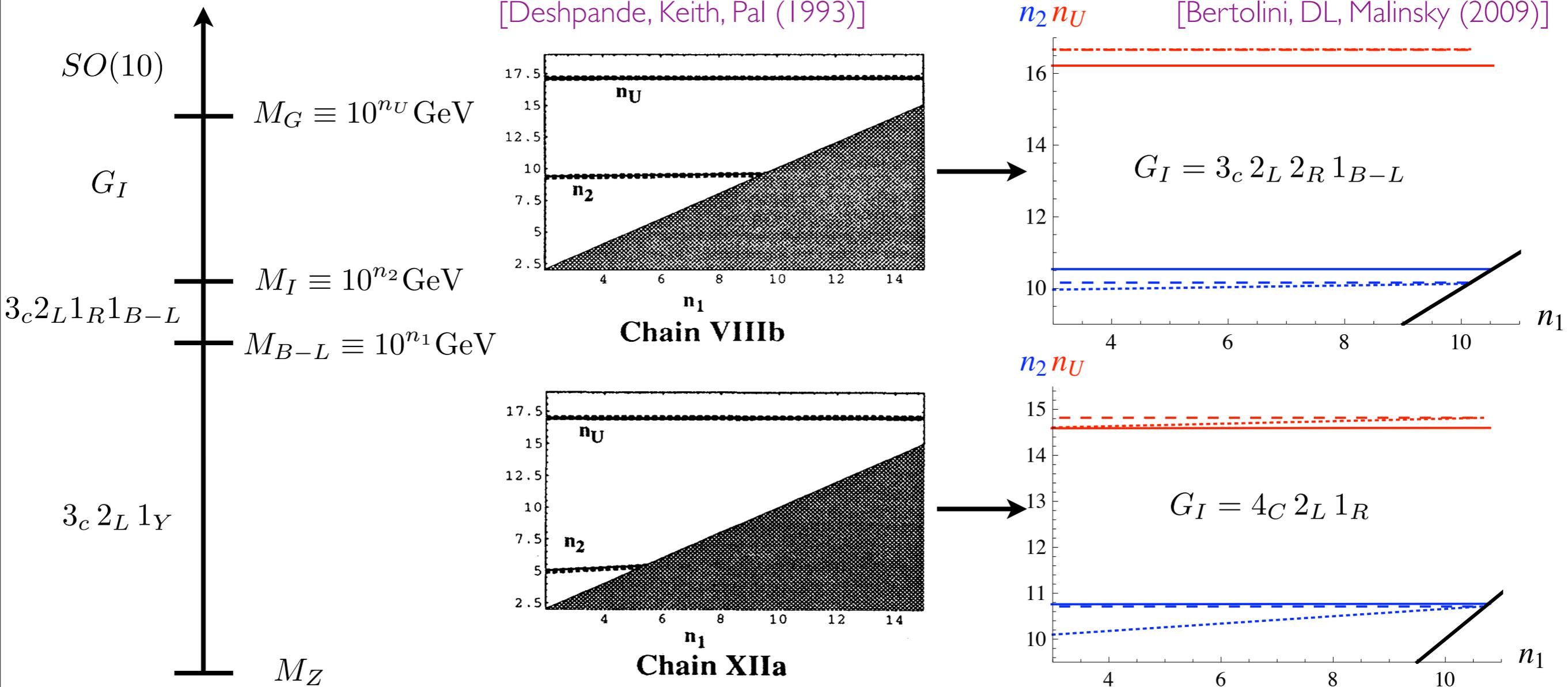
SUSY $SO(10)$ GUTs with rep.'s up to the adjoint do not provide a phenomenologically viable breaking to the SM (GUT-scale Little Hierarchy due to NR operators)

- The **flipped $SO(10)$** embedding offers the option of breaking the gauge symmetry to $SU(3)_C \otimes U(1)_Q$ at the **renormalizable** level and by means of a simple Higgs sector: $2 \times (16_H \oplus \overline{16}_H) \oplus 45_H$
- We made the case for a two-step breaking of an E_6 GUT realised in the vicinity of the Planck scale via an intermediate flipped $SO(10) \otimes U(1)$ stage

BACKUP SLIDES

IMPACT OF THE FULL TWO-LOOP ANALYSIS

General RG analysis of gauge coupling unification based on the Extended Survival Hypothesis (no detailed thresholds!)



- U(1) mixing makes the B-L scale essentially free (upper bound given by M_I)
- Two-loop effects tend to raise the M_I scale and lower the GUT scale
- **Sharp disagreement for chain XIIa:** the M_I scale is raised by 5 orders of magnitude !

MINIMAL FLIPPED $SO(10) \otimes U(1)$ SUSY HIGGS SECTOR

The most general renormalizable superpotential made of $2 \times (16_H + \overline{16}_H) \oplus 45_H$ is

$$W_H = \frac{\mu}{2} \text{Tr } 45^2 + \rho_{ij} 16_i \overline{16}_j + \tau_{ij} 16_i 45 \overline{16}_j \quad i, j = 1, 2 \quad \tau_{ij} \longrightarrow \tau_i \delta_{ij}$$

We require D- and F-flatness at the GUT scale

- The vacuum manifold reads

$$\begin{aligned} e_{1,2} &= r_{1,2} \cos \alpha_{1,2} e^{i\phi_{e_{1,2}}} & 8\mu\omega^+ &= \tau_1 r_1^2 \sin 2\alpha_1 e^{i(\phi_{e_1} - \phi_{\nu_1})} + \tau_2 r_2^2 \sin 2\alpha_2 e^{i(\phi_{e_2} - \phi_{\nu_2})} \\ \nu_{1,2} &= r_{1,2} \sin \alpha_{1,2} e^{i\phi_{\nu_{1,2}}} & 8\mu\omega^- &= \tau_1 r_1^2 \sin 2\alpha_1 e^{i(\phi_{\nu_1} - \phi_{e_1})} + \tau_2 r_2^2 \sin 2\alpha_2 e^{i(\phi_{\nu_2} - \phi_{e_2})} \\ \bar{e}_{1,2} &= r_{1,2} \cos \alpha_{1,2} e^{-i\phi_{e_{1,2}}} & 4\sqrt{2}\mu\omega_R &= \tau_1 r_1^2 \cos 2\alpha_1 + \tau_2 r_2^2 \cos 2\alpha_2, \\ \bar{\nu}_{1,2} &= r_{1,2} \sin \alpha_{1,2} e^{-i\phi_{\nu_{1,2}}} & 4\sqrt{2}\mu\omega_Y &= -\tau_1 r_1^2 - \tau_2 r_2^2 \end{aligned}$$

Where $r_{1,2}$ and $\alpha_{1,2}$ are fixed in terms of superpotential parameters ...

The inspection of the gauge boson spectrum reveals the following little groups

- $SU(5) \otimes U(1)$ for aligned $16 \oplus \overline{16}$ pairs $\alpha_1 = \alpha_2$ and $\phi_{\nu_1} - \phi_{\nu_2} = \phi_{e_1} - \phi_{e_2}$
- SM for misaligned $16 \oplus \overline{16}$'s pairs** $\alpha_1 \neq \alpha_2$ and/or $\phi_{\nu_1} - \phi_{\nu_2} \neq \phi_{e_1} - \phi_{e_2}$

MASS MATRICES (CHARGED FERMIONS)

After the EW symmetry breaking the mass matrices for the matter fields sharing the same unbroken $SU(3)_C \otimes U(1)_Q$ quantum numbers are

$$\hat{\nu}_H \equiv \nu_H / M_P$$

$$\hat{e}_H \equiv e_H / M_P$$

$$(U)(U^c)$$

$$M_u = Y_U v_u$$

$$(\mathbf{D}, \Delta)(\Delta^c, D^c)$$

$$M_d = \begin{pmatrix} Y_D \hat{\nu}_H v_d & Y_D \hat{e}_H v_d \\ Y_U e_H & Y_U \nu_H \end{pmatrix} \sim \mathcal{O} \begin{pmatrix} v & v \\ M_f & M_f \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{d}^c \\ \tilde{\Delta}^c \end{pmatrix} \equiv U_d \begin{pmatrix} \Delta^c \\ D^c \end{pmatrix}$$

$$M_d \rightarrow M_d U_d^\dagger \equiv M'_d \sim \mathcal{O} \begin{pmatrix} v & v \\ 0 & M_f \end{pmatrix}$$

$$(\Lambda^-, E)(\mathbf{E}^c, \Lambda^{c+})$$

$$M_e = \begin{pmatrix} Y_E \hat{\nu}_H v_d & Y_U e_H \\ Y_E \hat{e}_H v_d & Y_U \nu_H \end{pmatrix} \sim \mathcal{O} \begin{pmatrix} v & M_f \\ v & M_f \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{e} \\ \tilde{\Lambda}^- \end{pmatrix} \equiv U_e^* \begin{pmatrix} \Lambda^- \\ E \end{pmatrix}$$

$$M_e \rightarrow U_e M_e \equiv M'_e \sim \mathcal{O} \begin{pmatrix} v & 0 \\ v & M_f \end{pmatrix}$$

Up to a tiny $\mathcal{O}(v/M_f)$ rotation $(\mathbf{D})(\mathbf{d}^c)$ and $(\mathbf{e})(\mathbf{E}^c)$ correspond to the light d.o.f., while $(\Delta)(\tilde{\Delta}^c)$ and $(\tilde{\Lambda}^-)(\Lambda^{c+})$ get a super-heavy mass

MASS MATRICES (NEUTRINOS)

$(\Lambda^0, N, \Lambda^{c0}, N^c, S)$

$$M_\nu = \begin{pmatrix} 0 & 0 & Y_{Ue_H} & 0 & Y_{U\nu_u} \\ 0 & 0 & Y_{U\nu_H} & Y_{U\nu_u} & 0 \\ Y_{Ue_H} & Y_{U\nu_H} & Y_D \hat{\nu}_d \nu_d & 2Y_D \hat{\nu}_d \nu_H & 2Y_D \hat{\nu}_d e_H \\ 0 & Y_{U\nu_u} & 2Y_D \hat{\nu}_H \nu_d & Y_D \hat{\nu}_H \nu_H & 2Y_D \hat{\nu}_H e_H \\ Y_{U\nu_u} & 0 & 2Y_D \hat{e}_H \nu_d & 2Y_D \hat{e}_H \nu_H & Y_D \hat{e}_H e_H \end{pmatrix} \sim \mathcal{O} \begin{pmatrix} 0 & 0 & M_f & 0 & v \\ 0 & 0 & M_f & v & 0 \\ M_f & M_f & 0 & v & v \\ 0 & v & v & M_f^2/M_P & 2M_f^2/M_P \\ v & 0 & v & 2M_f^2/M_P & M_f^2/M_P \end{pmatrix}$$

Working in the one family approximation, for the **lightest neutrino** we get

$$m_\nu \sim \frac{(\nu_H^2 + e_H^2)^2 + 2e_H^2 \nu_H^2}{3e_H^2 \nu_H^2 (e_H^2 + \nu_H^2)} M_P v_u^2 \xrightarrow{e_H \sim \nu_H \sim M_f} m_\nu \sim \frac{v_u^2}{M_f^2/M_P} \sim 0.1 \text{ eV}$$

$$\nu \sim \frac{\nu_H}{\sqrt{\nu_H^2 + e_H^2}} \Lambda^0 - \frac{e_H}{\sqrt{\nu_H^2 + e_H^2}} N$$

$$\tilde{\Lambda}^0 \sim \frac{\nu_H}{\sqrt{\nu_H^2 + e_H^2}} \Lambda^0 + \frac{e_H}{\sqrt{\nu_H^2 + e_H^2}} N$$

Setting $v_u \sim v_d \sim 0$ and working in the basis $(\nu, \tilde{\Lambda}^0, \Lambda^{c0}, N^c, S)$ the **heavy spectrum** reads

$$M'_\nu \sim \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_f & 0 & 0 \\ 0 & M_f & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{M_f^2}{M_P} & 2\frac{M_f^2}{M_P} \\ 0 & 0 & 0 & 2\frac{M_f^2}{M_P} & \frac{M_f^2}{M_P} \end{pmatrix}$$



$$\begin{aligned} m_{\nu_{M_1}} &\sim -M_f^2/M_P & \nu_{M_1} &\sim \frac{1}{\sqrt{2}}(N^c - S), \\ m_{\nu_{M_2}} &\sim 3 \cdot M_f^2/M_P & \nu_{M_2} &\sim \frac{1}{\sqrt{2}}(N^c + S), \\ m_{\nu_{PD_1}} &\sim -M_f & \nu_{PD_1} &\sim \frac{1}{\sqrt{2}}(\tilde{\Lambda}^0 - \Lambda^{c0}), \\ m_{\nu_{PD_2}} &\sim M_f & \nu_{PD_2} &\sim \frac{1}{\sqrt{2}}(\tilde{\Lambda}^0 + \Lambda^{c0}), \end{aligned}$$

COMPARATIVE SUMMARY

SUSY vacua left invariant by the SM-singlet VEVs in various combinations of the lowest-dimensional representations in standard vs flipped $SO(10)$ and $E(6)$

Higgs superfields	Standard $SO(10)$		Flipped $SO(10) \otimes U(1)$	
	R	NR	R	NR
$16 \oplus \overline{16}$	$SO(10)$	$SU(5)$	$SO(10) \otimes U(1)$	$SU(5) \otimes U(1)$
$2 \times (16 \oplus \overline{16})$	$SO(10)$	$SU(5)$	$SO(10) \otimes U(1)$	SM
$45 \oplus 16 \oplus \overline{16}$	$SU(5)$	SM	$SU(5) \otimes U(1)$	$SM \otimes U(1)$
$45 \oplus 2 \times (16 \oplus \overline{16})$	$SU(5)$	SM	SM	SM
$2 \times 45 \oplus 16 \oplus \overline{16}$	$SU(5)$	SM	$SU(5) \otimes U(1)$	$SM \otimes U(1)$

Higgs superfields	R	NR
$27 \oplus \overline{27}$	E_6	$SO(10)$
$2 \times (27 \oplus \overline{27})$	E_6	$SU(5)$
$78 \oplus 27 \oplus \overline{27}$	$SO(10)$	$SM \otimes U(1)$
$78 \oplus 2 \times (27 \oplus \overline{27})$	SU(5)	SM
$2 \times 78 \oplus 2 \times (27 \oplus \overline{27})$	$SU(5)$	SM