Suppression of Flavor Violation in an A_4 Flavor Model for Quarks and Leptons in Warped Geometry

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Based on: 1)A.Kadosh and E.Pallante, JHEP08(2010)115

2)A.Kadosh and E.Pallante, [arXiv:1012.xxxx[hep-ph]]

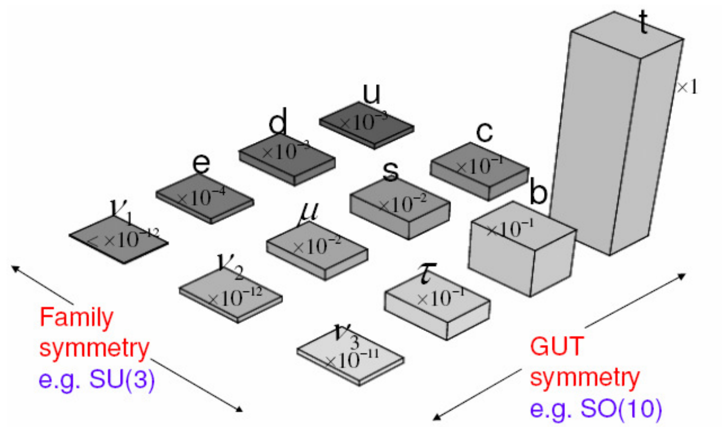
Outline

- Motivation
- The RS-A_4 setup
- Main Features
- Phenomenology and comparison with flavor anarchy
- Conclusions

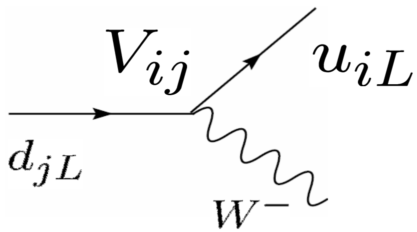
Motivation

Find a unified framework to account for the masses and mixing patterns of quarks and leptons.

Fermion mass hierarchy



Smallness and hierarchy of quark mixing angles



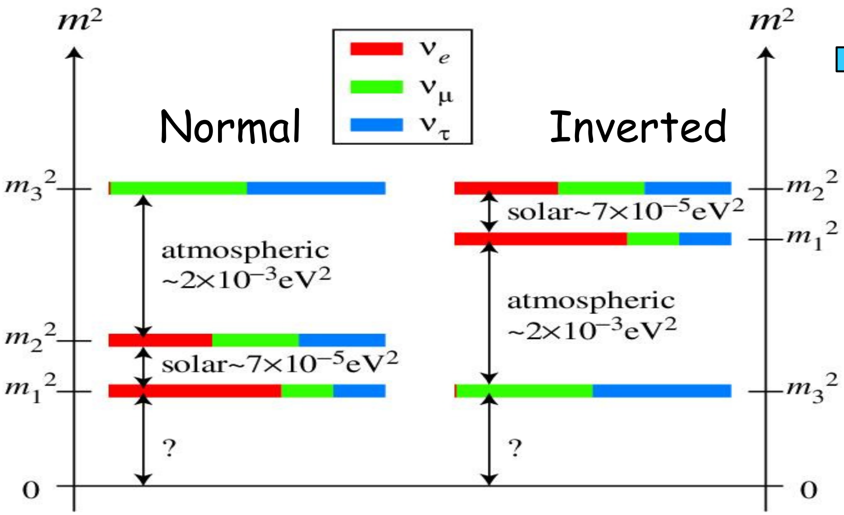
$$V_{ij} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$$\lambda_{CKM} \simeq 0.2257$$

Largeness of neutrino mixing angles and smallness of neutrino masses

TBM

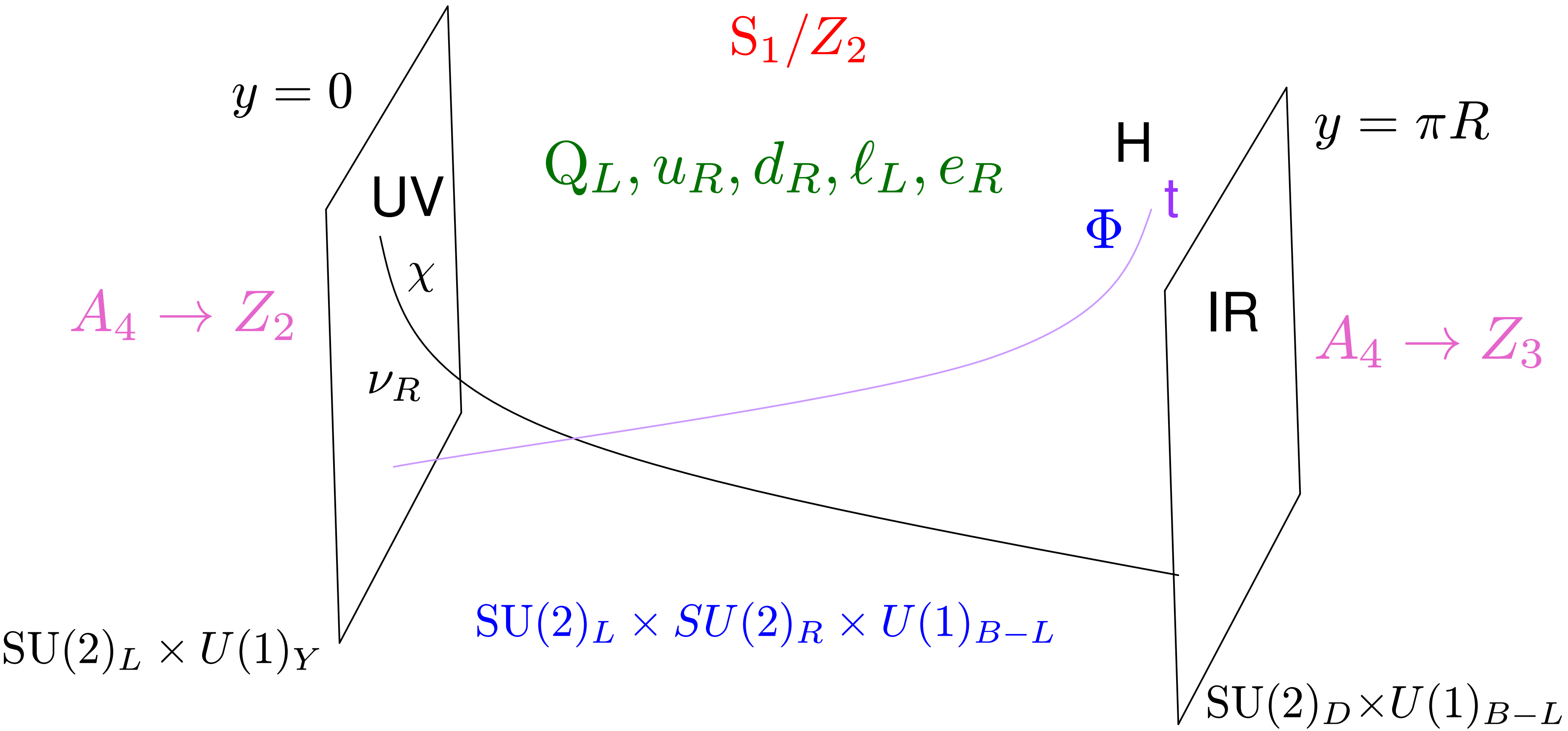
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



See-saw mechanism is the most elegant solution

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

A Warped Geometry Setup



$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dy^\nu + dy^2$$

The Yukawa Lagrangian

$$\mathcal{L}_{5D}^{Yuk} = \underbrace{\mathcal{L}_{LO}}_{\text{UV/IR}} + \underbrace{\mathcal{L}_{NLO}}_{\text{Cross brane}}$$

$$\frac{1}{\Lambda_{5D}^2}\bar{Q}_L \Phi H (u_R, d_R) \qquad \frac{1}{\Lambda_{5D}^{7/2}}\bar{Q}_L \Phi \chi H (u_R, d_R)$$

A_4 Assignments

$$\Phi \sim (1,1,1,0)\,(\underline{\mathbf{3}})\,, \quad \chi \sim (1,1,1,0)\,(\underline{\mathbf{3}})\,, \quad H\,(1,2,2,0)\,(\underline{\mathbf{1}})$$

$$Q_L \sim \left(3,2,1,\frac{1}{3}\right)(\underline{\mathbf{3}}) \qquad \ell_L \sim (1,2,1,-1)\,(\underline{\mathbf{3}})$$

$$u_R \oplus u'_R \oplus u''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \qquad \nu_R \sim (1,1,2,0)\,(\underline{\mathbf{3}})$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \quad e_R \oplus e'_R \oplus e''_R \sim (1,1,2,-1)\,(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

LO Results

SSB of A_4

$$\langle \Phi \rangle = (v_\Phi, v_\Phi, v_\Phi)$$
$$A_4 \to Z_3$$

$$\langle \chi \rangle = (0, v_\chi, 0)$$
$$A_4 \to Z_2$$

TBM neutrino mixing

$$V_L^{u,d} = U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

$$V_R^{u,d} = 1_{3\times3}$$

$$\omega = e^{2\pi i/3}$$

No quark mixing at LO!

NLO Corrections to the CKM Matrix

- Cross brane flavon interactions induce deviations of the CKM matrix from unity

$$M_q + \Delta M_q = U(\omega)\sqrt{3}\begin{pmatrix} \tilde{y}_{q1} v + (x_1^q + y_1^q)/3 & (x_2^q + y_2^q)/3 & (x_3^q + y_3^q)/3 \\ (x_1^q + \omega y_1^q)/3 & \tilde{y}'_{q2} v + (x_2^q + \omega y_2^q)/3 & (x_3^q + \omega y_3^q)/3 \\ (x_1^q + \omega^2 y_1^q)/3 & (x_2^q + \omega^2 y_2^q)/3 & \tilde{y}''_{q3} v + (x_3^q + \omega^2 y_3^q)/3 \end{pmatrix}$$

q=u,d

12 complex parameters

- Parameterizing V_{CKM} in terms of λ :

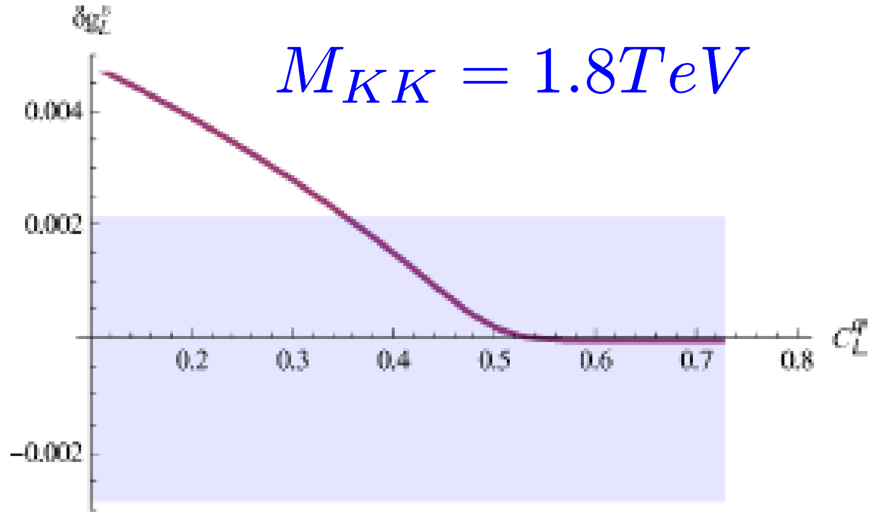
$$\mathcal{O}(x_i^{u,d}, y_i^{u,d}) \qquad V_{CKM} = \begin{pmatrix} 1 & a\lambda & b\lambda^3 \\ -a^*\lambda & 1 & c\lambda^2 \\ -b^*\lambda^3 & -c^*\lambda^2 & 1 \end{pmatrix}$$

$V_{ii} \neq 1$, $|V_{ub}| \neq |V_{td}|$ and phase structure $\mathcal{O}\left((x_i^{u,d})^2, (y_i^{u,d})^2\right)$

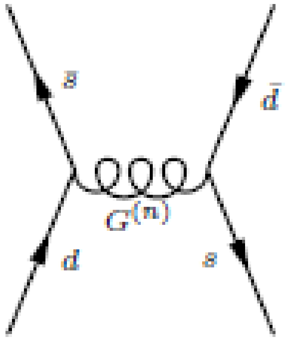
Main Features of RS-A_4 setup

Degenerate C_L

EWPM $\Rightarrow$ $Zb\bar{b}$



$\epsilon_K^{NP} = 0$



$\Rightarrow$ $M_{KK} \gtrsim 10\, TeV$

(In flavor anarchy)

Neutron EDM at 1-loop and HMFCNC $\left(C_{2,4}^{K,D}\right)$ at tree level, strongly suppressed.

Phenomenology-Dipole Operators

As a first step we work in the mass insertion approximation

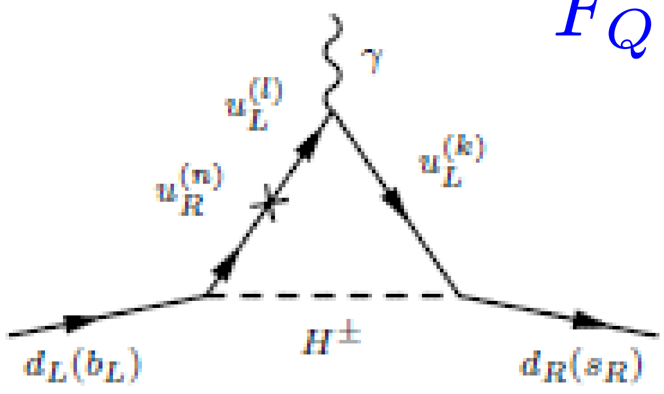
-Flavor part of amplitude in terms of spurions $F_Q, F_{u,d}, \hat{Y}_{u,d}$

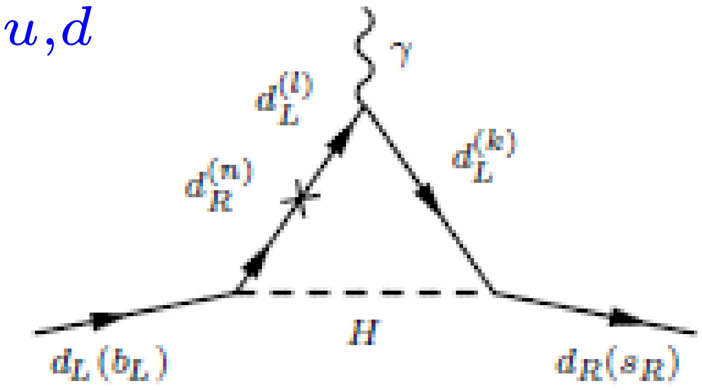
(Agashe, Soni, Perez 2004)

-IR Higgs vs. Bulk Higgs couplings $r_{nm}^{H\Phi(\chi)}(c_{Q_i}, c_{u_j,d_j}, \beta)$

$$O_7^\gamma = \bar{q}_L^i \sigma^{\mu\nu} F_{\mu\nu} q_R^j \qquad i = j = d \ (EDM)$$

$$F_Q Y_{u,d} Y_{u,d}^\dagger Y_d F_{u,d}$$





Dipole Operators (cont.)

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{m_{d_i} A^{1L} f_Q^2}{v^2 M_{KK}} \Big[V_R^{d\dagger} diag(f^2_{d,s,b}) (\hat{r}^d_{00})^{-1} \tilde{r}^d_{01}\, \tilde{r}^d_{11} (\hat{r}^d_{00})^{-1} V^d_R$$
$$\times diag(m^2_{d,s,b}) V_R^{d\dagger}\, (\hat{r}^d_{00})^{-1} \hat{r}^d_{10}\, V^d_R \Big]_{ij}$$

$\tilde{r}^{u,d}_{01}\tilde{r}^{u,d}_{11} = \hat{r}^{u,d}_{01}\hat{r}^{u,d}_{11} + \hat{r}^{u,d}_{01-+}\hat{r}^{u,d}_{1-+1}$ ➡ Overlap corrections

Various levels of Approximation

$(V_R^d)_{LO} = 1_{3\times3}$ ➡ EDM=0

$(V_R^d)_{NLO}$ + Degenerate Overlaps ➡ EDM=0

$(V_R^d)_{NLO}$ + Non degenerate Overlaps

↳ $EDM \sim \mathcal{O}\left((m_d/m_s) f_\chi^{u,d} \Delta r\right) \approx 10^{-29} e\cdot cm$

Dipole Operators (cont.)

Main drawback of spurion analysis $\Rightarrow$ Failure to account for the explicit coupling to the various types (BC) of KK modes.

Second step- diag. of the 1 gen. KK mass matrix

$$(\mathcal{A}_{ij})_D^{overlap} = \frac{\left(\sum_n (\hat{Y}_{KK}^{d_i})_{1n}^{mass} (\hat{Y}_{KK}^{d_j})_{n1}^{mass}\right)\Big|_{overlap}}{M_{KK}^{(n)d_j}}$$

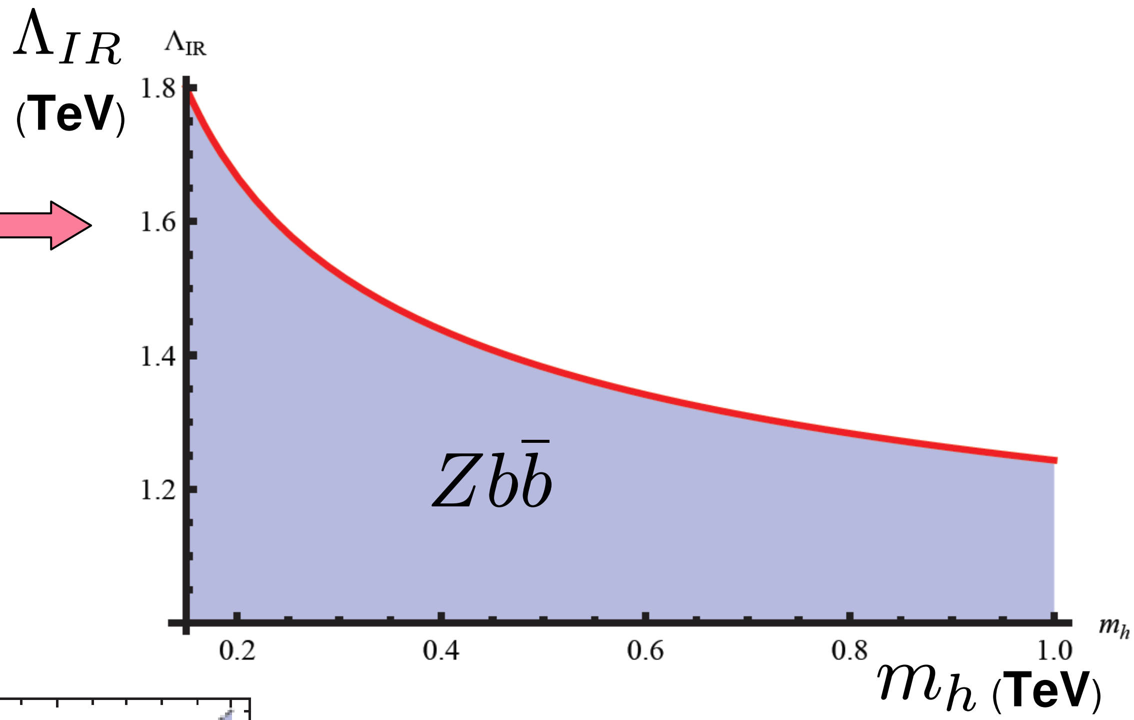
Generational mixing effects $\Rightarrow$ Mass insertion approx.

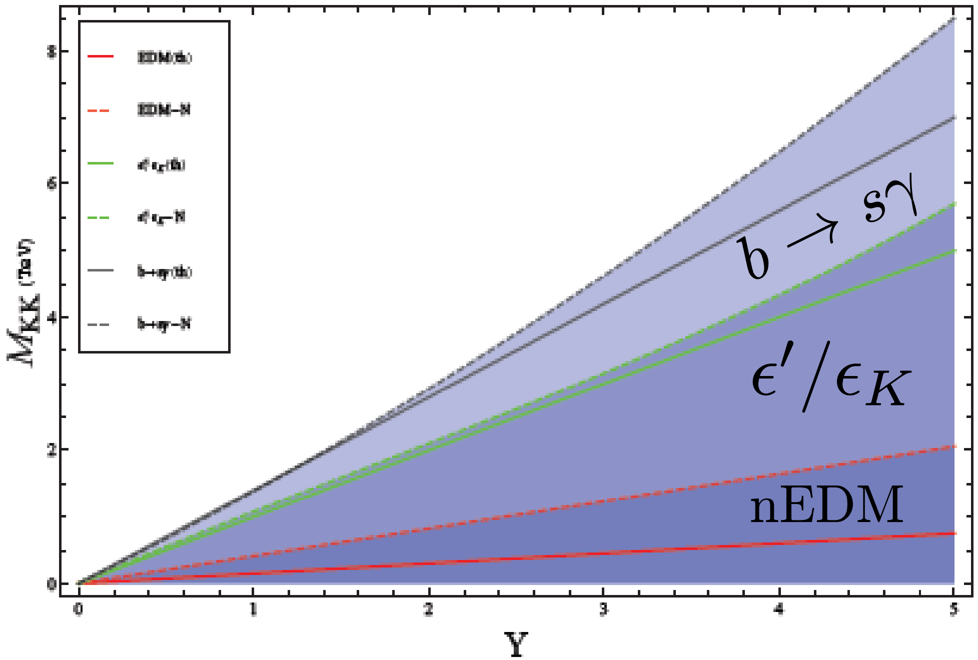
(Agashe-Azatov-Zhu 2009, Gedalia-Isidori-Perez 2009)

Third step- Approximate analytical and numerical diag. of 3 generations zero + 1^{st} KK mass matrix.

The constraint of Zbb with running of Higgs mass

(Casagrande, Neubert et al. , 2010)





Most significant constraints from dipole operators

Conclusions

RS-A_4 → Vacuum Alignment, Flavor Hierarchy
EW-Planck Hierarchy, CKM, TBM+…,
Neutrino masses.
EWPM constrain bulk masses!

Significant Relaxation of Pheno. constraints compared to flavor anarchy, due to degeneracy of C_L!!!

Possible extensions- P_{LR} extended Custodial Symm.

Larger flavor symmetries

"Soft Wall"

Questions

(. . .)

$$\mathcal{L}^{4D} \supset \hat{Y}^{u,d}_{ij} h^{(,*)}_{0(4D)} \Big[\psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^0_{u_j,d_j} f^{-1}_{u_j,d_j} r^H_{00}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^n_{u_j,d_j} r^H_{0n}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{n-+}_{u_j,d_j} r^H_{0n-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum_n \psi^{n\dagger}_{Q_i} \psi^0_{u_j,d_j} f^{-1}_{u_j,d_j} r^H_{n0}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_{n,m} \psi^{n\dagger}_{Q_i} \psi^m_{u_j,d_j} r^H_{nm}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum_{n,m} \psi^{n--}_{Q_i} (\psi^{m--}_{u_j,d_j})^\dagger r^H_{n^-m^-}(c_{Q_i}, c_{u_j,d_j}, \beta)$$
$$+ \sum_{n,m} \psi^{n\dagger}_{Q_i} \psi^{m-+}_{u_j,d_j} r^H_{nm-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum_{n,m} \psi^{n--}_{Q_i} (\psi^{m+-}_{u_j,d_j})^\dagger r^H_{n^-m^{+-}}(c_{Q_i}, c_{d_j,u_j}, \beta) \Big]$$

$$\hat{Y}^{u(d)}_{ij} = \frac{1}{v} r^{-1}_{00}(c_{Q_i}, c_{u_j(d_j)}, \beta) \left(F^{-1}_Q V^{u(d)}_L \mathrm{diag}(m_{u,c,t(d,s,b)}) V^{u(d)\dagger}_R F^{-1}_{u(d)} \right)_{ij}$$

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{A_{1L}}{v^2 M_{KK}} \Big[(V^d_L)^\dagger_{i\ell} (\hat{r}^d_{0n})_{\ell\ell_1} (\hat{r}^d_{00})^{-1}_{\ell\ell_1} \left(V^d_L diag(m_{d,s,b}) (V^d_R)^\dagger \mathrm{diag}(f^2_{d,s,b}) \right)_{\ell\ell_1}$$
$$\times (\hat{r}^d_{nm})_{\ell_2\ell_1} (\hat{r}^d_{00})^{-1}_{\ell_2\ell_1} \left(V^d_R diag(m_{d,s,b}) V^{d\dagger}_L \mathrm{diag}(f^2_{Q1,Q2,Q3}) \right)_{\ell_1\ell_2} (\hat{r}^d_{m0})_{\ell_2\ell_3}$$
$$\times (\hat{r}^d_{00})^{-1}_{\ell_2\ell_3} \left(V^d_L diag(m_{d,s,b}) (V^d_R)^\dagger \right)_{\ell_2\ell_3} (V^d_R)_{\ell_3 j} \Big] ,$$

A_4 Simplifications

$$\hat{r}^{u,d}_{00,10,01} = diag(r_{00,10,01}(c^L_q, c_{u_i,d_i}, \beta)) \qquad \hat{r}^{u,d}_{11} = diag(r_{11}(c_{u_i,d_i}, c^L_q, \beta))$$

$$\hat{r}^{u,d}_{01-+} = diag(r_{01-+}(c^L_q, c_{d_j,u_j}, \beta)) \quad \hat{r}^{u,d}_{1-+1} = diag(r_{1-+1}(c_{d_i,u_i}, c^L_q, \beta))$$

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{m_{d_i} A^{1L} f_Q^2}{v^2 M_{KK}} \Big[V_R^{d\dagger} \mathrm{d}iag(f^2_{d,s,b}) (\hat{r}^d_{00})^{-1} \tilde{r}^d_{01}\, \tilde{r}^d_{11} (\hat{r}^d_{00})^{-1} V^d_R\, \mathrm{d}iag(m_{d,s,b})$$

$$\times \mathrm{d}iag(m_{d,s,b}) V_R^{d\dagger} (\hat{r}^d_{00})^{-1} \hat{r}^d_{10}\, V^d_R \Big]_{ij},$$

$$B^{u,d}_P = \max\left((\hat{r}^{u,d}_{00})^{-3} (\hat{r}^{u,d}_{01} \hat{r}^{u,d}_{11} + \hat{r}^{u,d}_{01-+} \hat{r}^{u,d}_{1-+1})\, \hat{r}^{u,d}_{10} \right)$$

Near degeneracy
Of overlap corrections

$$(C^{d-type}_7)_{ij} = \frac{A^{1L} m_{d_i} m_{d_j} B^d_P}{v^2 M_{KK}} \Big[V_R^{d\dagger} \mathrm{d}iag(f^2_{d_1,d_2,d_3}) V^d_R \mathrm{d}iag(m_{d,s,b}) V_L^{d\dagger} \mathrm{d}iag(f^2_{Q_1,Q_2,Q_3}) V^d_L \Big]_{ij}$$

$$= \frac{A^{1L} f_Q^2 m_{d_i} m^2_{d_j} B^d_P}{v^2 M_{KK}} \sum_{n=1}^{3} (V^d_R)^*_{ni} (V^d_R)_{nj} f^2_{d_n}$$

1 generation KK Yukawa matrices

$$\mathrm{Y}^d_{KK} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \bar{d}_L^{(1^{--})} \\ \bar{Q}_L^{d(1)} \\ \bar{\tilde{d}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & \breve{y}_d^* r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & \breve{y}_u^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1^{--})} \\ d_R^{(1)} \\ \tilde{d}_R^{(1^{-+})} \end{pmatrix}$$

$$\hat{Y}^{d(h_-)}_{KK} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \bar{d}_L^{(1^{--})} \\ \bar{Q}_L^{d(1)} \\ \bar{\tilde{d}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} -\breve{y}_u f_q^{-1} f_u^{-1} r_{00} & 0 & -\breve{y}_u f_q^{-1} r_{01} & -\breve{y}_d f_q^{-1} r_{101} \\ 0 & \breve{y}_d^* r_{22} & 0 & 0 \\ -\breve{y}_u f_u^{-1} r_{10} & 0 & -\breve{y}_u r_{11} & -\breve{y}_d r_{111} \\ 0 & \breve{y}_u^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} u_R^{(0)} \\ Q_R^{u(1^{--})} \\ u_R^{(1)} \\ \tilde{u}_R^{(1^{-+})} \end{pmatrix}$$

$$\hat{Y}^{d(h_+)}_{KK} = \begin{pmatrix} \bar{Q}_L^{u(0)} \\ \bar{u}_L^{(1^{--})} \\ \bar{Q}_L^{u(1)} \\ \bar{\tilde{u}}_L^{(1^{+-})} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & -\breve{y}_u^* r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & -\breve{y}_d^* r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1^{--})} \\ d_R^{(1)} \\ \tilde{d}_R^{(1^{-+})} \end{pmatrix}$$

4X4 one gen. KK mass matrix

$$\frac{\hat{\mathrm{M}}_d^{KK}}{(M_{KK})} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \tilde{d}_L^{(1--)} \\ \bar{Q}_L^{d(1)} \\ \tilde{d}_L^{(1+-)} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} x & 0 & \breve{y}_d f_q^{-1} r_{01} x & \breve{y}_u f_q^{-1} r_{101} x \\ 0 & \breve{y}_d^* r_{22} x & 1 & 0 \\ \breve{y}_d f_d^{-1} r_{10} x & 1 & \breve{y}_d r_{11} x & \breve{y}_u r_{111} x \\ 0 & \breve{y}_u^* r_{222} x & 0 & 1 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1--)} \\ d_R^{(1)} \\ \tilde{d}_R^{(1-+)} \end{pmatrix}$$

$$\breve{y}_{u,d} \equiv (\hat{y}_{u,d}^{LO})_{11} v_\Phi^{4D} e^{k\pi R}/k$$

12X12 three gen. KK mass matrix

$$\hat{\mathrm{M}}_{Full}^D = M_{KK} \begin{pmatrix} \hat{\mathrm{M}}_d^{KK}/M_{KK} & x\hat{Y}_{KK}^s(\hat{y}_{12}^{LO}, f_s) & x\hat{Y}_{KK}^b(\hat{y}_{13}^{LO}, f_b) \\ x\hat{Y}_{KK}^d(\hat{y}_{21}^{LO}, f_d) & \hat{\mathrm{M}}_s^{KK}/M_{KK} & x\hat{Y}_{KK}^b(\hat{y}_{23}^{LO}, f_b) \\ x\hat{Y}_{KK}^d(\hat{y}_{31}^{LO}, f_d) & x\hat{Y}_{KK}^s(\hat{y}_{32}^{LO}, f_s) & \hat{\mathrm{M}}_b^{KK}/M_{KK} \end{pmatrix}$$

O_L^{sKK}

One example of KK diag. matrix

$$\begin{pmatrix} 1 & 0.94 f_Q^{-1} r_{01} \breve{y}_s x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x \\ -0.94 f_Q^{-1} r_{01} \breve{y}_s x & 1 & (6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x & -(6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x \\ \mathcal{O}(x^2) & (8.6(r_{11} + 8.1 r_{22}) \breve{y}_s & -\frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2(|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2(|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x \\ -f_Q^{-1} r_{101} \breve{y}_c^* x & \mathcal{O}(x^2) & \frac{1}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2(|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{1}{\sqrt{2}} - \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2(|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x \end{pmatrix}$$

12X12 additional A_4 rotation

$$\hat{\mathrm{O}}_{L,R}^{(U,D)A_4} = \begin{pmatrix} (V_{L,R}^{u,d})_{11} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{12} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{13} \times \tilde{\mathbb{1}}_{4\times4} \\ (V_{L,R}^{u,d})_{21} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{22} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{23} \times \tilde{\mathbb{1}}_{4\times4} \\ (V_{L,R}^{u,d})_{31} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{32} \times \tilde{\mathbb{1}}_{4\times4} & (V_{L,R}^{u,d})_{33} \times \tilde{\mathbb{1}}_{4\times4} \end{pmatrix}$$

Dynamical Completion Issues

- Vacuum Alignment – We will have to make sure that the scalar potential doesn't ruin the specific VEV structure we are interested in.
- Most importantly in any flavor model one should explain the origin of quark and lepton masses and their hierarchy (FN, GUT's, WED, UED etc.…).
- Ultimately, a dynamical origin for the A_4 symmetry should be supplemented.
- One of the possibilities is obtaining A_4 via compactification of a 6 dimensional flat space on an orbifold T_2/Z_2. The various fields reside on the four orbifold fixed points (Branes).

(Feruglio and Altarelli)

Off diagonal CKM elements

$$V_{us} = -V_{cd}^* \simeq \left((\tilde{x}_2^d + \tilde{y}_2^d) f_\chi^s - (\tilde{x}_2^u + \tilde{y}_2^u) f_\chi^c\right)$$

$$V_{cb} = -V_{ts}^* \simeq \left((\tilde{x}_3^d + \omega \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \omega \tilde{y}_3^u) f_\chi^t\right)$$

$$V_{ub} = -V_{td}^* \simeq \left((\tilde{x}_3^d + \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \tilde{y}_3^u) f_\chi^t\right)$$

$$\mathrm{V}_{CKM} = \begin{pmatrix} 1 & V_{us} & V_{ub} \\ -V_{us}^* & 1 & V_{cb} \\ -V_{ub}^* & -V_{cb}^* & 1 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} H & \tilde{H} \end{pmatrix} = \begin{pmatrix} h_0^* & h_+ \\ -h_+^* & h_0 \end{pmatrix} \qquad h_0(x,y) = v_H(\beta_H, y) + \sum_n h_0^{(n)}(x)\phi_n(y)$$

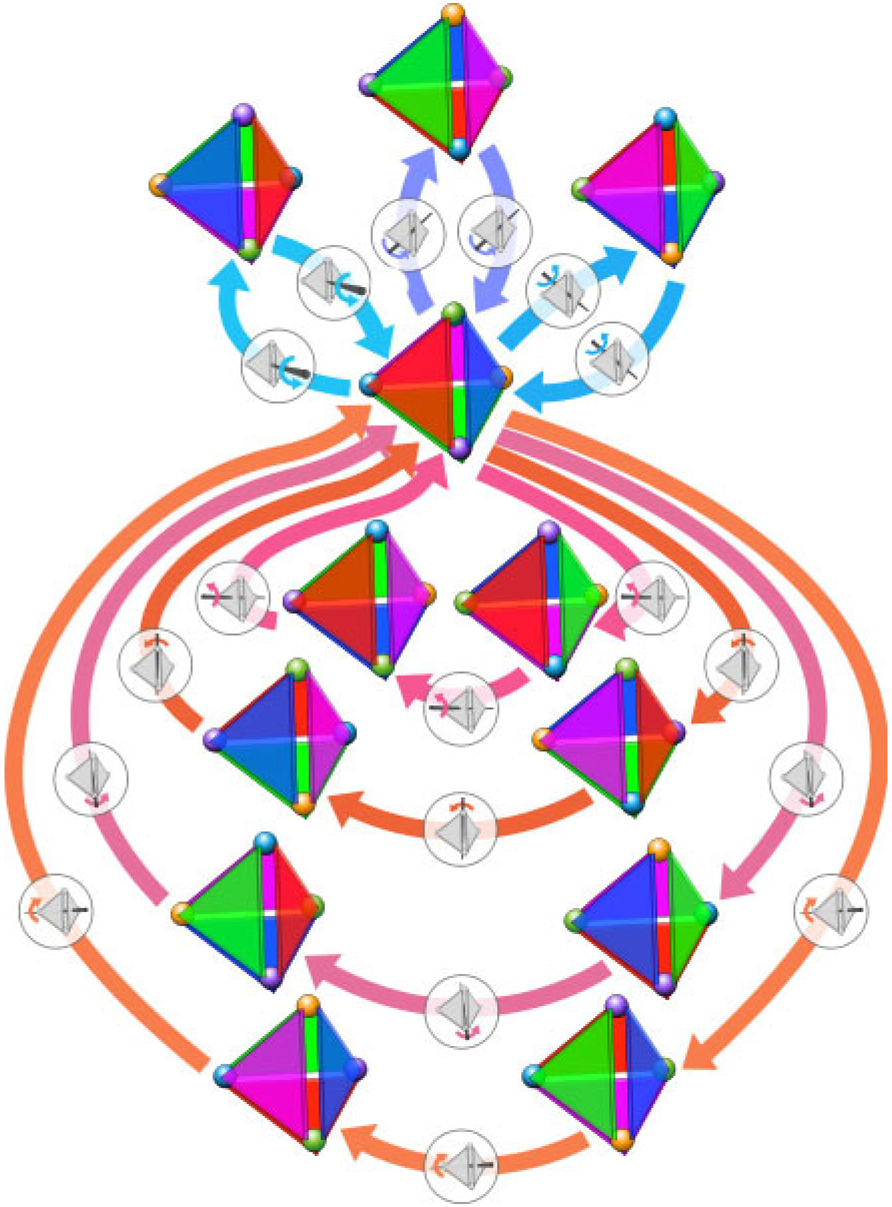
ZMA RH diag. Matrices

$$V_R^q = \begin{pmatrix} 1 & \Delta_1^q & \Delta_2^q \\ -(\Delta_1^q)^* & 1 & \Delta_3^q \\ -(\Delta_2^q)^* & -(\Delta_3^q)^* & 1 \end{pmatrix}$$

$$\Delta_1^q = \frac{m_{q1}}{m_{q2}} \left[f_X^{q_1} \left((\tilde{x}_1^q)^* + \omega^2 (\tilde{y}_1^q)^* \right) + f_X^{q_2} \left(\tilde{x}_2^q + \tilde{y}_2^q \right) \right]$$

$$\Delta_2^q = \frac{m_{q1}}{m_{q3}} \left[f_X^{q_1} \left((\tilde{x}_1^q)^* + \omega (\tilde{y}_1^q)^* \right) + f_X^{q_3} \left(\tilde{x}_3^q + \tilde{y}_3^q \right) \right]$$

$$\Delta_3^q = \frac{m_{q2}}{m_{q3}} \left[f_X^{q_2} \left((\tilde{x}_2^q)^* + \omega (\tilde{y}_2^q)^* \right) + f_X^{q_3} \left(\tilde{x}_3^q + \omega \tilde{y}_3^q \right) \right]$$



The Tetrahedral Group A_4

- A(4) is the group of even permutations of 4 objects
- It is also isomorphic to the symmetry group of a regular tetrahedron, and is a subgroup of SO(3)
- Other extensions include:

$$\mathrm{T}' \quad \Delta(27) \quad \S(81)$$

- Will be used to explain proximity of mixing in the lepton sector to TBM, and proximity of mixing in the quark sector to unity.
- Differs from other types of flavor models: "Anarchic", continuous flavor groups, GUT's, (SUSY),…

Some A(4) Basic properties:

•A(4) has one real triplet, $\underline{3}$ and three "singlets": $\underline{1}$, $\underline{1}'$ and $\underline{1}''$

$$\underline{3} \otimes \underline{3} = \underline{3}_s \oplus \underline{3}_a \oplus \underline{1} \oplus \underline{1}' \oplus \underline{1}'', \quad \text{and} \quad \underline{1}' \otimes \underline{1}' = \underline{1}''$$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_s} = (x_2 y_3 + x_3 y_2 \, , \, x_3 y_1 + x_1 y_3 \, , \, x_1 y_2 + x_2 y_1),$$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_a} = (x_2 y_3 - x_3 y_2 \, , \, x_3 y_1 - x_1 y_3 \, , \, x_1 y_2 - x_2 y_1),$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}} = x_1 y_1 + x_2 y_2 + x_3 y_3,$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}'} = x_1 y_1 + \omega \, x_2 y_2 + \omega^2 \, x_3 y_3, \qquad \omega = e^{i2\pi/3}$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}''} = x_1 y_1 + \omega^2 \, x_2 y_2 + \omega \, x_3 y_3,$$

A simple A(4) Model

(Ma, Feruglio, Altarelli, Babu, Volkas…)

•We assign the SM fermions to the following representations:

$$Q_L \sim \left(3,2,\tfrac{1}{3}\right)(\underline{\mathbf{3}}) \qquad \ell_L \sim (1,2,-1)(\underline{\mathbf{3}})$$

SU(3)xSU(2)xU(1)

Under A(4)

$$u_R \oplus u'_R \oplus u''_R \sim \left(3,1,\tfrac{4}{3}\right)(\underline{1}\oplus\underline{1}'\oplus\underline{1}'') \qquad \nu_R \sim (1,1,0)(\underline{\mathbf{3}})$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3,1,-\tfrac{2}{3}\right)(\underline{1}\oplus\underline{1}'\oplus\underline{1}'') \qquad e_R \oplus e'_R \oplus e''_R \sim (1,1,-2)(\underline{1}\oplus\underline{1}'\oplus\underline{1}'')$$

•The scalar sector of this model will be given by:

$$\Phi \sim (1,2,-1)(\underline{\mathbf{3}}), \quad \phi \sim (1,2,-1)(\underline{1}), \quad \chi \sim (1,1,0)(\underline{\mathbf{3}}).$$

•We will also need an additional U(1) symmetry which will be explicitly broken to Z_2 under which ϕ, χ and Q_L are odd and the rest of the fields are even.

•The Yukawa Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\rm Yuk} = &\ \lambda_u\ (\overline{Q}_L\Phi)_{\underline{1}}\, u_R + \lambda'_u(\overline{Q}_L\Phi)_{\underline{1}'}\, u''_R + \lambda''_u(\overline{Q}_L\Phi)_{\underline{1}''}\, u'_R + \\ &+\ \lambda_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}}\, d_R + \lambda'_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}'}\, d''_R + \lambda''_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}''}\, d'_R + \\ &+\ \lambda_\nu(\bar{\ell}_L\nu_R)_{\underline{1}}\,\phi + M\,[\overline{\nu}_R(\nu_R)^c]_{\underline{1}} + \lambda_\chi[\overline{\nu}_R(\nu_R)^c]_{\underline{3}_s}\cdot\chi + \\ &+\ \lambda_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}}\, e_R + \lambda'_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}'}\, e''_R + \lambda''_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}''}\, e'_R + h.c.\end{aligned}$$

•And the resulting mass matrix in each sector, f=(u,d,e):

$$\left(\overline{f}_{1L},\overline{f}_{2L},\overline{f}_{3L}\right)\begin{pmatrix}\lambda v_1 & \lambda' v_1 & \lambda'' v_1\\ \lambda v_2 & \omega\lambda' v_2 & \omega^2\lambda'' v_2\\ \lambda v_3 & \omega^2\lambda' v_3 & \omega\lambda'' v_3\end{pmatrix}\begin{pmatrix}f_R\\ f''_R\\ f'_R\end{pmatrix} + h.c.$$

The Neutrino Sector

- From the Yukawa Lagrangian we get that the Dirac and the bare Majorana mass matrices are proportional to the identity:

$$M_\nu^D = \lambda_\nu\, v_\phi\, 1 \equiv m_\nu^D\, 1 \quad \text{and} \quad M_{\nu Bare}^{Maj.} = M\mathbf{1}$$

The required non-trivial structure is supplied by the Yukawa coupling to the field, χ , which turns out to be:

$$\lambda_\chi \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R}\right) \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix}$$

•Inserting the VEV of χ the resulting 6x6 mass matrix is:

$$\begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & M & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & M & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & M \end{pmatrix} \qquad M_\chi \equiv \lambda_\chi v_\chi$$

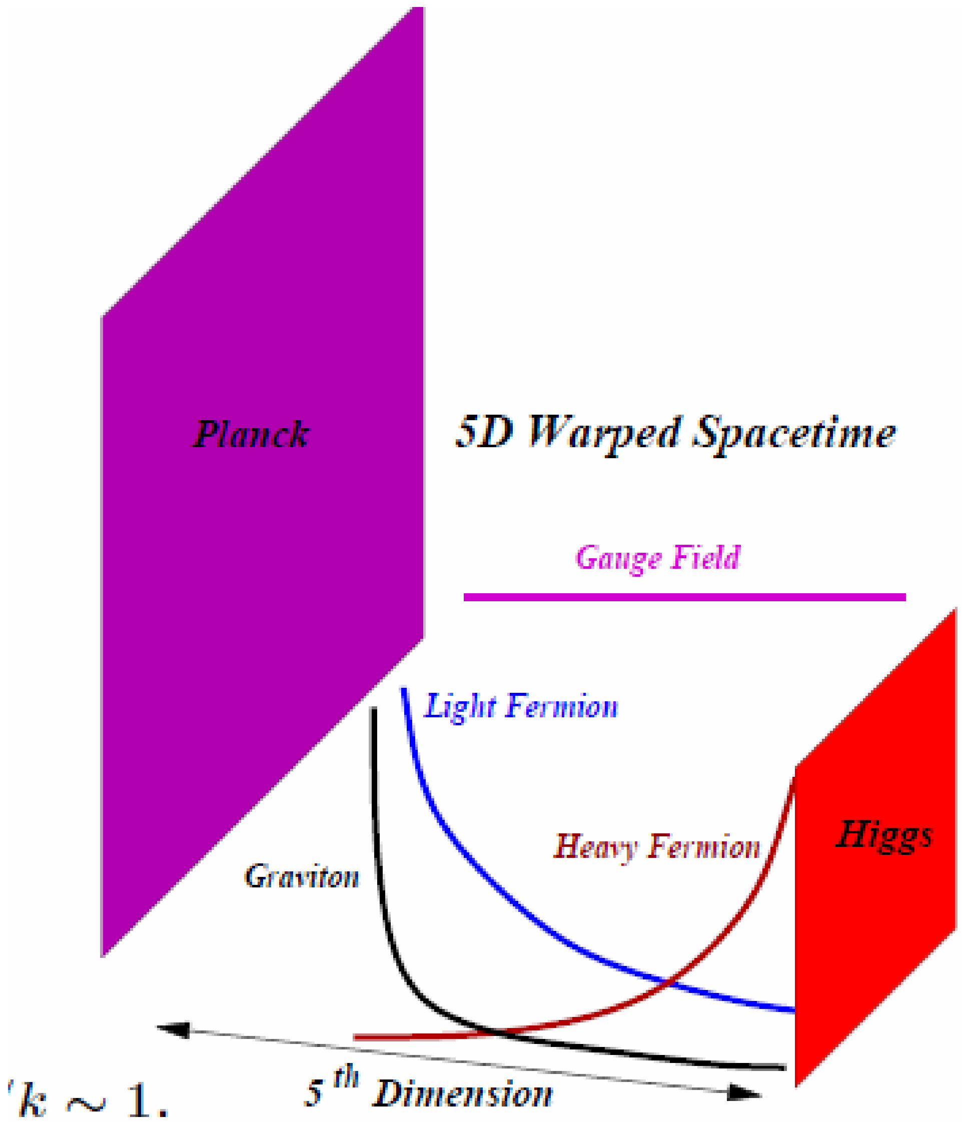
- In the see-saw limit, $|M|, |M_\chi| \gg m_\nu^D$ the effective 3x3 mass matrix for the light neutrinos is given by:

$$M_L = -M_\nu^D M_R^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{M} \begin{pmatrix} \frac{M^2}{M^2-M_\chi^2} & 0 & -\frac{M M_\chi}{M^2-M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{M M_\chi}{M^2-M_\chi^2} & 0 & \frac{M^2}{M^2-M_\chi^2} \end{pmatrix}$$

•The diagonalization matrix turns out to be : $V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$

•So the MNSP matrix at this order is Tri-bi-maximal:

$$V_{MNSP} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix}$$


Planck
5D Warped Spacetime
Gauge Field
Light Fermion
Higgs
Heavy Fermion
Graviton
'k ∼ 1.
5 th Dimension