TECHNISCHE UNIVERSITÄT MÜNCHEN

IAS TUM Institute for Advanced Study

Bimaximal Neutrino Mixing with Discrete Flavour Symmetries

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DISCRETE10: Symposium on Prospects in the Physics of Discrete Symmetries

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Outline

The Flavour Puzzle

The Tri-Bimaximal Mixing

The Bimaximal Mixing

Model Building

A Lepton Model in the SM Context

A Full Model in the Pati-Salam Context

Based on:

Guido Altarelli, Ferruccio Feruglio & LM,

Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry

JHEP 0905:020,2009

Reinier de Adelhart Toorop, Federica Bazzocchi & LM,

The Interplay Between GUT and Flavour Symmetries in a Pati-Salam x S_4 Model

JHEP 1008:001,2010

The Flavour Puzzle

The sources of the Flavour Puzzle are the Yukawa interactions:

$$\mathcal{L}_Y = (Y_e)_{ij}\, e_i^c H^\dagger \ell_j + (Y_d)_{ij}\, d_i^c H^\dagger q_j + (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + \frac{1}{2}(Y_\nu)_{ij} \frac{(\ell_i \widetilde{H}^*)\,(\widetilde{H}^\dagger \ell_j)}{\Lambda_L}$$

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Charged Fermions

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1$$

$$m_e : m_\mu : m_\tau \approx \lambda^{4\div 5} : \lambda^2 : 1$$

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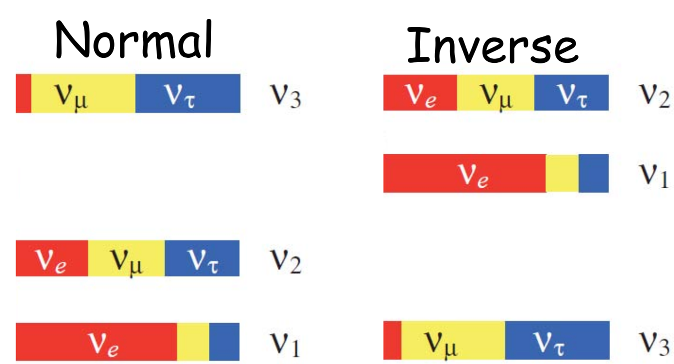
Neutrinos Solar and Atmospheric anomalies explained by neutrino oscillations:

$$m_\nu \lesssim \mathcal{O}(\mathrm{eV})$$

$$\Delta m^2_{sol} \equiv m_2^2 - m_1^2 \sim 7\times 10^{-5}\,\mathrm{eV}^2$$

$$\left|\Delta m^2_{atm}\right| \equiv \left|m_3^2 - m_1^2\right| \sim 2\times 10^{-3}\,\mathrm{eV}^2$$



The Mixing Matrices

CKM Matrix

$$V = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12})$$

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$$|V| = \begin{pmatrix} 0.97 & 0.23 & 0.0039 \\ 0.23 & 1 & 0.041 \\ 0.0081 & 0.038 & 1 \end{pmatrix}$$

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Wolfenstein parametrization

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx \theta_C \approx 0.23$$

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PMNS Matrix

Atmospheric

Reactor

Dirac

Solar

Majorana

$$U \quad = \quad R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12}) \cdot P$$

$$\begin{array}{rcll} \tan^2\theta_{12} & = & 0.444^{+0.039}_{-0.027} & (0.561 - 0.363) \\ \sin^2\theta_{23} & = & 0.466^{+0.073}_{-0.058} & (0.331 - 0.644) \\ \sin^2\theta_{13} & = & 0.009^{+0.013}_{-0.007} & (\leq 0.046) \end{array}$$

$\theta_{12} \approx 34^\circ$

$\theta_{23} \approx 43^\circ$

$\theta_{13} \approx 8^\circ$

The Tri-Bimaximal Pattern

[Harrison, Perkins & Scott; Zhi-Zhong Xing 2002]

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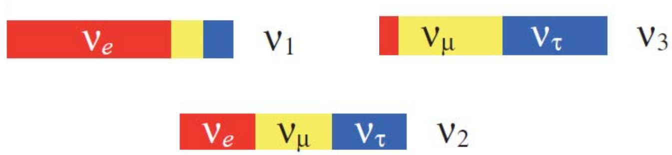

$$
U^{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}
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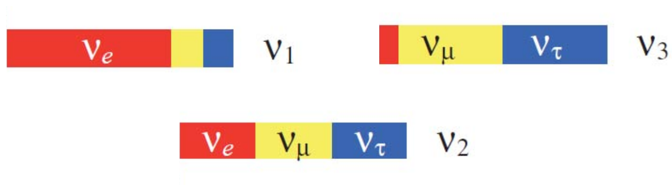

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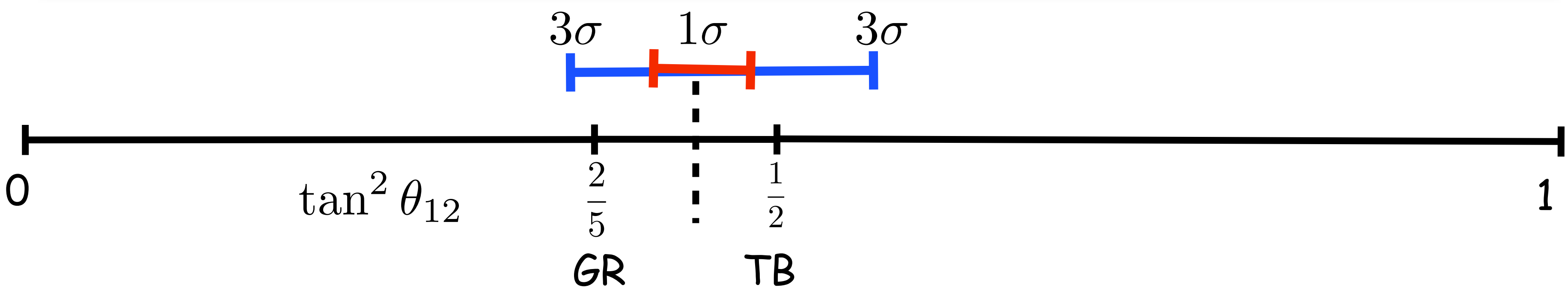
Mixing angles independent from mass eigenvalues vs. quark mixings $\lambda \simeq \sqrt{\frac{m_d}{m_s}}$

Neutrino Mass Patterns



<u>TRI-BIMAXIMAL (TB)</u>

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \tan^2\theta_{12} = \frac{1}{2} \quad \longrightarrow \quad \theta_{12} = 35.26^\circ$$

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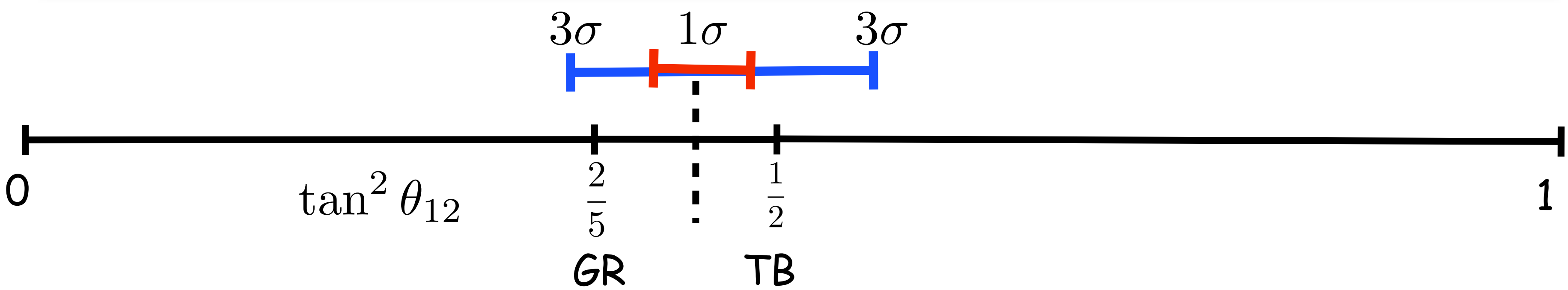

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<u>GOLDEN RATIO (GR)</u> [Kajiyama, Raidal & Strumia 2007]

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \tan\theta_{12} = \frac{1}{\phi} \quad \longrightarrow \quad \theta_{12} = 31.72^\circ$$

$$\phi \equiv \frac{1+\sqrt{5}}{2}$$

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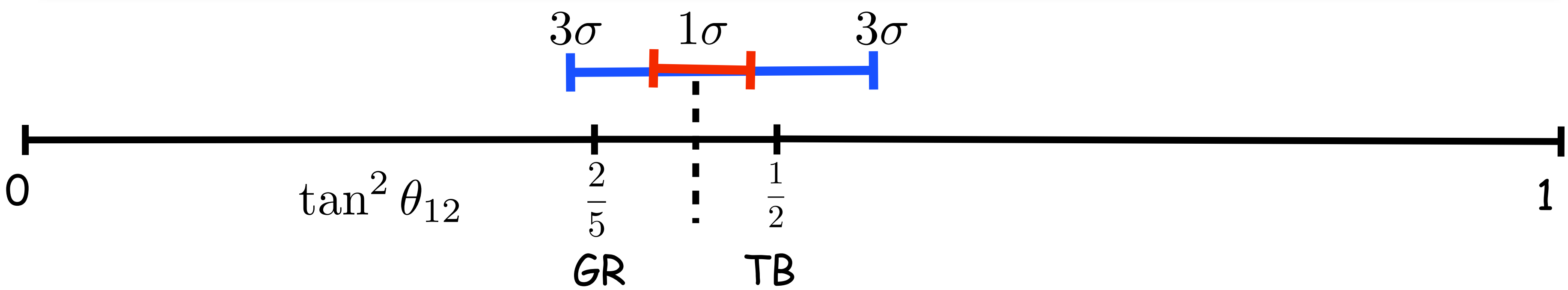

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The agreement of θ_{12} suggests that only tiny corrections $\left(\mathcal{O}(\theta_C^2)\right)$ are allowed

$\longrightarrow$ $\boxed{\theta_{13} \approx \mathcal{O}(\theta_C^2) \quad \text{is expected}}$

TB Mixing Accidental?

If θ_{13} is found close to its present upper bound $\theta_{13} \approx \theta_C$, this would imply that TB mixing is accidental.

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Quark-Lepton Complementarity: [Smirnov; Raidal; Minakata & Smirnov 2004]

$$\boxed{\pi/4 \approx \theta_{12} + \theta_C}$$

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$\longrightarrow$ Need of an alternative leading principle

Quark-Lepton Complementarity: [Smirnov; Raidal; Minakata & Smirnov 2004]

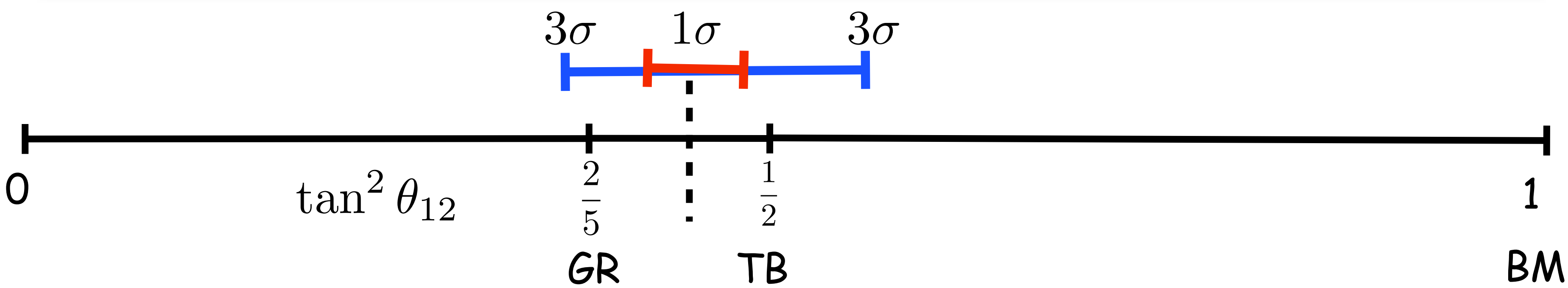
$$\boxed{\pi/4 \approx \theta_{12} + \theta_C}$$

This relation suggests a <u>maximal solar angle</u> in first approximation, corrected by θ_C

$$\Longrightarrow \quad \boxed{\theta_{12} \approx \pi/4 - \theta_C}$$

No compelling models with this feature!!!

Neutrino Mass Patterns (2)



BIMAXIMAL (BM) [Vissani 1997; Barger et al. 1998]

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \tan^2\theta_{12} = 1 \qquad \longrightarrow \qquad \theta_{12} = 45^\circ$$

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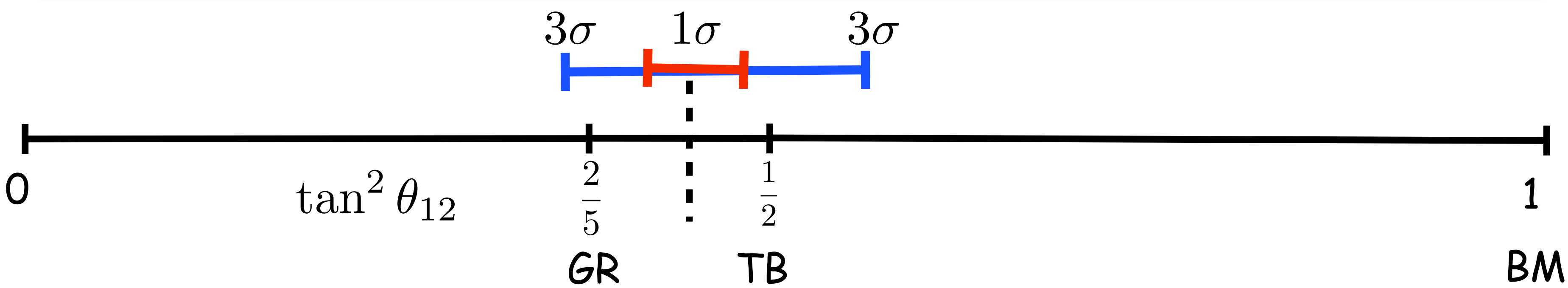

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Requires a large correction of $\mathcal{O}(\theta_C)$ to agree with the data:

$$\Rightarrow \quad \boxed{\theta_{12} \approx \pi/4 - \theta_C}$$

$$\Rightarrow \quad \boxed{\theta_{13} \approx \mathcal{O}(\theta_C)}$$ is expected

The Bimaximal Pattern

[Barger,Pakvasa,Weiler and Whisnant 1998]

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Mixings <u>independent</u> from mass eigenvalues

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Corrections needed to fit the data:

$$\begin{aligned} \sin^2\theta_{12} &\simeq 1/2 + \delta_{12} \\ \sin^2\theta_{23} &\simeq 1/2 + \delta_{23} \\ \sin\theta_{13} &\simeq \delta_{13} \end{aligned} \qquad \text{with} \qquad \left\{ \begin{aligned} \delta_{12} &\approx \theta_C \\ \delta_{23} &\ll \theta_C \\ \delta_{13} &\leq \theta_C \end{aligned} \right.$$

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Strategy:

at LO $U_\nu = U^{BM}$ and $U_\ell = 1 \longleftrightarrow U_{PMNS} = U_\ell^\dagger U_\nu = U^{BM}$

at NLO $U_{PMNS} = U^{BM} + \delta U$

Model Building

$$\sin^2\theta_{12} \simeq 1/2 + \delta_{12}$$
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many possibilities: our choice

$$\Rightarrow \quad U_{PMNS} = U_\ell^\dagger U_\nu$$

$$U_\nu = U^{BM}$$

$$U_\ell = \begin{pmatrix} 1 & \delta_{12} & \delta_{13} \\ -\delta_{12} & 1 & \delta_{23} \\ -\delta_{13} & -\delta_{23} & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & \theta_C & \theta_C \\ -\theta_C & 1 & \theta_C^2 \\ -\theta_C & -\theta_C^2 & 1 \end{pmatrix}$$

Lepton Model

[Altarelli, Feruglio & LM 2009]

We consider the flavour group S_4, the group of permutation of 4 objects.
It has 24 elements and 5 irreducible representations: 1, 1′, 1″, 2, 3, 3′.
The symmetry must be broken at low energy and then:

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\begin{tikzcd}
 & & S_4 \arrow[dl] \arrow[dr] & & \\
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The full symmetry is broken by a set of fields, the **flavons**, which are scalar under the SM group, but charged under S_4 . These fields live at very high energies (masses and VEVs are close to the GUT scale). They break the symmetry into different subgroups, thanks to misaligned VEVs.

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It has 24 elements and 5 irreducible representations: 1, 1′, 1″, 2, 3, 3′.
The symmetry must be broken at low energy and then:

keeps separated the two sectors

$$S_4 \times Z_4 \times U(1)_{FN}$$

explains the hierarchy

$$m_e \ll m_\mu \ll m_\tau$$

$$S_4 \times Z_4 \times U(1)_{FN} \xrightarrow{\langle \varphi_\ell \rangle} Z_4 \longrightarrow M_\ell^{\text{diag}}$$

$$S_4 \times Z_4 \times U(1)_{FN} \xrightarrow{\langle \varphi_\nu \rangle} Z_2 \longrightarrow M_\nu^{BM}$$

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	Matter fields					Higgses	Flavons				
	ℓ	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_ℓ	χ_ℓ	ξ_ν	φ_ν
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{3}$
Z_4	1	-1	$-i$	$-i$	1	1	1	i	i	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0

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$$\left.\begin{aligned} w_\ell &= y_\tau \frac{1}{\Lambda}\tau^c\,(\varphi_\ell \ell)\,h_d + y_\mu \frac{\theta}{\Lambda^2}\mu^c\,(\varphi_\ell \ell)'\,h_d + y_e \frac{\theta^2}{\Lambda^4}e^c\,(\varphi_\ell \varphi_\ell \ell)\,h_d \\ w_\nu &= y(\nu^c \ell)h_u + M\Lambda(\nu^c\nu^c) + x_a(\nu^c\nu^c\xi_\nu) + x_b(\nu^c\nu^c\varphi_\nu) \end{aligned}\right\}$$

Expansion in ϕ/Λ

	Matter fields					Higgses	Flavons				
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Z_4	1	-1	$-i$	$-i$	1	1	1	i	i	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0

$$w_\ell = y_\tau \frac{1}{\Lambda}\tau^c\,(\varphi_\ell \ell)\,h_d + y_\mu \frac{\theta}{\Lambda^2}\mu^c\,(\varphi_\ell\ell)'\,h_d + y_e\frac{\theta^2}{\Lambda^4}e^c\,(\varphi_\ell\varphi_\ell\ell)\,h_d$$

$$w_\nu = y(\nu^c\ell)h_u + M\Lambda(\nu^c\nu^c) + x_a(\nu^c\nu^c\xi_\nu) + x_b(\nu^c\nu^c\varphi_\nu)$$

Expansion in ϕ/Λ

vacuum alignment:

$$\frac{\langle\varphi_\ell\rangle}{\Lambda} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad \frac{\langle\chi_\ell\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \quad \frac{\langle\varphi_\nu\rangle}{\Lambda} = \begin{pmatrix} 0 \\ v' \\ -v' \end{pmatrix} \quad \frac{\langle\xi_\nu\rangle}{\Lambda} = v' \quad \frac{\langle\theta\rangle}{\Lambda} = t$$

$$\Rightarrow \quad M_\ell = \mathrm{diag}(y_e v^2 t^2,\, y_\mu v t,\, y_\tau v)v_d \qquad \begin{cases} t \sim 0.06 \\ v \sim 0.08 \end{cases}$$

$$\Rightarrow \quad M_\nu = -(m_\nu^D)^T M_N^{-1} m_\nu^D$$

diagonalized by BM

at **NLO**: charged leptons

$$m_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \xrightarrow{\text{NLO}} m_\ell = \begin{pmatrix} m_e & m_e v' & m_e v' \\ m_e v' & m_\mu & 0 \\ m_e v' & 0 & m_\tau \end{pmatrix}$$

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$$\begin{array}{rcl} \sin^2\theta_{12}^{BM} & = & 1/2 \\ \sin^2\theta_{23}^{BM} & = & 1/2 \\ \sin\theta_{13}^{BM} & = & 0 \end{array} \xrightarrow{\text{NLO}} \begin{array}{l} \sin^2\theta_{12} \simeq 1/2 + \mathcal{O}(v') \\ \sin^2\theta_{23} = 1/2 \\ \sin\theta_{13} \simeq \mathcal{O}(v') \end{array} \longrightarrow \boxed{v' \approx \mathcal{O}(\theta_C)}$$

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at **NNLO**: general corrections of $\mathcal{O}(v'^2) \longrightarrow \sin^2\theta_{23} \simeq 1/2 + \mathcal{O}(v'^2)$

$$\sin^2\theta_{12} \simeq 1/2 + \mathcal{O}(v')$$
$$\sin^2\theta_{23} = 1/2$$
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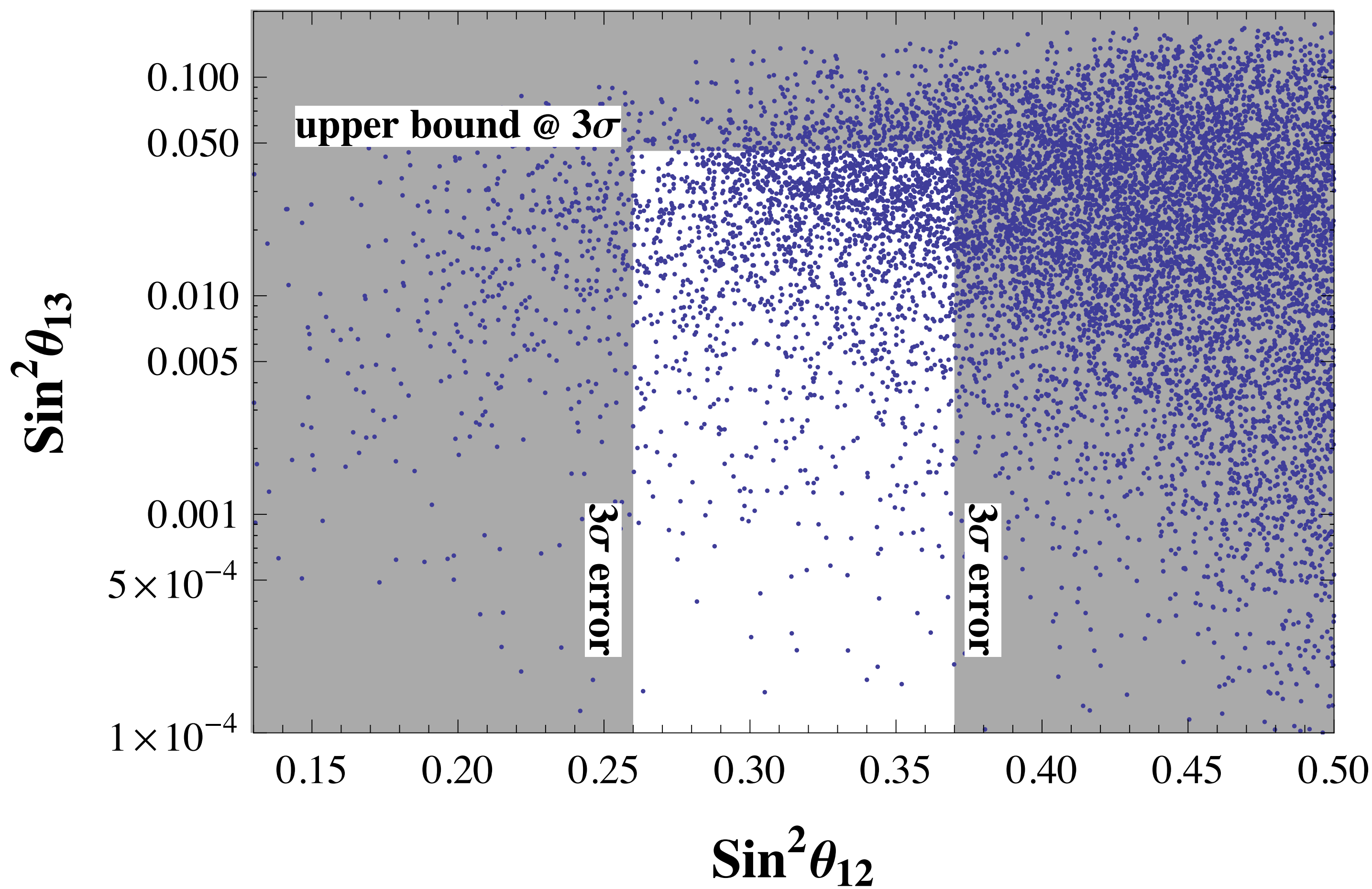
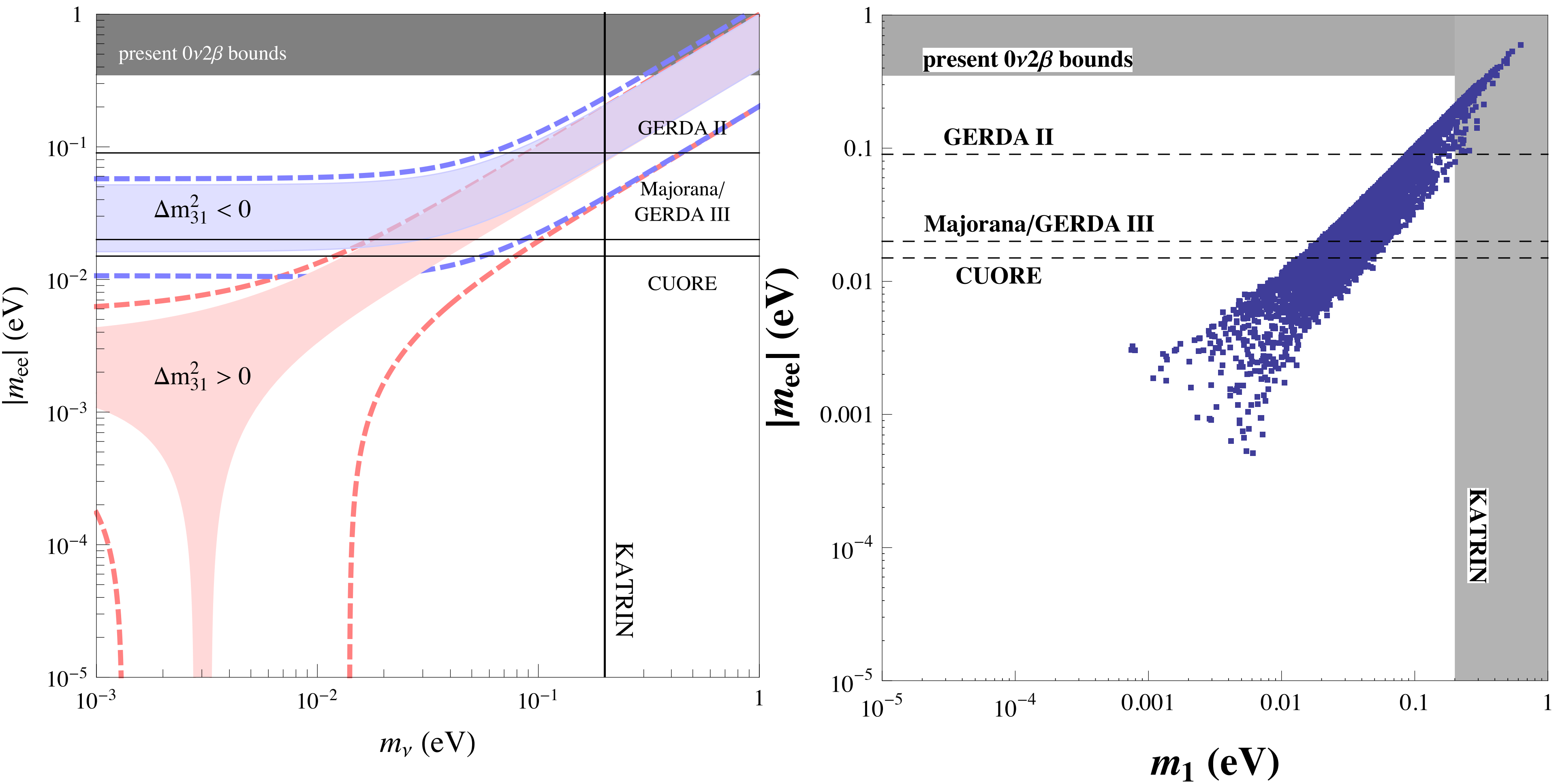
taking the parameters as random numbers in [0,2] and $v' = 0.15$





→ Normal ordering with moderate hierarchy or quasi degenerate spectrum

→ A lower bound of abour 0.1 meV on m_1 and on $|m_{ee}|$ is suggested

Extension to Quarks

Using the same setting as for leptons:

$$Q_L \sim \mathbf{3} \qquad U_R \sim \mathbf{1},\, \mathbf{1}' \qquad D_R \sim \mathbf{1},\, \mathbf{1}'$$

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$\Rightarrow$ More complicated setting!

Full Model in Pati-Salam

[de Adelhart Toorop, Bazzocchi & LM 2010]

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Complete Complementary Relations:

$$\left.\begin{aligned} \theta^\ell_{12} + \theta^q_{12} &\approx \pi/4 + \mathcal{O}(\theta_C^2) \\ \theta^\ell_{23} + \theta^q_{23} &\approx \pi/4 + \mathcal{O}(\theta_C^2) \end{aligned}\right\}$$

Common origin for CKM and PMNS matrices $\longrightarrow$ GUT's

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Our choice: <u>Pati-Salam</u> context keeping the previous flavour S4

all SM particles in two representations

$$F_L=\left(\begin{array}{ccc|c} u & u & u & \nu\\ d & d & d & e\end{array}\right)\qquad F^c=\left(\begin{array}{ccc|c} u^c & u^c & u^c & \nu^c\\ \hline d^c & d^c & d^c & e^c\end{array}\right)$$

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particular relations among the mass matrices

$$m_\ell=m_d\qquad\longrightarrow\qquad\boxed{U_e=V_d}$$

The PMNS matrix can be defined as:

$$U = R_{23}\left(-\frac{\pi}{4}\right) R_{13}(\lambda) R_{12}\left(\frac{\pi}{4}-\lambda\right) = \underbrace{\left(R_{23}\left(-\frac{\pi}{4}\right) R_{13}(\lambda) R_{12}(-\lambda)\right)}_{U_e^\dagger} \underbrace{R_{12}\left(\frac{\pi}{4}\right)}_{U_\nu}$$

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To recover the correct CKM we need

$$V_u = R_{23}\left(\frac{\pi}{4}\right) R_{13}(\lambda) R_{12}(\alpha\lambda)$$

$$\longrightarrow \quad V = \underbrace{\left(R_{12}(-\alpha\lambda) R_{13}(-\lambda) R_{23}\left(-\frac{\pi}{4}\right)\right)}_{V_u^\dagger} \underbrace{R_{23}\left(\frac{\pi}{4}\right) R_{13}(\lambda) R_{12}(\lambda))}_{V_d} \approx R_{12}(\lambda)$$

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The corrections will switch on the $R_{23}(\lambda^2)$ and the $R_{13}(\lambda^3)$

The model has many interesting predictions and postdictions:

- Bottom-tau unifications $|m_b| = |m_\tau|$
- Georgi-Jarlskog relation $|m_\mu| = 3|m_s|$
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- Neutrino masses with NH and IH spectrum
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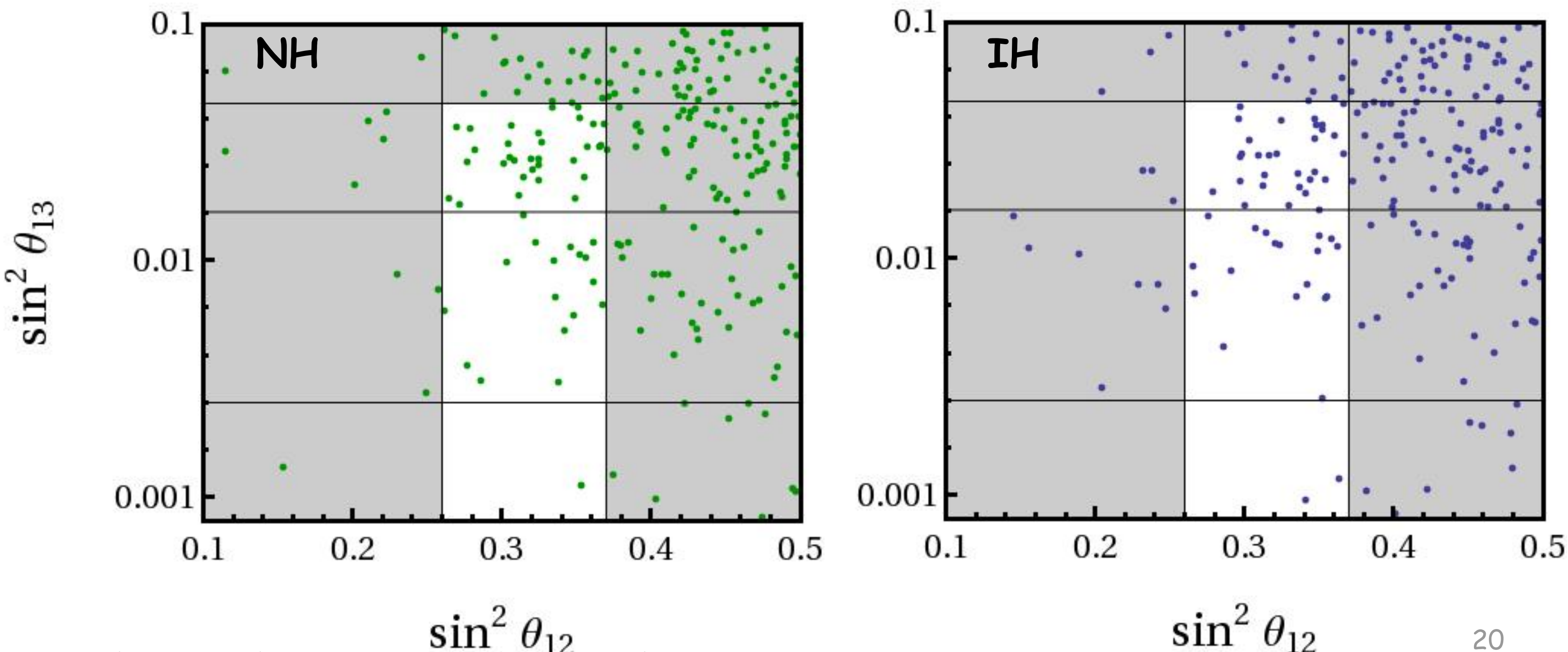

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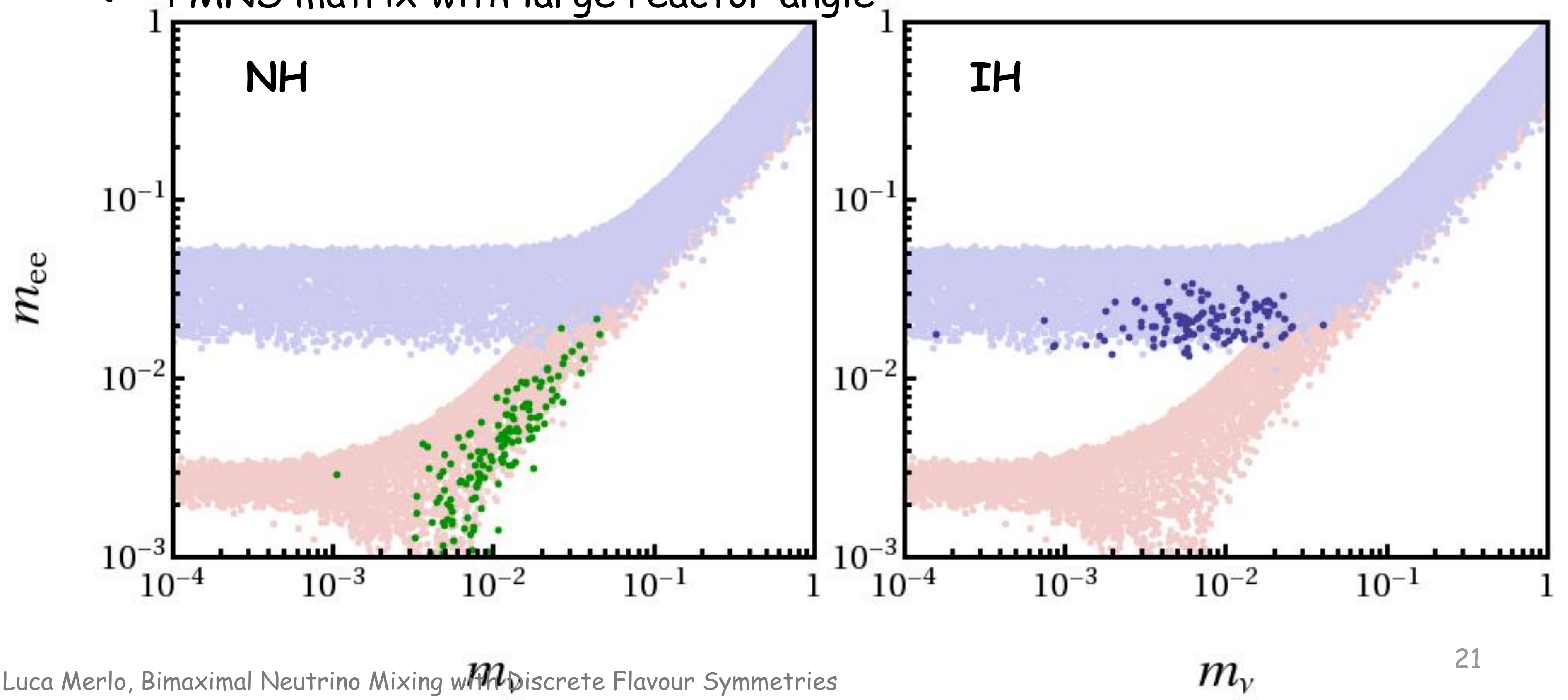

Conclusions

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- The reactor angle could be either **very small** or **close to its upper bound**

Tri-Bimaximal
with small corrections

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  - LO: exact neutrino BM mixing
  - NLO: corrections which correct the **solar** and the **reactor** angles **only**

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- Extension to the quark sector in the context of the Pati-Salam GUT

Thanks

Reactor Angle

[ISS Physics Working Group 2007]