UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

# *Fundamental Problems in Quantum Physics BELL*


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INFN
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*Consiglio di Sezione INFN*

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- **Involved units:**

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**Trento**

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**Physics Department** (open quantum systems)

Bassano Vacchini

Andrea Smirne

Nina Megier (AvH Fellow)

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**Mathematics Department** (field theory and general relativity)

Livio Pizzocchero

Davide Fermi (left, presently RTDa Roma 3)

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MILANO 1863

Mathematics Department (quantum probability)

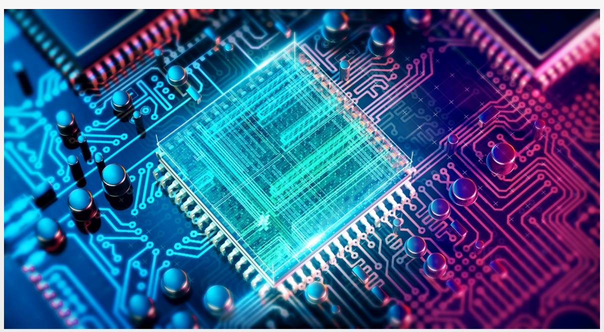
Alberto Barchielli

Alessandro Toigo

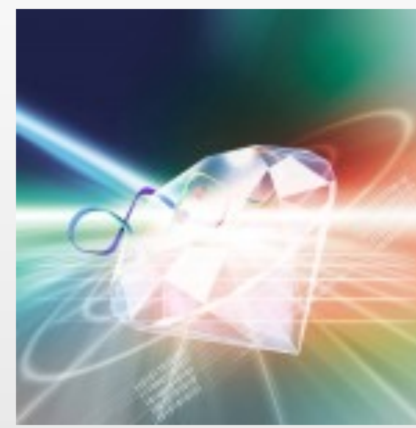
# Open quantum systems: from foundations to applications

- The systems we want to control and manipulate are subjected to interaction with the surrounding environment, i.e., they are open systems
- Quantum properties (such as entanglement and coherences) are particularly fragile under such an interaction

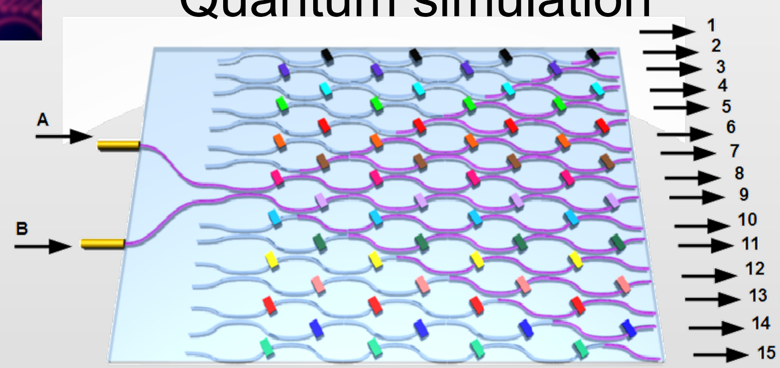
Quantum computation & communication



Quantum metrology and sensing



Quantum simulation

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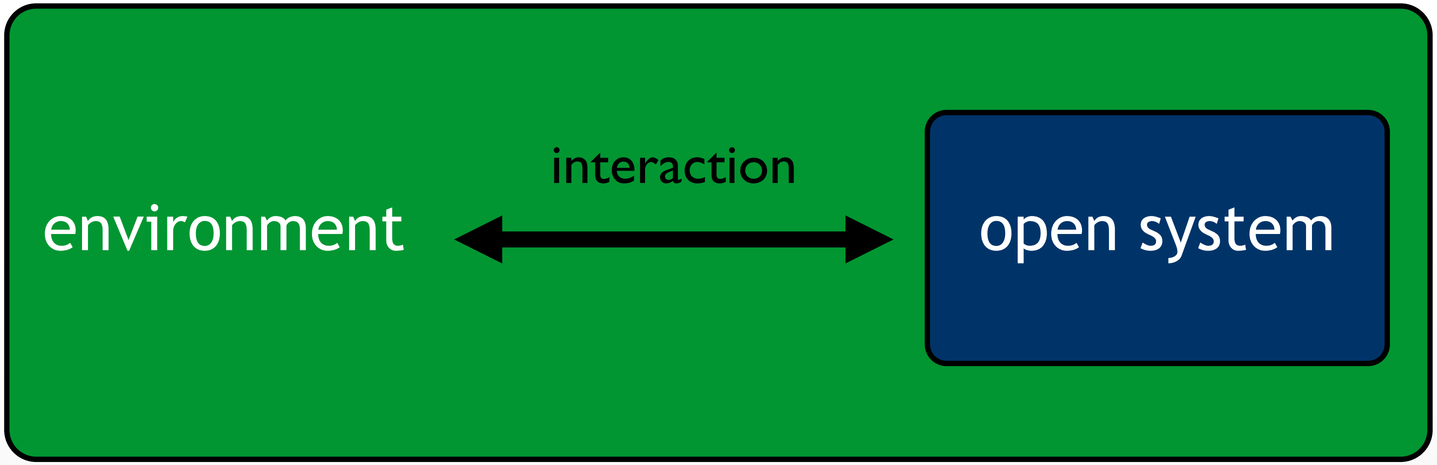
# Theoretical framework




## Bipartite setting

$$H = H_S + H_E + H_I$$

$$H \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_E) \qquad \rho_{SE} \in \mathcal{T}(\mathcal{H}_S \otimes \mathcal{H}_E)$$

**memory**

**Reduced dynamics**

$$\rho_S(0) \mapsto \rho_S(t) = \Phi(t)\rho_S(0)$$

**Correlations**

$$\rho_{SE}(t) \neq \rho_S(t) \otimes \rho_E(t)$$

[Davies, 1976; Alicki & Lendi, 1987; Breuer & Petruccione, 2002; Rivas & Huelga, 2012]

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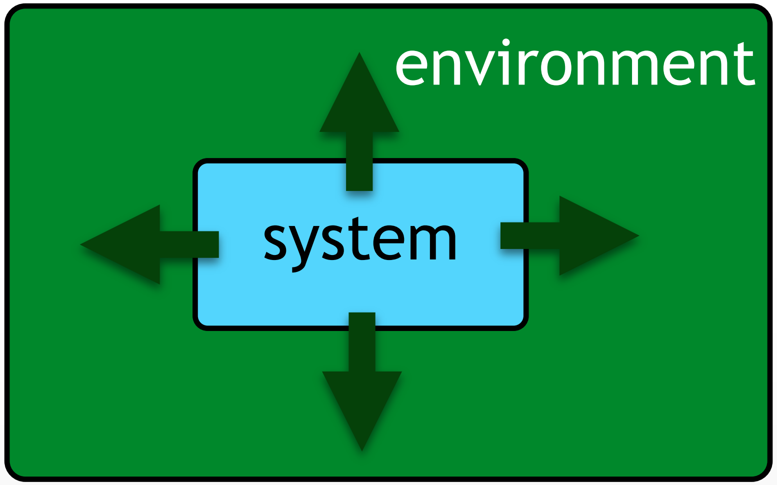
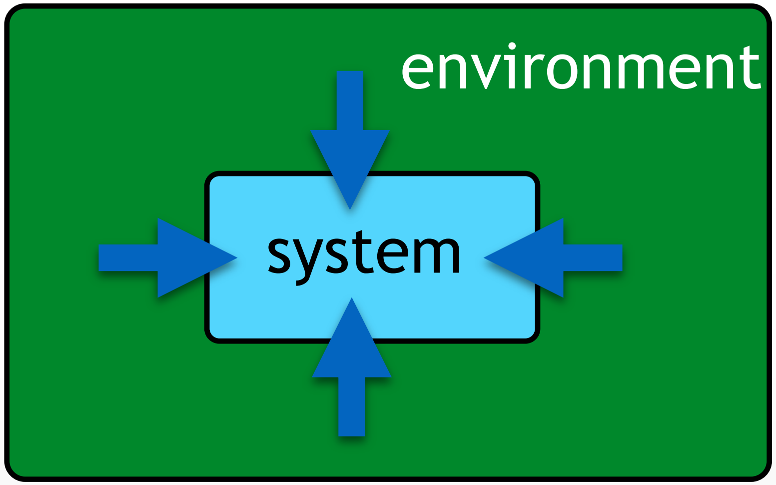
# Non-Markovianity – a physical picture




[Breuer, Laine & Piilo, PRL 2009; Breuer, Laine, Piilo & B.V. RMP 2016]

- Correlations imply bidirectional information flow: <u>memory effects</u>
- Quantitative foundation based on <u>entropic quantifiers</u>

  Megier, Smirne & Vacchini, arXiv 2101.02720 (2021), to appear in Phys. Rev. Lett.

- Variety of <u>analytical and numerical methods</u>

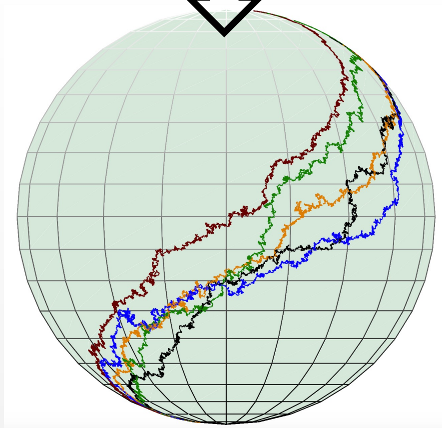
| | |
|---|---|
| Master equations | Path integrals |
| Numerical ab initio methods | Perturbative techniques |
| Stochastic methods | ....and many more! |

# Stochastic unraveling - Quantum jumps

$$\frac{d}{dt}\rho(t) = -i[H_S(t), \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left( L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t), \rho(t)\right\} \right)$$

Infinitely many possible mappings

<u>Stochastic trajectories</u> on the set of pure states

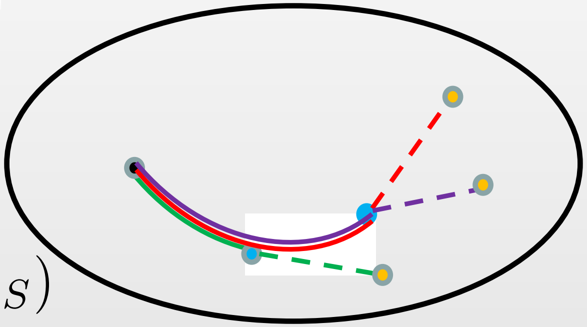


<u>Average</u> of the trajectories

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

- Deterministic evolution interrupted by random jumps

$\mathcal{T}(\mathcal{H}_S)$



- Quantum jumps experimentally observed in several platforms

Basche et al.Nature **373** (1995); Jelezko et al APL **81** (2002); Gleyzes et al Nature **446** (2007)

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# Rate operator quantum jumps

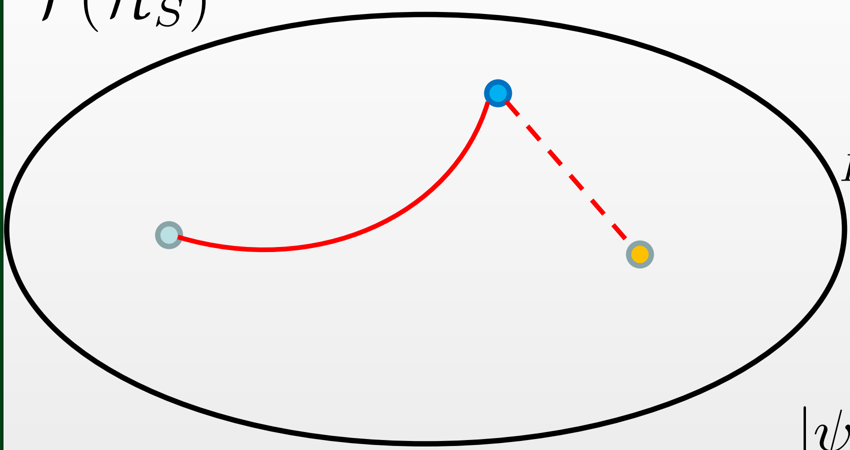
Rate Operator (RO)

$$W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha})|\psi(t)\rangle\langle\psi(t)|(L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$$

$$\ell_{\psi(t),\alpha} = \langle\psi(t)|L_\alpha(t)|\psi(t)\rangle$$

Smirne, Caiaffa & Piilo, Phys. Rev. Lett. **124**, 190402 (2020)

$\mathcal{T}(\mathcal{H}_S)$



- Deterministic evolution fixed by

$$H_{eff} = H_S - \frac{i}{2}\sum_{\alpha=1}^{n^2-1} c_\alpha \left(L^\dagger_\alpha L_\alpha - 2\ell^*_{\psi(t),\alpha} L_\alpha + |\ell_{\psi(t),\alpha}|^2\right)$$

- Interrupted by jumps

$$|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle \qquad p_j(t) = \lambda_{\psi(t),j}dt$$

RO eigenvectors and eigenvalues

➢ Positive probabilities guaranteed by a **positive RO**

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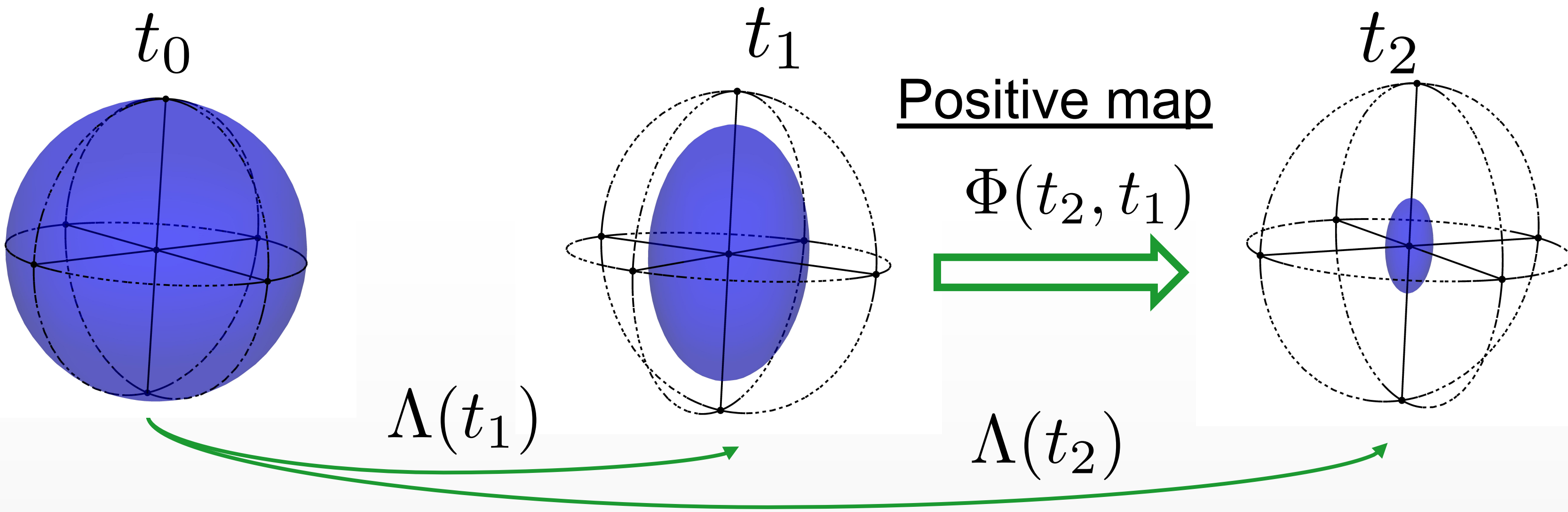
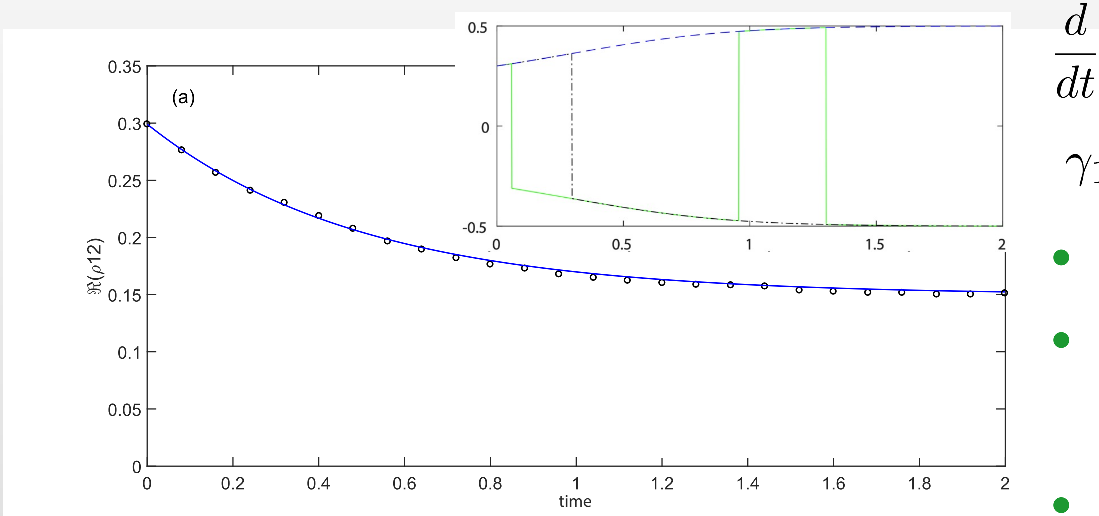
# Divisible dynamics




- Negative coefficients included – standard jumps do not apply




$$\frac{d}{dt}\rho(t) = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)\left[\sigma_k\rho(t)\sigma_k - \rho(t)\right]$$

$$\gamma_1 = \gamma_2 = 1 \qquad \underline{\gamma_3 = -\tanh t}$$

- Solid line: exact solution
- Circles: average over $10^4$ trajectories
- Inset: example trajectories

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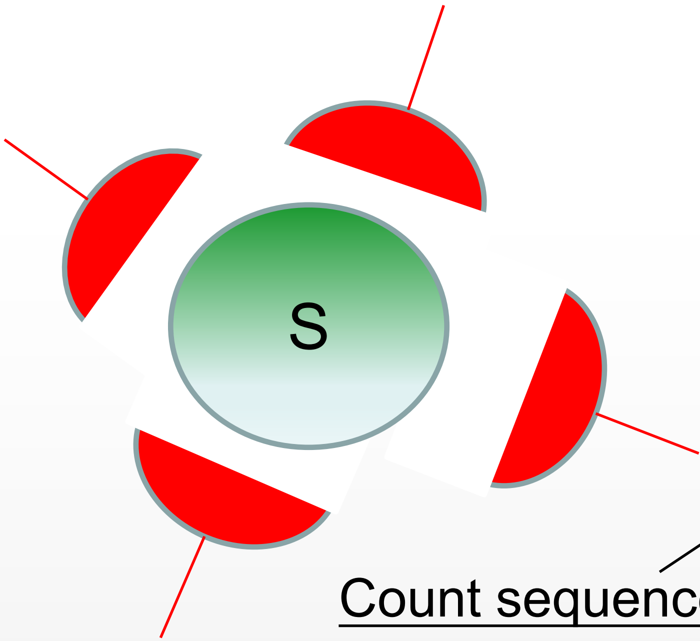
# Continuous measurement framework




The system is continuously monitored by *n* detectors: every dt two kinds of events

- No detector clicks: *null count* $\emptyset$
- The j-th detector clicks

$$\omega_t = (t_1, j_1; t_2, j_2; \dots t_m, j_m)$$

Count sequence: Instants and types of the (non-null) counts

It fixes the system state $|\psi(\omega_t)\rangle$

- Transformations and probabilities by postulates of quantum mechanics

$$\rho \mapsto \frac{\mathcal{I}_{\omega_t, j(\emptyset)}\rho}{\mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\right\}} \qquad p_{j(\emptyset)}(t) = \mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\right\}$$

If we identify the count sequence with the sequence of jumps, we get the same trajectories and associated probabilities as the unraveling!!

# Extension to fully general non-Markovian dynamics

$$W^J_{\psi(t)} = \sum_{j+} \lambda_{\psi(t),j+} |\varphi_{\psi(t),j+}\rangle\langle\varphi_{\psi(t),j+}| - \sum_{j^-} \left|\lambda_{\psi(t),j-}\right| |\varphi_{\psi(t),j-}\rangle\langle\varphi_{\psi(t),j-}|$$

Positive eigenvalues: as before $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j^+}\rangle \quad p_{j^+}(t) = \lambda_{\psi(t),j^+} dt$

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# Extension to fully general non-Markovian dynamics

$$W^J_{\psi(t)} = \sum_{j+} \lambda_{\psi(t),j+} |\varphi_{\psi(t),j+}\rangle\langle\varphi_{\psi(t),j+}| - \sum_{j-} |\lambda_{\psi(t),j-}|\, |\varphi_{\psi(t),j-}\rangle\langle\varphi_{\psi(t),j-}|$$

Positive eigenvalues: as before $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j+}\rangle \quad p_{j^+}(t) = \lambda_{\psi(t),j^+} dt$

Negative eigenvalues: we define **reversed jumps**, to avoid $p_{j^-}(t) < 0$

$$|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t),j-}\rangle \mapsto |\psi_{k'}(t)\rangle \quad p^{(k\to k')}_{j^-}(t) = \frac{N_{k'}(t)}{N_k(t)} |\lambda_{\psi_{k'}(t),j-}|\, dt$$

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# Extension to fully general non-Markovian dynamics

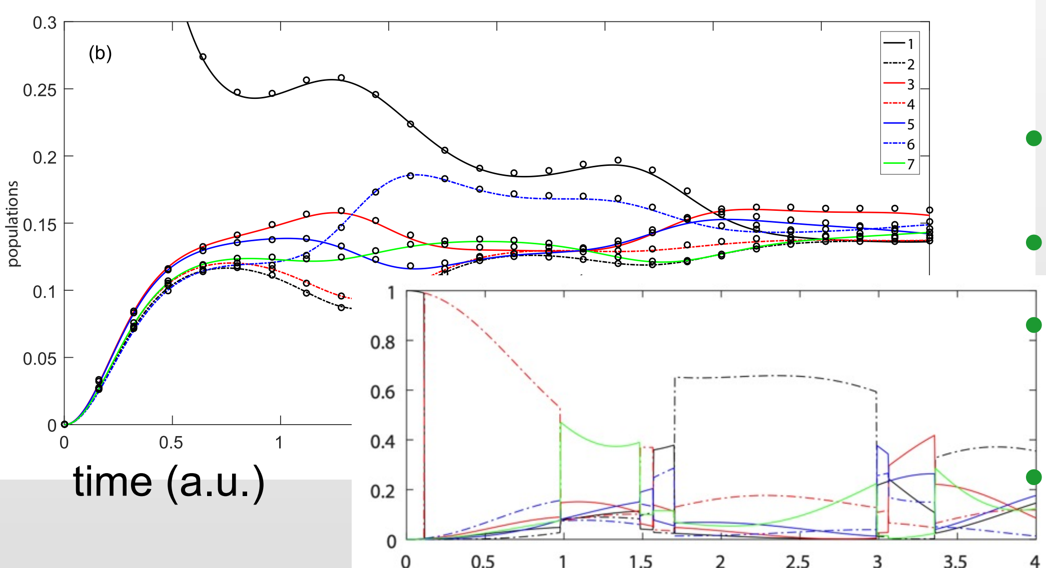
$$W^J_{\psi(t)} = \sum_{j^+} \lambda_{\psi(t),j^+} |\varphi_{\psi(t),j^+}\rangle\langle\varphi_{\psi(t),j^+}| - \sum_{j^-} |\lambda_{\psi(t),j^-}| \, |\varphi_{\psi(t),j^-}\rangle\langle\varphi_{\psi(t),j^-}|$$

Positive eigenvalues: as before $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j^+}\rangle \quad p_{j^+}(t) = \lambda_{\psi(t),j^+} dt$

Negative eigenvalues: we define **reversed jumps**, to avoid $p_{j^-}(t) < 0$

$\lambda_{\psi_1,j} > 0$: $|\psi_1\rangle \Downarrow |\psi_2\rangle = |\varphi_{\psi_1,j}\rangle$

$\lambda_{\psi_1,j} < 0$: $|\psi_1\rangle \Uparrow |\psi_2\rangle = |\varphi_{\psi_1,j}\rangle$




- 7 coupled sites: transport,..
- Different site populations
- Trajectory of populations with reversed jumps
- average $3 \times 10^4$ trajectories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# Collaborations

H.-P. Breuer - Uni Freiburg - Germany

M. Paternostro - Uni Belfast - United Kingdom

S. Campbell - University College Dublin - Ireland

F. Ciccarello, S. Lorenzo & G. Palma - Uni Palermo

J. Piilo & K. Luoma - Uni Turku - Finland

D. Chruscinski - Uni Torun - Poland

S. Huelga -Uni Ulm - Germany

# Publications of Milan unit (2019-2021)

| Authors | Article Title | Source Title |
|---|---|---|
| Smirne, A; Megier, N; Vacchini, B | On the connection between microscopic description and memory effects in open quantum syst | QUANTUM |
| Megier, N; Smirne, A; Vacchini, B | The interplay between local and non-local master equations: exact and approximated dynamics | NEW JOURNAL OF PHYSICS |
| Megier, N; Smirne, A; Vacchini, B | Evolution Equations for Quantum Semi-Markov Dynamics | ENTROPY |
| Pizzocchero, L | On the global stability of smooth solutions of the Navier-Stokes equations | APPLIED MATHEMATICS LETTERS |
| Milz, S; Egloff, D; Taranto, P; Theurer, | When Is a Non-Markovian Quantum Process Classical? | PHYSICAL REVIEW X |
| Fermi, D; Gengo, M; Pizzocchero, L | Integrable scalar cosmologies with matter and curvature | NUCLEAR PHYSICS B |
| Puebla, R; Smirne, A; Huelga, SF; Ple | Universal Anti-Kibble-Zurek Scaling in Fully Connected Systems | PHYSICAL REVIEW LETTERS |
| Carmeli, C; Heinosaari, T; Toigo, A | Quantum random access codes and incompatibility of measurements | EPL |
| Pizzocchero, L; Tassi, E | On approximate solutions of the equations of incompressible magnetohydrodynamics | NONLINEAR ANALYSIS-THEORY METHODS & |
| Cremona, F; Pizzocchero, L; Sarbach, | Gauge-invariant spherical linear perturbations of wormholes in Einstein gravity minimally couple | PHYSICAL REVIEW D |
| Hashimoto, K; Vacchini, B; Uchiyama, | Lower bounds for the mean dissipated heat in an open quantum system | PHYSICAL REVIEW A |
| Smirne, A; Caiaffa, M; Piilo, J | Rate Operator Unraveling for Open Quantum System Dynamics | PHYSICAL REVIEW LETTERS |
| Barchielli, A; Gregoratti, M | Entropic measurement uncertainty relations for all the infinite components of a spin vector | JOURNAL OF PHYSICS COMMUNICATIONS |
| Diaz, MG; Desef, B; Rosati, M; Egloff, | Accessible coherence in open quantum system dynamics | QUANTUM |
| Vacchini, B | Quantum renewal processes | SCIENTIFIC REPORTS |
| Carmeli, C; Heinosaari, T; Miyadera, T | Witnessing incompatibility of quantum channels | JOURNAL OF MATHEMATICAL PHYSICS |
| Cialdi, S; Benedetti, C; Tamascelli, D; | Experimental investigation of the effect of classical noise on quantum non-Markovian dynamics | PHYSICAL REVIEW A |
| Carmeli, C; Heinosaari, T; Miyadera, T | Noise-Disturbance Relation and the Galois Connection of Quantum Measurements | FOUNDATIONS OF PHYSICS |
| Carmeli, C; Cassinelli, G; Toigo, A | Constructing Extremal Compatible Quantum Observables by Means of Two Mutually Unbiased | FOUNDATIONS OF PHYSICS |
| Campbell, S; Popovic, M; Tamascelli, | Precursors of non-Markovianity | NEW JOURNAL OF PHYSICS |
| Campbell, S; Cakmak, B; Mustecaplio | Collisional unfolding of quantum Darwinism | PHYSICAL REVIEW A |
| Carmeli, C; Heinosaari, T; Toigo, A | Quantum Incompatibility Witnesses | PHYSICAL REVIEW LETTERS |
| Amato, G; Breuer, HP; Vacchini, B | Microscopic modeling of general time-dependent quantum Markov processes | PHYSICAL REVIEW A |
| Fermi, D; Gengo, M; Pizzocchero, L | On the Necessity of Phantom Fields for Solving the Horizon Problem in Scalar Cosmologies | UNIVERSE |
| Cakmak, B; Campbell, S; Vacchini, B; | Robust multipartite entanglement generation via a collision model | PHYSICAL REVIEW A |
| Cremona, F; Pirotta, F; Pizzocchero, L | On the linear instability of the Ellis-Bronnikov-Morris-Thorne wormhole | GENERAL RELATIVITY AND GRAVITATION |

# Spare slide: definition of the quantum instrument

Quantum instrument: map from the set of outcomes to CP maps on $S(\mathcal{H}_S)$

$$\mathcal{I}_{\omega_t,\emptyset}\rho = F_{\omega_t,\emptyset}\rho F^{\dagger}_{\omega_t,\emptyset} \qquad F_{\omega_t,\emptyset} = \left(Id - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\langle\psi(\omega_t)|$$

$$\mathcal{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^{\dagger}_{\omega_t,j}dt \qquad V_{\omega_t,j} = \sqrt{\lambda_{\psi(\omega_t),j}}|\varphi_{\psi(\omega_t),j}\rangle\langle\psi(\omega_t)|$$

- When applied to the state $|\psi(\omega_t)\rangle\langle\psi(\omega_t)|$ fixed by the count sequence $\omega_t$

$$\mathcal{I}_{\omega_t,j}|\psi(\omega_t)\rangle\langle\psi(\omega_t)| = V_{\omega_t,j}|\psi(\omega_t)\rangle\langle\psi(\omega_t)|V^{\dagger}_{\omega_t,j}dt \begin{cases} |\psi(\omega_t)\rangle\langle\psi(\omega_t)| \mapsto |\varphi_{\psi(\omega_t),j}\rangle\langle\varphi_{\psi(\omega_t),j}| \\ p_j(t) = \lambda_{\psi(\omega_t),j}dt \end{cases}$$

$$\mathcal{I}_{\omega_t,\emptyset}|\psi(\omega_t)\rangle\langle\psi(\omega_t)| = F_{\omega_t,\emptyset}|\psi(\omega_t)\rangle\langle\psi(\omega_t)|F^{\dagger}_{\omega_t,\emptyset} \begin{cases} |\psi(\omega_t)\rangle\langle\psi(\omega_t)| \mapsto \frac{1}{P(t)}\left(1 - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\langle\psi(\omega_t)|\left(1 + iH^{eff,\dagger}_{\omega_t}dt\right) \\ P(t) = 1 - \sum_{j=1}^{n} p_j(t) \end{cases}$$