Supernova neutrinos and oscillations

Where we are and where we need to go

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SNνD 2023

Neutrinos in Supernova

- Neutrinos are incredibly important in core-collapse supernovae.
  - A great deal of attention and effort is paid to how the neutrinos move energy / momentum / lepton number around – the transport.
  - Changing any property / interaction of the neutrinos often changes how the star explodes.

- Neutrinos are also the messengers which can tell us how the supernova explodes.
  - In 1987 we detected 20 neutrinos from a SN in the LMC which confirmed the basic paradigm that the core implodes.
  - If a SN occurs tomorrow in the Milky Way we will detect 10’s of thousands of neutrinos and be able to answer detailed questions.

Nuclear Physics / Astrophysics

- Progenitor and structure,
- Neutrino interactions with matter,
- Equation of State,
- Shock position / velocity,
- The SASI,
- The LESA,
- Nucleosynthesis conditions,
- ….

Neutrinos

- Neutrino mass ordering
- Number of ν flavors
- Self-interaction effects,
- MSW effects,
- Turbulence effects
- Non-standard interactions,
- Magnetic moments,
- ….

- SN neutrinos are fantastic probes of extreme physics.

Neutrino transport

- Using the mean field approximation, the neutrino distribution matrix *F* evolves according to the Quantum Kinetic Equations (QKEs)

$$\frac{\partial F}{\partial t} + \vec{v} \cdot \nabla F = -i[H, F] + C[F]$$

- *H* is the Hamiltonian, *C* the collision term

Sigl & Raffelt, Nuclear Physics B **406**, 423 (1993)

Volpe, Väänänen & Espinoza, PRD **87**, 113010 (2013)

Vlasenko, Fuller & Cirigliano, PRD **89** 105004 (2014)

Cirigliano, Fuller & Vlasenko, Physics Letters B **747**, 27 (2015)

Richers *et al*, PRD **99** 123014 (2019)

- The diagonal elements of *F* are the occupation numbers of the neutrino flavors, the off-diagonal are the coherences.
  - Classical supernova neutrino transport only follows the diagonal elements

- The neutrino Hamiltonian is made up of three terms:
  - the vacuum H_V term,
  - the matter potential H_M,
  - the self-interaction H_{SI},

- The vacuum term is

$$H_V = q + \frac{1}{2q} U_V \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_V^\dagger$$

  - q is the neutrino energy, m_1, m_2 and m_3 are the neutrino masses.
  - U_V is the mixing matrix parameterized by three mixing angles θ_{12}, θ_{13} and θ_{23} and a CP phase δ.
  - For antineutrinos $H_V \rightarrow H_V^*$

- In the presence of matter the neutrinos gain a potential energy.
  - For mixing between active flavors we only need consider the Charged Current potential.
- The matter Hamiltonian is

$$H_M = \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad V_{CC} = \sqrt{2}\, G_F\, n_e$$

  - For antineutrinos $H_M \rightarrow -H_M^*$.

- Beyond the Standard Model physics can modify the matter term.

Esteban-Pretel *et al*, PRD **81** 063003 (2010)

Stapleford *et al*, PRD **94** 093007 (2016)

- So many neutrinos are emitted in a supernova the Hamiltonian includes a term due to neutrino self-interactions.

- At a given location and time, the self-interaction Hamiltonian for left-handed neutrinos due to other left-handed neutrinos / right-handed antineutrinos is

$$H_{SI}(\hat{\boldsymbol{q}}) = \sqrt{2}\, G_F \int \frac{d^3 q'}{(2\pi)^3} \left(1 - \hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{q}}'\right) \left(F(\boldsymbol{q}') - \bar{F}^*(\boldsymbol{q}')\right)$$

  - For an antineutrino $H_{SI} \rightarrow -H_{SI}^*$.

- Beyond the Standard Model physics can modify this term too.

Blennow, Mirizzi & Serpico, PRD **78**, 113004 (2008)

Das, Dighe & Sen, JCAP **5** 051 (2017)

Yang & Kneller, PRD **97** 103018 (2018)

- There is a huge range of scales in the SN neutrino system.
  - The oscillation wavelength of neutrinos at the neutrinosphere is of order 10 microns: the core region of the supernova is ~few hundred km.

- Up until recently, all global calculations of flavor transformation in SN post-process ‘classical’ simulation data.

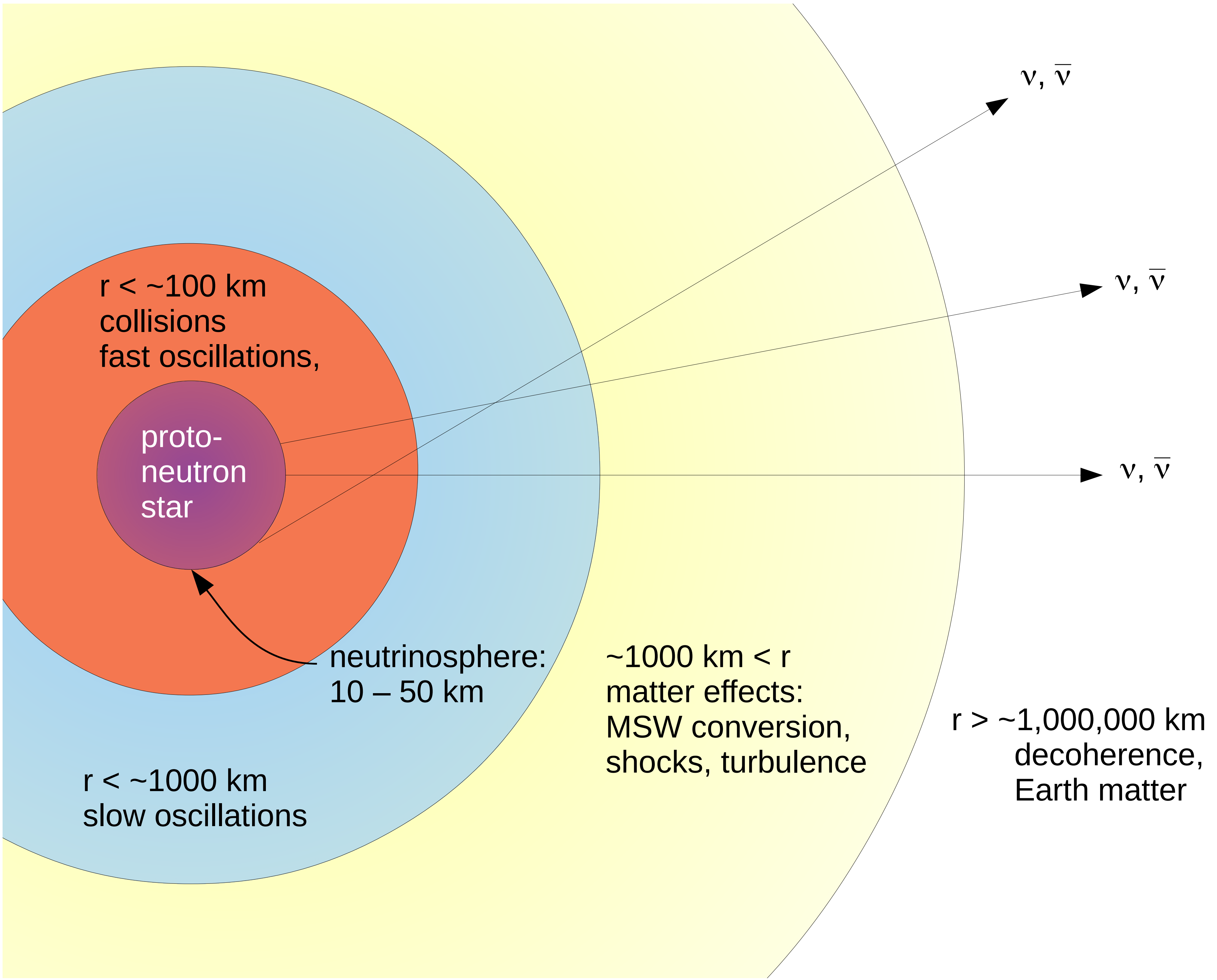
see Stapleford et al, PRD, **102,** 081301 (2020) and

Xiong et al PRD **107** 083016 (2023) for two exceptions

SN Oscilaltion Calculations

- Oscillation calculations can be divided into ‘local’ and ‘global’.
  - Local calculations consider small volumes (~1000 cm^3) and assume homogeneity.
  - Global calculations don’t assume homogeneity but make other assumptions / approximations (e.g. spherical symmetry, steady state)

- Most global calculations are based on the Bulb Model which assumes the emission occurs from a hard neutrinosphere.
- What has been found is that the flavor transformation occurs in several places due to different reasons.



ν, $\bar{\nu}$
ν, $\bar{\nu}$
ν, $\bar{\nu}$
r < ~100 km
collisions
fast oscillations,
proto-
neutron
star
neutrinosphere:
10 – 50 km
~1000 km < r
matter effects:
MSW conversion,
shocks, turbulence
r > ~1,000,000 km
decoherence,
Earth matter
r < ~1000 km
slow oscillations

- We don’t expect all flavor transformation ‘processes’ to be ‘active’ at all epochs of the neutrino emission.

- Of the various ‘processes’ that modify the neutrino spectra
  - Earth matter, decoherence and the dynamic MSW effect are well understood,
  - The theory for turbulence effects is known but we don’t know how much turbulence is present in a SN,
  - Self-interactions – especially fast oscillations and collisionally induced transformations – are active research.

Matter effects – dynamic MSW

- Beyond ~1000 km, the flavor transformation is due to matter.

Dighe & Smirnov, PRD **62** 033007 (2000)

Schirato & Fuller, arXiv:astro-ph/0205390

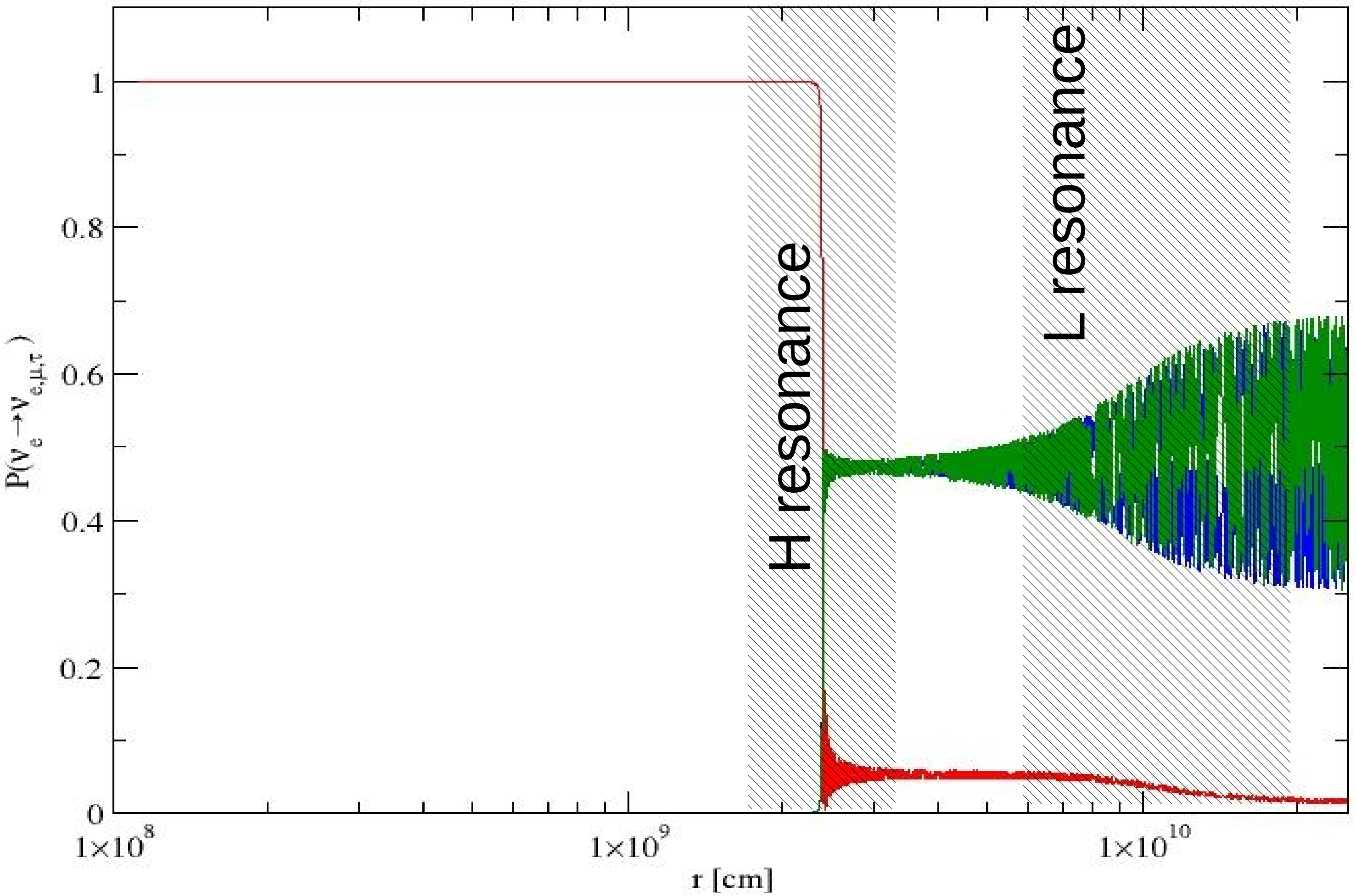
Fogli et al, PRD **68** 033005 (2003)

Fogli et al, JCAP **6** 012 (2006)

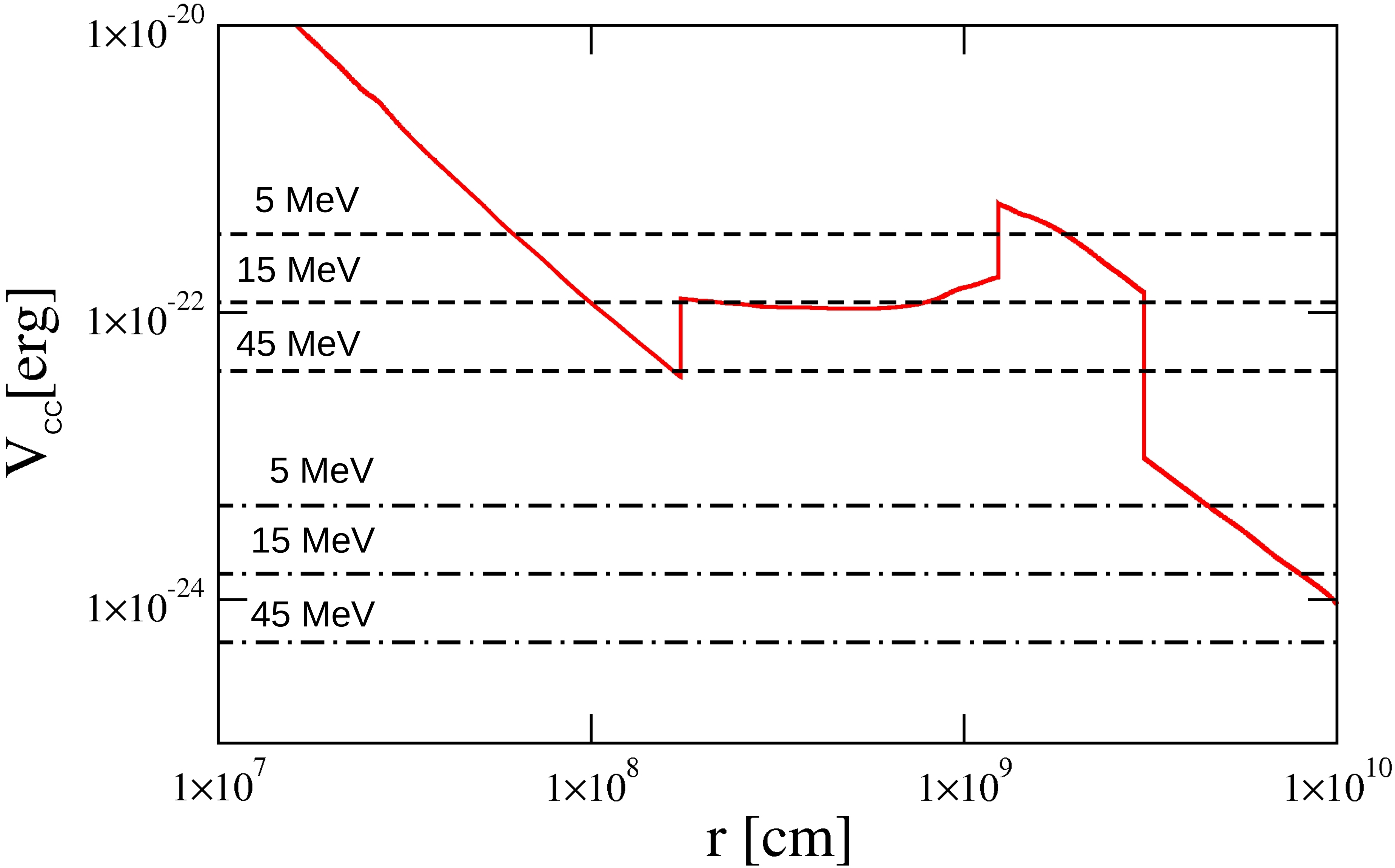
Kneller, McLaughlin & Brockman, PRD **77** 045023 (2008)

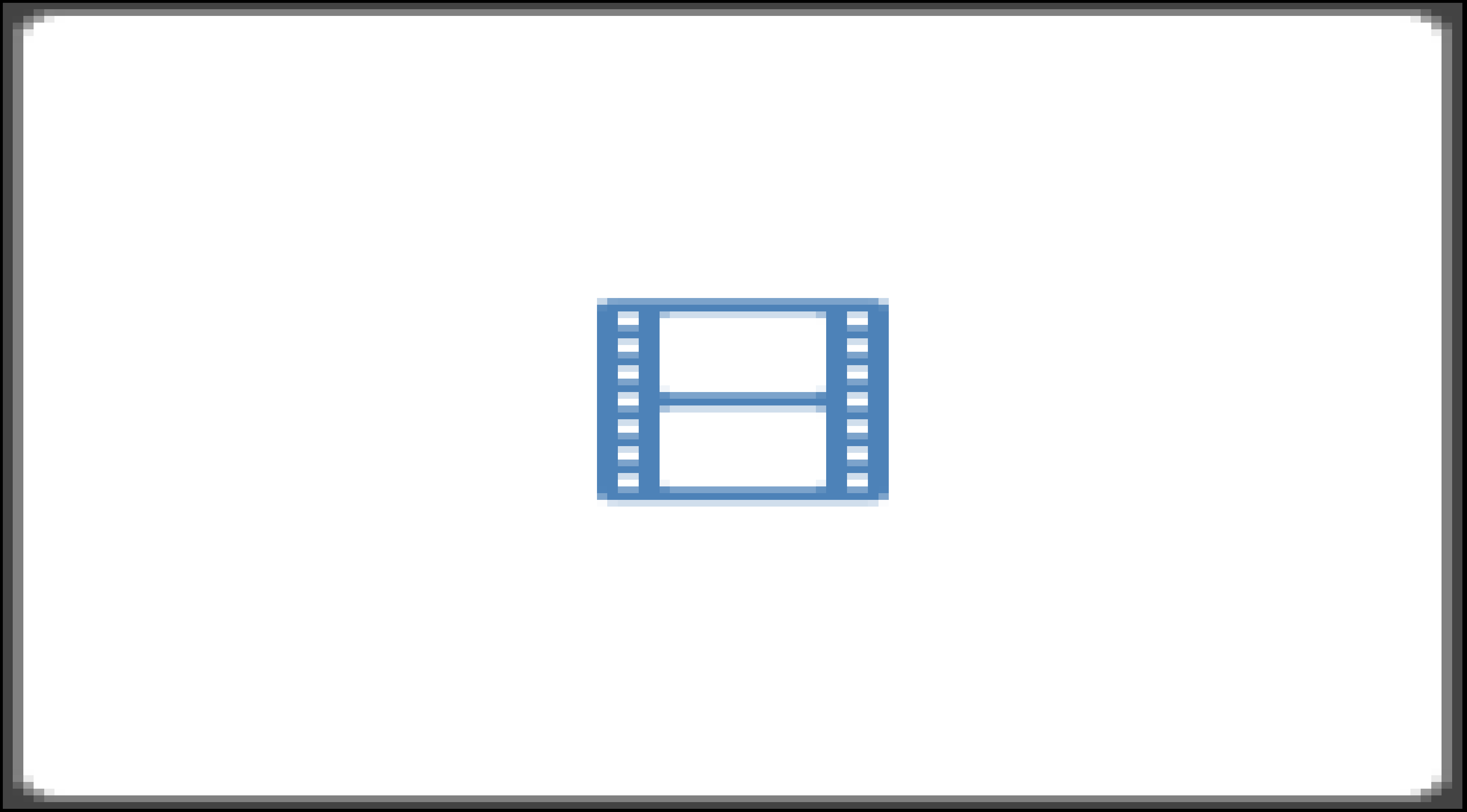
Lund & Kneller, PRD **88** 023008 (2013)

- Initially (up to ~1 second postbounce, sometimes longer) the flavor conversion is due to the adiabatic MSW effect.



- When the shock reaches the MSW resonance densities it modifies the adiabaticity of the flavor evolution.
- E.g. the MSW potential (with steepened shocks) t = 3 s using the $M = 10.8\ M_\odot$ model of Fischer et al, A&A, **517A**, 18F (2010).





Matter effects – turbulence

Patton, Kneller & McLaughlin, PRD **89** 073022 (2014)

Borriello et al JCAP **11** 030 (2014)

Capozzi *et al*, JCAP **4** 43 (2016)

Yang & Kneller, PRD **89** 073022 (2014)

- The presence of turbulence – random density fluctuations - produces another matter effect.
- Consider a 'smooth' density profile to which we add turbulence

$$\rho(r) = (1 + C(r))\langle \rho(r) \rangle$$

- where *C(r)* is a Gaussian random field.
- Realizations of *C* are constructed with a Fourier series.

$$C(r) \propto \sum_{n=1}^{N_q} \{A_n \cos(q_n r) + B_n \sin(q_n r)\}$$

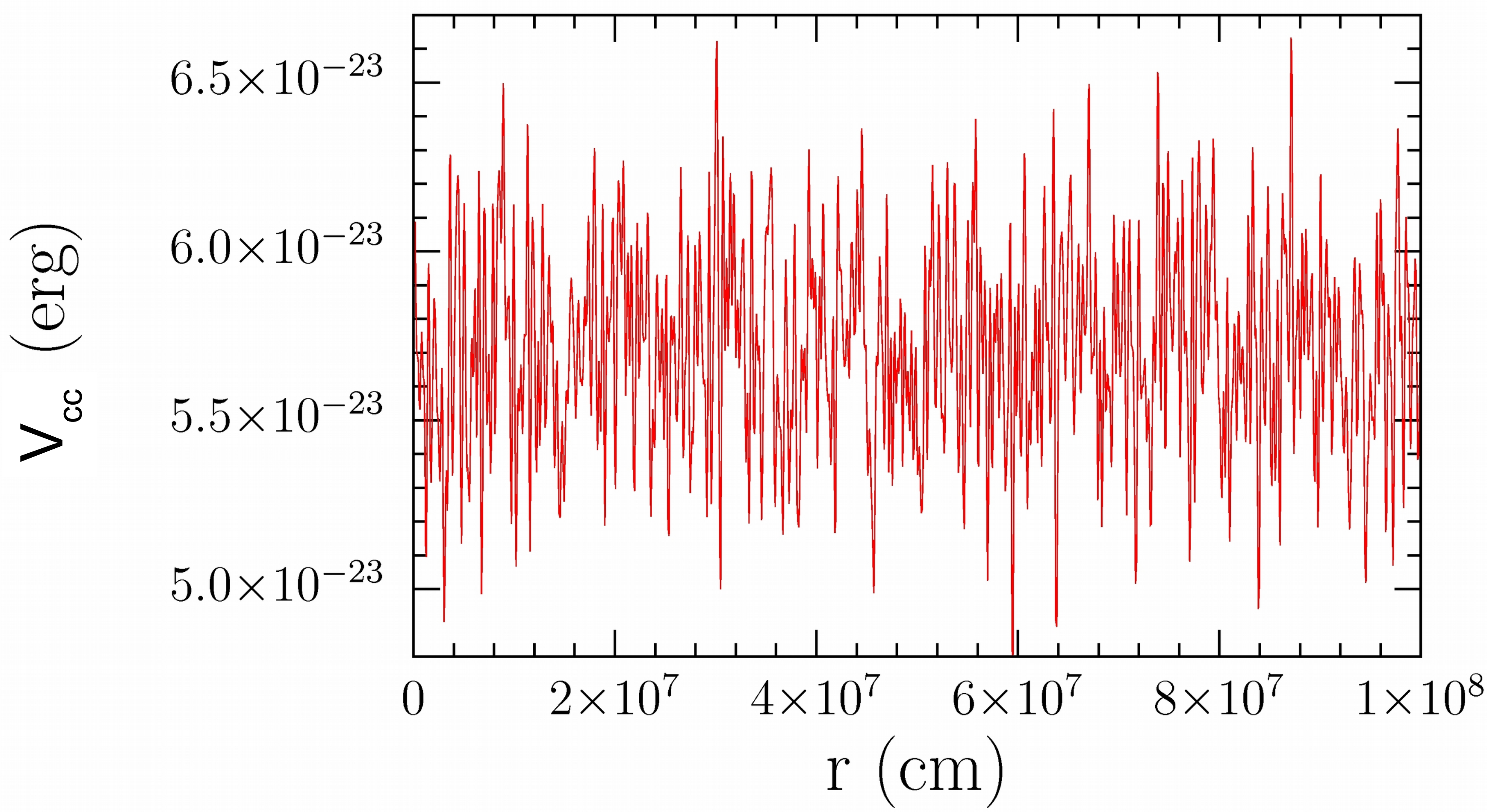
- Crunching through the math we eventually find the neutrino behaves like an illuminated atom.
  - Transitions between the eigenstates of the smooth underlying Hamiltonian are driven by the the Fourier modes which have frequencies that match the eigenvalue splitting.

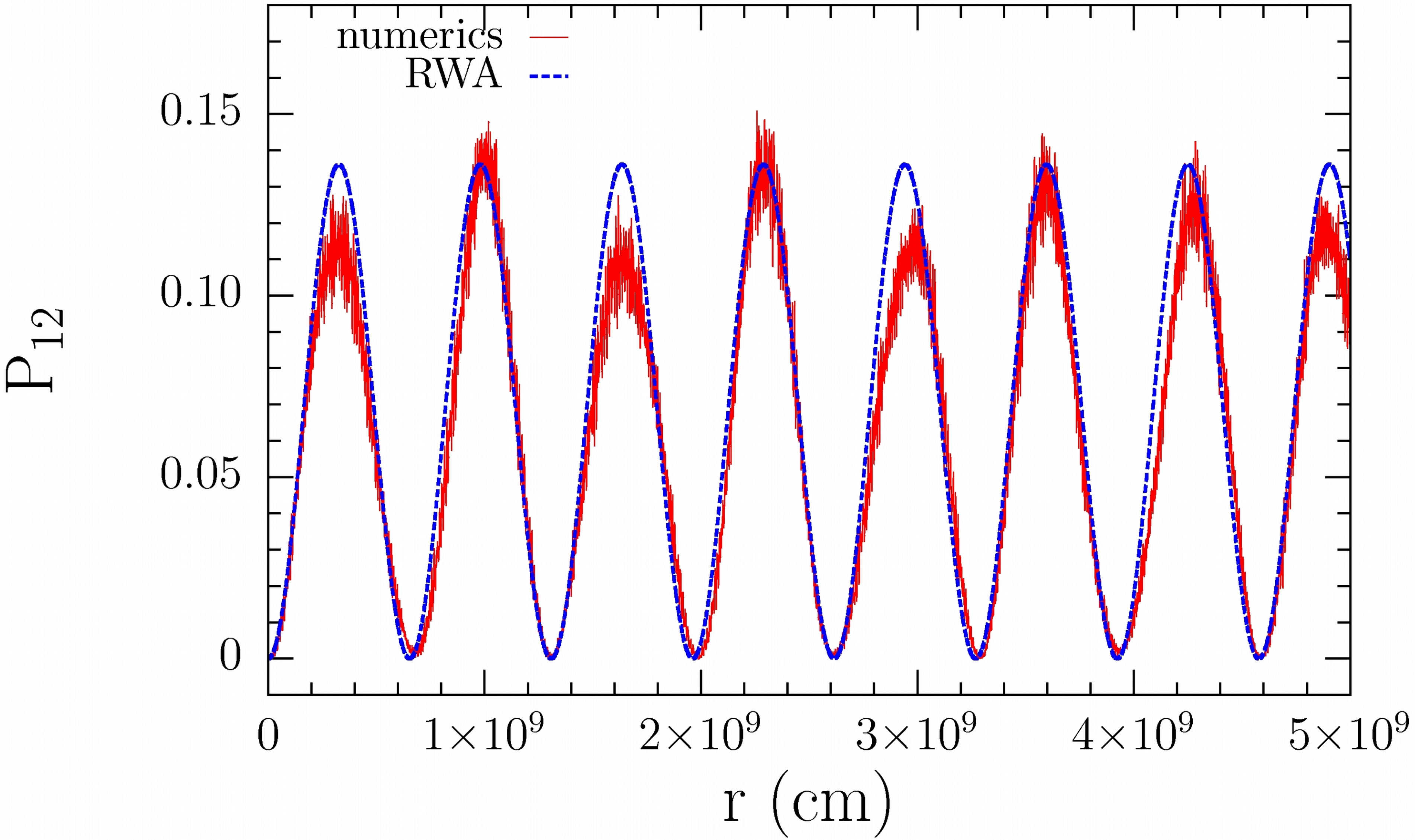
- Using the Rotating Wave Approximation, it is possible to derive an analytic solution.
- For two flavors the solution is particularly simple:

$$P_{12} = \frac{\kappa^2}{Q^2} \sin^2(Q r)$$

  - The quantities κ and Q are functions of the amplitudes {A}, {B} and wavenumbers {q} of the Fourier modes.

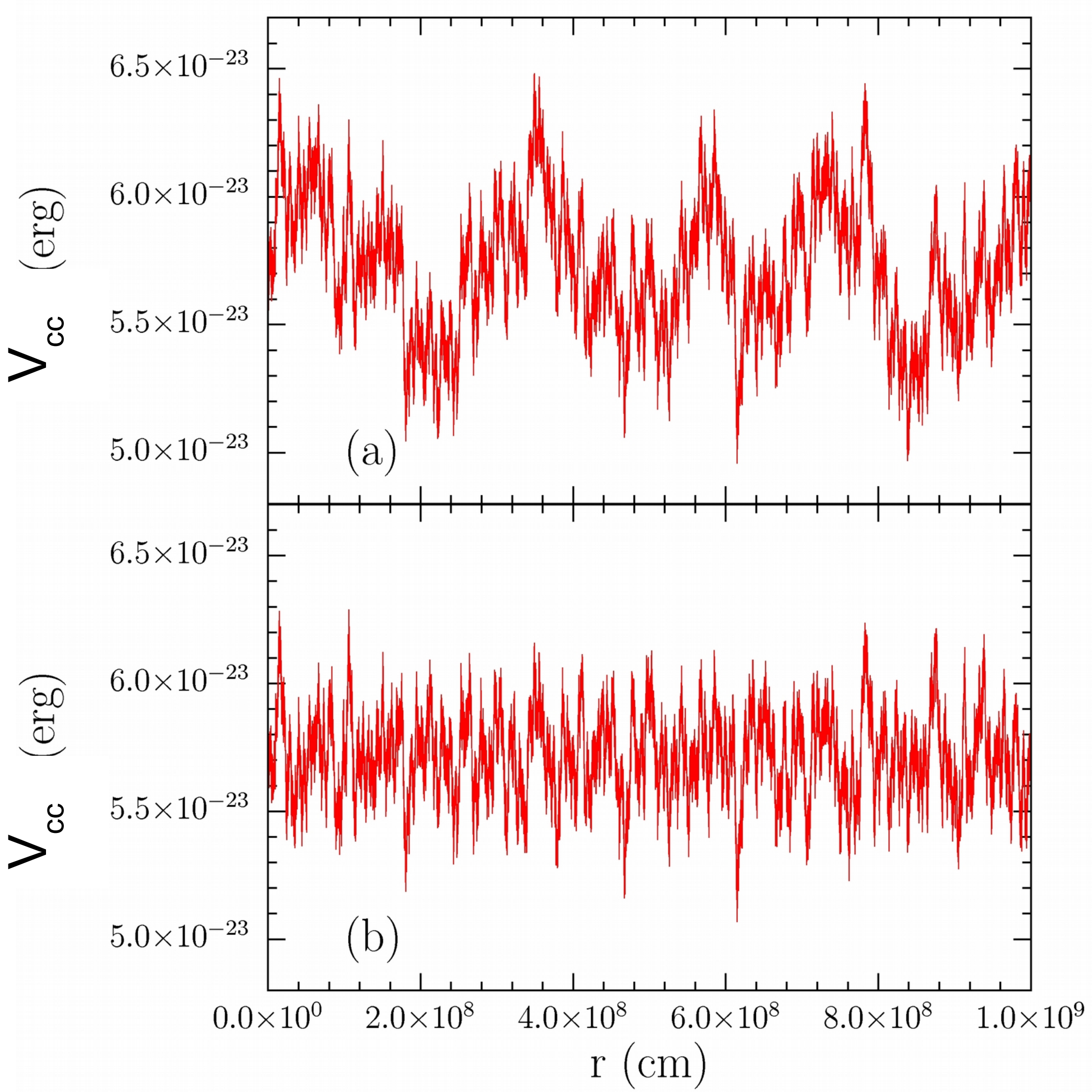
- E.g. a realization of turbulence created using 50 Fourier modes.





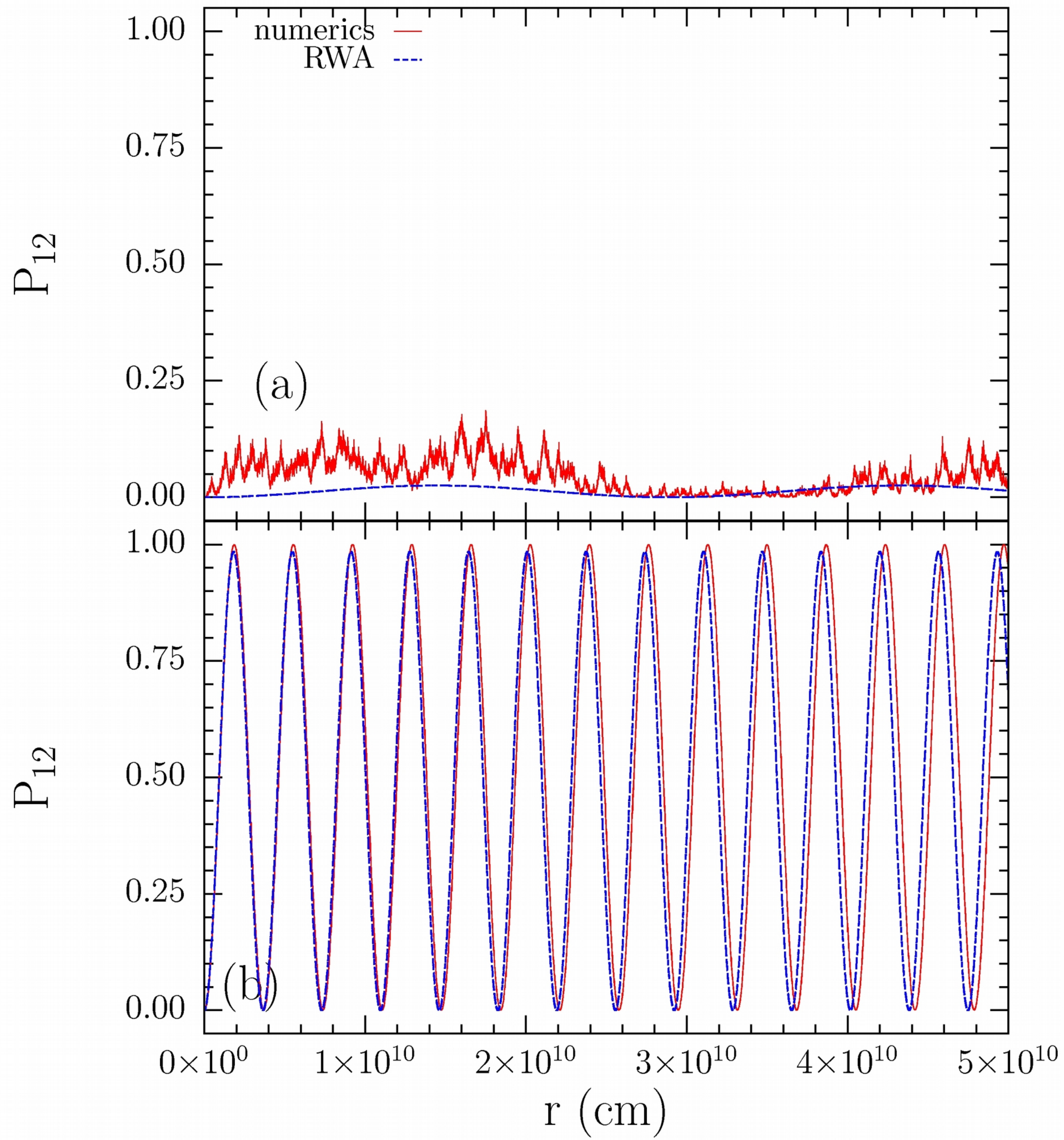
numerics
RWA
0.15
0.10
0.05
0
P_{12}
0
1×10^9
2×10^9
3×10^9
4×10^9
5×10^9
r (cm)

- The solution depends upon <u>all</u> the Fourier modes, not just the one on resonance.



40 modes:
one is resonant

35 modes: (5 longest wavelengths removed)
resonant mode still present



The presence of the five long wavelength modes suppress the transition.

- The effect of turbulence upon neutrinos depends upon six different lengthscales:
  - the cutoff scale – the longest wavelength Fourier mode
  - the dissipation scale – the shortest wavelength Fourier mode
  - the potential scale height – the distance over which the potential changes
  - the splitting scale – the wavelengths corresponding to the splitting between pairs of eigenvalues of the underlying Hamiltonian,
  - the transition scale – the wavelength of the transitions between pairs of neutrino eigenstates
  - the suppression scale – the shortest wavelength Fourier mode which sends the transition scale to infinity thereby suppressing transitions

- In order for turbulence to have an effect the 6 scales have to satisfy three conditions
  - the splitting scale must lie between the dissipation scale and the cutoff scale

$$\lambda_{diss} < \lambda_{split} < \lambda_{cut}$$

  - the transition scale must be smaller than the potential scale height

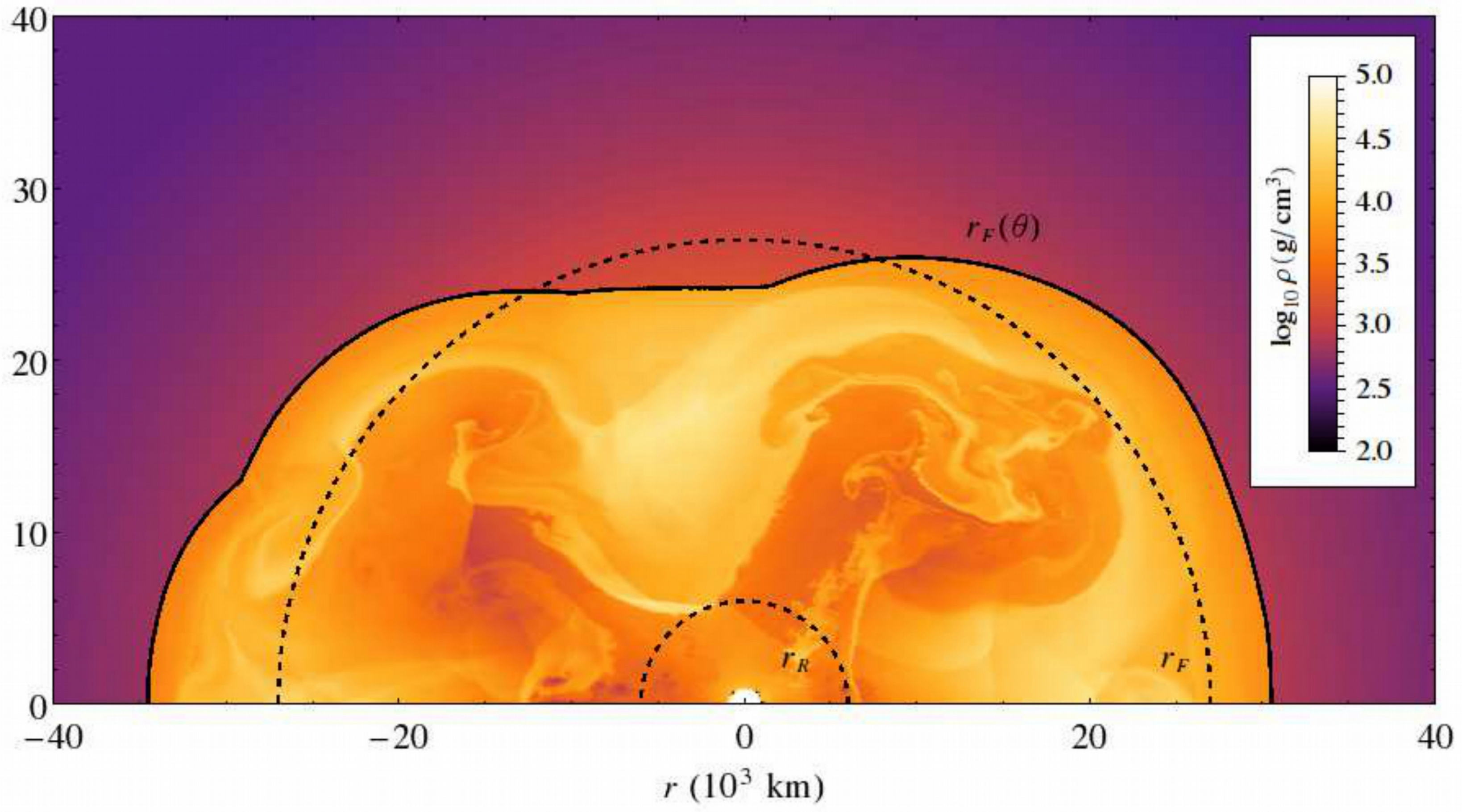
$$\lambda_{trans} < h_{scale}$$

  - the cutoff scale must be smaller than the suppression scale.

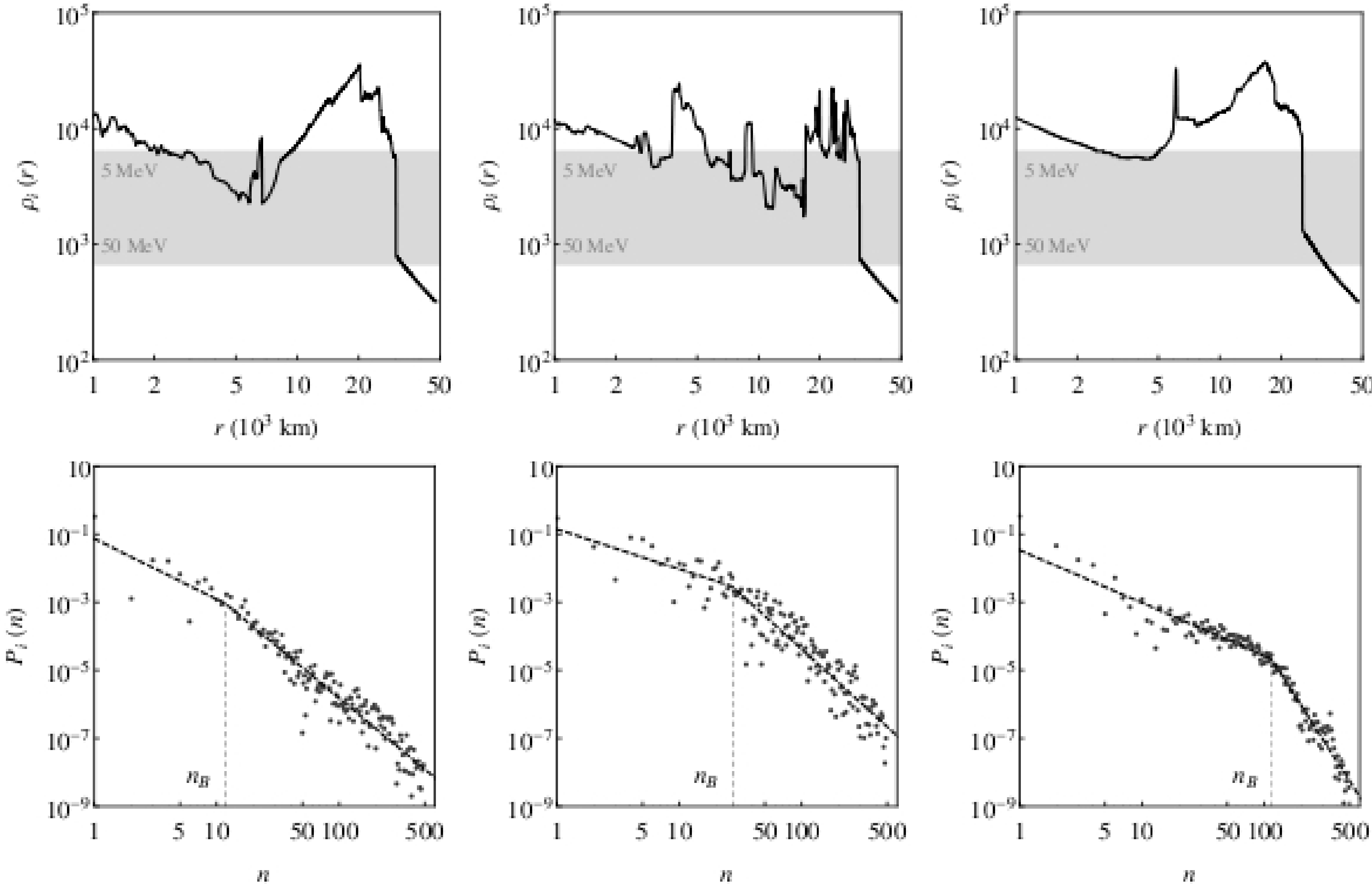
$$\lambda_{cut} < \lambda_{supp}$$

- Borriello et al examined the turbulence in a snapshot of a 2D simulation from the Garching group

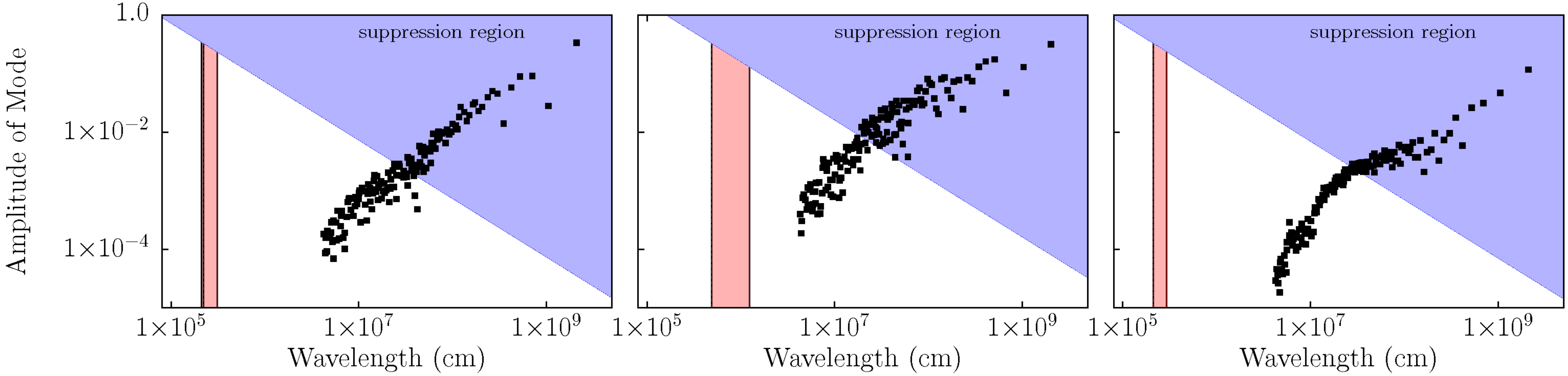
Borriello et al JCAP **11** 030 (2014)



- They computed the power spectrum along every ray



- We can overlay the power spectrum with the regions where transitions are induced, and the suppression region.

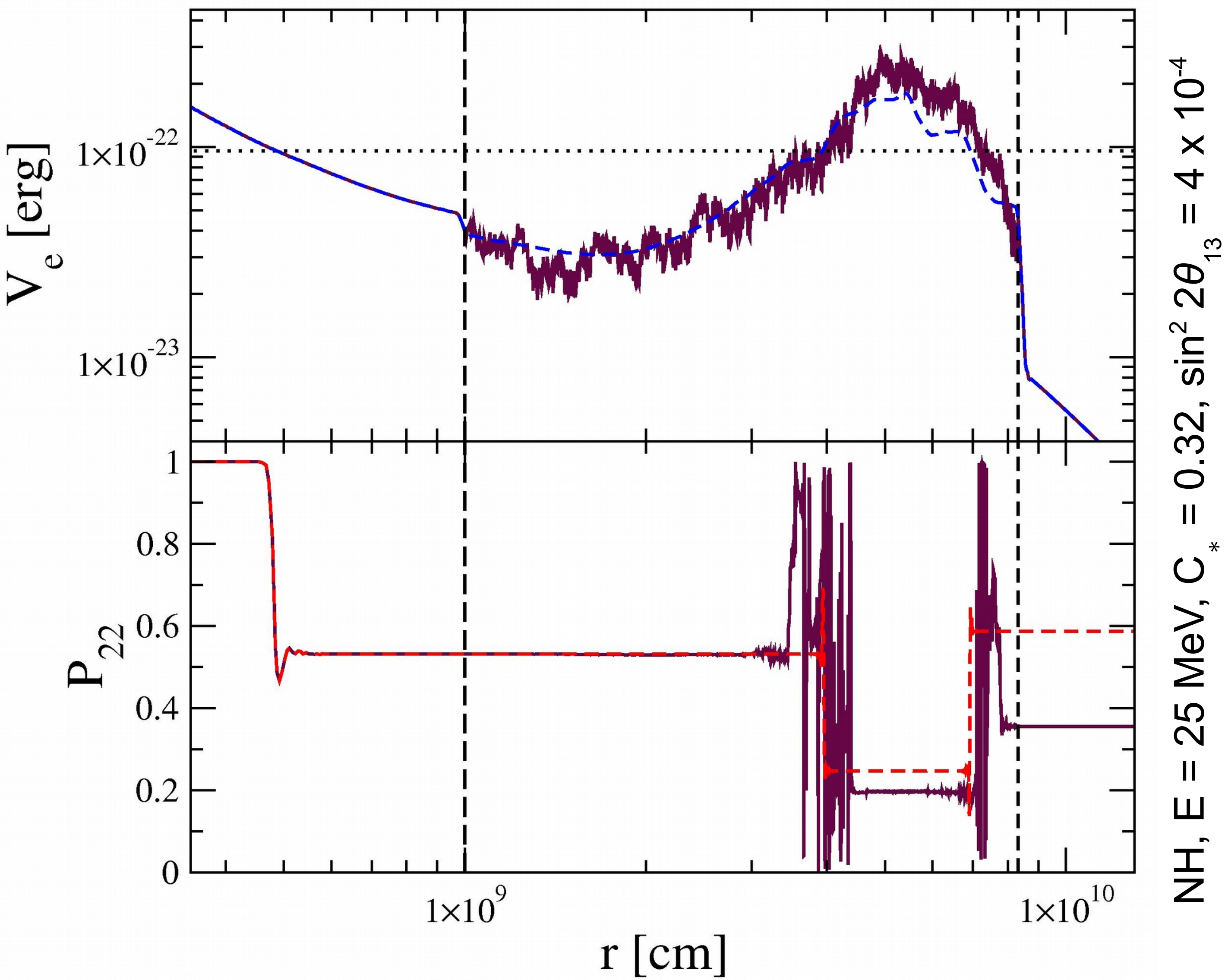


Patton, Kneller & McLaughlin, PRD **91** 025001 (2015)

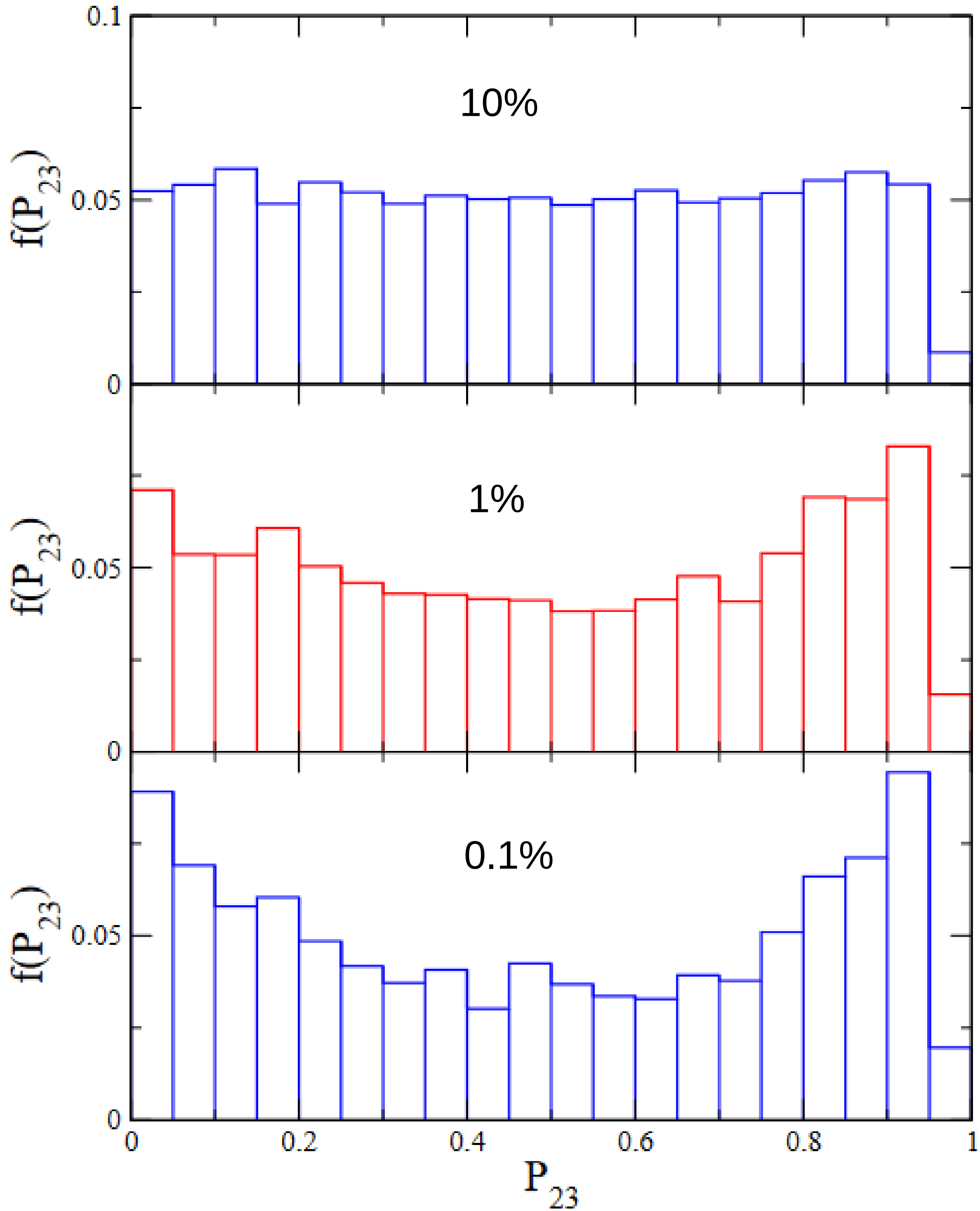
- There are no modes which induce resonances and lots of modes which suppress them.
- We expect no effect from the turbulence upon the neutrinos for this snapshot.

- We have to study the effect of turbulence by adding it to 1D profiles (which are turbulence free)

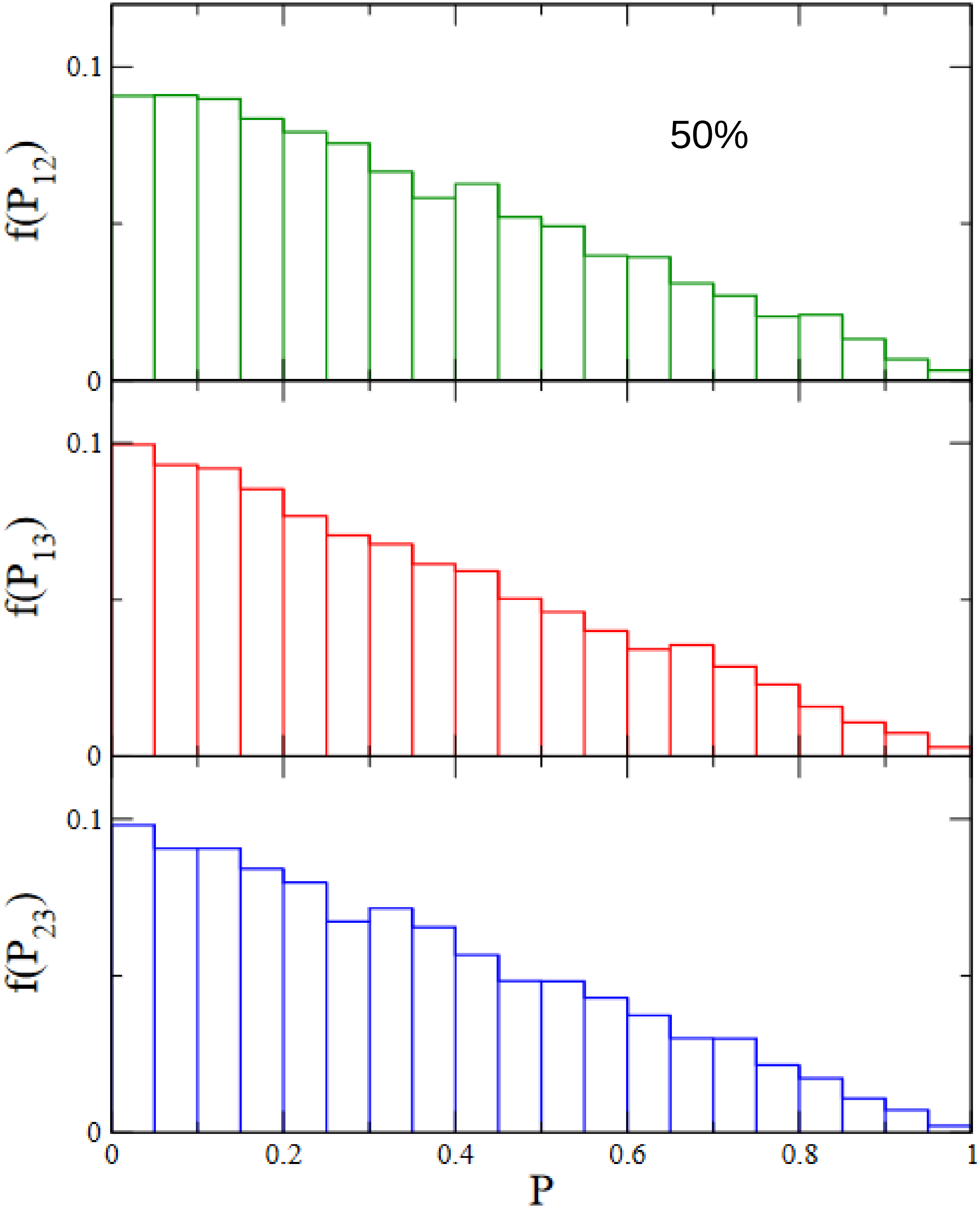
Kneller & Volpe, PRD **82** 123004 (2010)



For small amounts of turbulence we get 2 flavor depolarization.



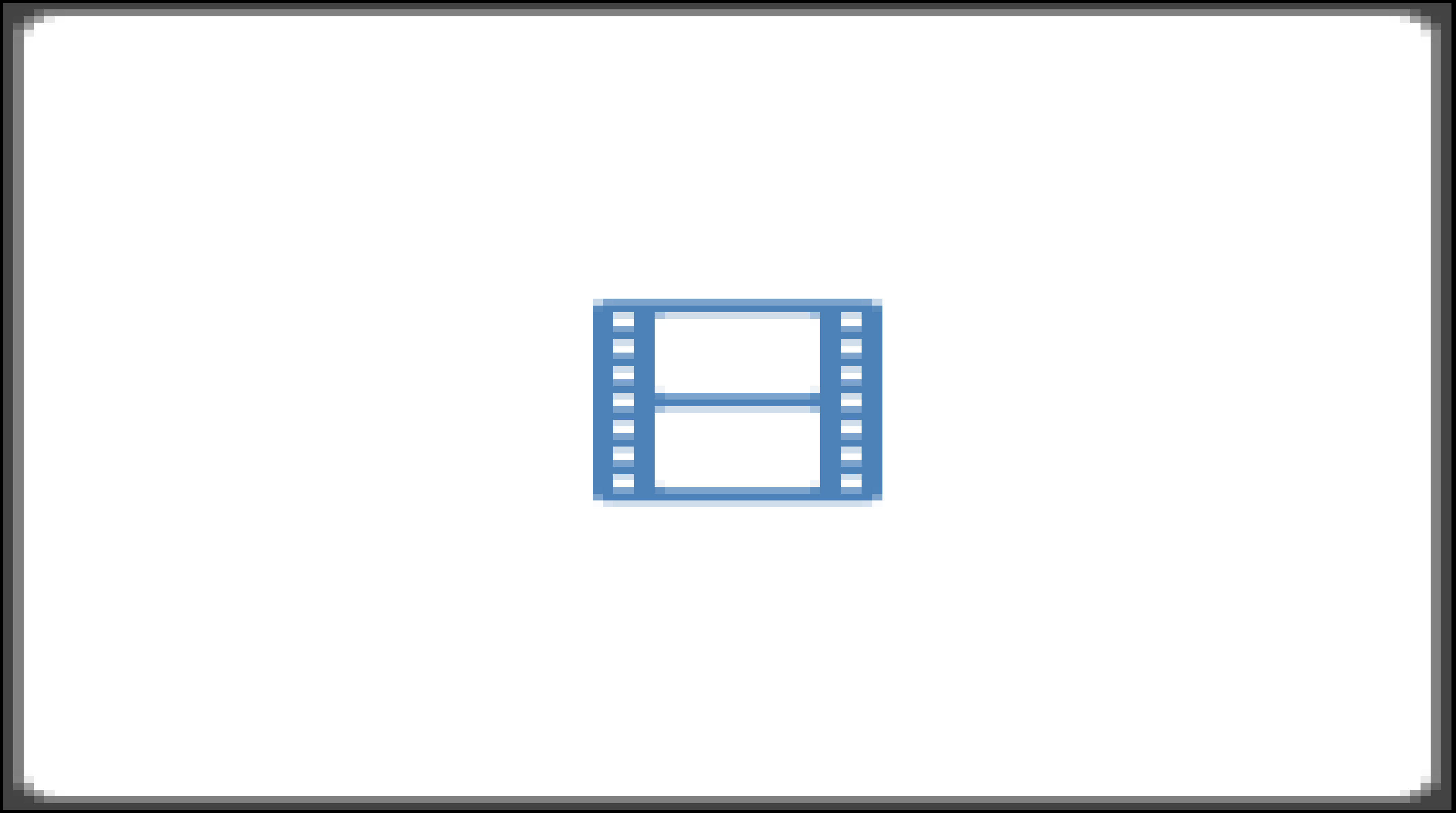
Kneller & Kabadi, PRD 92 013009 (2015)



For large amounts of turbulence we get 3 flavor depolarization.

- At the present time we don’t know the amplitude / spectrum of the turbulence around the MSW resonances because simulations don’t have the necessary spatial resolution.

- From studies where the turbulence is added to a 1D profile:
  - For large amplitude density fluctuations, > 30%, we expect 3 flavor depolarization
  - For small amplitude density fluctuations, < 30%, we expect 2 flavor depolarization.

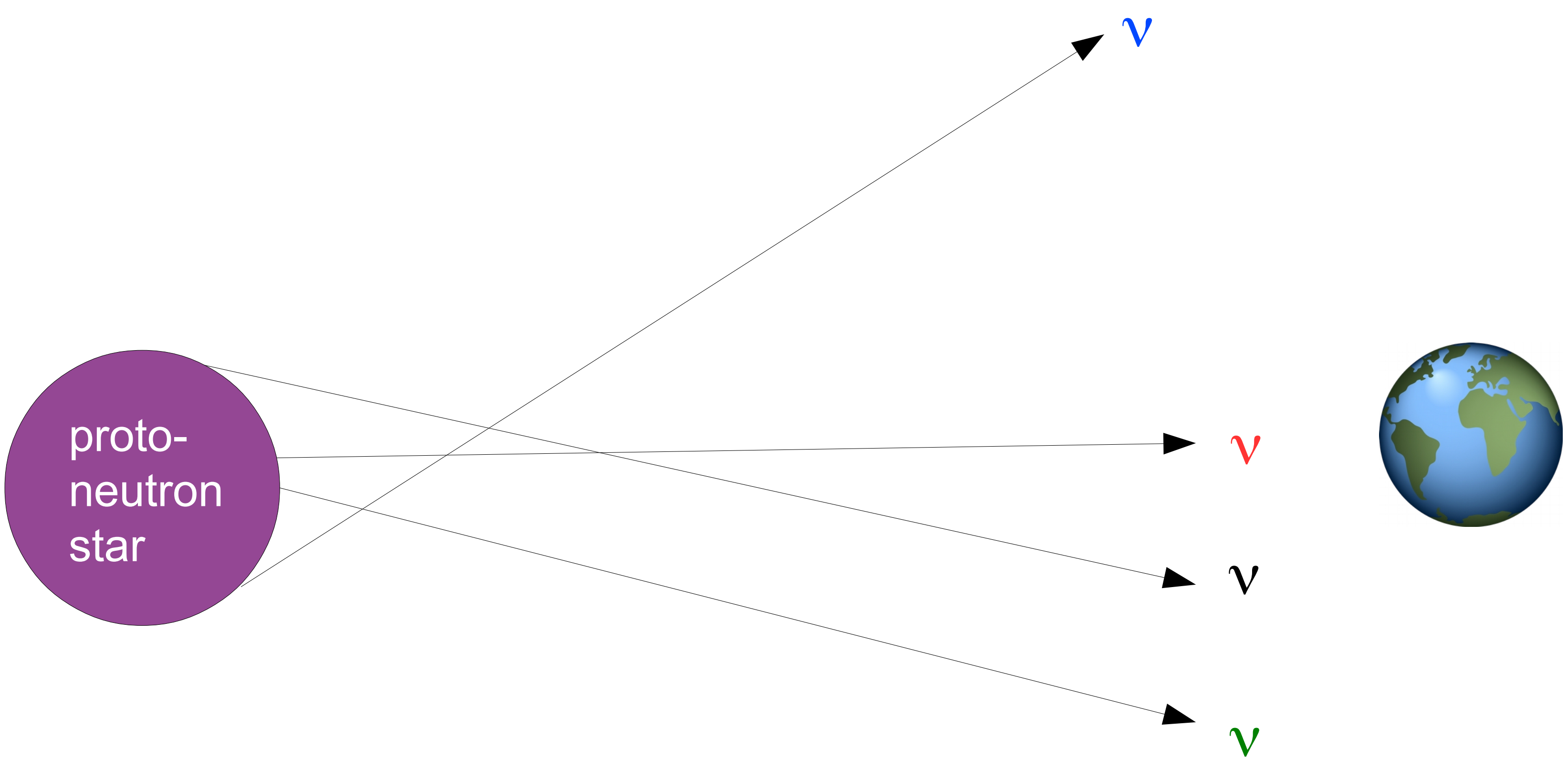


Self-Interactions - Slow Oscillations

- The current State-Of-The-Art for the slow oscillations of free-streaming neutrinos are so-called Multi-Angle calculations.

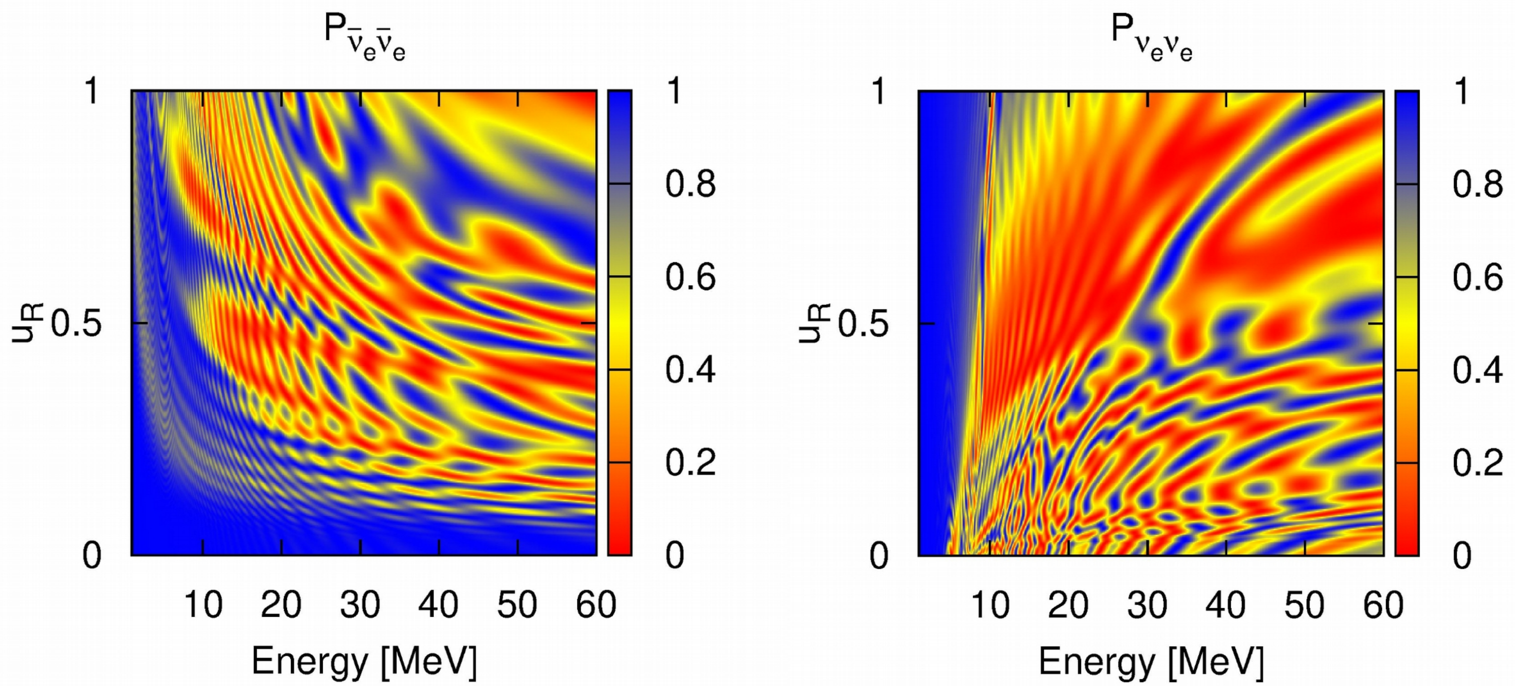
Duan et al PRL **97** 241101 (2006)

- In addition to treating the neutrinosphere as a hard surface with spherically symmetric neutrino emission.
  - There are no collisions or absorption/emission beyond the neutrinosphere.
  - The neutrino field is in steady state
  - The neutrino field has axial symmetry around the radial direction.
- This turns the neutrino transport into an initial-value problem.
- The imposed symmetries leave just two free variables:
  - The neutrino energy
  - The angle of emission at the neutrinosphere.



- The evolution of a single neutrino becomes dependent upon itself *and* every other neutrino emitted even if they never meet!

- E.g. using a snapshot at 0.7 s postbounce of the 10.8 $M_\odot$ simulation from Fischer *et al*, A&A **517** A80 (2010), and a half-isotropic angular distribution for the emission.



- This calculation took a few thousand core-hours.

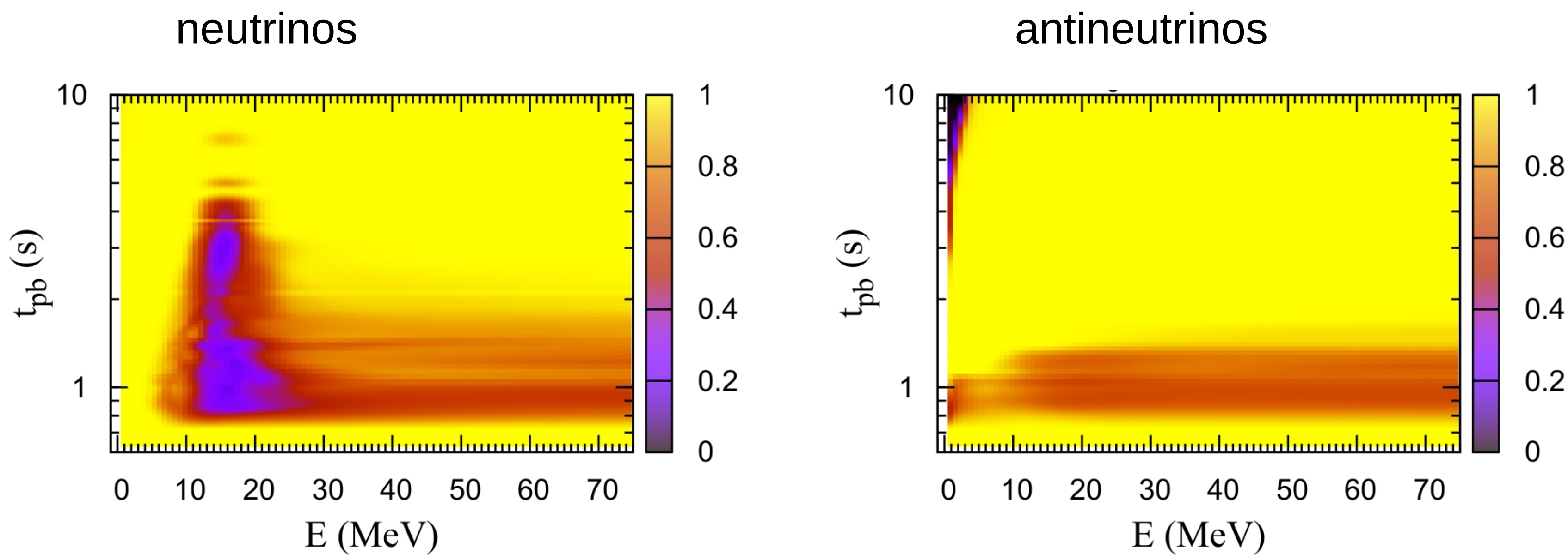


FIG. 6. The angle-averaged survival probabilities (a) $\langle P_{\nu_e \nu_e} \rangle_v$ and (b) $\langle P_{\bar{\nu}_e \bar{\nu}_e} \rangle_v$ as functions of E and emission time t_{em} (in terms of t_{pb}) at $r = 500$ km.

- Wu et al computed the time dependence of the flavor transformation due to ‘slow’ self-interactions for the 18 $M_\odot$ simulation from Fischer *et al*,

Wu et al, PRD **91**, 065016 (2015)

- Sasaki et al did the same for the first 300 ms of an ONeMg supernova.

Sasaki et al PRD **101**, 063027 (2020)

Self-Interactions - Fast Flavor Oscillations

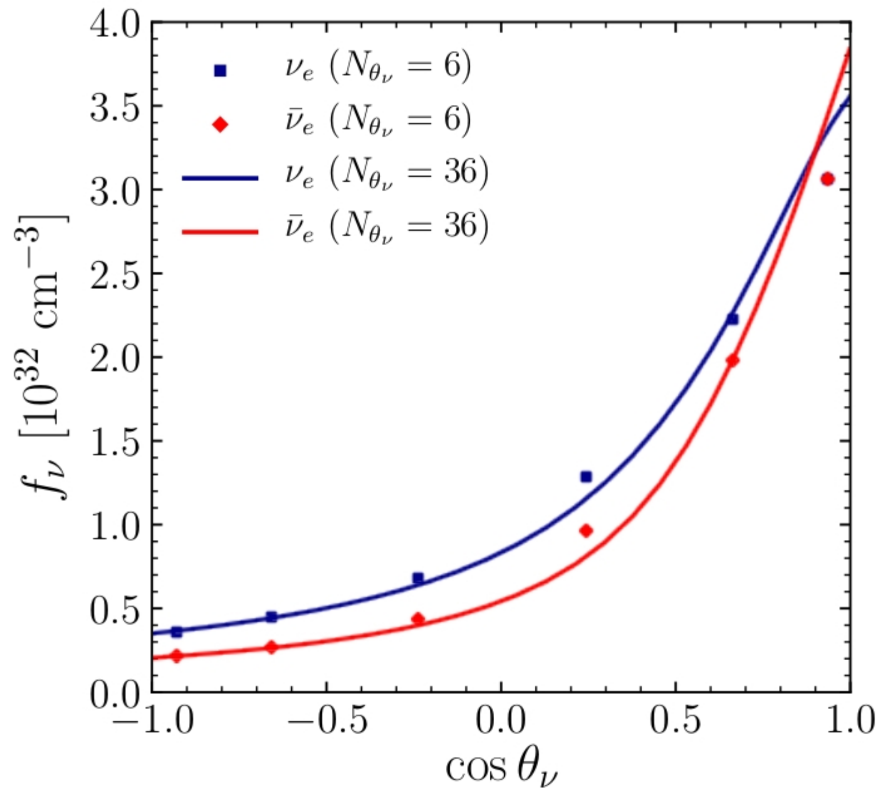
- Fast Flavor Oscillations (FFO) occur due to differences in the angular distribution of the neutrinos versus antineutrinos

Sawyer, PRD **72**, 045003 (2005),

Mirizzi & Serpico, PRL **108**, 231102 (2012)

Izaguirre, Raffelt & Tamborra, PRL **118**, 021101 (2017)

and many many more

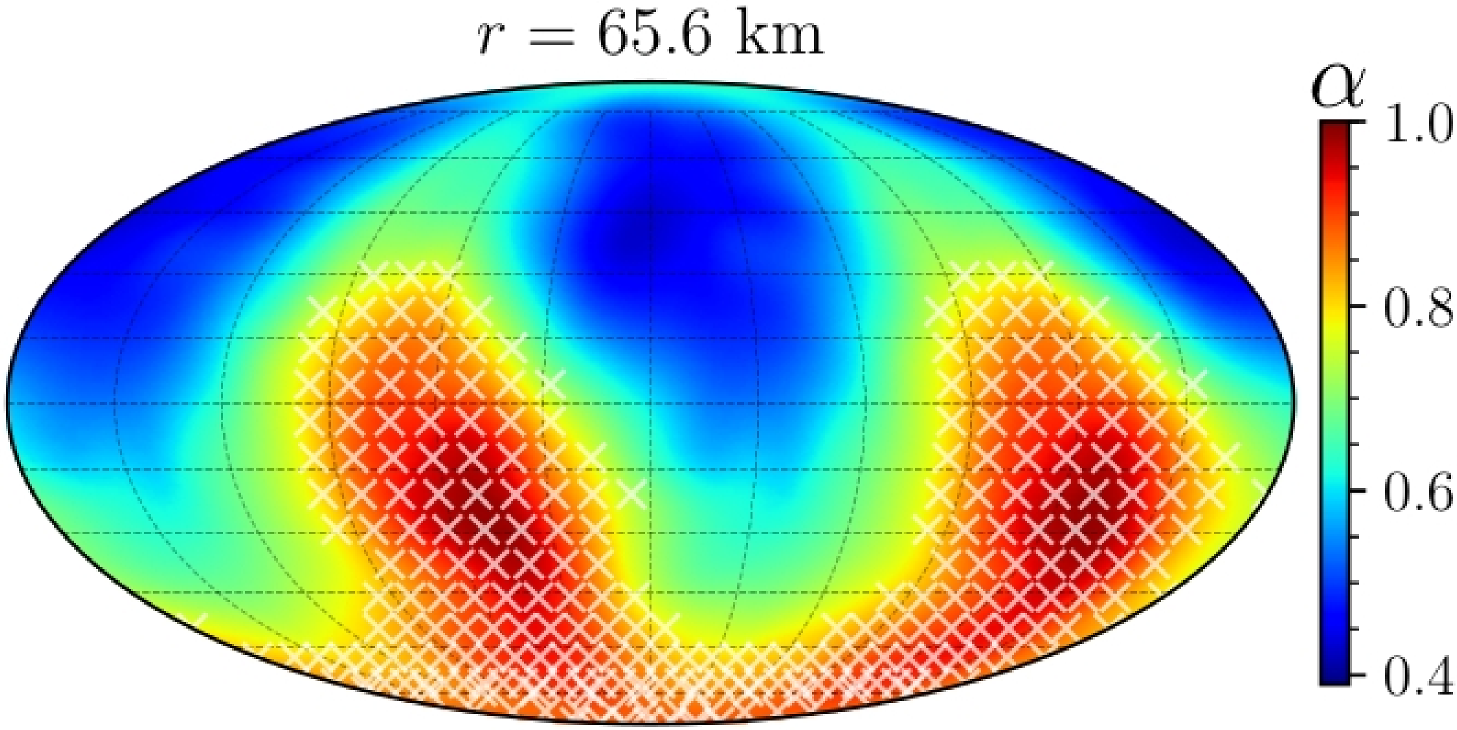


Abbar et al, PRD, **100** 043004 (2019)

- Abbar et al examined 2D and 3D simulations and found locations and times where FFO could occur.

Abbar et al, PRD, **100** 043004 (2019)

see also Nagakura et al, ApJ **886** 139 (2019)

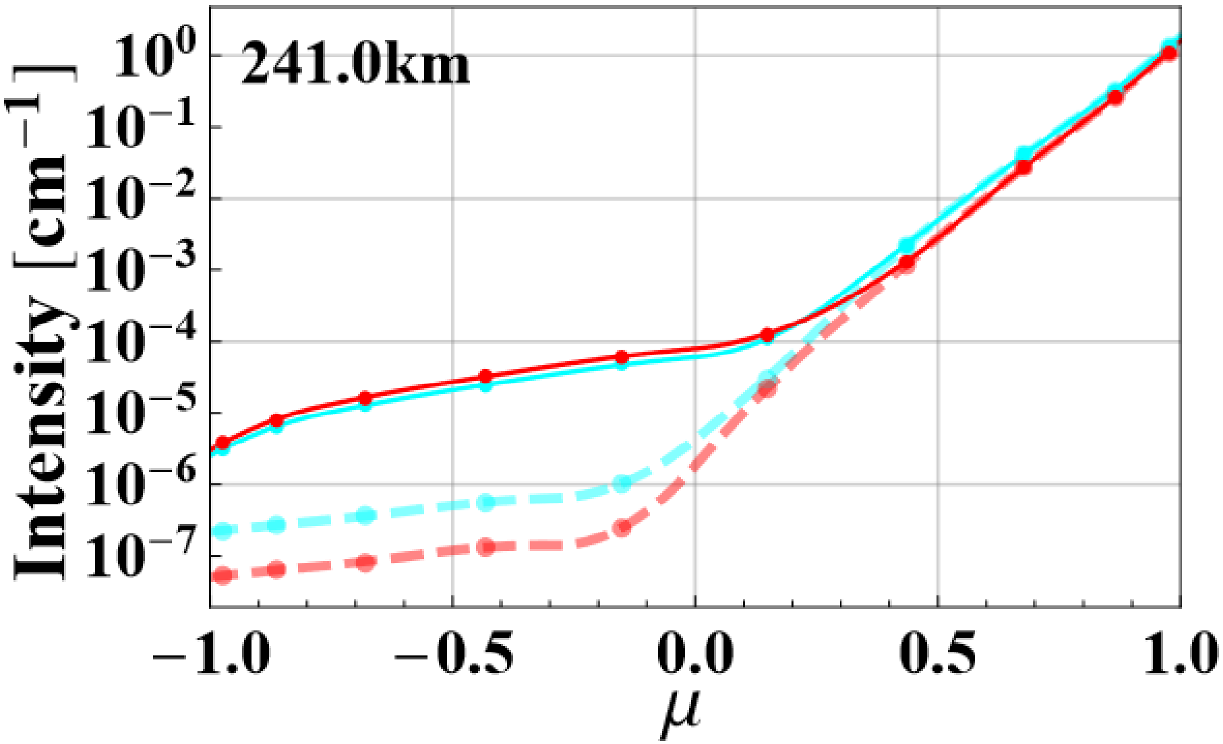


- A study by Tamborra et al of a 1D simulation did not find the angular crossings.

Tamborra et al, ApJ, **839** 132 (2017).

- Angular crossings in 1D were later found above the shock due to greater amount of scattering of the electron antineutrinos.

Morinaga et al PRR **2** 012046 (2020)



Beyond Post-Processing and the Bulb Model

- We need to go beyond post-processing and the Bulb model:
  - Neutrinospheres are not hard surfaces with uniform emission

    Hansen & Smirnov, JCAP, **10**, 027 (2019)
  - Collisions and emission are not negligible above the neutrinosphere (there are backwards going neutrinos)

    Cherry *et al*, PRL **108** 261104 (2012)

    Zaizen et al, JCAP, **06**, 011 (2020)
  - Supernovae are not spherically symmetric and it has also been shown that the symmetries can be spontaneously broken,

    Raffelt, Sarikas & Seixas PRL **111** 091101 (2013)
  - …
  - There is no feedback into the hydro.

Coupling oscillations and hydro

- Simulating a supernova using the QKEs for the neutrino transport will be super HARD
  - The spatial resolution will need to be of order μm, not km, time steps will be femtoseconds, not microseconds.
- A back-of-the-envelope estimate is that a 1D simulation with quantum neutrino transport would be $\sim 10^{16}$ more expensive than a classical simulation.
- It takes a classical 1D supernova simulation code a few hundred core-hours to run to 1 s postbounce.
- To make quantum supernova simulations feasible we will have to get creative: e.g. Nagakura & Zaizen rescaled the Hamiltonian down by a factor of 10^{-4} then extrapolated

Nagakura & Zaizen PRL **129** 261101 (2022)

see also Xiong et al PRD **107** 083016 (2023)

Neutrino oscillations with moments

- Many supernova simulation codes e.g. FLASH calculate the classical neutrino transport using angular moments.

- It is possible to generalize a classical moment to a quantum moment, and to do neutrino transformations with them.

Strack and Burrows, PRD **71** 093004 (2005)

Zhang and Burrows, PRD **88** 105009 (2013)

Myers et al, PRD **105** 123036 (2022)

Grohs et al, arXiv:2207.02214

- A quantum angular moment is defined as

$$M_n(q) = q \int F \cos^n \theta \; d\Omega$$

- where q is the energy of the neutrino, θ the angle relative to the radial direction, *F* is the neutrino distribution matrix
- The first few moments have well-known names
  - n = 0 is the (differential) energy density E_q
  - n = 1 is the (differential) radial component of the energy flux F_q
  - n = 2 is the ‘rr’ component of the (differential) pressure tensor P_q

- In spherical symmetry, the moments evolve according to

$$\frac{\partial E_q}{\partial t}+\frac{\partial F_q}{\partial r}+\frac{2F_q}{r}=-i[H_V+H_M+H_E,E_q]+i[H_F,F_q]$$

$$\frac{\partial F_q}{\partial t}+\frac{\partial P_q}{\partial r}+\frac{3P_q-E_q}{r}=-i[H_V+H_M+H_E,F_q]+i[H_F,P_q]$$

$$\vdots$$

  - the absorption / emission / collisions have been omitted, H_V is the vacuum Hamiltonian, H_M the matter Hamiltonian, H_E and H_F are the two contributions to the self-interaction,
- The infinite tower of equations can be truncated at what ever level one desires.
  - Typically one uses a one-moment (M0) or a two-moment (M1) truncation.
- We need an additional relationship between the moments in order to solve the equations.
- This relationship is called ‘The Closure’

Are moment-based approaches any good?

- We need to compare moment-based approaches to other methods e.g. Discrete Ordinates, Monte Carlo, or Particle-In-Cell

- We can make a comparison with ‘multi-angle calculations’ from the neutrino Bulb Model.
  - There is an analytic equation for the closure in the classical problem.

- We used a set of neutrino spectral parameters which produce a flavor instability close to the neutrinosphere.

	L [ergs/s]	$\langle E \rangle$ [MeV]	T [MeV]	η
ν_e	2.05×10^{49}	9.4	2.1	3.9
$\bar{\nu}_e$	2.55×10^{49}	13	3.5	2.3
ν_x	1.975×10^{48}	15.8	4.4	2.1
$\bar{\nu}_x$	1.975×10^{48}	15.8	4.4	2.1

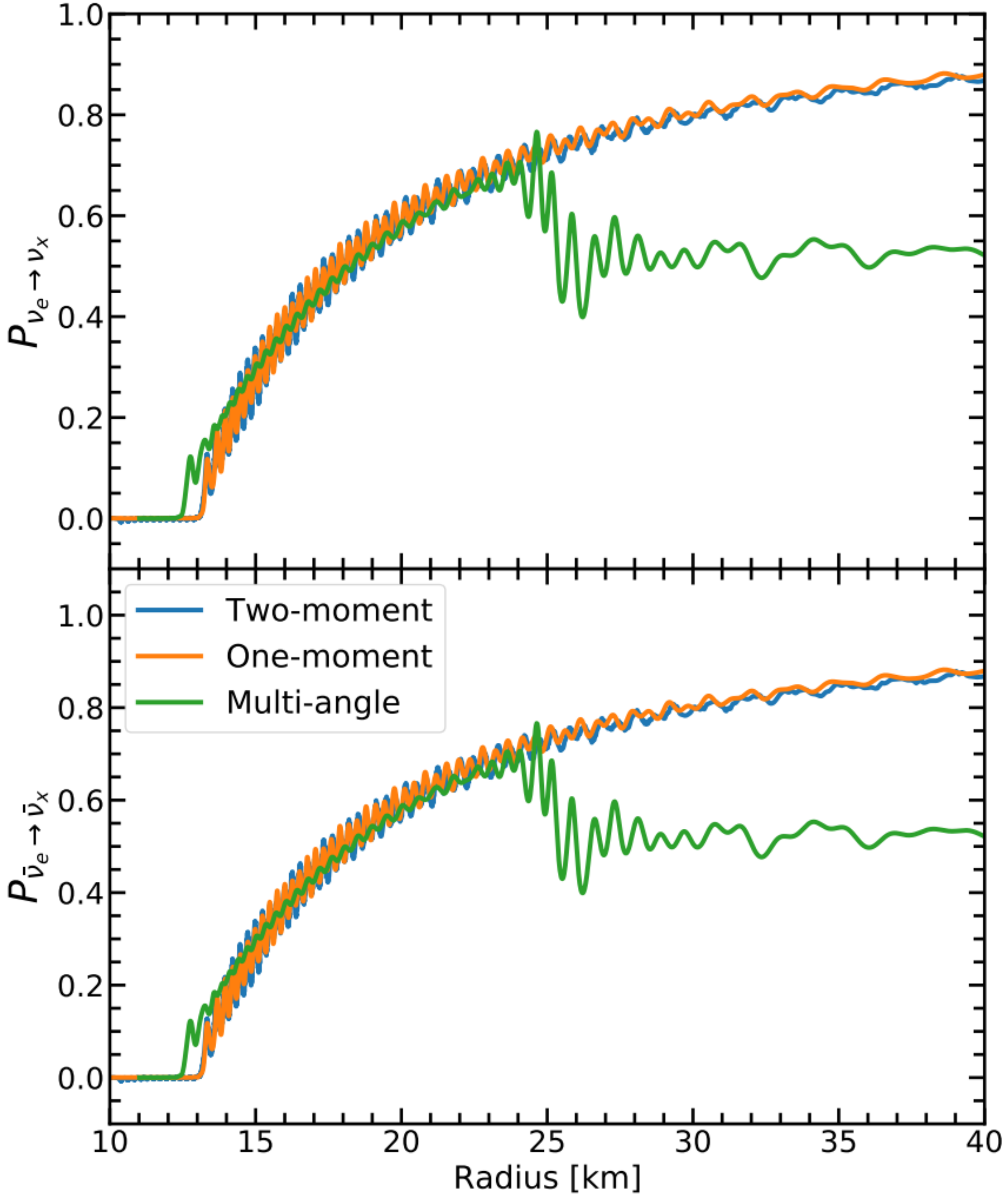
- For the M0 moment calculation, we use for the closure the analytic equation

$$F_q = \frac{\left(1 + \cos\theta_{max}\right)}{2} E_q$$

- where θ_{max} is the largest angle between the neutrino velocity vectors at some radius r, and the radial direction.

- For the M1 calculation the closure is again the analytic equation.

$$P_q = \frac{\left(1 + \cos\theta_{max} + \cos^2\theta_{max}\right)}{3} E_q$$

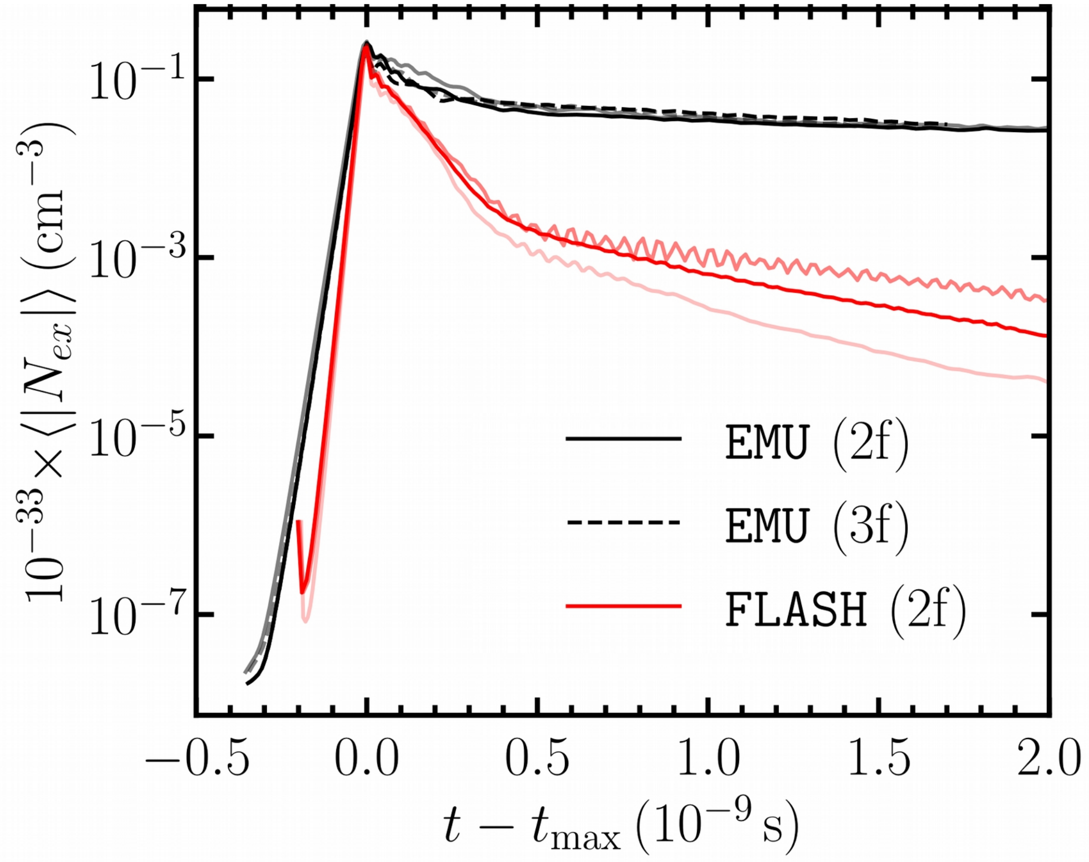


- The different approaches are in agreement about where the instability occurs.
- The multi-angle separates from the moments at ~23 km.

Myers et al, PRD **105** 123036 (2022)

The moment code is ~100 times faster than a multi-angle code.

- More recently we have looked at how well moments capture fast-flavor oscillations.
  - this is a demanding test: fast flavor oscillations depend upon angular distributions which is something the moments don't have.
- Surprisingly moments work well.



Grohs et al, arXiv:2207.02214

Summary

- While many things are understood, the theory of SN neutrino transformation is still a work in progress.

- How do we include oscillations in the simulations?