# Quantum systems: entanglement, simulations, information

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# A flash tour of quantum

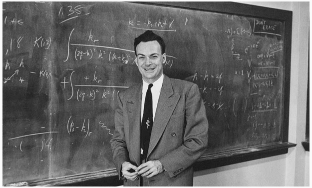
# "Nature isn't classical, dammit!"

Quantum mechanics is "a mathematical framework, or set of rules, for the construction of physical theories"[1] like quantum electrodynamics.

All theories that fit in this framework exhibit puzzling and novel phenomena, e.g. entanglement. The idea of making advantage of such phenomena for simulation of natural systems and information purposes dates back to the early 80s.

**Feynman's call**

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical."



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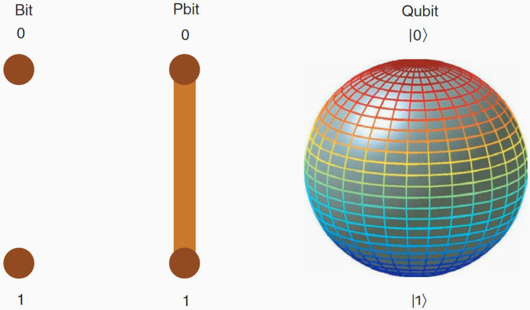
[1] M.A. Nielsen, I.L. Chuang (2000), *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press.

# Qubits

A qubit is any two-state quantum system. The rules of the game are (apparently) simple: a qubit is the linear (coherent) superposition of two quantum states $|0\rangle$, $|1\rangle$:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = 1;$$

by measuring the qubit in the basis $\{|0\rangle\,, |1\rangle\}$, we will find it in the state $|0\rangle$ (resp. $|1\rangle$) with probability $|\alpha|^2$ ($|\beta|^2$).



Original image from E. Knill et al., Los Alamos Science, no. 27 (2002).

**Bits and qubits**

Not to be confused with a classical probabilistic bit (pbit): quantumness is much more than just stochastic results! (e.g. measurement on different bases, superpositions, entanglement, etc.)

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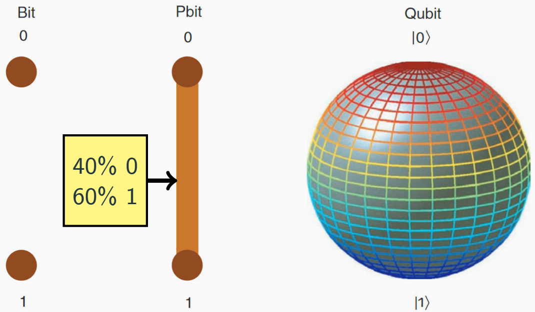

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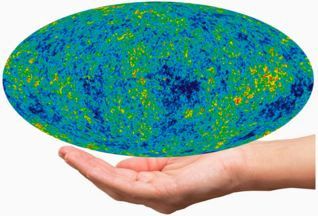
# “There’s lots of room in Hilbert space.”

Among the many novelties of qubits, it is particularly important to understand that a system of $n$ qubits carries a lot more information than a system of $n$ bits. Indeed:

- a system of $n$ bits can be in $2^n$ possible states, but we only need $n$ parameters (the bits themselves!) to characterize it;
- a system of $n$ qubits can be in any superposition of any basis of $2^n$ possible states, and thus we need $2^n - 1$ parameters to identify the state!

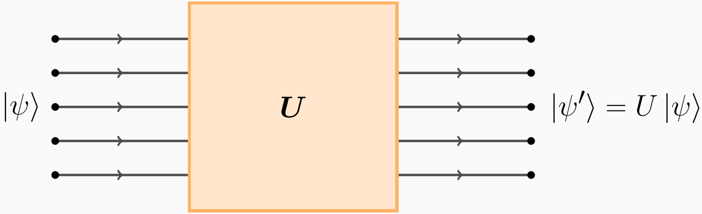
**“Hilbert space is is a big place.” (C. Caves)**

A state of a system of $n = 500$ qubits is described by a number of parameters larger than the estimated number of atoms in the Universe.



# Quantum simulation and computation

This enormous amount of information, together with the stronger correlations (entanglement) that can be realized between qubits, is something we would definitely like to make advantage of.



The ultimate goal: being able to implement any possible (unitary) operation on qubits (that is, to create quantum computers), and use them to support our main research lines.

**Is it really worth it?**

Are there classically untreatable problems that become feasible via quantum computation?

# Peter Shor's "bolt from the blue"

The answer is yes: the forefather of such results was found by Peter Shor in 1994.

**Factorization problem**

Given a large number $n$, find a couple of prime numbers $p$, $q$ such that $n = pq$.

At the classical level, this problem has the following properties:

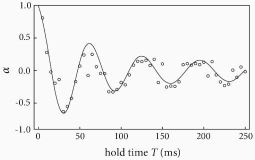
- given a (candidate) solution, it is fairly easy to show whether it is correct...
- ...but it is extremely hard to find it: the required number of operations is exponential in the $\log n$ of the number $n$ of digits.

Shor proved that a quantum computer could factorize numbers with a polynomial number of operations in $\log n$. The advantage for large numbers is startling (e.g.: 3 years versus $10^{10}$ years for a 400-digit number!) This is potentially useful for cryptography purposes.

# So, where is my quantum laptop?

There are important practical obstructions to overcome. To name a few:

- Decoherence and noise: the unavoidable interaction with the environment may break the coherence of our states, causing our qubits to decay and effectively behave as classical stochastic bits.
  - This effect is stronger for larger systems: Schrödinger's cat is either dead or alive.



R.P. Anderson et al., Phys. Rev. A **80**, 023603 (2009).

- Errors do occur when manipulating a large amount of (qu)bits, and more information implies more possible errors. For instance, the following two errors are possible:

$$\begin{cases} |0\rangle \to |1\rangle \\ |1\rangle \to |0\rangle \end{cases} , \qquad \begin{cases} |0\rangle \to |0\rangle \\ |1\rangle \to -|1\rangle \end{cases} ;$$

  while the first one, a bit-flip error, also occurs in classical information, the second one, a phase error, has no classical counterpart! $\Longrightarrow$ quantum error correction needed.

# What to do?

This is a formidable and interdisciplinary task, to be addressed from diverse (and intertwined!) points of view:

- theoretical tasks: modeling the qubit-environment interaction in different frameworks, e.g. atom-photon interactions in cavities and waveguides, developing resource theories, ...;
- technological tasks: projecting and building devices with "truly quantum" behavior, shielding them from decoherence, ...;
- informational tasks: characterizing problems that can be solved more conveniently via quantum algorithms than classical ones, developing quantum error-correction codes, creating and optimizing quantum error-correcting codes, ...

While a lot of progress has been done in the last years, there is still a lot to do!

# (Some of) our activities

# The QUANTUM group

**Staff members**

| | |
|---|---|
| Paolo Facchi | Francesco Pepe |
| Giuseppe Florio | Augusto Garuccio |
| Saverio Pascazio | Milena D'Angelo |

**Postdocs and PhD students**

| | |
|---|---|
| Davide Lonigro | Arturo Konderak |
| Daniele Amato | Rocco Maggi |
| Giuliano Angelone | |

# EU projects and QUANTUM flagship: QuantERA

- **PACE-IN: Photon-Atom Cooperative Effects at Interfaces**
  The overall objective of this proposal is to meet the critical challenge of studying, implementing and optimizing ground-breaking, dynamically-controlled interfaces between matter and light.
- **QuantHEP: Quantum Computing Solutions for High-Energy Physics**
  The key goal of project QuantHEP is to develop quantum algorithms as a solution to the increasingly challenging, and soon intractable, problem of analysing and simulating events from large particle-physics experiments.

QUANTERA

*This project has received funding from the European Union's*
*Horizon 2020 research and innovation programme*
*under Grant Agreement No 731473 and 101017733*

# An overview

1. Waveguide QED: characterization of bound states in diverse geometries;

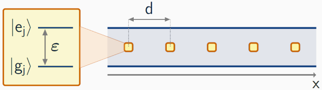
   Collaborators: A.D. Greentree (RMIT, Melbourne), M. S. Kim (Imperial Coll., London), T. Tufarelli (Nottingham);

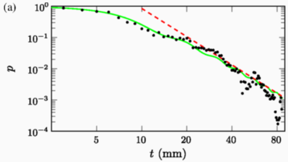
2. Nonexponential decay and unconventional regimes in quantum dynamics;

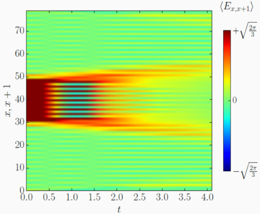
   Collaborators: H. Nakazato (Waseda, Tokyo), A. Crespi, R. Osellame (Pol. Milano), P. Mataloni, F. Sciarrino (Sapienza, Roma);

3. Quantum simulations of (1+1)-dimensional QED;

   Collaborators: M. Dalmonte (ICTP/SISSA), E. Ercolessi, G. Magnifico (Bologna), G. Marmo (Napoli), S. Notarnicola (Padova).

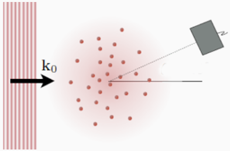






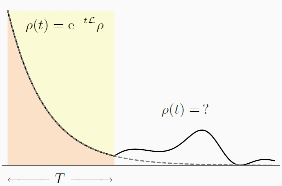
# An overview

4. Collective effects in cold atoms via random matrix theory.
   Collaborators: R. Kaiser (CNRS, Nice), F. D. Cunden (Bari).



5. Quantum electrodynamics of asymmetric systems.
   Collaborators: K. Słowik (UMK, Toruń);

6. Open quantum systems with delayed non-Markovianity;
   Collaborators: D. Burgarth (Macquarie, Sydney), M. Ligabò (Bari), K. Modi (Monash, Melbourne).

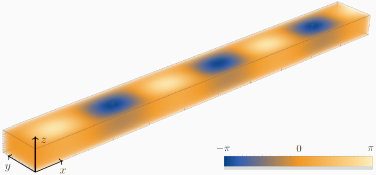


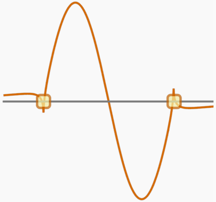
7. Quantum adiabaticity and conservation laws.
   Collaborators: D. Burgarth (Macquarie, Sydney), H. Nakazato, K. Yuasa (Waseda, Tokyo);

# Waveguide quantum electrodynamics

Two-level emitters interacting with an electromagnetic field embedded in a cavity or in a waveguide: a quickly developing field, which exhibit interesting nontrivial physics and promising applications, to name a few:

- Generation of atom-photon entangled states;
- Enhanced or inhibited correlated emission of photons;
- Couple of vacuum state and single-photon state as a qubit.





In particular, bound states in the continuum (BICs) can emerge when taking into account $n > 1$ atoms. Those are states whose energy would be sufficient to yield propagation, but that does not happen: the photon is trapped.

# Waveguide quantum electrodynamics: our activities

We have developed a nonperturbative formalism to characterize the BICs emerging in a regular array of $n$ quantum emitters in an infinite waveguide, and evaluated them in various cases:

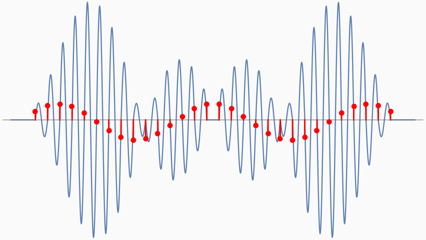
- $n = 2$: P. Facchi et al., Phys. Rev. A **94**, 043839 (2016); P. Facchi et al., J. Phys. Commun. **2**, 035006 (2018); P. Facchi et al., Phys. Rev. A **98**, 063823 (2018);
- $n = 3, 4$: P. Facchi et al., Phys. Rev. A **100**, 023834 (2019);
- Generic $n$: D. Lonigro et al., arXiv:2106.08213 [quant-ph] (2021).

A finite, closed waveguide has also been examined:

- D. Lonigro et al., arXiv:2103.10926 [quant-ph] (2021).

**Main message**

BICs are collective: all emitters have to retain a part of the excitation in order to steadily sustain the photon!

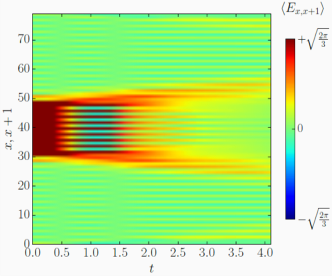


# Quantum simulations of real-time dynamics 1+1 QED

We have investigated the properties of a Schwinger-Weyl lattice model, i.e. (1+1)-dimensional QED discretized via a staggered-fermion $\mathbb{Z}_n$ model:

$$H = \sum_x \left( \tau(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.}) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right).$$

This model exhibits a rich phenomenology: vacuum instability with pair production, both spontaneous and induced by an external electric field, string-breaking mechanism, and a strong effect of confinement; its real-time dynamics has been studied.



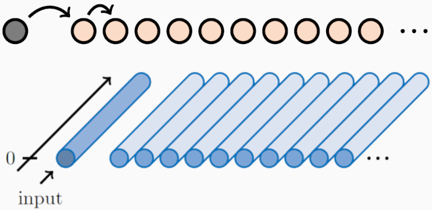
E. Ercolessi et al., Phys. Rev. D **98**, 074503 (2018);
G. Magnifico et al., Quantum **4**, 281 (2020).

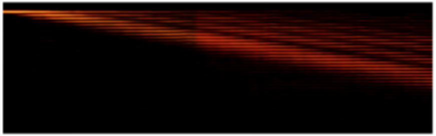
# Quantum simulation of decay

Unstable systems are often observed to decay exponentially, but quantum mechanics predicts deviations from the exponential decay. In particular, a large-time subexponential decay (e.g. power-law) is predicted under general assumptions, despite being hard to observe in most systems.

We studied power-law decay in a hopping model representing an array of femtosecond laser-written integrated waveguides. Experimental analyses (IFN-CNR Milano) confirmed our predictions.[1]

Another system exhibiting nonexponential decay is a Feshbach molecule, whose decay has been shown to follow a streched exponential law at large times.[2]





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[1] A. Crespi et al., Phys. Rev. Lett. 122, 130401 (2019).

[2] F.V. Pepe et al., Phys. Rev. A 101, 130401 (2020).