Standard Model and Beyond Standard Model Physics by QFT Methods

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Group Research

Who we are

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What we do

Flavour Physics

Anomalies between experimental data and Standard Model (SM) predictions:

- model independent analyses;
- BSM models (extended gauge symmetry).

Holography

Exploiting duality between gauge and gravity theories:

- applications to the equilibration of the quark-gluon plasma, to configurational entropy, to quantum chaos.

Spectroscopy

Understanding the structure of hadrons and interpreting newly discovered states.

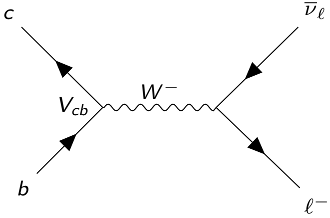
Group Research

Flavour anomalies

Flavour anomalies

Recent data show some convincing evidences of
Lepton Flavour Universality (LFU) violations

$b \to c\,\ell\,\bar{\nu}_\ell$

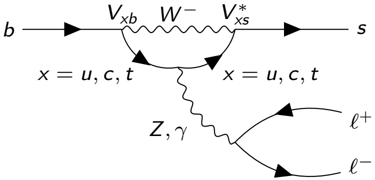


τ VS e, μ

W interacts equally with

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}$$

$b \to s\,\ell^-\,\ell^+$



μ VS e

γ and Z interact equally with e, μ, τ

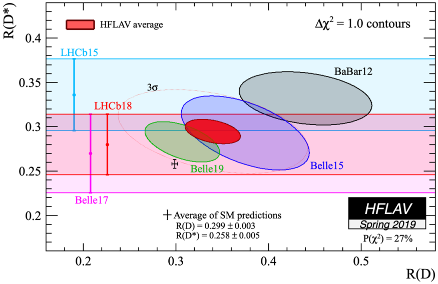
$b \to c\, \ell\, \bar{\nu}_\ell$

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\, \tau^-\, \bar{\nu}_\tau)}{\mathcal{B}(B \to D^{(*)}\, \ell^-\, \bar{\nu}_\ell)}\,, \quad \ell = e, \mu$$

$$R(D) = \begin{cases} 0.307 \pm 0.037\,(\text{stat}) \pm 0.016\,(\text{syst}) \\ 0.299 \pm 0.003 \end{cases}$$

$$R(D^*) = \begin{cases} 0.295 \pm 0.011\,(\text{stat}) \pm 0.008\,(\text{syst}) \\ 0.258 \pm 0.005 \end{cases}$$

Tiny theoretical error predictions!



$$R(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\, \tau^+\, \nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\, \mu^+\, \nu_\mu)}$$

$$R(J/\psi) = \begin{cases} 0.71 \pm 0.17\,(\text{stat}) \pm 0.18\,(\text{syst}) \\ 0.283 \pm 0.048 \end{cases}$$

$b \to s\,\ell^-\,\ell^+$

$$R_{K^{(*)}} = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma}{dq^2}(B \to K^{(*)}\,\mu^+\,\mu^-)dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma}{dq^2}(B \to K^{(*)}\,e^+\,e^-)dq^2} \qquad \text{and} \qquad [q^2] = [\mathrm{GeV}^2]$$

$$R_{K^+} = 0.846^{+0.054}_{-0.060}\,(\text{stat})^{-0.016}_{-0.014}\,(\text{syst}) \qquad \text{for} \qquad 1.1 < q^2 < 6\,,$$

$$R_{K^{*0}} = \begin{cases} 0.66^{+0.11}_{-0.07}\,(\text{stat}) \pm 0.03\,(\text{syst}) & \text{for} \quad 0.045 < q^2 < 1.1\,, \\ 0.69^{+0.11}_{-0.07}\,(\text{stat}) \pm 0.05\,(\text{syst}) & \text{for} \quad 1.1 < q^2 < 6\,, \end{cases}$$

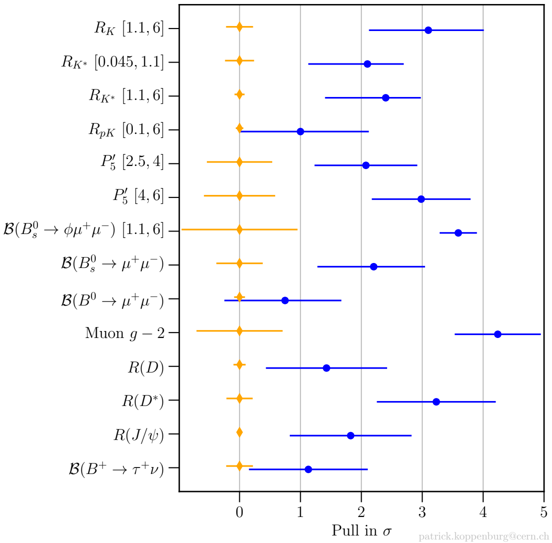
$$R_{K^*} = \begin{cases} 0.52^{+0.36}_{-0.26}\,(\text{stat}) \pm 0.05\,(\text{syst}) & \text{for} \quad 0.045 < q^2 < 1.1\,, \\ 0.90^{+0.27}_{-0.21}\,(\text{stat}) \pm 0.10\,(\text{syst}) & \text{for} \quad 0.1 < q^2 < 8\,, \end{cases}$$

The contribution from $B \to K\,J/\psi(\to \ell^+\,\ell^-)$
and from other intermediate resonances is under control

$$R_{K^*} = 1.18^{+0.52}_{-0.32}\,(\text{stat}) \pm 0.10\,(\text{syst}) \qquad \text{for} \qquad 15 < q^2 < 19\,,$$

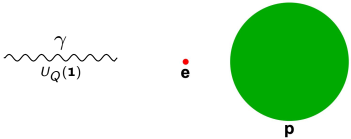
End-point spectrum

For all the ratios the SM predictions are close to 1!


R_K [1.1, 6]
R_{K^*} [0.045, 1.1]
R_{K^*} [1.1, 6]
R_{pK} [0.1, 6]
P_5' [2.5, 4]
P_5' [4, 6]
$\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-)$ [1.1, 6]
$\mathcal{B}(B_s^0 \to \mu^+\mu^-)$
$\mathcal{B}(B^0 \to \mu^+\mu^-)$
Muon $g-2$
$R(D)$
$R(D^*)$
$R(J/\psi)$
$\mathcal{B}(B^+ \to \tau^+\nu)$
0
1
2
3
4
5
Pull in σ
patrick.koppenburg@cern.ch

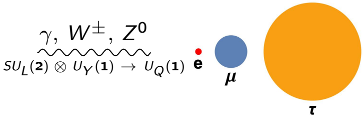
A digression on LFU

Let us back almost 100 years and suppose we can test matter only with long wave-length photons...



These two particles seem to be *identical copies* unless their masses

That is exactly the same (misleading) argument we use to infer LFU

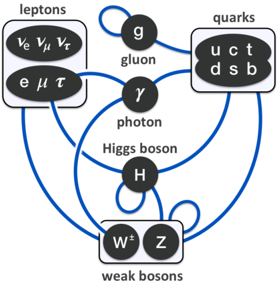


These three (families) of particles seem to be *identical copies* unless their masses

The SM quantum numbers of the three families could be an *accidental* low-energy property: the different families may have a very different behaviour at high energies, as signalled by their different masses (**Yukawa interaction!**).

Open questions

$$\mathcal{G}_{SM} : SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \xrightarrow{EWSB} SU_C(3) \otimes U_Q(1) \quad (1)$$



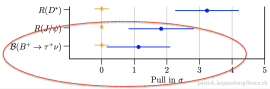
The tensions concern the Lepton Flavour Universality (LFU), in particular for

- Third lepton family in $b \to c$ channel.
- First and second generations in $b \to s$ transition.

LFU is *accidental symmetry* in SM, well verified in W and Z decays. Possibility:

- At higher energy scale ($> 246\,\mathrm{GeV}$) new interactions may exist which *look* at the three lepton families differently.

If the tensions are due to Physics Beyond the Standard Model (BSM), instead of being related to hadronic effects, they should be observed in a coherent way in other processes induced by the same transition ($b \to c$) or by others ($b \to u$).



The violation at tree level might require a new heavy mediator exchange!

Strategy

Model Independent Approach

Analysis of semileptonic decays for different quark transitions such as

- $b \to u$: $\begin{cases} \text{exclusive meson decays} \\ \text{inclusive meson and baryon (spin) decays} \end{cases}$
- $c \to s, d$: exclusive meson decays

starting from a general Hamiltonian structure

$$\mathcal{H}_{NP} = \underbrace{\mathcal{H}_{SM}}_{\gamma_\mu(1-\gamma_5)\otimes\gamma^\mu(1-\gamma_5)} + \sum_i \epsilon_i^X \underbrace{\mathcal{H}_X}_{\Gamma_M\otimes\Gamma^M}$$

The new (scalar, pseudoscalar, tensor) terms in the low-energy Hamiltonian encode the effects of BSM phenomena.

Predictions in a defined NP model

Study of rare decays $c \to u$ within 331 model

Exclusive $b \to u$ semileptonic decays

Exclusive $b \to u$ semileptonic decays

NP terms are weighted by complex lepton-flavour dependent couplings!

$$\mathcal{H}_{\rm eff}^{SM} = \frac{G_F\, V_{ub}}{\sqrt{2}} \left[\bar{\ell}\,\gamma^{\mu}\,(1-\gamma_5)\nu_\ell\right]\left[\bar{u}\,\gamma_\mu\,(1-\gamma_5)b\right] + \text{h.c.} \tag{2}$$

$$\begin{aligned}\mathcal{H}_{\rm eff}^{NP} = \frac{G_F\, V_{ub}}{\sqrt{2}} \Big[& (1+\epsilon_V^\ell)\,\left[\bar{\ell}\,\gamma^{\mu}\,(1-\gamma_5)\,\nu_\ell\right]\left[\bar{u}\,\gamma_\mu\,(1-\gamma_5)\,b\right] + \\ & + \epsilon_S^\ell\,\left[\bar{\ell}\,(1-\gamma_5)\nu_\ell\right]\left[\bar{u}\,b\right] + \epsilon_P^\ell\,\left[\bar{\ell}\,(1-\gamma_5)\nu_\ell\right]\left[\bar{u}\,\gamma_5\,b\right] + \\ & + \epsilon_T^\ell\,\left[\bar{\ell}\,\sigma^{\mu\nu}\,(1-\gamma_5)\,\nu_\ell\right]\left[\bar{u}\,\sigma_{\mu\nu}\,(1-\gamma_5)\,b\right]\Big] + \text{h.c.}\end{aligned} \tag{3}$$

	ϵ_V^ℓ	ϵ_S^ℓ	ϵ_P^ℓ	ϵ_T^ℓ
$B^- \to \ell^-\,\bar{\nu}_\ell$	✓		✓	
$\overline{B^0} \to \pi^+\ell^-\,\bar{\nu}_\ell$	✓	✓		✓
$\overline{B^0} \to \rho^+\,\ell^-\,\bar{\nu}_\ell$	✓		✓	✓
$\overline{B^0} \to a_1^+\,\ell^-\,\bar{\nu}_\ell$	✓	✓		✓

Exclusive $b \to u$ semileptonic decays

$B^- \to \ell^- \bar{\nu}_\ell$ decay rate

$$\Gamma = \frac{G_F^2\, |V_{ub}|^2\, f_B^2\, m_B^3}{8\,\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \underbrace{\left| \left(\frac{m_\ell}{m_B}\right)(1+\epsilon_V^\ell) + \frac{m_B}{m_b + m_u}\,\epsilon_P^\ell \right|^2}_{\epsilon_P^\ell \text{ removes the helicity suppression}} \tag{4}$$

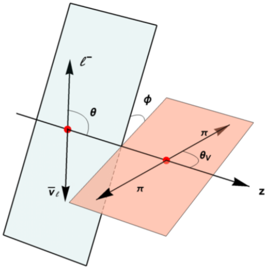
$\overline{B^0} \to \pi^+ \ell^- \bar{\nu}_\ell$ differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\, |V_{ub}|^2 \sqrt{\lambda}}{128\, m_B^3\, \pi^3\, q^2} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \Bigg\{ \left| m_\ell\,(1+\epsilon_V^\ell) + \frac{q^2}{m_b - m_u}\,\epsilon_S^\ell \right|^2 (m_B^2 - m_\pi^2)^2\, f_0^2 + \lambda \left[\frac{1}{3} \left| m_\ell\,(1+\epsilon_V^\ell)\, f_+ + \frac{4\,q^2}{m_B + m_\pi}\,\epsilon_T^\ell\, f_T \right|^2 + \frac{2\,q^2}{3} \left| (1+\epsilon_V^\ell)\, f_+ + \frac{4\, m_\ell}{m_B + m_\pi}\,\epsilon_T^\ell\, f_T \right|^2 \right] \Bigg\} \tag{5}$$

$$f_i \equiv f_i(q^2) \qquad \lambda \equiv \lambda(m_B^2, m_\pi^2, q^2) \tag{6}$$

Exclusive $b \to u$ semileptonic decays

$\overline{B^0} \to \rho^+(\to \pi\,\pi)\ell^-\,\bar{\nu}_\ell$ kinematics



- z is the flight direction of the B meson;
- θ_V is the angle between the pion flight direction and the z direction;
- θ is the angle between the lepton pair flight direction and the z direction;
- ϕ is the angle between the lepton plane and the hadron plane.

Exclusive $b \to u$ semileptonic decays

$\overline{B^0} \to \rho^+(\to \pi\,\pi)\ell^- \,\bar{\nu}_\ell$ fully differential distribution

$$\frac{d^4\Gamma(\overline{B} \to \rho(\to \pi\,\pi)\ell^- \,\bar{\nu}_\ell)}{dq^2\; d\cos\theta\; d\phi\; d\cos\theta_V} = \frac{3\,G_F^2\,|V_{ub}|^2\,\mathcal{B}(\rho \to \pi\,\pi)}{128\,m_B^2\,(2\,\pi)^4}\,|\overrightarrow{p_\rho}|\left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times$$
$$\times\,\{I_{1s}^\rho\,\sin^2\theta_V + I_{1c}^\rho\,\cos^2\theta_V + (I_{2s}^\rho\,\sin^2\theta_V + I_{2c}^\rho\,\cos^2\theta_V)\,\cos 2\theta +$$
$$+\,I_3^\rho\,\sin^2\theta_V\,sin^2\theta\,\cos 2\phi + I_4^\rho\,\sin 2\theta_V\,sin2\theta\,\cos\phi +$$
$$+\,I_5^\rho\,\sin 2\theta_V\,sin\theta\,\cos\phi + (I_{6s}^\rho\,\sin^2\theta_V + I_{6c}^\rho\,\cos^2\theta_V)\,\cos\theta +$$
$$+\,I_7^\rho\,\sin 2\theta_V\,sin\theta\,\sin\phi\} \tag{7}$$

Angular coefficient functions $(\epsilon_V^\ell, \not{\epsilon}_S^\ell, \epsilon_P^\ell, \epsilon_T^\ell)$

$$I_i = |1 + \epsilon_V^\ell|^2\,I_i^{SM} + |\epsilon_P^\ell|^2\,I_i^{NP,P} + |\epsilon_T^\ell|^2\,I_i^{NP,T} + 2\,\mathrm{Re}[\epsilon_P^\ell\,(1 + \epsilon_V^{\ell *})]\,I_i^{INT,P} +$$
$$+\,2\,\mathrm{Re}[\epsilon_T^\ell\,(1 + \epsilon_V^{\ell *})]\,I_i^{INT,T} + 2\,\mathrm{Re}[\epsilon_P^\ell\,\epsilon_T^{\ell *}]\,I_i^{INT,PT}\,, \qquad i = 1s, 1c, \ldots, 6c\,, \tag{8a}$$
$$I_7 = 2\,\mathrm{Im}[\epsilon_P^\ell\,(1 + \epsilon_V^{\ell *})]\,I_7^{INT,P} + 2\,\mathrm{Im}[\epsilon_T^\ell\,(1 + \epsilon_V^{\ell *})]\,I_7^{INT,T} + 2\,\mathrm{Im}[\epsilon_P^\ell\,\epsilon_T^{\ell *}]\,I_7^{INT,PT}\,. \tag{8b}$$

- Form Factors: A. Bharucha, D. M. Straub, and R. Zwicky, *$B \to V\ell^+\ell^-$ in the Standard Model from light-cone sum rules*, *JHEP* **08** (2016) 098, [arXiv:1503.05534].

Angular coefficient functions for $\overline{B^0} \to \rho^+(\to \pi\,\pi)\ell^- \,\bar{\nu}_\ell$

P. Colangelo, F. De Fazio, F. L., Phys.Rev.D 100 (2019) 7, 075037, arXiv:1906.07068

	SM
I_{1s}	$\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{1c}	$4m_\ell^2 H_t^2 + 2H_0^2(m_\ell^2 + q^2)$
I_{2s}	$-\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$
I_{2c}	$2H_0^2(m_\ell^2 - q^2)$
I_3	$2H_+H_-(m_\ell^2 - q^2)$
I_4	$H_0(H_+ + H_-)(m_\ell^2 - q^2)$
I_5	$-2H_t(H_+ + H_-)m_\ell^2 - 2H_0(H_+ - H_-)q^2$
I_{6s}	$2(H_+^2 - H_-^2)q^2$
I_{6c}	$-8H_tH_0m_\ell^2$
I_7	0

	NP, P	NP, T
I_{1s}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](3m_\ell^2 + q^2)$
I_{1c}	$4H_t^2 \frac{q^4}{(m_b+m_u)^2}$	$\frac{1}{8}(H_L^{NP})^2(m_\ell^2 + q^2)$
I_{2s}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](m_\ell^2 - q^2)$
I_{2c}	0	$-\frac{1}{8}(H_L^{NP})^2(m_\ell^2 - q^2)$
I_3	0	$-8H_+^{NP}H_-^{NP}(m_\ell^2 - q^2)$
I_4	0	$-\frac{1}{2}H_L^{NP}(H_+^{NP} + H_-^{NP})(m_\ell^2 - q^2)$
I_5	0	$-H_L^{NP}(H_+^{NP} - H_-^{NP})m_\ell^2$
I_{6s}	0	$8[(H_+^{NP})^2 - (H_-^{NP})^2]m_\ell^2$
$I_{6c,7}$	0	0

	INT, P	INT, T	INT, PT
I_{1s}	0	$-4[H_+^{NP}H_+ + H_-^{NP}H_-]m_\ell\sqrt{q^2}$	0
I_{1c}	$4H_t^2 \frac{m_\ell q^2}{m_b+m_u}$	$-H_L^{NP}H_0m_\ell\sqrt{q^2}$	0
$I_{2s,2c,3,4}$	0	0	0
I_5	$-H_t(H_+ + H_-)\frac{m_\ell q^2}{m_b+m_u}$	$\frac{1}{4}[H_L^{NP}(H_+ - H_-) + 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$2H_t(H_+^{NP} + H_-^{NP})\frac{(q^2)^{3/2}}{m_b+m_u}$
I_{6s}	0	$-4(H_+^{NP}H_+ - H_-^{NP}H_-)m_\ell\sqrt{q^2}$	0
I_{6c}	$-4H_tH_0\frac{m_\ell q^2}{m_b+m_u}$	$H_L^{NP}H_tm_\ell\sqrt{q^2}$	$H_L^{NP}H_t\frac{(q^2)^{3/2}}{m_b+m_u}$
I_7	$-H_t(H_+ - H_-)\frac{m_\ell q^2}{m_b+m_u}$	$\frac{1}{4}[H_L^{NP}(H_+ + H_-) - 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$2H_t(H_+^{NP} - H_-^{NP})\frac{(q^2)^{3/2}}{m_b+m_u}$

The H's are functions of the form factors (hadronic quantities).

Parameter constraints

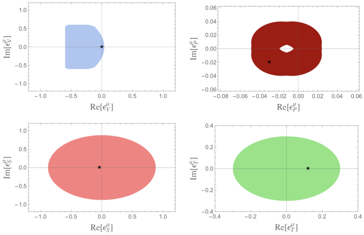
Experimental input constraining the NP parameters

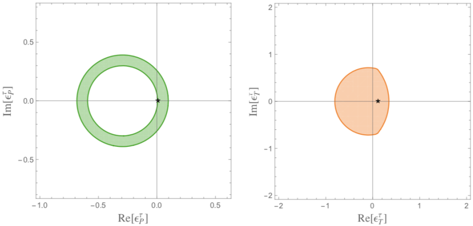
$$\mathcal{B}(B^- \to \mu^- \bar{\nu}_\mu) = (6.46 \pm 2.2 \pm 1.60) \times 10^{-7}$$
$$\mathcal{B}(\overline{B^0} \to \pi^+ \ell^- \bar{\nu}_\ell) = (1.50 \pm 0.06) \times 10^{-4}$$
$$\mathcal{B}(\overline{B^0} \to \rho^+ \ell^- \bar{\nu}_\ell) = (2.94 \pm 0.21) \times 10^{-4}$$

$$\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = (1.09 \pm 0.24) \times 10^{-4}$$
$$\mathcal{B}(\overline{B^0} \to \pi^+ \tau^- \bar{\nu}_\tau) < 2.5 \times 10^{-4}$$

- **Particle Data Group** Collaboration, M. Tanabashi et al., *Review of Particle Physics*, *Phys. Rev.* **D98** (2018) 030001.
- **Belle** Collaboration, A. Sibidanov et al., *Search for $B^0 \to \mu^- \bar{\nu}_\mu$ Decays at the Belle Experiment*, *Phys. Rev. Lett.* **121** (2018) 031801, [arXiv:1712.04123].
- **Belle** Collaboration, P. Hamer et al., *Search for $B^0 \to \pi^- \tau^+ \nu_\tau$ with hadronic tagging at Belle*, *Phys. Rev.* **D93** (2016) 032007, [arXiv:1509.06521].

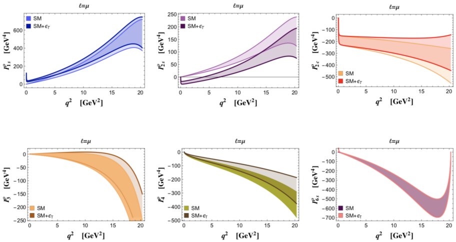
$$\chi^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{B}_i^{th} - \mathcal{B}_i^{exp}}{\Delta \mathcal{B}_i^{exp}} \right)^2 \quad \longrightarrow \quad \text{used to select benchmark points}$$



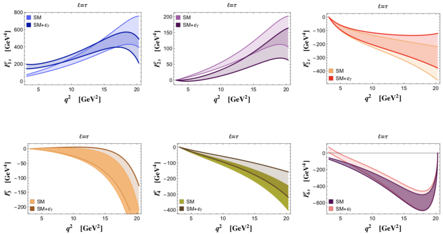


Observables in $\overline{B^0} \to \rho^+ \, \ell^- \, \bar{\nu}_\ell$

Angular coefficient functions (take contribution only from ϵ_T^μ)

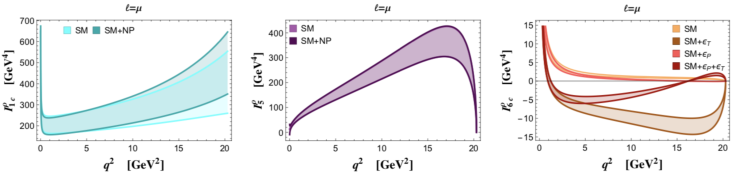


Angular coefficient functions (take contribution only from ϵ_T^τ)

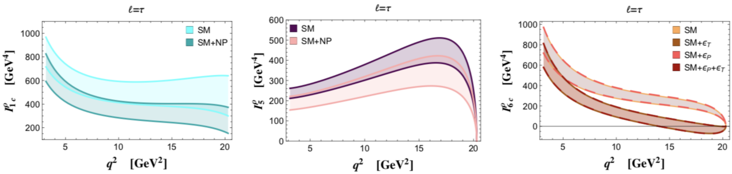


Observables in $\overline{B^0} \to \rho^+ \ell^- \bar{\nu}_\ell$

Angular coefficient functions (take contribution from ϵ_T^μ and ϵ_P^μ)

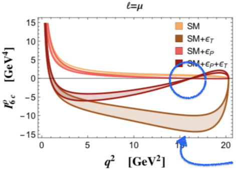


Angular coefficient functions (take contribution from ϵ_T^τ and ϵ_P^τ)



Observables in $\overline{B^0} \to \rho^+ \ell^- \bar{\nu}_\ell$

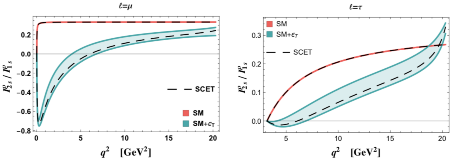
$I^{\rho}_{6c}(q^2)$



Observations!

- Depending on the ϵ^{ℓ}_{X}, the profile function may either change $\leftarrow$ or even have zeros $\bigcirc$.

$R^{\rho}_{2s/1s}(q^2) = I^{\rho}_{2s}(q^2)/I^{\rho}_{1s}(q^2)$



Observations!

- In the SM, the ratio is form factors independent, so that it is not affected by theoretical uncertainties;
- Introducing NP coefficients, there is one zero.

Observables in $\overline{B^0} \to \rho^+ \ell^- \bar{\nu}_\ell$

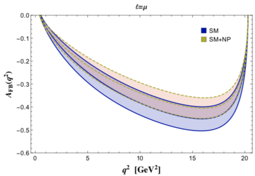
Forward-backward lepton asymmetries

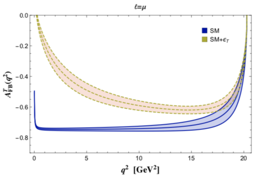
$$A_{FB}(q^2) = \left[\int_0^1 d\cos\theta \, \frac{d^2\Gamma}{dq^2\, d\cos\theta} - \int_{-1}^0 d\cos\theta \, \frac{d^2\Gamma}{dq^2\, d\cos\theta} \right] \Big/ \frac{d\Gamma}{dq^2} \tag{9}$$

$$A_{FB}(q^2) = \frac{3(I_{6c}^{\rho} + 2I_{6s}^{\rho})}{6I_{1c}^{\rho} + 12I_{1s}^{\rho} - 2I_{2c}^{\rho} - 4I_{2s}^{\rho}} \tag{10}$$

$$A_{FB}^{T}(q^2) = \frac{3I_{6s}^{\rho}}{6I_{1s}^{\rho} - 2I_{2s}^{\rho}} \tag{11}$$

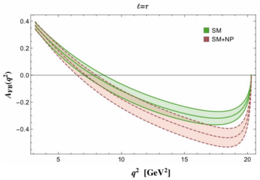
small deviation (NP)

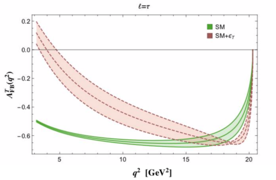




large deviation (only ϵ_T)

the position of the zero is modified (NP)





there is a zero that is absent in the SM (only ϵ_T)

Remarks...

... about the mode $\overline{B^0} \to a_1^+ \, \ell^- \, \bar{\nu}_\ell$

$$\frac{d^4\Gamma(\overline{B} \to a_1(\to \rho_{\|(\perp)}\, \pi)\ell^- \, \bar{\nu}_\ell)}{dq^2 \, d\cos\theta \, d\phi \, d\cos\theta_V} \sim f(\epsilon_V^\ell, \epsilon_S^\ell, \not{\epsilon}_P^\ell, \epsilon_T^\ell)$$

$$I_{i,\|}^X \neq I_{i,\perp}^X \quad \forall\, i \in \{\, 1s, 1c, \ldots, 7 \,\} \quad \text{and} \quad \forall\, X \in \{\, SM, INT, NP \,\}$$

Form factors (uncertanty of about 20%) from:

- R.-H. Li, C.-D. Lu, and W. Wang, *Transition form factors of B decays into p-wave axial-vector mesons in the perturbative QCD approach, Phys. Rev.* **D79** (2009) 034014, [arXiv:0901.0307].

Results

Similarly to $R(D^{(*)})$, it is defined

$$R_{M_u} = \frac{\mathcal{B}(B \to M_u \, \tau^- \, \bar{\nu}_\tau)}{\mathcal{B}(B \to M_u \, \ell^- \, \bar{\nu}_\ell)} \,, \quad \ell = e, \mu \,.$$

	SM	NP (benchmark point)
R_π	0.60 ± 0.01	0.75 ± 0.02
R_ρ	0.53 ± 0.02	0.49 ± 0.02
R_{a_1}	0.44 ± 0.07	0.67 ± 0.12

Remarks

- Leptonic/semileptonic decays induced by $b \to u$ transition are differently sensitive to the lepton-flavour dependent couplings.
- The fully differential angular distributions in $B \to \rho, a_1$ semileptonic decays allow to obtain several observable effects at the present experimental facilities.

Inclusive $b \to U = c, u$ semileptonic transitions: the case of Λ_b

Inclusive decay width

Calculation of inclusive semileptonic decays:

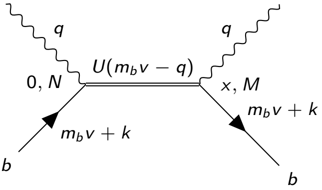
- exploits the large b-quark mass: Heavy Quark Expansion (HQE) $\rightarrow$ power series in $1/m_b$;
- small theoretical uncertainty:
  - requires few parameters to be fixed by the experiments;
  - avoids the problem of form factors (that affects exclusive modes).

Our analysis:

- Full consideration of the dependence on the spin s_μ of the decaying baryon at order $\mathcal{O}(m_b^{-3})$ (previous analyses partially done at order $\mathcal{O}(m_b^{-2})$).
- Inclusion of all the NP operators at the same order $\mathcal{O}(m_b^{-3})$ (previous analyses included NP only at the leading order).
- $m_\ell \neq 0$: evaluation of the fully differential rate considering non vanishing lepton masses.

Outline of the calculation

$$d\Gamma = \underbrace{d\Sigma}_{\text{phace space}} \frac{G_F^2\,|V_{Ub}|^2}{4M_H} \sum_{i,j} g_i^*\, g_j \underbrace{(W^{ij})_{MN}}_{\text{hadronic tensor}} \underbrace{(L^{ij})^{MN}}_{\text{leptonic tensor}}$$



$$(W^{ij})_{MN} = \frac{1}{\pi}\,\mathrm{Im}\left[(T^{ij})_{MN}\right] \qquad \text{(optical theorem)}$$

$$(T^{ij})_{MN} = i\int d^4x\, e^{iq\cdot x}\,\langle H_b|\mathbb{T}\left\{J^{(i)}_{q,M}{}^{\dagger}(x)\, J^{(j)}_{q,N}(0)\right\}|H_b\rangle$$

Operator product expansion (OPE) in the inverse b quark mass:

$$d\Gamma = d\Gamma_0 + \frac{\langle H_b|\bar{b}_v\,(iD)^2\,b_v|H_b\rangle}{m_b^2}\,d\Gamma_2 + \frac{\langle H_b|\bar{b}_v\,(iD)^3\,b_v|H_b\rangle}{m_b^3}\,d\Gamma_3 + \dots$$

The higher the order of the expansion, the greater the number of the parameters.

$$\mathcal{O}\left(\frac{1}{m_b^n}\right)\dots\begin{cases}\mathcal{O}\left(\frac{1}{m_b^3}\right)\begin{cases}\mathcal{O}\left(\frac{1}{m_b^2}\right)\begin{cases}-2\,M_H\,\hat{\mu}_\pi^2 = \langle H_b|\bar{b}_v\,iD^\mu\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\mu}_G^2 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ 2\,M_H\,\hat{\rho}_D^3 = \langle H_b|\bar{b}_v\,iD^\mu\,(iv\cdot D)\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\rho}_{LS}^3 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,(iv\cdot D)\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ \dots\end{cases}$$

For a heavy baryon, the dependence on the spin four-vector s_μ must be kept.

Results

P. Colangelo, F. De Fazio, F. L., JHEP 11 (2020) 032, arXiv:2006.13759

The whole calculation requires

$$\mathcal{M}^{\mu_1\mu_2\cdots\mu_n} \equiv \langle H_b|\bar{b}_v \, iD^{\mu_1} \, iD^{\mu_2} \dots iD^{\mu_n} \, b_v|H_b\rangle$$

never computed before for a polarized baryon.

Results

$$\mathcal{M}^{\rho\sigma\lambda} = M_H\left[\left(\frac{\hat\rho_D^3}{3}\,\Pi^{\rho\lambda}\,v^\sigma\,\mathbf{P}_+ + \frac{\hat\rho_{LS}^3}{6}\,v^\sigma\,i\,\epsilon^{\rho\lambda\alpha\beta}\,v_\alpha\,\mathbf{S}_\beta\right) - \left(\frac{\hat\rho_D^3}{3}\,\Pi^{\rho\lambda}\,v^\sigma\,s^\mu\,\mathbf{S}_\mu - \frac{\hat\rho_{LS}^3}{2}\,v^\sigma\,i\,\epsilon^{\rho\lambda\alpha\beta}\,v_\alpha\,s_\beta\,\mathbf{P}_+\right)\right]$$

$$\mathcal{M}^{\rho\sigma} = M_H\Bigg[\Bigg(\frac{\hat\mu_\pi^2}{3}\,\Pi^{\rho\sigma}\,\mathbf{P}_+ + \frac{\hat\mu_G^2}{6}\,i\,\epsilon^{\rho\sigma\alpha\beta}\,v_\alpha\,\mathbf{S}_\beta +$$
$$+\frac{\hat\rho_D^3+\hat\rho_{LS}^3}{24m_b}\left(\mathbf{4}\left(i\,\epsilon^{\rho\sigma\alpha\beta}\,v_\alpha\,\mathbf{S}_\beta - v^\rho\,v^\sigma\,\not{v}\right) + v^\rho\left(\mathbf{2}\,\gamma^\sigma + \not{v}\,\gamma^\sigma - \gamma^\sigma\,\not{v}\right) + v^\sigma\left(\mathbf{2}\,\gamma^\rho + \not{v}\,\gamma^\rho - \gamma^\rho\,\not{v}\right)\right)\Bigg) +$$
$$+\Bigg(-\frac{\hat\mu_\pi^2}{3}\,\Pi^{\rho\sigma}\,\mathbf{P}_+\,\not{s}\,\gamma_5 + \frac{\hat\mu_G^2}{2}\,i\,\epsilon^{\rho\sigma\alpha\beta}\,v_\alpha\,s_\beta\,\mathbf{P}_+ +$$
$$+\frac{\hat\rho_D^3}{12m_b}\left(\mathbf{6}\,i\,\epsilon^{\rho\sigma\alpha\beta}\,v_\alpha\,s_\beta + i\left(v^\rho\,\epsilon^{\sigma\mu\alpha\beta} - v^\sigma\,\epsilon^{\rho\mu\alpha\beta}\right)v_\alpha\,s_\beta\,\gamma_\mu +\right.$$
$$\left. + s^\rho\,v^\sigma\,\not{v}\,\gamma_5 + v^\rho\,s^\sigma\left(\mathbf{2}\,\gamma_5 + \not{v}\,\gamma_5\right) + \left(\mathbf{2}\,v^\rho\,v^\sigma\,\not{v} - v^\rho\,\gamma^\sigma - v^\sigma\,\gamma^\rho\right)\not{s}\,\gamma_5\right) +$$
$$+\frac{\hat\rho_{LS}^3}{8m_b}\left(\mathbf{4}\,i\,\epsilon^{\rho\sigma\alpha\beta}\,v_\alpha\,s_\beta + i\left(v^\rho\,\epsilon^{\sigma\mu\alpha\beta} - v^\sigma\,\epsilon^{\rho\mu\alpha\beta}\right)v_\alpha\,s_\beta\,\gamma_\mu +\right.$$
$$\left. + \left(s^\rho\,v^\sigma + v^\rho\,s^\sigma\right)\gamma_5 + \left(\mathbf{2}\,v^\rho\,v^\sigma\,\not{v} - v^\rho\,\gamma^\sigma - v^\sigma\,\gamma^\rho\right)\not{s}\,\gamma_5\right)\Bigg)\Bigg]$$

$$\mathcal{M}^{\rho} = M_H\Bigg[\left(\frac{\hat\mu_\pi^2-\hat\mu_G^2}{12m_b}\left(v^\rho\left(\mathbf{3}+\mathbf{5}\,\not{v}\right) - \mathbf{2}\,\gamma^\rho\right) - \frac{\hat\rho_D^3+\hat\rho_{LS}^3}{12m_b^2}\left(\mathbf{4}\,v^\rho\,\not{v} - \gamma^\rho\right)\right) +$$
$$+\Bigg(-\frac{\hat\mu_\pi^2}{12m_b}\left(\left(v^\rho\left(\mathbf{3}+\mathbf{5}\,\not{v}\right) - \mathbf{2}\,\gamma^\rho\right)\not{s}\,\gamma_5 + \mathbf{4}\,s^\rho\,\mathbf{P}_+\,\gamma_5\right) + \frac{\hat\mu_G^2}{4m_b}\left(\left(v^\rho\left(\mathbf{1}+\mathbf{2}\,\not{v}\right) - \gamma^\rho\right)\not{s}\,\gamma_5 + s^\rho\,\gamma_5\right) +$$
$$+\frac{\hat\rho_D^3}{12m_b^2}\left(\left(v^\rho\left(\mathbf{1}+\mathbf{4}\,\not{v}\right) - \mathbf{2}\,\gamma^\rho\right)\not{s}\,\gamma_5 + s^\rho\left(\mathbf{2}-\not{v}\right)\gamma_5\right) + \frac{\hat\rho_{LS}^3}{8m_b^2}\left(\left(\mathbf{3}\,v^\rho\,\not{v} - \gamma^\rho\right)\not{s}\,\gamma_5 + s^\rho\,\gamma_5\right)\Bigg)\Bigg]$$

$$\mathcal{M} = M_H\left[\left(\mathbf{P}_+ - \frac{\hat\mu_\pi^2-\hat\mu_G^2}{4m_b^2}\right) + \left(\mathbf{P}_+ + \frac{\hat\mu_\pi^2}{24m_b^2}\left(\mathbf{7}+\mathbf{5}\,\not{v}\right) - \frac{\hat\mu_G^2}{8m_b^2}\left(\mathbf{3}+\not{v}\right) - \frac{\hat\rho_D^3}{6m_b^3}\,\mathbf{P}_-\right)\not{s}\,\gamma_5\right]$$

Inclusive decay width

Fully differential decay distribution (H_b rest frame)

$$\frac{d^4\Gamma}{dE_\ell\, dq^2\, dq_0\, d\cos\theta_P}$$

- E_ℓ: lepton energy, with $p_\ell = (E_\ell, \boldsymbol{p}_\ell)$;
- q^2: dilepton invariant mass, with $q = (q_0, \boldsymbol{q})$;
- θ_P: angle between the hadron spin $\boldsymbol{s}$ and the lepton 3-momentum $\boldsymbol{p}_\ell$, i.e. $\cos\theta_P = \frac{\boldsymbol{s}\cdot\boldsymbol{p}_\ell}{|\boldsymbol{s}||\boldsymbol{p}_\ell|}$.

$$\Gamma(H_b \to X_U\, \ell^-\, \overline{\nu}_\ell) = \Gamma_0 \sum_{i,j} g_i^*\, g_j \left[\mathcal{C}_0^{(i,j)} + \frac{\hat{\mu}_\pi^2}{m_b^2}\, \mathcal{C}_{\hat{\mu}_\pi^2}^{(i,j)} + \frac{\hat{\mu}_G^2}{m_b^2}\, \mathcal{C}_{\hat{\mu}_G^2}^{(i,j)} + \frac{\hat{\rho}_D^3}{m_b^3}\, \mathcal{C}_{\hat{\rho}_D^3}^{(i,j)} + \frac{\hat{\rho}_{LS}^3}{m_b^3}\, \mathcal{C}_{\hat{\rho}_{LS}^3}^{(i,j)}\right]$$

$$\Gamma_0 = \frac{G_F^2 |V_{Ub}|^2 m_b^5}{192\pi^3}\,, \qquad \text{partonic term (free quark decay)}$$

The hadron polarization leads to new observables.
Polarized Λ_b can be produced in Z^0 decays $\to$ perspective measurements for *Future Circular Collider* (FCC) (leptonic).

Inclusive decay width

Definition of angular differential decay distributions and ratio

$$\frac{d\Gamma}{d\cos\theta_P} = A_\ell^U + B_\ell^U \cos\theta_P \qquad A_\ell^U = \frac{1}{2}\Gamma(H_b \to X_U\,\ell^-\,\overline{\nu}_\ell)$$

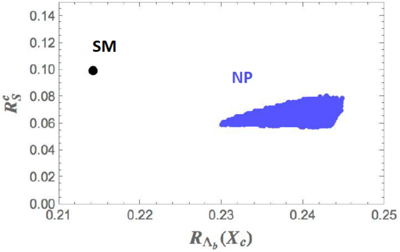
Analogously to $R(D^{(*)})$, the ratios are defined:

$$R_{\Lambda_b}(X_U) = \frac{\Gamma(\Lambda_b \to X_U\,\tau^-\,\overline{\nu}_\tau)}{\Gamma(\Lambda_b \to X_U\,\mu^-\,\overline{\nu}_\mu)} = \frac{A_\tau^U}{A_\mu^U}$$

$$R_S^U = \frac{B_\tau^U}{B_\mu^U}$$

	SM	NP
$R_{\Lambda_b}(X_u)$	0.234	0.238
$R_{\Lambda_b}(X_c)$	0.214	0.240
R_S^u	0.081	0.091
R_S^c	0.100	0.074

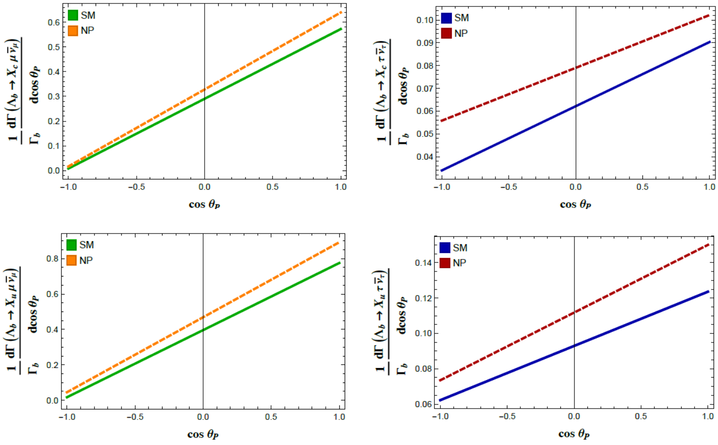
Correlation between $R_{\Lambda_b}(X_c)$ and R_S^c



Hardly measurable at LHC!
Hopefully realizable with high luminosity lepton machine

Normalized angular distribution: $\frac{1}{\Gamma_b}\frac{d\Gamma}{d\cos\theta_P}$

Linear dependence on $\cos\theta_P$:
NP modifies both the slope and the intercept.



The angular distribution for Λ_b, linear in $\cos\theta_P$, is particularly sensitive to NP.

Exclusive $c \to D = s, d$ semileptonic decays

$B_c \to B_{s,d}^{(*)}\,\bar{\ell}\,\nu_\ell$ applying Heavy Quark (HQ) Spin Symmetry

Infinite HQ mass limit $m_Q \gg \Lambda_{QCD}$: the HQ spin decouples $\Rightarrow$ HQ Spin Symmetry .

From 10 to 2 form factors: theoretical uncertainties greatly reduced!

$B_c \to P = B_{s,d}$

$$\langle P(p')|\bar{q}\,\Gamma\,Q|B_c(p)\rangle \sim [f_{+,0,T}]^{B_c\to P}(q^2)$$
$$\langle P(v,k)|\bar{q}\,\Gamma\,Q|B_c(v)\rangle \sim \Omega_{1,2}(y)$$

$$\Rightarrow \begin{cases} f_+ = \sqrt{\frac{m_P}{m_{B_c}}}\,[\Omega_1 + (m_{B_c} - m_P)\,a_0\,\Omega_2] \\ f_0 = \sqrt{\frac{m_P}{m_{B_c}}}\,\frac{1}{m_{B_c}^2 - m_P^2}\,[(m_{B_c}^2 - m_P^2 + q^2)\,\Omega_1 + \\ \qquad +(m_{B_c} + m_P)\,((m_{B_c} - m_P)^2 - q^2))\,a_0\,\Omega_2] \\ f_T = \sqrt{\frac{m_P}{m_{B_c}}}\,(m_{B_c} + m_P)\,a_0\,\Omega_2 \end{cases}$$

$B_c \to V = B_{s,d}^*$

$$\langle V(p',\epsilon)|\bar{q}\,\Gamma\,Q|B_c(p)\rangle \sim [V, A_{0,1,2}, T_{0,1,2}]^{B_c\to V}(q^2)$$
$$\langle V(v,k,\epsilon)|\bar{q}\,\Gamma\,Q|B_c(v)\rangle \sim \Omega_{1,2}(y)$$

$$\Rightarrow \begin{cases} V = \sqrt{\frac{m_V}{m_{B_c}}}\,(m_{B_c} + m_V)\,a_0\,\Omega_2 \\ A_0 = \frac{1}{2\sqrt{m_{B_c} m_V}}\,[2\,m_{B_c}\,\Omega_1 + (m_{B_c}^2 - m_V^2 + q^2)\,a_0\,\Omega_2] \\ A_1 = 2\,\frac{\sqrt{m_{B_c}\,m_V}}{m_{B_c} + m_V}\,\Omega_1 \\ A_2 = -\sqrt{\frac{m_V}{m_{B_c}}}\,(m_{B_c} + m_V)\,a_0\,\Omega_2 \\ T_0 = 0 \\ T_1 = 2\sqrt{\frac{m_V}{m_{B_c}}}\,[\Omega_1 - m_V\,a_0\,\Omega_2] \\ T_2 = 2\sqrt{m_{B_c}\,m_V}\,a_0\,\Omega_2 \end{cases}$$

Results in the SM...

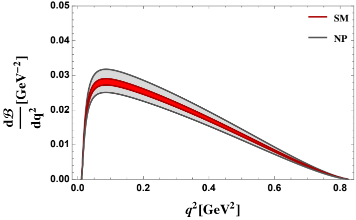
P. Colangelo, F. De Fazio, F. L., Phys.Rev.D 103 (2021) 7, 075019, arXiv:2102.05365

SM	$\mathcal{B}(B_c^+ \to B_a\, \ell^+\, \nu_\ell)$	$\mathcal{B}(B_c^+ \to B_a^*\, \ell^+\, \nu_\ell)$
$a = s, \ell = \mu$	$1.25(4) \times 10^{-2} \left(\frac{\lvert V_{cs}\rvert}{0.987}\right)^2$	$3.0(1) \times 10^{-2} \left(\frac{\lvert V_{cs}\rvert}{0.987}\right)^2$
$a = s, \ell = e$	$1.31(4) \times 10^{-2} \left(\frac{\lvert V_{cs}\rvert}{0.987}\right)^2$	$3.2(1) \times 10^{-2} \left(\frac{\lvert V_{cs}\rvert}{0.987}\right)^2$
$a = d, \ell = \mu$	$8.3(5) \times 10^{-4} \left(\frac{\lvert V_{cd}\rvert}{0.221}\right)^2$	$20(1) \times 10^{-4} \left(\frac{\lvert V_{cd}\rvert}{0.221}\right)^2$
$a = d, \ell = e$	$8.7(5) \times 10^{-4} \left(\frac{\lvert V_{cd}\rvert}{0.221}\right)^2$	$21(1) \times 10^{-4} \left(\frac{\lvert V_{cd}\rvert}{0.221}\right)^2$

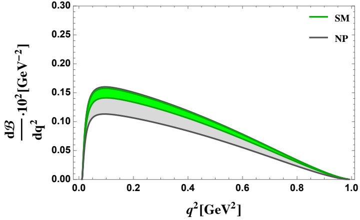
... and including NP

Differential Branching Ratios

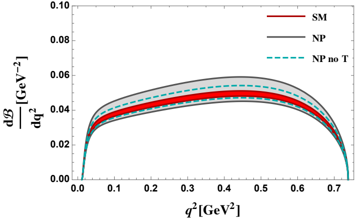
$B_c \to B_s\, \mu^+ \nu_\mu$



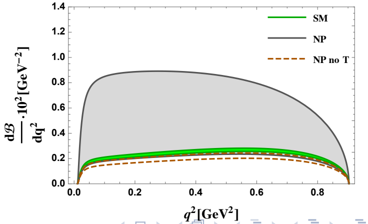
$B_c \to B_d\, \mu^+ \nu_\mu$



$B_c \to B_s^*\, \mu^+ \nu_\mu$



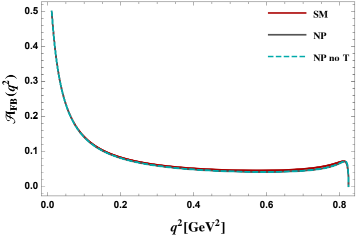
$B_c \to B_d^*\, \mu^+ \nu_\mu$



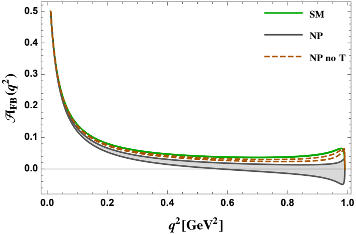
... and including NP

Forward-Backward Lepton Asymmetry

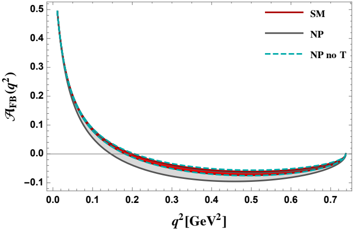
$B_c \to B_s\, \mu^+ \nu_\mu$



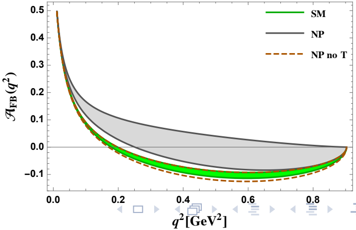
$B_c \to B_d\, \mu^+ \nu_\mu$



$B_c \to B_s^*\, \mu^+ \nu_\mu$

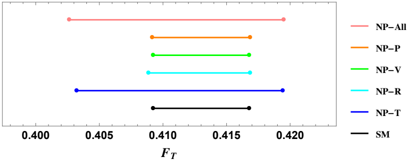


$B_c \to B_d^*\, \mu^+ \nu_\mu$

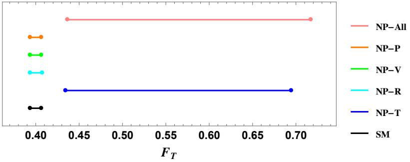


$$F_T = \frac{\Gamma_T}{\Gamma_T + \Gamma_L}$$

$B_c \to B_s^* \mu^+ \nu_\mu$



$B_c \to B_d^* \mu^+ \nu_\mu$



	$F_T(B_s^*)$	$F_T(B_d^*)$
SM	0.413(4)	0.400(10)
SM + T	0.411(8)	0.57(14)

- $F_T(B_s^*)$: it remains smaller than 1/2 when the NP operators are included, with the main effect due to the T operator
- $F_T(B_d^*)$: the integrated longitudinal width is larger than the transverse one in SM. The T operator may reverse such a hierarchy.

Group Research

Work in progress

Investigation of rare processes involving $c \to u$ transitions such as

- $B_c \to B_u^{(*)} \nu \bar{\nu}$
- $D^0 - \overline{D^0}$ mixing
- $D^0 \to \mu^+ \mu^-$
- CP asymmetry in D^0 decays

scrutinizing all the observables related to them.

Due to very efficient GIM cancellation these processes are strongly suppressed in the SM $\to$ deviations would be a clean signal of NP!

Analysis in a defined NP model: 331 model.

$$\downarrow$$

$$\text{Extended Gauge Group} \to \mathcal{G}_{331} : SU_C(3) \otimes SU_L(3) \otimes U_X(1)$$

In 331 the number of fermion generations is necessarily equal to the number of colors.

P. Colangelo, F. De Fazio, F. L., BARI-TH/21-728

A. J. Buras, P. Colangelo, F. De Fazio, F. L., BARI-TH/21-729

What else we did

Holography

F. Giannuzzi, S. Nicotri, *$U(1)_A$ axial anomaly, η', and topological susceptibility in the holographic soft-wall model*, arXiv:2105.00923

P. Colangelo, F. De Fazio, N. Losacco, *Chaos in a $Q\bar{Q}$ system at finite temperature and baryon density*, Phys.Rev.D 102 (2020) 7, 074016, arXiv:2007.06980

P. Colangelo, F. L., *Configurational Entropy can disentangle conventional hadrons from exotica*, Phys.Lett.B 788 (2019) 500-504, arXiv:1811.05272

Spectroscopy

F. Giannuzzi, *Heavy pentaquark spectroscopy in the diquark model*, Phys.Rev.D 99 (2019) 9, 094006, arXiv:1903.04430

S. Campanella, P. Colangelo, F. De Fazio, *Excited heavy meson decays to light vector mesons: implications for spectroscopy*, Phys.Rev.D 98 (2018) 11, 114028, arXiv:1810.04492

Group Research

Conclusions and perspectives

- LFU may be an accidental symmetry at low-energy, probably no more valid at higher energy scales;
- The difference among the three lepton families (existing only because of the Yukawa interaction in SM) could be related to another level of interaction (gauge bosons, scalar fields, ...).

- Important complementarity between direct and indirect NP searches (energy frontier and intensity frontier);
- Simultaneous explanations of flavour anomalies are an important goal of the present research;
- Many exciting physics opportunities at present and future facilities.

Still a lot of work to do.

Thank you for your attention!

Back-up

Group Research

Exclusive $b \to u$ decays

Inclusive $b \to U = u, c$ decays

Exclusive $c \to D = s, d$ decays

Work in progress

$B^- \to \ell^- \bar{\nu}_\ell$ decay rate

$$\Gamma = \frac{G_F^2 \, |V_{ub}|^2 \, f_B^2 \, m_B^3}{8\,\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \underbrace{\left| \left(\frac{m_\ell}{m_B}\right) (1 + \epsilon_V^\ell) + \frac{m_B}{m_b + m_u} \, \epsilon_P^\ell \right|^2}_{\epsilon_P^\ell \text{ removes the helicity suppression}}$$

$$\langle 0 | \bar{u} \, \gamma_\mu \, \gamma_5 \, b | B^-(p) \rangle = i \, f_B \, p_\mu \; ,$$

- f_B: **Flavour Lattice Averaging Group** Collaboration, S. Aoki et al., *FLAG Review 2019*, `arXiv:1902.08191`.

$\overline{B^0} \to \pi^+ \ell^- \bar{\nu}_\ell$ form factors

$$\langle \pi(p') | \bar{u}\, \gamma_\mu\, b | \overline{B}(p) \rangle = f_+(q^2) \left[p_\mu + p'_\mu - \frac{m_B^2 - m_\pi^2}{q^2}\, q_\mu \right] + f_0(q^2)\, \frac{m_B^2 - m_\pi^2}{q^2}\, q_\mu \;,$$

$$\langle \pi(p') | \bar{u}\, b | \overline{B}(p) \rangle = f_S(q^2) \;,$$

$$\langle \pi(p') | \bar{u}\, \sigma_{\mu\nu}\, b | \overline{B}(p) \rangle = -i\, \frac{2\, f_T(q^2)}{m_B + m_\pi}\, [p_\mu\, p'_\nu - p_\nu\, p'_\mu] \;,$$

$$\langle \pi(p') | \bar{u}\, \sigma_{\mu\nu}\, \gamma_5\, b | \overline{B}(p) \rangle = -\frac{2\, f_T(q^2)}{m_B + m_\pi}\, \epsilon_{\mu\nu\alpha\beta}\, p^\alpha\, p'^\beta \;,$$

$$f_S(q^2) = \frac{m_B^2 - m_\pi^2}{m_b - m_u}\, f_0(q^2) \;.$$

The parametrization of the form factors, with the condition $f_+(0) = f_0(0)$, is obtained fitting the Light-Cone QCD sum rule results in the range $m_e^2 \leq q^2 \leq 12\,\mathrm{GeV}^2$ and the lattice QCD results for $16\,\mathrm{GeV}^2 \leq q^2$.

- I. Sentitemsu Imsong, A. Khodjamirian, T. Mannel, and D. van Dyk, *Extrapolation and unitarity bounds for the $B \to \pi$ form factor, JHEP* **02** (2015) 126, [arXiv:1409.7816].
- A. Khodjamirian and A. V. Rusov, *$B_s \to K \ell \nu_\ell$ and $B_{(s)} \to \pi(K) \ell^+ \ell^-$ decays at large recoil and CKM matrix elements, JHEP* **08** (2017) 112, [arXiv:1703.04765].
- **Flavour Lattice Averaging Group** Collaboration, S. Aoki et al., *FLAG Review 2019*, arXiv:1902.08191.

$\overline{B^0} \to \pi^+ \ell^- \bar{\nu}_\ell$ form factors

$$f_{+,T}(t) = \frac{1}{1 - \frac{q^2}{m^2_{pole}}} \sum_{n=0}^{N-1} a_n \left[z(t)^n - \frac{n}{N}(-1)^{n-N} z(t)^N \right] \;, \quad m_{pole} = m_{B^*} \;,$$

$$f_0(t) = \sum_{n=0}^{N-1} a_n z(t)^n \;,$$

where

$$z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \;, \quad t_+ = (m_B + m_\pi)^2 \;, \quad t_0 = (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2 \;.$$

Kinematic ranges

For $\overline{B} \to \pi \, \mu^- \, \bar{\nu}_\mu \quad \Rightarrow \quad -0.279 \le z \le 0.283 \;,$

For $\overline{B} \to \pi \, \tau^- \, \bar{\nu}_\tau \quad \Rightarrow \quad -0.279 \le z \le 0.257 \;.$

	$f_+^{B\to\pi}$	$f_0^{B\to\pi}$	$f_T^{B\to\pi}$
a_0	0.416(20)	0.492(20)	0.400(21)
a_1	−0.430	−1.35	−0.50
a_2	0.114	2.50	0.00076
a_3			0.534

$\overline{B^0} \to \pi^+ \ell^- \bar{\nu}_\ell$ in Large Energy Limit

Replacing $q^2 = m_B^2 + m_\pi^2 - 2\, m_B\, E$,

$$\frac{d\Gamma}{dE} = \frac{G_F^2\, |V_{ub}|^2 \sqrt{\lambda}}{64\, m_B^2\, \pi^3\, q^2} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \xi_\pi^2(E) \times$$
$$\times \left\{ \left| m_\ell\, (1 + \epsilon_V^\ell) + \frac{q^2}{m_b - m_u}\, \epsilon_S^\ell \right|^2 (m_B^2 - m_\pi^2)^2 \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2}\right)^2 + \right.$$
$$\left. \lambda \left[\frac{1}{3} \left| m_\ell\, (1 + \epsilon_V^\ell) + \frac{4\, q^2}{m_B + m_\pi}\, \epsilon_T^\ell \right|^2 + \frac{2\, q^2}{3} \left| (1 + \epsilon_V^\ell) + \frac{4\, m_\ell}{m_B + m_\pi}\, \epsilon_T^\ell \right|^2 \right] \right\} ,$$

What are the effects?

- All the form factors are replaced by a single one

$$\{ f_0(q^2), f_+(q^2), f_T(q^2) \} \longrightarrow \xi_\pi(E) ,$$

- There exist observables free from theoretical uncertainties

$$\frac{dR(\pi)^{\ell\ell'}}{dE} = \frac{d\Gamma}{dE}(\overline{B} \to \pi^+ \ell^- \bar{\nu}_\ell) \Big/ \frac{d\Gamma}{dE}(\overline{B} \to \pi^+ \ell'^- \bar{\nu}_{\ell'}) .$$

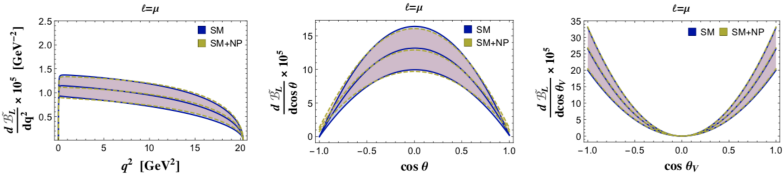
$\overline{B^0} \to \rho^+(\to \pi\,\pi)\ell^-\,\bar{\nu}_\ell$ fully differential distribution

$$\frac{d^4\Gamma(\overline{B} \to \rho(\to \pi\,\pi)\ell^-\,\bar{\nu}_\ell)}{dq^2\,d\cos\theta\,d\phi\,d\cos\theta_V} = \frac{3\,G_F^2\,|V_{ub}|^2\,\mathcal{B}(\rho \to \pi\,\pi)}{128\,m_B^2\,(2\,\pi)^4}\,|\overrightarrow{p_\rho}|\left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times$$
$$\times \{I_{1s}^\rho\,\sin^2\theta_V + I_{1c}^\rho\,\cos^2\theta_V + (I_{2s}^\rho\,\sin^2\theta_V + I_{2c}^\rho\,\cos^2\theta_V)\,\cos 2\theta +$$
$$+ I_3^\rho\,\sin^2\theta_V\,sin^2\theta\,\cos 2\phi + I_4^\rho\,\sin 2\theta_V\,sin 2\theta\,\cos\phi +$$
$$+ I_5^\rho\,\sin 2\theta_V\,sin\theta\,\cos\phi + (I_{6s}^\rho\,\sin^2\theta_V + I_{6c}^\rho\,\cos^2\theta_V)\,\cos\theta +$$
$$+ I_7^\rho\,\sin 2\theta_V\,sin\theta\,\sin\phi\}\,, \tag{12}$$

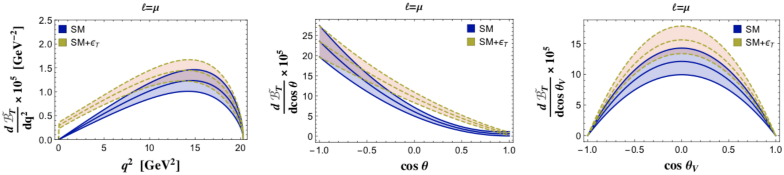
The angular coefficient functions depend on the form factors which mathematical behaviour is taken from

- Form Factors: A. Bharucha, D. M. Straub, and R. Zwicky, *$B \to V\ell^+\ell^-$ in the Standard Model from light-cone sum rules*, *JHEP* **08** (2016) 098, [arXiv:1503.05534].

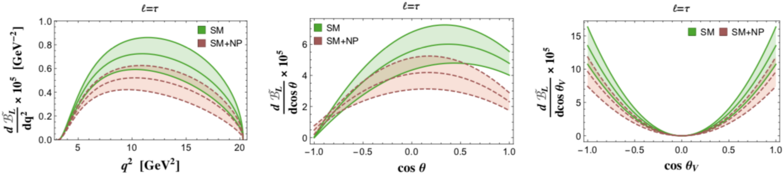
Longitudinal Differential Branching Ratios



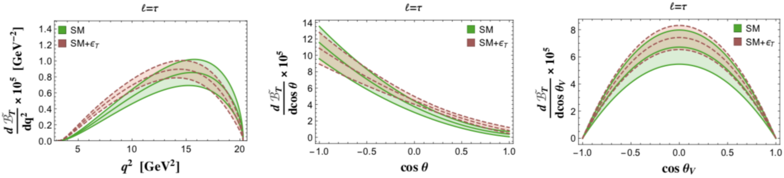
Transverse Differential Branching Ratios



Longitudinal Differential Branching Ratios



Transverse Differential Branching Ratios



Semileptonic modes

$\overline{B^0} \to a_1^+(\to \rho_{\parallel(\perp)}\,\pi)\ell^- \,\bar\nu_\ell$ Fully differential distribution

$$\frac{d^4\Gamma(\overline{B} \to a_1(\to \rho_{\parallel(\perp)}\,\pi)\ell^-\,\bar\nu_\ell)}{dq^2\, d\cos\theta\, d\phi\, d\cos\theta_V} = \frac{3\, G_F^2\, |V_{ub}|^2\, \mathcal{B}(a_1 \to \rho_{\parallel(\perp)}\,\pi)}{128\, m_B^2\, (2\,\pi)^4}\, |\overrightarrow{p_{a_1}}| \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times$$
$$\times \{ I^{a_1}_{1s,\parallel(\perp)} \sin^2\theta_V + I^{a_1}_{1c,\parallel(\perp)} (3 + \cos 2\theta_V) +$$
$$+ (I^{a_1}_{2s,\parallel(\perp)} \sin^2\theta_V + I^{a_1}_{2c,\parallel(\perp)} (3 + \cos 2\theta_V)) \cos 2\theta +$$
$$+ I^{a_1}_{3,\parallel(\perp)} \sin^2\theta_V\, sin^2\theta\, \cos 2\phi + I^{a_1}_{4,\parallel(\perp)} \sin 2\theta_V\, sin2\theta\, \cos\phi +$$
$$+ I^{a_1}_{5,\parallel(\perp)} \sin 2\theta_V\, sin\theta\, \cos\phi +$$
$$+ (I^{a_1}_{6s,\parallel(\perp)} \sin^2\theta_V + I^{a_1}_{6c,\parallel(\perp)} (3 + \cos 2\theta_V)) \cos\theta +$$
$$+ I^{a_1}_{7,\parallel(\perp)} \sin 2\theta_V\, sin\theta\, \sin\phi \} \,. \tag{13}$$

Angular coefficient functions $(\epsilon_V^\ell, \epsilon_S^\ell, \not{\epsilon}_P^\ell, \epsilon_T^\ell)$

$$I_i = |1 + \epsilon_V^\ell|^2\, I_i^{SM} + |\epsilon_S^\ell|^2\, I_i^{NP,S} + |\epsilon_T^\ell|^2\, I_i^{NP,T} + 2\,\mathrm{Re}[\epsilon_S^\ell\,(1 + \epsilon_V^{\ell *})]\, I_i^{INT,S} +$$
$$+ 2\,\mathrm{Re}[\epsilon_T^\ell\,(1 + \epsilon_V^{\ell *})]\, I_i^{INT,T} + 2\,\mathrm{Re}[\epsilon_S^\ell\, \epsilon_T^{\ell *}]\, I_i^{INT,ST}\,, \qquad i = 1s, 1c, \ldots, 6c\,, \tag{14a}$$
$$I_7 = 2\,\mathrm{Im}[\epsilon_S^\ell\,(1 + \epsilon_V^{\ell *})]\, I_7^{INT,S} + 2\,\mathrm{Im}[\epsilon_T^\ell\,(1 + \epsilon_V^{\ell *})]\, I_7^{INT,T} + 2\,\mathrm{Im}[\epsilon_S^\ell\, \epsilon_T^{\ell *}]\, I_7^{INT,ST}\,. \tag{14b}$$

Angular coefficient functions for $\overline{B^0} \to a_1^+(\to \rho_\parallel \pi)\ell^- \bar{\nu}_\ell$

P. Colangelo, F. De Fazio, F. L., Phys.Rev.D 100 (2019) 7, 075037, arXiv:1906.07068

	SM
I_{1s}	$\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{1c}	$4m_\ell^2 H_t^2 + 2H_0^2(m_\ell^2 + q^2)$
I_{2s}	$-\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$
I_{2c}	$2H_0^2(m_\ell^2 - q^2)$
I_3	$2H_+H_-(m_\ell^2 - q^2)$
I_4	$H_0(H_+ + H_-)(m_\ell^2 - q^2)$
I_5	$-2H_t(H_+ + H_-)m_\ell^2 - 2H_0(H_+ - H_-)q^2$
I_{6s}	$2(H_+^2 - H_-^2)q^2$
I_{6c}	$-8H_tH_0m_\ell^2$
I_7	0

	NP, S	NP, T
I_{1s}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](3m_\ell^2 + q^2)$
I_{1c}	$4H_t^2 \frac{q^4}{(m_b - m_u)^2}$	$\frac{1}{8}(H_L^{NP})^2(m_\ell^2 + q^2)$
I_{2s}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](m_\ell^2 - q^2)$
I_{2c}	0	$-\frac{1}{8}(H_L^{NP})^2(m_\ell^2 - q^2)$
I_3	0	$-8H_+^{NP}H_-^{NP}(m_\ell^2 - q^2)$
I_4	0	$-\frac{1}{2}H_L^{NP}(H_+^{NP} + H_-^{NP})(m_\ell^2 - q^2)$
I_5	0	$-H_L^{NP}(H_+^{NP} - H_-^{NP})m_\ell^2$
I_{6s}	0	$8[(H_+^{NP})^2 - (H_-^{NP})^2]m_\ell^2$
$I_{6c,7}$	0	0

	INT, S	INT, T	INT, ST
I_{1s}	0	$4[H_+^{NP}H_+ + H_-^{NP}H_-]m_\ell\sqrt{q^2}$	0
I_{1c}	$4H_t^2 \frac{m_\ell q^2}{m_b - m_u}$	$H_L^{NP}H_0 m_\ell \sqrt{q^2}$	0
$I_{2s,2c,3,4}$	0	0	0
I_5	$-H_t(H_+ + H_-)\frac{m_\ell q^2}{m_b - m_u}$	$-\frac{1}{4}[H_L^{NP}(H_+ - H_-) + 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$-2H_t(H_+^{NP} + H_-^{NP})\frac{(q^2)^{3/2}}{m_b - m_u}$
I_{6s}	0	$4(H_+^{NP}H_+ - H_-^{NP}H_-)m_\ell\sqrt{q^2}$	0
I_{6c}	$-4H_tH_0\frac{m_\ell q^2}{m_b - m_u}$	$-H_L^{NP}H_t m_\ell\sqrt{q^2}$	$-H_L^{NP}H_t\frac{(q^2)^{3/2}}{m_b - m_u}$
I_7	$-H_t(H_+ - H_-)\frac{m_\ell q^2}{m_b - m_u}$	$-\frac{1}{4}[H_L^{NP}(H_+ + H_-) - 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$-2H_t(H_+^{NP} - H_-^{NP})\frac{(q^2)^{3/2}}{m_b - m_u}$

Angular coefficient functions for $\overline{B^0} \to a_1^+(\to \rho_\perp \pi)\ell^- \bar{\nu}_\ell$

P. Colangelo, F. De Fazio, F. L., Phys.Rev.D 100 (2019) 7, 075037, arXiv:1906.07068

	SM
I_{1s}	$2H_t^2 m_\ell^2 + H_0^2(m_\ell^2 + q^2) + \frac{1}{4}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{1c}	$\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{2s}	$[H_0^2 - \frac{1}{4}(H_+^2 + H_-^2)](m_\ell^2 - q^2)$
I_{2c}	$-\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$
I_3	$-H_+H_-(m_\ell^2 - q^2)$
I_4	$-\frac{1}{2}H_0(H_+ + H_-)(m_\ell^2 - q^2)$
I_5	$H_t(H_+ + H_-)m_\ell^2 + H_0(H_+ - H_-)q^2$
I_{6s}	$-4H_tH_0 m_\ell^2 + (H_+^2 - H_-^2)q^2$
I_{6c}	$2(H_+^2 - H_-^2)q^2$
I_7	0

	NP, S	NP, T
I_{1s}	$2H_t^2 \frac{q^4}{(m_b - m_u)^2}$	$[(H_+^{NP})^2 + (H_-^{NP})^2](3m_\ell^2 + q^2) + \frac{1}{16}(H_L^{NP})^2(m_\ell^2 + q^2)$
I_{1c}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](3m_\ell^2 + q^2)$
I_{2s}	0	$[(H_+^{NP})^2 + (H_-^{NP})^2](m_\ell^2 - q^2) - \frac{1}{16}(H_L^{NP})^2(m_\ell^2 - q^2)$
I_{2c}	0	$2[(H_+^{NP})^2 + (H_-^{NP})^2](m_\ell^2 - q^2)$
I_3	0	$4H_+^{NP}H_-^{NP}(m_\ell^2 - q^2)$
I_4	0	$\frac{1}{4}H_L^{NP}(H_+^{NP} + H_-^{NP})(m_\ell^2 - q^2)$
I_5	0	$\frac{1}{2}H_L^{NP}(H_+^{NP} - H_-^{NP})m_\ell^2$
I_{6s}	0	$4[(H_+^{NP})^2 - (H_-^{NP})^2]m_\ell^2$
I_{6c}	0	$8[(H_+^{NP})^2 - (H_-^{NP})^2]m_\ell^2$
I_7	0	0

	INT, S	INT, T	INT, ST
I_{1s}	$2H_t^2 \frac{m_\ell q^2}{m_b - m_u}$	$\frac{1}{2}[4(H_+^{NP}H_+ + H_-^{NP}H_-) + H_L^{NP}H_0]m_\ell\sqrt{q^2}$	0
I_{1c}	0	$4(H_+^{NP}H_+ + H_-^{NP}H_-]m_\ell\sqrt{q^2})$	0
$I_{2s,2c,3,4}$	0	0	0
I_5	$\frac{1}{2}H_t(H_+ + H_-)\frac{m_\ell q^2}{m_b - m_u}$	$\frac{1}{8}[H_L^{NP}(H_+ - H_-) + 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$H_t(H_+^{NP} + H_-^{NP})\frac{(q^2)^{3/2}}{m_b - m_u}$
I_{6s}	$-2H_tH_0\frac{m_\ell q^2}{m_b - m_u}$	$-\frac{1}{2}[-4(H_+^{NP}H_+ - H_-^{NP}H_-) + H_L^{NP}H_t]m_\ell\sqrt{q^2}$	$-H_tH_L^{NP}\frac{(q^2)^{3/2}}{2(m_b - m_u)}$
I_{6c}	0	$4(H_+^{NP}H_+ - H_-^{NP}H_-)m_\ell\sqrt{q^2}$	0
I_7	$\frac{1}{2}H_t(H_+ - H_-)\frac{m_\ell q^2}{m_b - m_u}$	$\frac{1}{8}[H_L^{NP}(H_+ + H_-) - 8H_+^{NP}(H_t + H_0) + 8H_-^{NP}(H_t - H_0)]m_\ell\sqrt{q^2}$	$H_t(H_+^{NP} - H_-^{NP})\frac{(q^2)^{3/2}}{m_b - m_u}$

Group Research

Exclusive $b \to u$ decays

Inclusive $b \to U = u, c$ decays

Exclusive $c \to D = s, d$ decays

Work in progress

Differential phase space $d\Sigma$

1. $\frac{1}{(2\pi)^5}\, d^4q\, \delta^4(q - p_\ell - p_\nu)\, \frac{d^3p_\ell}{2E_\ell}\, \frac{d^3p_\nu}{2E_\nu}$

   $d^4q = \frac{|\boldsymbol{q}|\, dq_0\, dq^2}{2}\, d\Omega_q\,, \qquad \frac{d^3p_\ell}{2E_\ell} = \frac{|\boldsymbol{p}_\ell|\, dE_\ell}{2}\, d\Omega_\ell\,, \qquad \frac{d^3p_\nu}{2E_\nu} = d^4p_\nu\, \theta(E_\nu)\, \delta(p_\nu^2)\,,$

2. $\frac{1}{4(2\pi)^5}\, |\boldsymbol{q}|\, dq_0\, dq^2\, d\Omega_q\, \delta^4(q - p_\ell - p_\nu)\, |\boldsymbol{p}_\ell|\, dE_\ell\, d\Omega_\ell\, d^4p_\nu\, \theta(E_\nu)\, \delta(p_\nu^2)$

   $q = p_\ell + p_\nu\,, \qquad p_\ell = (E_\ell, \boldsymbol{p}_\ell)\,, \qquad q = (q_0, \boldsymbol{q})\,,$

3. $\frac{1}{4(2\pi)^5}\, |\boldsymbol{q}|\, dq_0\, dq^2\, d\Omega_q\, |\boldsymbol{p}_\ell|\, dE_\ell\, d\Omega_\ell\, \theta(q_0 - E_\ell)\, \delta(q^2 + m_\ell^2 - 2\, q \cdot p_\ell)$

   $\delta(a\,x + b) = \frac{1}{|a|}\, \delta\left(x + \frac{b}{a}\right)\,, \qquad q \cdot p_\ell = q_0\, E_\ell - |\boldsymbol{q}|\, |\boldsymbol{p}_\ell|\, \cos\alpha_{q\ell}$

4. $\frac{1}{8(2\pi)^5}\, dq_0\, dq^2\, d\Omega_q\, dE_\ell\, d\Omega_\ell\, \theta(q_0 - E_\ell)\, \delta\left(\cos\alpha_{q\ell} - \frac{2q_0\, E_\ell - (q^2 + m_\ell^2)}{2|\boldsymbol{q}|\, |\boldsymbol{p}_\ell|}\right)$

$$\int_a^b dx\, \delta(x-y)\, g(x) = \int_{-\infty}^{+\infty} dx\, \delta(x-y)\, \theta(b-x)\, \theta(x-a)\, g(x) = \theta(b-y)\, \theta(y-a)\, g(y)$$

Part I: tensor $(T^{ij})_{MN}$

1. $i \int d^4x \, e^{iq\cdot x} \, \langle H_b | \mathbb{T} \left[J^{(i)}_{q,M}{}^{\dagger}(x) \, J^{(j)}_{q,N}(0) \right] | H_b \rangle$

   hadron current: $J^{(j)}_{q,N}(y) = \overline{U}(0) \, \Gamma^{(j)}_{q,N} \, b(0)$

2. $i \int d^4x \, e^{iq\cdot x} \, \langle H_b | \mathbb{T} \left[\overline{b}(x) \, \overline{\Gamma}^{(i)}_{q,M} \, U(x) \, \overline{U}(0) \, \Gamma^{(j)}_{q,N} \, b(0) \right] | H_b \rangle$

   QCD quark fields: $b(x) = e^{i m_b \, v\cdot x} \, b_v(x)$

3. $i \int d^4x \, e^{-i(m_b \, v - q)\cdot x} \, \langle H_b | \mathbb{T} \left[\overline{b}_v(x) \, \overline{\Gamma}^{(i)}_{q,M} \, U(x) \, \overline{U}(0) \, \Gamma^{(j)}_{q,N} \, b_v(0) \right] | H_b \rangle$

   residual small momentum: $b_v(x) = e^{ik\cdot x} \, b_v(0)$

4. $i \int d^4x \, e^{-i(m_b \, v + k - q)\cdot x} \, \langle H_b | \mathbb{T} \left[\overline{b}_v(0) \, \overline{\Gamma}^{(i)}_{q,M} \, U(x) \, \overline{U}(0) \, \Gamma^{(j)}_{q,N} \, b_v(0) \right] | H_b \rangle$

   linearity of integral

5. $\langle H_b | \overline{b}_v(0) \, \overline{\Gamma}^{(i)}_{q,M} \left(i \int d^4x \, e^{-i(m_b \, v + k - q)\cdot x} \, \mathbb{T} \left[U(x) \, \overline{U}(0) \right] \right) \Gamma^{(j)}_{q,N} \, b_v(0) | H_b \rangle$

   definition of propagator

6. $\langle H_b | \overline{b}_v(0) \, \overline{\Gamma}^{(i)}_{q,M} \, \frac{1}{m_b \not{v} + \not{k} - \not{q} - m_U} \, \Gamma^{(j)}_{q,N} \, b_v(0) | H_b \rangle$

   series expansion with respect to $\frac{|k^\mu|}{m_b} \sim \frac{\Lambda_{QCD}}{m_b} \ll 1$

7. $\sum_{n=0}^{+\infty} (-1)^n \, \langle H_b | \overline{b}_v(0) \, \overline{\Gamma}^{(i)}_{q,M} \, \frac{1}{\not{p}_U - m_U} \left(i \not{D} \, \frac{1}{\not{p}_U - m_U} \right)^n \Gamma^{(j)}_{q,N} \, b_v(0) | H_b \rangle$

   $p_U^\mu = m_b \, v^\mu - q^\mu$ and $k \to i \, D$ QCD covariant derivative

8. $\sum_{n=0}^{+\infty} (-1)^n \, \langle H_b | \overline{b}_v(0) \, \overline{\Gamma}^{(i)}_{q,M} \, (\not{p}_U + m_U) \, (i \not{D} \, (\not{p}_U + m_U))^n \, \Gamma^{(j)}_{q,N} \, b_v(0) | H_b \rangle \; \frac{1}{\Delta_0^{n+1}}$

   $\Delta_0 = p_U^2 - m_U^2 + i \, \varepsilon$

Part II: tensor $(W^{ij})_{MN} = \frac{1}{\pi}\,\mathrm{Im}\Big[(T^{ij})_{MN}\Big]$

9. $\sum_{n=0}^{+\infty} \frac{1}{n!} \langle H_b|\bar{b}_v(0)\,\bar{\Gamma}^{(i)}_{q,M}\,(\not{p}_U + m_U)\,(i\not{D}\,(\not{p}_U + m_U))^n\,\Gamma^{(j)}_{q,N}\,b_v(0)|H_b\rangle\ \delta^{(n)}(\Delta_0)$

   Cauchy theorem: $\frac{1}{\Delta_0^{n+1}} = \frac{(-1)^n}{n!}\,\frac{\partial^n}{\partial\Delta_0^n}\left[\mathrm{P.V.}\left(\frac{1}{\Delta_0}\right) + i\,\pi\,\delta(\Delta_0)\right]$

10. $\sum_{n=0}^{+\infty} \frac{1}{n!}\,\mathrm{Tr}\left[\bar{\Gamma}^{(i)}_{q,M}\,(\not{p}_U + m_U)\,\prod_{k=1}^{n}\,[\gamma_{\mu_k}\,(\not{p}_U + m_U)]\,\Gamma^{(j)}_{q,N}\,\mathcal{M}^{\mu_1\mu_2\cdots\mu_n}\right]\delta^{(n)}(\Delta_0)$

    trace formalism: $\langle H_b|\bar{b}_v(0)\,\bar{\Gamma}^{(i)}_{q,M}\,(\not{p}_U + m_U)\,(i\not{D}\,(\not{p}_U + m_U))^n\,\Gamma^{(j)}_{q,N}\,b_v(0)|H_b\rangle =$

    $\left[\bar{\Gamma}^{(i)}_{q,M}\,(\not{p}_U + m_U)\,\prod_{k=1}^{n}\,[\gamma_{\mu_k}\,(\not{p}_U + m_U)]\,\Gamma^{(j)}_{q,N}\right]_{ab}\underbrace{\langle H_b|\bar{b}_v\,iD^{\mu_1}\,iD^{\mu_2}\,\ldots\,iD^{\mu_n}\,b_v|H_b\rangle_{ba}}_{(\mathcal{M}^{\mu_1\mu_2\cdots\mu_n})_{ba}}$

Tools for $\langle H_b|\bar{b}_v\,iD^{\mu_1}\,iD^{\mu_2}\,\ldots\,iD^{\mu_n}\,b_v|H_b\rangle_{ba} = (\mathcal{M}^{\mu_1\mu_2\cdots\mu_n})_{ba}$

$$\Pi^{\alpha\beta} = g^{\alpha\beta} - v^\alpha\,v^\beta\,, \qquad P_\pm = \frac{1 \pm \not{v}}{2}\,, \qquad S_\mu = P_+\,\gamma_\mu\,\gamma_5\,P_+\,.$$

Boundaries $q_0 \to q^2 \to E_\ell$

$$E_\ell + \frac{q^2 - m_\ell^2}{2m_\ell} E_\ell^- \leq q_0 \leq E_\ell + \frac{q^2 - m_\ell^2}{2m_\ell} E_\ell^+ \,, \qquad E_\ell^\pm \equiv E_\ell \pm \sqrt{E_\ell^2 - m_\ell^2} \,,$$

$$\frac{E_\ell^-}{m_b - E_\ell^-} (m_b^2 - m_U^2 - m_b E_\ell^-) \leq q^2 \leq \frac{E_\ell^+}{m_b - E_\ell^+} (m_b^2 - m_U^2 - m_b E_\ell^+) \,,$$

$$m_\ell \leq E_\ell \leq \frac{m_b^2 - m_U^2 + m_\ell^2}{2m_b} \,.$$

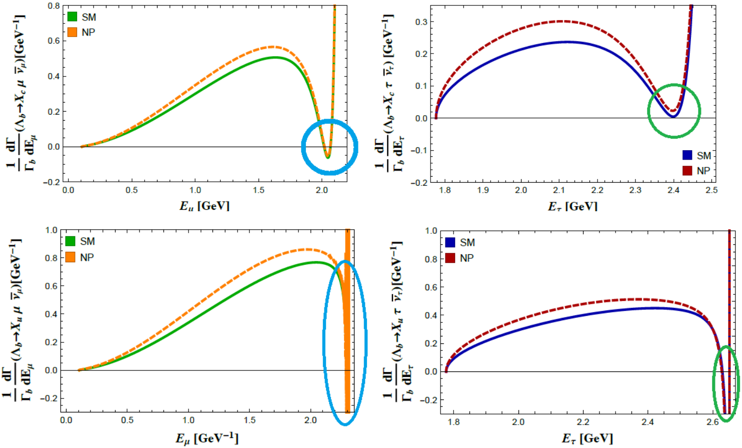
Boundaries $E_\ell \to q_0 \to q^2$

$$\frac{q_0 (q^2 + m_\ell^2) - \sqrt{q_0^2 - q^2}\,(q^2 - m_\ell^2)}{2q^2} \leq E_\ell \leq \frac{q_0 (q^2 + m_\ell^2) + \sqrt{q_0^2 - q^2}\,(q^2 - m_\ell^2)}{2q^2} \,,$$

$$\sqrt{q^2} \leq q_0 \leq m_b - m_U \,,$$

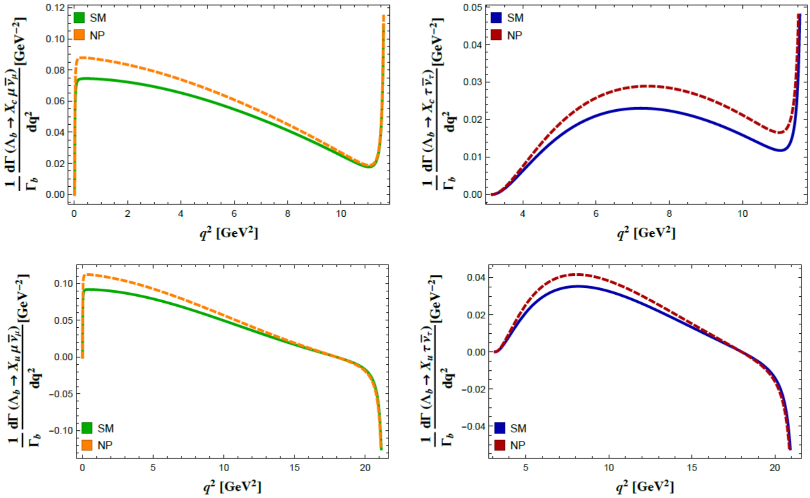
$$m_\ell^2 \leq q^2 \leq (m_b - m_U)^2 \,.$$

Normalized charged lepton energy spectra $\frac{1}{\Gamma_b}\frac{d\Gamma}{dE_\ell}(\Lambda_b \to X_U\, \ell^-\, \overline{\nu}_\ell)[\mathrm{GeV}^{-1}]$



Singularities $\mapsto E_\ell^{\max} \Leftrightarrow$ U propagator on-shell!

Normalized dilepton invariant mass q^2 distributions $\frac{1}{\Gamma_b}\frac{d\Gamma}{dq^2}(\Lambda_b \to X_U\, \ell^-\, \overline{\nu}_\ell)[\mathrm{GeV}^{-2}]$



Group Research

Exclusive $b \to u$ decays

Inclusive $b \to U = u, c$ decays

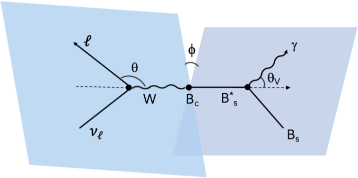
Exclusive $c \to D = s, d$ decays

Work in progress

Heavy Quark Spin Symmetry

In the infinite heavy quark mass limit $m_Q \gg \Lambda_{QCD}$ the QCD Lagrangian exhibits a heavy quark (HQ) spin symmetry, with the decoupling of the heavy quark spin from gluons.

In the semileptonic $B_c \to B_a^{(*)}$ $(a = s, d)$ decays induced by the $c \to s, d$ transition, since $m_c \ll m_b$ the energy released to the final hadronic system is much smaller than m_b. The b quark remains almost unaffected, so that the final meson keeps the same B_c four-velocity v.



- B_c meson rest frame;
- θ_V is the angle between the B_s^* flight direction and the photon direction;
- θ is the angle between the lepton pair flight direction and the W flight direction;
- ϕ is the angle between the lepton plane and the hadron plane.

Kinematics

$$p = m_{B_c}\, v\ , \qquad p' = m_{B_a}\, v' = m_{B_a}\, v + k\ , \qquad q = p - p' = (m_{B_c} - m_{B_a})\, v - k\ , \qquad v \cdot k \sim \mathcal{O}(m_b^{-1})\ .$$

Heavy Quark Spin Symmetry

The heavy pseudoscalar and the vector mesons are collected in doublets, the two components of which represent states differing only for the orientation of the heavy quark spin.

$$\begin{pmatrix} B_c^+ \\ B_c^{*+} \end{pmatrix} : H^{c\bar{b}} = \frac{1+\not{v}}{2}\left[B_c^{*\mu}\,\gamma_\mu - B_c\,\gamma_5\right]\frac{1-\not{v}}{2} \qquad \begin{pmatrix} B_a \\ B_a^* \end{pmatrix} : H^{\bar{b}} = \left[B_a^{*\mu}\,\gamma_\mu - B_a\,\gamma_5\right]\frac{1-\not{v}}{2}$$

$$H^{c\bar{b}} \to S_c\, H^{c\bar{b}}\, S_b^\dagger$$
$$H_a^{\bar{b}} \to \left(U\, H^{\bar{b}}\right)_a S_b^\dagger$$
$$\left[\not{v}, H^{(c)\bar{b}}\right] = 2\, H^{(c)\bar{b}}$$
$$\left\{\not{v}, H^{(c)\bar{b}}\right\} = 0$$

$S_{c,b} \in$ heavy quark spin transformations

$U_a \in$ light quark $SU_F(3)$ transformations

Matrix element (invariant under rotations of the $\bar{b}$ spin)

$$\langle B_a^{(*)}(v,k)|\bar{q}\,\Gamma\, Q|B_c(v)\rangle = -\sqrt{m_{B_c}\, m_{B_a^{(*)}}}\,\mathrm{Tr}\left[\bar{H}_a^{(b)}\,\Omega_a(v, a_0\, k)\,\Gamma\, H^{(c\bar{b})}\right]$$

$$\bar{H}_a = \gamma^0\, H_a^\dagger\, \gamma^0 \qquad \text{and} \qquad \Omega_a(v, a_0\, k) = \Omega_{1a} + \not{k}\, a_0\, \Omega_{2a}$$

dimensionless nonperturbative functions

$B_c \to V(\to P\gamma)\bar{\ell}\nu_\ell$ fully differential distribution

$$\frac{d^4\Gamma(B_c \to V(\to P\gamma)\bar{\ell}\nu_\ell)}{dq^2\, d\cos\theta\, d\phi\, d\cos\theta_V} = N_\gamma\, |\overrightarrow{p_V}| \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \{ I_{1s}\, \sin^2\theta_V + I_{1c}\,(3+\cos 2\theta_V) + \\ + (I_{2s}\, \sin^2\theta_V + I_{2c}\,(3+\cos 2\theta_V))\, \cos 2\theta + \\ + I_3\, \sin^2\theta_V\, sin^2\theta\, \cos 2\phi + I_4\, \sin 2\theta_V\, sin2\theta\, \cos\phi + \\ + I_5\, \sin 2\theta_V\, sin\theta\, \cos\phi + (I_{6s}\, \sin^2\theta_V + I_{6c}\,(3+\cos 2\theta_V))\, \cos\theta + \\ + I_7\, \sin 2\theta_V\, sin\theta\, \sin\phi + I_8\, \sin 2\theta_V\, sin2\theta\, \sin\phi + I_9\, \sin^2\theta_V\, sin^2\theta\, \sin 2\phi \}\ , \tag{15}$$

$$N_\gamma = \frac{3\, G_F^2\, |V_{CKM}|^2\, \mathcal{B}(V \to P\gamma)}{128(2\pi)^4\, m_{B_c}^2} \qquad |\overrightarrow{p_V}| = \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}}{2m_{B_c}}$$

Angular coefficient functions $(\epsilon_V^\ell, \epsilon_R^\ell, \epsilon_P^\ell, \epsilon_T^\ell)$

$$I_i = |1+\epsilon_V^\ell|^2\, I_i^{SM} + |\epsilon_R^\ell|^2\, I_i^{NP,R} + |\epsilon_P^\ell|^2\, I_i^{NP,P} + |\epsilon_T^\ell|^2\, I_i^{NP,T} + \\ + 2\,\mathrm{Re}[\epsilon_R^\ell\,(1+\epsilon_V^{\ell*})]\, I_i^{INT,R} + 2\,\mathrm{Re}[\epsilon_P^\ell\,(1+\epsilon_V^{\ell*})]\, I_i^{INT,P} + 2\,\mathrm{Re}[\epsilon_T^\ell\,(1+\epsilon_V^{\ell*})]\, I_i^{INT,T} + \\ + 2\,\mathrm{Re}[\epsilon_R^\ell\,\epsilon_P^{\ell*}]\, I_i^{INT,RP} + 2\,\mathrm{Re}[\epsilon_R^\ell\,\epsilon_T^{\ell*}]\, I_i^{INT,RT} + 2\,\mathrm{Re}[\epsilon_P^\ell\,\epsilon_T^{\ell*}]\, I_i^{INT,PT}\ , \qquad i = 1s, 1c, \ldots, 6c\ , \tag{16a}$$

$$I_7 = 2\,\mathrm{Im}[\epsilon_R^\ell\,(1+\epsilon_V^{\ell*})]\, I_7^{INT,R} + 2\,\mathrm{Im}[\epsilon_P^\ell\,(1+\epsilon_V^{\ell*})]\, I_7^{INT,P} + 2\,\mathrm{Im}[\epsilon_T^\ell\,(1+\epsilon_V^{\ell*})]\, I_7^{INT,T} + \\ + 2\,\mathrm{Im}[\epsilon_R^\ell\,\epsilon_P^{\ell*}]\, I_7^{INT,RP} + 2\,\mathrm{Im}[\epsilon_R^\ell\,\epsilon_T^{\ell*}]\, I_7^{INT,RT} + 2\,\mathrm{Im}[\epsilon_P^\ell\,\epsilon_T^{\ell*}]\, I_7^{INT,PT}\ , \tag{16b}$$

$$I_i = 2\,\mathrm{Im}[\epsilon_R^\ell\,(1+\epsilon_V^{\ell*})]\, I_i^{INT,R}\ , \qquad i = 8, 9\ . \tag{16c}$$

Helicity amplitudes for $B_c \to V(\to P\,\gamma)\bar{\ell}\,\nu_\ell$

P. Colangelo, F. De Fazio, F. L., Phys.Rev.D 103 (2021) 7, 075019, arXiv:2102.05365

SM

$$H_0 = \frac{(m_{B_c}+m_V)^2\,(m_{B_c}^2-m_V^2-q^2)\,A_1-\lambda\,A_2}{2m_V(m_{B_c}+m_V)\sqrt{q^2}} = \sqrt{\frac{m_{B_c}}{m_V}}\,\frac{m_{B_c}^2-m_V^2-q^2}{\sqrt{q^2}}\,\Omega_1+\frac{\lambda}{2\sqrt{m_{B_c}^2\,m_V,q^2}}\,a_0\,\Omega_2$$

$$H_\pm = \frac{(m_{B_c}+m_V)^2\,A_1\mp\sqrt{\lambda}\,V}{m_{B_c}+m_V} = \sqrt{\frac{m_V}{m_{B_c}}}\left[2\,m_{B_c}\,\Omega_1\mp\sqrt{\lambda}\,a_0\,\Omega_2\right]$$

$$H_t = -\sqrt{\frac{\lambda}{q^2}}\,A_0 = -\frac{1}{2}\sqrt{\frac{\lambda}{m_{B_c}\,m_V\,q^2}}\left[2\,m_{B_c}\,\Omega_1+(m_{B_c}^2-m_V^2+q^2)\,a_0\,\Omega_2\right]$$

$$\lambda = \lambda(m_{B_c}^2, m_V^2, q^2)\,.$$

NP

$$H_\pm^{NP} = \frac{(m_{B_c}^2-m_V^2\pm\sqrt{\lambda})\,(T_1+T_2)+q^2\,(T_1-T_2)}{\sqrt{q^2}} =$$

$$= 2\sqrt{\frac{m_V}{m_{B_c}q^2}}\left[(m_{B_c}^2-m_V^2+q^2\pm\sqrt{\lambda})\,\Omega_1+\left((m_{B_c}+m_V)\left((m_{B_c}-m_V)^2-q^2\right)\pm(m_{B_c}-m_V)\sqrt{\lambda}\right)a_0\,\Omega_2\right]$$

$$H_L^{NP} = 4\left[\frac{\lambda}{m_V(m_{B_c}+m_V)^2}\,T_0+2\,\frac{m_{B_c}^2+m_V^2-q^2}{m_V}\,T_1+4\,m_V\,T_2\right] =$$

$$= \frac{16}{\sqrt{m_{B_c}m_V}}\left[(m_{B_c}^2+m_V^2-q^2)\,\Omega_1-m_V\left((m_{B_c}-m_V)^2-q^2\right)a_0\,\Omega_2\right]$$

Angular coefficients

	SM
I_{1s}	$2m_\ell^2 H_t^2 + H_0^2(m_\ell^2 + q^2)$
I_{1c}	$\frac{1}{8}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{2s}	$H_0^2(m_\ell^2 - q^2)$
I_{2c}	$\frac{1}{8}(H_+^2 + H_-^2)(q^2 - m_\ell^2)$
I_3	$H_+H_-(q^2 - m_\ell^2)$
I_4	$\frac{1}{2}H_0(H_+ + H_-)(q^2 - m_\ell^2)$
I_5	$H_t(H_+ + H_-)m_\ell^2 + H_0(H_+ - H_-)q^2$
I_{6s}	$-4H_tH_0m_\ell^2$
I_{6c}	$\frac{1}{2}(H_+^2 - H_-^2)q^2$
$I_{7,8,9}$	0

	NP, P	INT, P
I_{1s}	$2H_t^2 \frac{q^4}{(m_Q+m_q)^2}$	$2H_t^2 \frac{m_\ell q^2}{m_Q+m_q}$
$I_{1c,2s,2c,3,4,6c,8,9}$	0	0
I_5	0	$H_t(H_+ + H_-)\frac{m_\ell q^2}{2(m_Q+m_q)}$
I_{6s}	0	$-2H_tH_0 \frac{m_\ell q^2}{m_Q+m_q}$
I_7	0	$H_t(H_+ - H_-)\frac{m_\ell q^2}{2(m_Q+m_q)}$

	NP, R	INT, R
I_{1s}	$2m_\ell^2 H_t^2 + H_0^2(m_\ell^2 + q^2)$	$-2m_\ell^2 H_t^2 - H_0^2(m_\ell^2 + q^2)$
I_{1c}	$\frac{1}{8}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$	$-\frac{1}{4}H_+H_-(m_\ell^2 + 3q^2)$
I_{2s}	$H_0^2(m_\ell^2 - q^2)$	$-H_0^2(m_\ell^2 - q^2)$
I_{2c}	$\frac{1}{8}(H_+^2 + H_-^2)(q^2 - m_\ell^2)$	$\frac{1}{4}H_+H_-(m_\ell^2 - q^2)$
I_3	$H_+H_-(q^2 - m_\ell^2)$	$\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$
I_4	$\frac{1}{2}H_0(H_+ + H_-)(q^2 - m_\ell^2)$	$\frac{1}{2}H_0(H_+ + H_-)(m_\ell^2 - q^2)$
I_5	$H_t(H_+ + H_-)m_\ell^2 - H_0(H_+ - H_-)q^2$	$-H_t(H_+ + H_-)m_\ell^2$
I_{6s}	$-4H_tH_0m_\ell^2$	$4H_tH_0m_\ell^2$
I_{6c}	$-\frac{1}{2}(H_+^2 - H_-^2)q^2$	0
I_7	0	$-H_t(H_+ - H_-)m_\ell^2$
I_8	0	$\frac{1}{2}H_0(H_+ - H_-)(m_\ell^2 - q^2)$
I_9	0	$\frac{1}{2}(H_+^2 - H_-^2)(m_\ell^2 - q^2)$

Angular coefficients

	NP, T	INT, T
I_{1s}	$\frac{1}{16}(H_L^{NP})^2(q^2+m_\ell^2)$	$-\frac{1}{2}H_L^{NP}H_0 m_\ell\sqrt{q^2}$
I_{1c}	$\frac{1}{2}((H_+^{NP})^2+(H_-^{NP})^2)(3m_\ell^2+q^2)$	$-(H_+^{NP}H_+ + H_-^{NP}H_-)m_\ell\sqrt{q^2}$
I_{2s}	$\frac{1}{16}(H_L^{NP})^2(q^2-m_\ell^2)$	0
I_{2c}	$\frac{1}{2}((H_+^{NP})^2+(H_-^{NP})^2)(m_\ell^2-q^2)$	0
I_3	$-4H_+^{NP}H_-^{NP}(q^2-m_\ell^2)$	0
I_4	$-\frac{1}{4}H_L^{NP}(H_+^{NP}+H_-^{NP})(q^2-m_\ell^2)$	0
I_5	$\frac{1}{2}H_L^{NP}(H_+^{NP}-H_-^{NP})m_\ell^2$	$-\frac{1}{8}[H_L^{NP}(H_+-H_-)+8H_+^{NP}(H_t+H_0)+8H_-^{NP}(H_t-H_0)]m_\ell\sqrt{q^2}$
I_{6s}	0	$\frac{1}{2}H_L^{NP}H_t m_\ell\sqrt{q^2}$
I_{6c}	$2((H_+^{NP})^2-(H_-^{NP})^2)m_\ell^2$	$-(H_+^{NP}H_+ - H_-^{NP}H_-)m_\ell\sqrt{q^2}$
I_7	0	$-\frac{1}{8}[H_L^{NP}(H_++H_-)-8H_+^{NP}(H_t+H_0)+8H_-^{NP}(H_t-H_0)]m_\ell\sqrt{q^2}$
$I_{8,9}$	0	0

	INT, PR	INT, RT	INT, PT
I_{1s}	$-2H_t^2\frac{m_\ell q^2}{m_Q+m_q}$	$\frac{1}{2}H_0H_L^{NP}m_\ell\sqrt{q^2}$	0
I_{1c}	0	$(H_+^{NP}H_- + H_-^{NP}H_+)m_\ell\sqrt{q^2}$	0
$I_{2s,2c,3,4,8,9}$	0	0	0
I_5	$-H_t(H_++H_-)\frac{m_\ell q^2}{2(m_Q+m_q)}$	$\frac{1}{8}[H_L^{NP}(H_--H_+)+8H_+^{NP}(H_t+H_0)+8H_-^{NP}(H_t-H_0)]m_\ell\sqrt{q^2}$	$-H_t(H_+^{NP}+H_-^{NP})\frac{(q^2)^{3/2}}{m_Q+m_q}$
I_{6s}	$2H_tH_0\frac{m_\ell q^2}{m_Q+m_q}$	$-\frac{1}{2}H_tH_L^{NP}m_\ell\sqrt{q^2}$	$H_tH_L^{NP}\frac{(q^2)^{3/2}}{2(m_Q+m_q)}$
I_{6c}	0	$(H_+^{NP}H_- - H_-^{NP}H_+)m_\ell\sqrt{q^2}$	0
I_7	$H_t(H_+-H_-)\frac{m_\ell q^2}{2(m_Q+m_q)}$	$-\frac{1}{8}[H_L^{NP}(H_-+H_+)-8H_+^{NP}(H_t+H_0)+8H_-^{NP}(H_t-H_0)]m_\ell\sqrt{q^2}$	$-H_t(H_+^{NP}-H_-^{NP})\frac{(q^2)^{3/2}}{m_Q+m_q}$

Group Research

Exclusive $b \to u$ decays

Inclusive $b \to U = u, c$ decays

Exclusive $c \to D = s, d$ decays

Work in progress

About 331 models: general features

P. Frampton, PRL 69 (92) 2889
F. Pisano & V. Pleitez, PRD 46 (92) 410

$$\mathcal{G}_{331} : SU_C(3) \otimes SU_L(3) \otimes U_X(1)$$

$\Downarrow$ **Spontaneously broken to**

$$SU_C(3) \otimes SU_L(2) \otimes U_X(1)$$

$\Downarrow$ **Spontaneously broken to**

$$SU_C(3) \otimes U_Q(1)$$

requirement of anomaly cancellation $+$
asymptotic freedom of QCD $=$

$$N_{generations} = N_{colours}$$

Fundamental Relation

$$Q = T_3 + \beta\, T_8 + X$$

Key parameter:
defines the variant of the model

$$\beta = \frac{n}{\sqrt{3}} \qquad \text{with} \qquad n \in \mathbb{Z}$$

New gauge bosons have integer charge

New Gauge Bosons

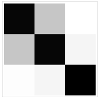
A new heavy Z' mediates tree level FCNC in the quark sector

About 331 models: quark mixing

SM

Quark mass eigenstates defined upon rotation through two unitary matrices U_L and V_L, with $V_{CKM} = U_L^\dagger V_L$ entering in decay amplitudes.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$



Gradient representation of V_{CKM} values well known by experiments

331 model

In contrast to SM only one of them can be traded for V_{CKM}, the other one enters in Z' couplings to quarks

$$V_L = \begin{pmatrix} \tilde{c}_{12}\,\tilde{c}_{13} & \tilde{s}_{12}\,\tilde{c}_{23}\,e^{i\,\delta_3} - \tilde{c}_{12}\,\tilde{s}_{13}\,\tilde{s}_{23}\,e^{i\,(\delta_1-\delta_2)} & \tilde{c}_{12}\,\tilde{c}_{23}\,\tilde{s}_{13}\,e^{i\,\delta_1} + \tilde{s}_{12}\,\tilde{s}_{23}\,e^{i\,(\delta_2+\delta_3)} \\ -\tilde{c}_{13}\,\tilde{s}_{12}\,e^{-i\,\delta_3} & \tilde{c}_{12}\,\tilde{c}_{23} + \tilde{s}_{12}\,\tilde{s}_{13}\,\tilde{s}_{23}\,e^{i\,(\delta_1-\delta_2-\delta_3)} & -\tilde{s}_{12}\,\tilde{s}_{13}\,\tilde{c}_{23}\,e^{i\,(\delta_1-\delta_3)} - \tilde{c}_{12}\,\tilde{s}_{23}\,e^{i\delta_2} \\ -\tilde{s}_{13}\,e^{-i\,\delta_1} & -\tilde{c}_{13}\,\tilde{s}_{23}\,e^{-i\,\delta_2} & \tilde{c}_{13}\,\tilde{c}_{23} \end{pmatrix}$$

Constraints from the model

FCNC involve only left-handed quarks (fermions in general)

In the case of B_d, B_s, K systems there are only four parameters: $\tilde{s}_{13}$, $\tilde{s}_{23}$, δ_1 and δ_2

	$\tilde{s}_{13}$	$\tilde{s}_{23}$	δ_1	δ_2	$\delta_1 - \delta_2$
B_d	✓		✓		
B_s		✓		✓	
K	✓	✓			✓

- stringent correlations among observables
- $U_L = V_L \cdot V_{CKM}^\dagger$

	$\tilde{s}_{13}$	$\tilde{s}_{23}$	δ_1	δ_2	$\delta_1 - \delta_2$
D	✓	✓	✓	✓	
B_c	✓	✓	✓	✓	