Falling into Gargantua: gravitational waves from extreme-mass ratio inspirals

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Ph.D. seminars, 09 June 2021

Outline of the presentation

1. Gravitational waves from astrophysical binaries
2. Extreme mass-ratio inspirals: an overview
3. Extreme mass-ratio inspirals with a spinning secondary

Gravitational waves from astrophysical binaries

Gravitational waves (GW): what are we talking about?!?

Weak perturbation $h_{\mu\nu}$ of a background space-time $g^0_{\mu\nu}$

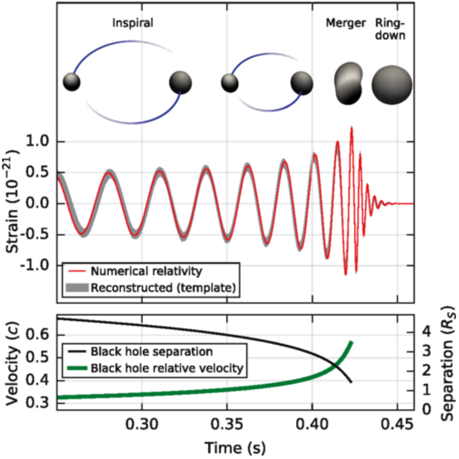
$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$$

Far for sources, $g^0_{\mu\nu} = \eta_{\mu\nu}$ (flat space-time).
Properties of gravitational waves (GW)

- propagate in vacuum at the speed of light
- rank 2 tensor perturbation
- two physical polarizations, $h_\times$ and h_+
- extremely weak: prefactor $G/c^4 \sim 8 \cdot 10^{-55} s^2/(\mathrm{kg} \cdot \mathrm{m})$
- quadrupolar radiation

Astrophysical sources of GW

GW events detected come from mergers of astrophysical compact binaries



Credit: Abbott+, PRL 116, 061102 (2016)

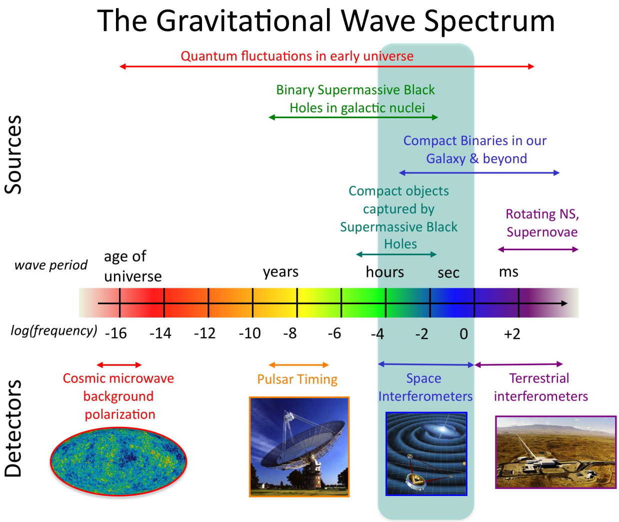
GW signal divided in 3 part:

- inspiral
- merger
- ringdown

LIGO/Virgo detected 50 events:

- mass ratio $q \lesssim 1/2$
- circular, equatorial orbits
- signal last $\mathcal{O}(10)s$
- frequency signal $\mathcal{O}(10)$Hz

Astrophysical sources of GW



Credit: NASA Goddard Space Flight Center

Extreme mass-ratio inspirals: an overview

Falling into Gargantua

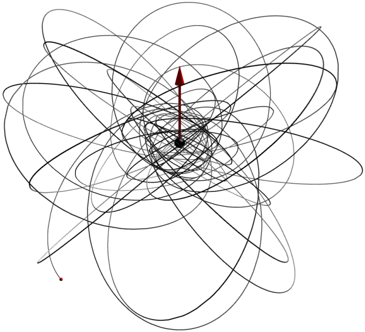
Falling into Gargantua

"The Life of Gargantua and of Pantagruel" by François Rabelais

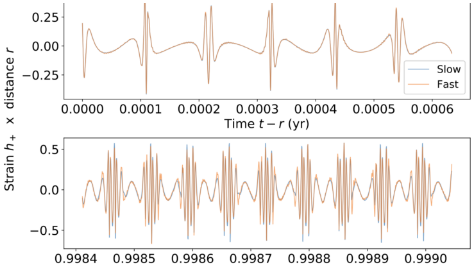
Falling into Gargantua

Extreme-mass ratio inspirals (EMRIs)

- stellar-mass object captured by a supermassive black hole
- observable by future space-borne observatories (LISA, TianQin)
- complicated orbits (eccentric, inclined)
- long, rich waveform with $\sim 10^5$ cycles (remain in band for years!)



Credit: Maarten van de Meent



Credit: Niels Warburton

Science with EMRIs

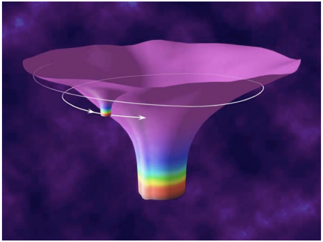
- binary parameters measurable with excellent accuracy
- possible to localize the sources
- possible to probe environment (accretion disk)
- test extension of General Relativity (additional boson particles)
- test the Kerr paradigm: is the primary a black hole?

Modelling GW emission in EMRIs

Solve the Einstein field equations?

$$G_{\mu\nu}[g_{\rho\sigma}] = 8\pi T_{\mu\nu}[g_{\rho\sigma}]$$

Nope! Computationally too expensive for EMRIs
Typical mass ratio $q = \mu/M = 10^{-4} - 10^{-6} \implies$ expand everything in q



Credit: Wikipedia

$$g_{\mu\nu} = g^0_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu} + \mathcal{O}(q^3)$$

$g^0_{\mu\nu}$ is the black hole metric primary

Equations of motion

$$v^\beta \nabla_\beta p^\alpha = 0 + qF^\alpha_{(1)} + q^2 F^\alpha_{(2)} + \mathcal{O}(q^3)$$

Need to evolve both equations of motion and ODEs for $h^{(1)}_{\mu\nu}, h^{(2)}_{\mu\nu}$

Extreme mass-ratio inspirals with a spinning secondary

GW emission for spins aligned, circular equatorial orbits

$$\sigma := \frac{S}{\mu M} = \frac{S}{\mu^2} q = \chi q \qquad \text{with } q \ll \sigma \ll 1 \text{ in EMRIs}$$

Special orbit

- Circular equatorial orbits
- Secondary spin χ (anti)-aligned to primary spin a

$\implies$ dynamics fixed by $E = E(r, \sigma), J_z = J_z(r, \sigma), \Omega = \Omega(r, \sigma)$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -q^2 \mathcal{F}(r, \sigma) \left(\frac{\mathrm{d}E}{\mathrm{d}r} \right)^{-1} \qquad \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \Omega(r(t), \sigma)$$

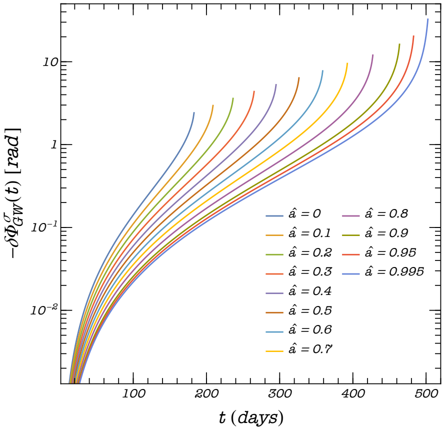
$$h_+ = \frac{\mu}{D} \sum_{\ell,m} [\mathrm{Re}\, \mathcal{A}_{\ell m}(t, \sigma) \cos(m\Phi) + \mathrm{Im}\, \mathcal{A}_{\ell m}(t, \sigma) \sin(m\Phi)]$$

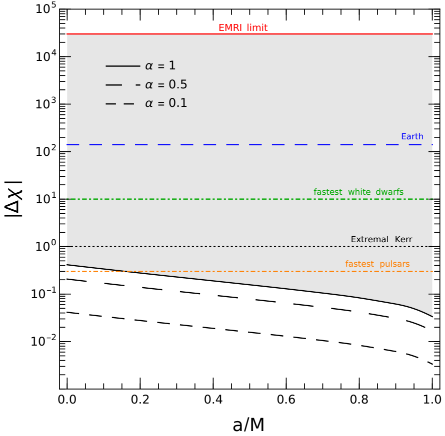
$$h_\times = \frac{\mu}{D} \sum_{\ell,m} [\mathrm{Re}\, \mathcal{A}_{\ell m}(t, \sigma) \sin(m\Phi) - \mathrm{Im}\, \mathcal{A}_{\ell m}(t, \sigma) \cos(m\Phi)]$$

Energy fluxes $\mathcal{F}$ and amplitudes $\mathcal{A}_{\ell m}$ obtained by solving 2 decoupled ODEs

Phase correction due to the secondary spin

Almost all information contained in GW phase $\Phi_{GW} = m\Phi$
Dephasing $\sim$ 1rad is relevant $\implies$ bug (systematics) or feature (physics)?





Piovano+, Phys. Rev. D 102, 024041 (2020), PLB 811 135860 (2020)

Parameter estimation with Fisher matrices

In the high SNR limit

$$\sigma^2_{x_i} = \Sigma_{ii} = (\Gamma^{-1})_{ii} \qquad \text{covariance matrix}$$

$$\Gamma_{ij} = \sum_{\alpha=I,II} \left(\frac{d\tilde{h}_\alpha}{dx^i} \middle| \frac{d\tilde{h}_\alpha}{dx^j} \right)_{\vec{x}=\vec{x}_0} \qquad \text{Fisher matrix}$$

$$(p_\alpha|q_\alpha) = 4\mathrm{Re}\int_{f_\mathrm{min}}^{f_\mathrm{max}} \frac{df}{S_n(f)} \tilde{p}^*_\alpha(f)\tilde{q}_\alpha(f) \qquad \text{scalar product}$$

11 parameters for circular equatorial orbits with spins (anti)aligned

- intrinsic: masses $(\ln\mu, \ln M)$, spins (a, χ), r_0
- extrinsic: ϕ_0, angles $(\vartheta_S, \varphi_S, \vartheta_K, \varphi_K)$, distance $\ln D$

Γ_{ij} is ill-conditioned: small error in $\Gamma_{ij} \implies$ large error in Σ_{ij}

Some cases required waveforms with >90 digits for stable Γ_{ij} and Σ_{ij} !

Fisher-matrix errors neglecting the secondary spin

Masses: $M = 10^6 M_\odot, \mu = 10 M_\odot$. Spins: $a = 0.9, \chi = 0$. 1 year observation.

ℓ	$\ln M$	$\ln \mu$	a/M	r_0/M	ϕ_0
2	-3.24	-3.53	-4.15	-4.45	0.48
2+3	-3.25	-3.54	-4.16	-4.46	-0.52
2+3+4	-3.25	-3.55	-4.16	-4.46	-0.53

Table: $\log_{10}$ of the errors.

ℓ	$\ln D$	$\Delta\Omega_S$	$\Delta\Omega_K$
2	-0.33	7.9×10^{-4}	2.5
2+3	-1.33	7.3×10^{-4}	1.3×10^{-2}
2+3+4	-1.34	7.2×10^{-4}	1.1×10^{-2}

Table: $\log_{10}$ of the error on $\ln D$ and errors on the sky location of the source ($\Delta\Omega_S$) and primary spin ($\Delta\Omega_K$) .

Piovano+, arXiv:2105.07083

Fisher-matrix errors including the secondary spin

$M = 10^6 M_\odot, \chi = 1$. Sky location fixed. 1 year observation

a/M	$\mu/M_\odot$	prior	$\ln M$	$\ln \mu$	a/M	χ	r_0/M	ϕ_0
0.9	10	no	-2.26	-2.41	-2.66	2.85	-3.88	0.48
		yes	-3.24	-3.53	-4.14	0.48	-4.45	0.48
	100	no	-2.20	-2.39	-2.78	1.66	-4.14	-0.015
		yes	-3.30	-3.52	-4.32	0.064	-4.93	-0.024
0.99	10	no	-2.81	-2.96	-4.55	1.98	-3.89	0.47
		yes	-3.51	-3.76	-4.67	0.52	-4.32	0.47
	100	no	-2.14	-2.33	-3.39	1.21	-3.75	-0.12
		yes	-3.01	-3.22	-4.03	0.11	-4.50	-0.12

Table: $\log_{10}$ of the errors with and without imposing a prior on χ.
Only $\ell = 2$ is included.

- Secondary spin is not measurable
- With no prior on χ, correlations spoil errors on intrinsic parameters

Piovano+, arXiv:2105.07083

Conclusions and future work

We estimated errors on binary parameters for fully relativistic waveforms with a Fisher matrix approach

Summary:

- Our analysis confirm results obtained with kludge waveforms
- Secondary spin not measurable due to correlations

Future work:

- Consider eccentric and inclined orbits
- Include secondary spin in fast-waveform? (Challenging)
- More sophisticated statistical analysis? (MonteCarlo)
- Include conservative effects and spin evolution into the dynamics

Thank you for you attention!