Reflecting
the Holographic Correspondence

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PhD seminars, 26 May 2021

The Holographic Principle

$$S_{BH} = \frac{A}{4\,G}$$

Information encoded in the area A of the event horizon

G. ‘t Hooft, *Dimensional reduction in Quantum Gravity,* 1993, gr-qc/9310026

L. Susskind, *The world as a hologram*, 1994, hep-th/9409089

AdS/CFT

Theory of gravity
defined on an
Anti de Sitter space

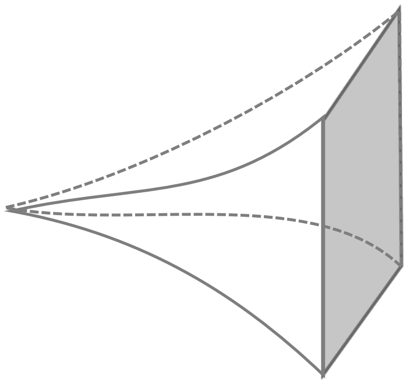
↔

Supersymmetric Conformal
Gauge Theory
on the boundary of
the AdS space

J. Maldacena, *The large N limit of superconformal field theories and supergravity*, 1997, hep-th/9711200

(Super-)Gravity side: $AdS_5 \times S^5$

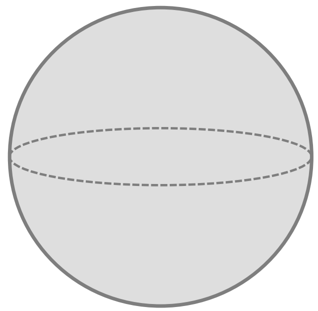
$$ds^2 = \underbrace{\frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S^5}$$



Negative cosmological constant

AdS_5 isometry group: SO(2,4)

Conformal flat boundary



S^5 isometry group: SO(6)

$\int_{S^5} F_5 = N$ (in Type IIB)

Boundary side: (S) CFT_4

Supercharges turn fermions into bosons, e.g. gauge field $A_\mu \leftrightarrow$ gaugino λ

Fields organised in multiplets, e.g. vector multiplet (λ, A_μ) if $\mathcal{N} = 1$
vector multiplet $(\varphi, \lambda, A_\mu)$ if $\mathcal{N} = 2$
chiral multiplet (ϕ, ψ) if $\mathcal{N} = 1$

Global R-symmetry, $U(1)_R$ if $\mathcal{N} = 1$,
$SU(4)_R \simeq SO(6)$ if $\mathcal{N} = 4$

CFT_4 symmetry: Poincaré transformation and dilatation, $SO(2,4)$ group

AdS/CFT

$AdS_5 x S^5$ (with N unit of flux)		$\mathcal{N} = 4$ $SCFT_4$ SU(N)
AdS isometry SO(2,4)	⟷	SO(2,4) conformal symmetry
Sphere isometry SO(6)	⟷	SU(4)≃SO(6) R-symmetry

$$\frac{R^4}{l_s^4} = g_{YM}^2 N = \lambda$$

$$4\pi g_s = g_{YM}^2$$

Low curvature, weak gravity	⟷	Large N, strong coupling

The conjecture

AdS field $\phi(z,x)$ as source for CFT operator $\mathcal{O}(x)$

$$Z_{CFT}[\varphi(x)] = \int \mathcal{D}[\phi] e^{-S[\phi]+\int d^4x\, \varphi(x)\mathcal{O}(x)}\Big|_{\phi(z\to 0,x)=\varphi(x)} = e^{-S_{string}}$$

Test: $\mathcal{O}_k \sim \mathrm{Tr}\left(\phi^1 \dots \phi^k\right)$, $\langle \mathcal{O}_k(x)\mathcal{O}_k(y)\rangle \sim \frac{1}{(x-y)^{2k}}$,

$$\langle \mathcal{O}_k(x)\mathcal{O}_k(y)\mathcal{O}_k(z)\rangle \sim \frac{k^{3/2}}{N(x-y)^{2k}(y-z)^{2k}(x-z)^{2k}}$$

E. Witten, *Anti-de Sitter space and holography*, 1998, hep-th/9802150
S. Gubser, I. Klebanov, A. Polyakov, *Gauge theory correlators from noncritical string theory*, 1998, hep-th/9802109

AdS/CFT

$AdS_5 x S^5$ (with N unit of flux) ⟷ $\mathcal{N} = 4$ $SCFT_4$ SU(N)

AdS ⟷ conformal symmetry

Sphere ⟷ Global symmetry

Deform the sphere: global symmetry changes

$AdS_5 x Y_5$ ⟷ $\mathcal{N} = 1$ $SCFT_4$ G

Y_5 a suitable (for String Theory) 5-dim space

G = Gauge group

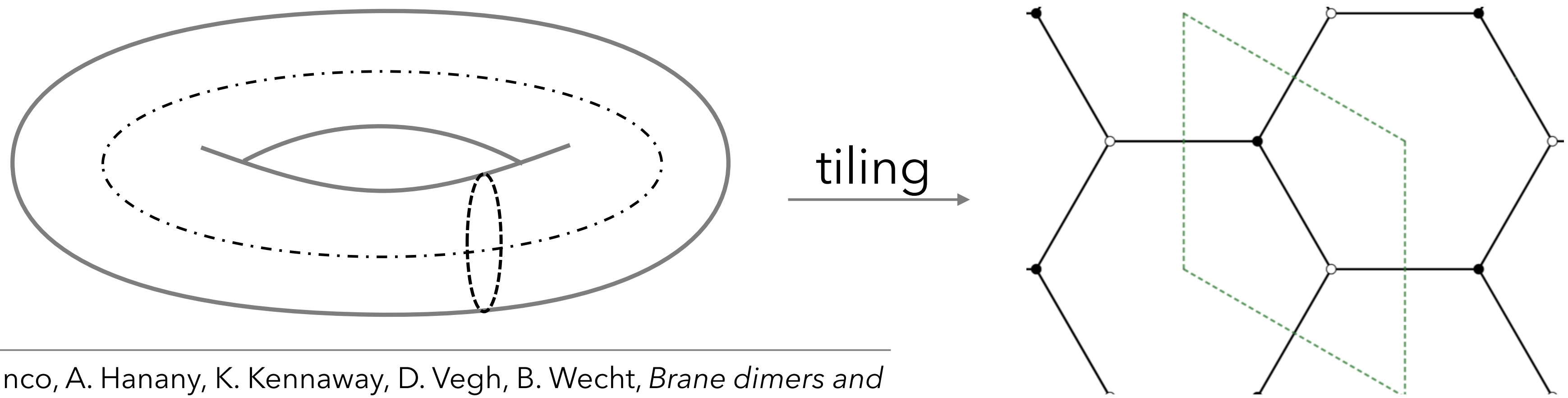
D. Morrison, R. Plesser, *Nonspherical horizons. 1.*, 1998, hep-th/9810201

Toric Geometry

$$\underbrace{ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2}_{AdS_5} + \underbrace{R^2 ds_Y^2}_{Y^5}$$

(transverse space: $\frac{R^2}{r^2}dr^2 + R^2 ds_Y^2$)

We have more control when isometries of transverse space are $U(1)^2 \times U(1)_R$



S. Franco, A. Hanany, K. Kennaway, D. Vegh, B. Wecht, *Brane dimers and quiver gauge theories*, 2005, hep-th/0504110

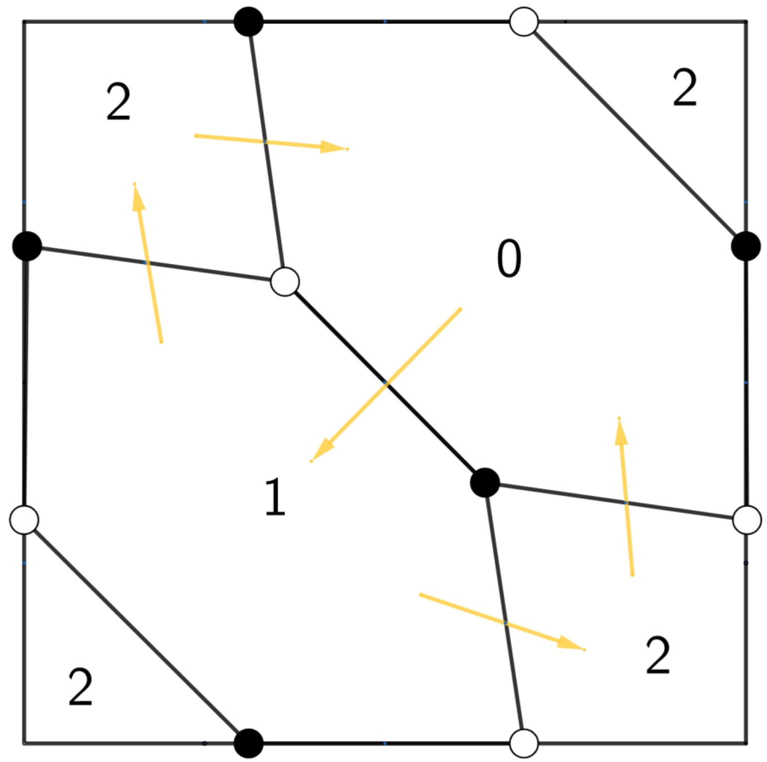
Toric Geometry

Dictionary:

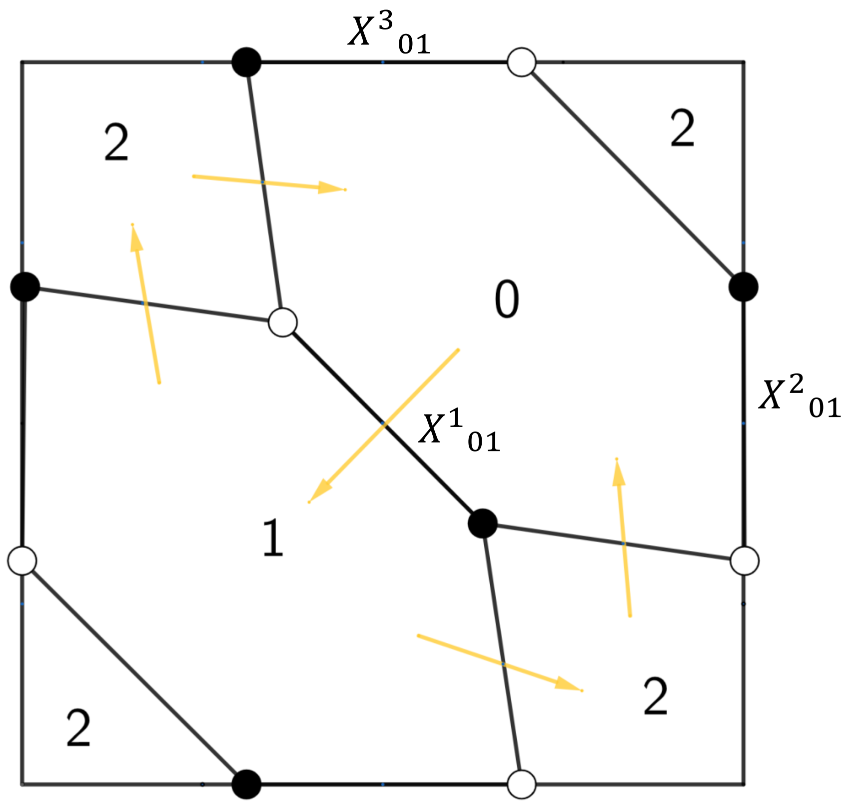
Face → Gauge group SU(N)

Edge → Matter field

White (black) node → +(-) interaction term

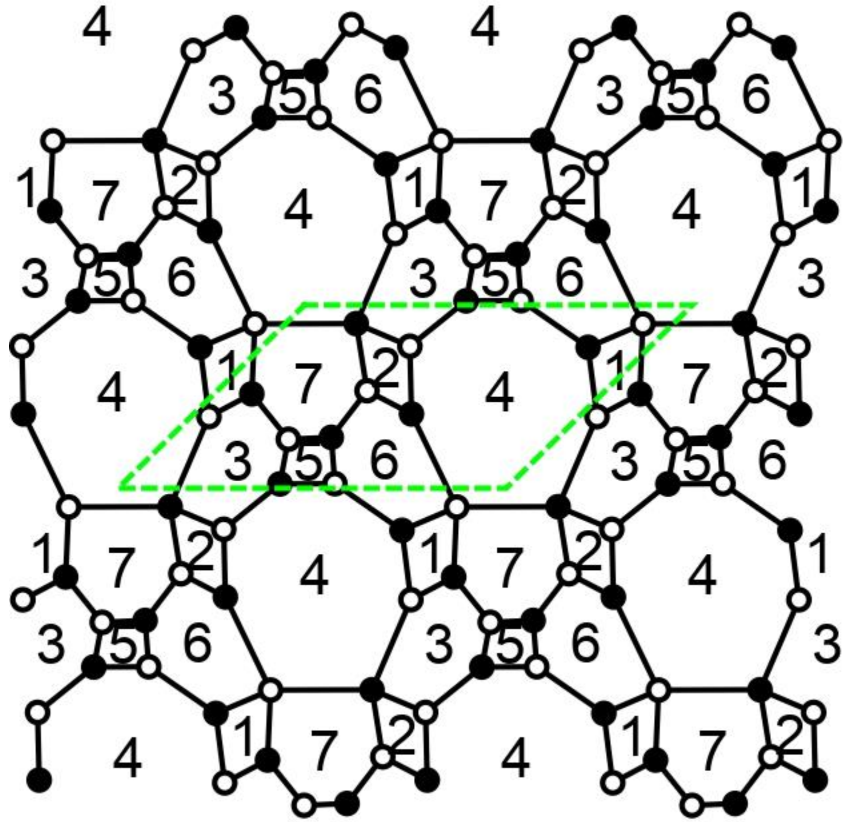


Toric Geometry



$$
\begin{aligned}
W &= X^1{}_{01}X^2{}_{12}\,X^3{}_{20} - X^1{}_{01}X^3{}_{12}\,X^2{}_{20} \\
&\quad + X^2{}_{01}X^3{}_{12}\,X^1{}_{20} - X^2{}_{01}X^1{}_{12}\,X^3{}_{20} \\
&\quad + X^3{}_{01}X^1{}_{12}\,X^2{}_{20} - X^3{}_{01}X^2{}_{12}\,X^1{}_{20} \\
&= \varepsilon_{ijk}X^i{}_{01}X^j{}_{12}\,X^k{}_{20}
\end{aligned}
$$

Toric Geometry



(Don't) try this at home

Image from: A. Hanany, R. Seong, *Brane Tilings and Reflexive Polygons,* 2012, hep-th/1201.2614

a-maximization

$$a = \frac{3}{32}\left(3\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right)$$

'Counting' of degrees of freedom, a-theorem $a_{UV} > a_{IR}$

Non-R abelian global symmetry mix with $U(1)_R$

R-charges uniquely determined at the maximum of a, a_{SCFT}:

$$\frac{3}{2} R_{\mathcal{O}} = \Delta = 1 + \frac{1}{2}\gamma_{\mathcal{O}}$$

B. Wecht, K. Intriligator, *The Exact superconformal R symmetry maximizes a*, 2003, hep-th/0304128

a-maximization

$$a_{SCFT} = \max\left\{\frac{3}{32}\left(3\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right)\right\}$$

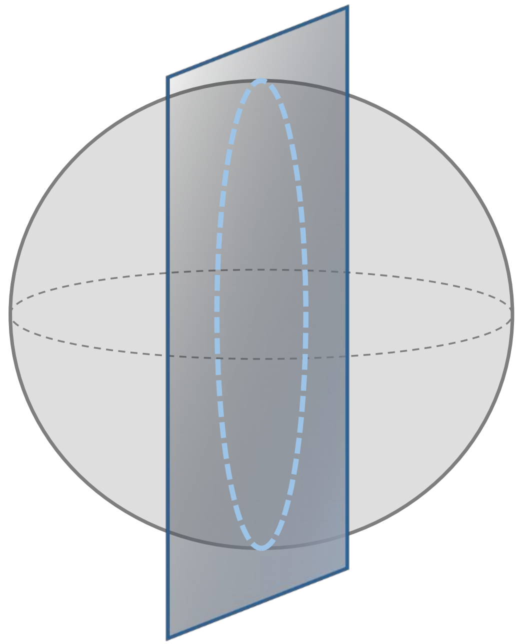
Relation to geometry

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + \overbrace{\frac{R^2}{r^2}dr^2 + R^2 ds_Y^2}^{\text{transverse space}}$$

$$\mathrm{Vol}(Y, r) \sim \frac{1}{a_{SCFT}}$$

S. Gubser, *Einstein manifolds and conformal field theories,* 1998, hep-th/9807164

Orientifold projection

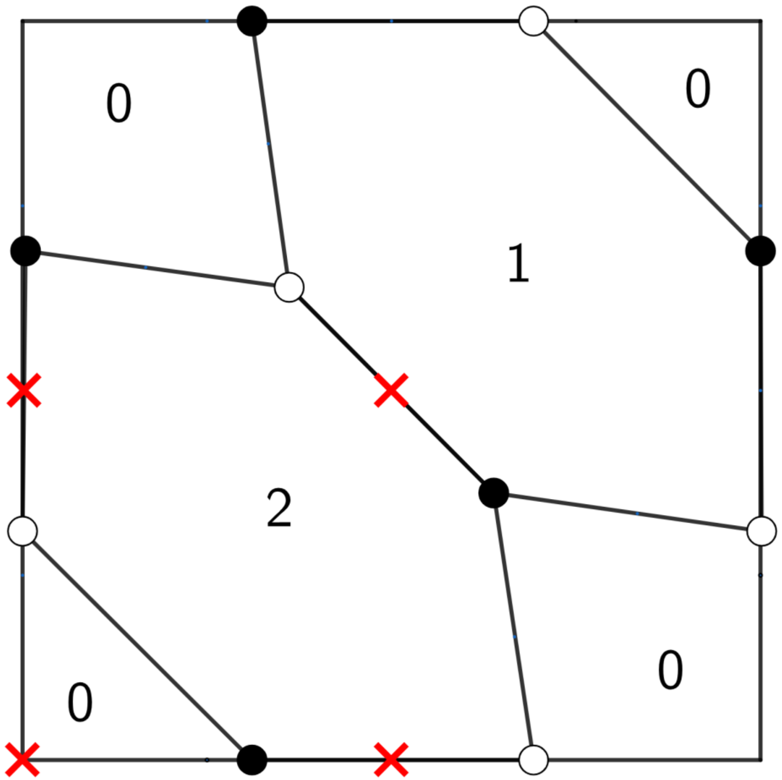


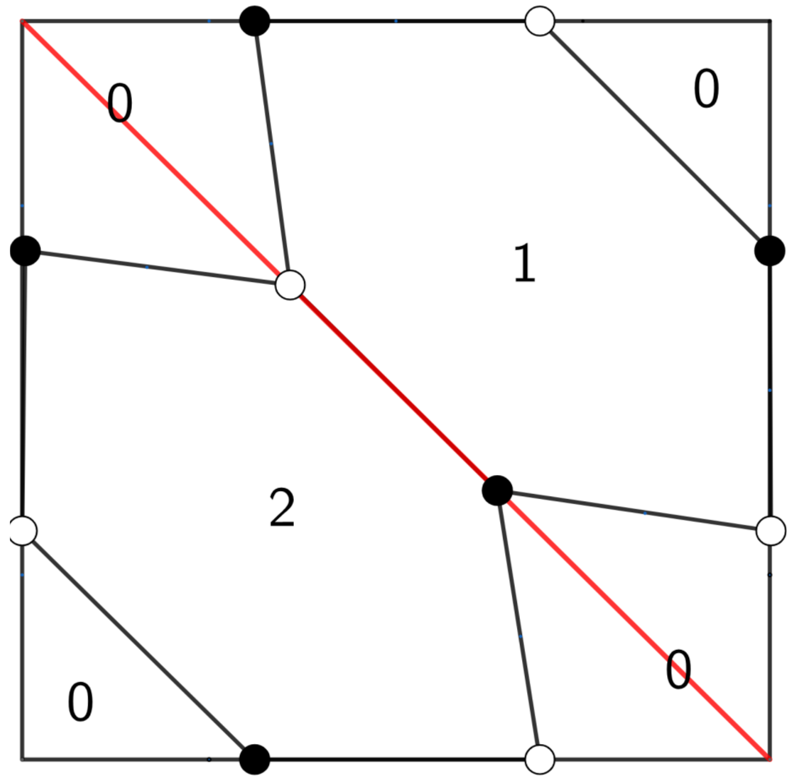
$\mathbb{Z}_2$ involution of the transverse space
Orientation of strings reversed

Why the orientifold Ω?
- It breaks (some) of the supersymmetries;
- It reduces the spectrum;
- It allows for SO, Sp gauge groups and tensorial matter;

A. Sagnotti, *Open strings and their symmetry groups,* 1987, hep-th/0208020

Orientifold projection





S. Franco, A. Hanany, D. Krefl, J. Park, A. Uranga, D. Vegh, *Dimers and orientifolds*, 2007, hep-th/0707.0298

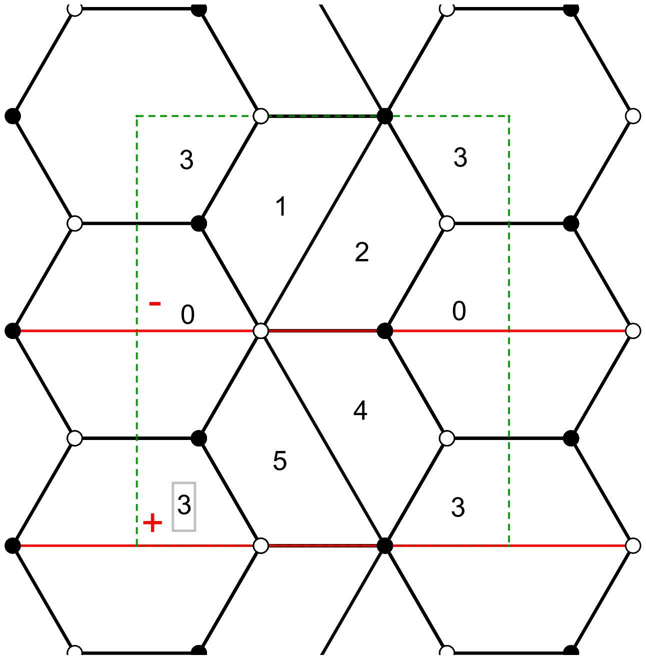
Orientifold projection

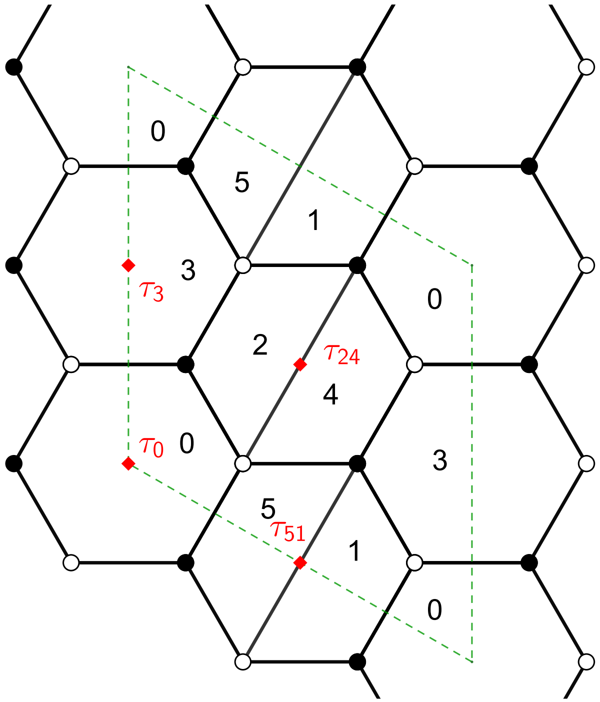
What is the fate of the conformal point after the involution Ω?

1. $\frac{a^\Omega}{a} = \frac{1}{2}$
2. a^Ω has no maximum
3. $\frac{a^\Omega}{a} < \frac{1}{2}$ $\longleftarrow$ New!

A. Antinucci, SM, F. Riccioni, *Infrared duality in unoriented Pseudo del Pezzo*, 2020, hep-th/2007.14749

A new (unoriented) IR duality





a^{Ω} charges match

Different parent, same orientifolded theories

Conclusion

- The AdS/CFT correspondence is geometrical in nature
- Engineering SCFT theories by deforming the space transverse to AdS
- The orientifold may yield new SCFTs, by breaking the global symmetry
- A new IR duality is found

Fun fact: the sphere was not a sphere, equator larger than meridian.
Too lazy to fix but not enough to write this

Thank you