

Carrier Envelope Phase signatures in Thomson scattering Spectrum



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Education

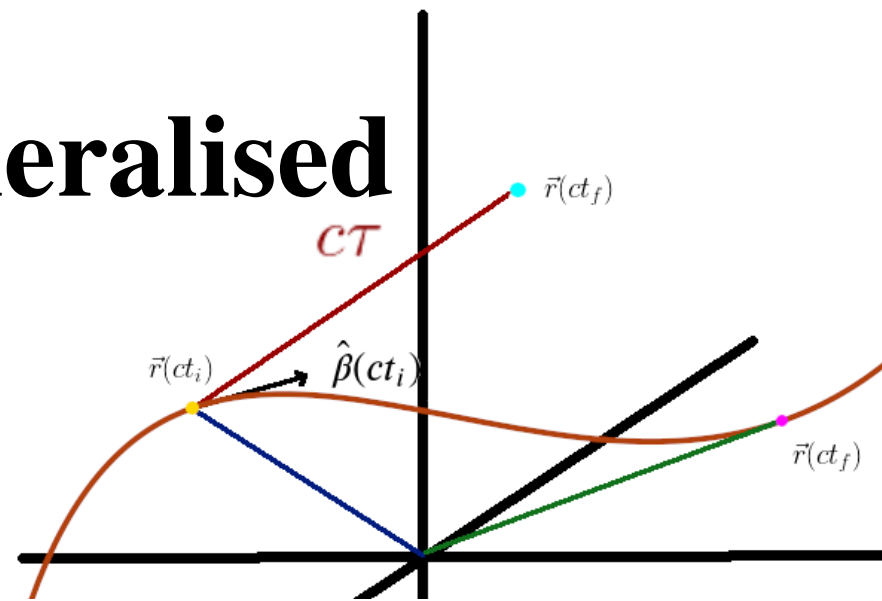
- B.Sc. Engineering Physics
Fontys University of Applied Sciences,
Eindhoven, The Netherlands
- Pre-master Applied Physics,
Eindhoven University of Technology,
Eindhoven, The Netherlands
- M.Sc. Photonics
Friedrich Schiller Universität,
Jena, Germany
- (Current) PhD Accelerator Physics,
Università degli Studi di Roma La Sapienza,
Roma, Italy

Supervisors: Vittoria Petrillo, Luca Serafini



PhD Topics: most generalised

Lienard Wiechert potentials



$$\vec{E}(ct, \vec{r}(ct)) = q \left(\frac{\hat{n} - \vec{\beta}}{\gamma^2 (1 - \hat{n}\vec{\beta})^3 c\tau^2} + \frac{\hat{n} \times \left((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}} \right)}{(1 - \hat{n}\vec{\beta})^3 c\tau} \right) \Big|_{ct_{ret}}$$

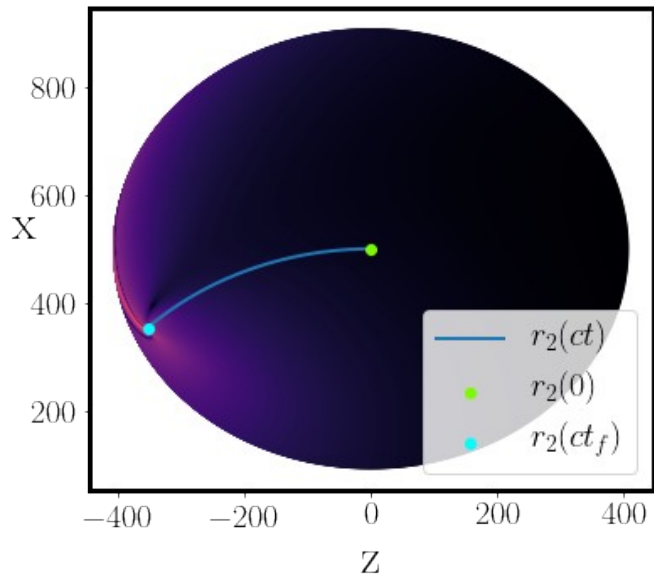
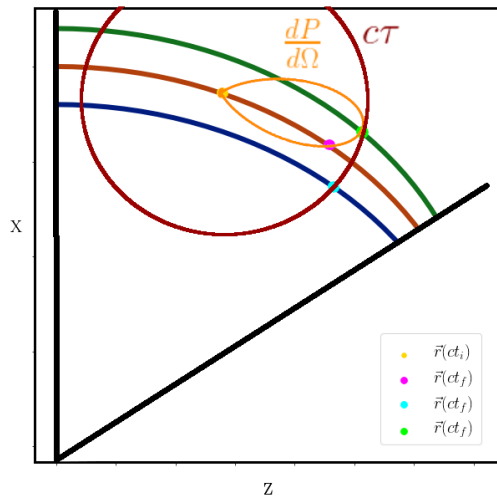
From Wikipedia, the free encyclopedia

The **Liénard-Wiechert potentials** describe the classical [electromagnetic](#) effect of a moving [electric point charge](#) in terms of a [vector potential](#) and a [scalar potential](#) in the [Lorenz gauge](#). Built directly from [Maxwell's equations](#), these describe the complete, [relativistically](#) correct, time-varying [electromagnetic field](#) for a [point charge](#) in arbitrary motion, but are not corrected for [quantum-mechanical](#) effects. [Electromagnetic radiation](#) in the form of [waves](#) can be obtained from these potentials. These expressions were developed in part by [Alfred-Marie Liénard](#) in 1898^[1] and independently by [Emil Wiechert](#) in 1900.^{[2][3]}

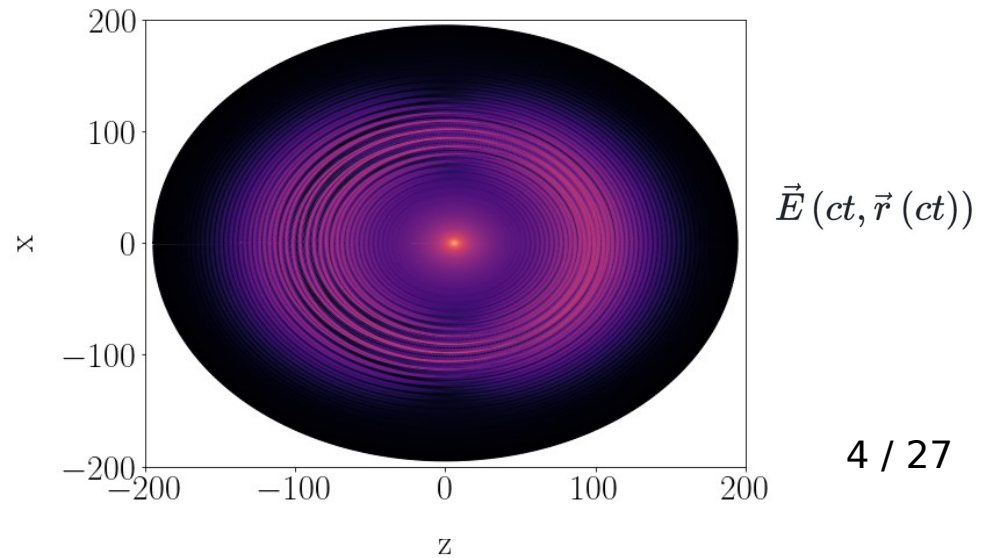
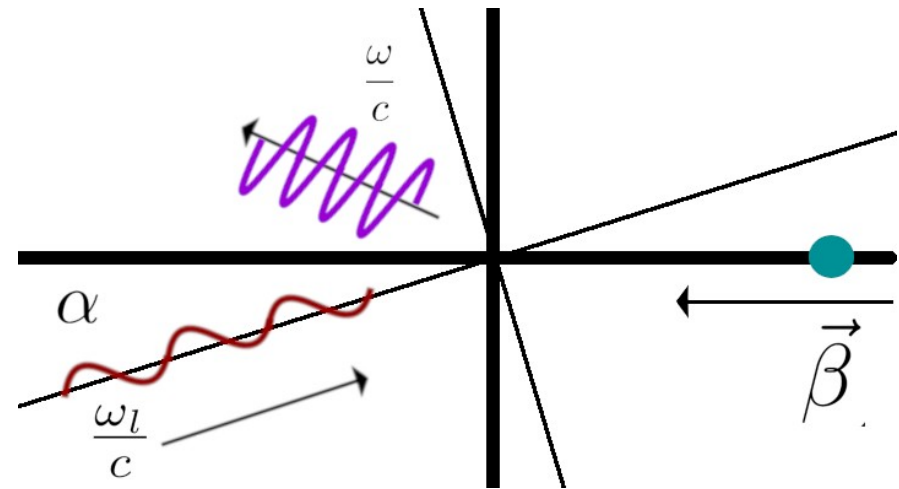
In 1905, Poincaré was the first to recognize that the transformation has the properties of a [mathematical group](#), and named it after Lorentz.^[9] Later in the same year [Albert Einstein](#) published what is now called [special relativity](#), by deriving the Lorentz transformation under the assumptions of the [principle of relativity](#) and the constancy of the speed of light in any [inertial reference frame](#), and by abandoning the mechanistic aether as unnecessary.^[10]

PhD Topics

Retarded time
Particle interaction



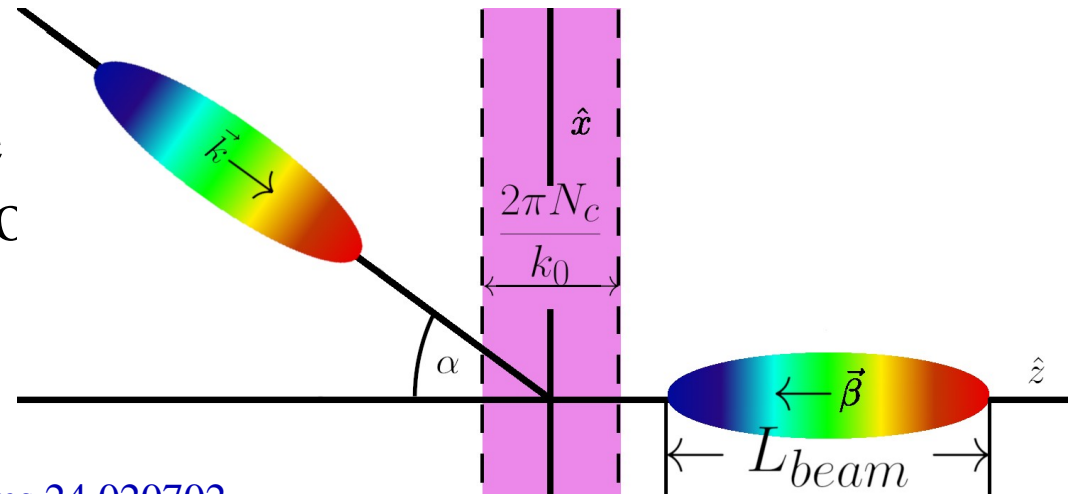
Scattering
Manipulation of laser pulse



PhD Topics : Thomson scattering

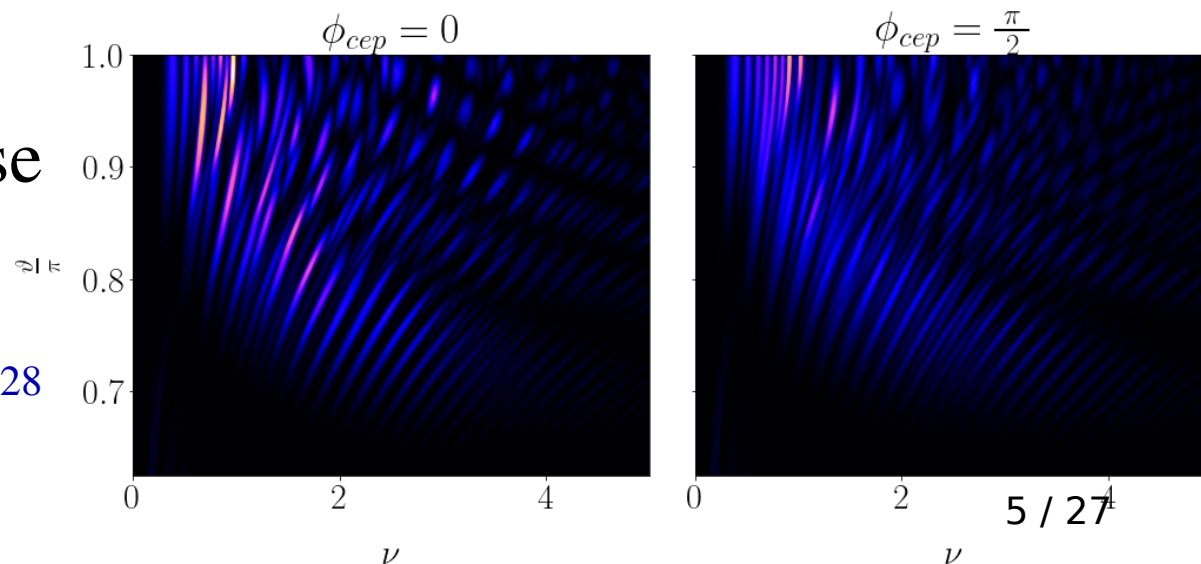
- Modulation of laser pulse frequency for compensating energy spread electrons

Ruijter et al
<https://doi.org/10.1103/PhysRevAccelBeams.24.020702>



- Carrier Envelope Phase (CEP) dependency

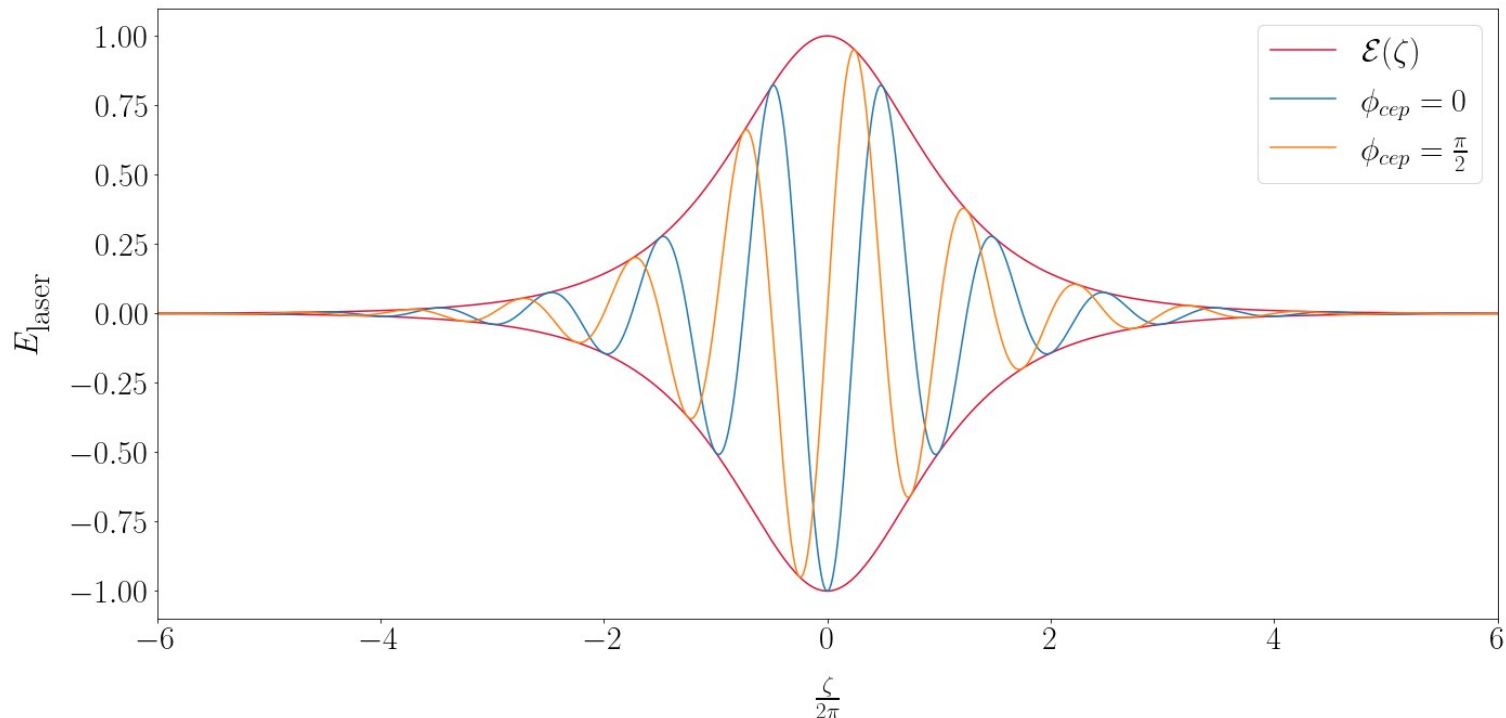
Ruijter et al
<https://doi.org/10.3390/cryst11050528>



CEP : what's the deal?

Short pulses ~ Length comparable to wavelength

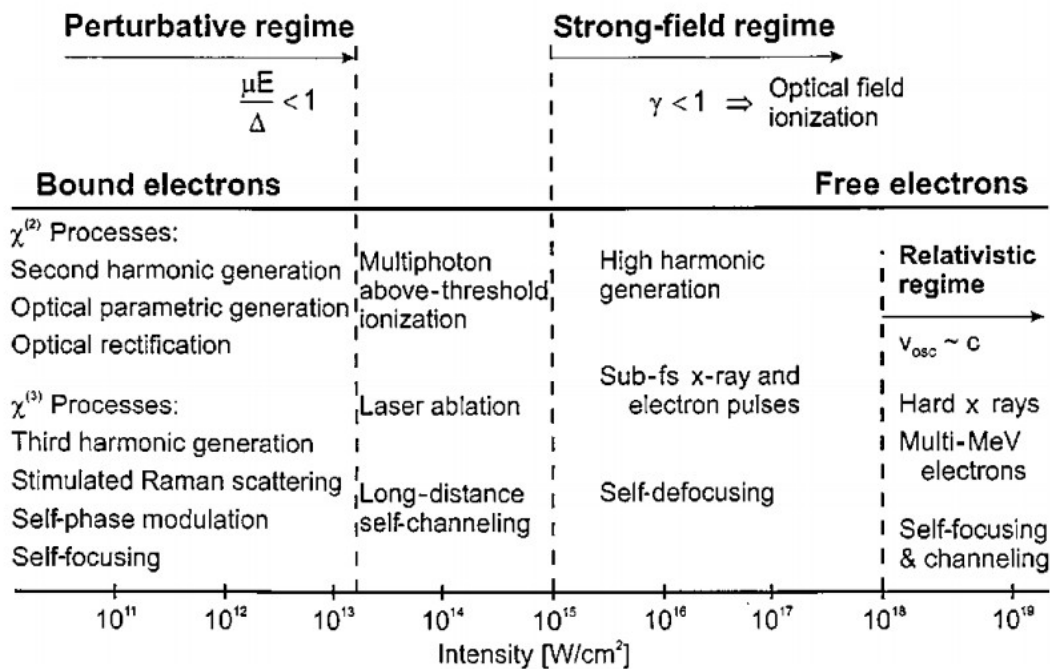
Strong dependency CEP on the shape of the electric field near maximum
Temporal profile



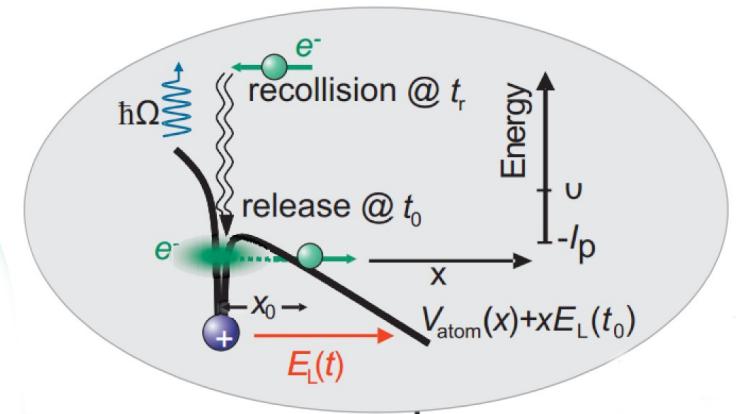
CEP : what's the deal?

- Interesting physics in the short pulse regime

Regimes of Nonlinear Optics



T. Brabec & F. Krausz (2000)
<https://doi.org/10.1103/RevModPhys.72.545>



F. Krausz & M. Ivanov (2009)
<https://doi.org/10.1103/RevModPhys.81.163>

- CEP measurements for Intensities $< 10^{15}$ W/cm²

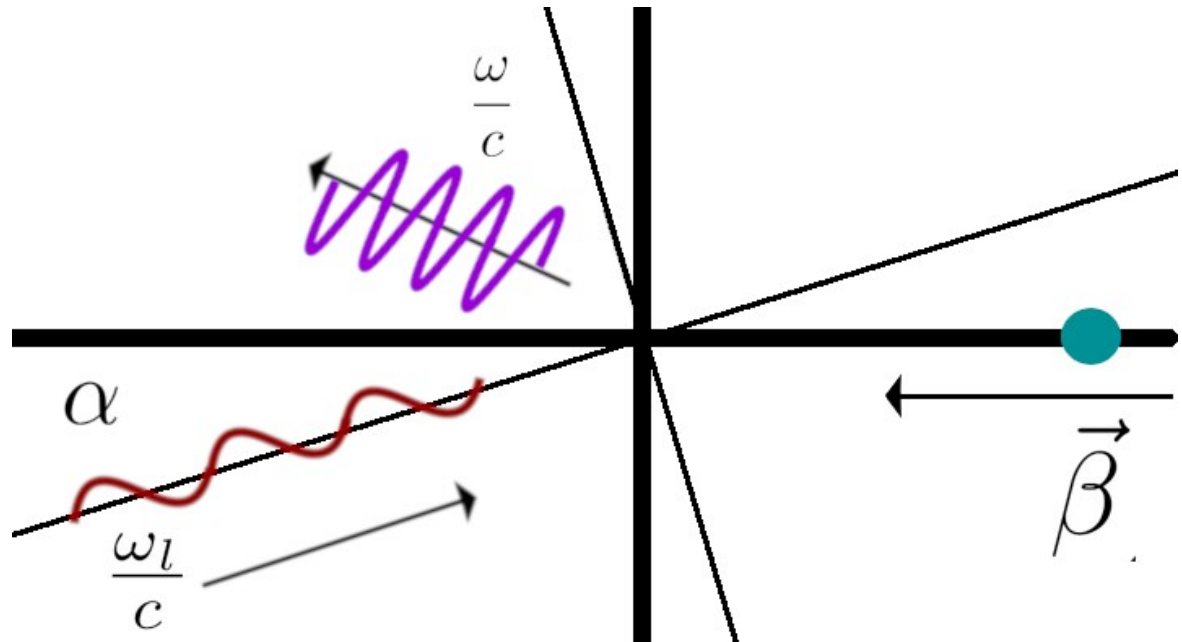
G.G. Paulus et al
<https://doi.org/10.1103/PhysRevLett.91.253004>

S. Fukahori
<https://doi.org/10.1103/PhysRevA.95.053410>

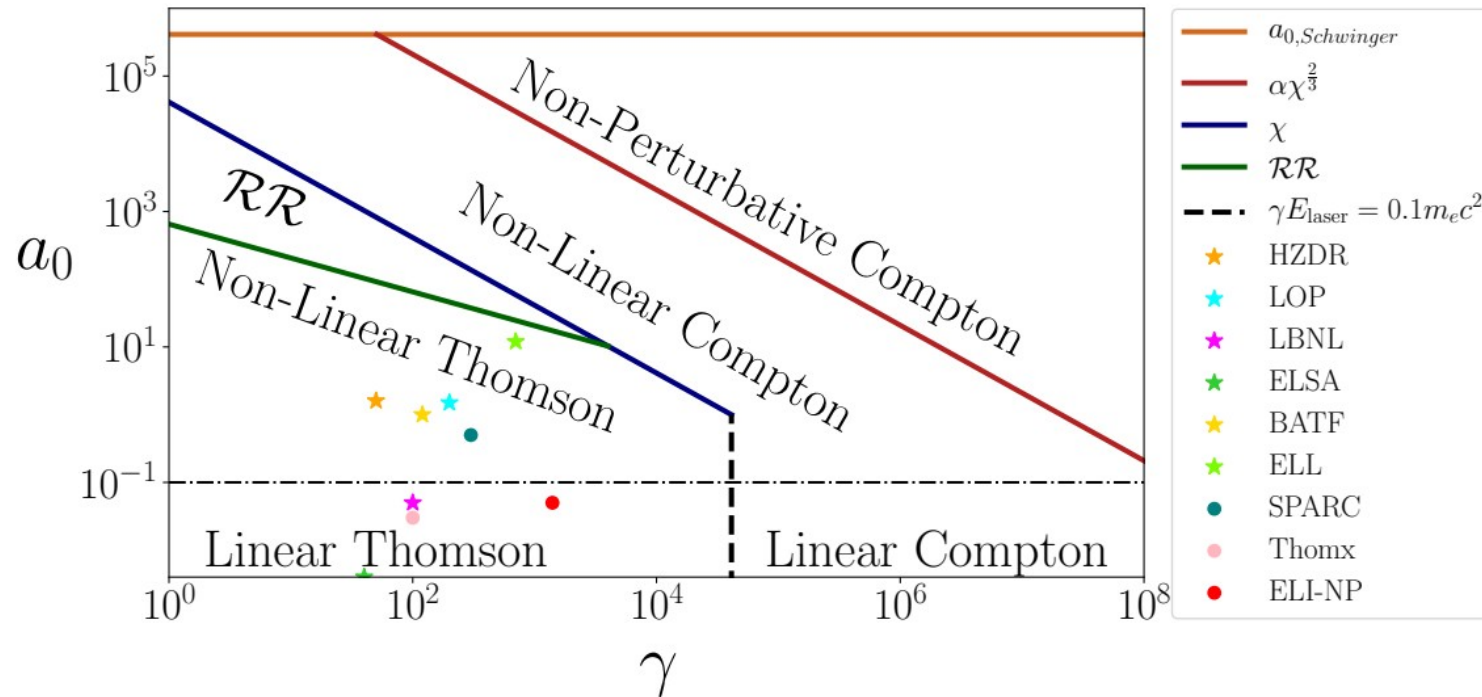
CEP : what's the deal?

Thomson scattering as diagnostic tool:

- Intensity profile of laser pulse
- CEP measurement



Energy regimes scattering processes

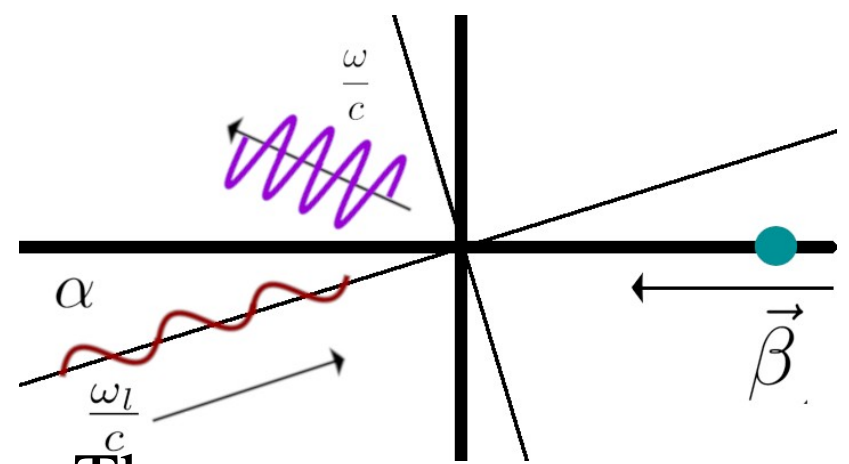


- J. Schwinger, Phys. Rev. 82, (5) 1951
- A. Fedotov, IOP: Conf. Ser. 826, 2017
- A.I. Nikishov and V.I. Ritus, Sov. Phys. JETP 19, 1964
- Y. Hadad et al, Phys. Rev. D 82, (9) 2010
- A. Di Piazza, Letters in Mathematical Physics 83, (3) 2008
- M. Ruijter et al, IOP 51, (22) 2018

B. Terzic et al

<https://iopscience.iop.org/article/10.1209/0295-5075/126/12003/pdf>

Thomson scattering



What does the linear & non-linear Thomson regime require?

- Energy loss particle due 1 photon collision

$$\frac{\hbar\omega_{lab}\gamma}{mc^2} \ll 1$$

- Energy loss particle due field strength

$$\frac{\hbar\omega_{lab}\gamma a_0}{mc^2} \ll 1$$

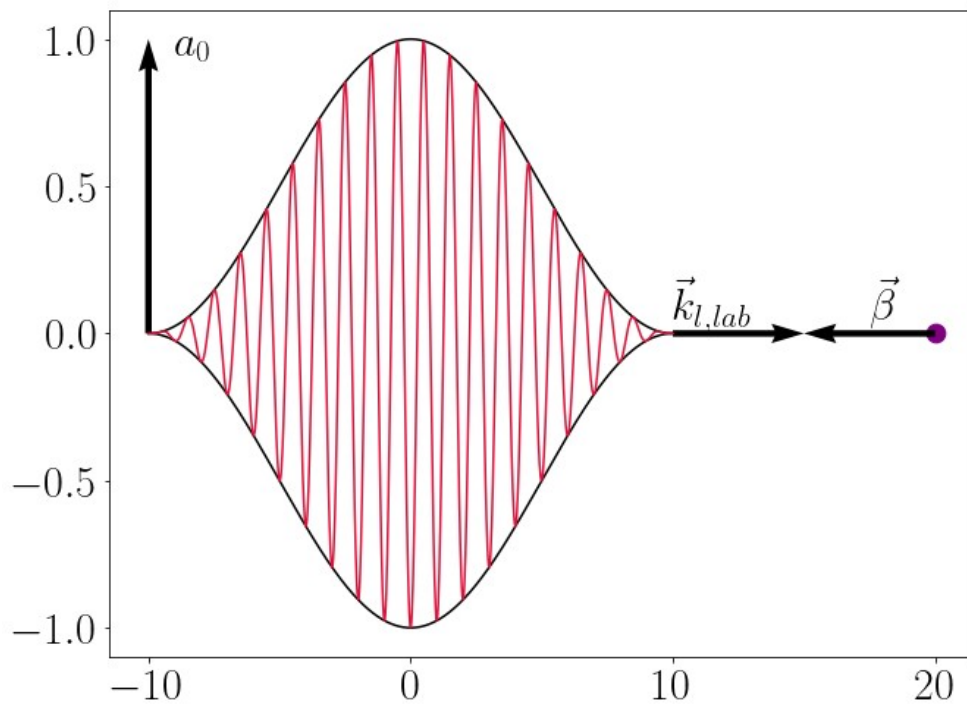
- Energy loss particle during all interaction

$$\tilde{\mathcal{E}}_{rad} a_0^2 N_c \ll 1$$

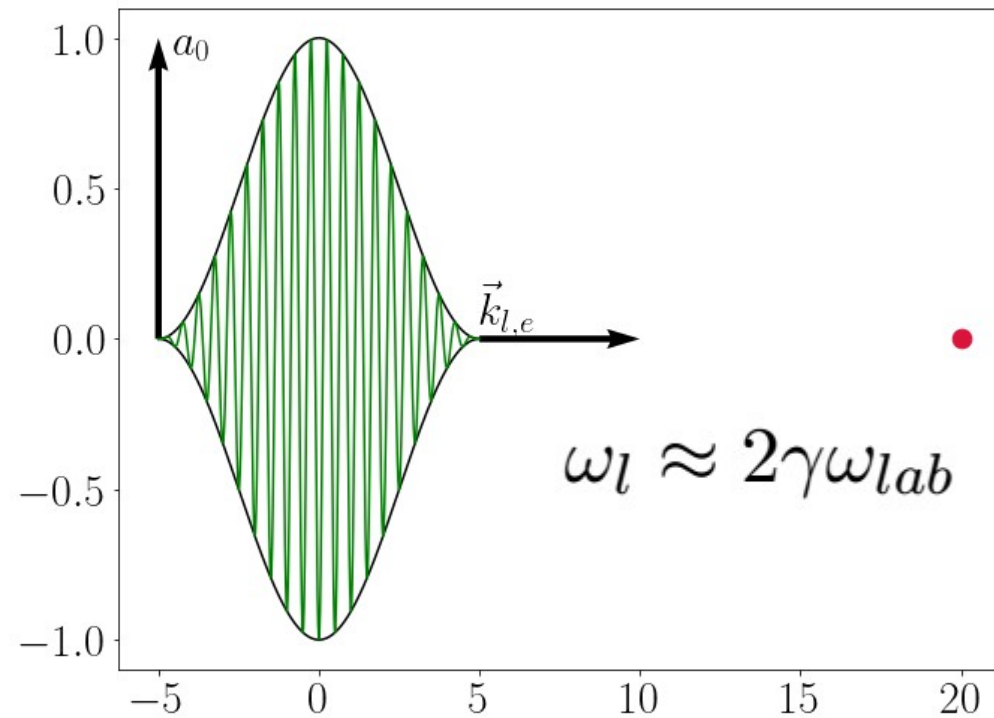
Solving the motion

Special relativity: No frame of reference is special

lab frame



frame e initially at rest

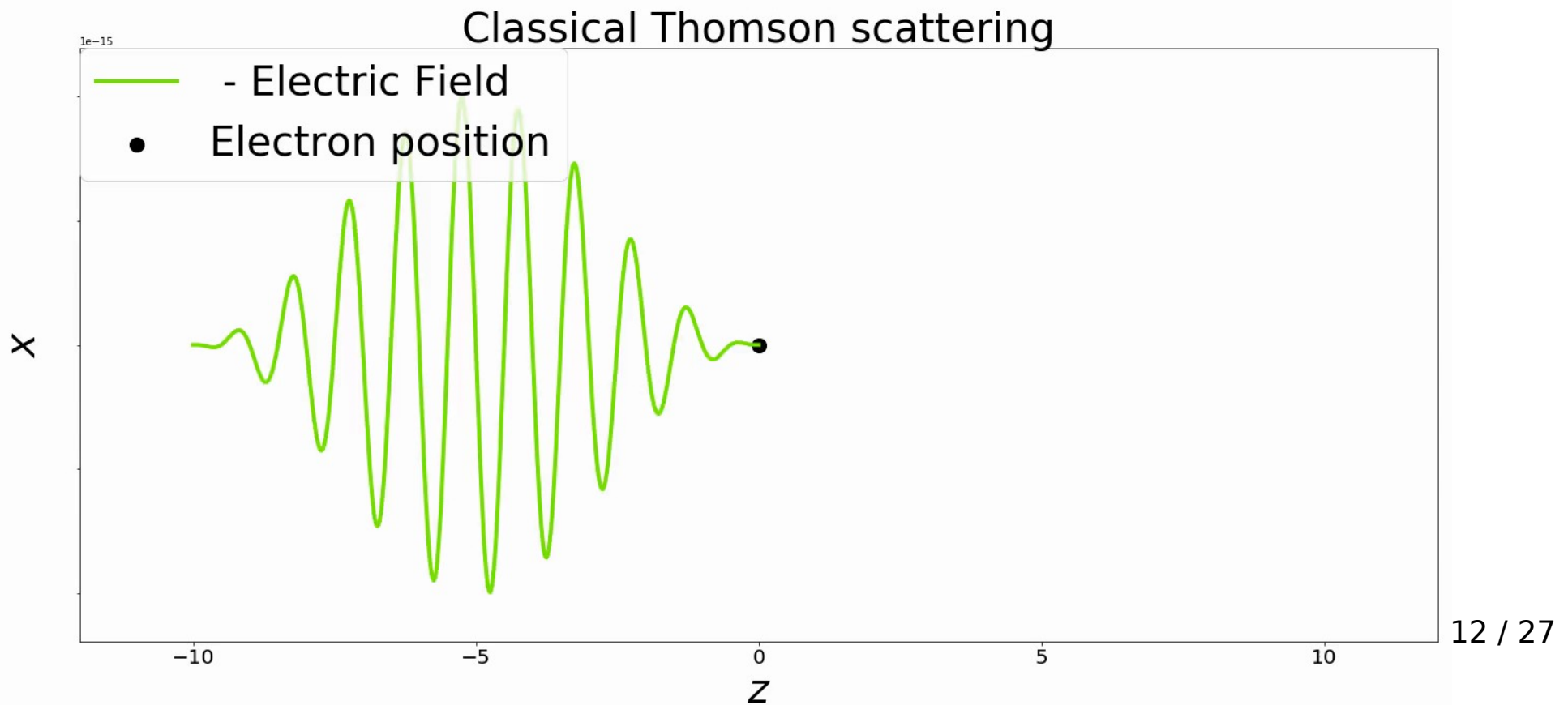


Lorentz invariants:

$$a_0 \quad N_c$$

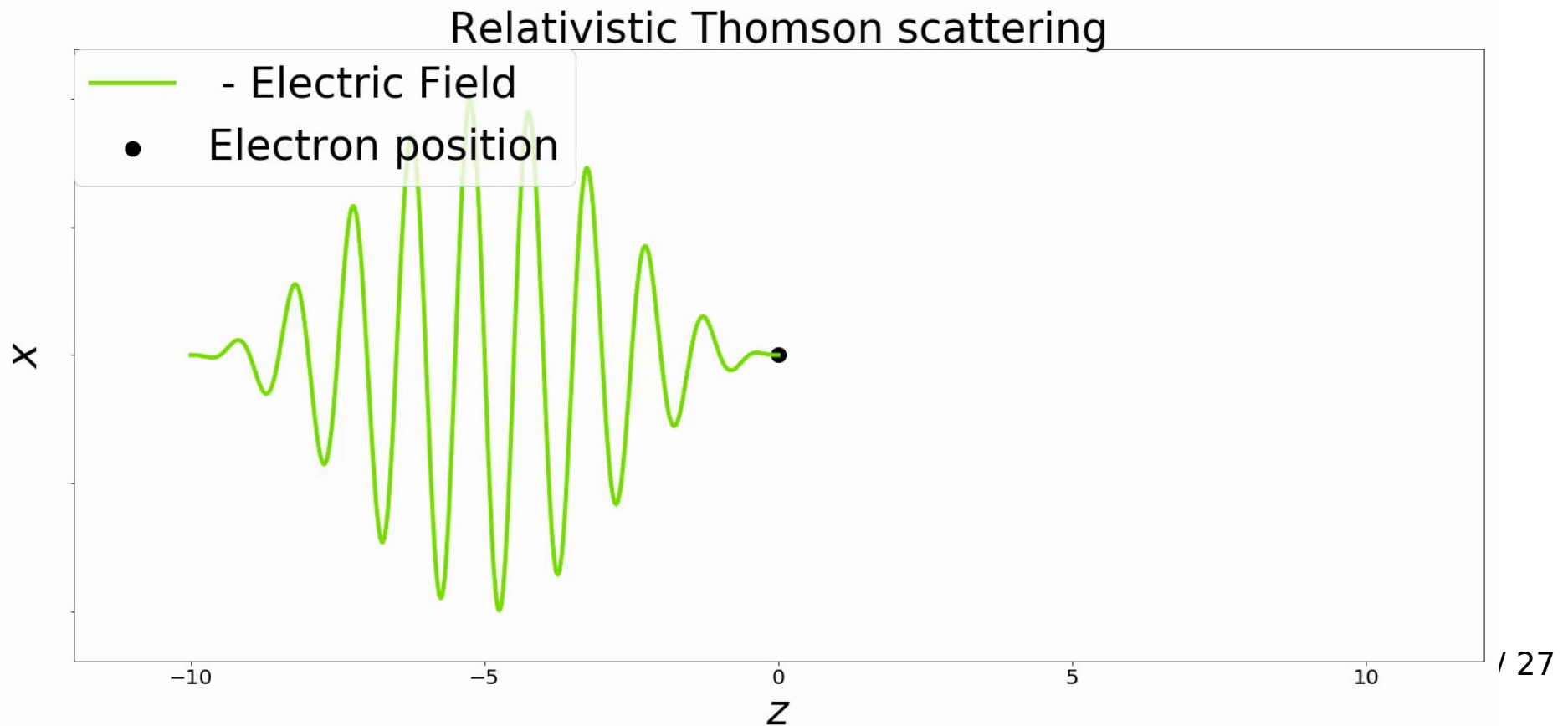
Solving the motion

Low intensity: $a_0 \ll 1$



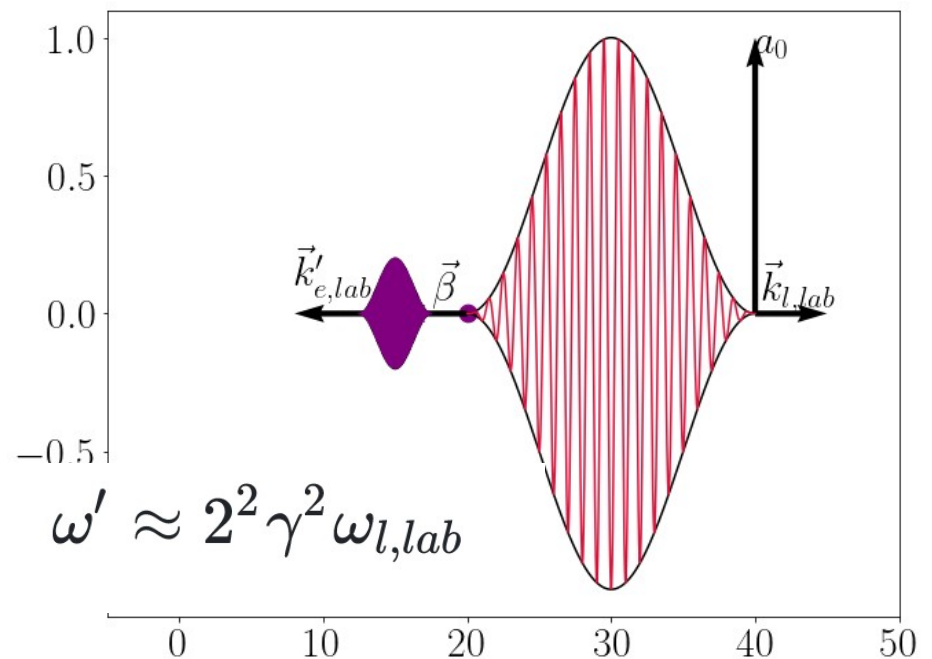
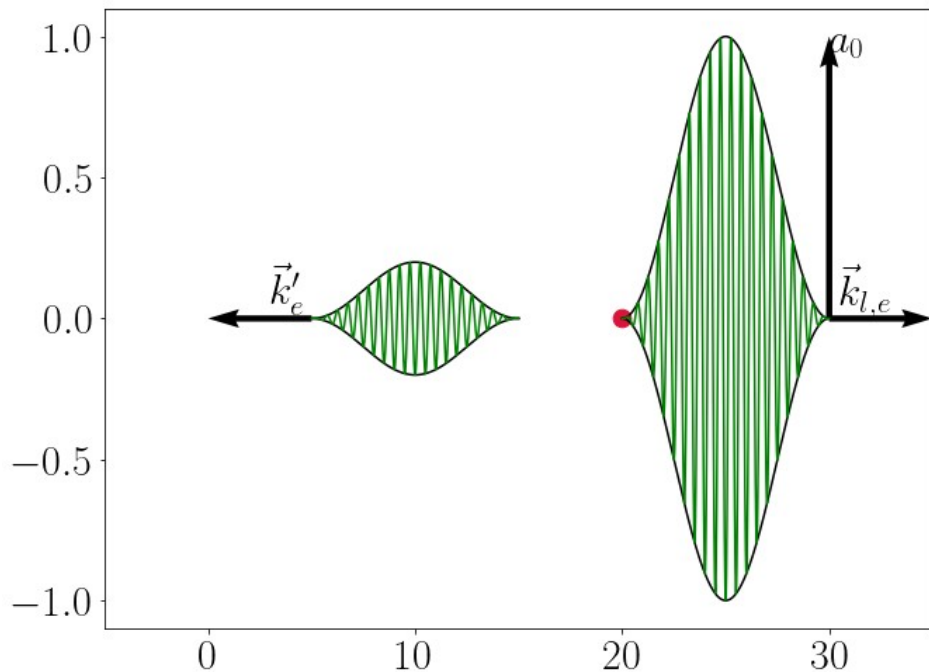
Solving the motion

High intensity: $a_0 > 1$



Solving the motion

Lorentz back transform gives second Doppler shift

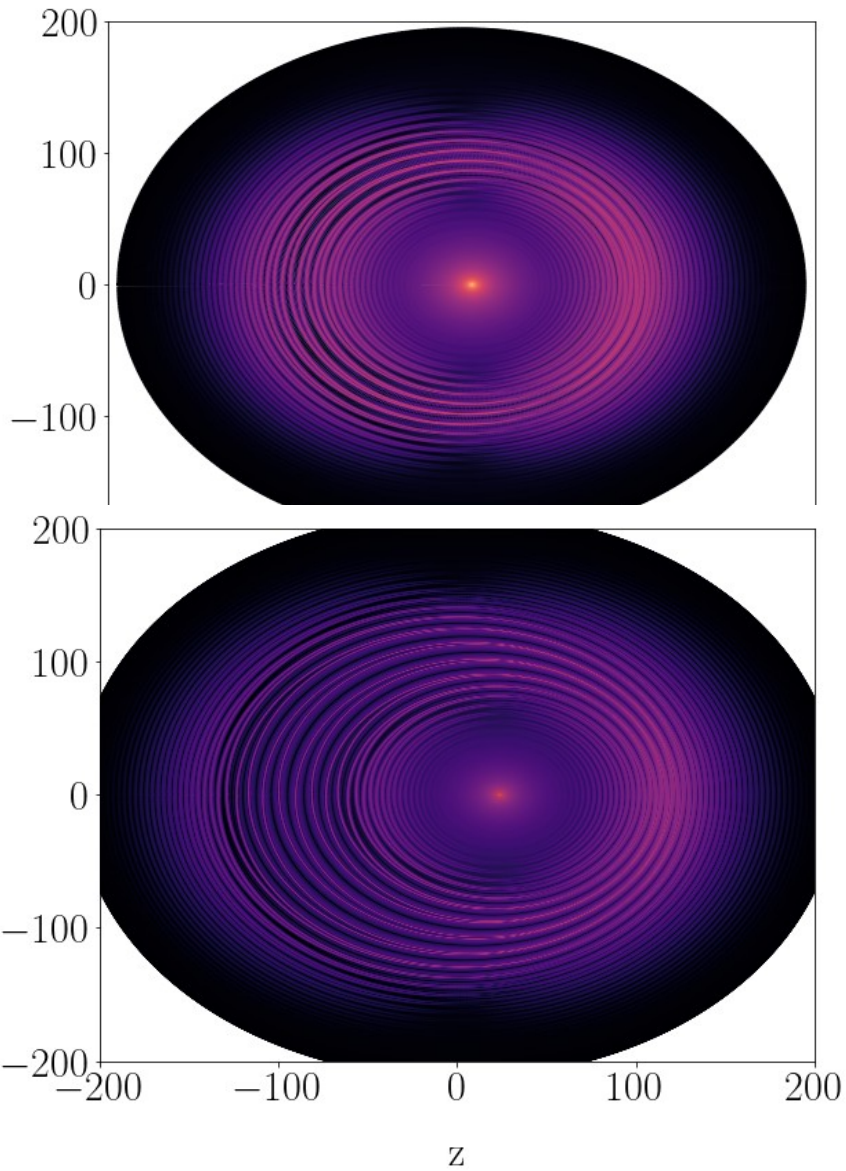


$$\omega' \approx 2^2 \gamma^2 \omega_{l,lab}$$

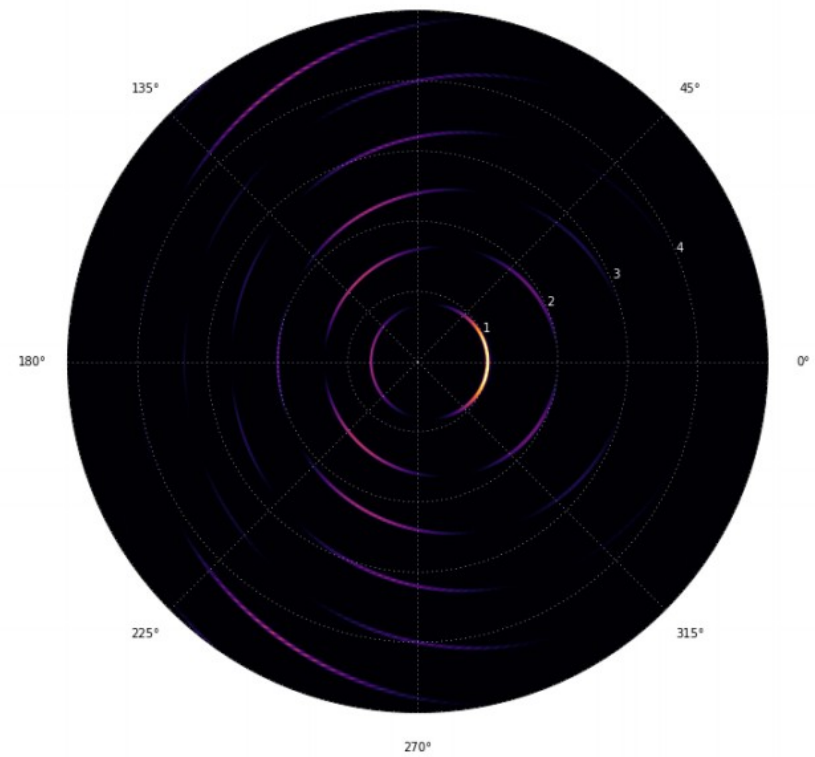
$$\nu = \frac{\omega}{2^2 \gamma^2 \omega_{l,lab}}$$

Emitted radiation

$$\vec{E}(ct, \vec{r}(ct))$$



$$\frac{d^2 I}{d\omega d\Omega}$$



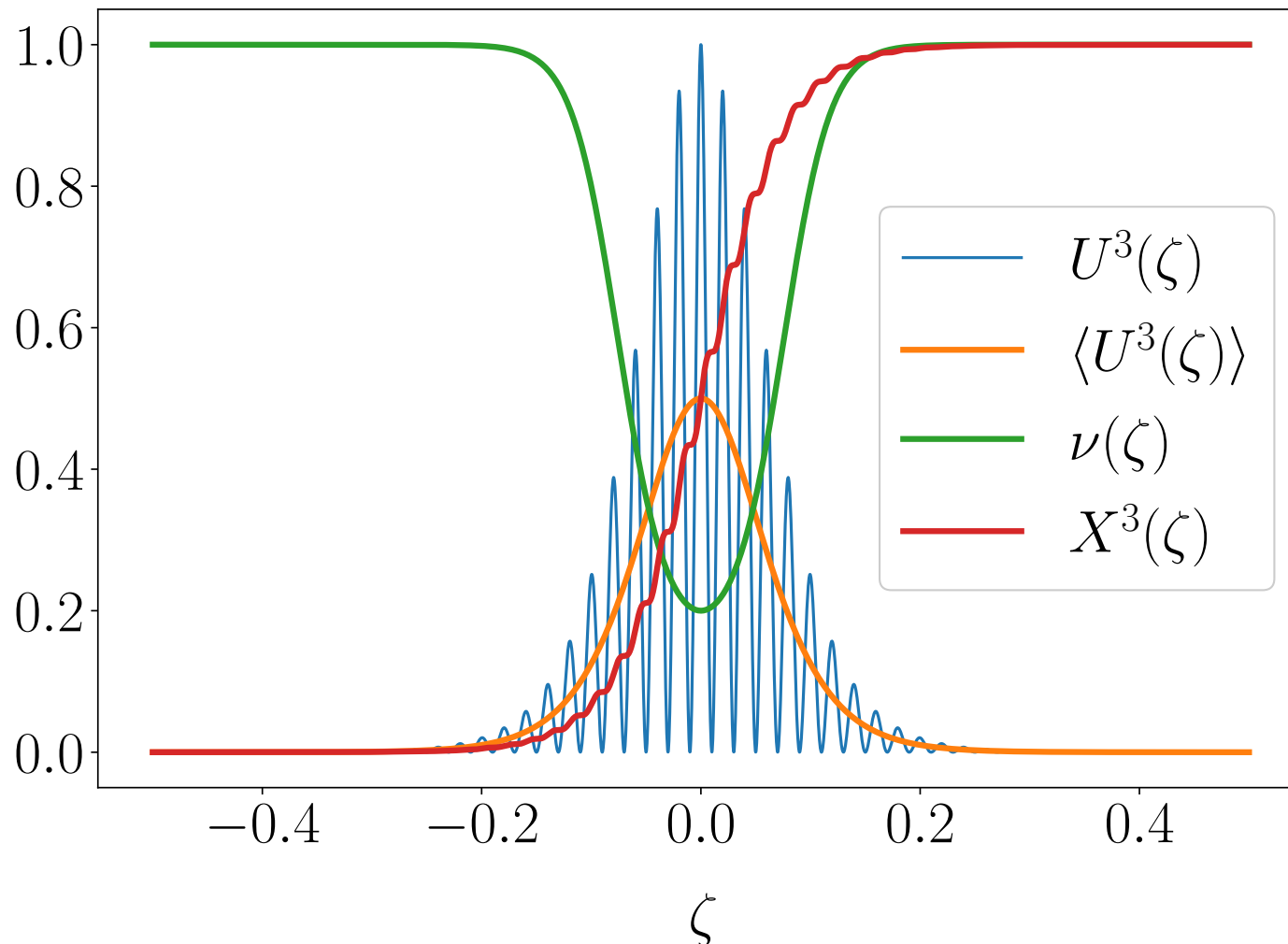
Emitted radiation from motion

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \omega \int_{-\infty}^{\infty} dt \hat{n} \times \hat{n} \times \vec{\beta}_i \exp \left[i \frac{\omega}{c} (ct - \hat{n} \cdot \vec{r}_i) \right] \right|^2$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi c} \left| \sum_{\mathcal{H}} \int d\zeta G_{\mathcal{H}} (\zeta, \nu) \exp [iF_{\mathcal{H}} (\zeta, \nu)] \right|^2$$

Emitted radiation from motion

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi c} \left| \sum_{\mathcal{H}} \int d\zeta G_{\mathcal{H}}(\zeta, \nu) \exp[iF_{\mathcal{H}}(\zeta, \nu)] \right|^2$$



$$\frac{d}{d\zeta} F_{\mathcal{H}} = \mathcal{H} + \nu (1 + \langle U^3 \rangle)$$

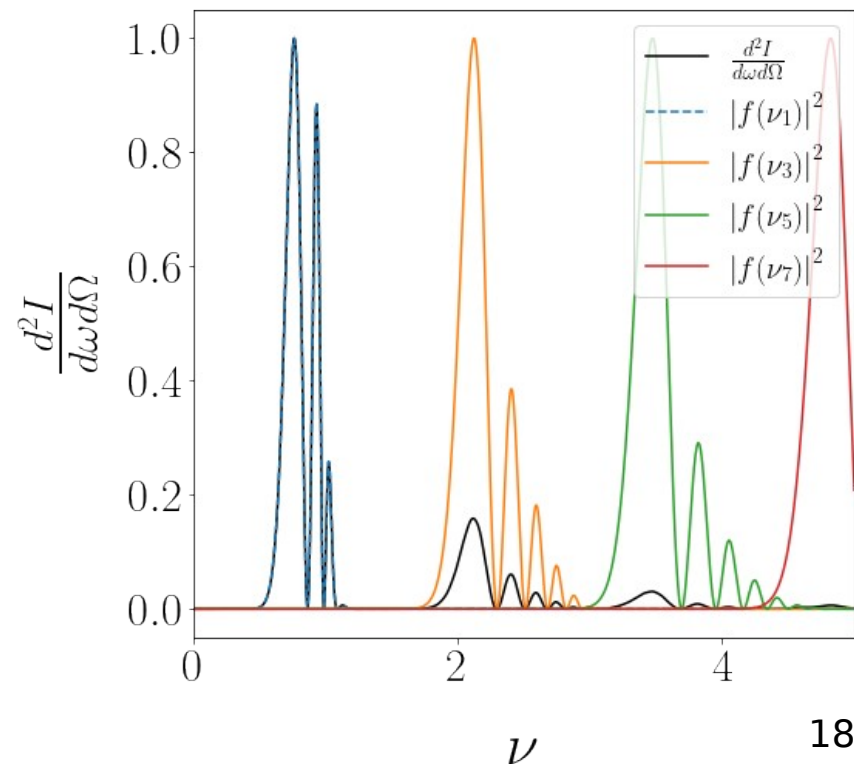
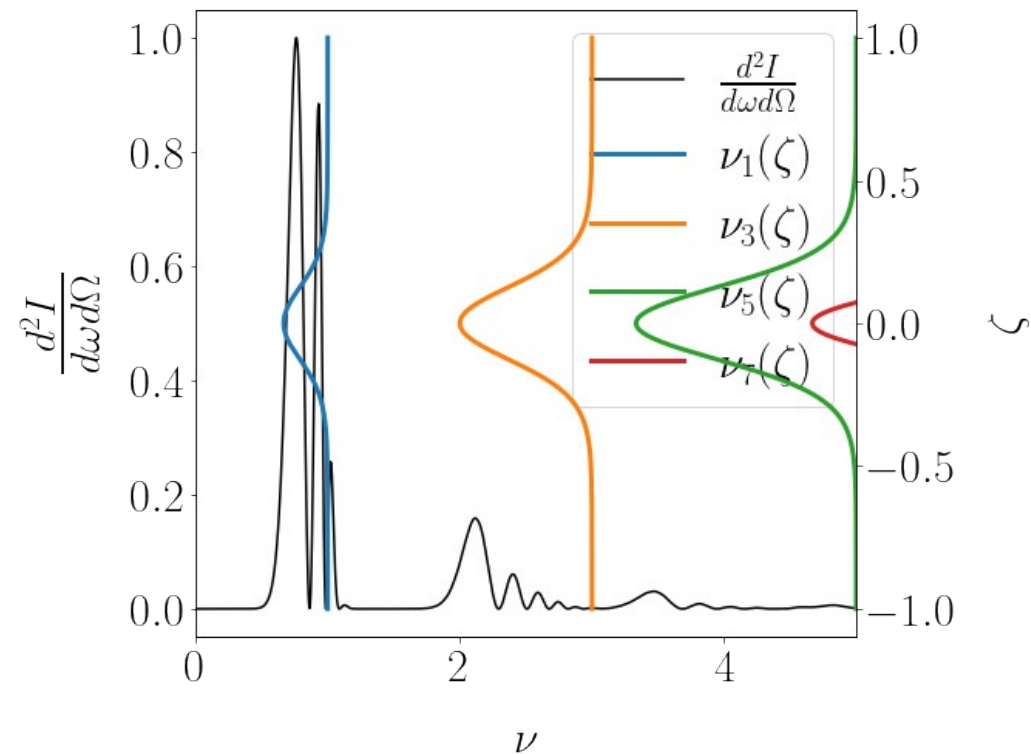
Analytical description of emitted radiation:
 S. G. Rykovanov et al. -
 Phys. Rev. Accel. Beams 19,
 030701 (2016)

$$G_{\mathcal{H}} \propto \exp[i\mathcal{H}\phi_{cep}]$$

Single electron – on-axis radiation

$$a_0 \ll 1 \quad |f_{\nu_1}|^2 + |f_{\nu_3}|^2 + \dots$$

CEP: global phase, not visible

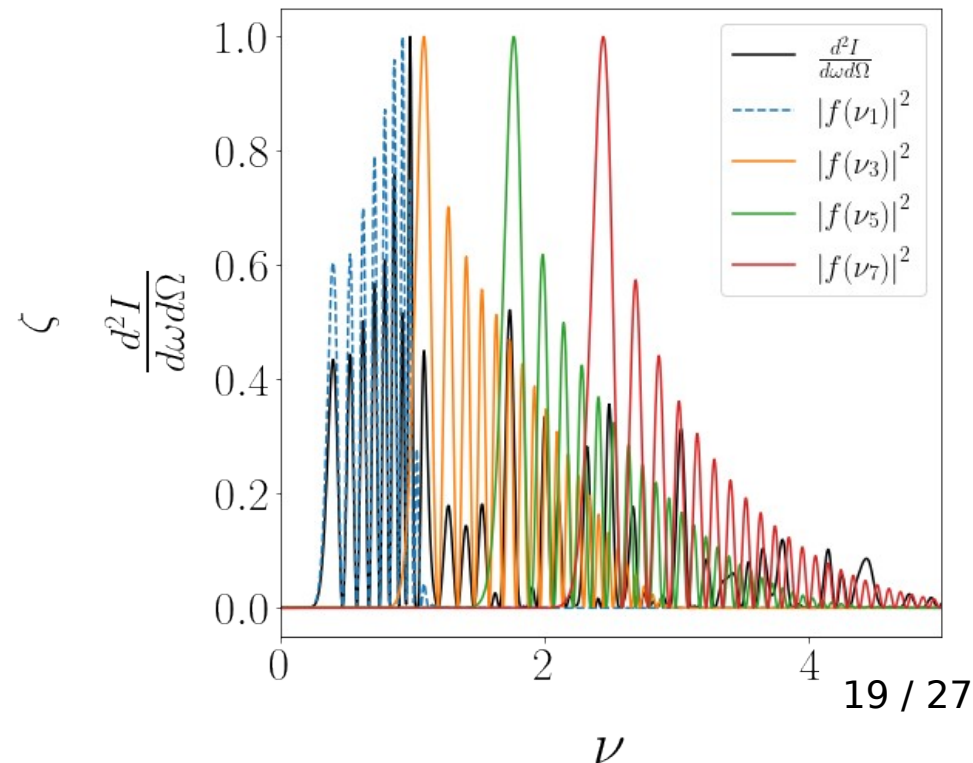
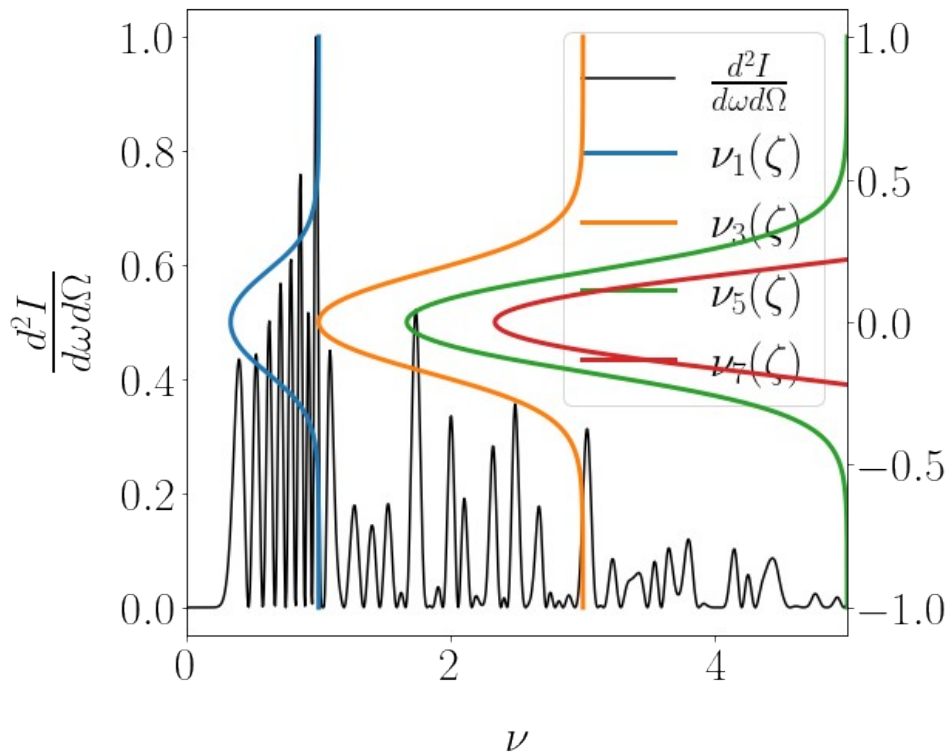


Single electron – on-axis radiation

$a_0 \sim 1$: Harmonics start to overlap

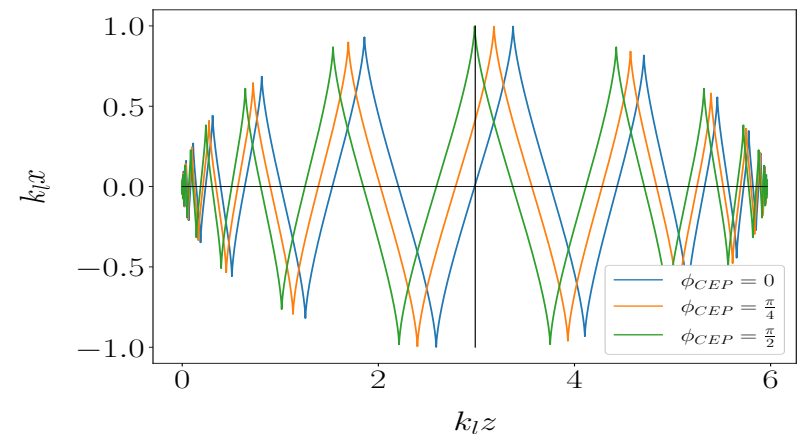
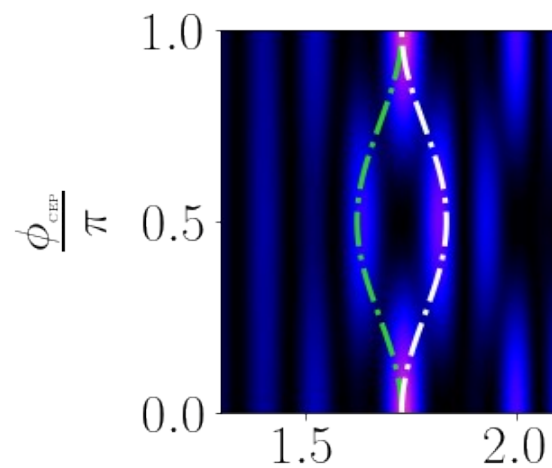
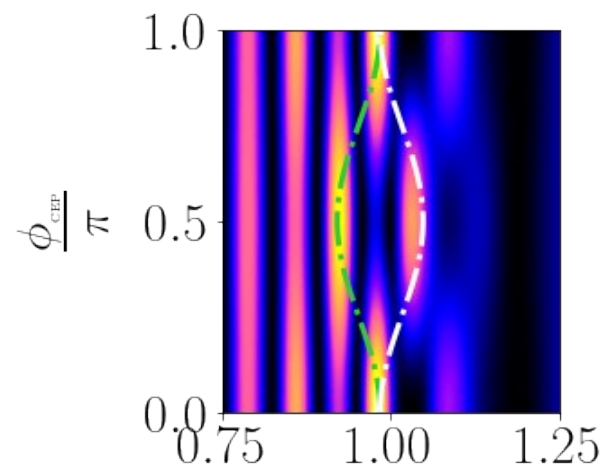
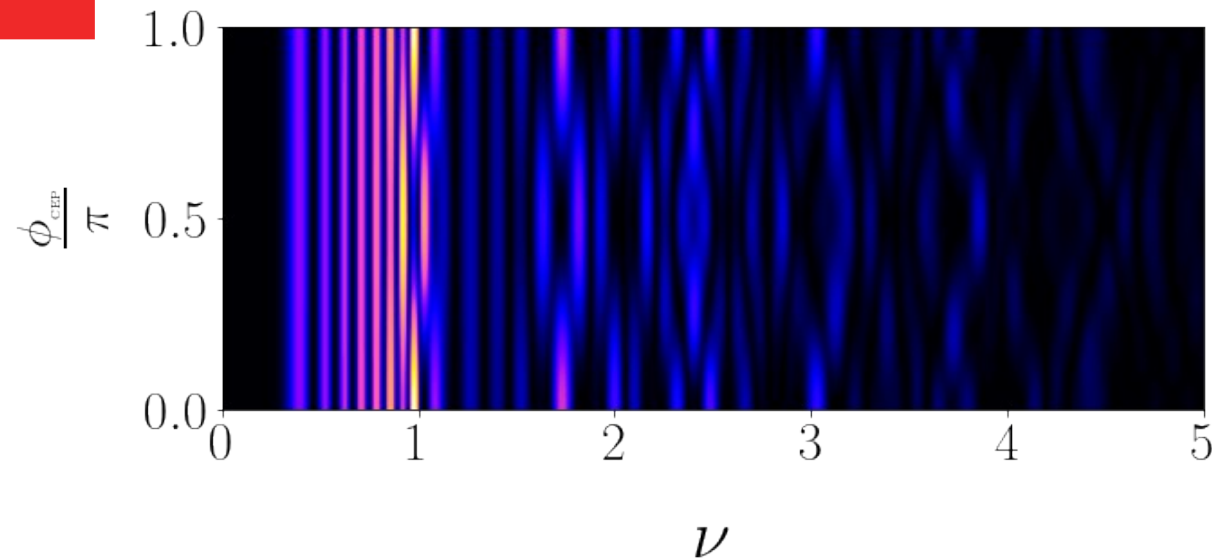
$$|f_{\nu_1}|^2 + 2\text{Re}(f_{\nu_1}f_{\nu_3}^*) + |f_{\nu_3}|^2 + \dots$$

$$a_{0\Delta\mathcal{H}} = \sqrt{2 \frac{\mathcal{H}_j - \mathcal{H}_i}{\mathcal{H}_i}}$$



Single electron – on-axis radiation

- Amplitude of a harmonic peak changes
→ position of the peak scales with

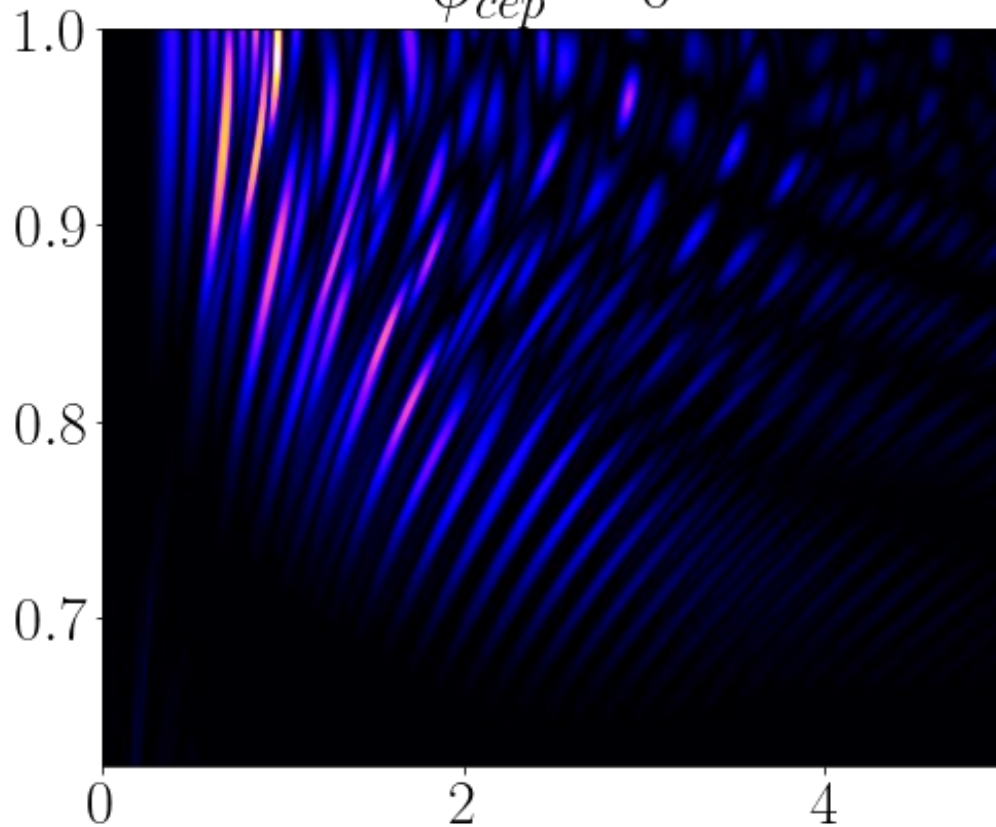


$$v_{\mathcal{H},cep} \propto \frac{\mathcal{H}}{1 + \frac{a_0^2}{2}} \left(1 \pm \frac{\sin^2(\phi_{cep})}{\pi N_c} \right)$$

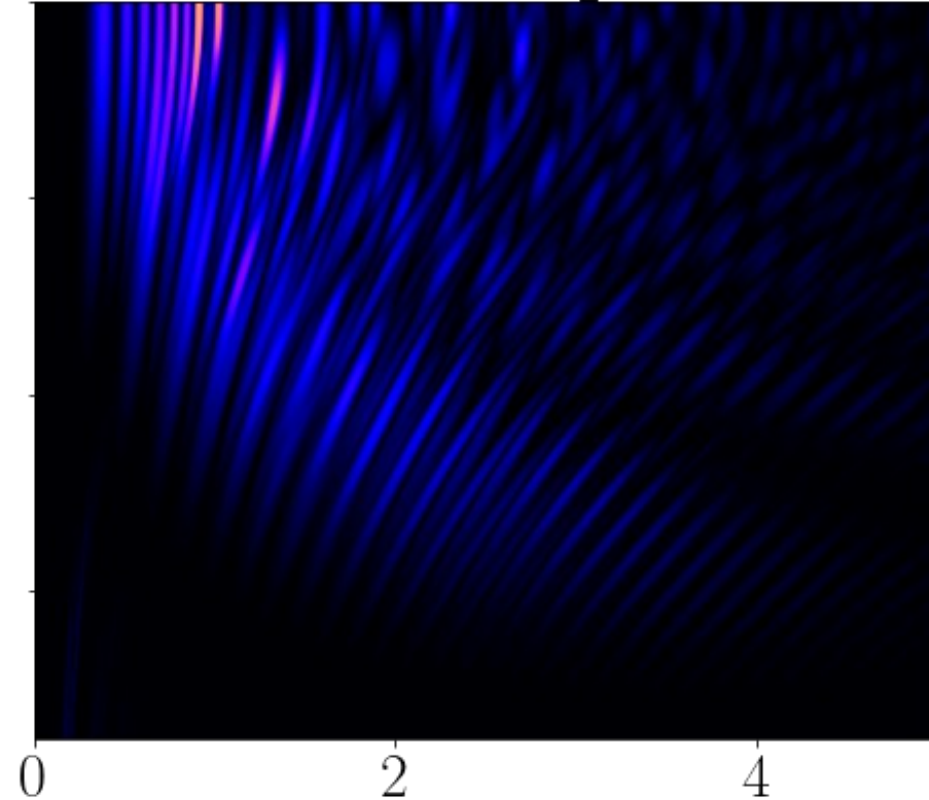
Single electron – off axis

- Linear polarization

$$\phi_{cep} = 0$$



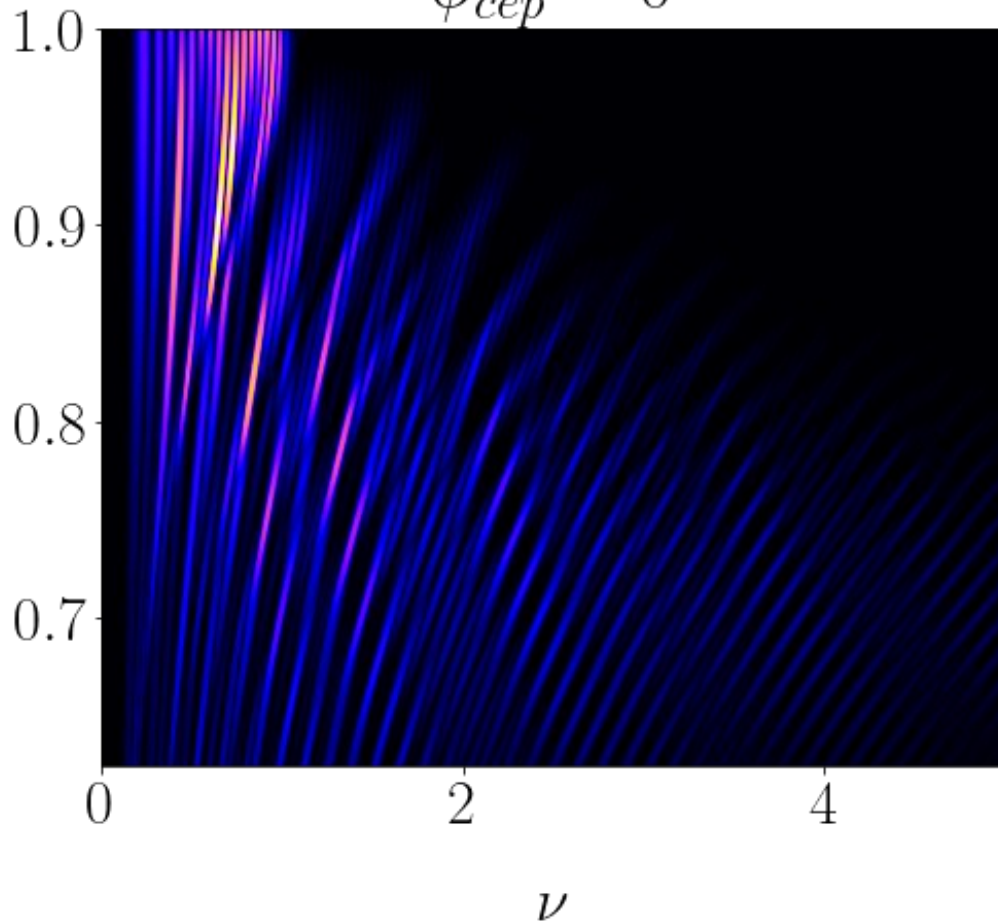
$$\phi_{cep} = \frac{\pi}{2}$$



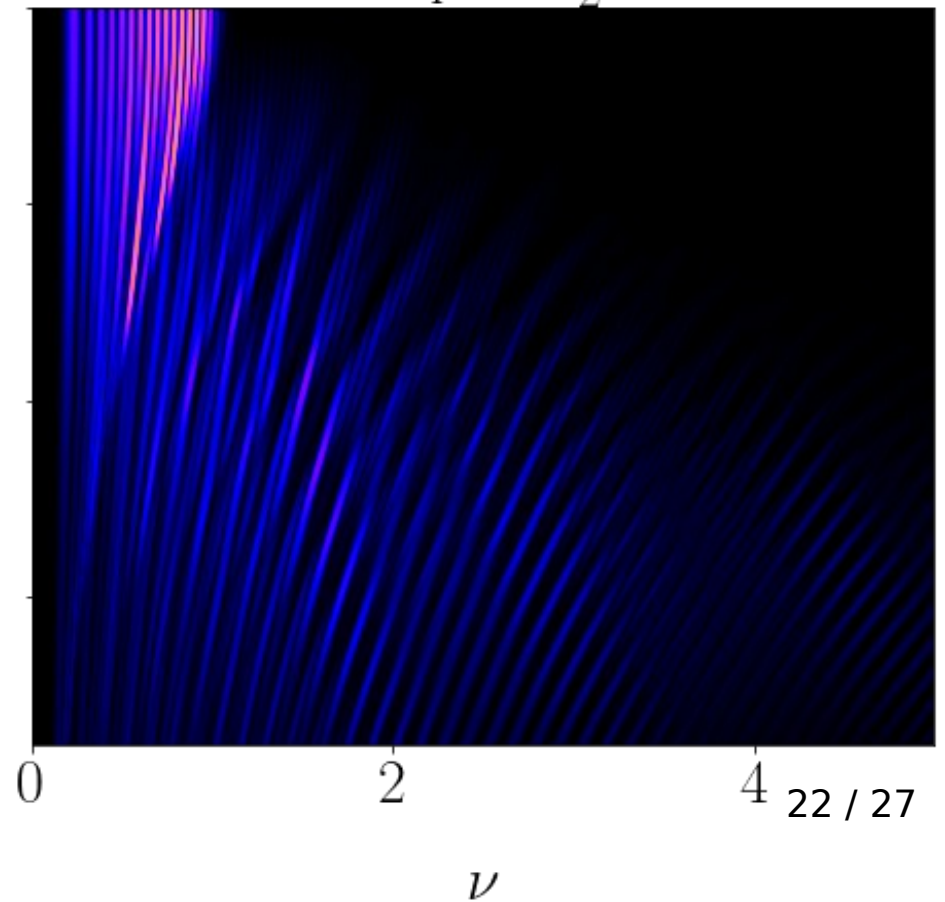
Single electron – off axis

- Also for circular polarization effect visible: harmonics overlap off axis!

$$\phi_{cep} = 0$$



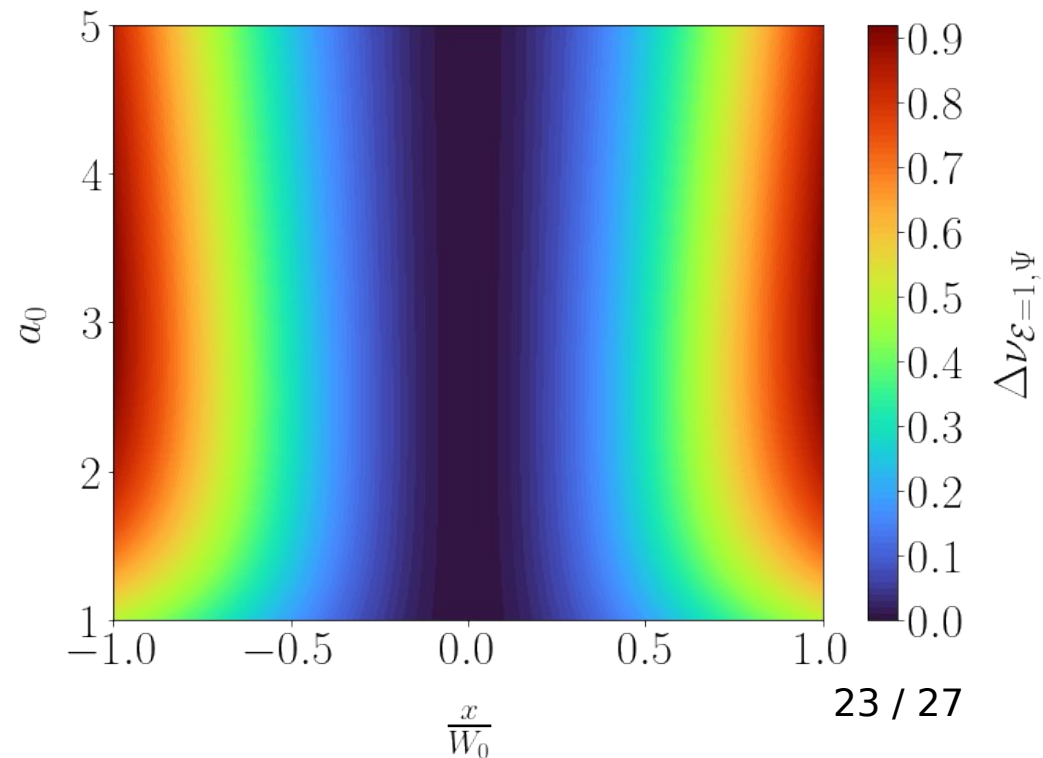
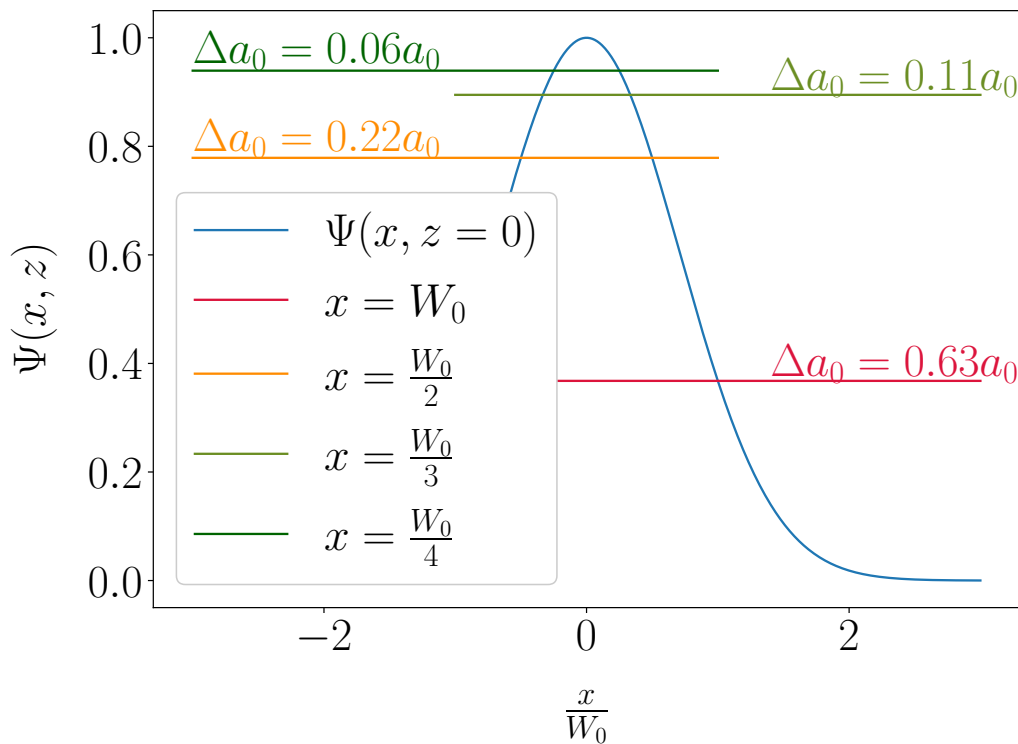
$$\phi_{cep} = \frac{\pi}{2}$$



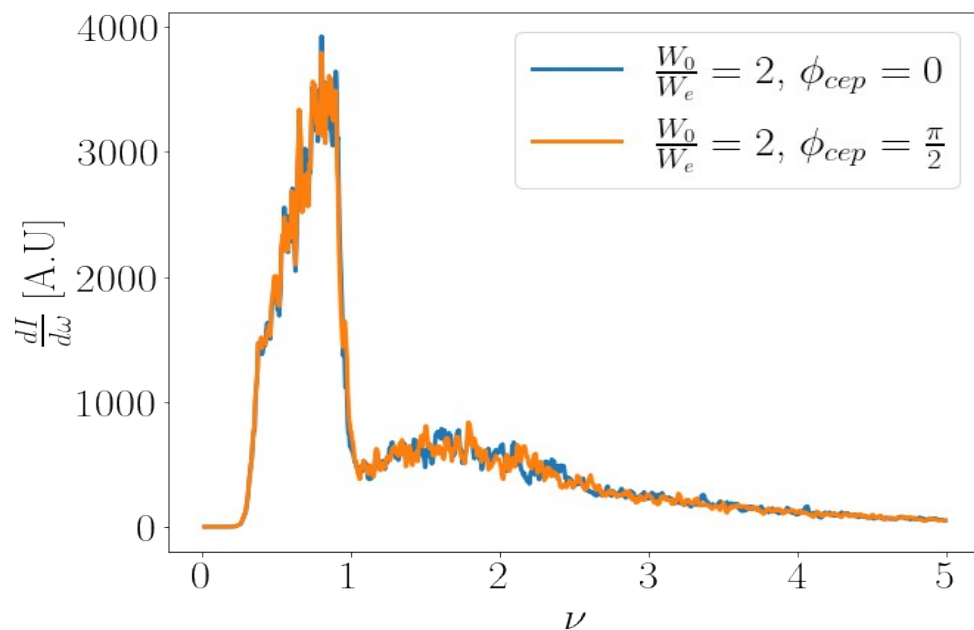
Bunch – 3D pulse

Electrons experience different intensities depending on their transverse position

$$\Delta v_{\mathcal{E}=1,\Psi} = \frac{\frac{a_0^2}{2}}{1 + \frac{a_0^2}{2}} \frac{1 - \Psi^2}{1 + \frac{a_0^2 \Psi^2}{2}}$$

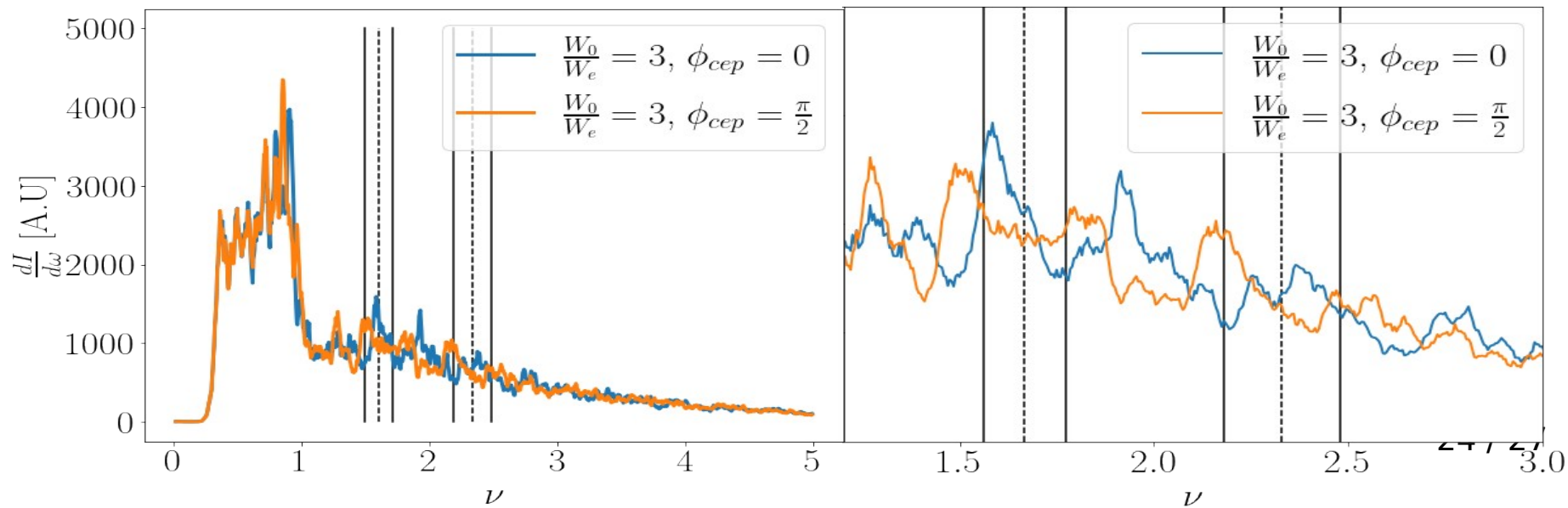


Bunch – 3D pulse



$$\nu_{\mathcal{H},cep} \propto \frac{\mathcal{H}}{1 + \frac{a_0^2}{2}} \left(1 \pm \frac{\sin^2(\phi_{cep})}{\pi N_c} \right)$$

5th and 7th harmonics



Conclusion

- Analytical description how CEP influences Thomson spectrum:

$$\text{interval for } \phi_{cep} : \left[0, \frac{\pi}{2}\right] + k\pi$$

- Single electron – Plane wave

- Shift harmonic peaks
$$v_{\mathcal{H},cep} \propto \frac{\mathcal{H}}{1 + \frac{a_0^2}{2}} \left(1 \pm \frac{\sin^2(\phi_{cep})}{\pi N_c}\right)$$

- Electron bunch – 3D pulse

- Ratio laser – electron beam waist to ommit nonlinear broadening
$$\frac{W_0}{W_e} \geq 3$$

Gaussian Laser Beams with Radial Polarization

Kirk T. McDonald

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

(March 14, 2000; updated December 2, 2012)

$$\nabla^2 \mathbf{A} = \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2}.$$

$$A_j(\mathbf{r}, t) = \psi(r_\perp, z)g(\varphi)e^{i\varphi},$$

$$\nabla^2 \psi + 2ik \frac{\partial \psi}{\partial z} \left(1 - \frac{ig'}{g}\right) = 0,$$

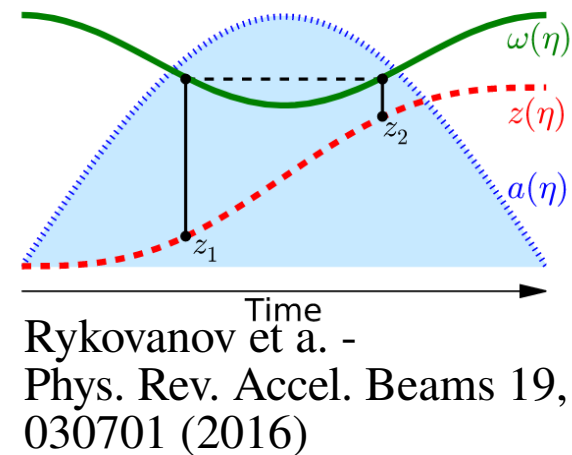
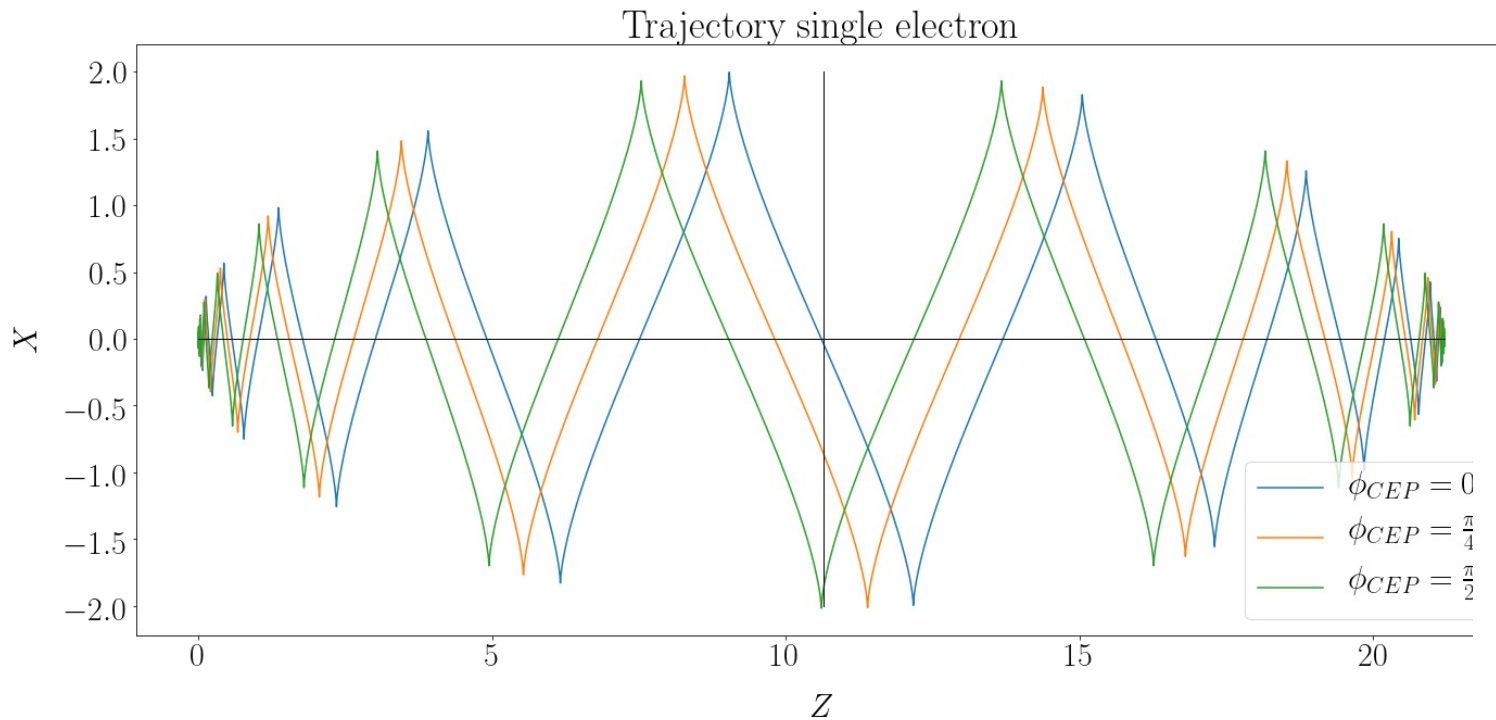
For a pulsed laser beam, g must obey

$$\left| \frac{g'}{g} \right| \ll 1$$

A more appropriate form for a pulsed beam is a hyperbolic secant soliton):

$$g(\varphi) = \operatorname{sech} \left(\frac{\varphi}{\varphi_0} \right).$$

CEP Dependence on Trajectory



- Rectangular envelope function is not affected
- Short pulses: single/few maxima of cosine near peak intensity \rightarrow large effect CEP on acceleration
- Long pulses: multiple maxima of cosine near peak intensity \rightarrow minor effect of CEP