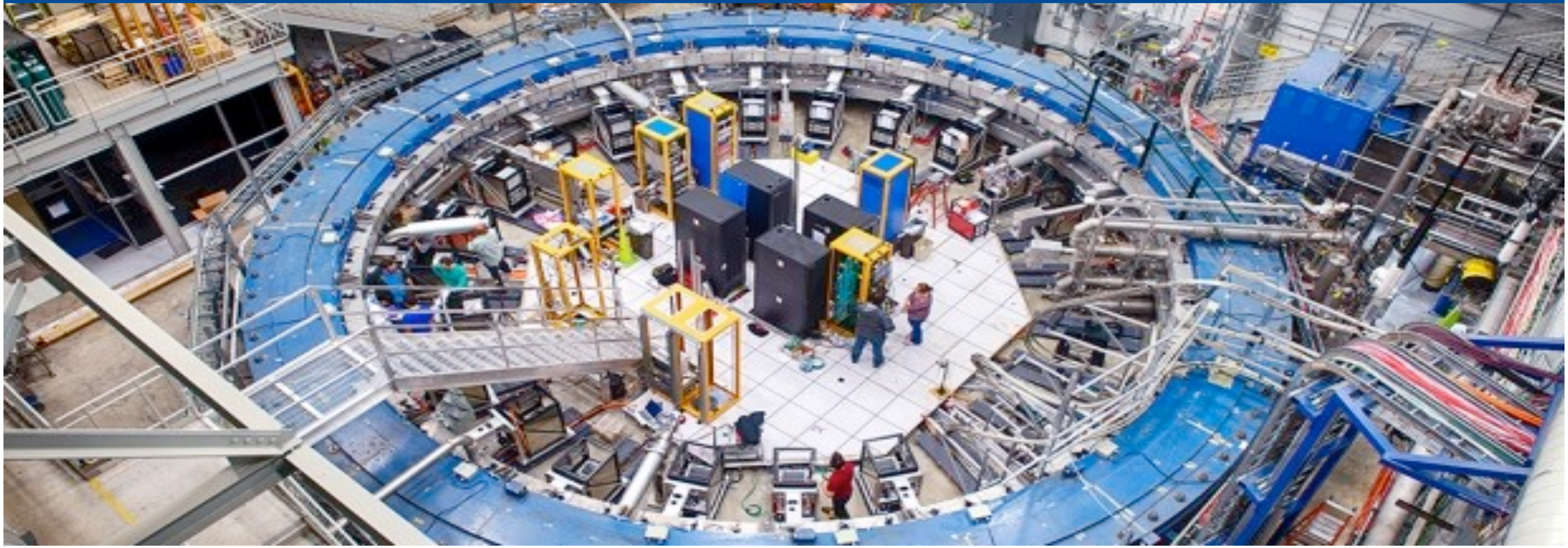


Measuring the muon magnetic anomaly a_μ with the Muon g-2 experiment at Fermilab

INFN Genova Colloquium – 27 April 2021

Marco Incagli – INFN Pisa

What is "g-2"?

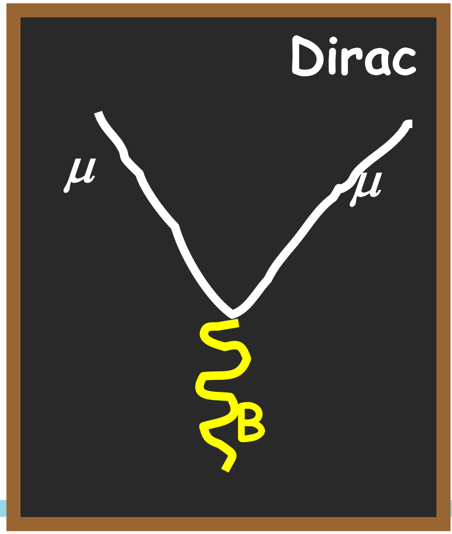
$$\vec{\mu}_p = -g_p \frac{e}{2m_p} \vec{S}$$

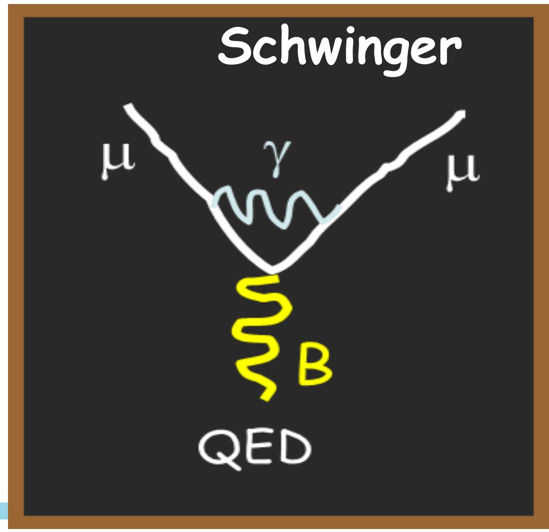
$$a_p = \frac{g_p - 2}{2}$$

- ***g*$_P$** : proportionality between spin and magnetic moment for particle P
- ***a*$_P$** : magnetic anomaly
- ***a*$_P$** = 0 at tree level (*purely Dirac particle*)

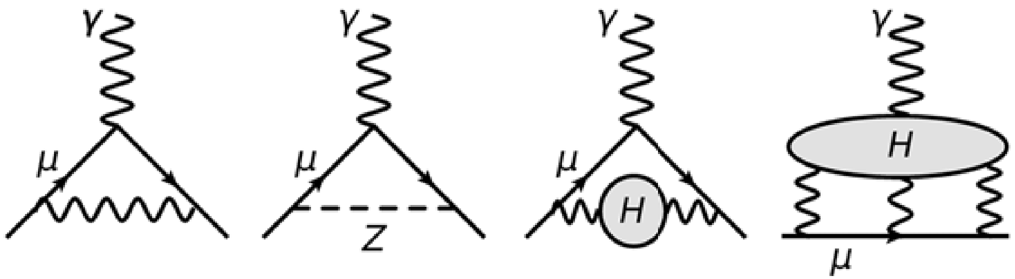
- Using modern language, the term (g-2)/2 reflects the magnitude of the Feynmann diagrams beyond leading order

$$a = 0 + \alpha/2\pi + \ldots$$





Contributions to a_μ



+ ?

QED	Weak	HLO	HLbL	New Physics.
Known	Known	Data	Models/Lattice	?

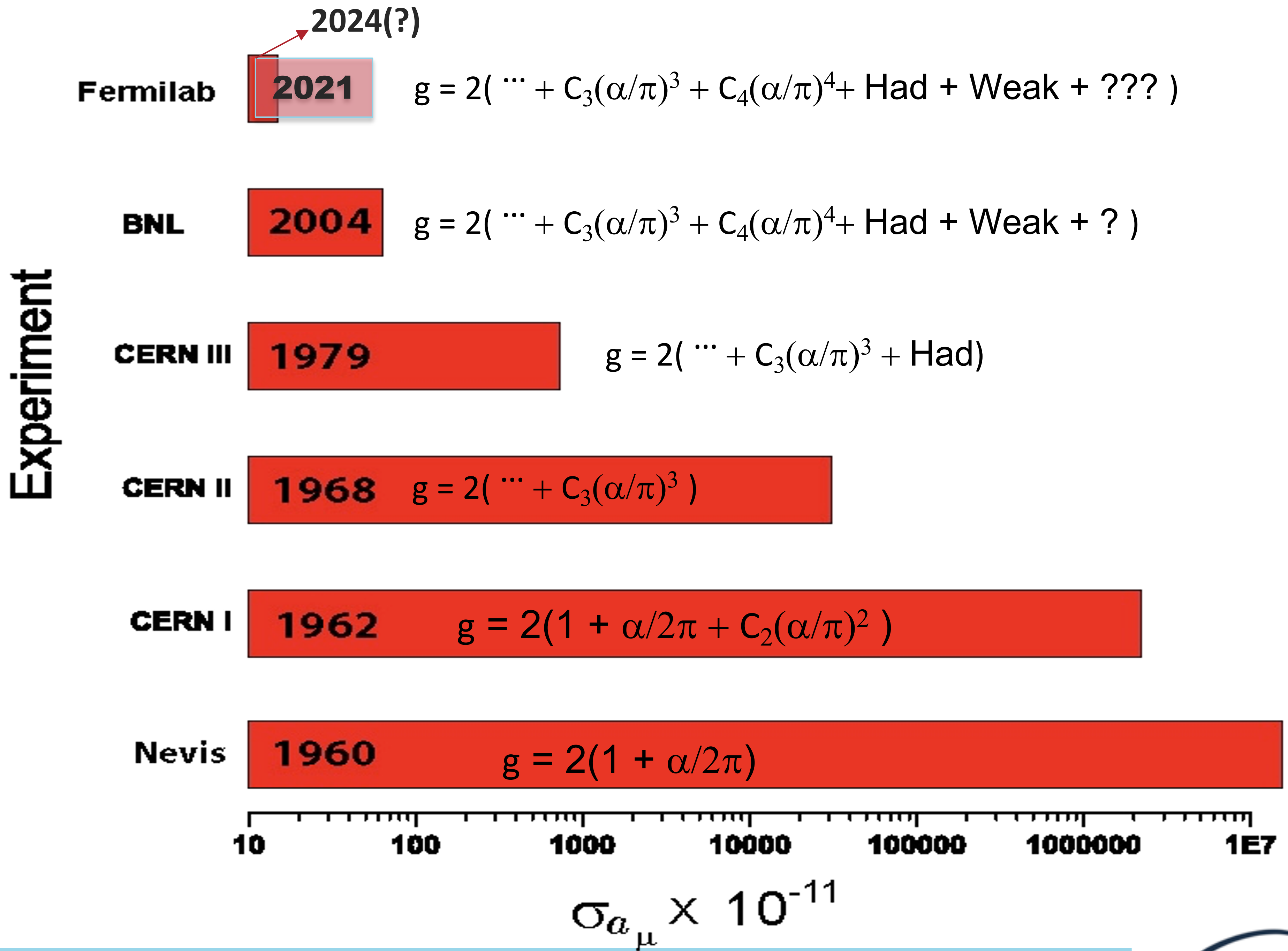
HLbL = Hadronic Light by Light = Hadronic higher order

	VALUE ($\times 10^{-10}$) UNITS
QED ($\gamma + \ell$)	$11\,658\,471.8951 \pm 0.0009 \pm 0.0019 \pm 0.0007 \pm 0.0077_\alpha$
HVP(lo) Davier17	692.6 ± 3.33
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	$11\,659\,181.7 \pm 4.2$

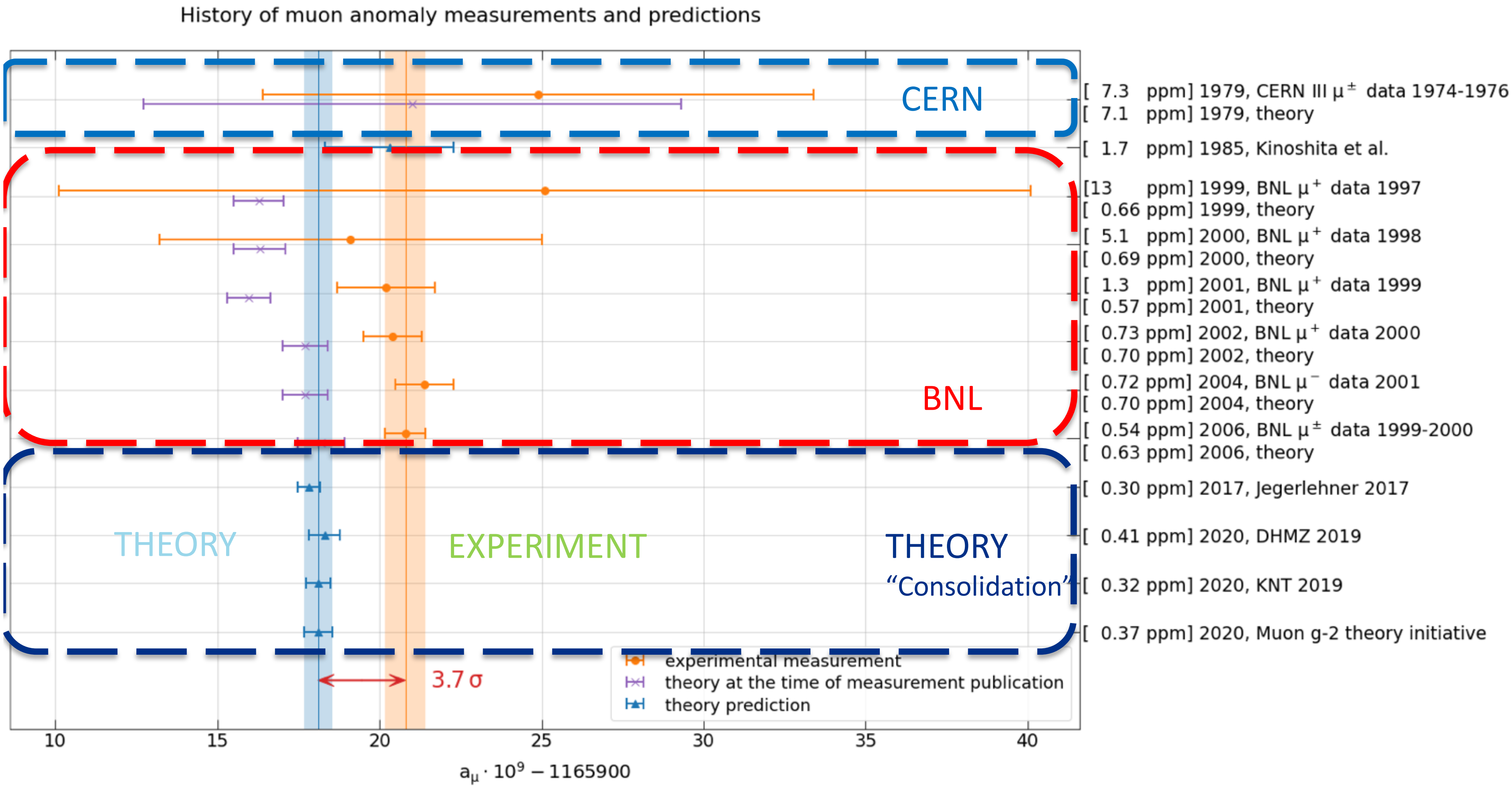
686 ppt !!!

Theory initiative White Paper (arXiv 2006:08443)

$a_\mu = (1\,116\,591\,810 \pm 43)\times 10^{-11} \rightarrow 370$ ppb


2024(?)
Fermilab
2021
g = 2(⋯ + C3(α/π)3 + C4(α/π)4+ Had + Weak + ???)
BNL
2004
g = 2(⋯ + C3(α/π)3 + C4(α/π)4+ Had + Weak + ?)
Experiment
CERN III
1979
g = 2(⋯ + C3(α/π)3 + Had)
CERN II
1968
g = 2(⋯ + C3(α/π)3)
CERN I
1962
g = 2(1 + α/2π + C2(α/π)2)
Nevis
1960
g = 2(1 + α/2π)
10
100
1000
10000
100000
1000000
1E7
σaμ X 10-11

A rich history of g-2 Theory and measurements



Tension between theory and experiment

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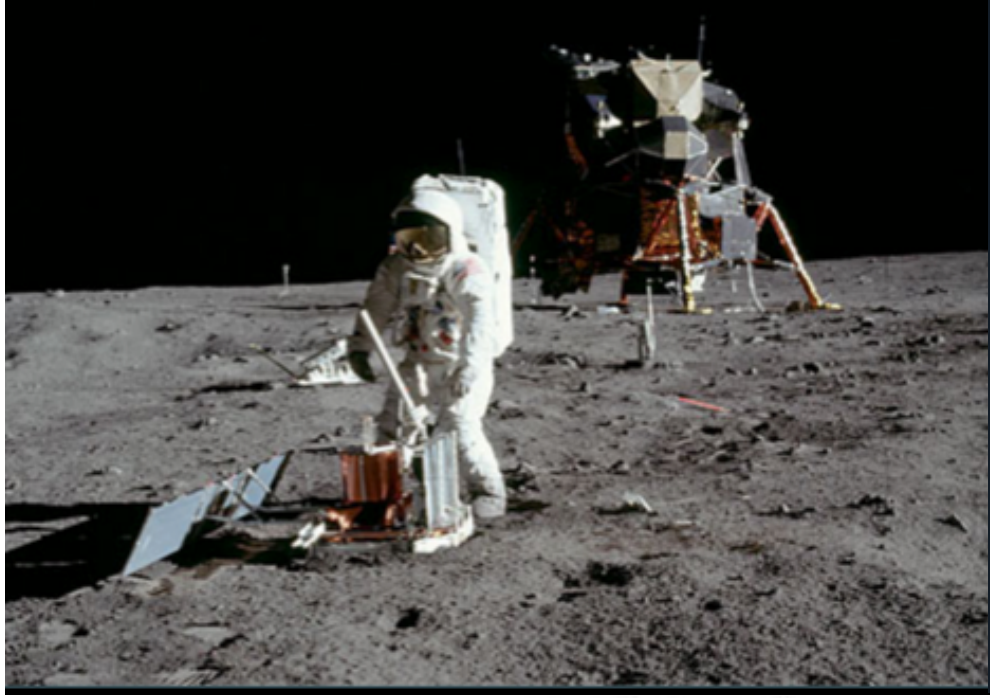
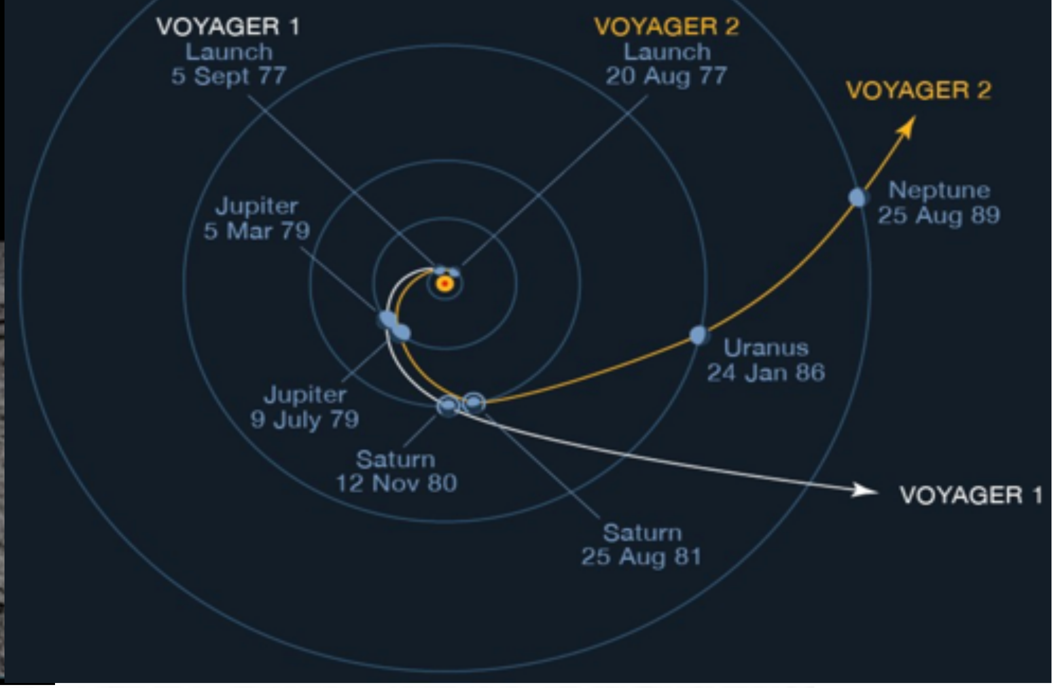
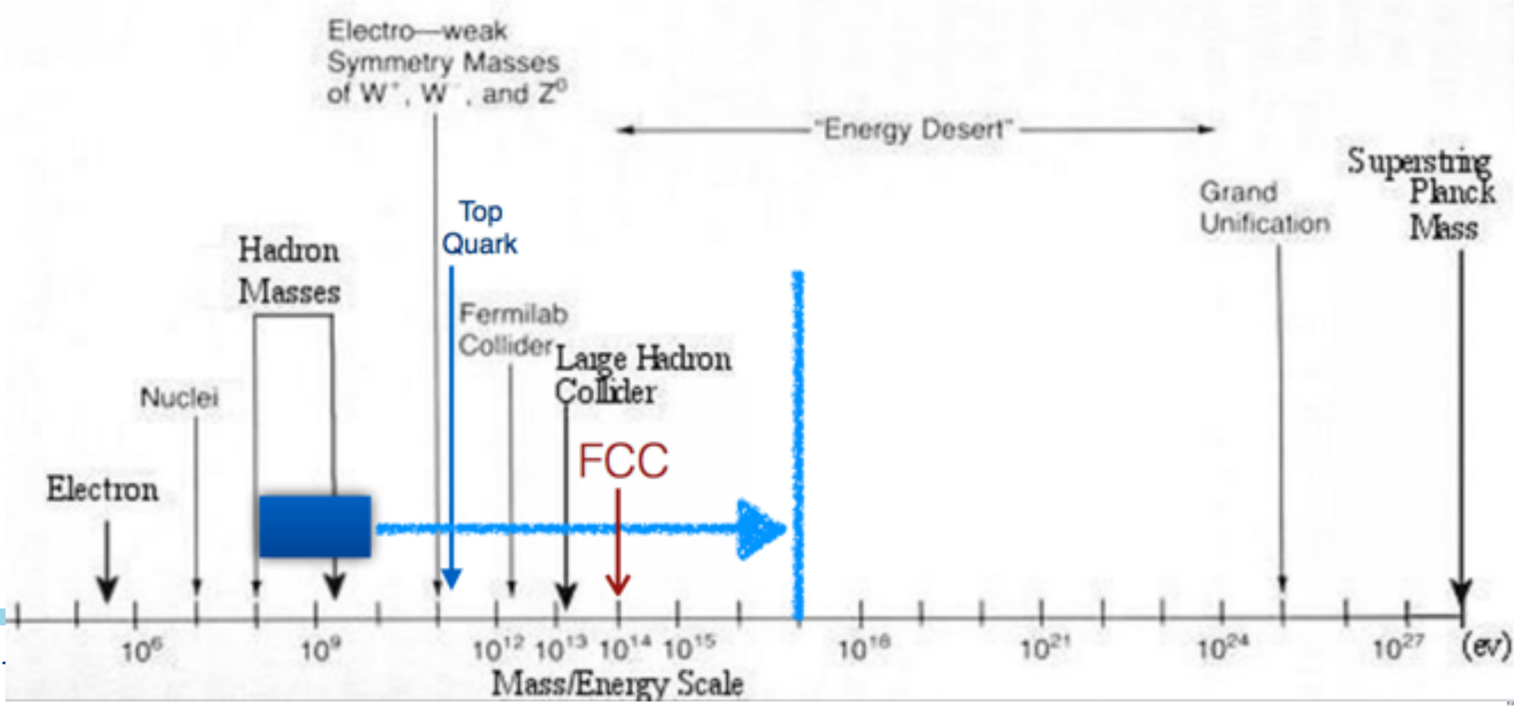
HIGH ENERGY: systematic search



HIGH PRECISION: launching probes





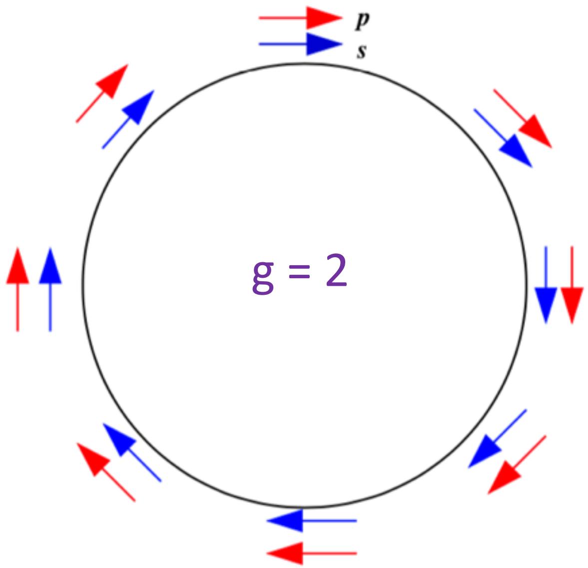
The Fundamental Experimental Principle

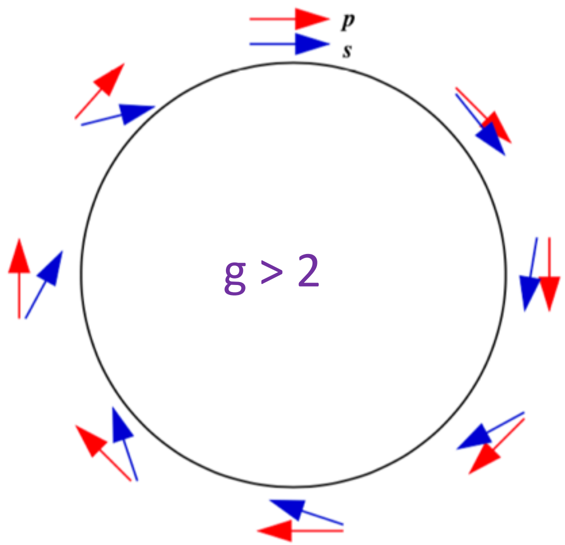
- Difference between spin precession and cyclotron revolution for a muon (charged particle with spin) in a magnetic field*:

$$\omega_a = \omega_s - \omega_c = g\frac{e}{2m}B - \frac{e}{m}B = \frac{g-2}{2}\frac{e}{m}B = a_\mu \frac{e}{m}B$$

*__s__ and __p__ are assumed to be in a plane perpendicular to __B__

- simple classical calculation
- the relativistic approach provides the same result





From particle to beam

- It is not possible to follow the spin of a single muon, we can only have an *information of the spin direction when the muon is produced and when it decays* (see next slide)
- Need a **beam of muons** →
  - focusing elements: using *electrostatic quadrupoles*
  - betatron oscillations around ideal trajectory
- Additional terms in the muon precession complex

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

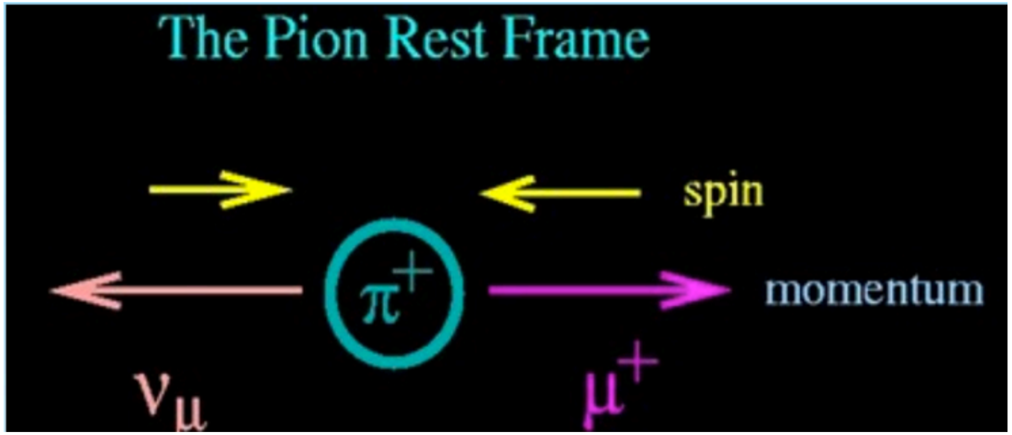
Term due to ElectroStatic Quadrupoles (ESQ)

Term due to beam vertical oscillations: pitch correction

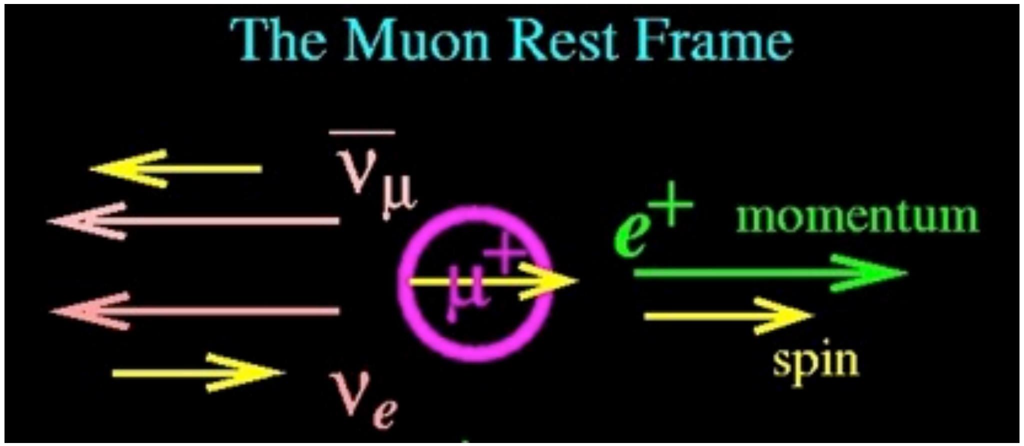
Becomes ~ 0 at magic $\gamma \sim 29.3$ or $p \sim 3.1$ GeV/c

How do we measure the spin direction?

- Use V-A structure of weak decays to build a polarized beam...



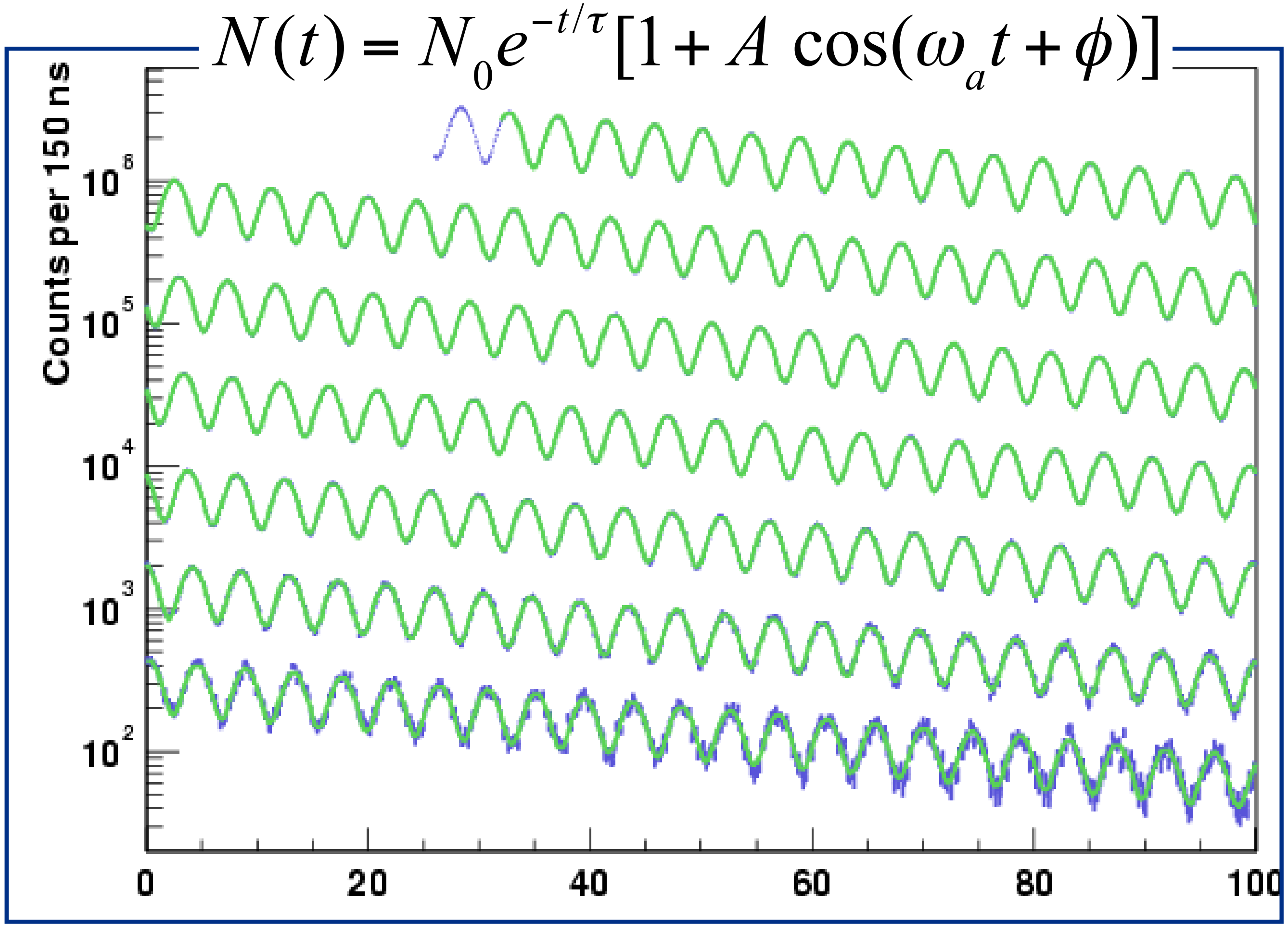
- ... and to measure the muon polarization looking for energetic positrons



Measuring the spin precession

- The number of observed positrons above a threshold energy oscillates with the $\omega_a/2\pi$ frequency due to spin precession

$$N(t) = N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)]$$



time (μsec)

- exponential decay modulated by spin precession
- *note that the x-axis "wraps up" every 100 μsec for a total of ~700 μs → ~11 muon lifetimes*

Extracting a_μ(simplified)

$$\boldsymbol{\omega_a = a_\mu (e/m)B \rightarrow a_\mu = \omega_a (m/eB)}$$

by expressing B in terms of the (shielded) proton precession frequency ($B = \hbar\omega'_p/2\mu'_p$):

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \cdot \frac{\mu'_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2} = R'_\mu \cdot \frac{\mu'_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

What we measure

External (precise) data

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

ratio of muon to proton precession in the same magnetic dipole field

$\tilde{\omega}'_p$ = (shielded) Proton Larmor angular velocity **weighted for the muon distribution**

Extracting a_μ(more precise) left as reference

$$\frac{\mu_e(H)}{\mu'_p(T)}$$ Measured to 10.5 ppb accuracy at *reference temp.* $T_r = 34.7^oC$
Metrologia **13**, 179 (1977)

$$\frac{m_\mu}{m_e}$$ Known to 22 ppb from muonium hyperfine splitting
Phys. Rev. Lett. **82**, 711 (1999)

$$\frac{\mu_e}{\mu_e(H)}$$ Bound-state QED (exact)
Rev. Mod. Phys. **88** 035009 (2016)

$$\frac{g_e}{2}$$ Measured to 0.28 ppt
Phys. Rev. A **83**, 052122 (2011)

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

Total uncertainty of 25 ppb

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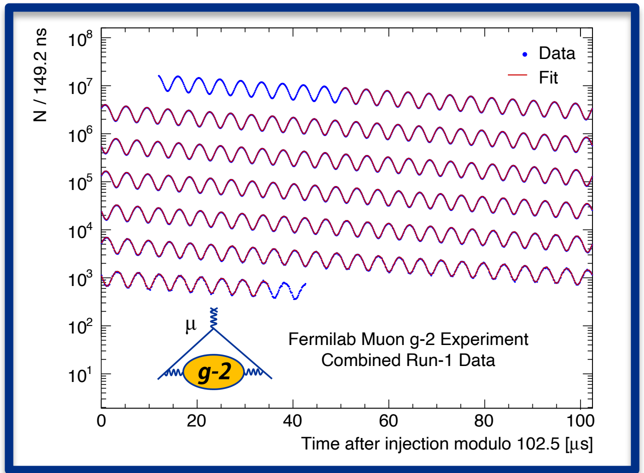
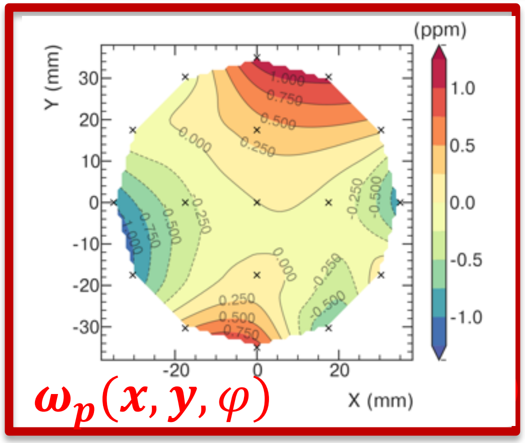
The key ingredients

ω_a=muon spin precession respect to momentum (in B field)

ω_a



$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \sim$$



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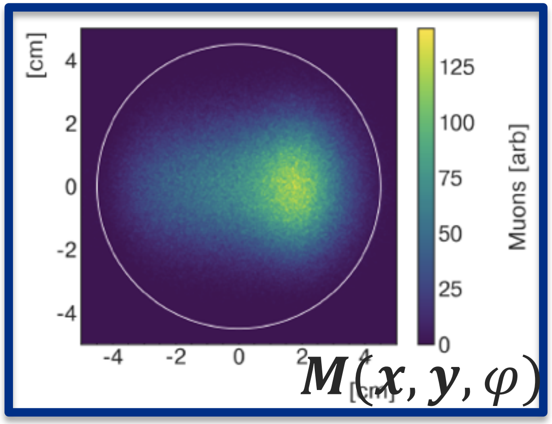
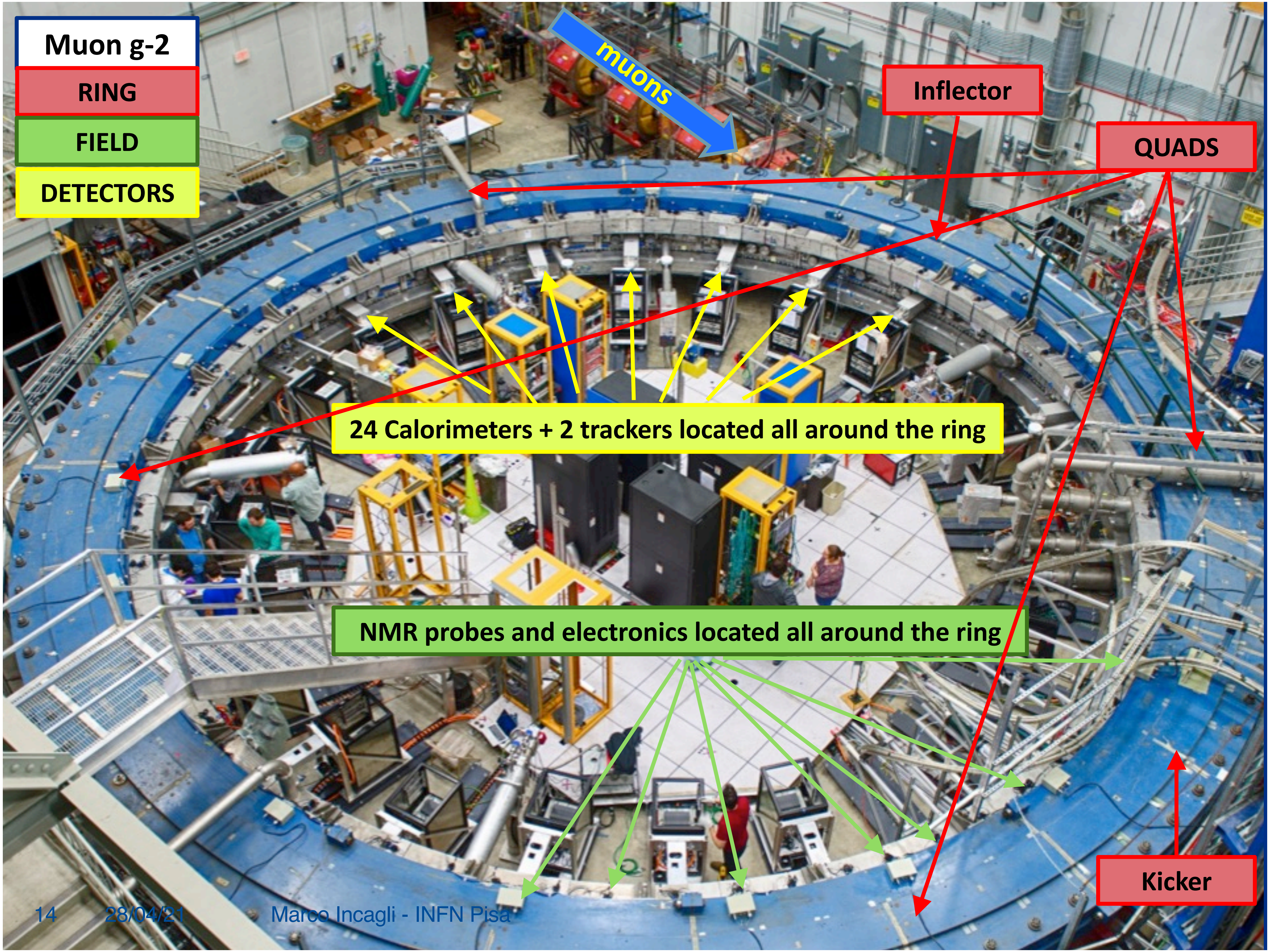


$$\tilde{\omega}_p' = \omega_p'(x, y, \varphi) \cdot M(x, y, \varphi)$$

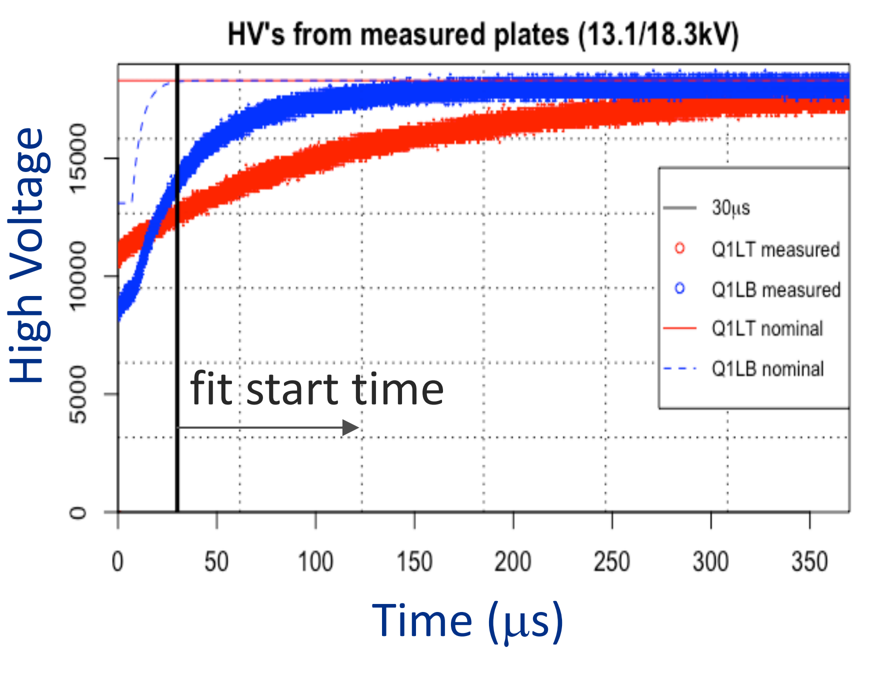
ω_p=proton precession frequency

M=muon spatial distribution

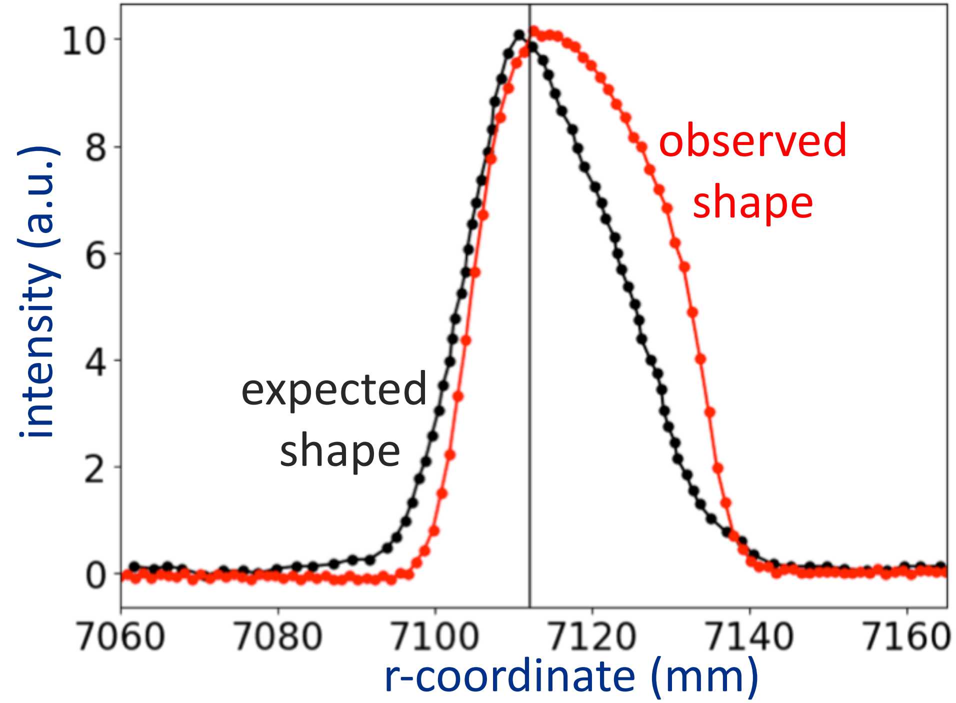

Muon g-2
RING
FIELD
DETECTORS
muons
Inflector
QUADS
24 Calorimeters + 2 trackers located all around the ring
NMR probes and electronics located all around the ring
Kicker

Problems related to Storage Ring in Run1

- Two main problems observed in Run-1, fixed in Run-2:
  - bad resistors (QUADRUPOLES)
  - low current (KICKER)



Two resistors providing HV to the vertical plates of a QUADRUPOLE had a slow recovery time
The beam slowly moved vertically during fill.



Beam radial profile: ring acceptance $\Delta p = \pm 0.5\%$
Asymmetry due to not perfect KICK

Additional (important) corrections

- On top of the *key ingredients* additional corrections due to beam dynamics and transient fields have to be included

Corrections due to beam dynamics

$$R'_\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \cdot \omega'_p(x,y,\varphi) \cdot M(x,y,\varphi) \cdot (1 + B_k + B_q)} \right)$$

Corrections due to transient magnetic fields

Four articles on arXiv for details

PRAB

Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab

T. Albahri,[30] A. Anastasi°,[10] K. Badgley,[7] S. Baeßler,[36,a] I. Bailey,[17,b] V. A. Baranov,[15] E. Barlas-Yucel,[28]

PRA

Magnetic Field Measurement and Analysis for the Muon $g-2$ Experiment at Fermilab

T. Albahri,[39] A. Anastasi,[11,a] K. Badgley,[7] S. Baeßler,[47,b] I. Bailey,[19,c] V. A. Baranov,[17] E. Barlas-Yucel,[37]
T. Barrett,[6] F. Bedeschi,[11] M. Berz,[20] M. Bhattacharya,[43] H. P. Binney,[48] P. Bloom,[21] J. Bono,[7] E. Bottalico,[11,32]

PRD

Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ experiment

T. Albahri,[39] A. Anastasi,[11,a] A. Anisenkov,[4,b] K. Badgley,[7] S. Baeßler,[47,c] I. Bailey,[19,d] V. A. Baranov,[17]

PRL

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

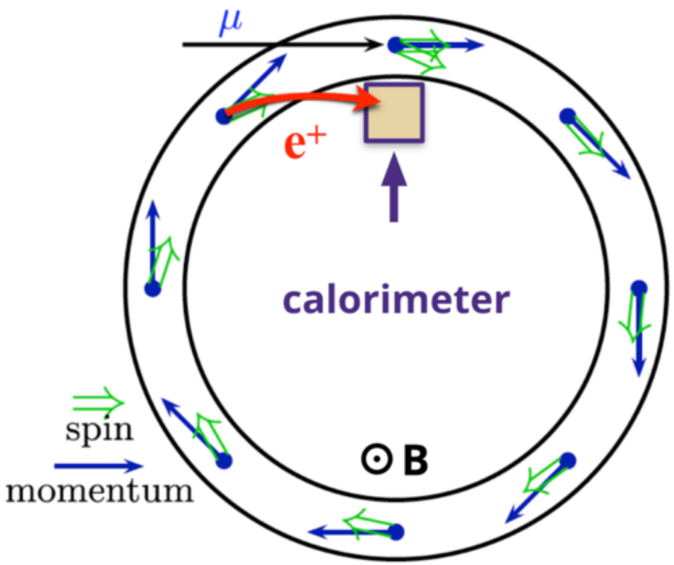
B. Abi,[44] T. Albahri,[39] S. Al-Kilani,[36] D. Allspach,[7] L. P. Alonzi,[48] A. Anastasi,[11,a] A. Anisenkov,[4,b] F. Azfar,[44] K. Badgley,[7] S. Baeßler,[47,c] I. Bailey,[19,d] V. A. Baranov,[17] E. Barlas-Yucel,[37] T. Barrett,[6] E. Barzi,[7] A. Basti,[11,32] F. Bedeschi,[11] A. Behnke,[22] M. Berz,[20] M. Bhattacharya,[43] H. P. Binney,[48] R. Bjorkquist,[6] P. Bloom,[21] J. Bono,[7] E. Bottalico,[11,32] T. Bowcock,[39] D. Boyden,[22] G. Cantatore,[13,34] R. M. Carey,[2] J. Carroll,[39] B. C. K. Casey,[7] D. Cauz,[35,8] S. Ceravolo,[9] R. Chakraborty,[38] S. P. Chang,[18,5] A. Chapelain,[6] S. Chappa,[7] S. Charity,[7] R. Chislett,[36] J. Choi,[5] Z. Chu,[26,e] T. E. Chupp,[42] M. E. Convery,[7] A. Conway,[41] G. Corradi,[9] S. Corrodi,[1] L. Cotrozzi,[11,32] J. D. Crnkovic,[3,37,43] S. Dabagov,[9,f] P. M. De Lurgio,[1] P. T. Debevec,[37] S. Di Falco,[11] P. Di Meo,[10] G. Di Sciascio,[12] R. Di Stefano,[10,30] B. Drendel,[7] A. Driutti,[35,13,38] V. N. Duginov,[17] M. Eads,[22] N. Eggert,[6] A. Epps,[22] J. Esquivel,[7] M. Farooq,[42] R. Fatemi,[38] C. Ferrari,[11,14] M. Fertl,[48,16] A. Fiedler,[22] A. T. Fienberg,[48] A. Fioretti,[11,14] D. Flay,[41] S. B. Foster,[2] H. Friedsam,[7] E. Frlež,[47] N. S. Froemming,[48,22] J. Fry,[47] C. Fu,[26,e] C. Gabbanini,[11,14] M. D. Galati,[11,32] S. Ganguly,[37,7] A. Garcia,[48] D. E. Gastler,[2] J. George,[41] L. K. Gibbons,[6] A. Gioiosa,[29,11] K. L. Giovanetti,[15] P. Girotti,[11,32] W. Gohn,[38] T. Gorringe,[38] J. Grange,[1,42] S. Grant,[36] F. Gray,[24] S. Haciomeroglu,[5] D. Hahn,[7] T. Halewood-Leagas,[39] D. Hampai,[9] F. Han,[38] E. Hazen,[2] J. Hempstead,[48] S. Henry,[44] A. T. Herrod,[39,d] D. W. Hertzog,[48] G. Hesketh,[36] A. Hibbert,[39] Z. Hodge,[48] J. L. Holzbauer,[43] K. W. Hong,[47] R. Hong,[1,38] M. Iacovacci,[10,31] M. Incagli,[11] C. Johnstone,[7] J. A. Johnstone,[7] P. Kammel,[48] M. Kargiantoulakis,[7] M. Karuza,[13,45] J. Kaspar,[48] D. Kawall,[41] L. Kelton,[38] A. Keshavarzi,[40] D. Kessler,[41] K. S. Khaw,[27,26,48,e] Z. Khechadoorian,[6] N. V. Khomutov,[17] B. Kiburg,[7] M. Kiburg,[7,21] O. Kim,[18,5] S. C. Kim,[6] Y. I. Kim,[5] B. King,[39,a] N. Kinnaird,[2] M. Korostelev,[19,d] I. Kourbanis,[7] E. Kraegeloh,[42] V. A. Krylov,[17] A. Kuchibhotla,[37] N. A. Kuchinskiy,[17] K. R. Labe,[6] J. LaBounty,[48] M. Lancaster,[40] M. J. Lee,[5] S. Lee,[5] S. Leo,[37] B. Li,[26,1,e] D. Li,[26,g] L. Li,[26,e] I. Logashenko,[4,b] A. Lorente Campos,[38] A. Lucà,[7] G. Lukicov,[36] G. Luo,[22] A. Lusiani,[11,25] A. L. Lyon,[7] B. MacCoy,[48] R. Madrak,[7] K. Makino,[20] F. Marignetti,[10,30] S. Mastroianni,[10] S. Maxfield,[39] M. McEvoy,[22] W. Merritt,[7] A. A. Mikhailichenko,[6,a] J. P. Miller,[2] S. Miozzi,[12] J. P. Morgan,[7] W. M. Morse,[3] J. Mott,[2,7] E. Motuk,[36] A. Nath,[10,31] D. Newton,[39,h] H. Nguyen,[7] M. Oberling,[1] R. Osofsky,[48] J.-F. Ostiguy,[7] S. Park,[5] G. Pauletta,[35,8] G. M. Piacentino,[29,12] R. N. Pilato,[11,32] K. T. Pitts,[37] B. Plaster,[38] D. Počanić,[47] N. Pohlman,[22] C. C. Polly,[7] M. Popovic,[7] J. Price,[39] B. Quinn,[43] N. Raha,[11] S. Ramachandran,[1] E. Ramberg,[7] N. T. Rider,[6] J. L. Ritchie,[46] B. L. Roberts,[2] D. L. Rubin,[6] L. Santi,[35,8] D. Sathyan,[2] H. Schellman,[23,i] C. Schlesier,[37] A. Schreckenberger,[46,2,37] Y. K. Semertzidis,[5,18] Y. M. Shatunov,[4] D. Shemyakin,[4,b] M. Shenk,[22] D. Sim,[39] M. W. Smith,[48,11] A. Smith,[39] A. K. Soha,[7] M. Sorbara,[12,33] D. Stöckinger,[28] J. Stapleton,[7] D. Still,[7] C. Stoughton,[7] D. Stratakis,[7] C. Strohman,[6] T. Stuttard,[36] H. E. Swanson,[48] G. Sweetmore,[40] D. A. Sweigart,[6] M. J. Syphers,[22,7] D. A. Tarazona,[20] T. Teubner,[39] A. E. Tewsley-Booth,[42] K. Thomson,[39] V. Tishchenko,[3] N. H. Tran,[2] W. Turner,[39] E. Valetov,[20,19,27,d] D. Vasilkova,[36] G. Venanzoni,[11] V. P. Volnykh,[17] T. Walton,[7] M. Warren,[36] A. Weisskopf,[20] L. Welty-Rieger,[7] M. Whitley,[39] P. Winter,[1] A. Wolski,[39,d] M. Wormald,[39] W. Wu,[43] and C. Yoshikawa[7]
(The Muon $g-2$ Collaboration)

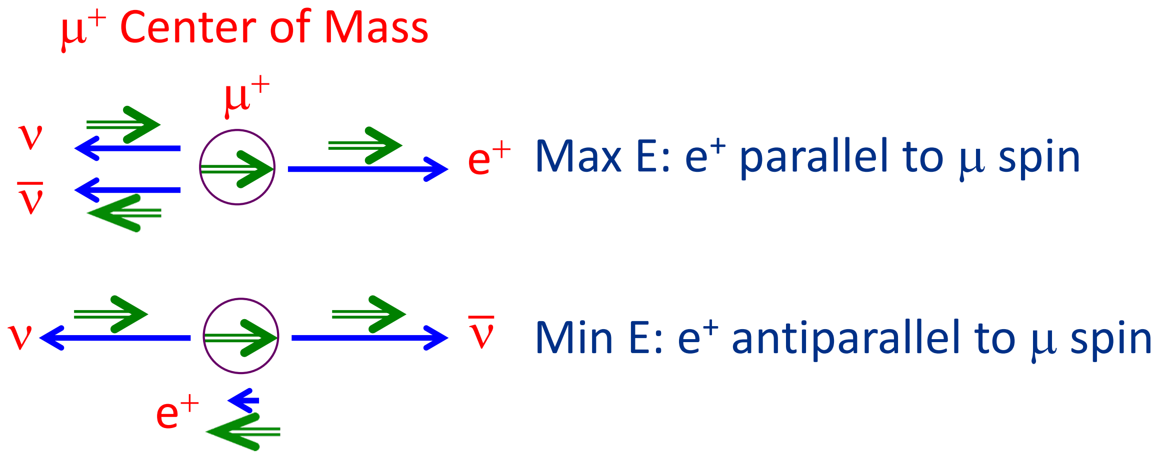
Beam Dynamics

Proton Precession

Muon Precession

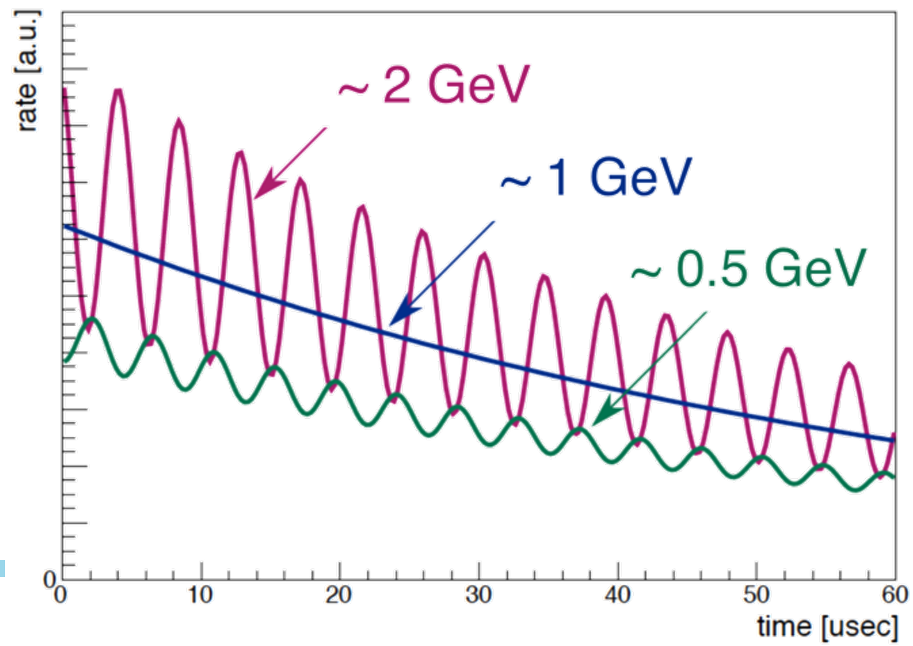
ω_a principle of measurement





$$N(t) = N_0 e^{-t/\tau}\left(1 + A\cos(\omega_a t + \varphi)\right)$$

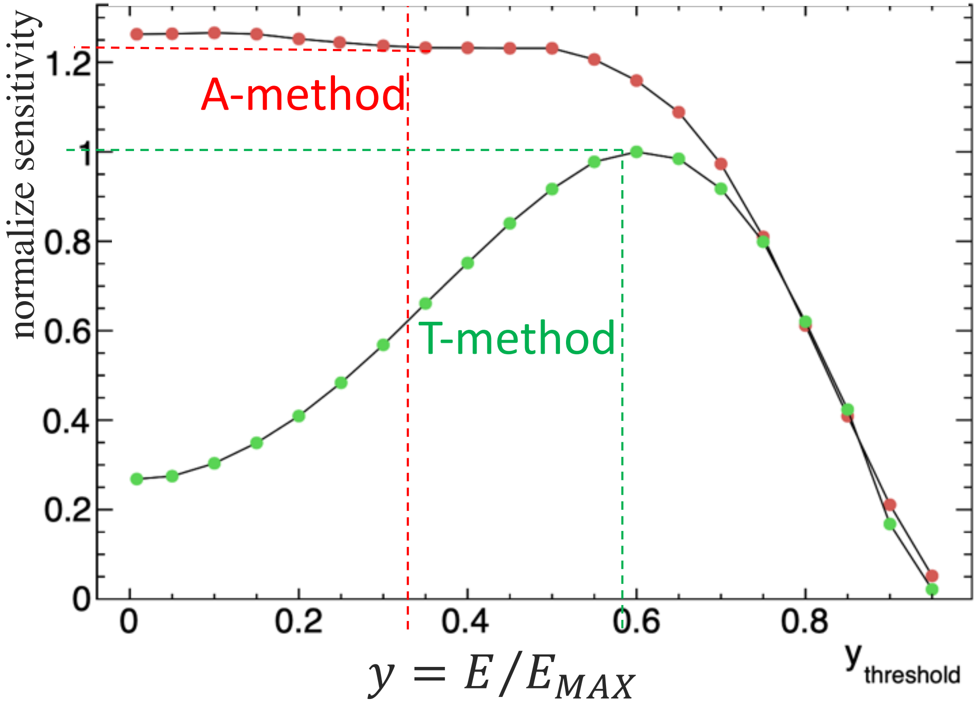
- positron emission correlated with muon spin direction
- correlation depends on *positron energy* : <u>the Asymmetry A(E) can be positive, null or negative</u>



Optimizing the statistical sensitivity

$$N(t) = N_0 e^{-t/\tau}\left(1 + A\cos(\omega_a t + \varphi)\right)$$

- **T-method** = counting positrons above E_{thr} vs time
  - by decreasing the threshold the asymmetry decreases but the number of events increases → *max sensitivity for E_{thr}~1.7 GeV*
- **A-method** = each positron is *weighted by the value of its asymmetry A(E)*
  - optimize statistical sensitivity



$y = E/E_{MAX}$

$(E_{MAX} = 3.1GeV)$

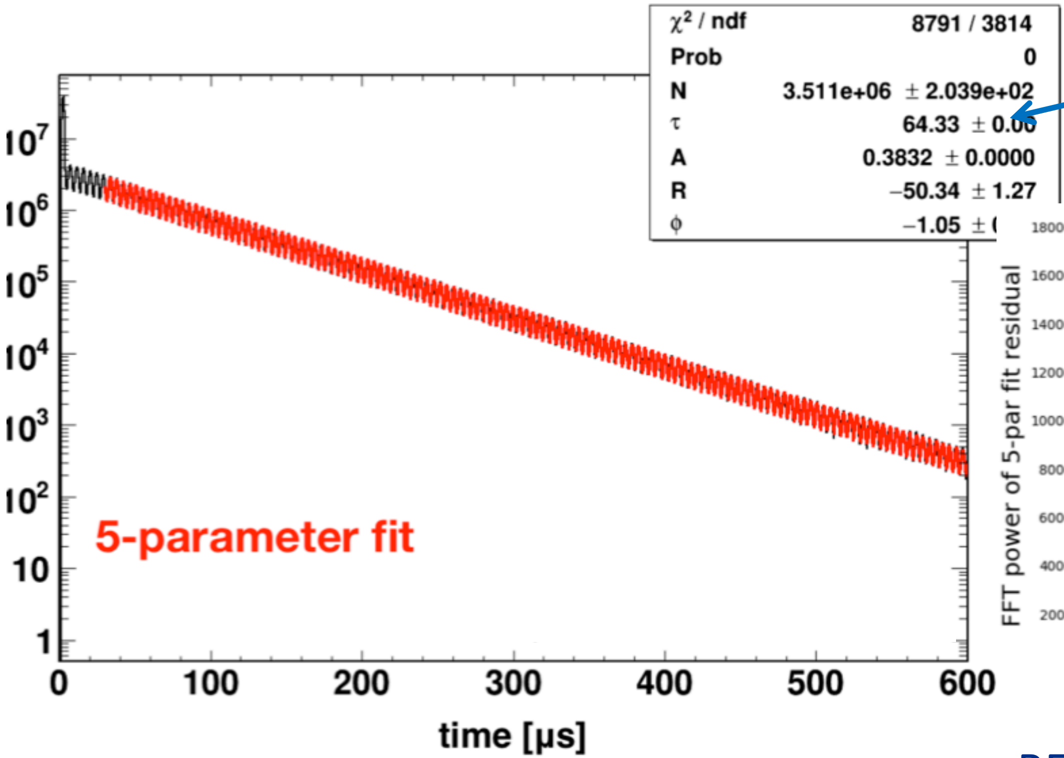
- In theory the A-method can use *all decay positrons*, in practice, due to calorimeter acceptance and to low A(E) value at low energies, only positrons with *E_{thr}>1.1 GeV (y>0.3) are used*

Measuring ω_a : 5 parameters fit function

- Fit with simple positron oscillation:

$$N_{ideal}(t) = N_0 \exp(-t/\tau_{\mu}) \left[1 + A\cos(\omega_a t + \varphi)\right]$$

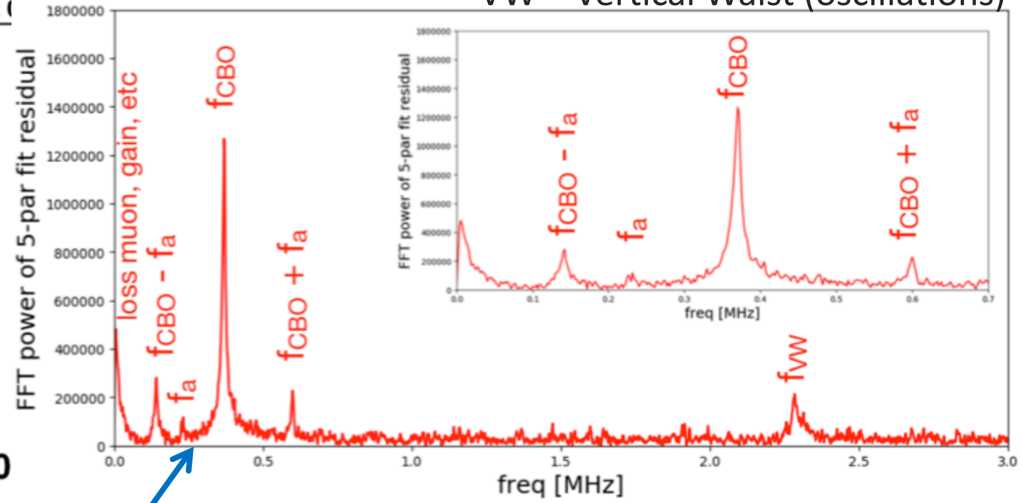
- This simple fit is clearly not sufficient and well defined resonances are observed in the residuals



muon lifetime: $\gamma\tau_\mu$ = 64.35 μsec

CBO = Coherent Betatron Ocillations

VW = Vertical Waist (oscillations)



RESIDUALS (in frequency space)

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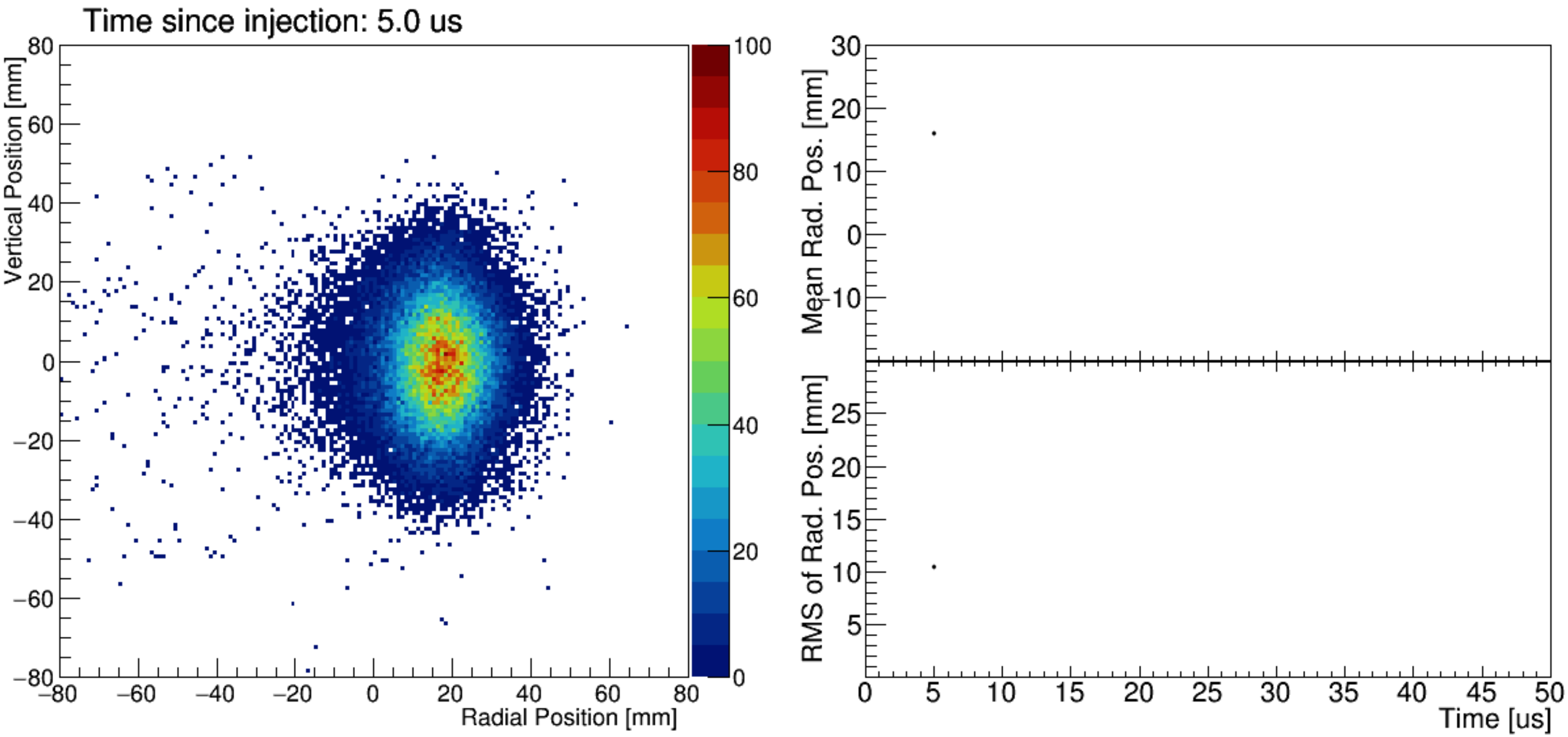
Beam oscillations

- Beam oscillations are accurately measured by the tracker and folded into the fit function



The complete 22 parameters fit function

ω_y, ω_{VW} vertical oscillations

ω_{CBO}, ω_{2CBO} radial oscillations

$$N_0\, e^{-\frac{t}{\gamma\tau}}\left(1 + A \cdot A_{BO}(t)\cos(\omega_a t + \phi \cdot \phi_{BO}(t))\right) \cdot N_{\rm CBO}(t) \cdot N_{\rm VW}(t) \cdot N_y(t) \cdot N_{\rm 2CBO}(t) \cdot J(t)$$

$$A_{\rm BO}(t) = 1 + A_A \cos(\omega_{\rm CBO}(t) + \phi_A)e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$\phi_{\rm BO}(t) = 1 + A_\phi \cos(\omega_{\rm CBO}(t) + \phi_\phi)e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$N_{\rm CBO}(t) = 1 + A_{\rm CBO}\cos(\omega_{\rm CBO}(t) + \phi_{\rm CBO})e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$N_{\rm 2CBO}(t) = 1 + A_{\rm 2CBO}\cos(2\omega_{\rm CBO}(t) + \phi_{\rm 2CBO})e^{-\frac{t}{2\tau_{\rm CBO}}}$$

$$N_{\rm VW}(t) = 1 + A_{\rm VW}\cos(\omega_{\rm VW}(t)t + \phi_{\rm VW})e^{-\frac{t}{\tau_{\rm VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t)t + \phi_y)e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue= fixed parameters

$$J(t) = 1 - k_{LM}\int_{t_0}^{t}\Lambda(t)dt$$

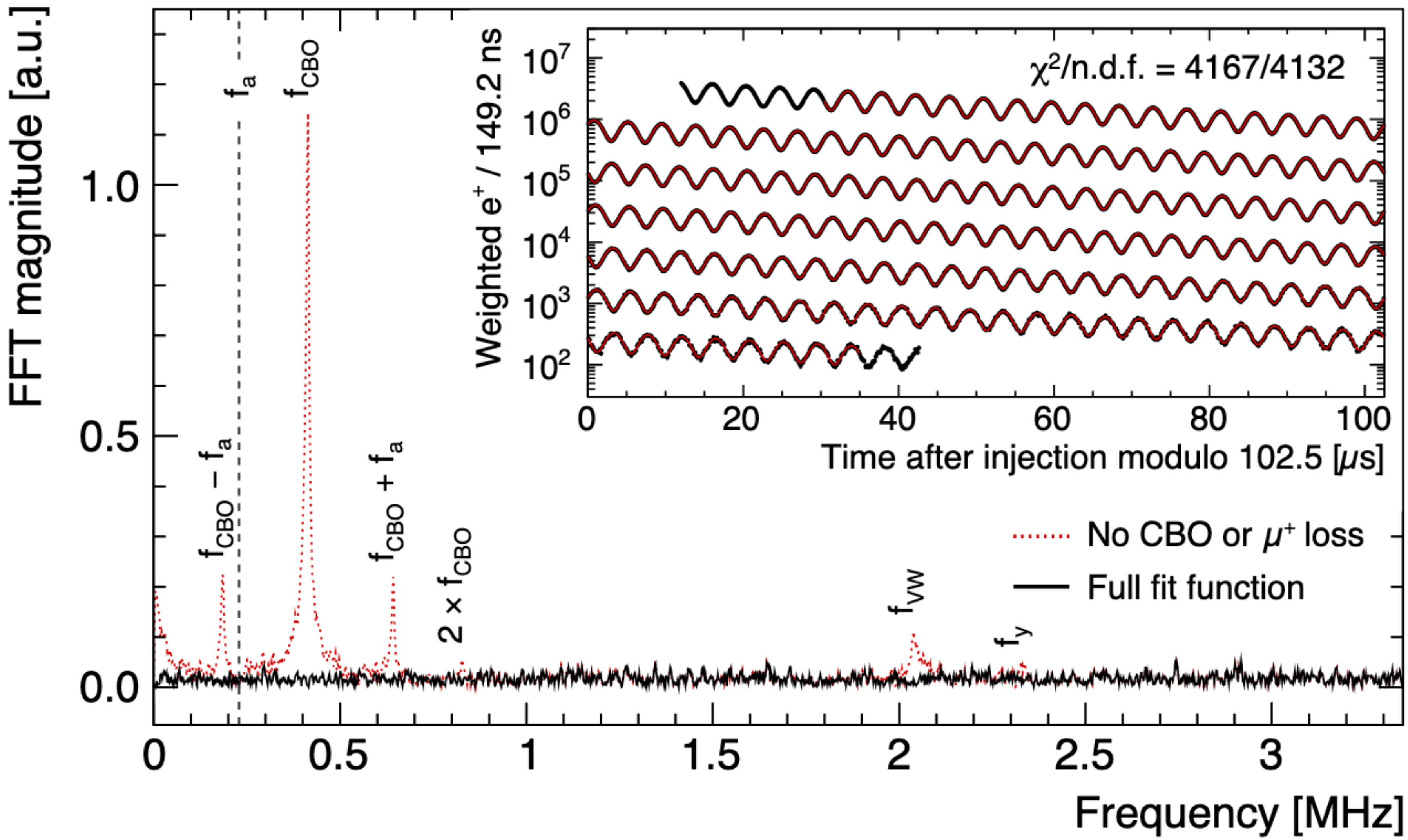
Lost muons (μ hitting collimators)

$$\omega_{\rm CBO}(t) = \omega_0 t + Ae^{-\frac{t}{\tau_A}} + Be^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F\omega_{\rm CBO(t)}\sqrt{2\omega_c/F\omega_{\rm CBO}(t) - 1}$$

$$\omega_{\rm VW}(t) = \omega_c - 2\omega_y(t)$$

Final fit



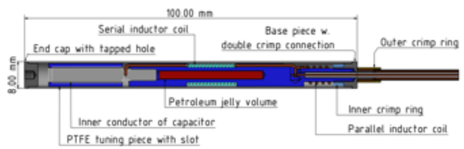
FFT magnitude [a.u.]
1.0
0.5
0.0
f_a
f_{CBO}
$f_{CBO} - f_a$
$f_{CBO} + f_a$
$2 \times f_{CBO}$
f_{VW}
f_y
0
0.5
1
1.5
2
2.5
3
Frequency [MHz]
No CBO or μ^+ loss
Full fit function
Weighted e^+ / 149.2 ns
10^7
10^6
10^5
10^4
10^3
10^2
χ^2/n.d.f. = 4167/4132
0
20
40
60
80
100
Time after injection modulo 102.5 [μs]

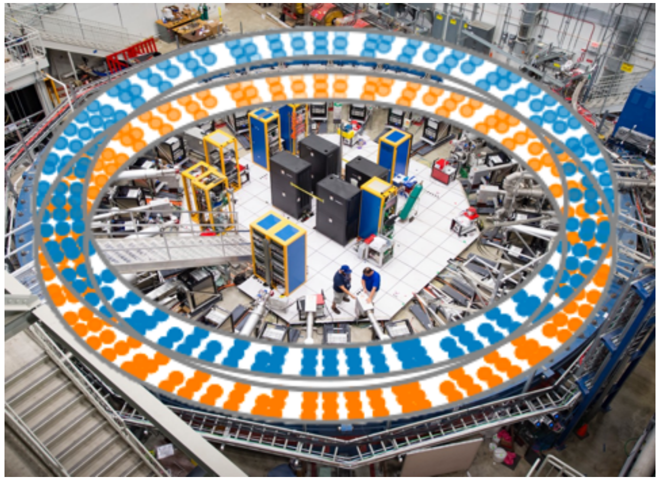
We also need B to determine a_μ

$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$

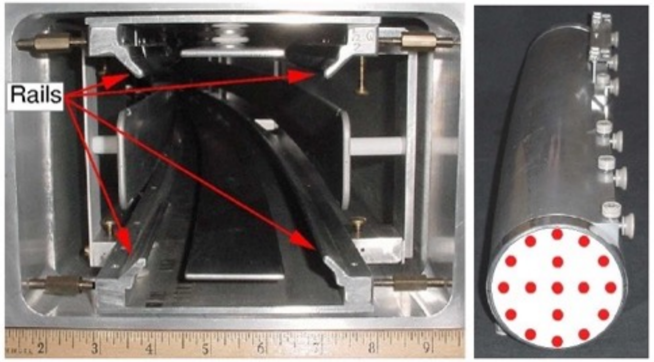
- Use NMR to find B-field in terms of proton precession frequency ω_p (comagnetometer)

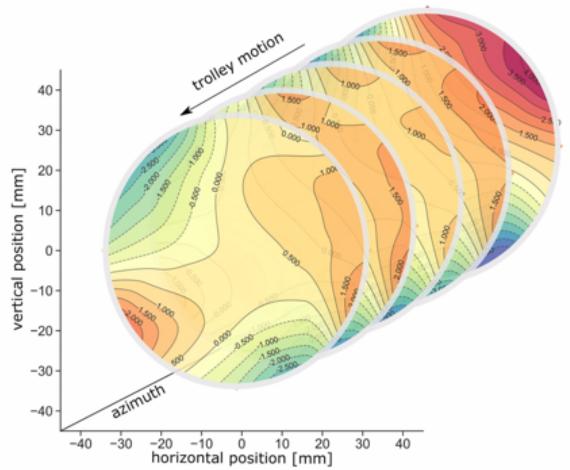
378 fixed probes monitor 24/7



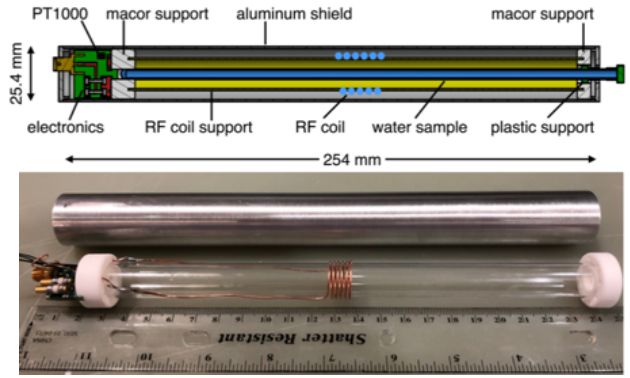


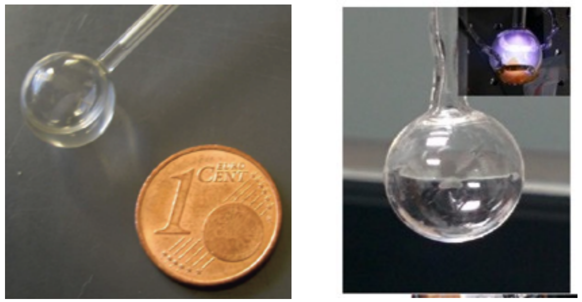
NMR trolley maps field every 3 days





Trolley cross-calibrated to absolute probes





Absolute probes all cross-calibrated at ANL test magnet

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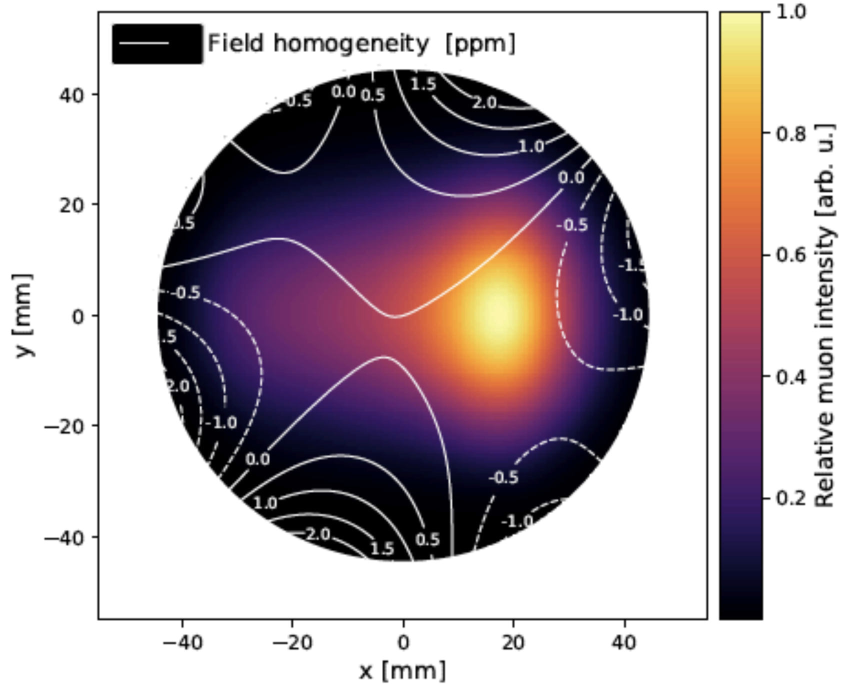
$\omega'_p \rightarrow \tilde{\omega}'_p$: muon weighted average

- Need field experienced by muons
- Use tracker to recostruct e+ track and muon vertex in storage ring
- Use beam dynamics models, tuned to the tracker data, to extrapolate the distribution all around the ring
- Systematic uncertainties mostly due to Beam Dynamics models used for extrapolation, to field map and to tracker alignment

$$\boldsymbol{\delta_{\tilde{\omega}'_p} \sim 56\ ppb}$$

Muon’s view of a tracker



Final uncertainties from Run 1

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	–	434
ω_a^m (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p(x,y,\phi) \times M(x,y,\phi) \rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu'_p(34.7°)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

- 462 ppb overall error
  - 434 ppb statistical
  - 157 ppb systematic
    - 25 ppb external inputs
- Results for Run 1 are vastly dominated by statistical error
- At 157 ppb systematic error
  - Nearly half of BNL
  - Not quite to 100 ppb goal

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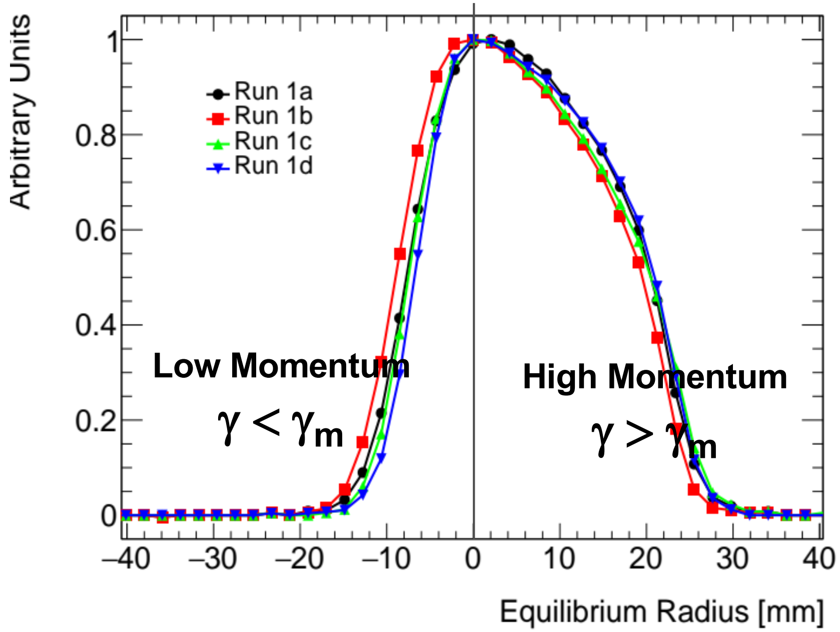
B_k	-27	37
B_q	-17	92
$\mu_p'(34.7°)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

(..) = 0 for $\gamma = \gamma_{magic}$

- Correction terms larger than total error
- Dominated by Electric Field corrections C_E
- Related to non-centered radial distribution
- Fixed in Run-2



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$g_e/2$	–	0
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Total fundamental factors	–	25
Totals	544	462

- Two largest systematics:
  - *phase acceptance*
  - *quadrupole field*

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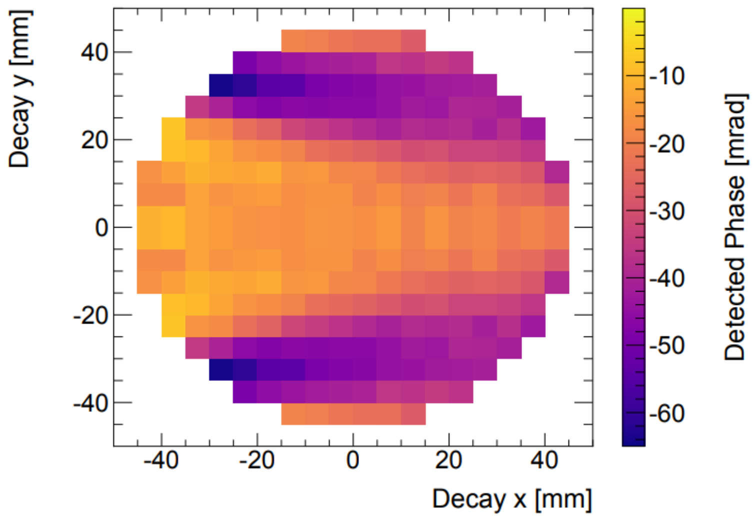
C_{PA} – Phase acceptance error

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

- But if the phase of the muon population changes in time :

$$\cos(\omega_a t + \phi(t)) \approx \cos(\omega_a t + \phi' t + \phi_0) = \cos((\omega_a + \phi')t + \phi_0)$$

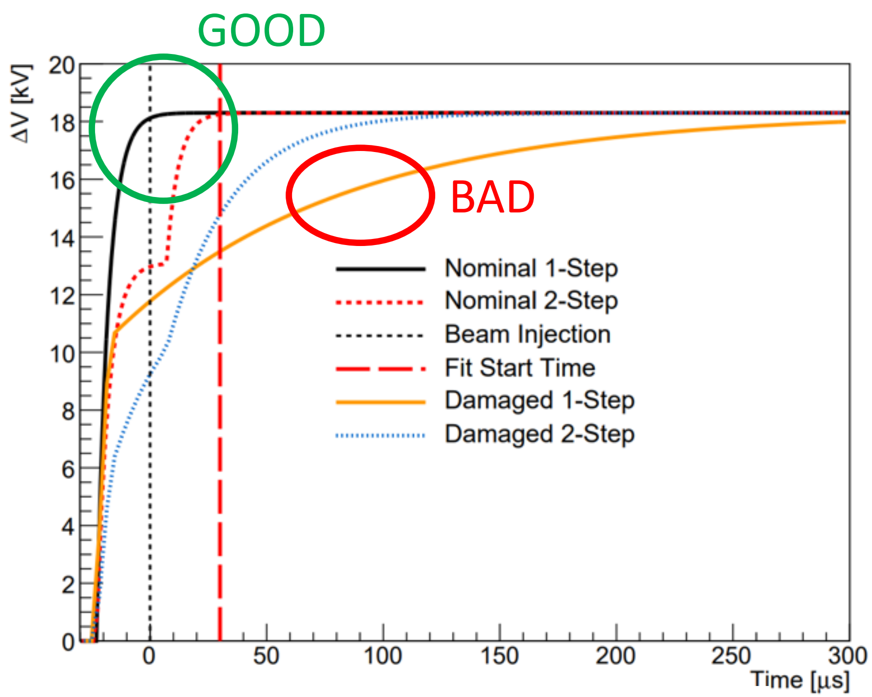
- The extracted ω_a is shifted by ϕ' !

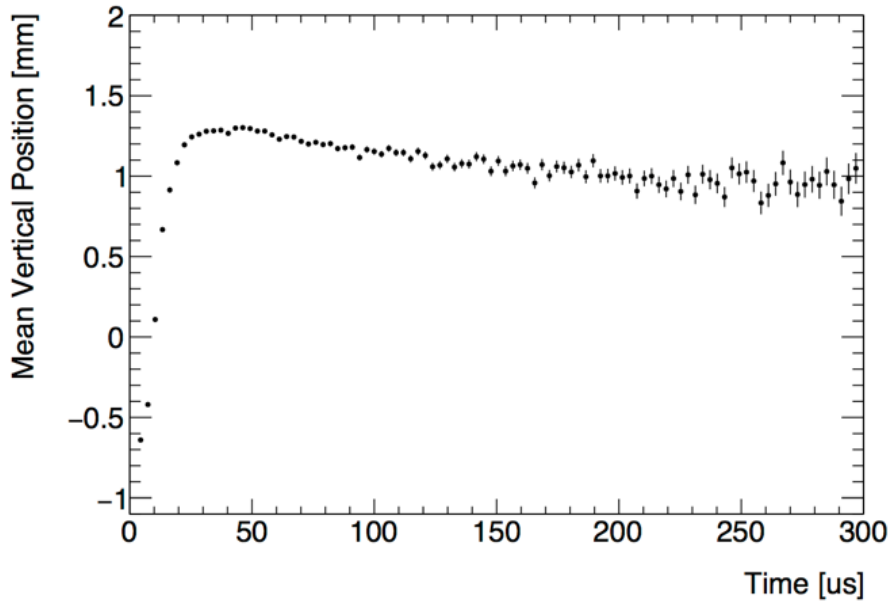


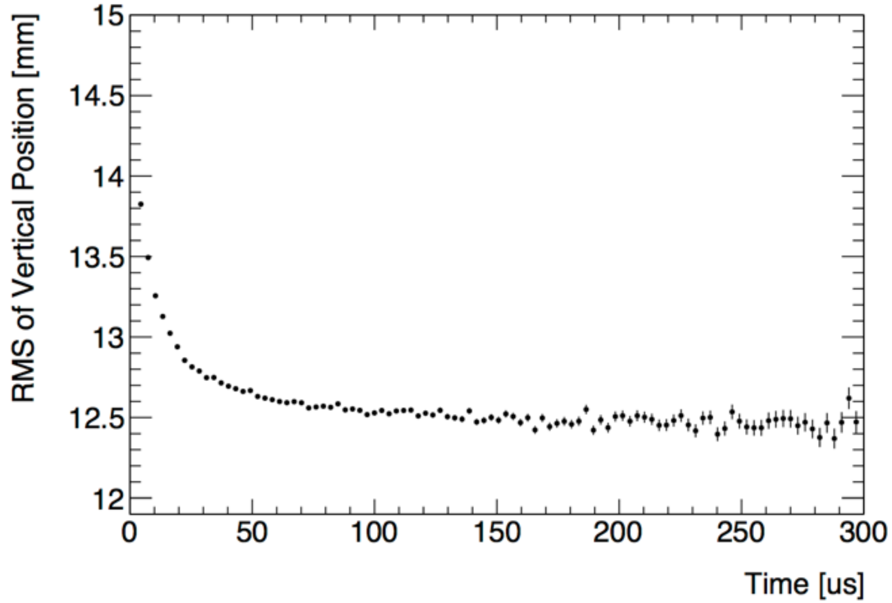
- The decay positrons we detect carry a particular phase
- That phase depends on muon decay position (x,y) and energy E
- Not a big issue if the muon distribution remains stable in the gap

C_{PA} – Phase acceptance error

- HV resistors failed → changing E-field → beam vertical mean and width changed
- $C_{PA} = -158\,ppb,\ \delta_{PA} = 75\,ppb$
- Faulty resistors fixed before Run-2





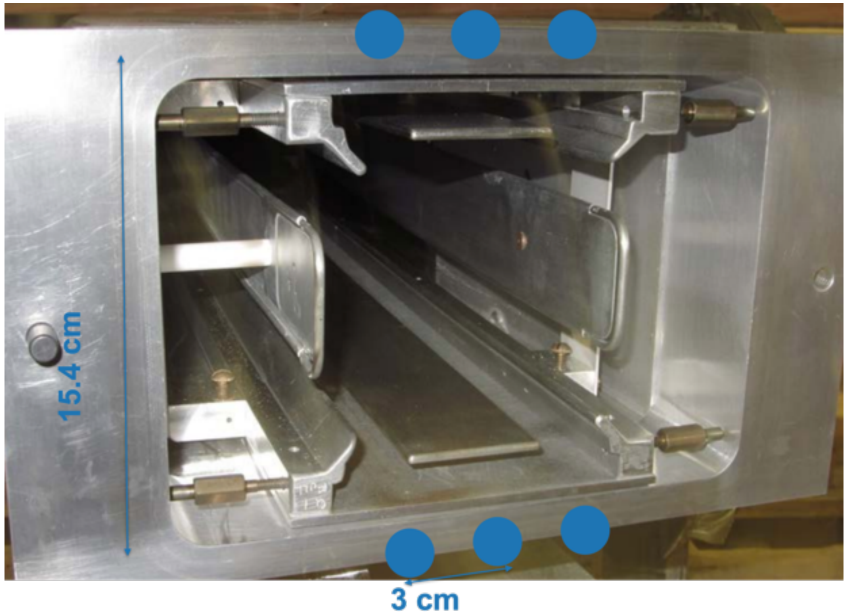


ElectroStatic Quadrupoles transient field B_q

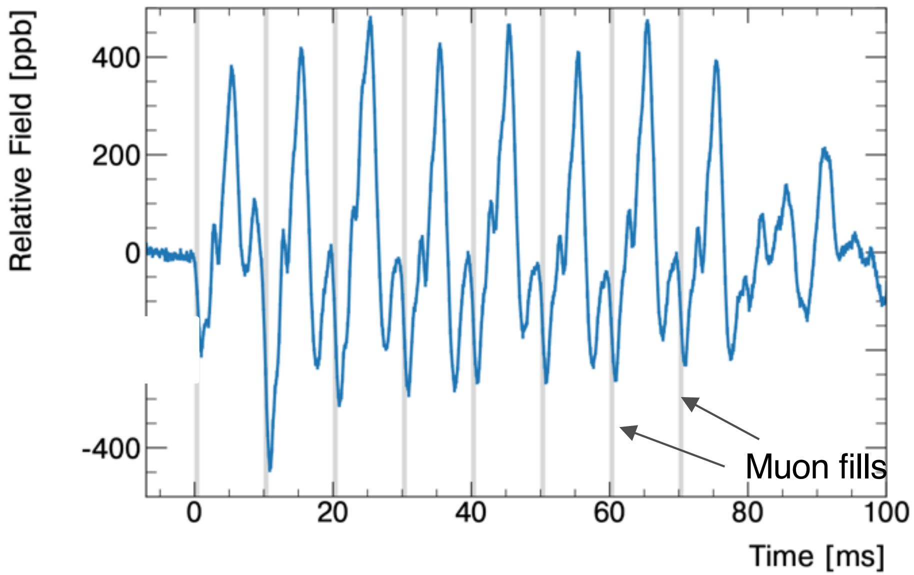
- The ESQ are charged/discharged every muon fill (700ms)
- The electric pulse induces mechanical vibrations in the plates which generate magnetic perturbations
- Special NMR probes measured B_q at several positions

$$B_q \sim 20\ ppb, \delta_{B_q} \sim 90\ ppb$$

- Uncertainty dominated by limited number of measurements in Run-1, reduced in Run-2 by more measurements



Quad Plates inside Vacuum Chamber



The blinding

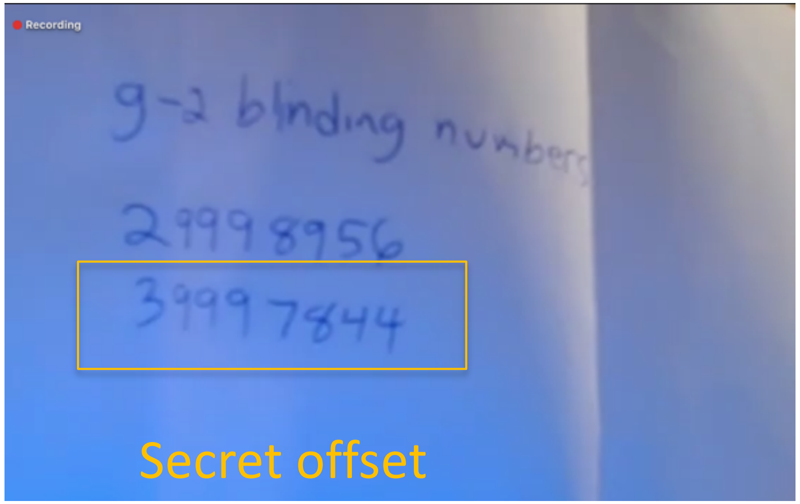
$$R'_\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \cdot \omega'_p(x,y,\varphi) \cdot M(x,y,\varphi) \cdot (1 + B_k + B_q)} \right)$$

- Clock frequency f_{clock} uncalibrated by Joe Lykken and Greg Bock (FNAL Directorate) Feb 22 2018
  - stop in each week to check clock and sealing
- Secret envelopes kept until physics analysis complete and ready to be revealed Feb 25 2021

a_μ: Unblinding

On February 25 the collaboration met for the unblinding:

1) The *box* (envelope) was opened
2) The number was plugged into two independent programs
3) And the result was....



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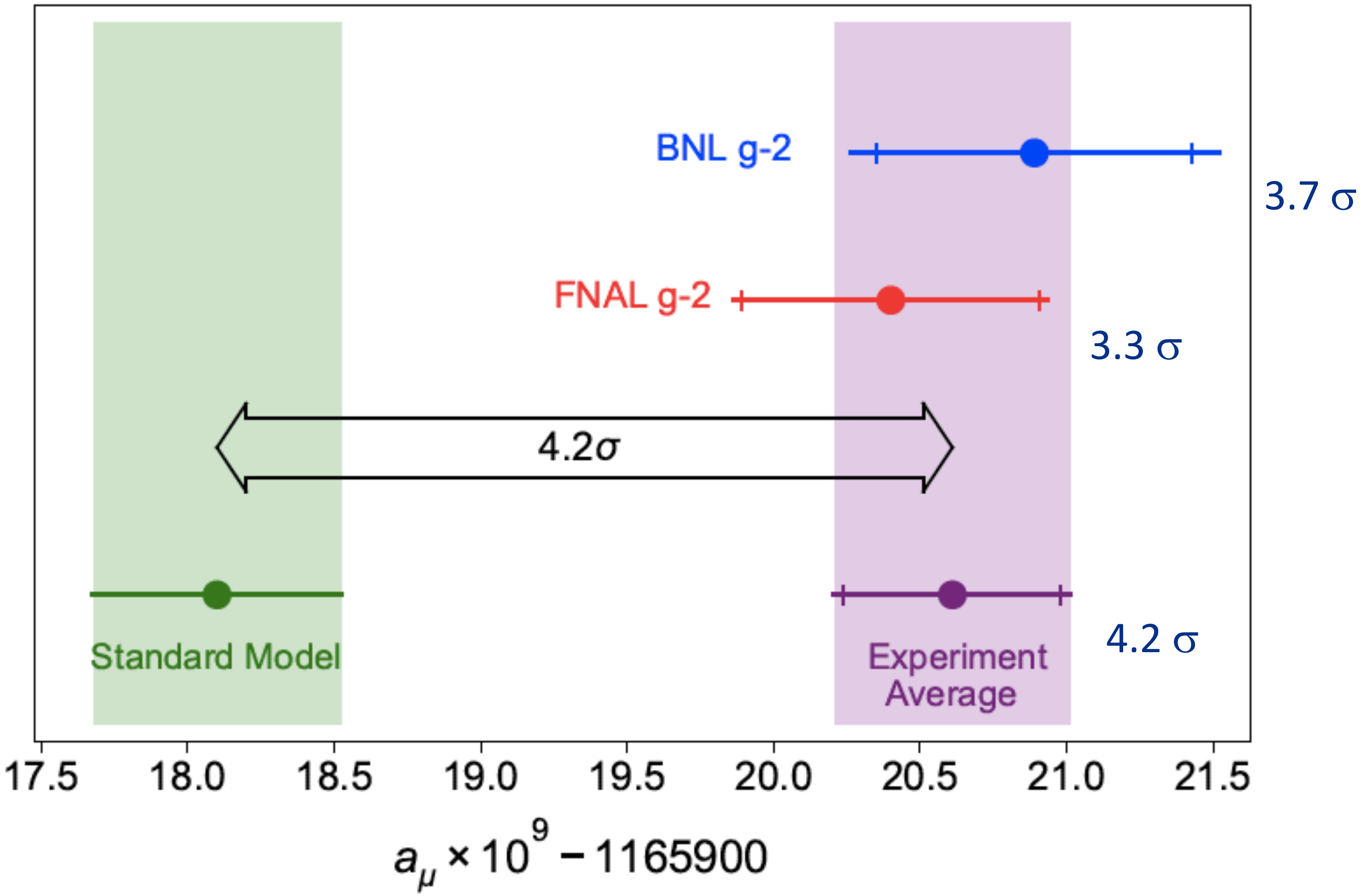
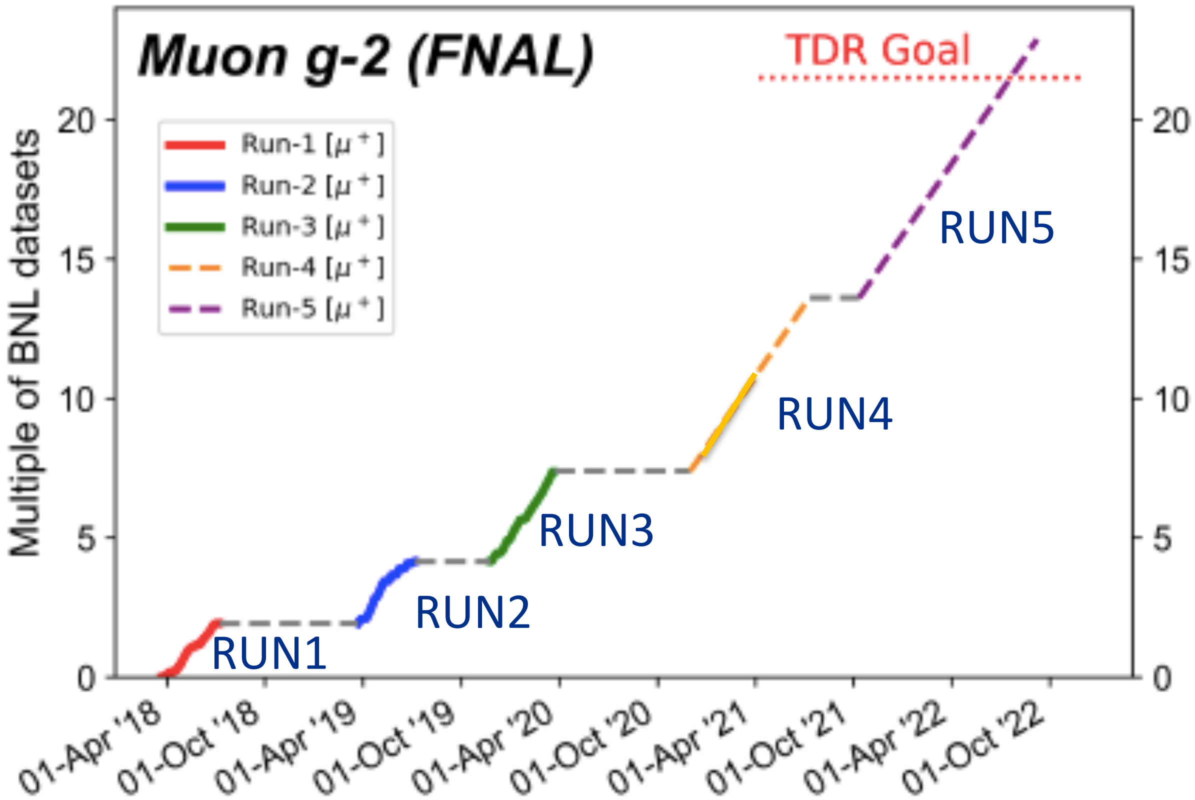
Are we ready to unblind?

a_μ: Unblinding and result



The future of E989 *Muon g-2 (Fnal)*

- RUN-1 is only 6% of the final dataset
- Analysis of RUN-2/3 in progress (factor ~2 in precision)
- RUN-4 (November 2020-July 2021) is expected to bring the statistics to ~10 times the Run-1 dataset

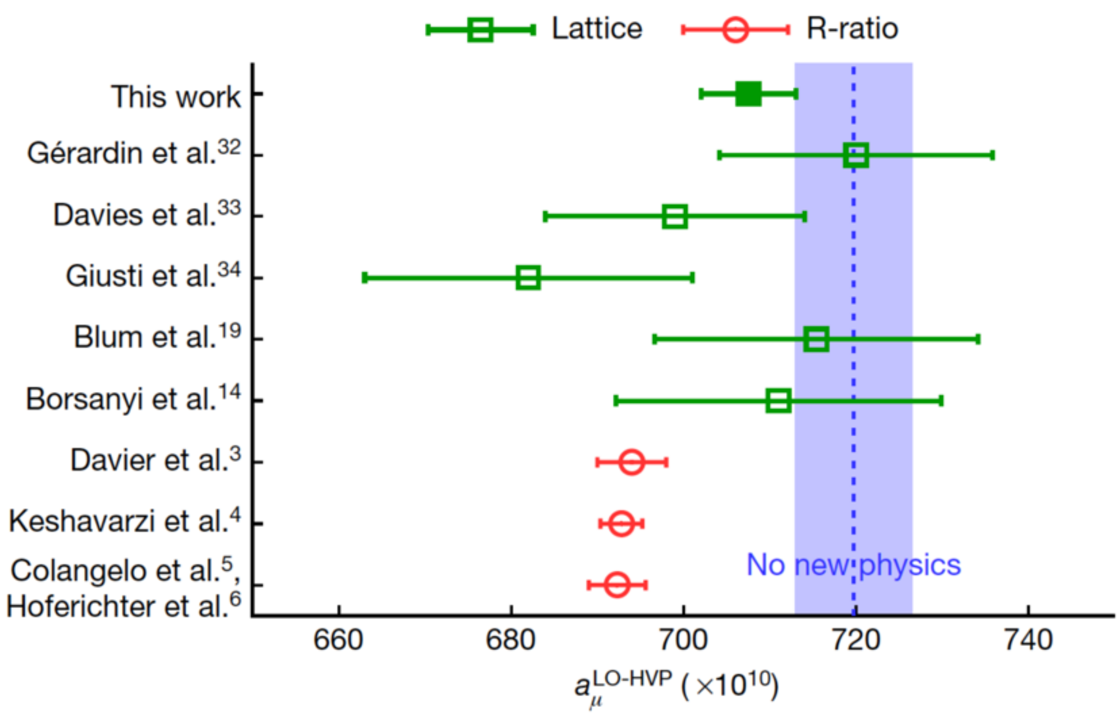


- RUN-5 in 2021-2022 should allow to achieve the project goal which will allow to reduce by a factor ~4 current total error

A new prediction based on lattice

- a recent (published April 7!) result has reduced the uncertainty of the lattice calculation from 2-3% to 0.8%!
- breakthrough which requires further scrutiny
- uncertainty similar to the result obtained with the dispersion relation (0.6%)
- the lattice result is much more in agreement with the experimental result

Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision: $a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst} \times 10^{-11}$ (55). **Some tension with dispersive evaluations.** BMWc 2021



Borsanyi et al (BMWc), Nature 2021

- The theoretical error is dominated by the Hadronic Vacuum Polarization (HVP) diagram

Contribution	$a_\mu \times 10^{11}$	$\Delta a_\mu \times 10^{11}$
QED	116584718.931	0.104
Electroweak	153.6	1.0
HVP(White Paper)	6845	40
HLbL(White Paper)	92	18
SM(White Paper)	116591810	43
Experiment	116592061	41
Experiment−SM(White Paper)	251	59

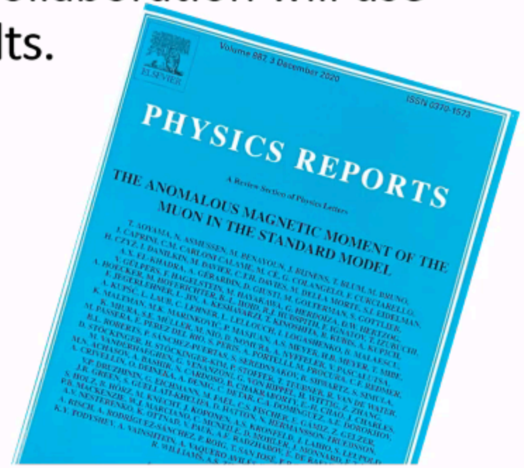
- Result of the "*theory initiative*": ~130 theoriticians, had several meetings over 4 years, produced in June 2020, a White Paper which is our benchmark (ArXiv 2006.04822)

The Theory Initiative

- ~130 physicists collaborated for 3 years [June 2017 – June 2020] in seven workshops to produce a reference number for am to be used by FNAL g-2 experiment as a benchmark

Muon g-2 Theory Initiative defines SM benchmark value that our collaboration will use for comparison. We don't "pick and choose" other individual results.

Group photo from the Seattle workshop in September 2019, https://indico.fnal.gov/event/21626/



Organizers:
Aida El-Khadra
Martin Hoferichter
DWH

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
WP20 → Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	Eq. (8.14)	279(76)	

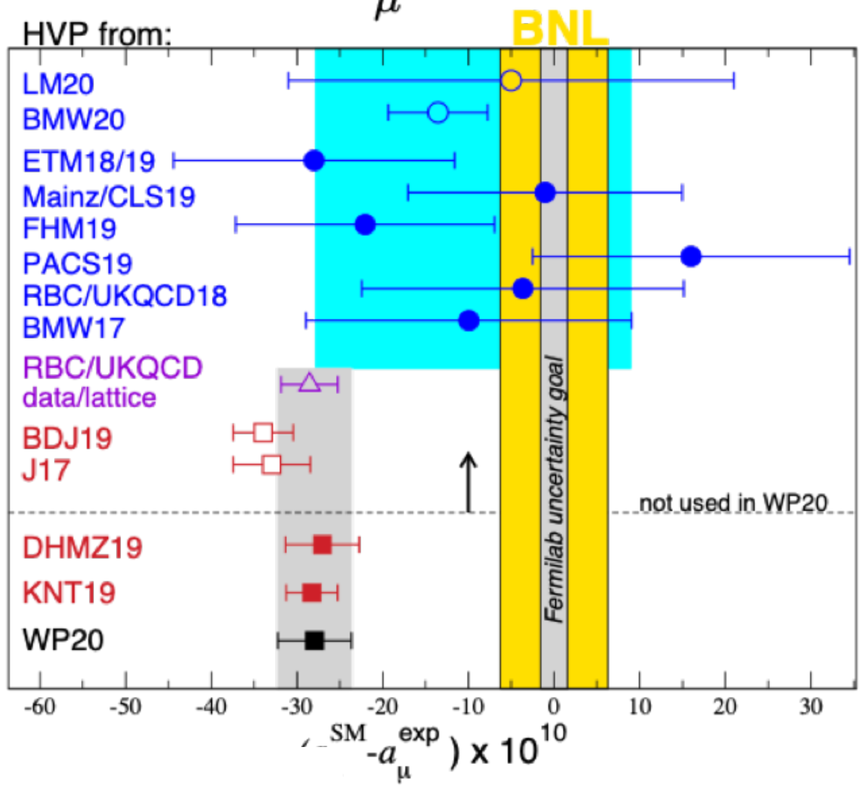
The result

- From the White Paper:

For HVP, the current uncertainties in lattice calculations are too large to perform a similar average and the future confrontation of phenomenology and lattice QCD crucially depends on the outcome of forthcoming lattice studies. For this reason, we adopt [the data-driven evaluations of HVP] as our final estimate

$$a_\mu^{\mathrm{HVP}} + [a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}]$$

$$a_\mu^{\mathrm{SM}}$$



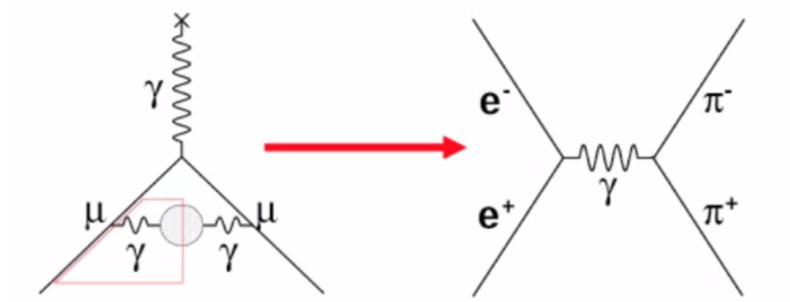
3. Lattice QCD calculations of HVP

T. Blum, M. Bruno, M. Cè, C. T. H. Davies, M. Della Morte, A. X. El-Khadra, D. Giusti, Steven Gottlieb, V. Gülpers, G. Herdoíza, T. Izubuchi, C. Lehner, L. Lellouch, M. K. Marinković, A. S. Meyer, K. Miura, A. Portelli, S. Simula, R. Van de Water, G. von Hippel, H. Wittig

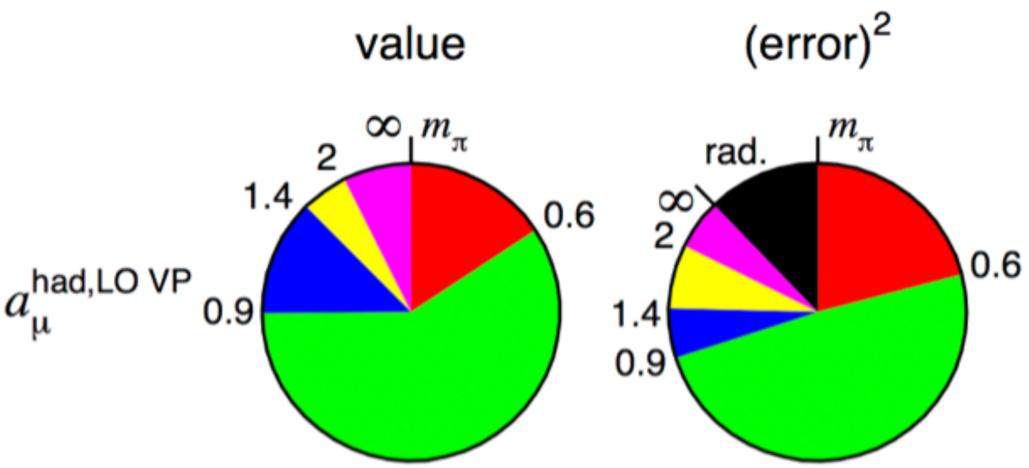
(Section 3 of the White Paper)

a_μ^{HLO} : dispersion integral

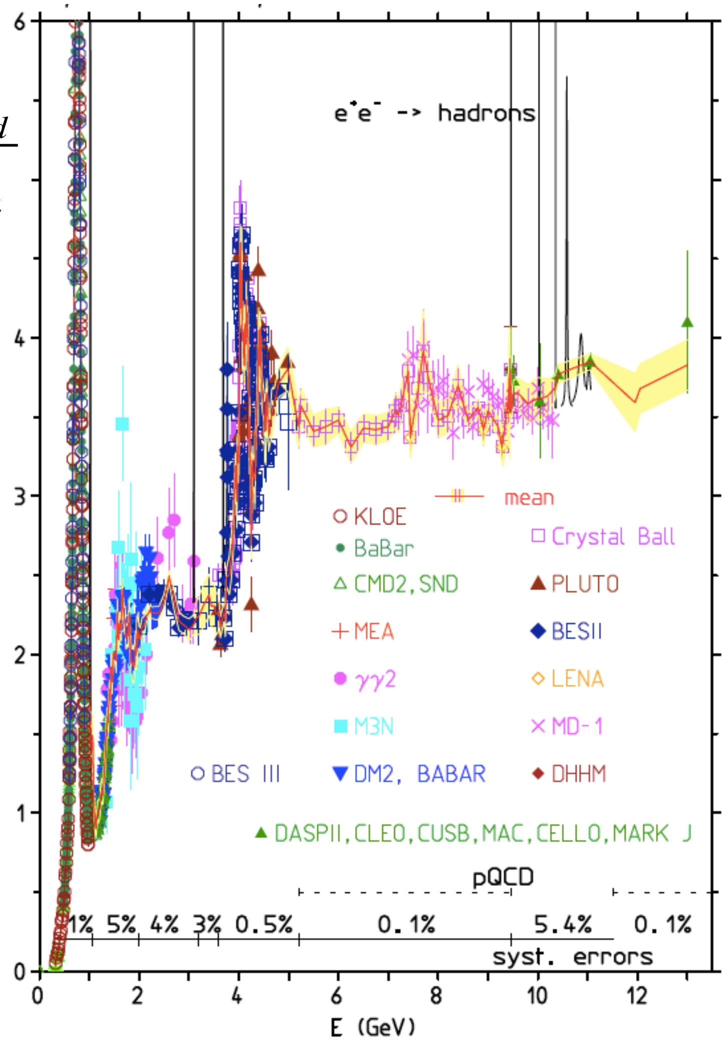
$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty}\sigma_{e^+e^-\to hadr}(s)K(s)\,ds$$



- Kernel function: $K(s) \propto {}^{1}/_{s}$
- Due to the 1/s term, the low energies most important



$$R = \frac{\sigma_{had}}{\sigma^0_{\mu\mu}}$$



Hadronic cross section and $\alpha(M_Z)$

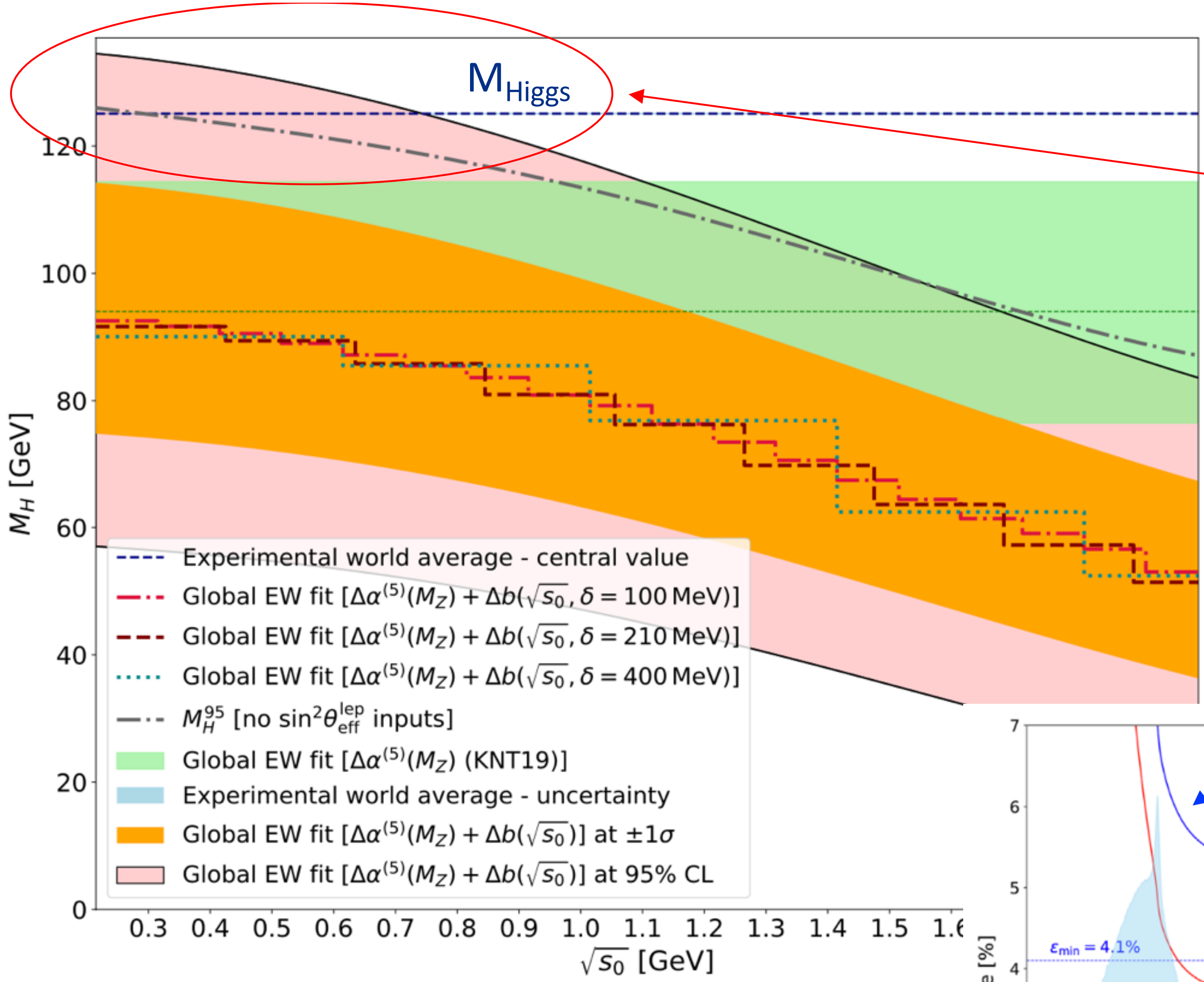
- To cope with experimental data (with the BMW21 lattice data) σ(s) has to increase by 5-8% (3-6%)
- An upward shift of σ(s) introduces an increase of $\Delta\alpha_{had}(M_Z)$
- This increase has the side effect of modifying the outcome of the global ElectroWeak fit, with consequences on the W and Higgs masses

$a_\mu^{HLO} \rightarrow$

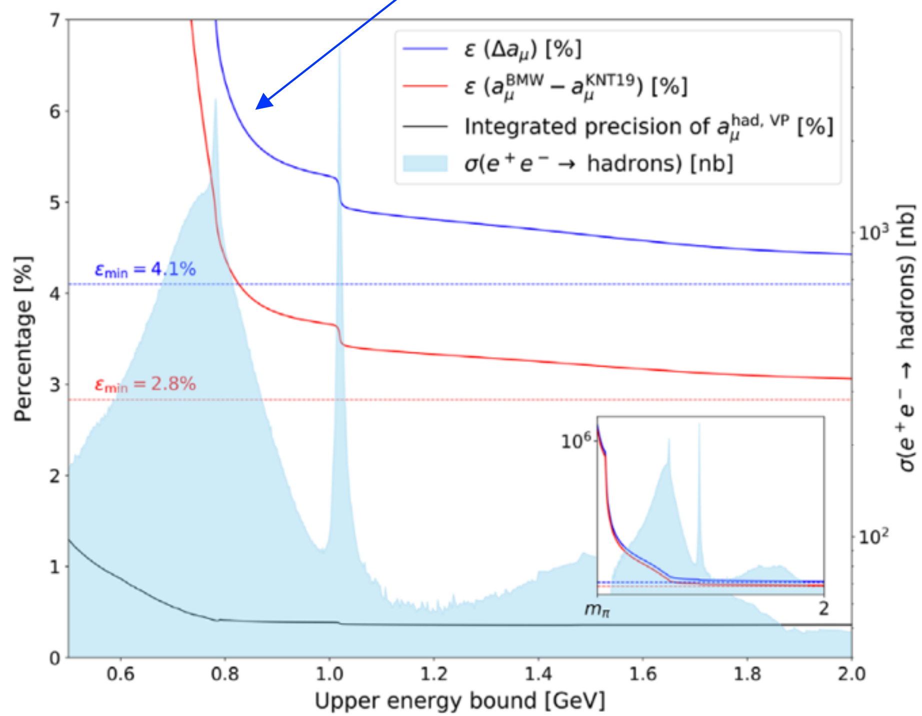
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$



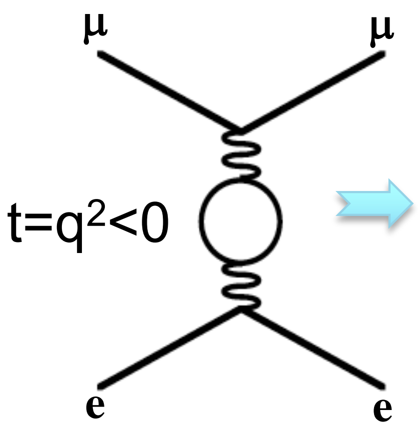
- the "allowed region", even with some tension, is $\sqrt{s} < 1\ GeV$
- the cross section would have to be increased by 5-6 % (blue line)



- typical error in the measurement of the $\pi\pi$ cross section for $\sqrt{s} < 1\ GeV$ is ~0.5 %

a_μ^{HAD} from μ + e → μ + e scattering

- NEW IDEA:
  - measure a_μ^{HAD} using *μe→μe* (t-channel) instead of ee→ππ !



$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2 \Rightarrow \alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha_{LEP}(t)-\Delta\alpha_{HAD}(t)}$$

$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}(x)dx$$

$$t = \frac{x^2 m_\mu}{x-1} < 0; \quad (0 \le x < 1);$$

t=0 (at $x=0$), t=-∞ (at $x=1$)

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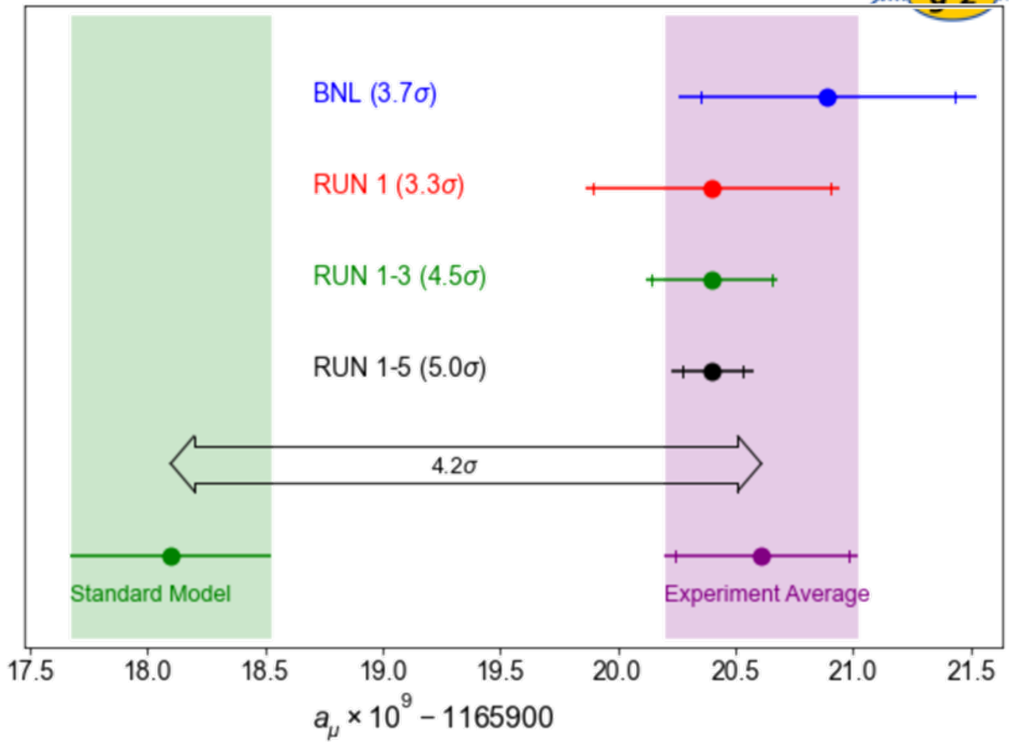
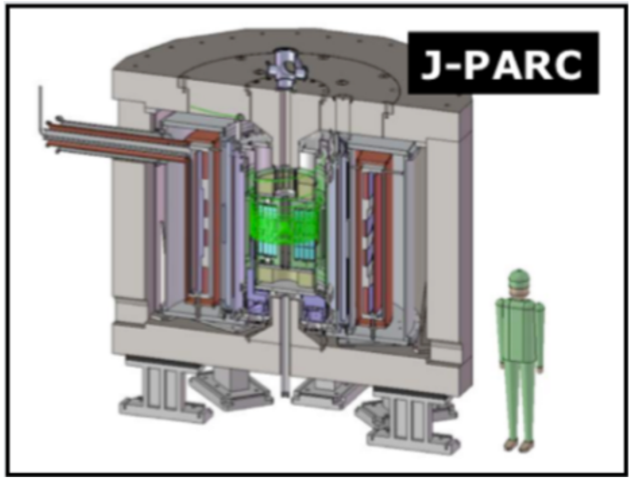
Conclusions: experiment

1. FNAL Muon g-2 :
   - ***a_μ* measured at 0.46 ppm**
   - data available already to reduce errore by x2
   - collecting data in 2021 and 2022 to reduce error by x4

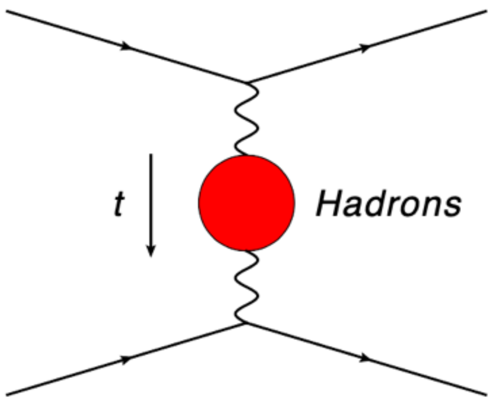


2. A new type of experiment projected at J-Parc using low energy muons (p~300 MeV/c)
   - new technique
   - timing unclear



Conclusions: theory

1. close scrutiny of lattice calculations to establish its solidity
   – how to reconcile it with dispersion approach and Standard Model global EW fit?
2. Use the dispersive approach with t-channel data (*electron-muon* scattering), instead of the standard s-channel
   – new experiment proposed at CERN: Muone (mu-on-e scattering)



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!