g-2 orienteering

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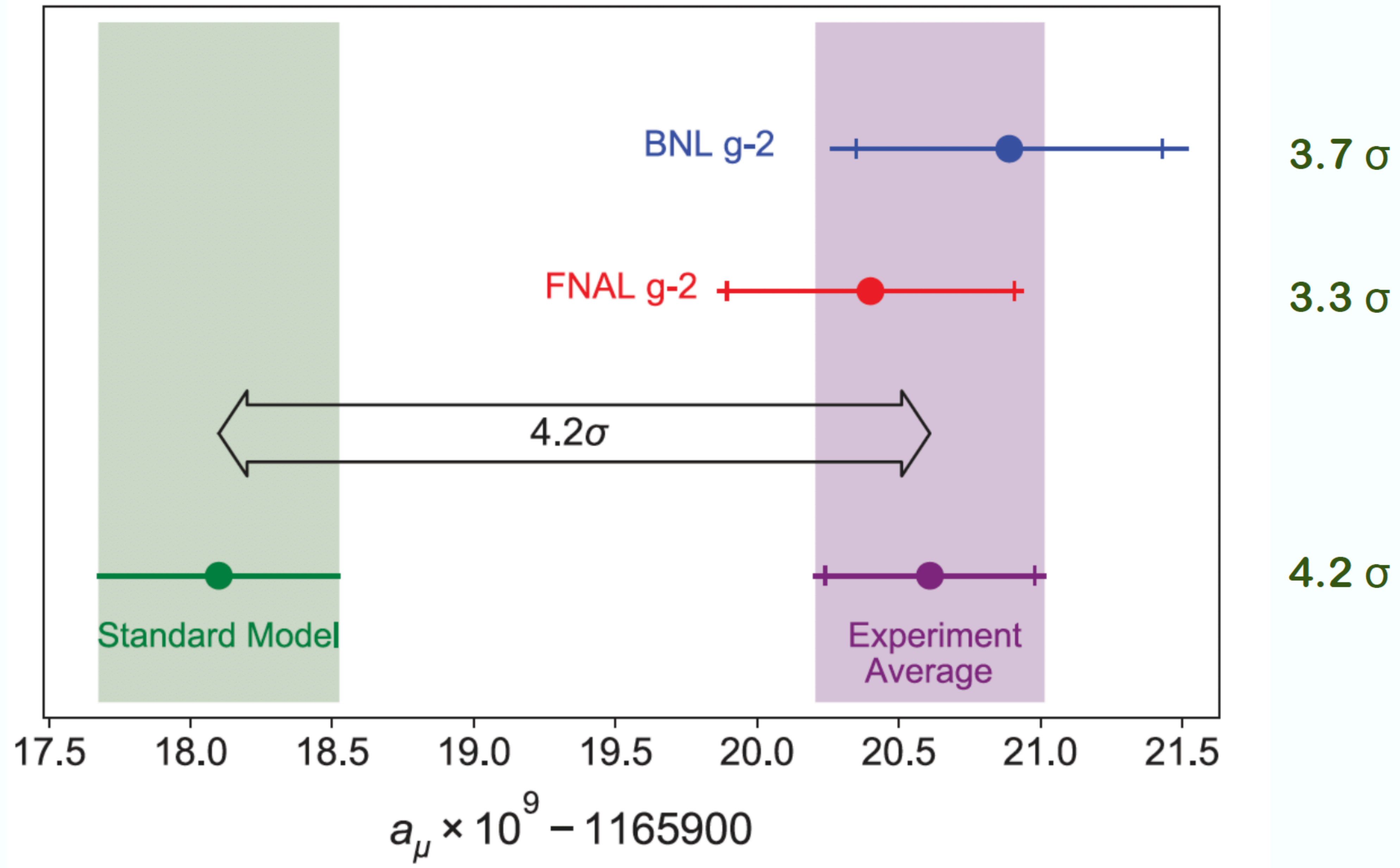
News

1 month ago

New York Times

A Tiny Particle's Wobble Could Upend the Known Laws of Physics

Experiments with particles known as muons suggest that there are forms of matter and energy vital to the nature and evolution of the cosmos that are not yet known to science.



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

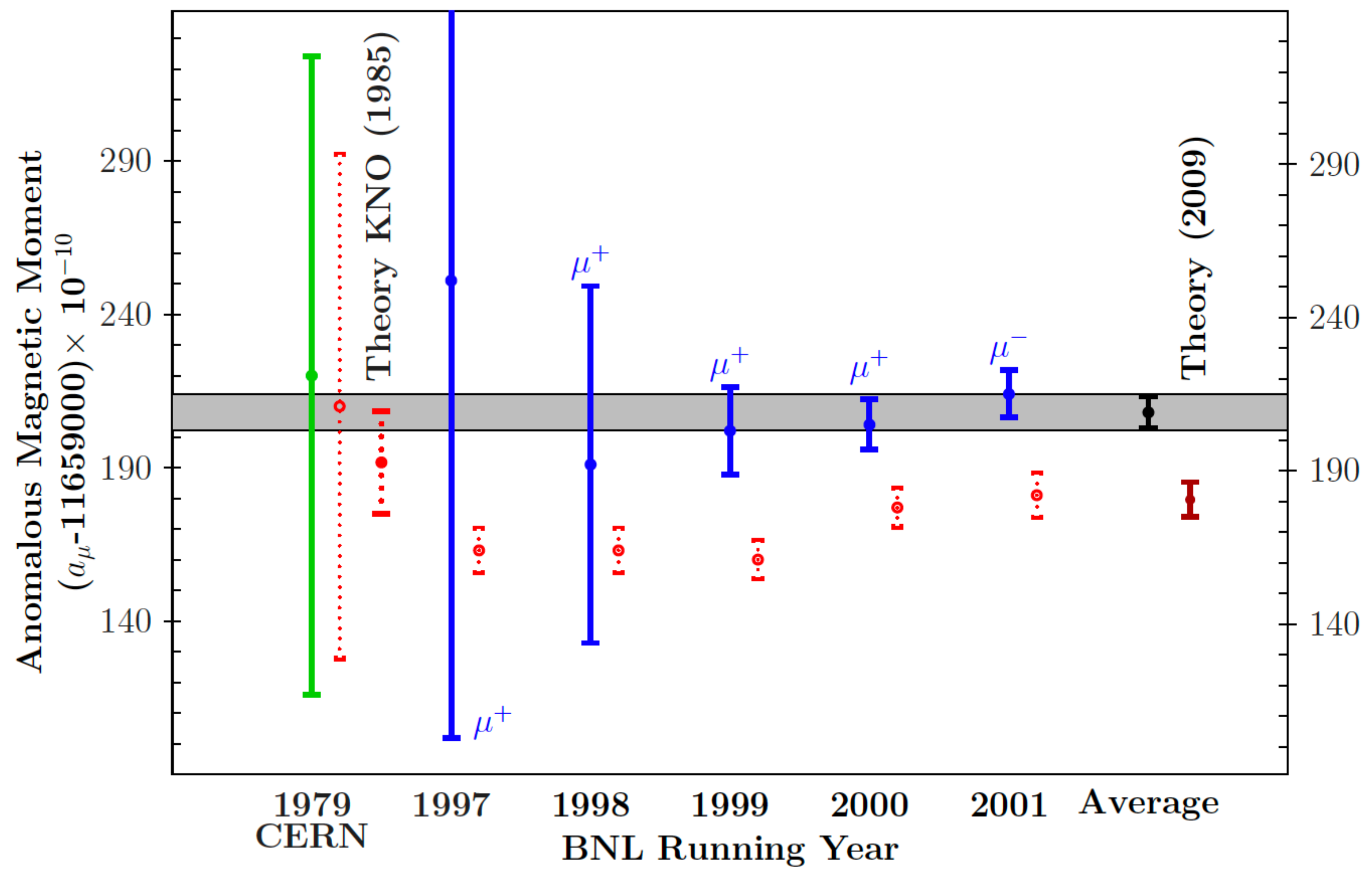
$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- **FNAL aims at 16 $\times$ 10^{-11}. First 3 runs completed, 4th in progress.**
- **Muon $g-2$ proposal at J-PARC: Phase-1 with similar BNL precision.**

Preliminaries

(g-2) histo & sociology

- (g-2) is one of the most famous topics in physics: It is like the Monna Lisa of Quantum Field Theory.
- A lot of instant theory interest with about 100 papers on the arXiv in the last month.
- Disclaimer: I am not a g-2 expert! Consider this as an attempt to trigger interesting questions and to motivate more in-depth analyses…
- Main references:
    - [Fred Jegerlehner,Andreas Nyffeler, Phys. Rept. 477 (2009)]
    - [Giudice, Paradisi, Passera, JHEP 11 (2012) 113]
- Many thanks to Paride Paradisi (PD) for material/slides and many discussions.



Citation Summary

Exclude self-citations

	Citeable	Published
Papers	95	4
Citations	455	22
h-index	12	2
Citations/paper (avg)	4.8	5.5

g-2 orienteering
Plan

- Basics and status of TH vs EXP
- New Physics Strategies
- Main lines of exploration and selected examples
- Discussion

(g-2) : Introduction

The easy-peasy stuff

g-2 : Basics
From the Dirac equation...

The Dirac equation implies:

$$(i\not{D} - m)\psi \Longrightarrow \left(D_\mu^2 + m^2 + \tfrac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}\right)\psi = 0 \Rightarrow \frac{(H - eA_0)^2}{2m}\psi = \left(\frac{m}{2} + \frac{(\vec{p} - e\vec{A})^2}{2m} - 2\frac{e}{2m}\vec{B}\cdot\vec{S}\right)\psi$$

In general, the interactions of magnetic and electric dipole moments are

$$\mathcal{H} = -\vec{\mu}_m \cdot \vec{B} - \vec{d}_e \cdot \vec{E}$$

as $\vec{\mu}_m$, $\vec{d}_e$ and $\vec{B}$ are axial vectors $\Rightarrow$ $P\vec{v} = -\vec{v}$, $T\vec{v} = \vec{v}$; $P\vec{a} = \vec{a}$, $T\vec{a} = -\vec{a}$

$\Rightarrow$ P and/or T conservation $\Rightarrow$ no $\vec{d}_e$

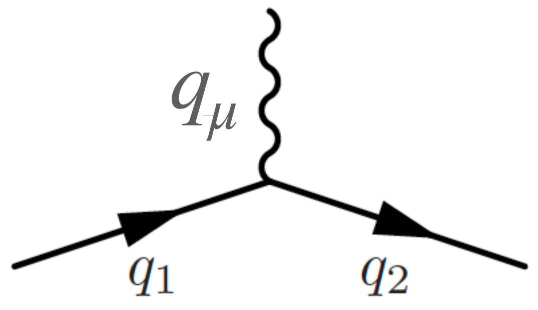
$\Rightarrow$ Standard Model $\Rightarrow$ $\vec{d}_e$ is extremely small

$$\vec{\mu}_m = g\,Q\,\mu_0\,\frac{\vec{\sigma}}{2}\,, \quad \vec{d}_e = \eta\,Q\,\mu_0\,\frac{\vec{\sigma}}{2} \qquad \text{with } \mu_0 = \frac{e}{2m} \qquad \Rightarrow \quad g = 2, \quad \eta = 0$$

g-2 : Basics

…to quantum field theory…

$$i\mathcal{M}_0^\mu =$$



$$= -ie\bar{u}(q_2)\gamma^\mu u(q_1) = -i\left[e\left(\frac{q_1^\mu + q_2^\mu}{2m}\right)\bar{u}(q_2)\,u(q_1) + \frac{e}{2m} i\bar{u}(q_2)\,q_\mu\,\sigma^{\mu\nu}u(q_1)\right]$$

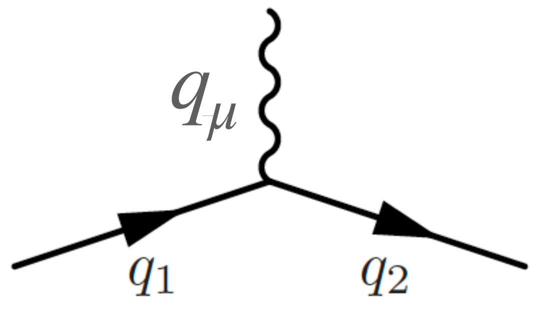
Gordon decomposition

$$= -i\left[e\left(\frac{q_1^\mu + q_2^\mu}{2m}\right)\bar{u}_L(q_2)\,u_R(q_1) + \frac{ie}{2m}\frac{g}{2}\bar{u}_L(q_2)\,q_\mu\sigma^{\mu\nu}u_R(q_1) + h.c.\right]$$

$g = 2$

g-2 : Basics

...to quantum field theory...

$i\mathcal{M}_0^\mu =$ 

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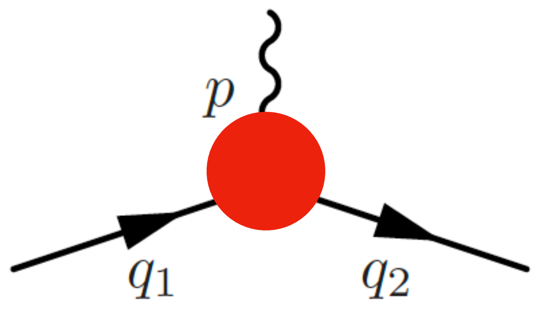
$i\mathcal{M}^\mu =$ 

$$= (-ie)\bar{u}(q_2)\left[F_1(q^2)\;\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}q_\mu\;F_2(q^2)\right]u(q_1)$$

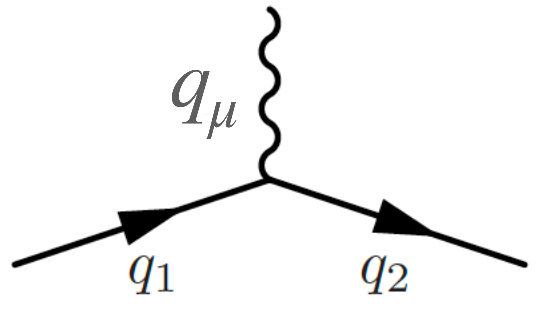
Lorentz Invariance.
Charge conservation $\Rightarrow F_1(0) = 1$

$$= -i\left[e\left(\frac{q_1^\mu + q_2^\mu}{2m}\right)\bar{u}_L(q_2)\,u_R(q_1)F_1(q^2) + \frac{ie}{2m}\bar{u}_L(q_2)\,q_\mu\sigma^{\mu\nu}u_R(q_1)(F_1(q^2) + F_2(q^2)) + h.c.\right]$$

$$\frac{g}{2} = 1 + F_2(0) \Rightarrow g = 2 + 2a_\mu = 2 + 2F_2(0) \quad \Rightarrow a_\mu = F_2(0)$$

g-2 : Basics

...to quantum field theory...

$i\mathcal{M}_0^\mu =$ 

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Gordon decomposition

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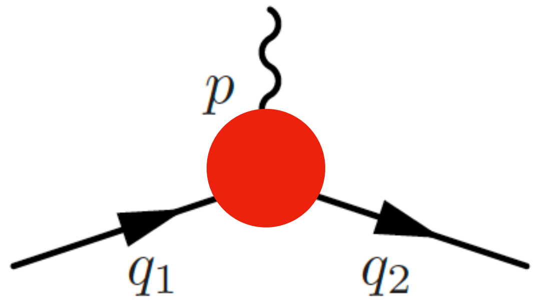
$i\mathcal{M}^\mu =$ 

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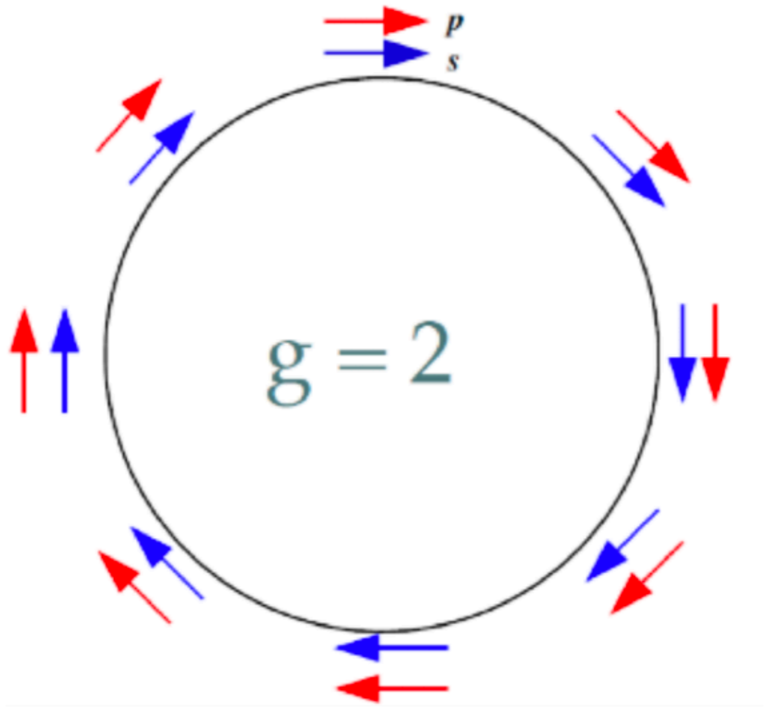
$$= -i\left[e\left(\frac{q_1^\mu + q_2^\mu}{2m}\right)\bar{u}_L(q_2)\,u_R(q_1)F_1(q^2) + \frac{ie}{2m}\bar{u}_L(q_2)\,q_\mu\sigma^{\mu\nu}u_R(q_1)(F_1(q^2) + F_2(q^2)) + h.c.\right]$$

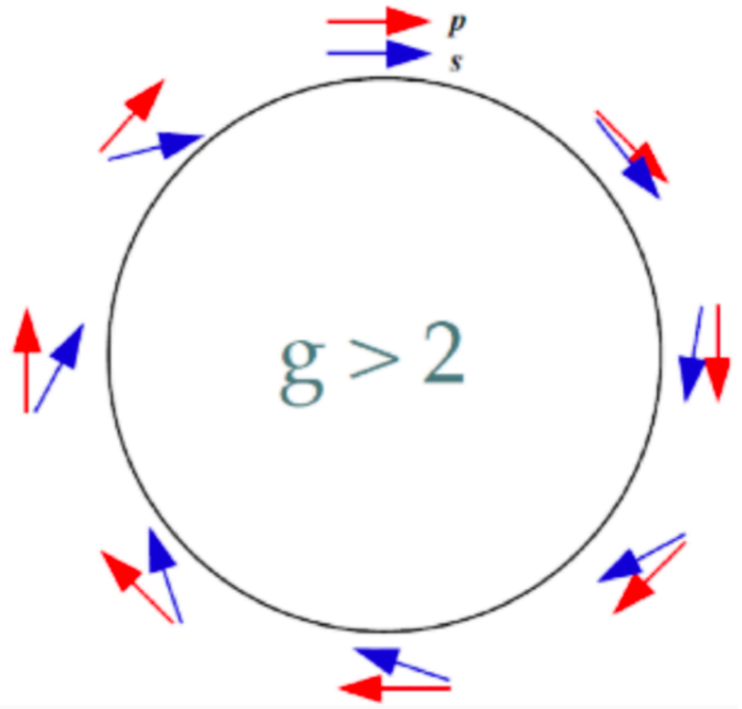
$$\frac{g}{2} = 1 + F_2(0) \Rightarrow g = 2 + 2a_\mu = 2 + 2F_2(0) \quad \Rightarrow a_\mu = \frac{g-2}{2} = F_2(0)$$



$$\omega_c = \frac{eB}{m\gamma}$$

$$\omega_s = g\frac{eB}{2m} + \frac{eB}{m\gamma}(1-\gamma)$$

$$\omega_a = \left(\frac{g-2}{2}\right)\frac{eB}{m} = a_\mu\frac{eB}{m}$$



g-2 : Basics
...to the effective interaction in the SM...

Loop corrections from QED can therefore be parametrized by

$$\Rightarrow \qquad \delta\mathcal{L}_{\rm eff}^{\rm AMM} = -\,a_\mu\,\frac{e}{4m}\,\{\bar\psi_L(x)\,\sigma^{\mu\nu}F_{\mu\nu}(x)\,\psi_R(x)+\bar\psi_R(x)\,\sigma^{\mu\nu}F_{\mu\nu}(x)\,\psi_L(x)\}$$

Considering generic loops in the SM and in BSM without the constraints of P, T invariance one obtains

$$\Gamma_\mu(q^2) = -ie\left\{\gamma_\mu\left[F_{1V}(q^2)+F_{1A}(q^2)\gamma_5\right]+\frac{\sigma_{\mu\nu}}{2m_f}q^\nu\left[iF_{2V}(q^2)+F_{2A}(q^2)\gamma_5\right]\right\}$$

$$F_{1V}(0) = Q_f,$$
$$F_{2V}(0) = a_f\,Q_f,$$
$$F_{2A}(0) = d_f\,\frac{2m_f}{e},$$

which can be obtained from $([c_f] = M^{-1})$:

$$\Rightarrow \qquad \delta\mathscr{L}_{\rm eff}^{\rm AMM} = \frac{c_f}{2}\bar\psi_L\sigma^{\mu\nu}F_{\mu\nu}\psi_R + h.c.$$

$$c_f = a_f\frac{Q_f e}{2m_f} - id_f$$

g-2 : Basics

...to the effective interaction in the BSM...

Going beyond the SM and allowing for flavor violation one can parametrize the contributions to g-2

$$\mathscr{L}_{\rm eff}^{\rm NP} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau \qquad ([A_{\ell\ell}] = M^{-2})$$

which describes dipole transitions ($\ell \to \ell'\gamma$) in the leptonic sector (ex. $\frac{\mathrm{BR}(\ell\to\ell'\gamma)}{\mathrm{BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right)$) and gives

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

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Note 1:

The connection with lepton flavor violation arises because this is the first term in the effective lagrangian that allows for an EM transitions between fermions with different masses. No dim=4 FCNC currents among fermions of different masses are allowed in an unbroken gauge theory.

g-2 : Basics

…to the effective interaction in the BSM…

Going beyond the SM and allowing for flavor violation one can parametrize the contributions to g-2

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Note 2:
The form above is not manifestly SU(2) x U(1) symmetric. The corresponding SMEFT operators are

$$\mathscr{L}_{\rm SMEFT}^{\rm Dipole} = \frac{c_{eB}}{\Lambda^2}\bar L\sigma^{\mu\nu}e'_R\Phi B_{\mu\nu} + \frac{c_{eW}}{\Lambda^2}\bar L\sigma^{\mu\nu}\sigma^I e'_R\Phi W^I_{\mu\nu} + h.c.$$

which give rise to the same term as in $\mathscr{L}_{\rm eff}^{\rm NP}$ upon the Higgs field acquiring a vev (and accounting for (B,W) mixing).

g-2 : The theory budget
QED

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
Experiment (E989 – Run I)	116592040(54)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

QED corrections

Long history of contributions also from Bologna (Remiddi e Laporta).
Key result of 4 loops as it is order $300 \cdot 10^{-11}$. 5-loops is relevant for the electron…

$a_\mu^{\text{QED}} = (1/2)\,(\alpha/\pi)$ [Schwinger, 1948]

$+\,0.765857426\ (16)\ (\alpha/\pi)^2$

[Sommerfield; Petermann; Suura&Wichmann '57; Elend '66]

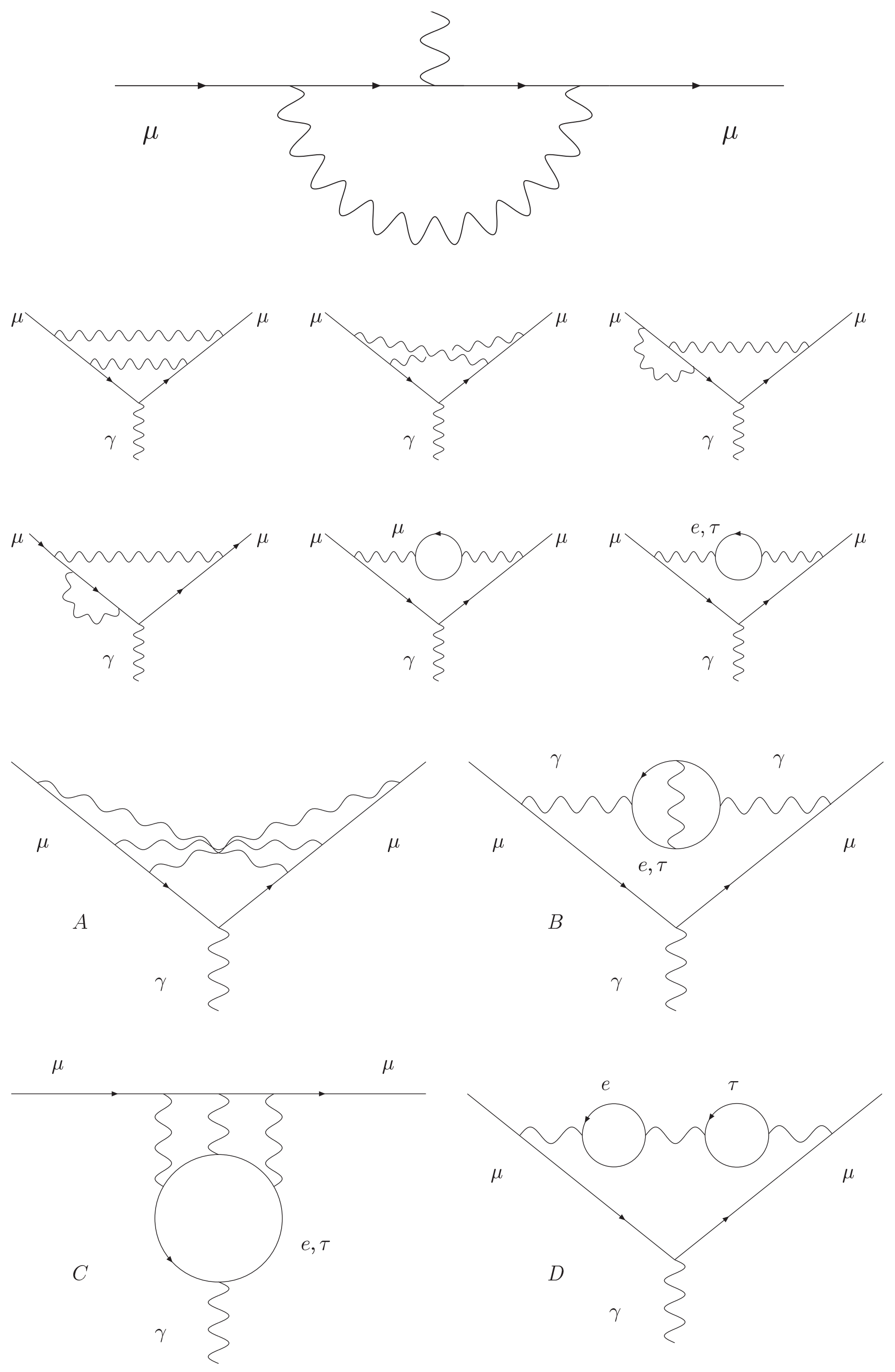
$+\,24.05050988\ (28)\ (\alpha/\pi)^3$

[Remiddi, Laporta, Barbieri...; Czarnecki, Skrzypek '99]

$+\,130.8780\ (60)\ (\alpha/\pi)^4$

[Kinoshita et al. '81-'15; Steinhauser et al. '13-'16; Laporta '17]

$+\,750.86\ (88)\ (\alpha/\pi)^5$ [Kinoshita et al. '90-'19]



a_μ^{QED} = 116584718.931 (19)(100)(23) x 10^{-11}

mainly from 4-loop coeff. unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018 WP20 value

[WP20 ≡ T. Aoyama *et al.*, Phys. Rept. '20]

g-2 : The theory budget
EW

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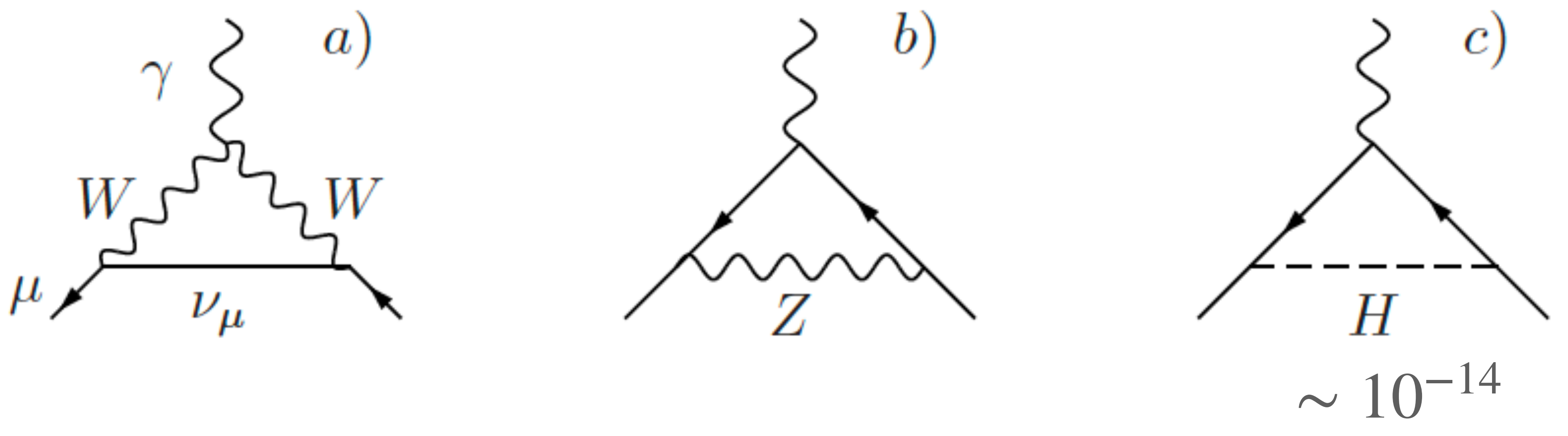


EW corrections

At one loop

$$a_\mu^{(2)\,\rm EW}(W) = \frac{\sqrt{2}G_\mu m_\mu^2}{16\pi^2}\,\frac{10}{3} \simeq +388.70(0)\times 10^{-11}\,,$$

$$a_\mu^{(2)\,\rm EW}(Z) = \frac{\sqrt{2}G_\mu m_\mu^2}{16\pi^2}\,\frac{(-1+4\,s_W^2)^2-5}{3} \simeq -193.89(2)\times 10^{-11}$$

Order of magnitude ~ to the discrepancy. Cancellations take place.

Template for New Physics at the EW scale.

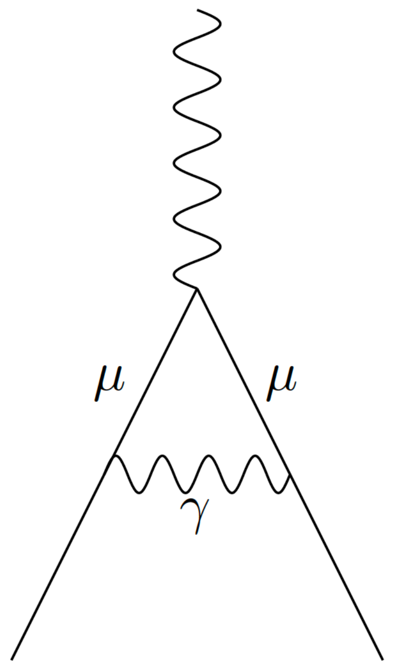
$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 200\times 10^{-11}$$

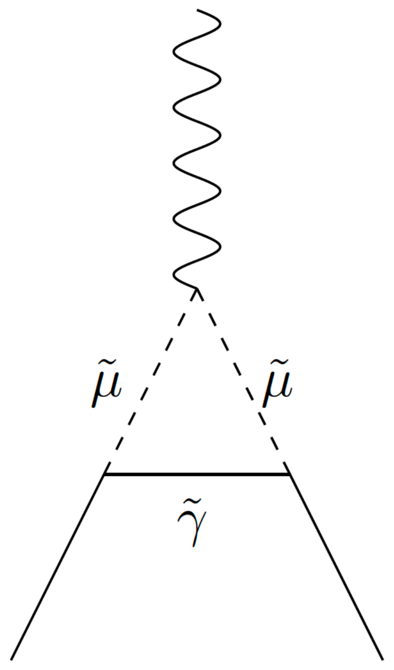
Two-loop known and also other leading terms beyond.

g-2 : The theory budget

Two tidbits

$$= \frac{\alpha}{\pi} \int_0^1 \mathrm{d}x \, (1-x) \; = \; \frac{\alpha}{\pi} \, \frac{1}{2} \, ; \qquad -2\frac{1}{2} \, \frac{\alpha}{\pi} \int_0^1 \mathrm{d}x \, (1-x) \; = -\frac{\alpha}{\pi} \, \frac{1}{2} \, ;$$



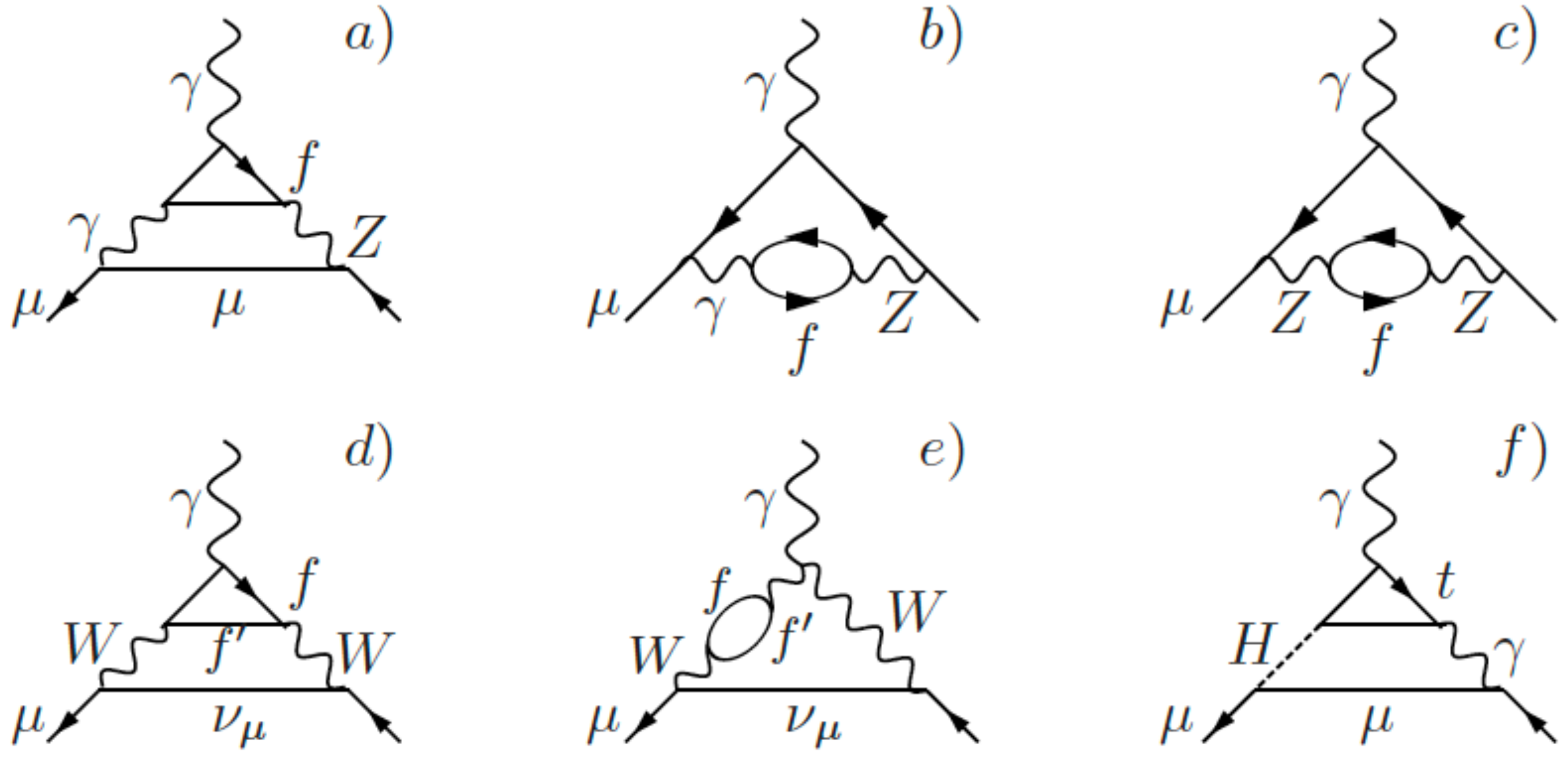


$$\frac{\alpha}{2\pi} \quad + \quad -\frac{\alpha}{2\pi} \quad =0$$

Exact SUSY $\Rightarrow$ no anomalous magnetic moment

[S. Ferrara and E. Remiddi, Phys. Lett. B53 (1974) 347.]

$$a_\mu^{(4)\,\mathrm{EW}}([f]) \simeq \mathcal{K}_2 \, 2T_{3f} N_{cf} Q_f^2 \left[3 \ln \frac{M_Z^2}{m_{f'}^2} + C_f \right] ,$$



$$\sum\nolimits_f N_{cf} Q_f^2 a_f = 0$$ Anomaly cancellation condition

Cancellation of anomalies $\Rightarrow$ no $\log M_Z^2/m_f^2$ enhanced terms!

$$a_\mu^{(4)\,\mathrm{EW}}([\tau, b, t]) = -\mathcal{K}_2 \left[\frac{8}{3} \ln \frac{m_t^2}{M_Z^2} - \frac{2}{9} \frac{M_Z^2}{m_t^2} \left(\ln \frac{m_t^2}{M_Z^2} + \frac{5}{3} \right) + \ln \frac{M_Z^2}{m_b^2} + 3 \ln \frac{M_Z^2}{m_\tau^2} - \frac{8}{3} + \; \cdots \; \right]$$
$$\simeq -\mathcal{K}_2 \; \times 30.3(3) \simeq -8.21(10) \times 10^{-11} \, .$$

g-2 : The theory budget

HVP

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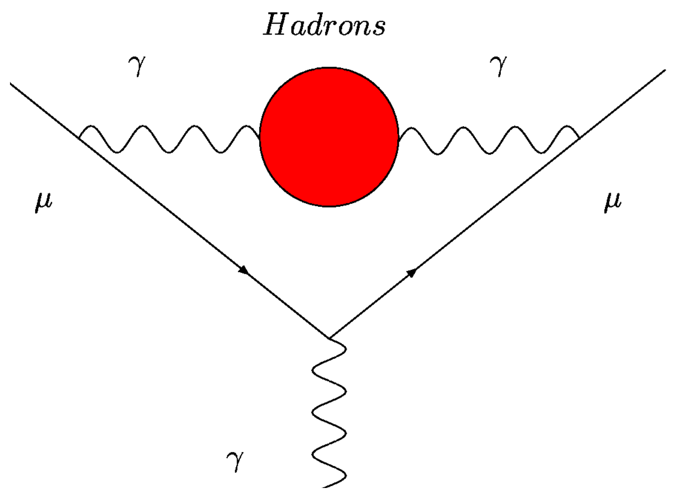
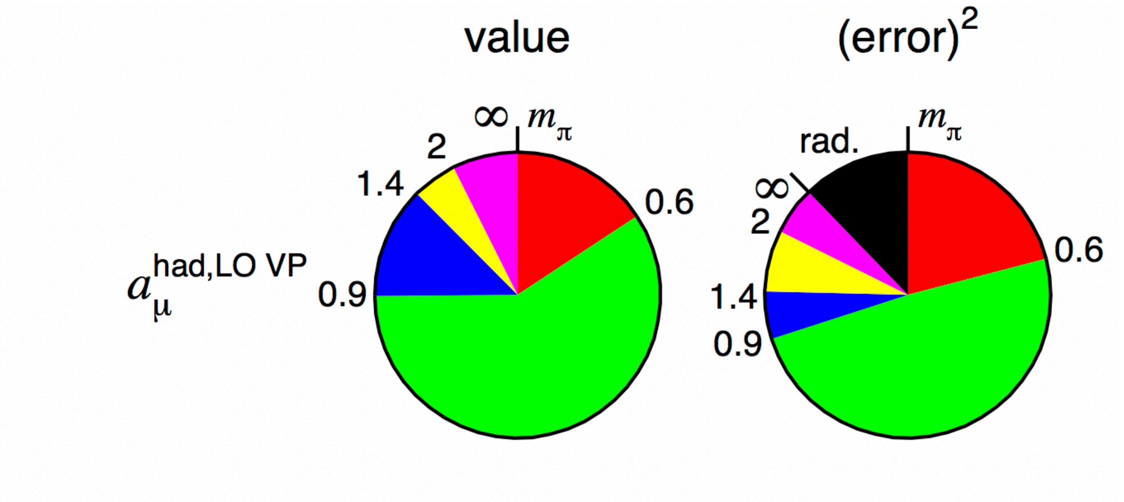
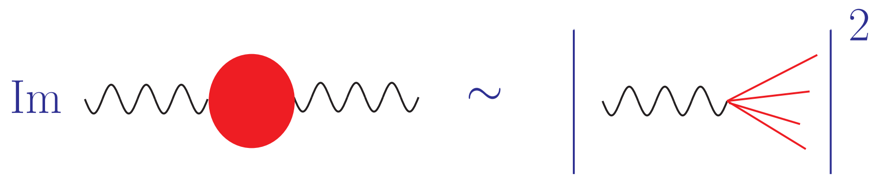
Vacuum polarization



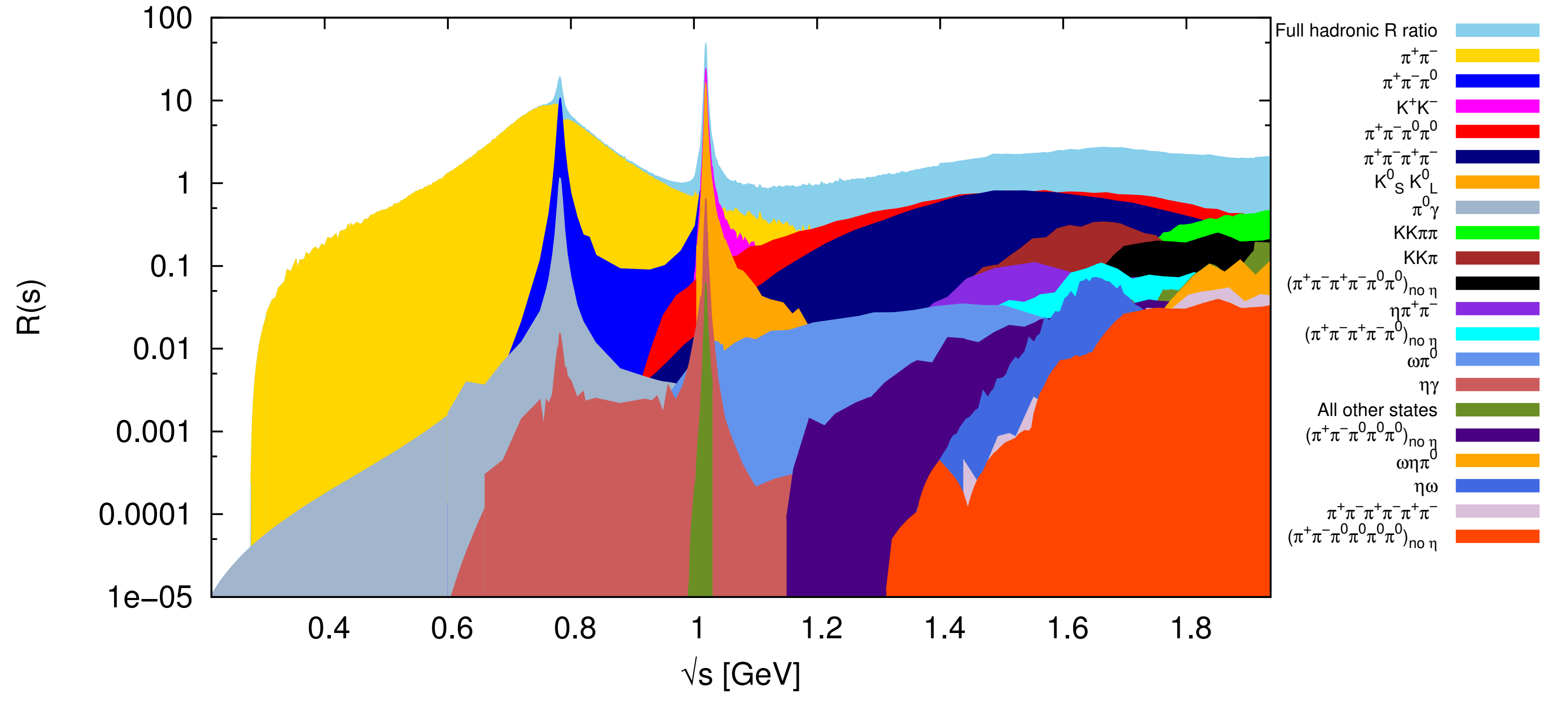


Keshavarzi, Nomura, Teubner 2018



$\sim \quad \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$

$$a_\mu^{\rm HLO} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\,K(s)\,R(s) \qquad R(s) = \frac{\sigma(e^+e^- \to had)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



$$a^{\rm HLO}_{\mu,e^+e^-} = 6931(40) \times 10^{-11}\,(0.6\%) \ \text{[WP20]}$$

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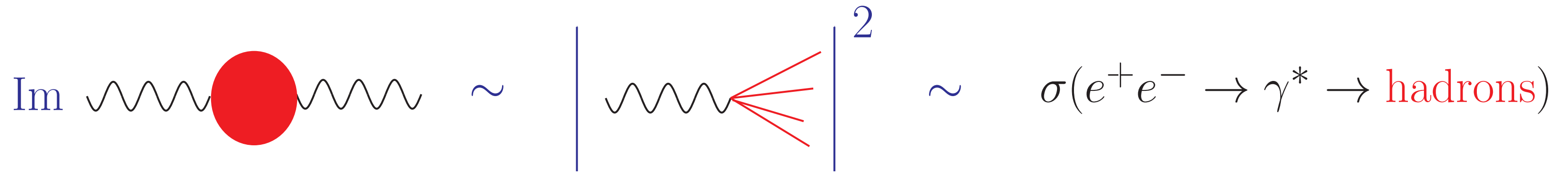
Vacuum polarization

- **Can Δa_μ be due to missing contributions in $\sigma(e^+e^- \to had)$?**
  - An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ defined by: $\alpha^{-1}(M_Z) = \alpha^{-1}\left[1 - \Delta\alpha(M_Z) - \Delta\alpha^{(5)}_{\rm had}(M_Z) - \Delta\alpha_{\rm top}(M_Z)\right]$

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\,\sigma(s)\,, \qquad \Delta\alpha^{(5)}_{\rm had} = \frac{M_Z^2}{4\pi\alpha^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$



- **A change in $\sigma(e^+e^- \to had)$ is strongly disfavoured by:**
  - **EW-fit for $\sqrt{s} \gtrsim 1$ GeV** [Marciano, Passera, Sirlin, '08, Keshavarzi, Marciano, Passera, Sirlin, '20, Crivellin, Hoferichter, Manzari, Montull, '20]. A shift of $\sigma(e^+e^- \to had)$ to accomodate the Δa_μ anomaly would necessarely require new physics to show up in the EW-fit!
  - **Experimental data on $e^+e^- \to \pi^+\pi^-$ for $\sqrt{s} \lesssim 1$ GeV** [Colangelo, Hoferichter, Stoffer, '21]
- **A check of the BMW results by other lattice QCD (LQCD) coll. is worth.**
- **LQCD coll. should provide $\Delta\alpha^{\rm LQCD}_{\rm had}$ to be compared with $\Delta\alpha^{e^+e^-}_{\rm had}$.**

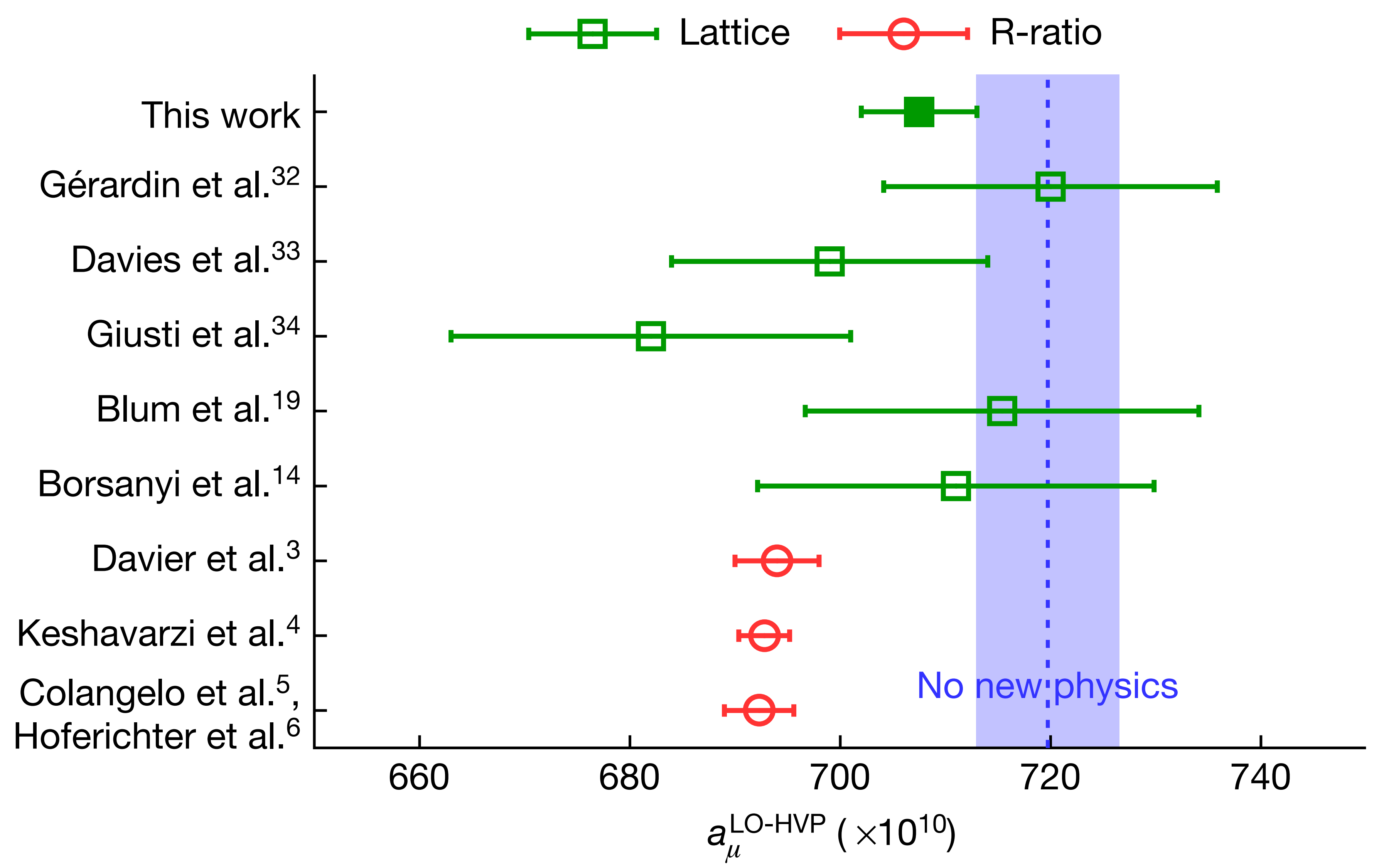
g-2 : The theory budget

HVP

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
Experiment (E989 – Run I)	116592040(54)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

Vacuum polarization

- Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision: $a^{\mathrm{HLO}}_{\mu,\mathrm{LQCD}} = 7075(23)_{\mathrm{stat}}(50)_{\mathrm{syst}} \times 10^{-11}$ [Borsanyi et al., Nature 2021].



- BMW results weakens the long-standing muon $g-2$ discrepancy but it shows a tension with dispersive evaluations of $a^{\mathrm{HLO}}_{\mu,e^+e^-} = 6931(40) \times 10^{-11}$.

g-2 : The theory budget
HVP

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
Experiment (E989 – Run I)	116592040(54)
QED	116 584 718.931(104)
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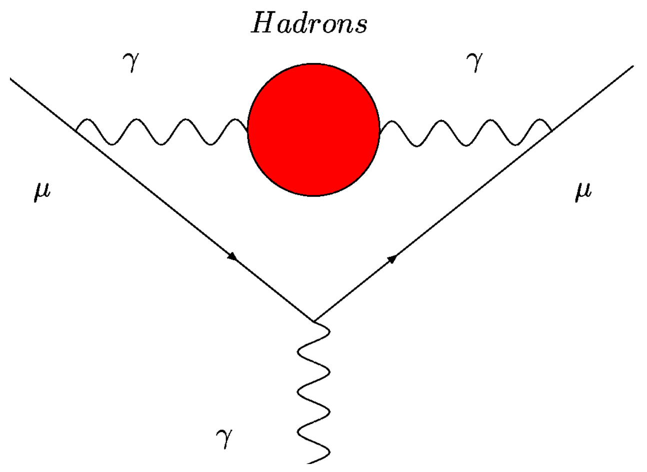
Vacuum polarization

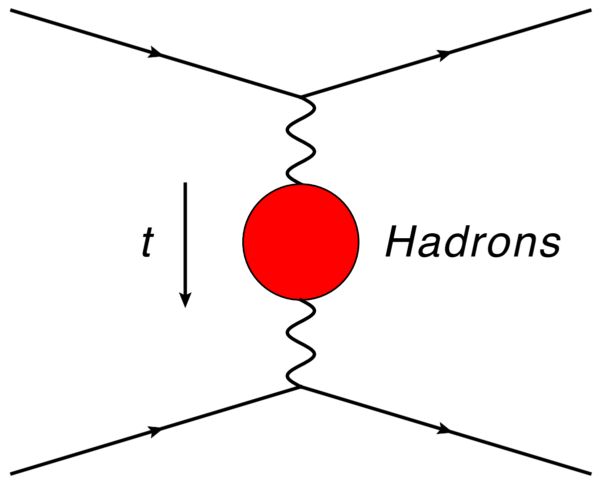
- The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\text{had}}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, simply exchanging the x and s integrations:



$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

[Carloni Calame, Passera, Trentadue, Venanzoni, 2015]

g-2 : The theory budget

HVP

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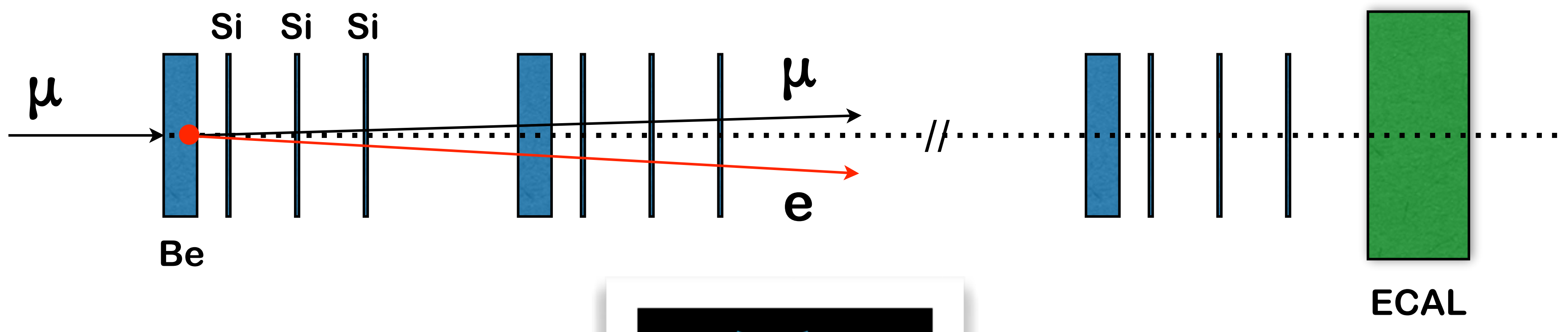
Vacuum polarization

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

g-2 : The theory budget

Light-by-light scattering

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
Experiment (E989 – Run I)	116592040(54)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
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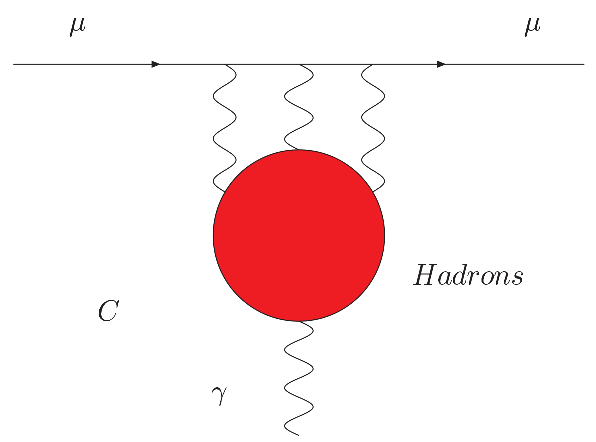
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

Light-by-light

- **Hadronic light-by-light at $\mathcal{O}(\alpha^3)$**
  - Significant improvements thanks to data-driven dispersive approach
    [Colangelo, Hoferichter, Procura, Stoffer '14-'17; Pauk, Vanderhaeghen '14]
  - Lattice: RBC: $82(35) \times 10^{-11}$ [1911.08123] Mainz: $110(15) \times 10^{-11}$ [2104.02632]

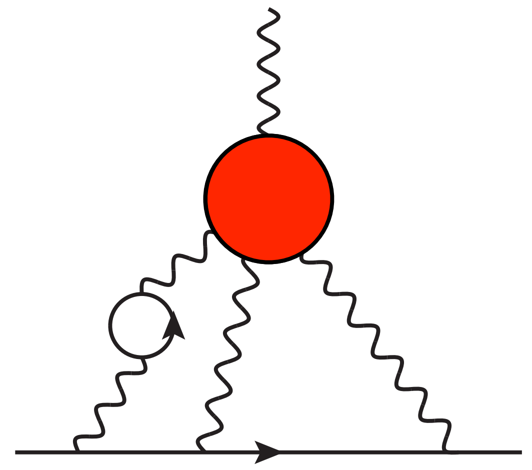
$a_\mu^{\text{HNLO}}(\text{lbl}) =$ 80 (40) x 10^{-11} Knecht & Nyffeler '02
= 136 (25) x 10^{-11} Melnikov & Vainshtein '03
= 105 (26) x 10^{-11} Prades, de Rafael, Vainshtein '09
= 100 (29) x 10^{-11} Jegerlehner, arXiv:1705.00263
= 92 (19) x 10^{-11} WP20 (phenomenology)



- **Hadronic light-by-light at $\mathcal{O}(\alpha^4)$**

$a_\mu^{\text{HNNLO}}(\text{lbl}) = 2\ (1) \times 10^{-11}$

[Colangelo, Hoferichter, Nyffeler, Passera, Stoffer '14]



g-2 : The theory budget

Δa_μ

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
Experiment (E989 – Run I)	116592040(54)
QED	116 584 718.931(104)
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HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

Difference

- **Status of the muon $a_\mu \equiv \frac{g_\mu - 2}{2}$ as of April 7^{th}**

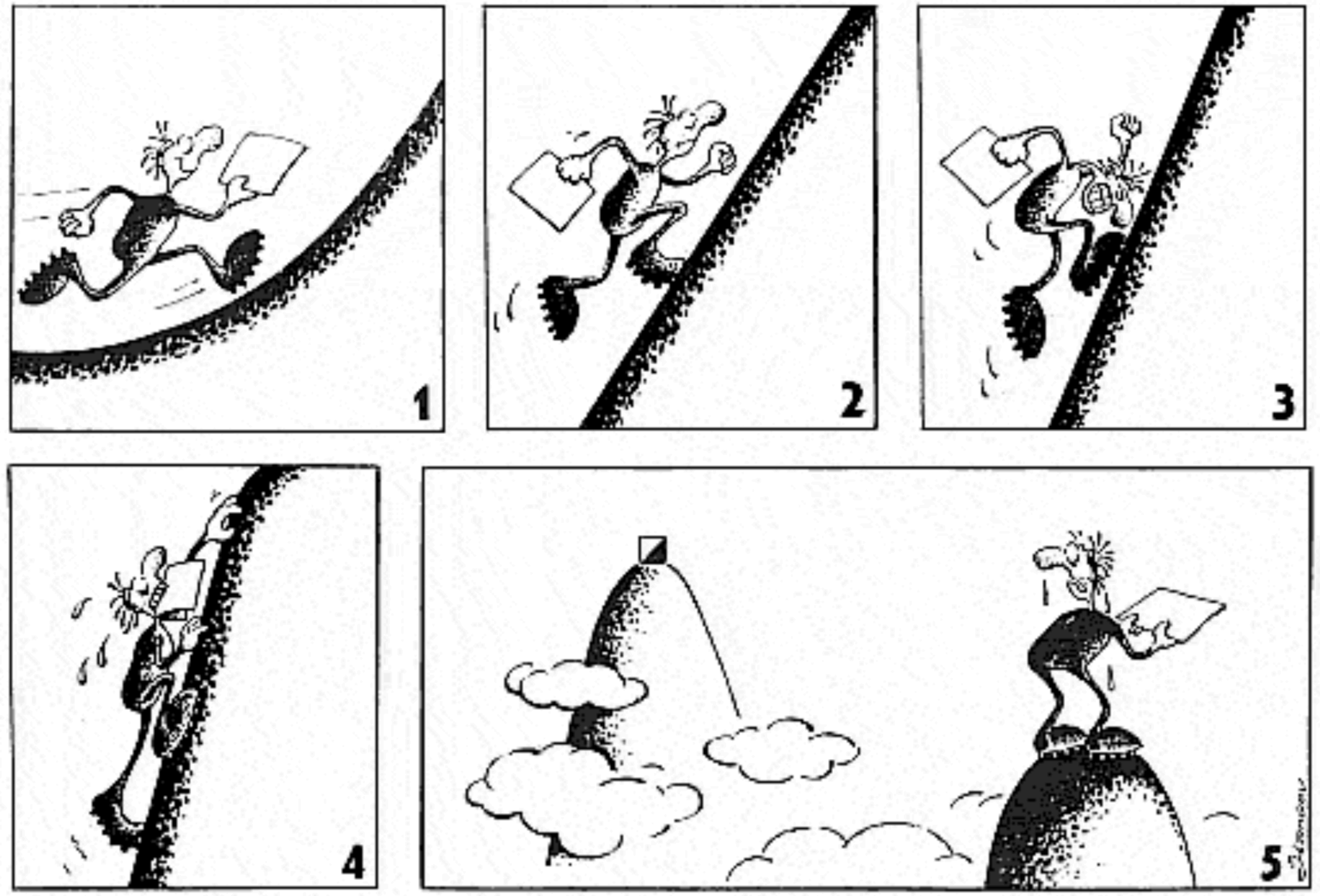
$$a_\mu^{\text{EXP}} = 116592061(41) \times 10^{-11} \text{ [BNL + FNAL]}$$
$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11} \text{ [WP20]}$$

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} \equiv a_\mu^{\text{NP}} = 251\,(59) \times 10^{-11} \qquad \textbf{(4.2}\sigma\textbf{ discrepancy!)}$$

$$\underbrace{(0.1)_{\text{QED}}\,,\quad (1)_{\text{EW}}\,,\quad (18)_{\text{HLbL}}\,,\quad (40)_{\text{HVP}}\,,}_{(43)_{\text{TH}}}\quad (41)_{\delta a_\mu^{\text{EXP}}}\,.$$

- Hadronic uncertainties (HLbL & HVP) are very hard to improve.
- $\delta a_\mu^{\text{EXP}} \approx 16 \times 10^{-11}$ by the E989 Muon $g-2$ exp. in a few years.

g-2 : New Physics
Strategies & general features



Strategies

Scales

- a_μ is a loop effect and just … one number:

$$\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} \equiv a_\mu^{\mathrm{NP}} = (251 \pm 59) \times 10^{-11}$$

- Two main strategies: top-down and bottom-up. In both cases need to correlate it with other searches/measurements/observations/constraints/anomalies.

Strategies
Scales

- a_μ is a loop effect and just … one number:

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (251 \pm 59) \times 10^{-11}$$

- Two main strategies: top-down and bottom-up. In both cases need to correlate it with other searches/measurements/observations/constraints/anomalies.

- For example:

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 200 \times 10^{-11}$$

WIMP miracle:

Obtaining the correct abundance of dark matter today via thermal production requires a self-annihilation cross section of

$$\boxed{\langle \sigma_{\chi\chi} v \rangle \approx \frac{g^4}{16\pi m_\chi^2} \stackrel{!}{=} \frac{1.7 \cdot 10^{-9}}{\mathrm{GeV}^2}}$$

$$\boxed{\Omega_\chi h^2 \approx 0.12 \, \frac{x_{\rm dec}}{23} \, \frac{\sqrt{g_{\rm eff}}}{10} \, \frac{2.04 \cdot 10^{-26}\mathrm{cm}^3/\mathrm{s}}{\langle \sigma_{\chi\chi} v \rangle}}$$

which is roughly what is expected for a new particle in the 100 GeV mass range that interacts via the electroweak force.

WIMP-(g-2) miracle squared!

Strategies

Scales

- a_μ is a loop effect and just … one number:

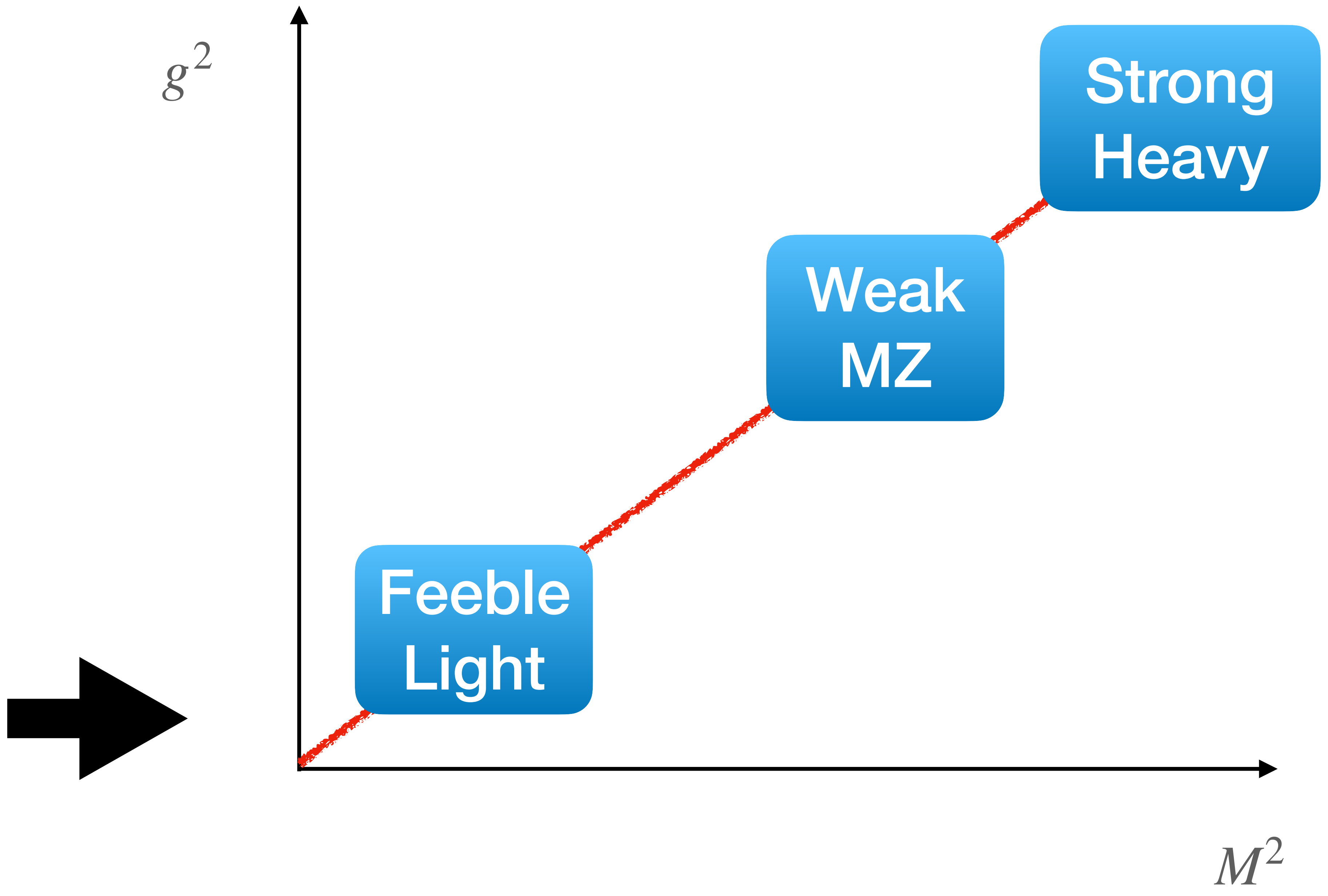
$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (251 \pm 59) \times 10^{-11}$$

- Two main strategies: top-down and bottom-up. In both cases need to correlate it with other searches/measurements/observations/constraints/anomalies.

- For example:

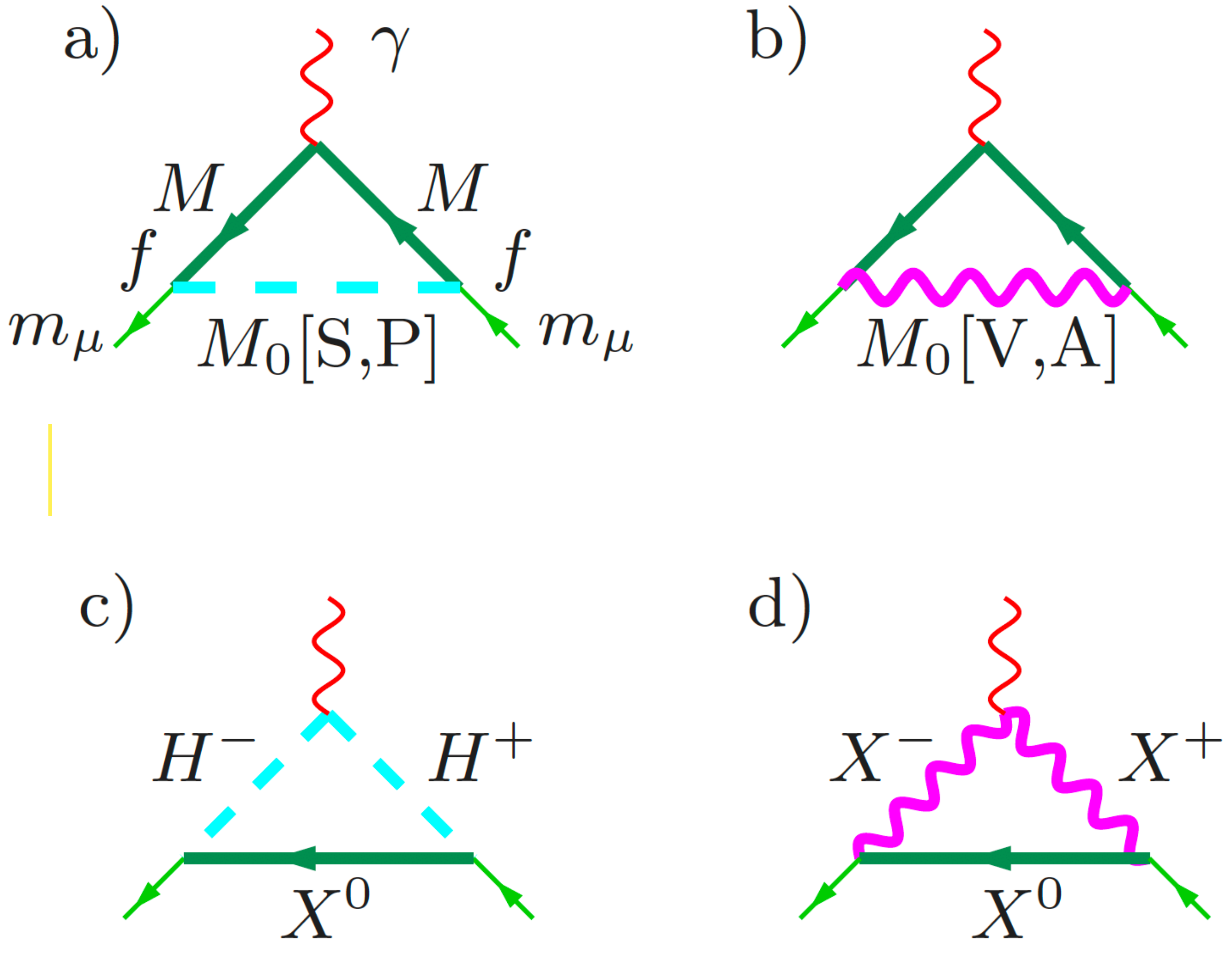
$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 200 \times 10^{-11}$$

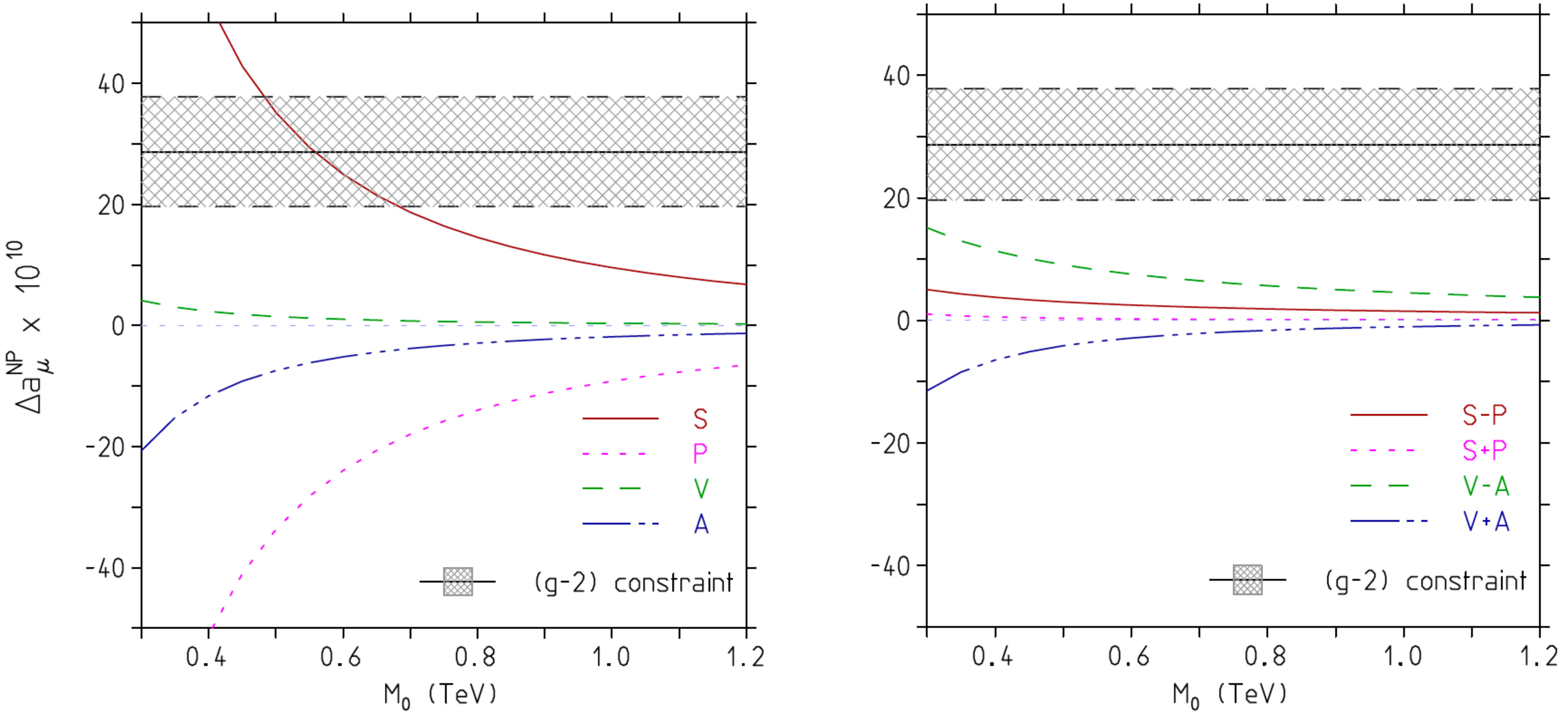
- More in general
  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.



General features

Cancellations





(a) Case: $m_\mu = M \ll M_0$

(b) Case: $m_\mu \ll M_0 = M$

(c) Case M₀=0 => $a_\mu^\phi = \frac{3y_\phi^2}{16\pi^2}$, $a_\mu^\pi = -\frac{y_\pi^2}{16\pi^2}$. $\Rightarrow a_\mu^S = -3a_\mu^P$ [Perez, 2021]

Very simple 1-loop formulas exist that can be used to estimate effects of any NP model.

General features

Naive vs. enhanced

Let us consider now some new particles with typical mass $\Lambda_{\rm NP}$ and couplings g^L_ℓ and g^R_ℓ to left- and right-handed leptons ℓ, respectively. The one-loop new-physics contribution to the amplitude $A_{\ell\ell'}$ is then of the form

$$A_{\ell\ell'} = \frac{1}{(4\pi\,\Lambda_{\rm NP})^2}\left[\left(g^L_{\ell k}\, g^{L*}_{\ell' k} + g^R_{\ell k}\, g^{R*}_{\ell' k}\right) f_1(x_k) + \frac{v}{m_\ell}\left(g^L_{\ell k}\, g^{R*}_{\ell' k}\right) f_2(x_k)\right],$$

and therefore

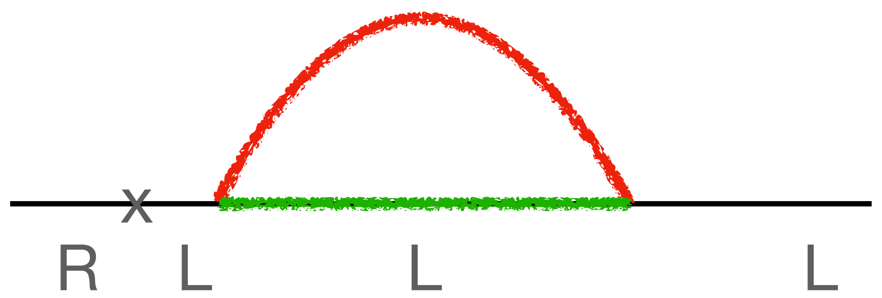
$$\Delta a_\ell = \frac{2m_\ell^2}{(4\pi\,\Lambda_{\rm NP})^2}\left[\left(|g^L_{\ell k}|^2 + |g^R_{\ell k}|^2\right) f_1(x_k) + \frac{v}{m_\ell}{\rm Re}\left(g^L_{\ell k} g^{R*}_{\ell k}\right) f_2(x_k)\right].$$

$$\frac{{\rm BR}(\ell \to \ell'\gamma)}{{\rm BR}(\ell \to \ell'\nu_\ell\bar\nu_{\ell'})} = \frac{48\pi\alpha G_F^{-2}}{(4\pi\Lambda_{\rm NP})^4}\left(\left|\left(g^L_{\ell k}\, g^{L*}_{\ell' k} + g^R_{\ell k}\, g^{R*}_{\ell' k}\right) f_1(x_k) + \frac{v}{m_\ell} g^L_{\ell k}\, g^{R*}_{\ell' k} f_2(x_k)\right|^2 + \ell \leftrightarrow \ell'\right),$$

$$\frac{d_\ell}{e} = \frac{v}{(4\pi\,\Lambda_{\rm NP})^2}\,{\rm Im}\left(g^L_{\ell k}\, g^{R*}_{\ell k}\right) f_2(x_k)\,.$$

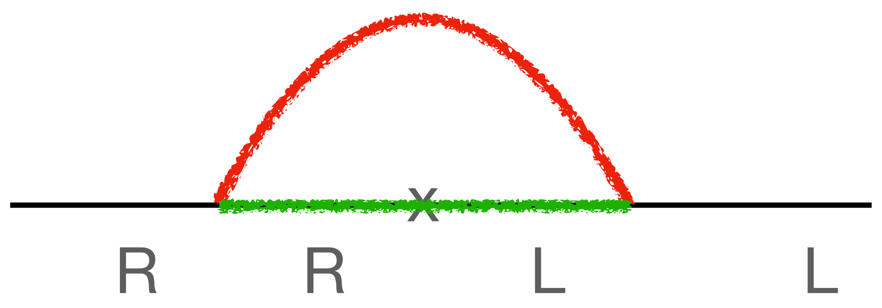
On general grounds, one would expect that, in concrete NP scenarios, Δa_ℓ, d_ℓ and ${\rm BR}(\ell \to \ell'\gamma)$, are correlated. In practice, their correlations depend on the unknown flavor and CP structure of the couplings g^L and g^R, and thus we cannot draw any firm conclusion. In the following, we will point out the general conditions that have to be fulfilled by any NP theory in order to account for large effects in Δa_ℓ while satisfying the constraints from d_ℓ and ${\rm BR}(\ell \to \ell'\gamma)$.

f_1



"normal" -> naive

f_2



"enhanced"

With $f_{1,2}$ we indicate loop functions which depend on ratios (x_k) of unknown masses of the new particles contributing to the amplitude $\ell \to \ell'\gamma$, and k is a lepton flavor index. In the term proportional to f_1, the chiral flip required by the dipole transition occurs through a mass insertion in the external lepton line. In the term proportional to f_2, the mass insertion is in the internal line of some new particle, thus explaining the parametric factor v/m_ℓ. Although f_2 must be proportional to the lepton Yukawa coupling, as a consequence of chiral symmetry, in practice this term can become very sizeable whenever a new large coupling leads to a chiral enhancement.

General features

Naive scaling

In a broad class of theories beyond the SM, $g_\ell^{L,R}$ and f_1 are flavor universal (*i.e.* are the same for any ℓ) and f_2 vanishes, such that

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}.$$

NAIVE SCALING

$$\frac{m_e^2}{m_\mu^2} = 2.5 \cdot 10^{-5} \qquad \leftarrow \mu \rightarrow \qquad \frac{m_\tau^2}{m_\mu^2} = 2.8 \cdot 10^3$$

General features
LFC and LFV

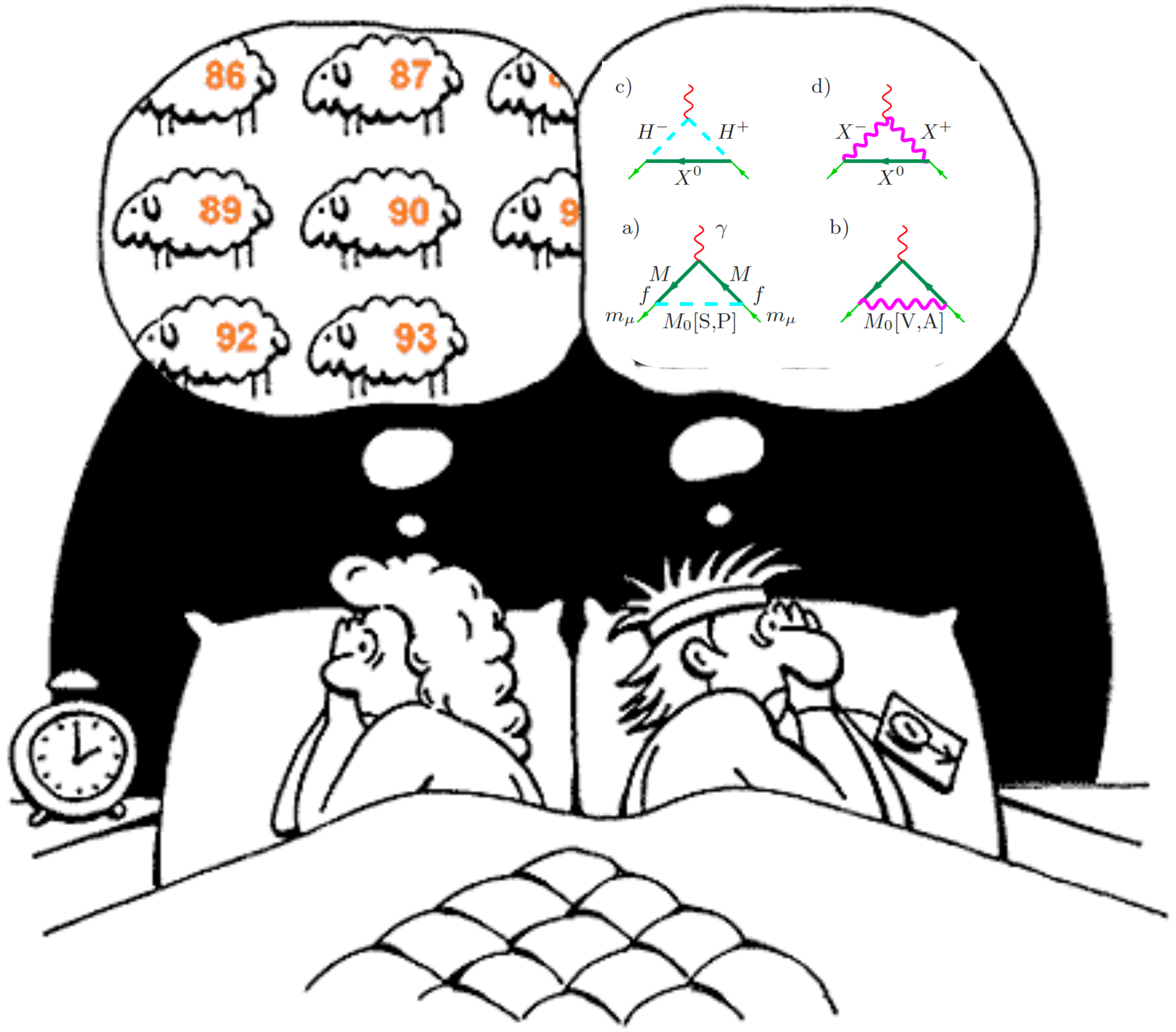
Two classes of models, Flavor conserving and flavor violating. For example in SUSY:

1. Lepton flavor conserving (LFC) case. The charged slepton mass matrix violates the global non-abelian flavor symmetry, but preserves $U(1)^3$. This case is characterized by non-degenerate sleptons ($m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$) but vanishing mixing angles because of an exact alignment, which ensures that Yukawa couplings and the slepton mass matrix can be simultaneously diagonalized in the same basis.

2. Lepton flavor violating (LFV) case. The slepton mass matrix fully breaks flavor symmetry up to U(1) lepton number, generating mixing angles that allow for flavor transitions. Lepton flavor violating processes, such as $\mu \to e\gamma$, provide stringent constraints on this case. However, because of flavor transitions, a_e and a_μ can receive new large contributions proportional to m_τ (from a chiral flip in the internal line of the loop diagram), giving a new source of non-naive scaling.

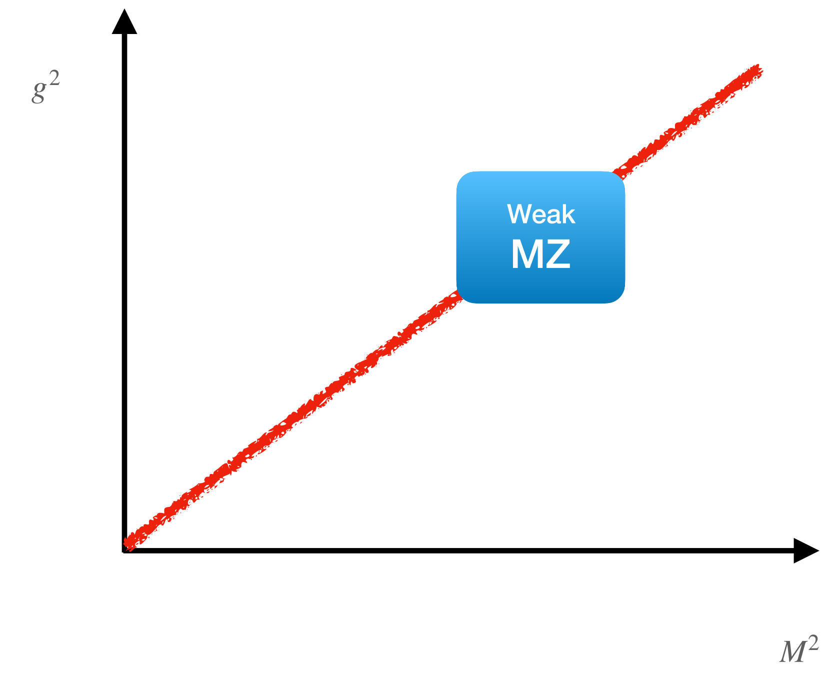
g-2 : New Physics

Examples



SUSY

LFC and LFV



$$\Delta a_\ell^{\rm LFC} = \frac{5\alpha_2}{48\pi}\frac{m_\ell^2}{m_{\tilde\ell}^2}\tan\beta + \frac{\alpha_Y}{24\pi}\frac{m_\ell^2}{m_{\tilde\ell}^3}A_\ell$$

$$\approx 3\times10^{-9}\left(\frac{m_\ell}{m_\mu}\right)^2\left(\frac{100\ {\rm GeV}}{m_{\tilde\ell}}\right)^2\left[\left(\frac{\tan\beta}{3}\right)+0.1\left(\frac{A_\ell}{3m_{\tilde\ell}}\right)\right].$$

$$\Delta a_e \approx \Delta a_\mu \frac{m_e^2}{m_\mu^2}\frac{m_{\tilde\mu}^2}{m_{\tilde e}^2} \approx \frac{m_{\tilde\mu}^2}{m_{\tilde e}^2}\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)10^{-13},$$

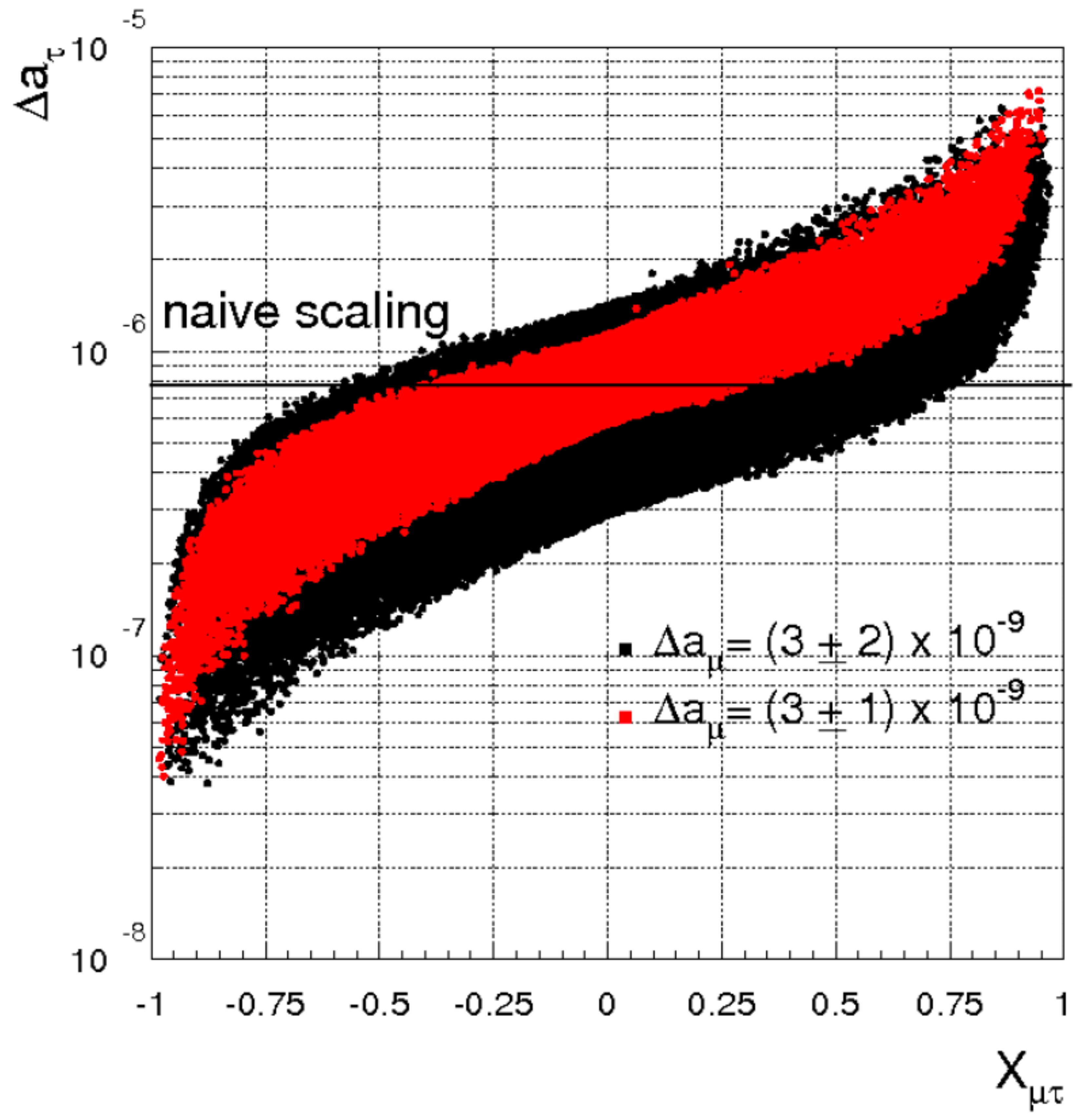
$$\Delta a_\tau \approx \Delta a_\mu \frac{m_\tau^2}{m_\mu^2}\frac{m_{\tilde\mu}^2}{m_{\tilde\tau}^2} \approx \frac{m_{\tilde\mu}^2}{m_{\tilde\tau}^2}\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)10^{-6}.$$

Higgs enhanced

$$\Delta a_\ell^{\rm LFV} \simeq \frac{\alpha_1}{\pi}\left(\frac{m_\ell m_\tau}{m_{\tilde\ell}^2}\right)\tan\beta\,\frac{{\rm Re}\left(\mu M_1 \delta_{RR}^{\ell\tau}\delta_{LL}^{\tau\ell}\right)}{m_{\tilde\ell}^2}\, l_n(x_1) \qquad \ell = e,\mu\,.$$

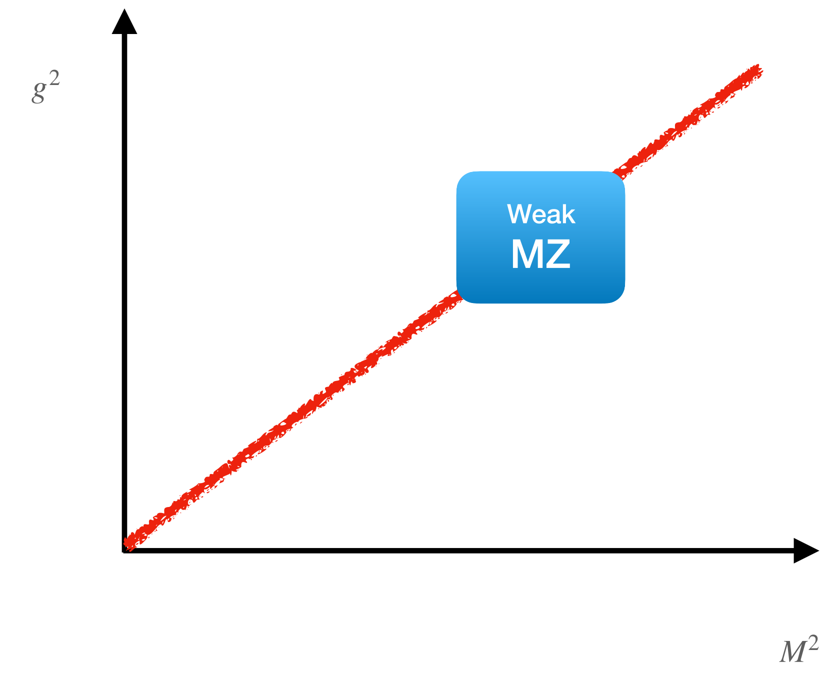
$$\Delta a_\ell^{\rm LFV} \approx 5\times10^{-13}\left(\frac{m_\ell}{m_e}\right)\left(\frac{\tan\beta}{30}\right)\left(\frac{2\,{\rm TeV}}{m_{\tilde\ell}}\right)^2\left(\frac{\mu/m_{\tilde\ell}}{2}\right)\left(\frac{\delta_{RR}^{\ell\tau}}{0.5}\right)\left(\frac{\delta_{LL}^{\tau\ell}}{0.5}\right)$$

f2 enhanced v/ml



SUSY

g-2 vs LHC



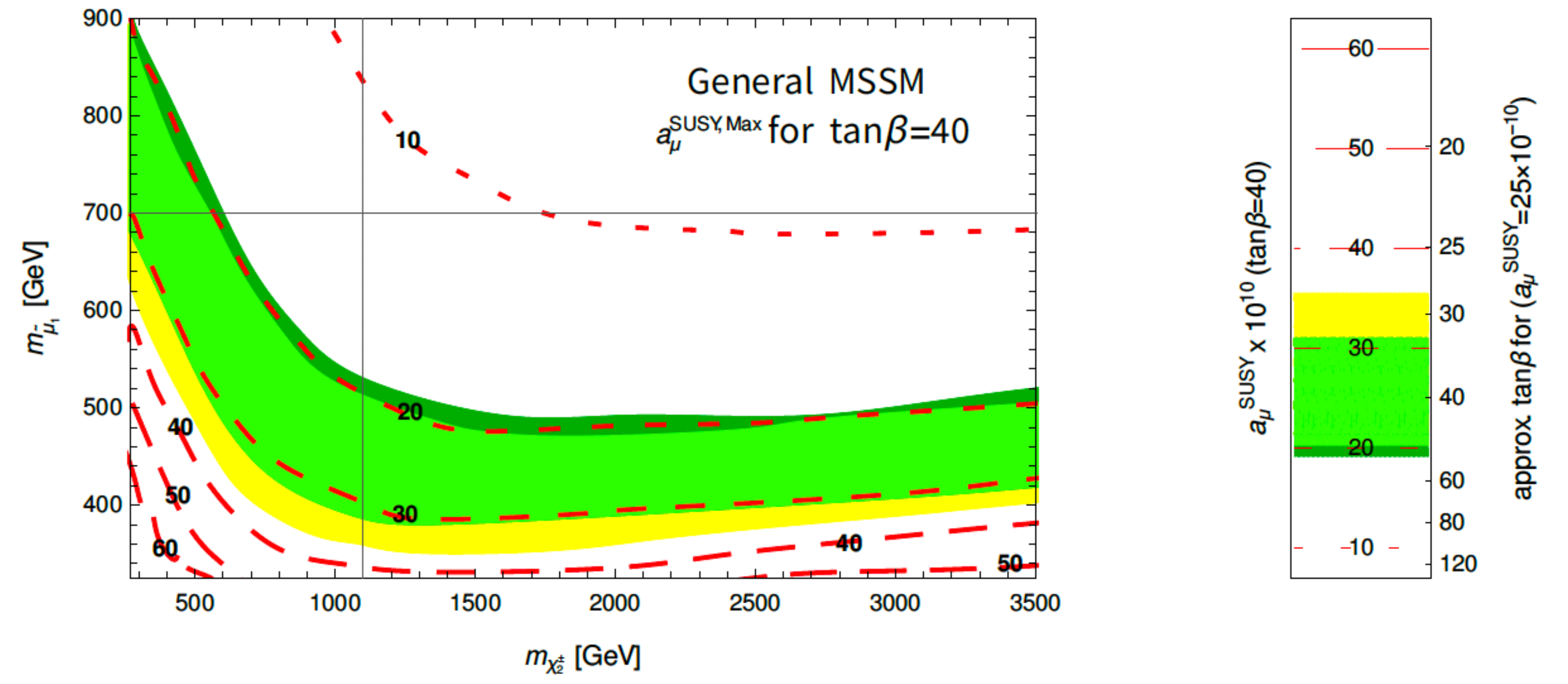
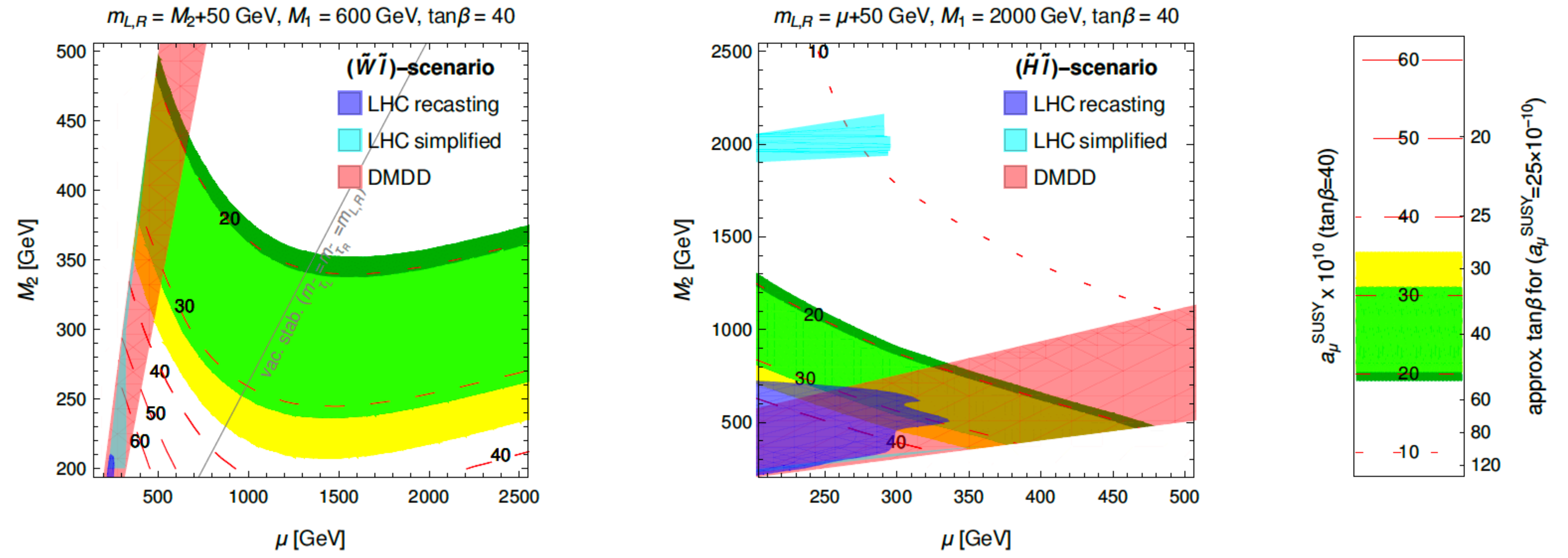


Figure 15: The theoretical maximum MSSM contribution $a_\mu^{\rm SUSY,Max}$ for $\tan\beta = 40$ in the plane of the heaviest chargino and the lightest smuon mass. (For each point in the plane, the actual value of the MSSM contribution can take any value between 0 and $\pm a_\mu^{\rm SUSY,Max}$, depending on the signs of parameters and details such as other masses and mixings.) The yellow/green coloured regions show where $a_\mu^{\rm SUSY,Max}$ (for $\tan\beta = 40$) is within the 1σ bands corresponding to the BNL and new deviations $\Delta a_\mu^{\rm BNL}$ and Δa_μ^{2021}, see Eqs. (6,7), and their overlap. The red dashed contour lines can be interpreted in two ways. Firstly, they directly correspond to certain values of $a_\mu^{\rm SUSY,Max}$ for $\tan\beta = 40$, as indicated in the left axis of the legend plot. Secondly, thanks to the approximate linearity in $\tan\beta$, each contour can be used to estimate the required $\tan\beta$ value for which $a_\mu^{\rm SUSY,Max}$ agrees with the deviation Δa_μ^{2021} (keeping other input parameters fixed). These $\tan\beta$ values can be read off from the right axis of the legend plot (the values are approximate since the linearity is not exact). As an example of the reinterpretation we take the point $m_{\chi_2^\pm} = 1750$ GeV and $m_{\tilde\mu_1} = 700$ GeV. For $\tan\beta = 40$ we get $a_\mu^{\rm SUSY,Max} = 10 \times 10^{-10}$. The required $\tan\beta$ value to get Δa_μ^{2021} would be around 100, as read off from the right axis. The results for $a_\mu^{\rm SUSY,Max}$ were obtained from a scan using GM2Calc [45] in which all relevant SUSY masses are varied independently between 100 GeV and 4 TeV. The black lines indicate the maximum LHC reach for charginos and sleptons of 1100 and 700 GeV reported in Refs. [473, 476], respectively.



(a) (b)

Figure 17: (a) $(\tilde W\tilde l)$-scenario. (b) $(\tilde H\tilde l)$-scenario. For parameter values see the plots and the text. The red dashed contours correspond to values of $a_\mu^{\rm SUSY}$ as indicated in the legend on the right; the yellow/green coloured regions correspond to the 1σ bands corresponding to the BNL deviation (6) and the new deviation including FNAL (7), and their overlap. For the $\tan\beta$-reinterpretation see caption of Fig. 15. The red shaded region is excluded by dark matter direct detection if the LSP is assumed stable; the blue shaded regions correspond to the limits from the LHC recasting, see Fig. 20 for details. The cyan shaded region corresponds to the additional LHC limits implemented in a simplified way; in both plots the slepton search (59), Ref. [476] excludes a narrow strip at small μ and M_2, where the slepton–LSP mass splitting is largest. In the the right plot the compressed-mass searches of Ref. [514] exclude another small region at large M_2, which enters the LSP mass via mixing. The thin solid gray line corresponds to the vacuum stability constraint of Ref. [176]; it applies in case the left- and right-handed stau-masses are set equal to the smuon/selectron masses and excludes the points to its right, i.e. with larger μ.

Very extensive review of the SUSY scenarios available: [Athron et al, 2021]

g-2 of the electron

Naive scaling

- **Testing the muon $g-2$ anomaly through the electron $g-2$**

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \qquad \Longleftrightarrow \qquad \Delta a_e = \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 0.7\times 10^{-13}$$

  - a_e has never played a role in testing NP effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which was is the most precise value of α up to 2018!
  - The situation has now changed thanks to th. and exp. progresses.
  - α can be extracted from atomic physics and a_e used to perform NP tests!

- **Status of Δa_e as of 2012**

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM} = -9.2\,(8.1)\times 10^{-13},$$
$$\delta a_e \times 10^{13}: \quad (0.6)_{\rm QED4},\ (0.4)_{\rm QED5},\ (0.2)_{\rm HAD},\ (7.6)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

  - The errors from QED4 and QED5 will be reduced soon to 0.1×10^{-13} [Kinoshita]
  - We expect a reduction of $\delta a_e^{\rm EXP}$ to a part in 10^{-13} (or better). [Gabrielse]
  - Work is also in progress for a significant reduction of $\delta\alpha$. [Nez]

- **Status of Δa_e as of 2018: 2.4σ discrepancy** [Parker et al., Science, '18]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm Berkeley}) = -8.8\,(3.6)\times 10^{-13}$$
$$\delta a_e \times 10^{13}: \quad (0.1)_{\rm QED5},\ (0.1)_{\rm HAD},\ (2.3)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

- **Status of Δa_e as of 2020: 1.6σ discrepancy** [Morel et al., Nature, '20]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm LKB2020}) = 4.8\,(3.0)\times 10^{-13}$$
$$\delta a_e \times 10^{13}: \quad (0.1)_{\rm QED5},\ (0.1)_{\rm HAD},\ (0.9)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

- **$\Delta a_e \lesssim 10^{-13}$ is not too far! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.** [Giudice, P.P, & Passera, '12]

g-2 of the tau

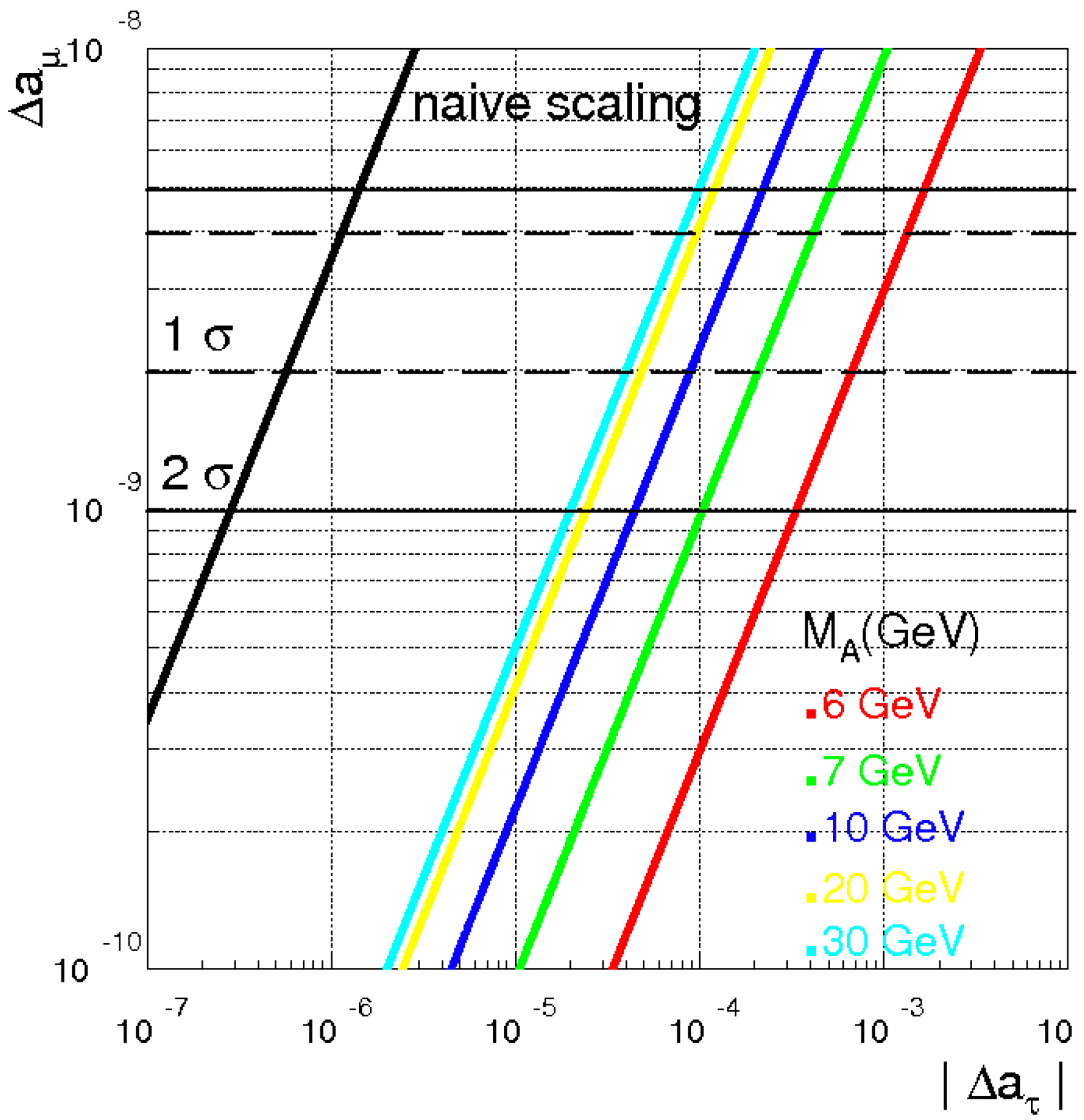
Naive vs enhancement with taus

The present experimental sensitivity of the τ lepton $g{-}2$ is only 10^{-2}. In fact, while the SM prediction for a_τ is precisely known [38],

$$a_\tau^{\rm SM} = 117\,721\,(5) \times 10^{-8}, \tag{33}$$

the very short lifetime of this lepton (2.9×10^{-13} s) makes it very difficult to determine its anomalous magnetic moment by measuring its spin precession in a magnetic field, like in the electron and muon $g{-}2$ experiments. Instead, experiments focused on high-precision measurements of τ pair production in various high-energy processes and comparison of the measured cross sections with the SM predictions.

If an NP contribution were of the same order of magnitude as that of the EW, from a purely theoretical point of view, the $g-2$ of the τ would provide a much cleaner test of the presence (or absence) of such NP effects than the muon one.



The present PDG limit on the τ $g{-}2$ was derived by the DELPHI collaboration from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross section measurements at LEP2: $-0.052 < a_\tau^{\rm EXP} < 0.013$ at 95% confidence level [39].

Naive scaling at work

LFV decays and EDMs

- $\mathrm{BR}(\ell_i \to \ell_j\gamma)$ vs. $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3\times 10^{-13}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4\times 10^{-8}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- EDMs vs. $(g-2)_\mu$

$$d_e \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 10^{-29}\left(\frac{\phi_e^{CPV}}{10^{-5}}\right) e\,\mathrm{cm}\,,$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\,\phi_\mu^{CPV}\; e\,\mathrm{cm}\,.$$

- Main messages:
  - $\Delta a_\mu \approx (3\pm 1)\times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ are still allowed!**

Comments:

LFV contributions to Δa_ℓ can be particularly important as they are typically chirally enhanced by m_τ/m_ℓ, as we will discuss in the case of supersymmetry. Such a chiral enhancement is not effective in τ LFV processes and therefore, in this case, it might be possible to keep $\mathrm{BR}(\tau \to \ell\gamma)$ under control, while generating large effects especially for Δa_e.

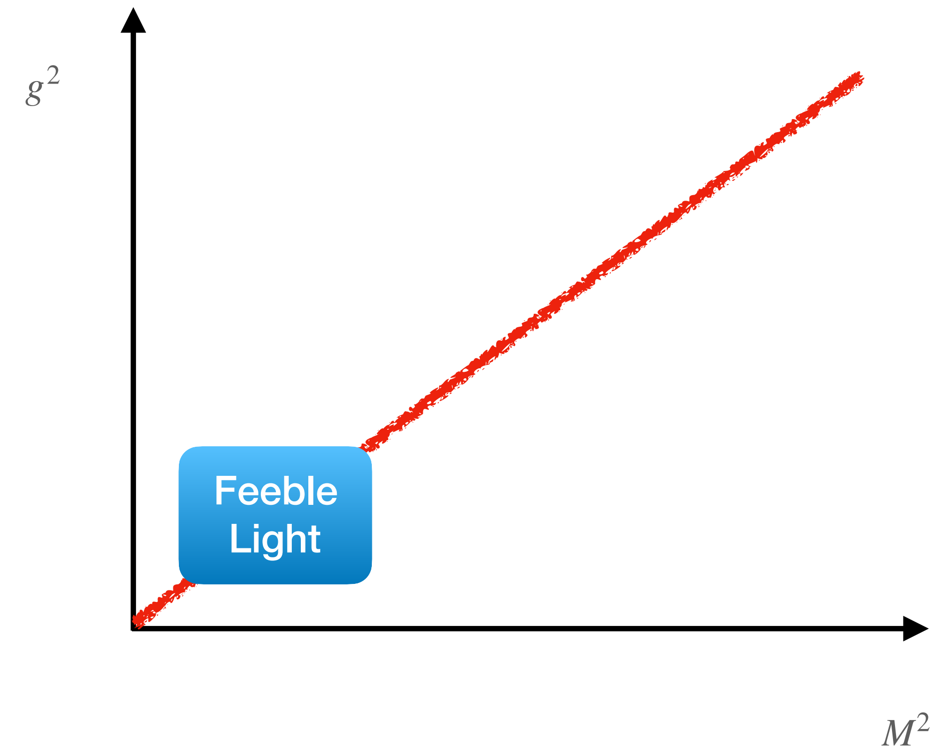
From eq. (31) we learn that an explanation of the Δa_μ anomaly implies, for a natural CPV phase $\phi_\mu \sim \mathcal{O}(1)$, a *model-independent* upper bound on $d_\mu \lesssim 3\times 10^{-22}\, e$ cm which is still far from the current bound $d_\mu \lesssim 10^{-18}\, e$ cm, but well within the expected future sensitivity $d_\mu < 10^{-24}\, e$ cm [37]. Therefore, any experimental effort to improve the resolution on d_μ would be valuable.

On the other hand, the electron EDM imposes a bound on the corresponding CPV phase ϕ_e at the level of 10^{-3}, if NS is at work. Such a condition could be realized for instance if ϕ_e is generated radiatively while ϕ_μ arises already at the tree level. Going beyond NS, one could also envisage scenarios where the electronic dipoles are suppressed compared to the muonic dipoles because of hierarchical couplings $g_e^{L,R} \ll g_\mu^{L,R}$, as it happens for instance in a multi-Higgs doublet model where $g_\ell^{L,R}$ are related to Yukawa couplings. In general, as shown by eq. (30), the EDMs (but not Δa_μ) vanish if $g^L = g^R$ as it could arise in a left-right symmetric theory.

The tilt gives rise to an oscillating vertical component of the muon polarization and may be detected by recording separately the electrons which strike the counters above and below the mid–plane of the ring. This measurement has been performed in the last CERN experiment on $g-2$. The result $d_\mu = (3.7\pm 3.4)\times 10^{-19}\, e\cdot$ cm showed that it is negligibly small. The present experimental bound is $d_\mu < 2.7\times 10^{-19}\, e\cdot$ cm while the SM estimate is $d_\mu \sim 3.2\times 10^{-25}\, e\cdot$ cm. One thus may safely assume d_μ to be too small to be able to affect the extraction of a_μ.

Feeble and light

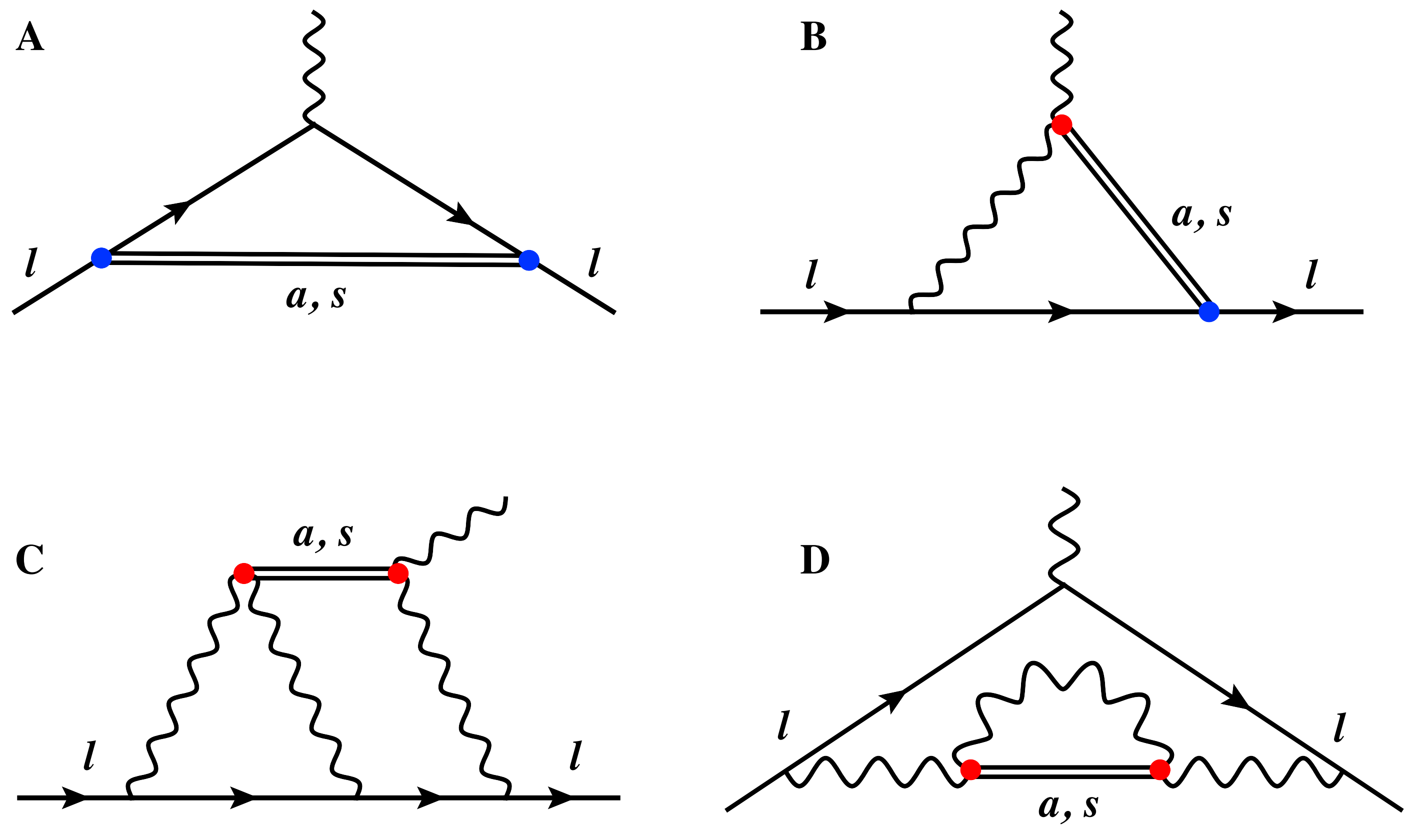
ALPS



Let us consider an axion-like particle:

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2} \frac{\partial^\nu a}{\Lambda} \bar{\mu} \gamma_\nu \gamma_5 \mu$$

A pseudo-scalar in the vertex correction A) gives a negative contributions to Δa_μ. However, the B) and two loop contributions can be positive:



$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \left[\frac{12\alpha^3}{\pi} C_{\gamma\gamma}^2 \ln^2 \frac{\Lambda^2}{m_\mu^2} - \frac{(c_{\mu\mu})^2}{16\pi^2} h_1\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi} c_{\mu\mu} C_{\gamma\gamma} \ln \frac{\Lambda^2}{m_\mu^2} \right]$$

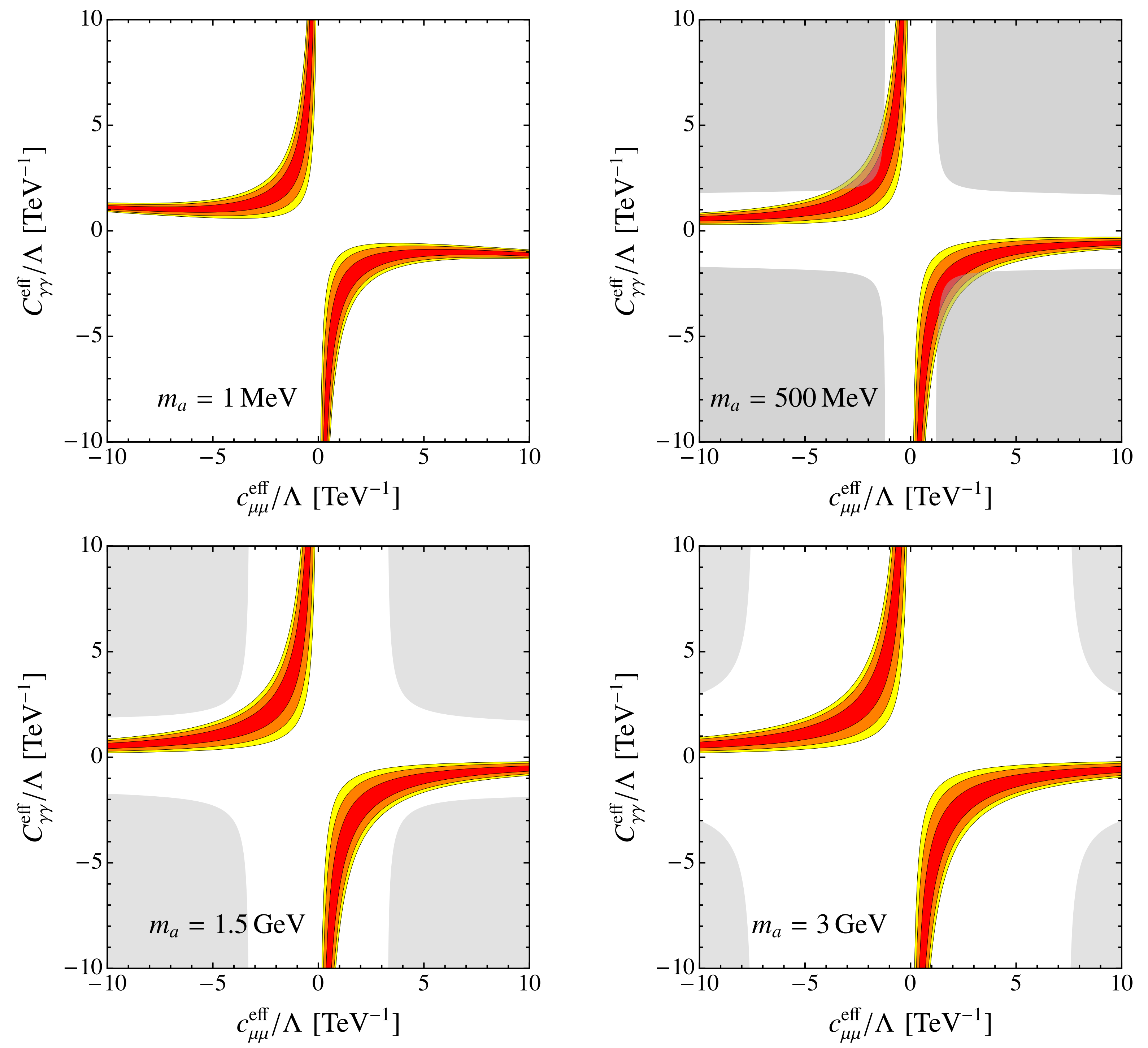
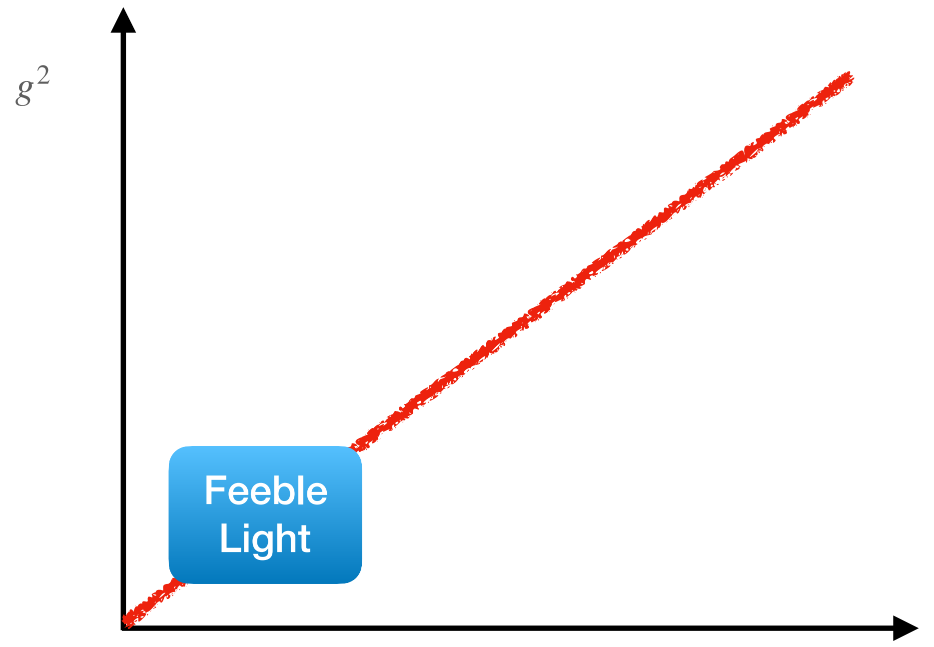


Figure: Δa_μ regions favoured at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by the BaBar search $e^+e^- \to \mu^+\mu^- + \mu^+\mu^-$ [Bauer, Neubert, Thamm, '17]

Feeble and light

Hidden photons



$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4}X_{\alpha\beta}X^{\alpha\beta} - \frac{\epsilon_Y}{2}B_{\alpha\beta}X^{\alpha\beta} - g_x\, j^X_\alpha\, X^\alpha - \frac{M_X^2}{2}X_\alpha X^\alpha$$

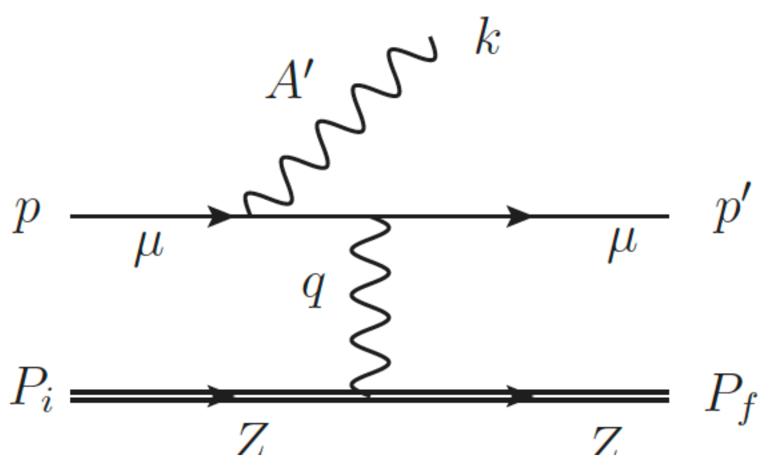
$$j^{\mu-\tau}_\alpha = \bar{L}_2\gamma_\alpha L_2 + \bar{\mu}_R\gamma_\alpha\mu_R - \bar{L}_3\gamma_\alpha L_3 - \bar{\tau}_R\gamma_\alpha\tau_R\,,$$

$$j^\mu_\alpha = \bar{L}_2\gamma_\alpha L_2 + \bar{\mu}_R\gamma_\alpha\mu_R + \sum_\psi Q_\psi\bar{\psi}\gamma_\alpha\psi\,,$$

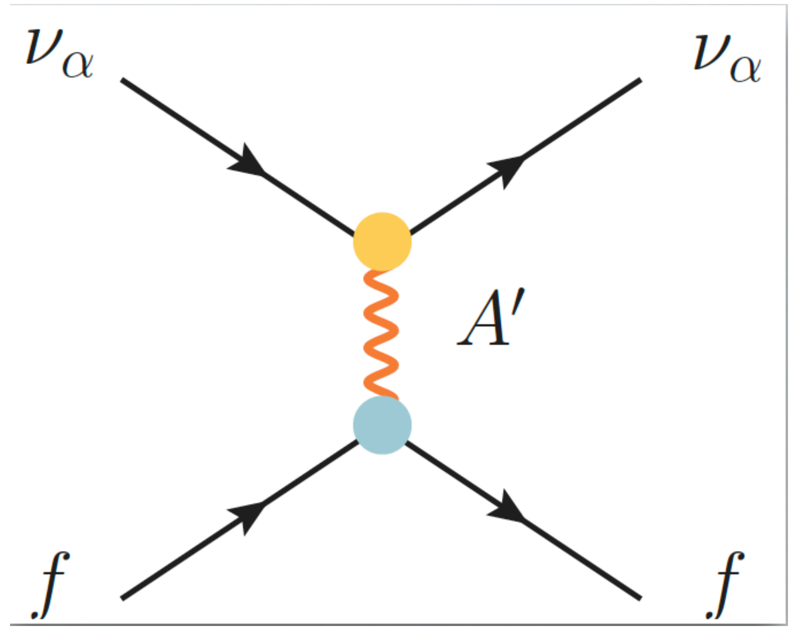
$$\epsilon_{\mu\tau} \approx \frac{e\,g_{\mu\tau}}{6\pi^2}\log\left(\frac{m_\mu}{m_\tau}\right) \approx -\frac{g_{\mu\tau}}{70}\,,$$

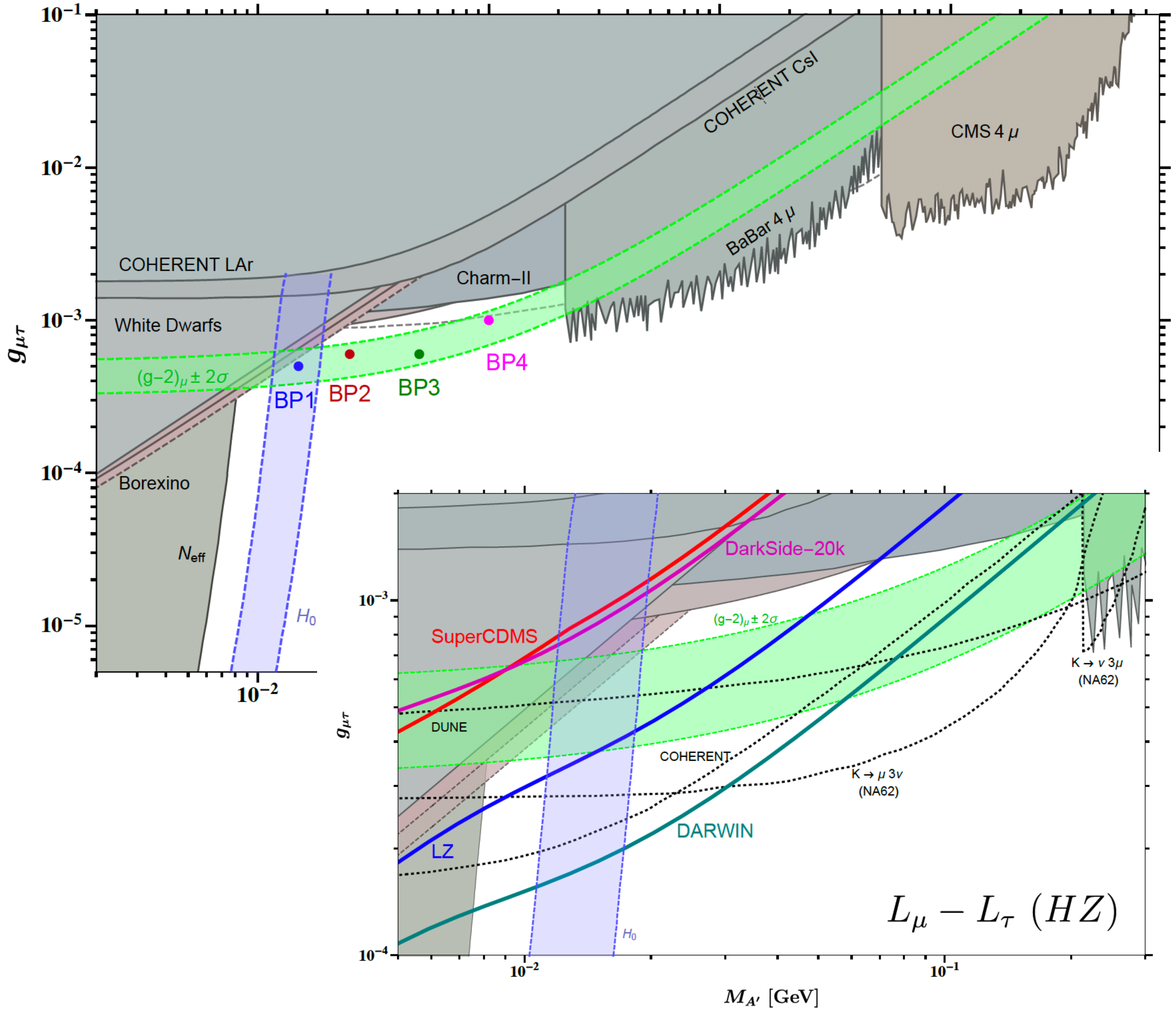
$$\epsilon_\mu \approx \frac{e\,g_\mu}{6\pi^2}\log\left(\frac{m_\mu}{M_{\rm NP}}\right) \approx -Log\ g_\mu\,,$$

$$\Delta a_\mu = Q_\mu^{x^2}\,\frac{\alpha_x}{\pi}\int_0^1 {\rm d}u\,\frac{u^2(1-u)}{u^2+\frac{(1-u)}{x_\mu^2}}$$



NA64mu

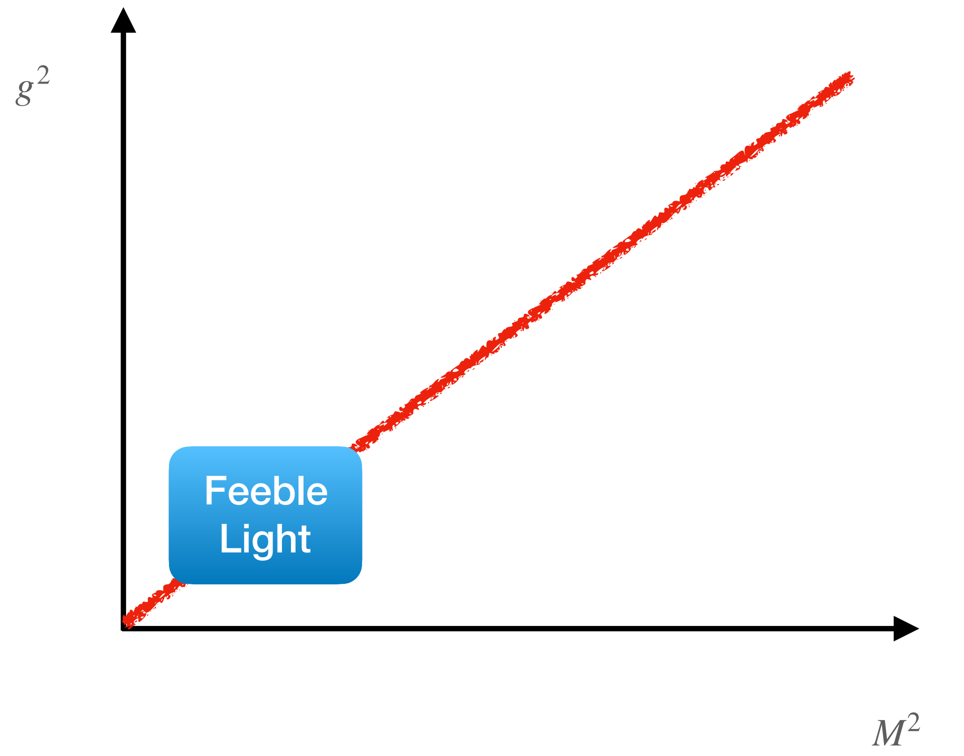




[Amaral & al, 2021]

Feeble and light

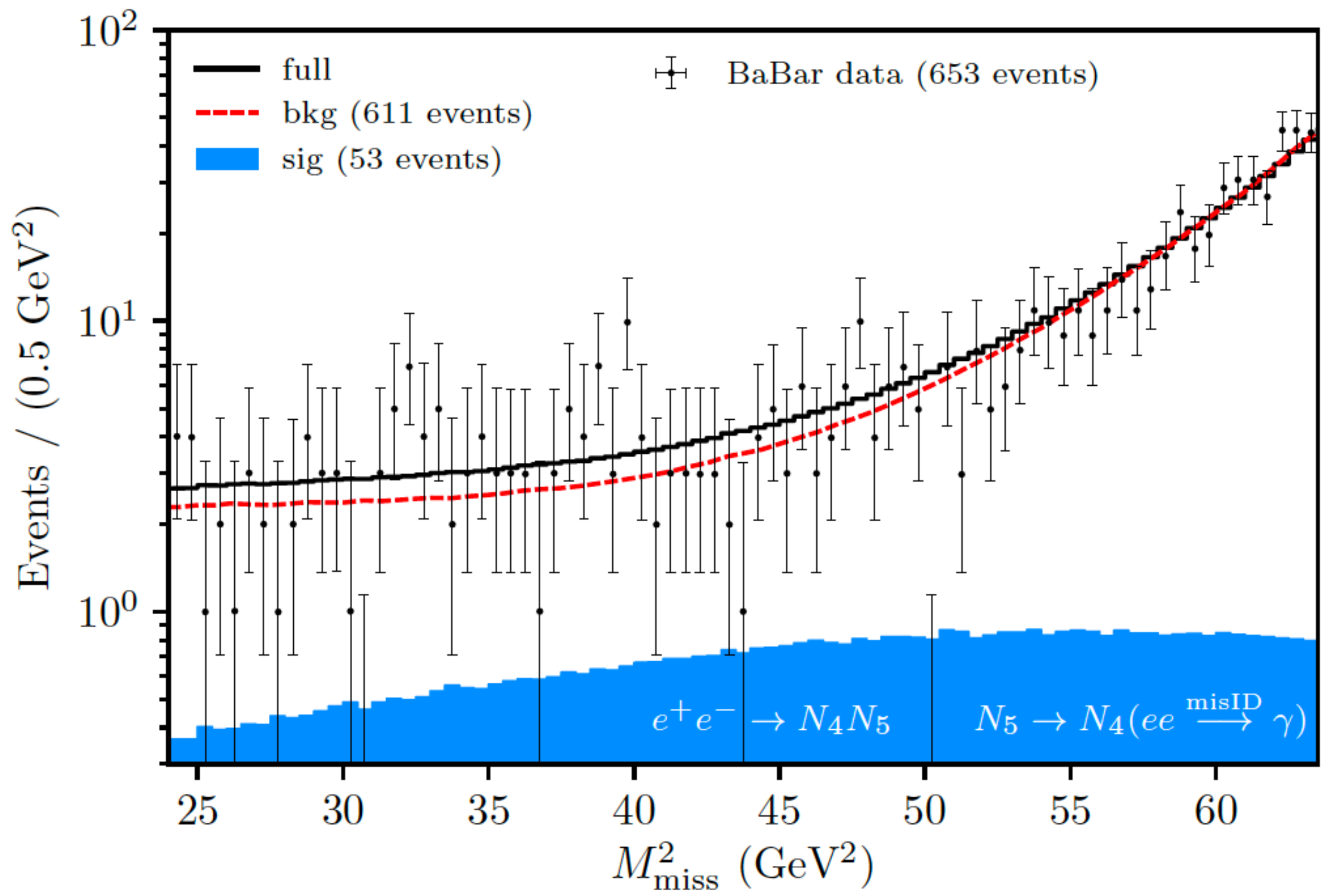
Hidden photons & miniBooNE anomaly



$$\mathcal{L} \supset \mathcal{L}_{\rm SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\sin\chi}{2} X_{\mu\nu} B^{\mu\nu} \qquad (1)$$
$$+ (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) - \lambda_{\Phi H} |H|^2 |\Phi|^2$$
$$+ \overline{\widehat{\nu}_N} i \partial\!\!\!/ \widehat{\nu}_N + \overline{\widehat{\nu}_D} i D\!\!\!\!/_X \widehat{\nu}_D - \Bigg[(\overline{L}\widetilde{H}) Y \widehat{\nu}_N^c + \frac{1}{2} \overline{\widehat{\nu}_N} M_N \widehat{\nu}_N^c$$
$$+ \overline{\widehat{\nu}_N} \left(Y_L \widehat{\nu}_{D_L}^c \Phi + Y_R \widehat{\nu}_{D_R} \Phi^* \right) + \overline{\widehat{\nu}_D} M_X \widehat{\nu}_D + \text{h.c.} \Bigg],$$

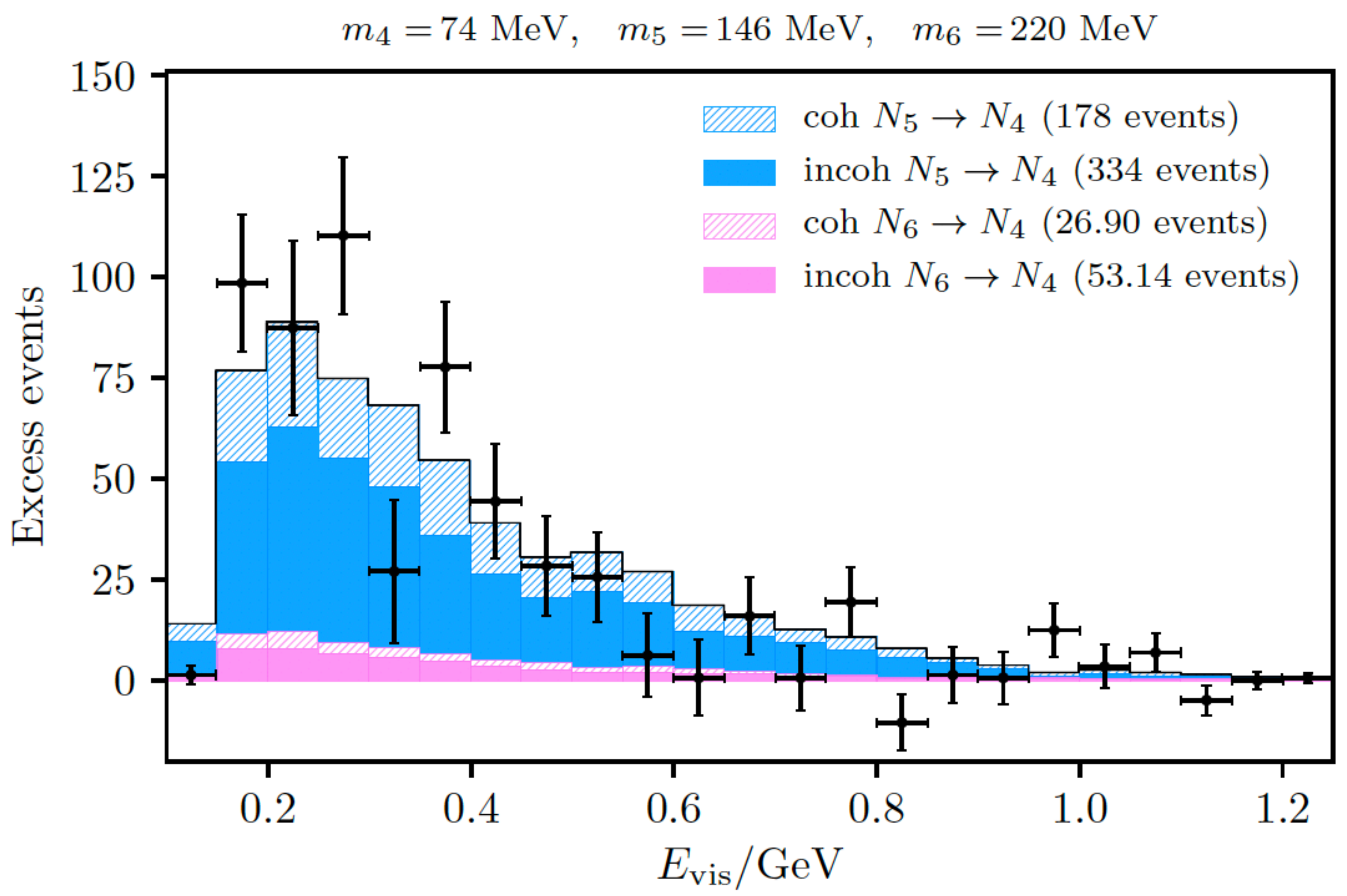
$$\mathcal{L}_{\nu-\text{mass}} = \frac{1}{2} \overline{\widehat{\nu}_f^c} \begin{pmatrix} 0 & M_D & 0 & 0 \\ M_D^T & M_N & \Lambda_L & \Lambda_R \\ 0 & \Lambda_L^T & 0 & M_X \\ 0 & \Lambda_R^T & M_X^T & 0 \end{pmatrix} \widehat{\nu}_f + \text{ h.c.},$$

BP	MB	Δa_μ	BB	Acc	α_D	m_3 /eV	m_4 /MeV	m_5 /MeV	m_6 /MeV	$\lvert V_{43}\rvert^2$ /10^{-8}	$\lvert V_{53}\rvert^2$ /10^{-8}	$\lvert V_{63}\rvert^2$ /10^{-8}	$\mathcal{B}(Z' \to N_j N_k)$/% 44	45	46	55	56	66	$c\tau^0$/cm N_4	N_5	N_6
A	✓	✓	✓	(✓)	0.39	0.05	35	120	185	0	22.2	0	0	5.4	0	0	95	0	1.6×10^{13}	3.0	0.26
B	✓	✓	✓	✓	0.32	0.05	74	146	220	13.6	26.5	123	0.15	11	0.48	1.6	86	0.59	1.1×10^{7}	2.2	0.14



Pseudo-monophotons at BaBar – The dominant production of dark particles in e^+e^- colliders is s-channel pair production of HNLs due to the large values of $\alpha_D \varepsilon^2$. In particular, the process

$$e^+e^- \to Z'^*(\text{or } \Upsilon(nS)) \to N_4(N_5 \to N_4 e^+ e^-), \qquad (4)$$



MiniBooNE excess – MiniBooNE is a mineral oil Cherenkov detector in a predominantly ν_μ beam with $\langle E_\nu \rangle \simeq 800$ MeV. Recent results with improved background analysis and larger statistics [78] report an excess of 560.6 ± 119.6 (77.4 ± 28.5) e-like events in ν $(\overline{\nu})$ mode.

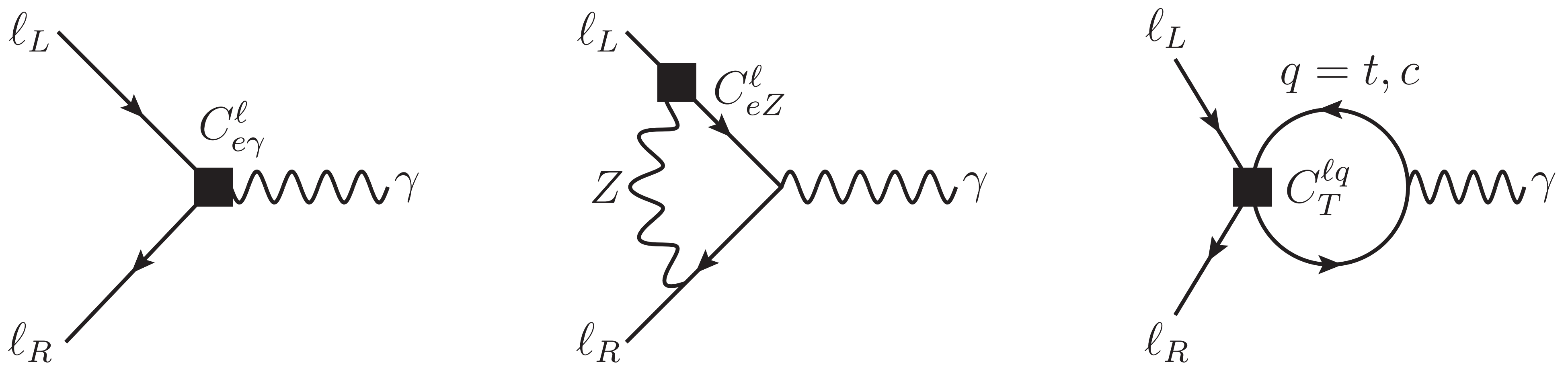
We propose that the MiniBooNE excess arises from the decay products of HNLs produced in ν_μ upscattering inside the detector,

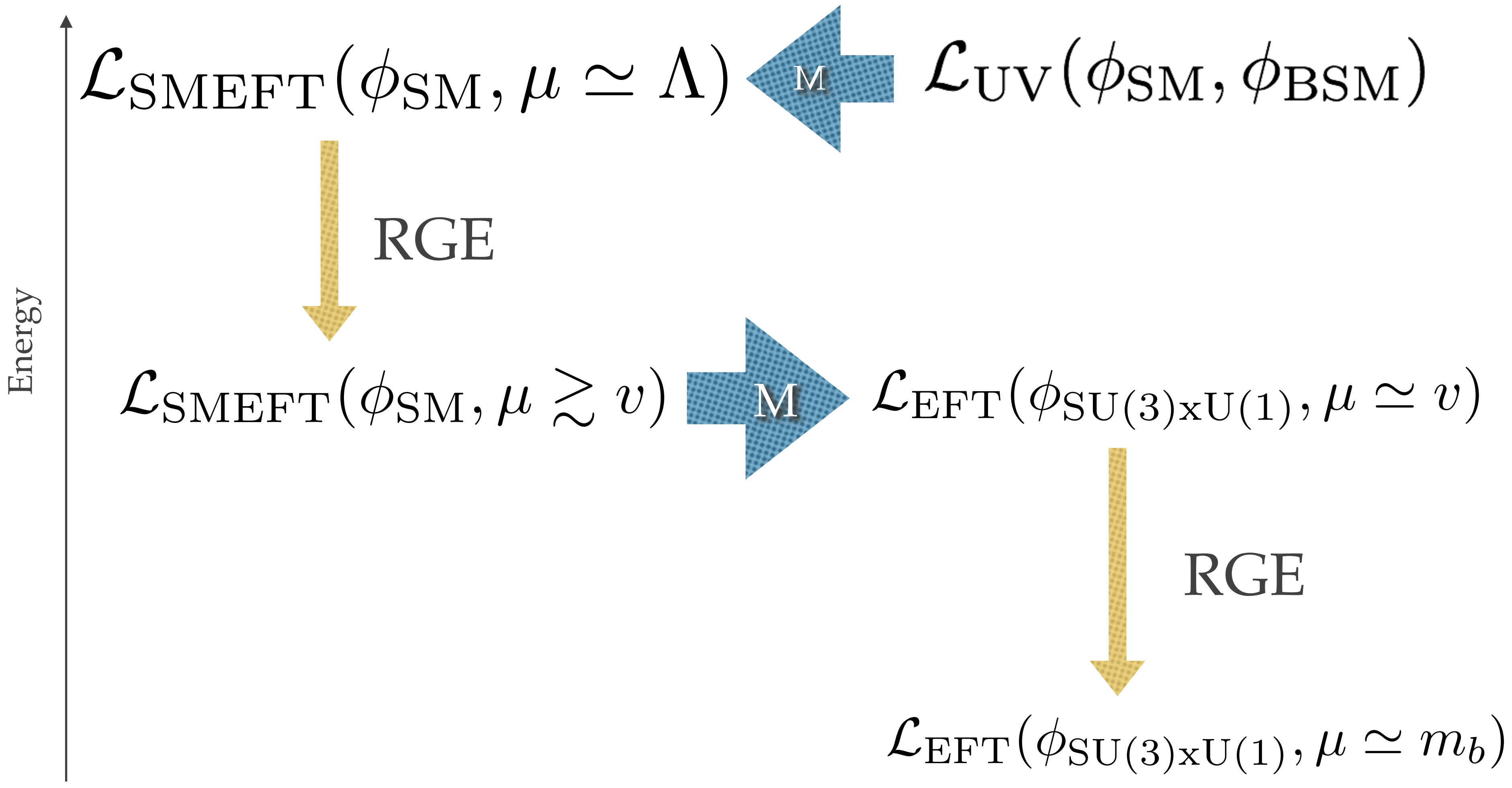
$$\nu_\mu + \mathcal{H} \to (N_{6,5} \to N_4 + e^+ + e^-) + \mathcal{H}, \qquad (6)$$

[Abdullahi, Hostert, Pascoli 2021]

EFT approach
SMEFT

$$\mathcal{L} = \sum_{V=B,W} \frac{C^{\ell}_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_{T}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$





$$\Delta a_\ell \simeq \frac{4 m_\ell^2}{e\Lambda^2} \frac{v}{m_\ell} \left(C^{\ell}_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^{\ell}_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell^2}{\pi^2} \frac{m_q}{m_\ell} \frac{C^{\ell q}_{T}}{\Lambda^2} \log \frac{\Lambda}{m_q}$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{250\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu}_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{50\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu}_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{100\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu t}_{T}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{10\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu c}_{T}|$$

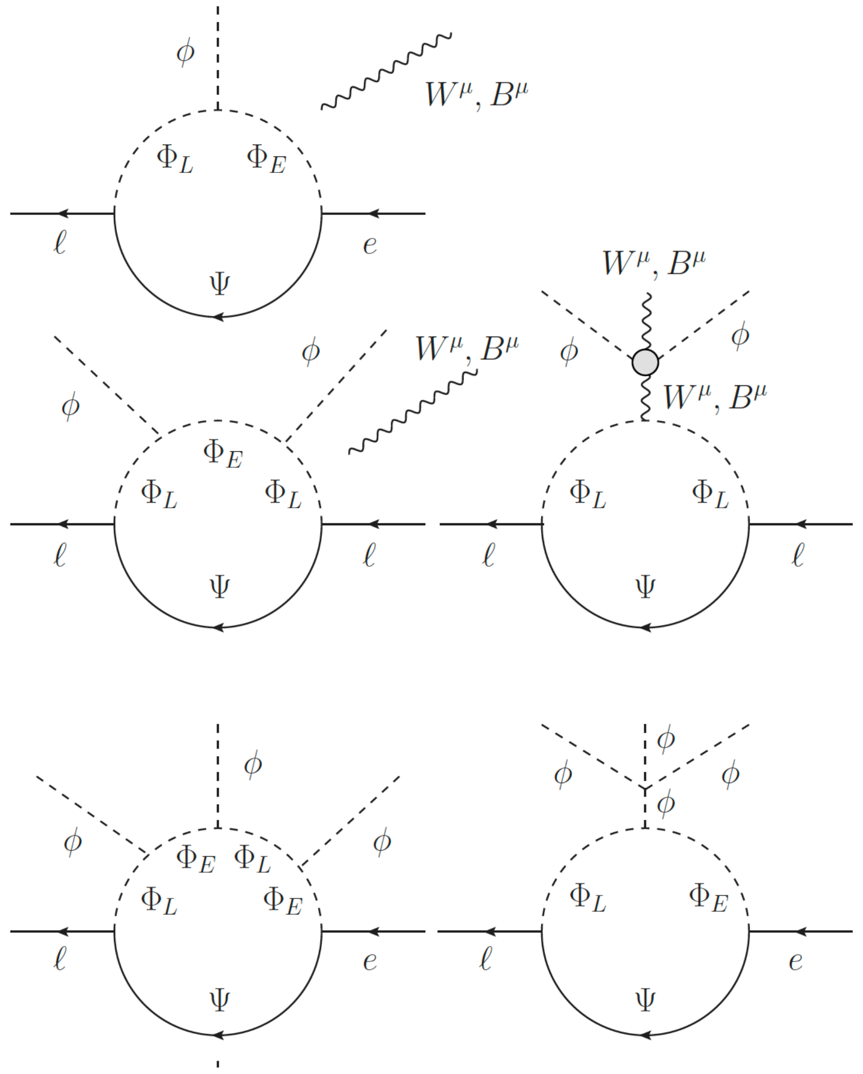
- **Strongly coupled NP:** $C^{\mu}_{e\gamma}, C^{\mu t}_{T} \sim g^2_{\mathrm{NP}}/16\pi^2 \lesssim 1$ implying $\Lambda \lesssim$ *few* x 100 TeV, beyond the direct production reach of any foreseen collider.
- **Weakly coupled NP:** $C^{\mu}_{e\gamma}, C^{\mu t}_{T} \lesssim 1/16\pi^2$ implying $\Lambda \lesssim 20$ TeV maybe within the direct production reach of a very high-energy Muon Collider

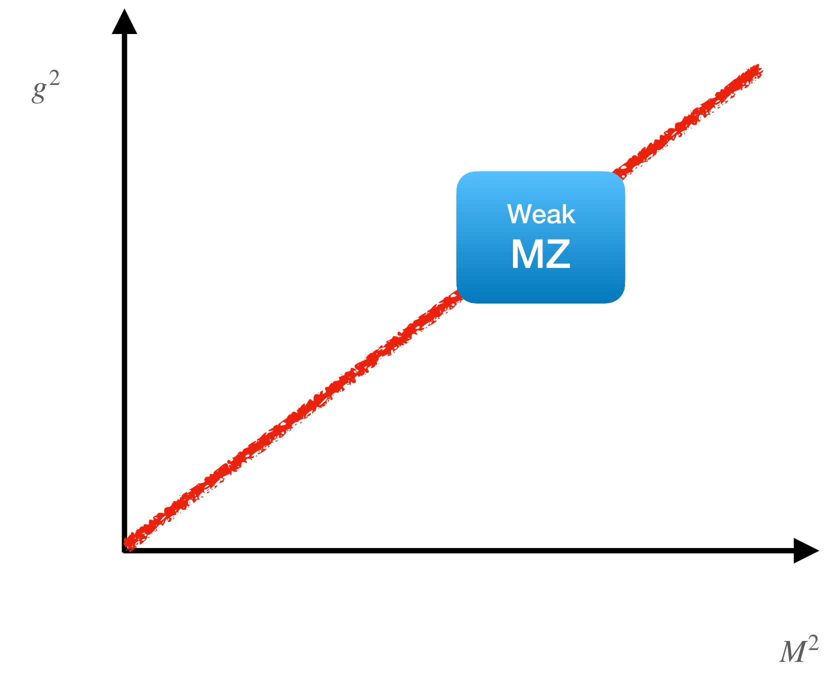
EFT : top down

Chirally enhanced UV model

There are two classes of models that display chiral enhancement for a_μ: (I) two scalars $\Phi_{L,E}$ and one fermion Ψ and (II) two fermions $\Psi_{L,E}$ and one scalar Φ.[2] We define the Lagrangians in these two cases as

$$\begin{aligned}\mathcal{L}_{\rm I} &= \lambda_L^{\rm I}\,\bar{\ell}\Psi\Phi_L + \lambda_E^{\rm I}\,\bar{e}\Psi\Phi_E + A\,\Phi_L^\dagger\Phi_E\phi,\\ \mathcal{L}_{\rm II} &= \lambda_L^{\rm II}\,\bar{\ell}\Psi_L\Phi + \lambda_E^{\rm II}\,\bar{e}\Psi_E\Phi + \kappa\,\bar{\Psi}_L\Psi_E\phi,\end{aligned} \tag{3}$$





$$\mathcal{L}_{\rm UV}(\phi_{\rm SM}, \phi_{\rm BSM}) \xrightarrow{\rm M} \mathcal{L}_{\rm SMEFT}(\phi_{\rm SM}, \mu \simeq \Lambda)$$

$$\begin{aligned}Q_{e\phi} &= \phi^\dagger\phi\,\bar{\ell}e\phi, & Q_{\phi e} &= \phi^\dagger i\overleftrightarrow{D}^\mu\phi\,\bar{e}\gamma_\mu e,\\ Q_{eB} &= \bar{\ell}\sigma_{\mu\nu}e\phi\,B^{\mu\nu}, & Q_{\phi\ell}^{(1)} &= \phi^\dagger i\overleftrightarrow{D}^\mu\phi\,\bar{\ell}\gamma_\mu\ell,\\ Q_{eW} &= \bar{\ell}\sigma_{\mu\nu}e\tau^I\phi\,W_I^{\mu\nu}, & Q_{\phi\ell}^{(3)} &= \phi^\dagger i\overleftrightarrow{D}_I^\mu\phi\,\bar{\ell}\tau^I\gamma_\mu\ell,\end{aligned}$$

RGE

The effective operators in the SMEFT Lagrangian that directly generate $(g-2)_\mu$, $h\to\mu\mu$, $Z\to\mu\mu,\nu\nu$, and $W\to\mu\nu$ (after EW symmetry breaking) are [77]

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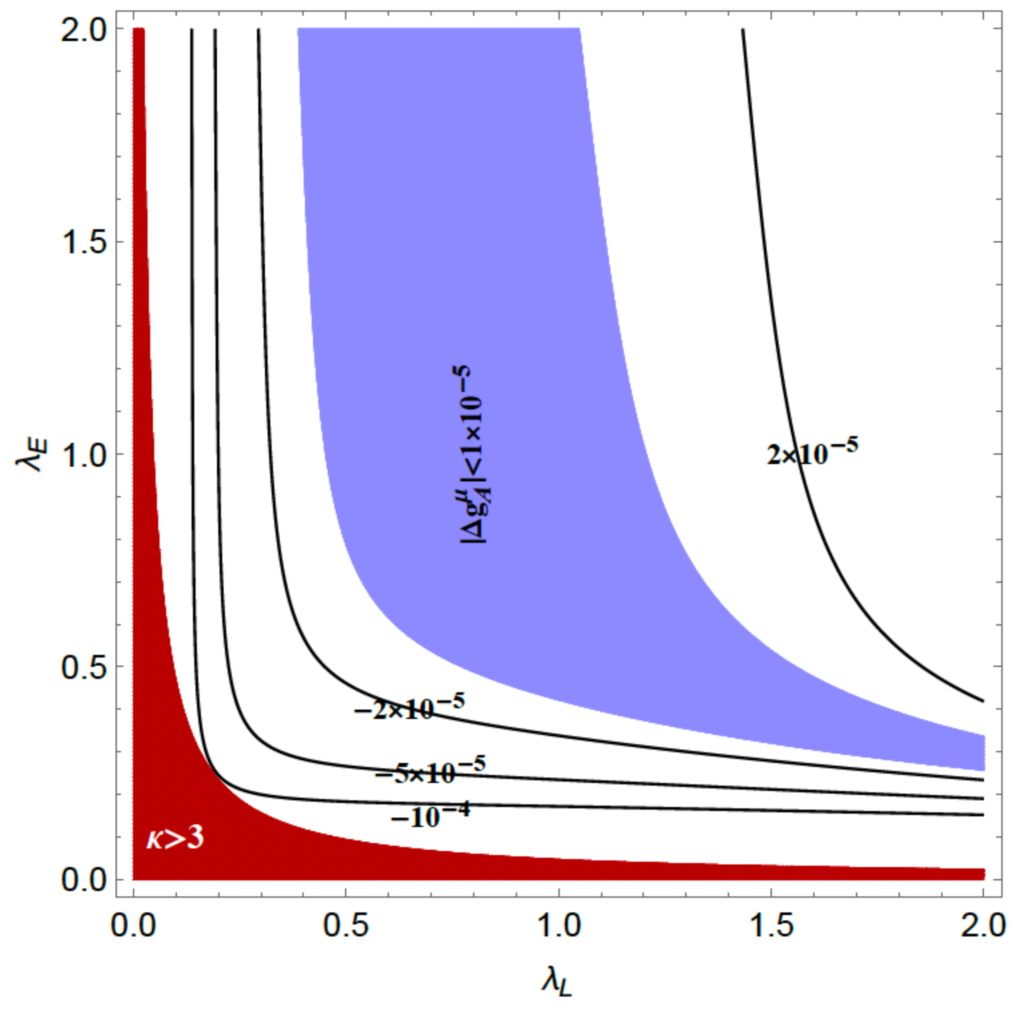
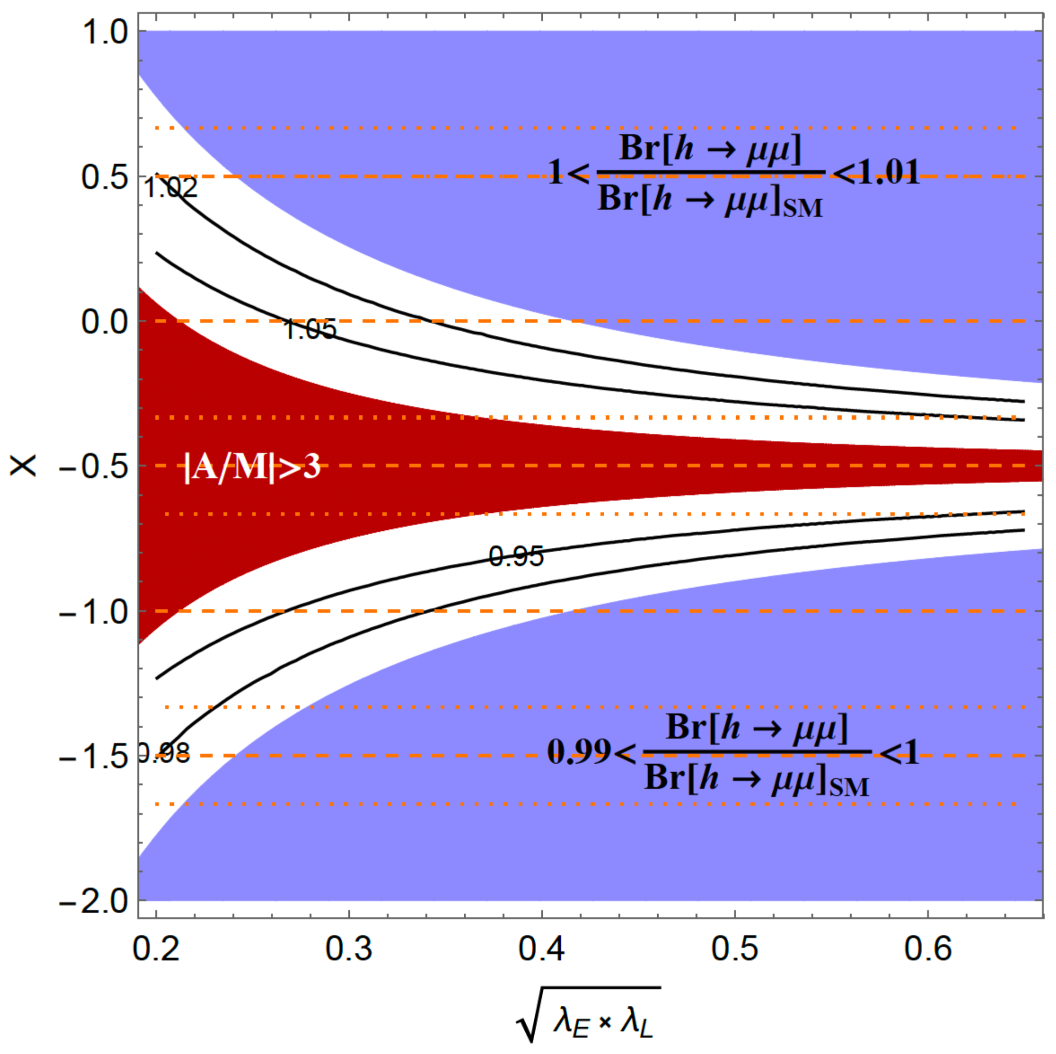


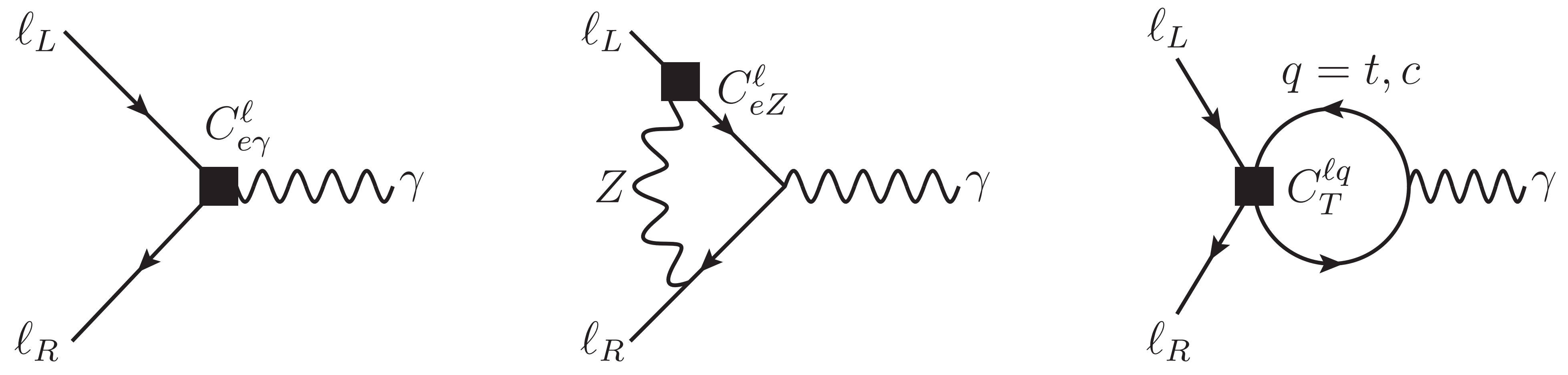


[Crivellin, Hoferichter, 2021]

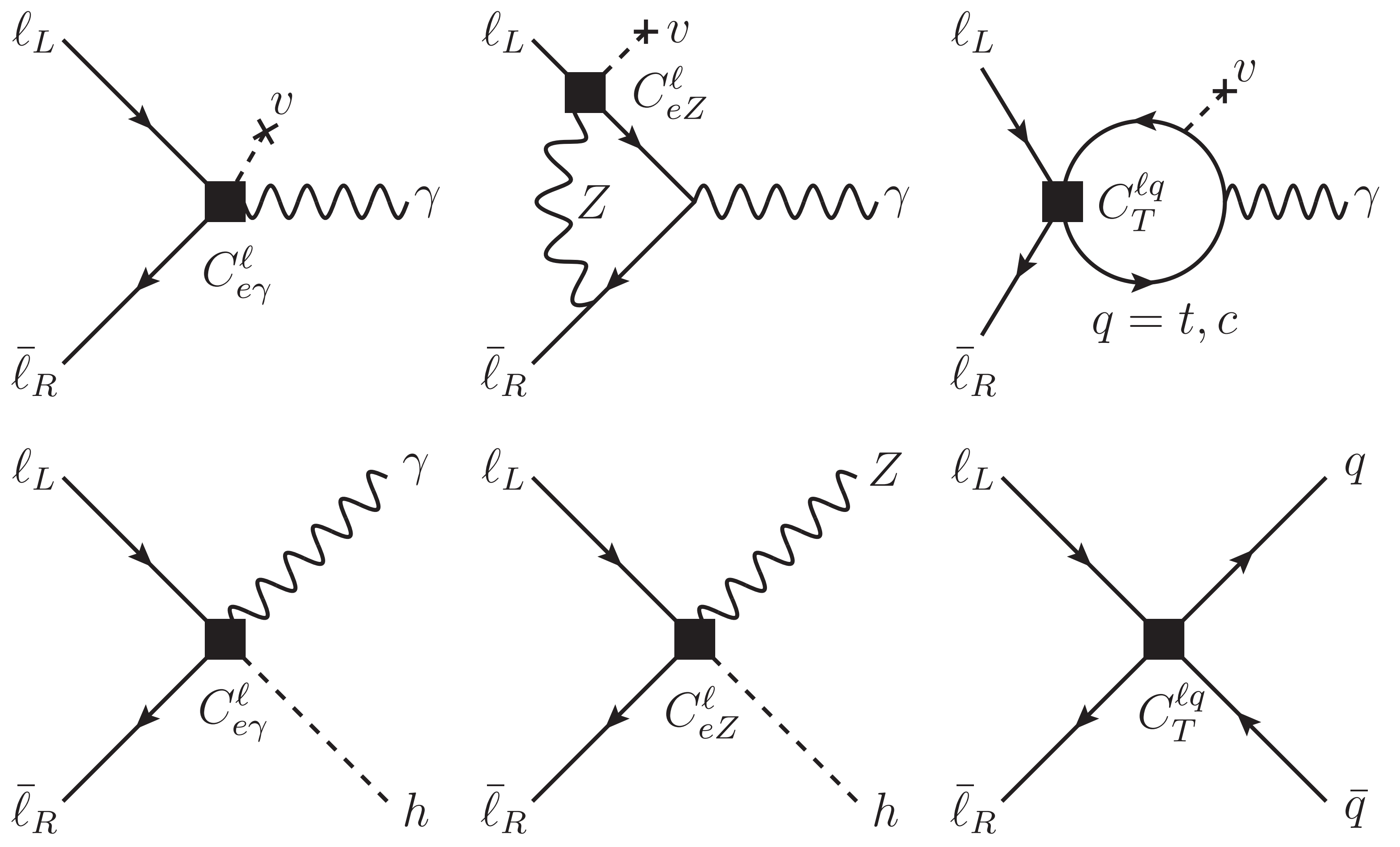
EFT : bottom up

(g-2)$_\mu$ vs muon collider

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C_T^{\ell q}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



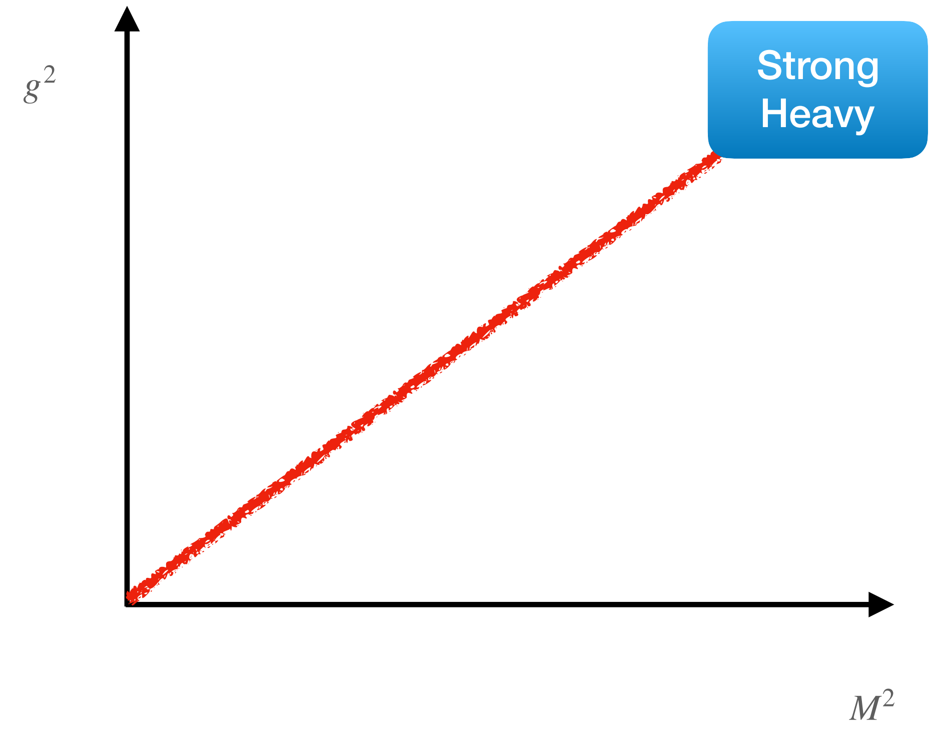
$$\Delta a_\ell \simeq \frac{4 m_\ell^2}{e\Lambda^2} \frac{v}{m_\ell} \left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^\ell_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell^2}{\pi^2} \frac{m_q}{m_\ell} \frac{C_T^{\ell q}}{\Lambda^2} \log \frac{\Lambda}{m_q}$$



$$\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T} \quad \Longleftrightarrow \quad \sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4} |C_{eV,T}|^2 \quad (f = e\gamma, eZ, q\bar{q})$$

At high energy $\sigma_{\mu\mu\to f}$ can compete with Δa_μ to test the very same NP!

[Buttazzo & Paradisi, 2020]



EFT : bottom up

$(g\text{-}2)_\mu$ vs muon collider

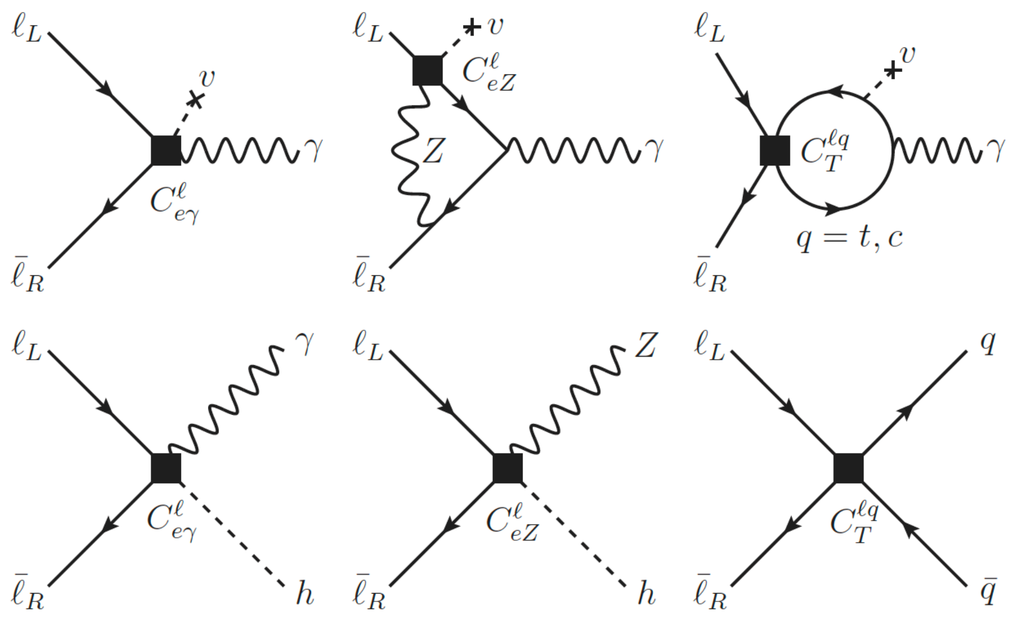
- **Connecting $\mu^+\mu^- \to (h\gamma, Zh, t\bar{t}, c\bar{c})$ with Δa_μ**

$$\sigma^{\rm cut}_{\mu\mu\to h\gamma} \approx 0.5\,\text{ab}\left(\frac{\sqrt{s}}{30\,\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to Zh} \approx 38\,\text{ab}\left(\frac{\sqrt{s}}{10\,\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to t\bar{t}} \approx 58\,\text{ab}\left(\frac{\sqrt{s}}{10\,\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to c\bar{c}} \approx 100\,\text{fb}\left(\frac{\sqrt{s}}{3\,\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$$



- **Δa_μ predictions in the SMEFT**

$$\frac{|\Delta a_\mu|}{3\times10^{-9}} \approx \left(\frac{250\,\text{TeV}}{\Lambda}\right)^2 |C^\mu_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3\times10^{-9}} \approx \left(\frac{50\,\text{TeV}}{\Lambda}\right)^2 |C^\mu_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3\times10^{-9}} \approx \left(\frac{100\,\text{TeV}}{\Lambda}\right)^2 |C^{\mu t}_T| \qquad \frac{|\Delta a_\mu|}{3\times10^{-9}} \approx \left(\frac{10\,\text{TeV}}{\Lambda}\right)^2 |C^{\mu c}_T|$$

- **SM irreducible background**

$$\sigma^{\rm SM,cut}_{\mu\mu\to Z\gamma} \approx 82\,\text{ab}\left(\frac{30\,\text{TeV}}{\sqrt{s}}\right)^2 \qquad \sigma^{\rm SM}_{\mu\mu\to Zh} \approx 122\,\text{ab}\left(\frac{10\,\text{TeV}}{\sqrt{s}}\right)^2$$

$$\sigma^{\rm SM}_{\mu\mu\to t\bar{t}} \approx 1.7\,\text{fb}\left(\frac{10\,\text{TeV}}{\sqrt{s}}\right)^2 \qquad \sigma^{\rm SM}_{\mu\mu\to c\bar{c}} \approx 19\,\text{fb}\left(\frac{3\,\text{TeV}}{\sqrt{s}}\right)^2$$

[Buttazzo & Paradisi, 2020]

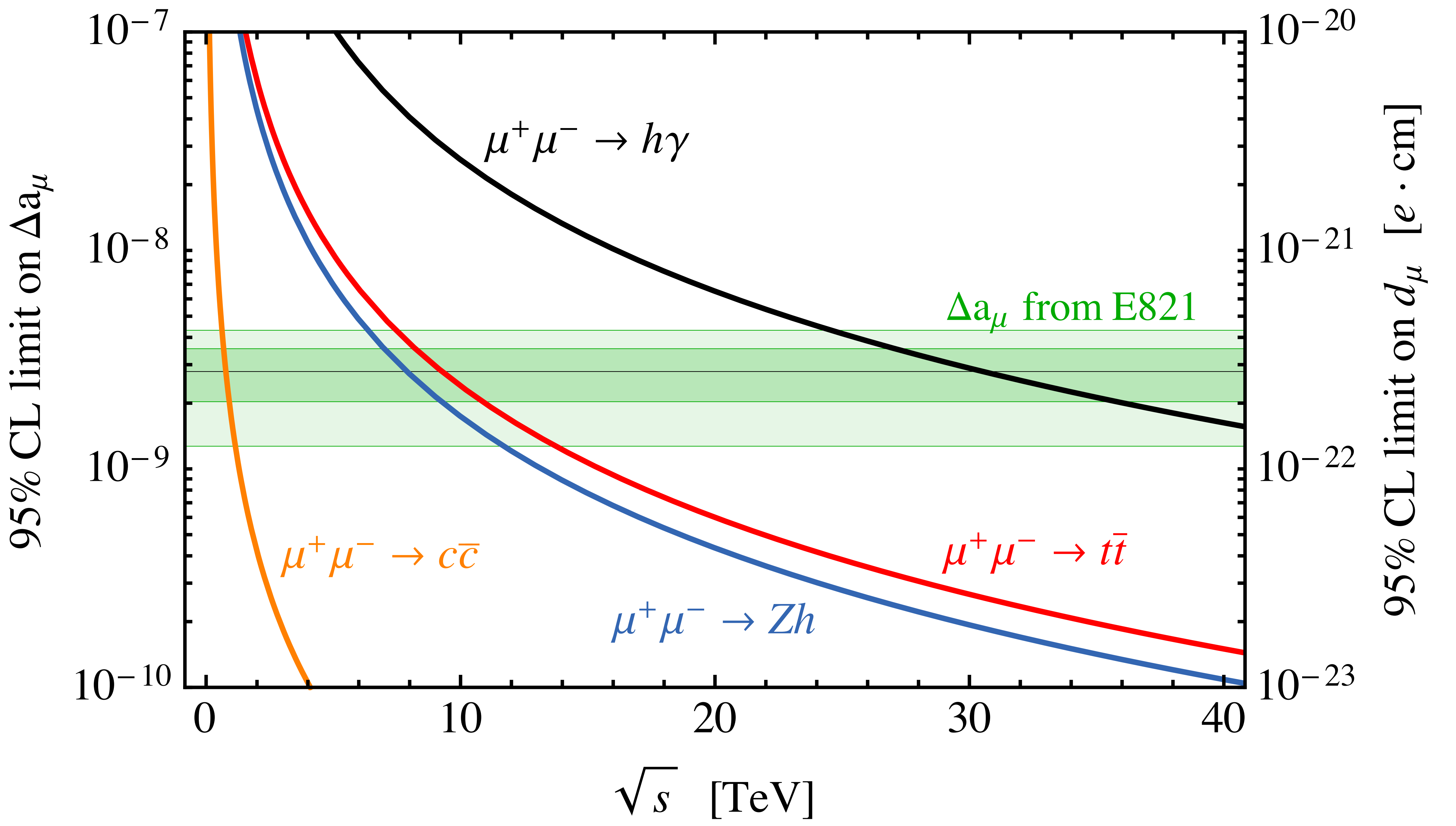


Figure: 95% C.L. reach on Δa_μ, as well as on the muon EDM d_μ, as a function of $\sqrt{s}$ from various processes for the reference integrated luminosity $\mathcal{L} = (\sqrt{s}/10\,\text{TeV})^2 \times 10\,\text{ab}^{-1}$.

$$d_\mu = \frac{\Delta a_\mu \tan\phi_\mu}{2m_\mu}\, e \simeq 3\times10^{-22}\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)\tan\phi_\mu\; e\,\text{cm}$$

Connecting with anomalies

B-Anomalies

"Why is it when something happens it is ALWAYS you, muons?" (Falkowski, blogger physicist)

- Is there a mounting evidence of violation of lepton flavor universality?
- Is $(g\text{-}2)_\mu$ flavour violating?
- Are the $(g\text{-}2)_\mu$ and flavor anomalies related?

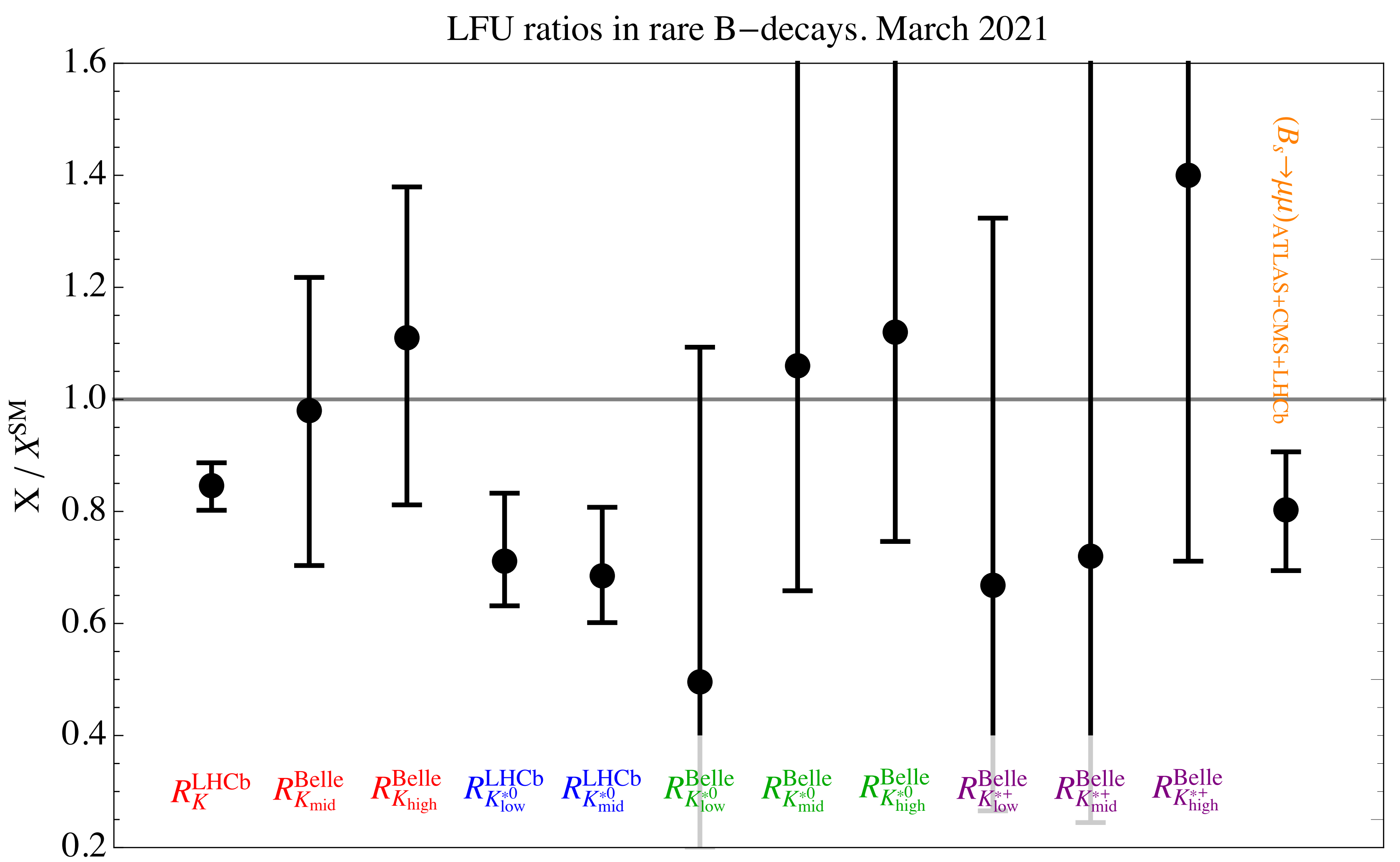
"Coincidences mean you're on the right path." (Van Booy, writer)

g-2 : 1M$ question

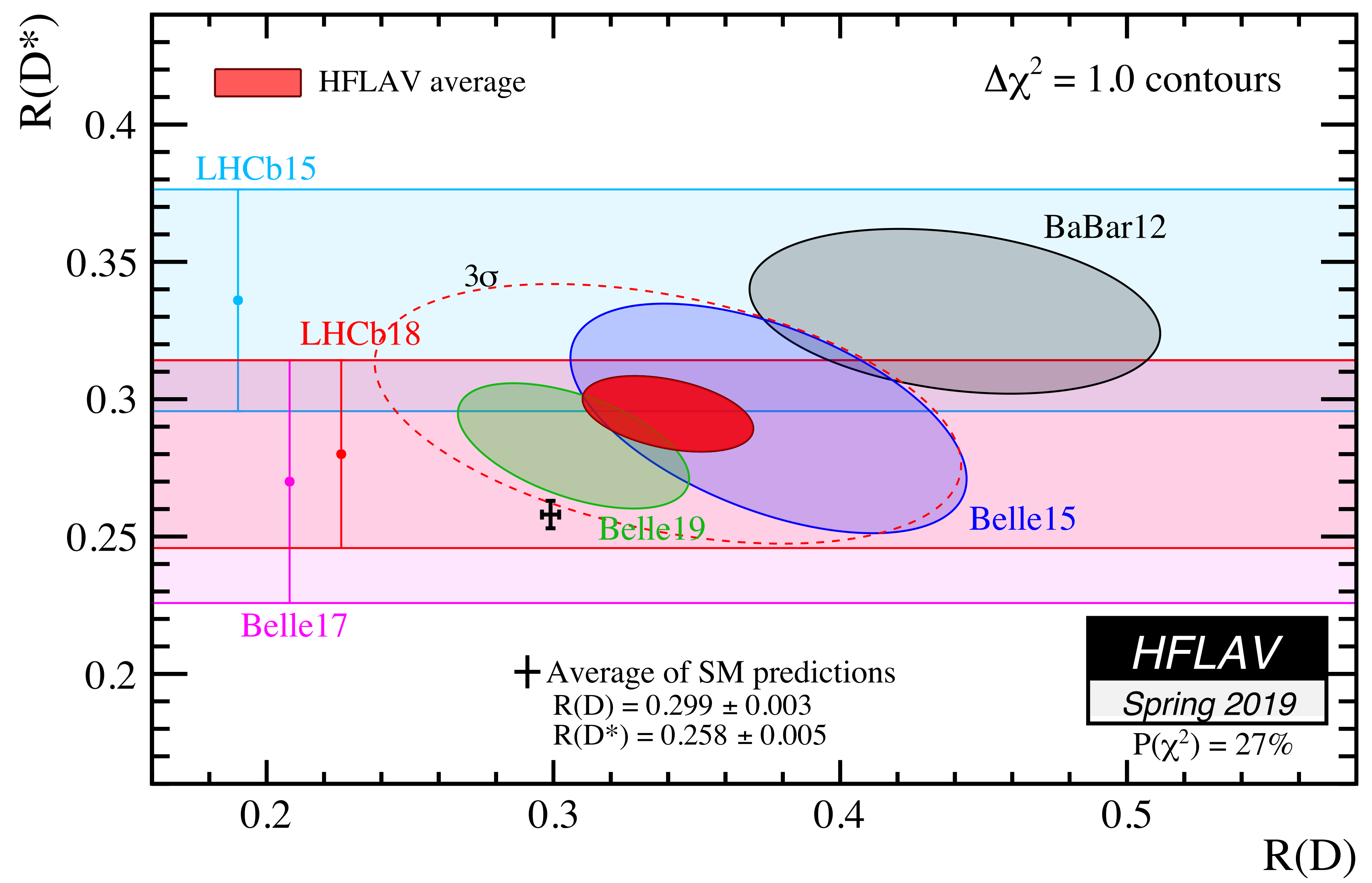
B-Anomalies

$$R_{K^{(*)}} = \left.\frac{\mathcal{B}(B \to K^{(*)}\mu\mu)}{\mathcal{B}(B \to K^{(*)}ee)}\right|_{q^2\in[q^2_{\min},q^2_{\max}]} \quad \& \quad R^{\rm exp}_{K^{(*)}} < R^{\rm SM}_{K^{(*)}}$$

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar\nu)}{\mathcal{B}(B \to D^{(*)}\ell\bar\nu)}_{\ell\in(e,\mu)} \quad \& \quad R^{\rm exp}_{D^{(*)}} > R^{\rm SM}_{D^{(*)}}$$



Marzocca4Moriond21 based on https://arxiv.org/abs/1903.10954



$$R_K^{[1.1,6]} = 0.847(42)^{\rm LHCb} \quad \text{vs} \quad R_K^{[1,6]} = 1.00(1)^{\rm SM}$$

$$R_{K^*}^{[1.1,6]} = 0.71(10)^{\rm LHCb} \quad \text{vs} \quad R_{K^*}^{[1,6]} = 1.00(1)^{\rm SM}$$

$$\text{Exp}: \quad R_D = 0.340\pm0.030\,, \quad R_{D^*} = 0.295\pm0.014$$

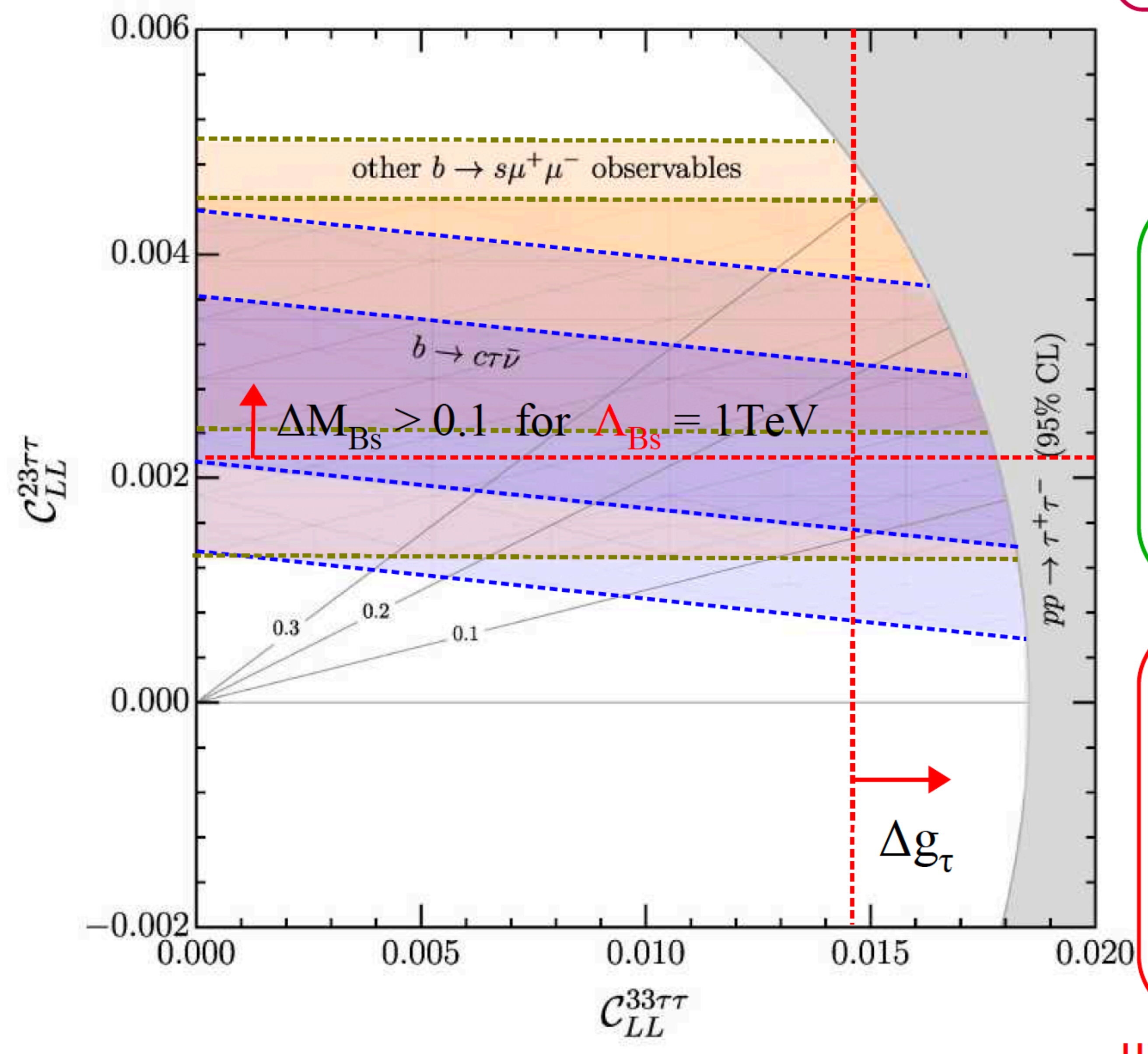
$$\text{SM}: \quad R_D^{\rm SM} = 0.293\pm0.008\,, \quad R_{D^*}^{\rm SM} = 0.257\pm0.003$$

g-2 : 1M$ question

B-Anomalies

EFT considerations

$$(\bar{q}_L^i \gamma_\mu \ell_L^\alpha)(\bar{\ell}_L^\beta \gamma_\mu q_L^j) = \frac{1}{2}\left[\mathcal{O}_{\ell q}^{(1)} + \mathcal{O}_{\ell q}^{(3)}\right]^{ij\alpha\beta}$$



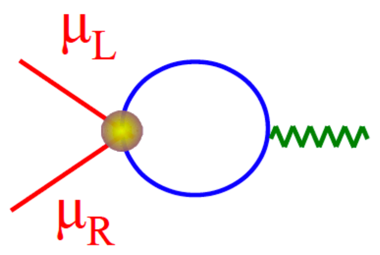
Pattern emerging from data in 2 ↔ 3 sector:

- ✔ ~ 10^{-1} for each 2nd gen. q_L or l_L
  → $|C^{23\mu\mu}| \sim 10^{-3}\ |C^{33\tau\tau}|$
  → $|V_{ts}| \sim 0.4\times10^{-1}$
- ✔ Nice consistency among the two sets of anomalies

Additional ~10^{-2} (~loop) suppression for

- ✖ Four-quarks (ΔF=2)
- ✖ Four-leptons ($\tau\to\mu\nu\nu$)
- ✖ Semi-leptonic $O^{(1\text{-}3)}$ ($b\to s\nu\nu$)

N.B.: with this sets of operators → tiny contribution to $a_\mu = (g\text{-}2)_\mu/2$

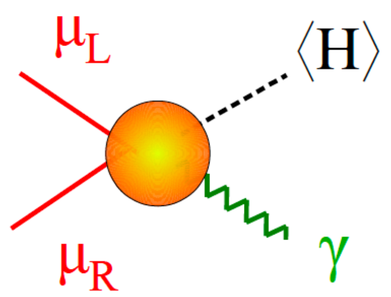


$\Delta a_\mu \ll a_\mu^{\text{SM-EW}}$

Worries

There are of course still several worries, and here the personal view becomes even more relevant.... So, let me mention a few of them:

- Not easy to reconcile the $(g\text{-}2)_\mu$ anomaly with both flavor anomalies and, more generally, with models with a "natural" flavor structure (↔ Y_{SM}). Is $(g\text{-}2)_\mu$ suggesting something a different way?



Maybe.... examples of recent "attempts":

- $a_\mu \oplus R_K$ with special role of muons [$U(1)_{B\text{-}3L_\mu} \subset G$] Greljo, Stangl, Thomsen '21
- $a_\mu \oplus R_K \oplus R_D$ with 2 scalars [$S_1+\phi^+$] and peculiar flavor struct. Marzocca, Trifinopoulos '21

But... $(g\text{-}2)_\mu$ is more "flexible" (*no generation change, necessary loop-level*)
→ could come from light NP: no obvious connection to the flavor anomalies

Flavour (Anomalies)

New Physics in the t-channel

R(K^{(*)}) $\longrightarrow \sim \dfrac{g_\mu V_{ts}}{\Lambda^2}(\bar{b}_L\gamma_\alpha s_L)(\bar{\mu}_L\gamma^\alpha\mu_L)$

$\Lambda/\sqrt{g_\mu} \sim 7$ TeV

CKM-like flavor structure

SM gauge invariance $SU(2)_L$

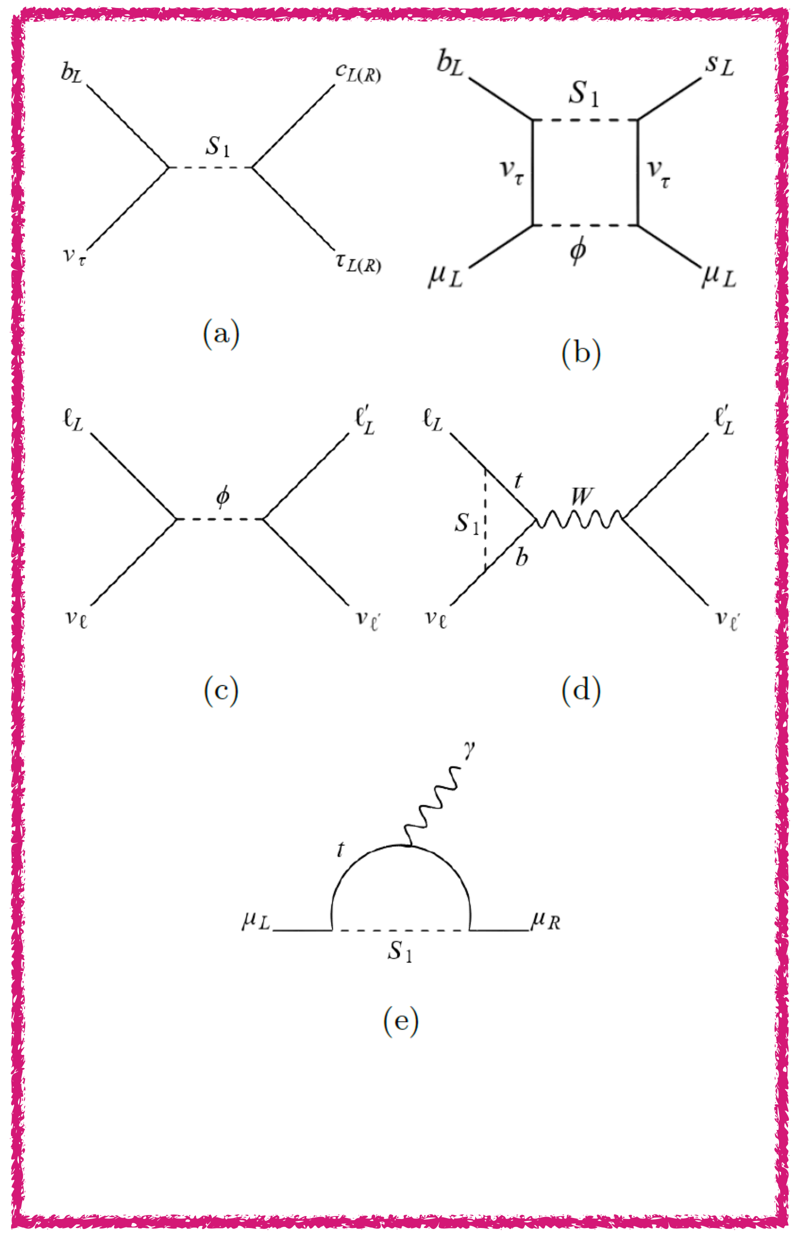
$$C_T\,(\bar{Q}^i_L\gamma_\mu\sigma^a Q^j_L)(\bar{L}^\alpha_L\gamma^\mu\sigma^a L^\beta_L) + C_S\,(\bar{Q}^i_L\gamma_\mu Q^j_L)(\bar{L}^\alpha_L\gamma^\mu L^\beta_L) \sim \frac{g_\mu V_{cb}}{\Lambda^2}(\bar{b}_L\gamma_\alpha c_L)(\bar{\nu}^\mu_L\gamma^\alpha\mu_L)$$

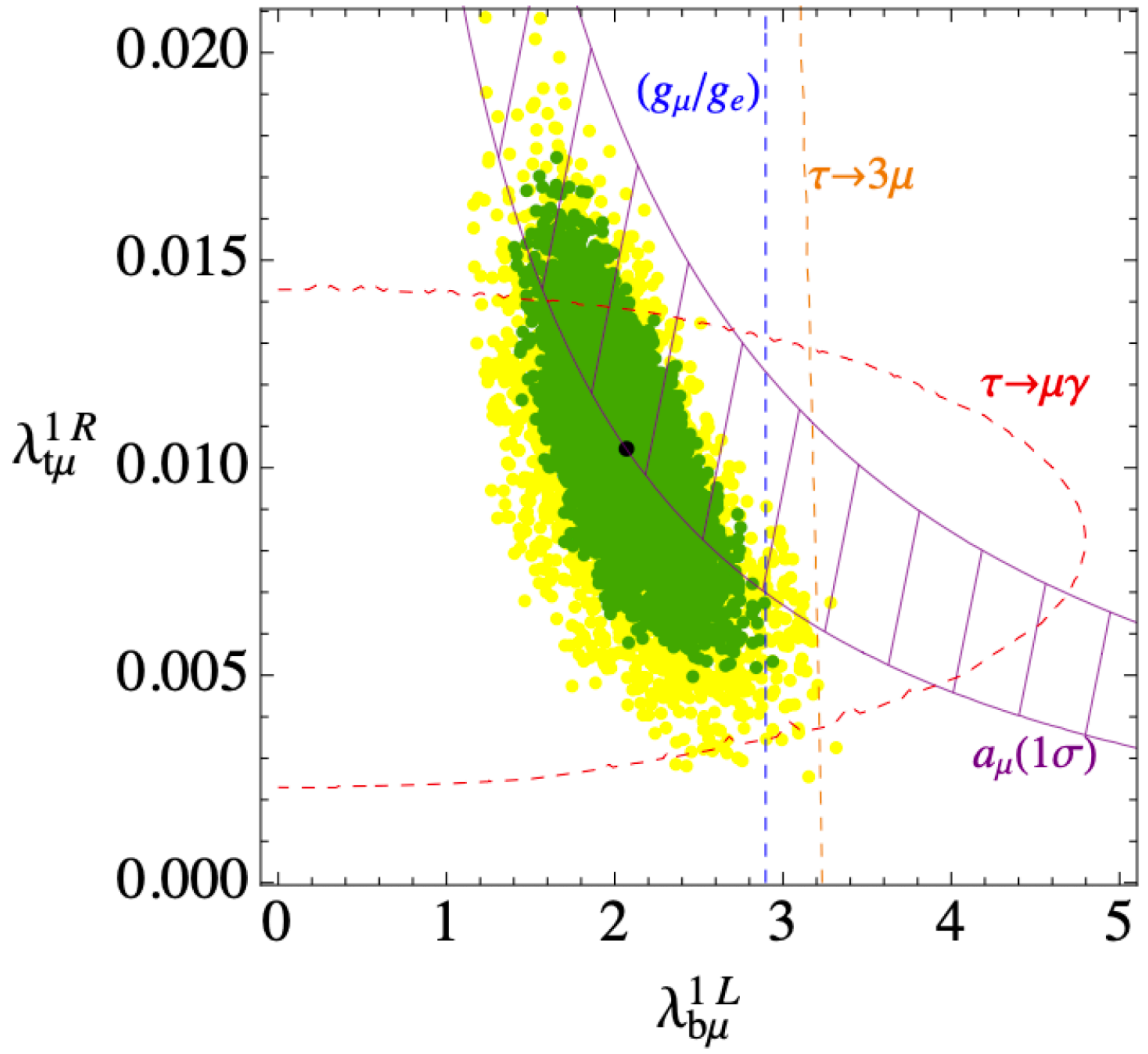
Lepton flavor extension $\quad g_e \ll g_\mu \ll g_\tau$

$$\sim \frac{g_\tau V_{cb}}{\Lambda^2}(\bar{b}_L\gamma_\alpha c_L)(\bar{\nu}^\tau_L\gamma^\alpha\tau_L)$$

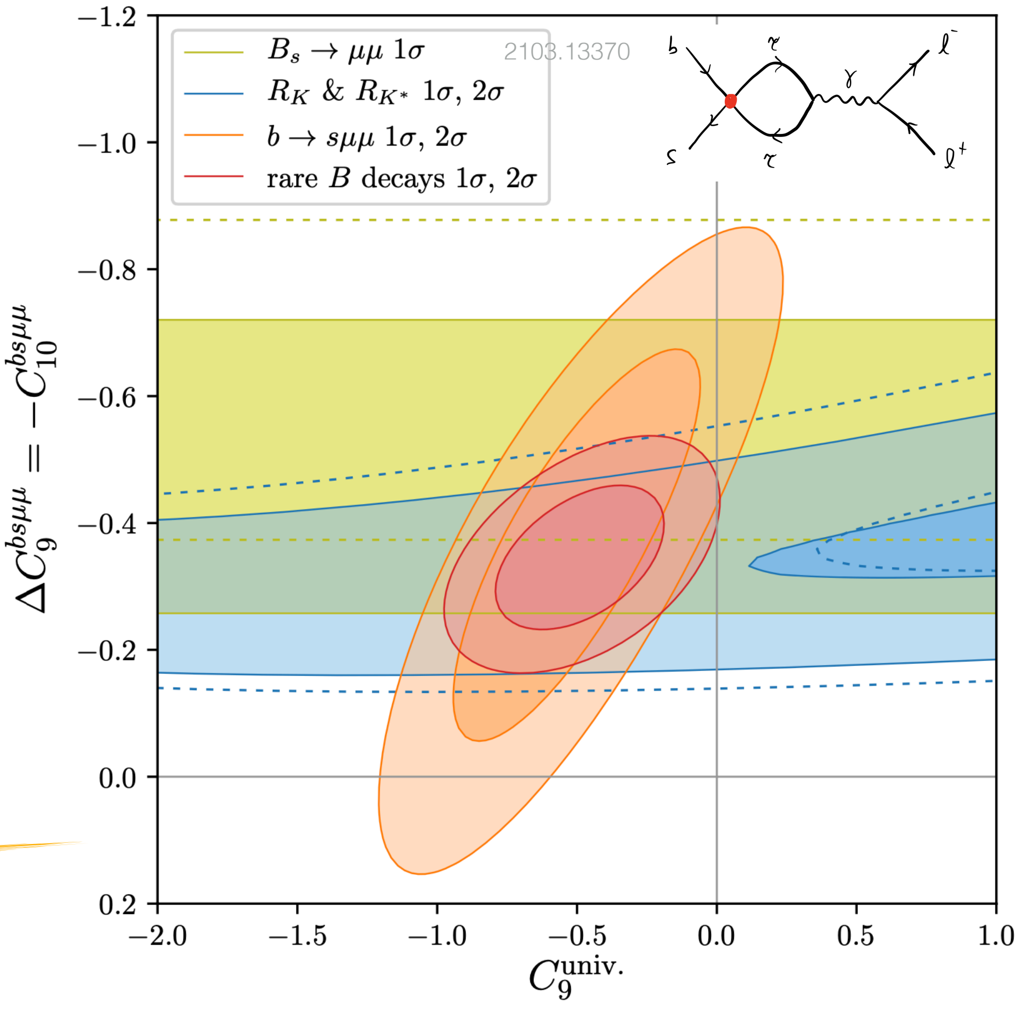
R(D^{(*)})

$\Lambda/\sqrt{g_\tau} \sim 1$ TeV





Algueró et al. 1809.08447



$m_t \gg m_c, m_u$
$m_b \gg m_s, m_d$
$m_\tau \gg m_\mu, m_e$

Approximate flavor global symmetry of the Standard Model

$$G_F = U(2)_q \times U(2)_\ell \times U(2)_u \times U(2)_d \times U(2)_e \qquad \psi_i = (\underbrace{\psi_1\ \psi_2}_{2}\ \underbrace{\psi_3}_{1})$$

$+$

$$\mathcal{L}_{int} \sim \left(\lambda^{1L}_{ij}\, q^i_L\, \varepsilon\, \ell^j_L + \lambda^{1R}_{ij}\, u^i_R\, e^j_R\right) S_1 + \lambda^{3L}_{ij}\, q^i_L\, \varepsilon\sigma^A\, \ell^j_L\, S^A_3 + h.c.$$

[Marzocca & Trifinopoulos, 2021]

g-2 : Conclusions
A few lessons

- $\Delta a_\mu > 0$ at 4σ, confirmed by new FNAL experiment.
- Borderline control of hadronic uncertainties leaves open the possibility that the deviation might be resolved with the SM.
- However, it is stimulating to entertain the possibility that NP might be responsible for the deviation.
- Two main strategies: top-down and bottom-up. In both cases need to correlate it with other searches/measurements/observations/constraints/anomalies.
- Is the $(g\text{-}2)_\mu$ anomaly related to lepton flavor violation?
- What is next?

Hoping you feel better than this guy..thanks!

g-2 : More questions
Please add to the list

- Bologna experimental activities. Opportunities? Challenges? New ideas and proposals?
- ….
- ….
- ….
- ….
- ….
- ….
- ….