Muon g–2: theory vs experiment

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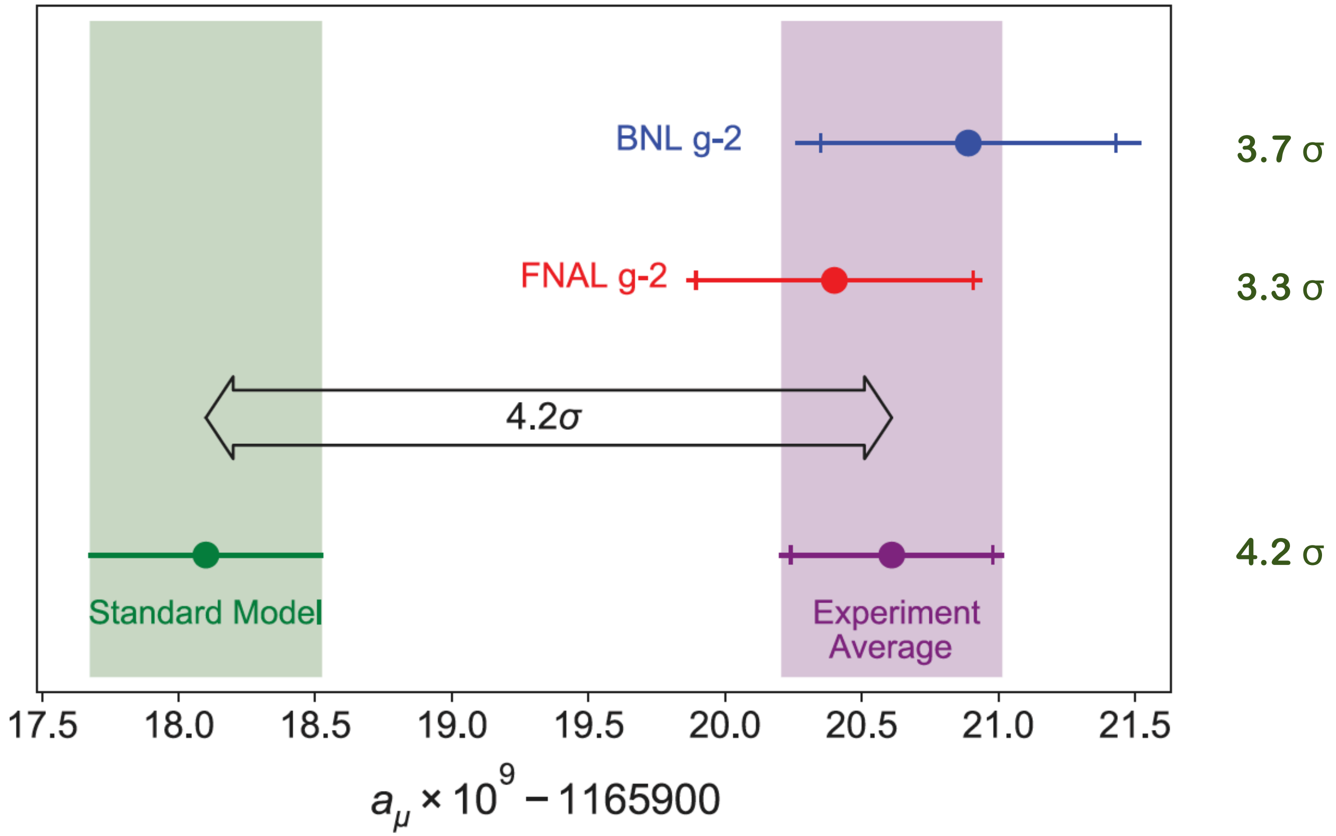
Muon g-2 mini-workshop
Pisa — April 14th 2021

Muon g-2: FNAL confirms BNL



a_μ^{EXP} = (116592089 ± 63) x 10^{-11} [0.54ppm] BNL E821

a_μ^{EXP} = (116592040 ± 54) x 10^{-11} [0.46ppm] FNAL E989 Run 1

a_μ^{EXP} = (116592061 ± 41) x 10^{-11} [0.35ppm] WA

Muon g-2: the Standard Model prediction

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

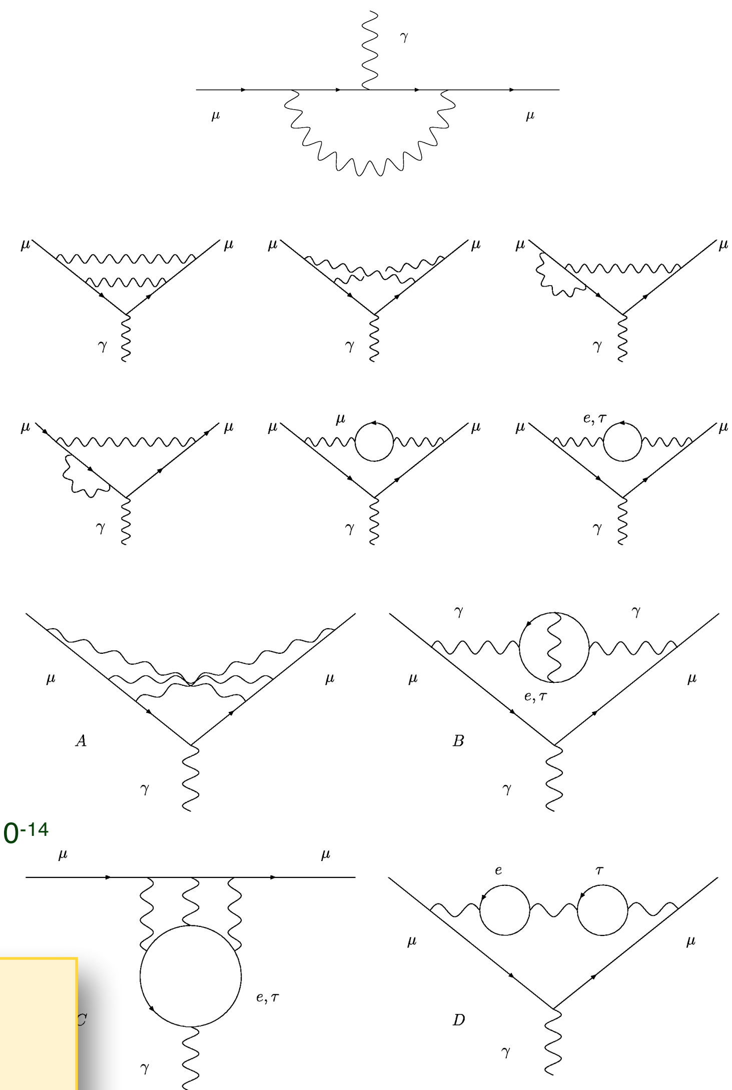
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek ’99; MP ’04;
Friot, Greynat & de Rafael ’05, Ananthanarayan, Friot, Ghosh 2020

$+ \; 130.8780\,(60)\,(\alpha/\pi)^4$

Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term) COMPLETED^2!

$+ \; 750.86\,(88)\,(\alpha/\pi)^5$ COMPLETED!

Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible $\delta a_\mu \sim 6\times10^{-14}$



Adding up, we get:

$a_\mu^{QED} = 116584718.931\,(19)(100)(23) \times 10^{-11}$

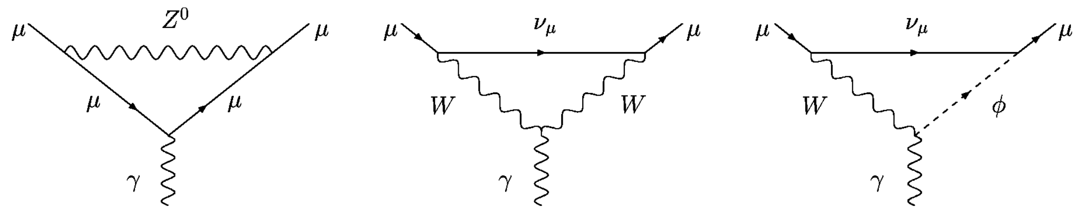
mainly from 4-loop coeff. unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018 WP20 value

Shift down in $a_\mu^{QED} \sim 1.4 \times 10^{-12}$ with new LKB Paris α(Rb) value (Morel et al 2020)

The electroweak contribution

- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

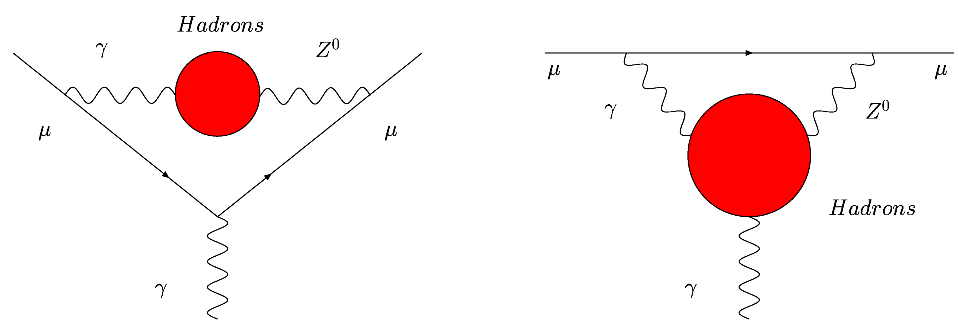
- **One-loop plus higher-order terms:**

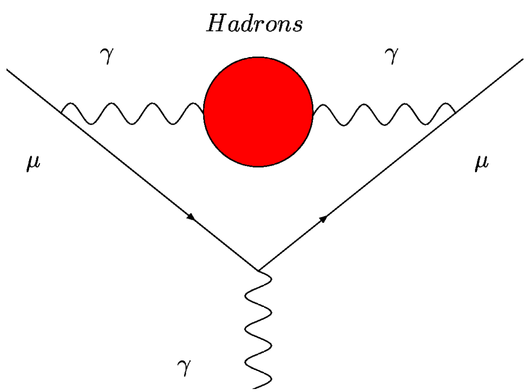
$a_\mu^{\rm EW} = 153.6\ (1.0) \times 10^{-11}$

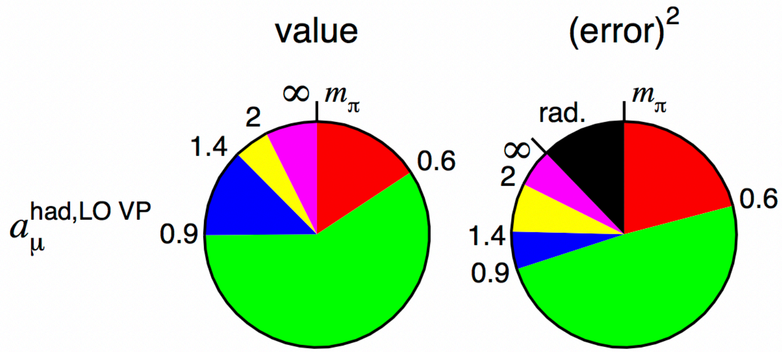
Hadronic loop uncertainties (and 3-loop nonleading logs).

WP20 value

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.







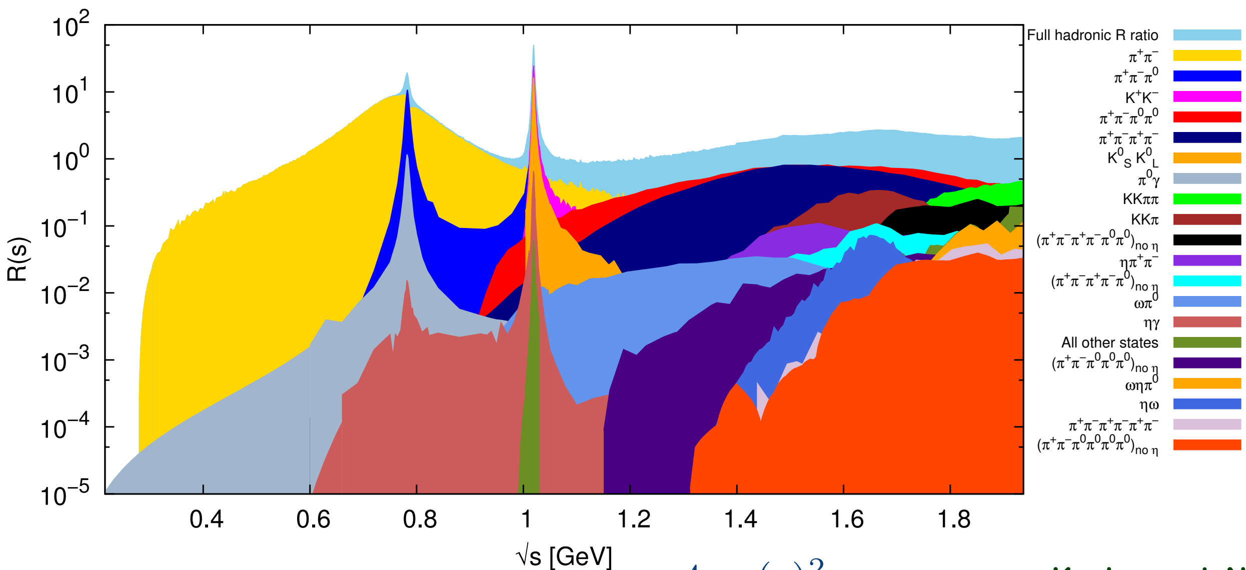
Keshavarzi, Nomura, Teubner 2018

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\,\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s) \qquad K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)(s/m^2)}$$

a_μ^{HLO} = 6895 (33) x 10^{-11}	F. Jegerlehner, arXiv:1711.06089
= 6939 (40) x 10^{-11}	Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921
= 6928 (24) x 10^{-11}	Keshavarzi, Nomura, Teubner, arXiv:1911.00367
= 6931 (40) x 10^{-11} (0.6%)	WP20 value

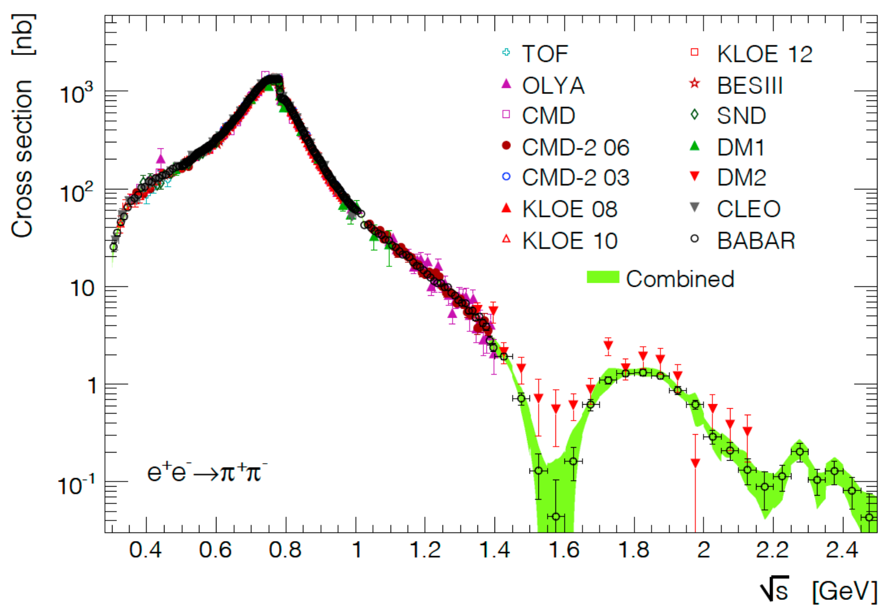
- **WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHHKS**
  Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19
- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585

The low-energy hadronic cross section



$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

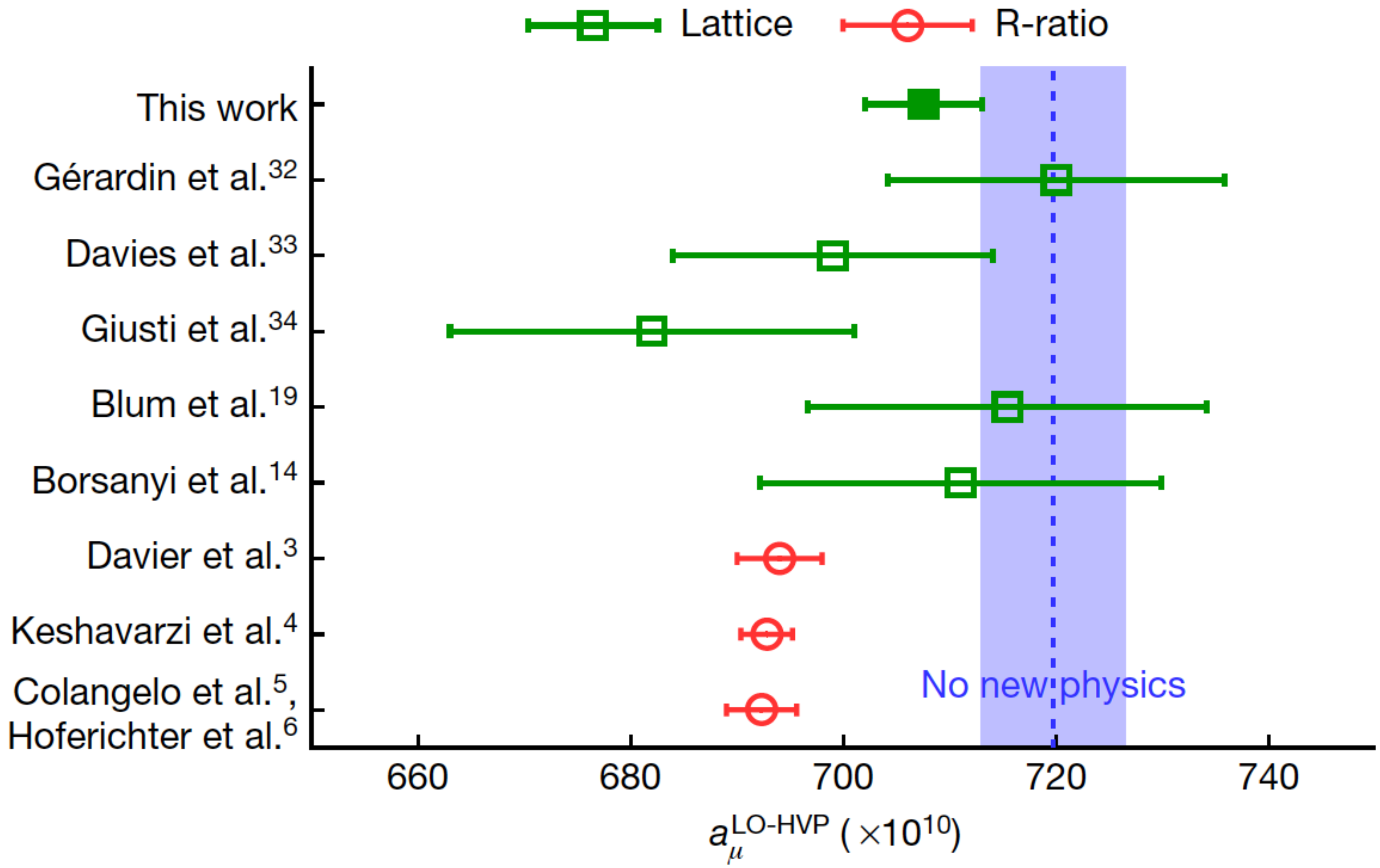
Keshavarzi, Nomura Teubner
PRD 2018



Davier, Hoecker, Malaescu, Zhang
EPJC 2020

The hadronic LO contribution from lattice QCD

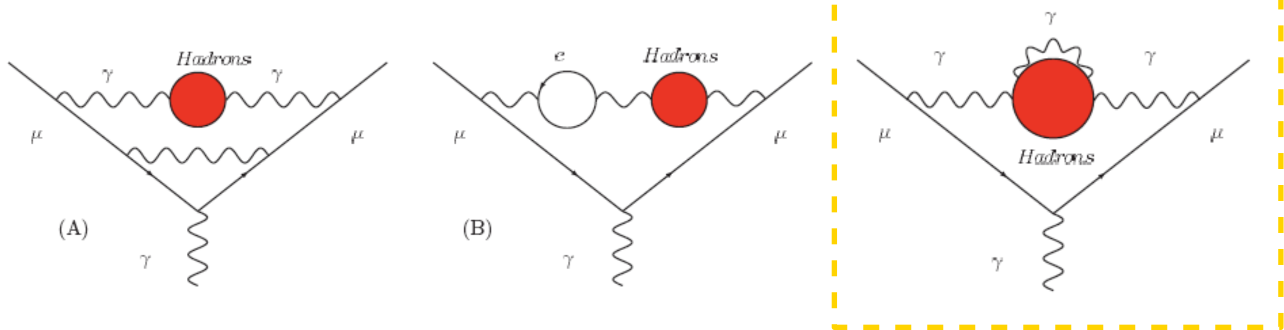
- **Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:** $a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst} \times 10^{-11}$ (the two errors together give 55). **Some tension with dispersive evaluations.** BMWc 2021



Borsanyi et al (BMWc), Nature 2021

- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:

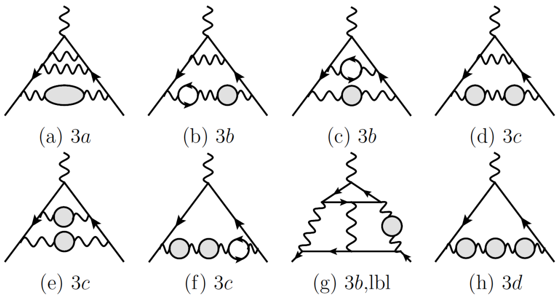
Already included in a_μ^{HLO}



$$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$$

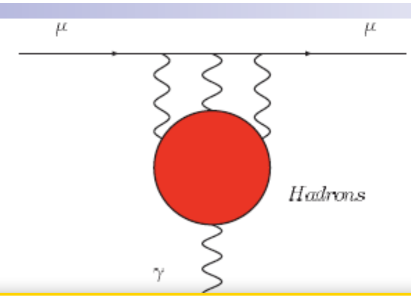
Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:



$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014



- **Hadronic light-by-light at O(α³)**
  - **This term had a troubled life! But nowadays:**

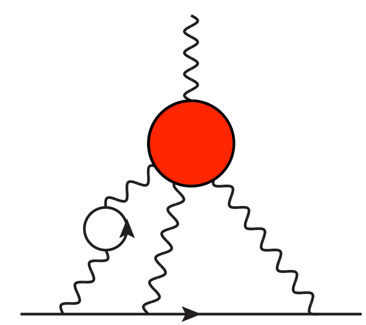
$a_\mu^{HNLO}(lbl) =$	$80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$=$	$136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$=$	$105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$=$	$100\ (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263
$=$	$92\ (19) \times 10^{-11}$	WP20 (phenomenology)

  - **Significant improvements due to data-driven dispersive approach.**
    Colangelo, Hoferichter, Procura, Stoffer, 2014–17; Pauk, Vanderhaeghen 2014.
  - **Lattice: RBC: $82(35)\times10^{-11}$** 1911.08123 **Mainz: $110(15)\times10^{-11}$** 2104.02632

- **Hadronic light-by-light at O(α⁴)**

$$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20



Muon g-2: the discrepancy

Comparing the SM prediction with the measured muon g-2 value:

$a_\mu^{EXP} = 116592061\ (41) \times 10^{-11}$ BNL+FNAL

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ WP20

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 251\ (59) \times 10^{-11}$ 4.2 σ

Is Δa_μ due to new physics beyond the SM? Could be due to:

- NP at the weak scale and weakly coupled to SM particles
- NP very heavy and strongly coupled to SM particles
- NP very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

- Can Δa_μ be due to missing contributions in the hadronic σ(s)?
- An upward shift of σ(s) also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

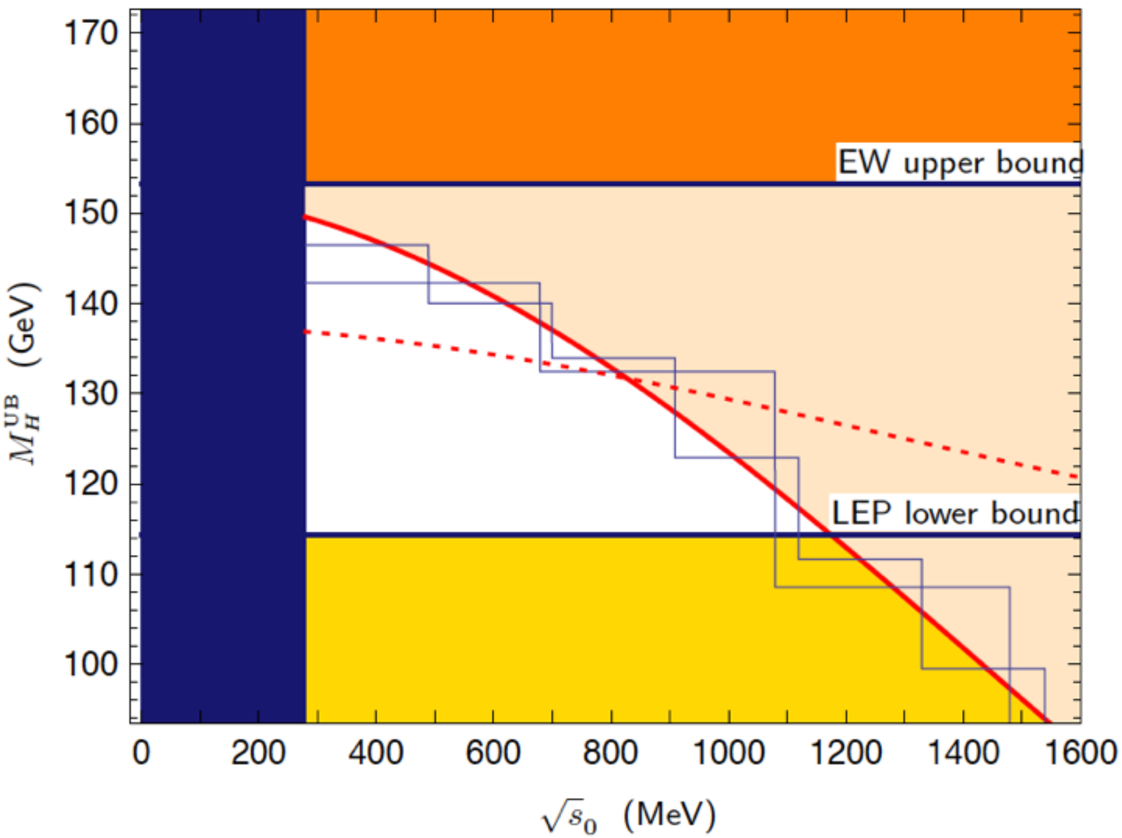
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

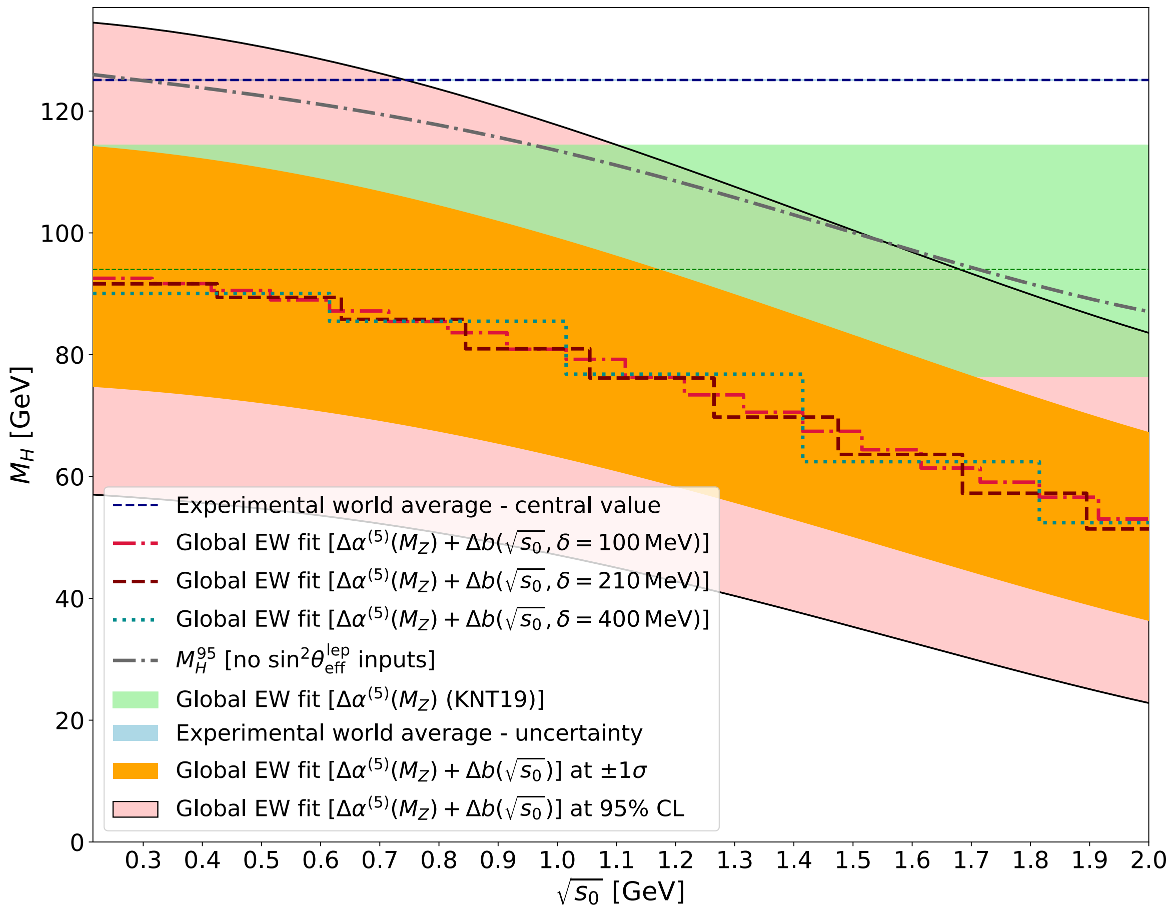
ϵ>0, in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to fix Δa_μ ?



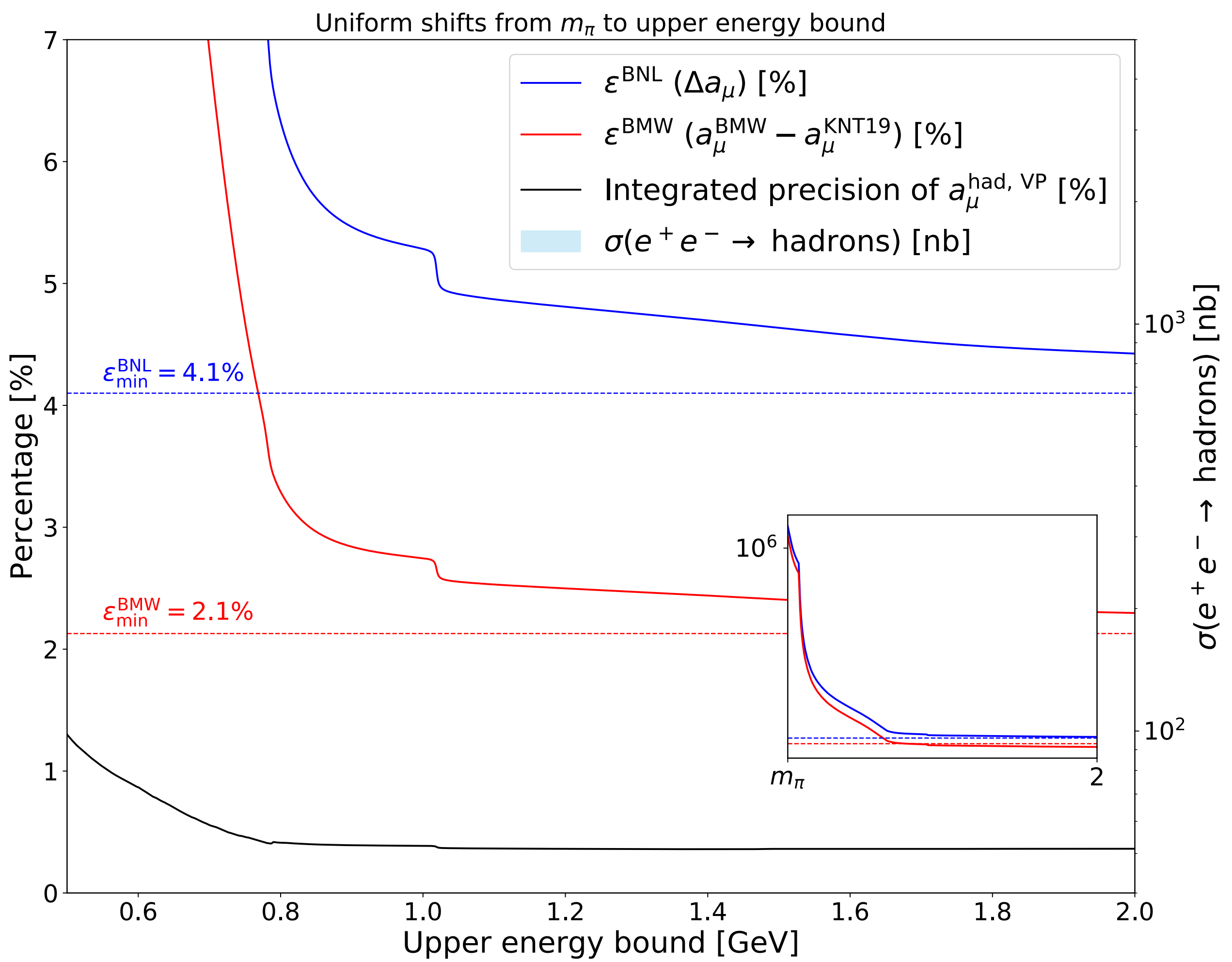
Marciano, MP, Sirlin, PRD 2008



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?



Shifts below ~1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

Electron g-2: SM vs experiment

- **Using the best determinations of α (which differ by 5.4σ!):**

  α = 1/137.035 999 046 (27) [Cs 2018]

  α = 1/137.035 999 206 (11) [Rb 2020]

$$a_e^{SM} = 115\,965\,218\,16.16\,(0.11)\,(0.08)\,(2.28) \times 10^{-13} \quad [Cs18]$$
$$= 115\,965\,218\,02.64\,(0.11)\,(0.08)\,(0.93) \times 10^{-13} \quad [Rb20]$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

$$a_e^{EXP} = 115\,965\,218\,07.3\,(2.8) \times 10^{-13}$$ Hanneke et al, PRL 2008

- **The (EXP – SM) difference is:**

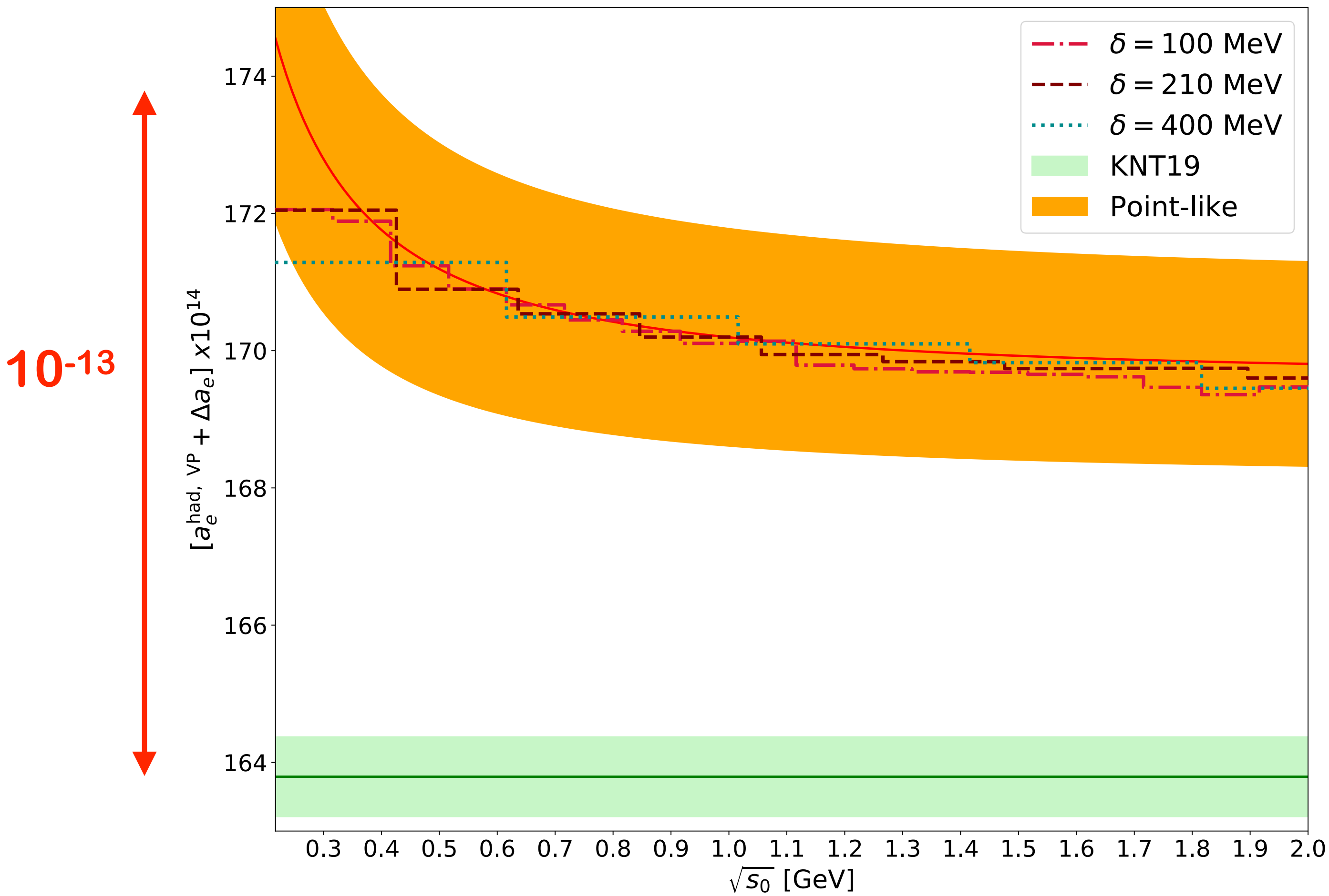
$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -\,8.9\,(3.6) \times 10^{-13}\ [2.5\sigma]\quad [Cs18]$$
$$= +\,4.7\,(3.0) \times 10^{-13}\ [1.6\sigma]\quad [Rb20]$$

QED 5-loop: $a_e^{QED5} = 4.6 \times 10^{-13}$

- **NP sensitivity limited only by the experimental errors in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector.**

Giudice, Paradisi, MP 2012

Shift of the electron g-2

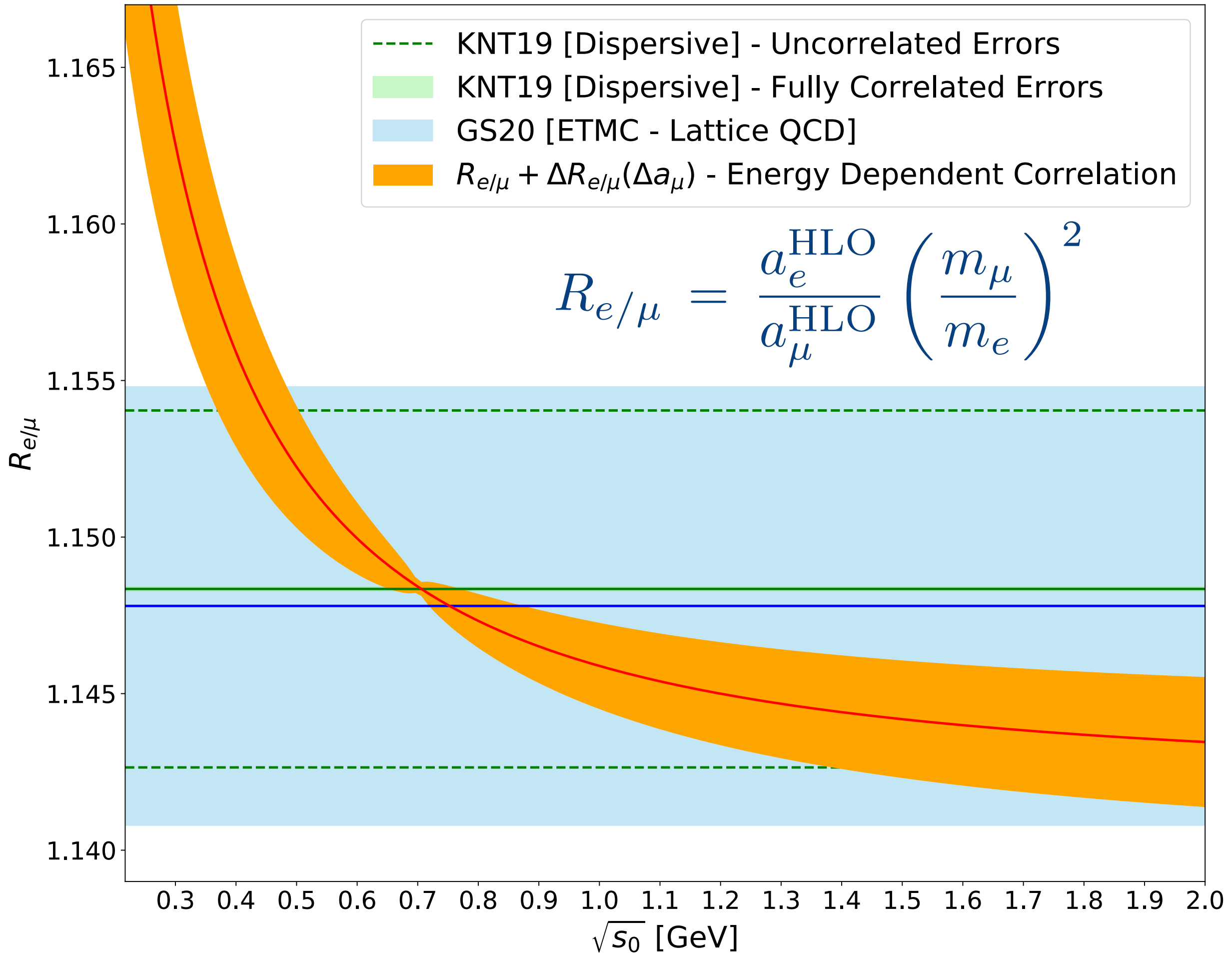


Shifts $\Delta\sigma(s)$ to fix Δa_μ only slightly change Δa_e

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

$a_\mu \Longleftrightarrow \Delta\alpha$: recent related literature

- Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits," arXiv:2003.04886.

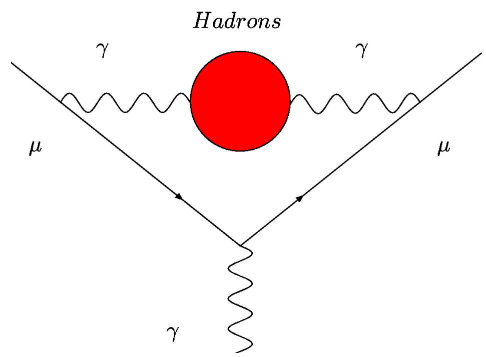
- Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.

- Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit," arXiv:2008.08107.

- Colangelo, Hoferichter and Stoffer, "Constraints on the two-pion contribution to hadronic vacuum polarization," arXiv:2010.07943.

The MUonE project

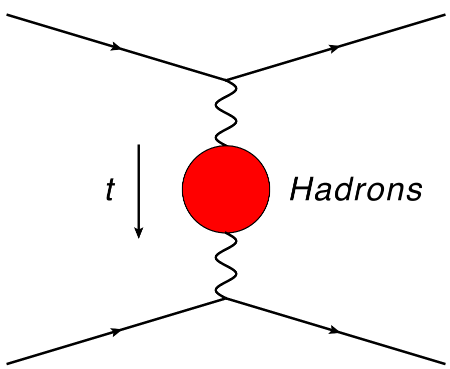
- The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:



$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\mathrm{had}}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- Alternatively, simply exchanging the x and s integrations:



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

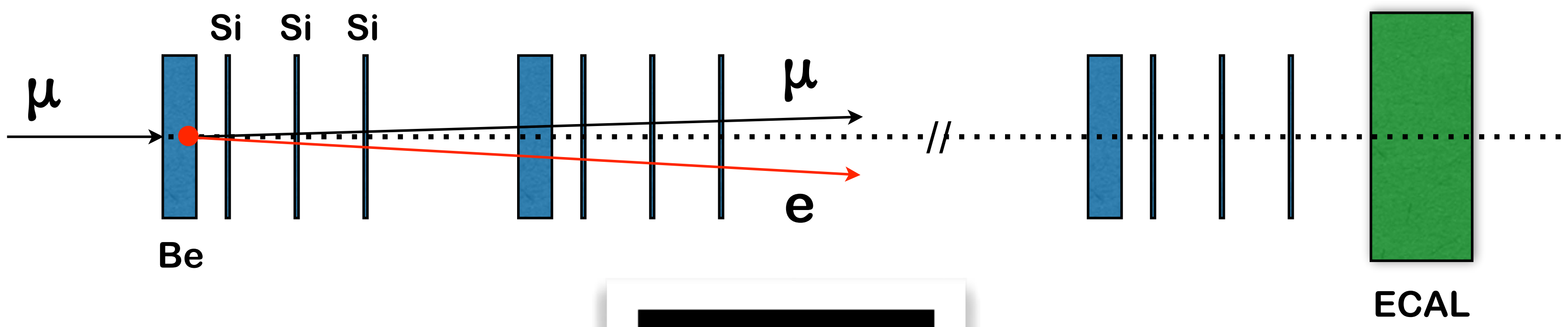
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

MUonE — Status

- Statistics: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2-3 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7$ nb^{-1}.

- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie ~20×10^{-11}.

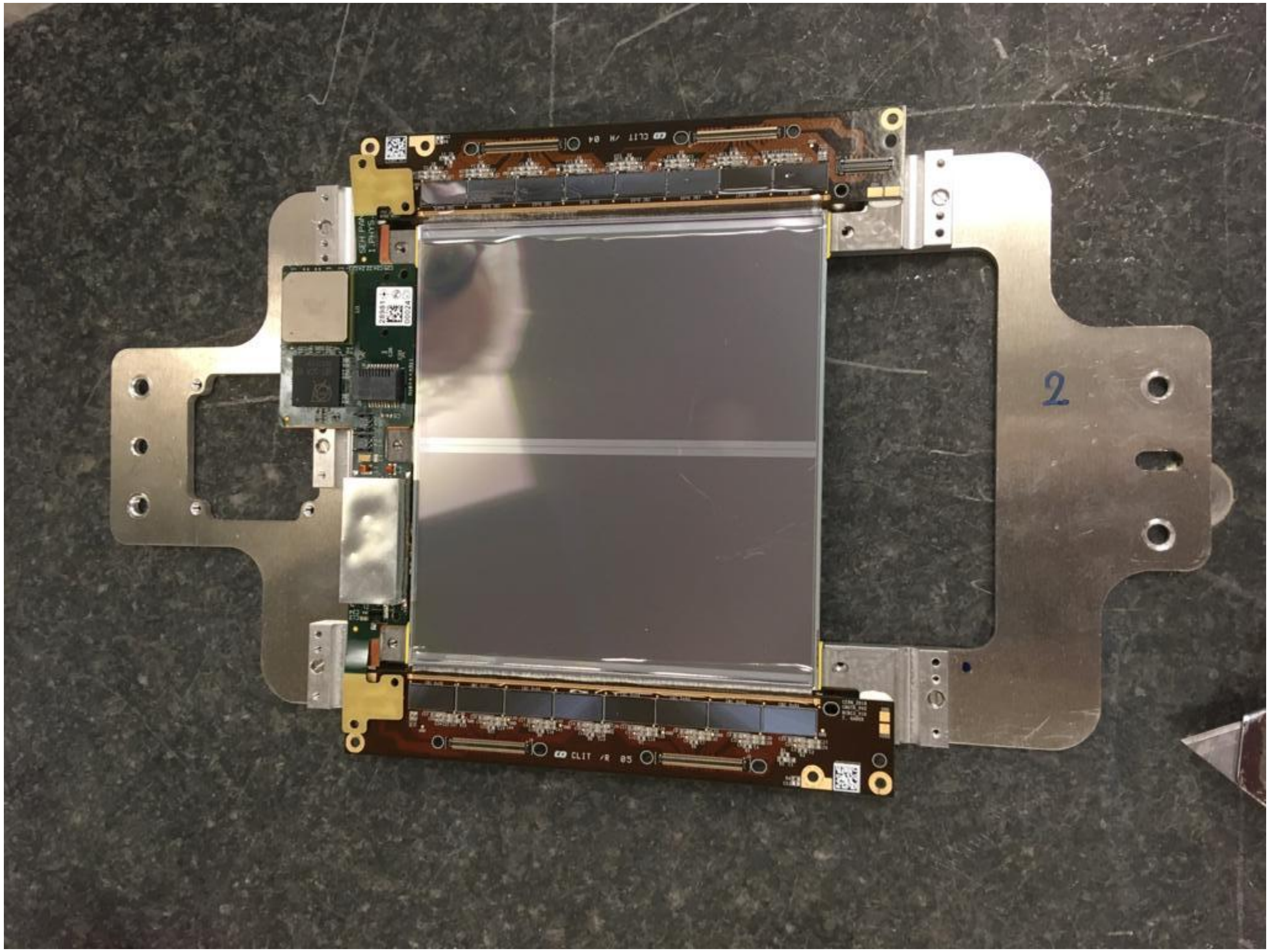
- Systematic effects must be known at ≲ 10ppm!

- Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111

- LoI submitted to CERN SPSC in 2019: Test run approved for 2021.

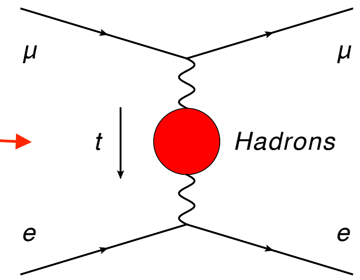
- Full-statistics run hopefully in 2022–24.

MUonE — Progress



MUonE — Theory

- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at $\lesssim$ 10ppm!



- **Fully differential fixed-order MC @ NLO ready** Pavia and PSI 2018-19
- **NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude close to completion.** Padova 2017 - present
- **Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation** Pavia and PSI 2020
- **NNLO hadronic effects computed** Padova and KIT 2019
- **Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes** PSI 2019-present
- **New Physics extracting $\Delta\alpha_{had}(t)$ at MUonE?** Padova and Heidelberg 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result.
- The discrepancy between experiment and SM increases to 4.2σ.
- The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.
- Is the present Δa_μ discrepancy due to missed contributions in the hadronic $\sigma(s)$?

  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- Leading hadronic contribution to a_μ: dispersive vs lattice? MUonE will provide a new independent determination alternative to both.